MICROSCOPIC DERIVATION OF THE GINZBURG-LANDAU EQUATIONS IN THE THEORY OF SUPERCONDUCTIVITY

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It is shown that the phenomenological Ginzburg–Landau equations follow from the theory of superconductivity in the London temperature region in the neighborhood of $T_C$. In these equations there occurs, however, twice the electronic charge; this is related to the physical meaning of $\Psi(x)$ as the wave function for Cooper pairs. The constant $\kappa$ turns out to be small. The problem of the surface energy for the boundary between the normal and superconducting phases in the neighborhood of $T_C$ is discussed.

It is well known that the behavior of superconductors in a magnetic field in the neighborhood of the critical temperature is qualitatively well described by Ginzburg and Landau’s phenomenological theory. We shall show in the present paper that equations of the same type as those in Ginzburg and Landau’s theory follow indeed from the theory of superconductivity.

We shall start from the equations for the thermodynamic Green functions which we obtained earlier. These equations are in a magnetic field of the form

$$\left\{ -\frac{\partial}{\partial t} + \frac{1}{2m} \left( \frac{\partial}{\partial r} - i e A(r) \right)^2 + \mu \right\} G(x, x')$$

shall do the same in the following. One verifies easily that such a cut-off corresponds to a “spread out” in Eq. (1') over a distance of the order $\hbar v/\omega$.

Let us go over to the Fourier components of the Green functions $G(x, x')$ and $F^+(x, x')$ with respect to the difference of the variables $u = \tau - \tau'$, for instance,

$$G(r, r'; u) = T \sum_n e^{-i\omega_n u} \mathcal{G}_n(r, r'),$$

$$\mathcal{G}_n(r, r') = \frac{1}{2} \int_{-\hbar v}^{\hbar v} e^{i\omega_n u} G(r, r'; u) du,$$

where $\omega_n = \pi(2n + 1)\hbar v/\omega$. Equa-
Striped Ground States in the Hubbard model

\[ H = - \sum_{\langle ij \rangle, \sigma} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}. \]

• Brief Intro to the Hubbard model
• Four powerful simulation methods
• Results for \( U=8, \ 1/8 \) doping:
  - Consensus on the phase of the ground state
  - Stripes with nearly degenerate wavelengths
• Some additional DMRG results for pairing on 4 and 5 leg cylinders
We studied a wide range of doping, $U/t$, temperature—in thermodynamic limit. For most of parameter space, there was good agreement in energies and other properties.

For parameters relevant to the cuprates ($U/t \sim 8$)—the toughest to simulate—the methods disagreed on the nature of the ground state.

A smaller group of us then decided to focus on this toughest region to see if we could resolve differences.
Stripe order in the underdoped region of the two-dimensional Hubbard model

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T=0, U=8, 1/8 doping
maximum uncertainty in the phase, maximum inhomogeneity

What is the ground state phase?

- Density matrix renormalization group (DMRG)
  - Weakness: finite cylinder size
- Density Matrix Embedding theory (DMET)
  - Weakness: finite cluster size
- Infinite projected entangled pair states (IPEPS)
  - Weakness: finite bond dimension and extrapolation
- Constrained Path Monte Carlo (CPMC, AFQMC)
  - Weakness: Constraint based on trial wavefunction

A key aspect of the work is that the weaknesses are very different. Thus, if multiple methods agree, we can have high confidence we have the right answer.

By focusing on one point in the phase diagram, we were all able to improve our results substantially.
Energy extrapolated to thermodynamic limit

Overall uncertainty almost an order of magnitude reduced from previous benchmark

Error bars neglect systematic errors—that is what we need the comparison for.
Uniform state versus stripes

- DMET and iPEPS both can be forced to give uniform states:
  - DMET has a cluster size. For a 2x2 cluster, no stripe patterns can form
  - iPEPS similarly has a cluster that is repeated to infinity. A 2x2 cluster cannot have stripes
  - DMRG always gives stripes. Currently no way to force a uniform state. CPMC also gives stripes as lowest energy state.

Both methods’ uniform states show d-wave pairing.
Each is higher in energy than a striped state.
New iPEPS Energy Extrapolation method (Corboz)

Uniform vs diagonal stripes vs. vertical stripes

Vertical stripes with different fillings/spacings
Vertical stripes: filling, wavelength

Filled Stripe

\[ f = 1 \]
\[ \lambda = 8 \]

The magnetic wavelength is \( 2 \lambda \)

Half filled Stripe

\[ f = 1/2 \]
\[ \lambda = 4 \]

Filled stripes were found with Hartree Fock in late 80's—but HF may not be not accurate

Half-filled stripes were found in some cuprates in the mid 90's. A few years later, DMRG on the t-J model showed half-filled stripes (White & Scalapino)

Zaanen

Tranquada

\[ E = -6.1321 \]
\[ m = 40 \]

DMRG on the t-J model—formation of two half-filled stripes
We find a remarkable near-degeneracy for states with different stripe wavelengths, with $\lambda=8$ very slightly lower in energy, and $\lambda=4$ significantly higher.

The near degeneracy likely points towards disordered stripes and/or fluctuating stripes.
Pairing and stripes “intertwined” on Lx4 cylinders (DMRG)

Red numbers are stripe fillings on a 16x4 system

Response to a uniform d-wave pairing field
D=0.05
To force a lengthwise stripe, chemical potential on bottom leg and $t_x = 1.2 \, t_y$

The pairing response is much weaker on a single long stripe than at the boundary between different stripe fillings.

Chia-Min Chung
Conclusions

- We are closing in on solutions to Hubbard models with numerical simulations!
- Using four different methods with very different uncontrolled errors, we have converged to a consistent general picture of the Hubbard ground state at $U=8$, near $1/8$ doping
- The system exhibits d-wave pairing but intertwined with stripes. The biggest response occurs at the transition between different stripe fillings. (Disordered or fluctuating stripes…)