Quantum matter without quasiparticles

Frontiers in Many Body Physics: Memorial for Lev Petrovich Gor’kov
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Ubiquitous “Strange”, “Bad”, or “Incoherent”, metal has a resistivity, \( \rho \), which obeys

\[ \rho \sim T, \]

and

in some cases \( \rho \gg \frac{h}{e^2} \)

(in two dimensions), where \( \frac{h}{e^2} \) is the quantum unit of resistance.
Strange metals just got stranger…

B-linear magnetoresistance!? 


P. Giraldo-Gallo et. al., arXiv:1705.05806
Strange metals just got stranger...
Scaling between B and T !?

\[ \rho(H, T) - \rho(0, 0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma \]

Theories of metallic states without quasiparticles without disorder

- Breakdown of quasiparticles arises from long-wavelength coupling of electrons to some bosonic collective mode. In all cases this can be written in terms of a continuum theory with a conserved momentum. **The critical theory has zero resistance, even though the electron quasiparticles do not exist.**

- Need to add irrelevant (umklapp) effects to obtain a non-zero resistivity, but this not yield a large linear-in-$T$ resistivity.
Theories of metallic states without quasiparticles in the presence of disorder

- Well-known perturbative theory of disordered metals has 2 classes of known fixed points, the insulator at strong disorder, and the metal at weak disorder. The latter state has long-lived, extended quasiparticle excitations (which are not plane waves).

- **Needed:** a metallic fixed point at intermediate disorder and strong interactions without quasiparticle excitations. Although disorder is present, it largely self-averages at long scales.
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- **SYK models**
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
Place electrons randomly on some sites
Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
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Entangle electrons pairwise randomly
This describes both a strange metal and a black hole!
The SYK model
(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large $N$ limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. 53, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;kl} c_{i}^{\dagger} c_{j}^{\dagger} c_{k} c_{\ell} - \mu \sum_{i} c_{i}^{\dagger} c_{i}$$

$$c_{i} c_{j} + c_{j} c_{i} = 0 , \quad c_{i} c_{j}^{\dagger} + c_{j}^{\dagger} c_{i} = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_{i} c_{i}^{\dagger} c_{i}$$

$U_{ij;kl}$ are independent random variables with $\overline{U_{ij;kl}} = 0$ and $|U_{ij;kl}|^{2} = U^{2}$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
The SYK model

Feynman graph expansion in $J_{ij...}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} , \
\Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^{-}) = Q.$$
The SYK model

- $T = 0$ fermion Green’s function is singular:

\[ G(\tau) \sim \frac{1}{\sqrt{\tau}} \text{ at large } \tau. \]

(Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

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- $T > 0$ Green’s function has conformal invariance, and is ‘dephased’ at the characteristic scale $\sim k_B T / \hbar$, which is independent of $U$.

  $$G \sim e^{-2\pi \mathcal{E} T \tau} \left(\frac{T}{\sin(\pi k_B T \tau / \hbar)}\right)^{1/2}$$

  $\mathcal{E}$ measures particle-hole asymmetry.

A. Georges and O. Parcollet PRB 59, 5341 (1999)
S. Sachdev, PRX, 5, 041025 (2015)
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- The last property indicates $\tau_{\text{eq}} \sim \hbar / (k_B T)$, and this has been found in a recent numerical study.
The SYK model

So the Green’s functions display thermal ‘damping’ at a scale set by $T$ alone, which is independent of $U$. 

\[ G^R(\omega)G^A(\omega) \]

\[ -\text{Re}G^R(\omega) \]

\[ -\text{Im}G^R(\omega) \]

Green’s functions away from half-filling
Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents

\[
H = \sum_x \sum_{i<j, k<l} U_{ijkl,x} c_i^{\dagger} c_j^{\dagger} c_k c_l + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_i^{\dagger} c_j c_{i,x} c_{j,x'}
\]

\[
|U_{ijkl}|^2 = \frac{2U^2}{N^3} \quad \quad |t_{ij,xx'}|^2 = t_0^2 / N.
\]
Large $N$ equations

$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),$$

$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$
Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models
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- There is a low coherence scale $E_c \sim t_0^2/U$, with SYK criticality at $T > E_c$ and heavy Fermi liquid behavior for $T < E_c$. 

See also A. Georges and O. Parcollet PRB 59, 5341 (1999)
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- There is a low coherence scale $E_c \sim t_0^2 / U$, with SYK criticality at $T > E_c$ and heavy Fermi liquid behavior for $T < E_c$.

- From the Kubo formula, we have the conductivity

$$\text{Re}[\sigma(\omega)] \propto t_0^2 \int d\Omega \frac{f(\omega + \Omega) - f(\Omega)}{\omega} A(\Omega) A(\omega + \Omega)$$

where $A(\omega) = \text{Im}[G^R(\omega)]$.

At $T > E_c$, using $A(\omega) \sim \omega^{-1/2} F(\omega/T)$, this yields the bad metal behavior

$$\sigma \sim \frac{e^2}{h} \frac{t_0^2}{U} \frac{1}{T} \quad ; \quad \rho \sim \left( \frac{h}{e^2} \right) \frac{T}{E_c}$$
arXiv:1712.05026
Title: Magnetotransport in a model of a disordered strange metal
Authors: Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev
Infecting a Fermi liquid and making it SYK

Mobile electrons (c, green) interacting with SYK quantum dots (f, blue) with exchange interactions. This yields the first model agreeing with magnetotransport in strange metals!

\[
H = -t \sum_{\langle rr' \rangle; i=1}^{M} (c_{r'i}^\dagger c_{r'i} + \text{h.c.}) - \mu_c \sum_{r; i=1}^{M} c_{ri}^\dagger c_{ri} - \mu \sum_{r; i=1}^{N} f_{ri}^\dagger f_{ri} \\
+ \frac{1}{NM^{1/2}} \sum_{r; i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl}^r f_{ri}^\dagger f_{rj}^\dagger c_{rk} c_{rl} + \frac{1}{N^{3/2}} \sum_{r; i,j,k,l=1}^{N} J_{ijkl}^r f_{ri}^\dagger f_{rj}^\dagger f_{rk} f_{rl}.
\]


Similar results in non-random models by Y. Werman, D. Chowdhury, T. Senthil, and E. Berg, to appear
Infected a Fermi liquid and making it SYK

\[ \Sigma(\tau - \tau') = -J^2 G^2(\tau - \tau')G(\tau' - \tau) - \frac{M}{N} g^2 G(\tau - \tau')G^c(\tau - \tau')G^c(\tau' - \tau), \]

\[ G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)}, \quad (f \text{ electrons}) \]

\[ \Sigma^c(\tau - \tau') = -g^2 G^c(\tau - \tau')G(\tau - \tau')G(\tau' - \tau), \]

\[ G^c(i\omega_n) = \sum_k \frac{1}{i\omega_n - \epsilon_k + \mu_c - \Sigma^c(i\omega_n)}. \quad (c \text{ electrons}) \]

Exactly solvable in the large \( N,M \) limits!

- Low-\( T \) phase: \( c \) electrons form a Marginal Fermi-liquid (MFL), \( f \) electrons are local SYK models

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Infecting a Fermi liquid and making it SYK

\[ G_c(\tau) = -\frac{C_c}{\sqrt{1 + e^{-4\pi \mathcal{E}_c}}} \left( \frac{T}{\sin(\pi T \tau)} \right)^{1/2} e^{-2\pi \mathcal{E}_c T \tau}, \]
\[ G(\tau) = -\frac{C}{\sqrt{1 + e^{-4\pi \mathcal{E}}}} \left( \frac{T}{\sin(\pi T \tau)} \right)^{1/2} e^{-2\pi \mathcal{E} T \tau}, \quad 0 \leq \tau < \beta \]

- High-\( T \) phase: \( c \) electrons form an “incoherent metal” (IM), with local Green’s function, and no notion of momentum; \( f \) electrons remain local SYK models.

Infecting a Fermi liquid and making it SYK

- Low-\(T\) phase: \(c\) electrons form a Marginal Fermi-liquid (MFL), \(f\) electrons are local SYK models

\[
\Sigma^c(i\omega_n) = \frac{ig^2\nu(0)T}{2J \cosh^{1/2}(2\pi E)\pi^{3/2}} \left( \frac{\omega_n}{T} \ln \left( \frac{2\pi Te^{\gamma_E-1}}{J} \right) + \frac{\omega_n}{T} \psi \left( \frac{\omega_n}{2\pi T} \right) + \pi \right),
\]

\[
\Sigma^c(i\omega_n) \rightarrow \frac{ig^2\nu(0)}{2J \cosh^{1/2}(2\pi E)\pi^{3/2}} \omega_n \ln \left( \frac{|\omega_n|e^{\gamma_E-1}}{J} \right), \quad |\omega_n| \gg T \quad (\nu(0) \sim 1/t)
\]

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Linear-in-\(T\) resistivity

Both the MFL and the IM are not translationally-invariant and have linear-in-\(T\) resistivities!

\[
\sigma_0^\text{MFL} = 0.120251 \times MT^{-1} J \times \left( \frac{v_F^2}{g^2} \right) \cosh^{1/2}(2\pi \mathcal{E}). \quad (v_F \sim t)
\]

\[
\sigma_0^\text{IM} = (\pi^{1/2}/8) \times MT^{-1} J \times \left( \frac{\Lambda}{\nu(0)g^2} \right) \frac{\cosh^{1/2}(2\pi \mathcal{E})}{\cosh(2\pi \mathcal{E}_c)}.
\]

[Can be obtained straightforwardly from Kubo formula in the large-\(N,M\) limits]

The IM is also a “Bad metal” with \(\sigma_0^\text{IM} \ll 1\)

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Magnetotransport: Marginal-Fermi liquid

- Thanks to large $N,M$, we can also exactly derive the linear-response Boltzmann equation for non-quantizing magnetic fields...

$$(1 - \partial_\omega \text{Re}[\Sigma^c_R(\omega)]) \partial_t \delta n(t, k, \omega) + v_F \hat{k} \cdot \mathbf{E}(t) n'_f(\omega) + v_F (\hat{k} \times \mathbf{B}) \cdot \nabla_k \delta n(t, k, \omega) = 2\delta n(t, k, \omega) \text{Im}[\Sigma^c_R(\omega)],$$

$$(\mathbf{B} = eBa^2/\hbar) \text{ (i.e. flux per unit cell)}$$

$$\sigma_{L}^{\text{MFL}} = M \frac{v_F^2 \nu(0)}{16T} \int_{-\infty}^{\infty} \frac{dE_1}{2\pi} \text{sech}^2 \left( \frac{E_1}{2T} \right) \frac{-\text{Im}[\Sigma^c_R(E_1)]}{\text{Im}[\Sigma^c_R(E_1)]^2 + (v_F/(2k_F))^2 \mathbf{B}^2},$$

$$\sigma_{H}^{\text{MFL}} = -M \frac{v_F^2 \nu(0)}{16T} \int_{-\infty}^{\infty} \frac{dE_1}{2\pi} \text{sech}^2 \left( \frac{E_1}{2T} \right) \frac{(v_F/(2k_F))\mathbf{B}}{\text{Im}[\Sigma^c_R(E_1)]^2 + (v_F/(2k_F))^2 \mathbf{B}^2}.$$

$$\sigma_{L}^{\text{MFL}} \sim T^{-1} s_L((v_F/k_F)(\mathbf{B}/T)), \quad \sigma_{H}^{\text{MFL}} \sim -\mathbf{B} T^{-2} s_H((v_F/k_F)(\mathbf{B}/T)).$$

$$s_{L,H}(x \to \infty) \propto 1/x^2, \quad s_{L,H}(x \to 0) \propto x^0.$$

Scaling between magnetic field and temperature in orbital magnetotransport!
Magnetotransport with mesoscopic homogeneity

- No macroscopic momentum, so equations describing charge transport are just

\[ \nabla \cdot \mathbf{I}(x) = 0, \quad \mathbf{I}(x) = \sigma(x) \cdot \mathbf{E}(x), \quad \mathbf{E}(x) = -\nabla \Phi(x). \]


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\[ \nabla \cdot \mathbf{I}(\mathbf{x}) = 0, \quad \mathbf{I}(\mathbf{x}) = \sigma(\mathbf{x}) \cdot \mathbf{E}(\mathbf{x}), \quad \mathbf{E}(\mathbf{x}) = -\nabla \Phi(\mathbf{x}). \]

- Current path length increases linearly with \( B \) at large \( B \) due to local Hall effect, which causes the global resistance to increase linearly with \( B \) at large \( B \).

\[ \text{Figure 3} \text{ Visualization of currents and voltages at large magnetic field in a } 10 \times 10 \text{ random network of disks with radii 1 (arbitrary units), where the potential difference } U = -1 \text{ V. The black arrows represent the currents, and arrow size depicts the magnitude of the current. The major current path is perpendicular to the applied voltage for a significant proportion of the time, which implies that the magnetoresistance is provided internally by the Hall effect, which is therefore linear in } H. \]
Solvable toy model: two-component disorder

- Two types of domains $a, b$ with different carrier densities and lifetimes randomly distributed in approximately equal fractions over sample.

- Effective medium equations can be solved exactly

$$
\left( I + \frac{\sigma_a - \sigma^e}{2\sigma_L^e} \right)^{-1} \cdot (\sigma^a - \sigma^e) + \left( I + \frac{\sigma_b - \sigma^e}{2\sigma_L^e} \right)^{-1} \cdot (\sigma^b - \sigma^e) = 0.
$$

\[ \rho_L^e \equiv \frac{\sigma_L^e}{\sigma_L^e + \sigma_H^e} = \frac{\sqrt{(B/m)^2 \left( \gamma_a \sigma_{0a}^{MFL} - \gamma_b \sigma_{0b}^{MFL} \right)^2 + \gamma_a^2 \gamma_b^2 \left( \sigma_{0a}^{MFL} + \sigma_{0b}^{MFL} \right)^2}}{\gamma_a \gamma_b \left( \sigma_{0a}^{MFL} \sigma_{0b}^{MFL} \right)^{1/2} \left( \sigma_{0a}^{MFL} + \sigma_{0b}^{MFL} \right)}, \]

\[ \rho_H^e \equiv -\frac{\sigma_H^e / B}{\sigma_L^e + \sigma_H^e} = \frac{\gamma_a + \gamma_b}{m \gamma_a \gamma_b \left( \sigma_{0a}^{MFL} + \sigma_{0b}^{MFL} \right)} \cdot (m = k_F / v_F \sim 1/t) \]

$\gamma_{a,b} \sim T$ (i.e. effective transport scattering rates)

\[ \rho_L^e \sim \sqrt{c_1 T^2 + c_2 B^2} \]

Scaling between $B$ and $T$ at microscopic orbital level has been transferred to global MR!
Magnetotransport in strange metals

- Engineered a model of a Fermi surface coupled to SYK quantum dots which leads to a marginal Fermi liquid with a linear-in-$T$ resistance, with a magnetoresistance which scales as $B \sim T$. Higher temperatures lead to an incoherent metal with a local Green's function and a linear-in-$T$ resistance, but negligible magnetoresistance.
Engineered a model of a Fermi surface coupled to SYK quantum dots which leads to a marginal Fermi liquid with a linear-in-\( T \) resistance, with a magnetoresistance which scales as \( B \sim T \).

Mesoscopic disorder then leads to linear-in-\( B \) magnetoresistance, and a combined dependence which scales as \( \sim \sqrt{B^2 + T^2} \).
Engineered a model of a Fermi surface coupled to SYK quantum dots which leads to a marginal Fermi liquid with a linear-in-$T$ resistance, with a magnetoaresistance which scales as $B \sim T$.

Mesoscopic disorder then leads to linear-in-$B$ magnetoaresistance, and a combined dependence which scales as $\sim \sqrt{B^2 + T^2}$.

Higher temperatures lead to an incoherent metal with a local Green’s function and a linear-in-$T$ resistance, but negligible magnetoaresistance.
• This simple two-component model describes a new state of matter which is realized by electrons in the presence of strong interactions and disorder.

• Can such a model be realized as a fixed-point of a generic theory of strongly-interacting electrons in the presence of disorder?

• Can we start from a single-band Hubbard model with disorder, and end up with such two-band fixed point, with emergent local conservation laws?
Electrons in doped silicon appear to separate into two components: localized spin moments and itinerant electrons.
