Measuring entanglement in synthetic quantum systems

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Theory Winter School, Tallahassee, Florida
Jan 8-12, 2018
Entanglement in Many-body Systems

- **Resource for quantum information processing**

- **Novel states of matter:**
  Order beyond simple broken symmetry

  **Example** - Topological order, spin liquid, fractional quantum Hall - characterized by quantum entanglement!

- **Quantum criticality**

- **Quantum dynamics** ...

- **Challenge:** Entanglement not detected in traditional CM experiments
Outline

• Recap – preparing entangled states with ion qubits/spins
• State tomography
• Witness operators
• Replica method – measuring second Renyi entropy
Preparing entangled states with ions

1. Initialize the qubits to a (product) state
2. Evolve the state under single qubit unitary rotations and laser-induced phonon mediated spin-spin interactions [digital ‘circuit’ of logic gates or analog Hamiltonian evolution]
3. **Measurement** – unitary rotation of measurement basis + Spin dependent fluorescence
State Tomography

Reconstruct the entire density matrix

$$|\phi\downarrow + \rangle = \frac{1}{\sqrt{2}} \left( |00\rangle + |11\rangle \right)$$

No. of qubits, $N = 2$, # measurements = 1800 ($\sim 3^{\uparrow N}$), total time taken = 40 sec

State Tomography

Reconstruct the entire density matrix

\[ |W_N\rangle = (|D\cdots D S\rangle + |D\cdots D S D\rangle + |D\cdots D S D D\rangle + \cdots + |S D\cdots D\rangle)/\sqrt{N} \]

No. of qubits, \( N = 8 \), # measurements \( > 656,100 \) (\( \sim 3^N \)), total time taken = 10 hrs.

Witness operators

Make your most educated guess!

\[ \langle W \rangle < 0 \quad \Rightarrow \quad \text{has entanglement of the particular kind!} \]

\[
W_{\text{GHZ}} = \frac{1}{2} - |\text{GHZ}\rangle \langle \text{GHZ}| = \frac{9}{4} - \hat{J}_x^2 - \sigma_\phi^{(1)} \sigma_\phi^{(2)} \sigma_\phi^{(3)}
\]

\[ |\text{GHZ}\rangle = |\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle \]

Kim, K et al
Quantum Simulation: Platforms

Trapped ions

Neutral atoms in optical lattices

Superconducting circuits

Photonic networks

NV defects in diamonds
Physics Today 67(10), 38(2014)
Entanglement

Itinerant many-body systems
Philipp Preiss  
→ Jochim Lab, Heidelberg

Matthew Rispoli

Alex Ruichao Ma  
→ Simon lab, Chicago

Eric Tai

Markus Greiner

**Theory:**
Andrew Daley
Hannes Pichler
Peter Zoller
Dieter Jaksch …

Alex Lukin
Previous work on single site addressability in lattices:
Detecting single atoms in large spacing lattices (D. Weiss) and 1D standing waves (D. Meschede), Electron Microscope (H. Ott), Absorption imaging (J. Steinhauer), single trap (P. Grangier, Weinfurter/Weber), few site resolution (C. Chin), See also: Sherson et al., Nature 467, 68 (2010)
Quantum gas microscope

Hologram for projecting optical lattice

High resolution imaging

High aperture objective

NA=0.8

2D quantum gas of Rb-87 in optical lattice
... and the whole apparatus
Superfluid Mott insulator

\[ H = -J \sum_{\langle i, j \rangle} (a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2} \sum_i n_i (n_i - 1) \]

Bose Hubbard Model

Projecting arbitrary potential landscapes

2D quantum gas of Rb-87 in optical lattice

Image: EKB Technologies

Thesis: P. Zupancic (LMU/Harvard, 2014)
Arbitrary beam shaping

Weitenberg et al., *Nature* 471, 319-324 (2011)
Zupancic, P., Master’s Thesis, LMU Munich/Harvard 2013

High-order Laguerre Modes
A bottom-up system for neutral atoms

(Single shot image)
Single-Particle Bloch oscillations

Single-Particle Bloch oscillations

- Temporal period $T_B = \frac{2\pi}{F}$, spatial width $L_B = \frac{4J}{F}$

- Delocalized over \sim 14 sites = 10\textmu m.

- Revival probability 96(3)\%

Entanglement in Many-body Systems

Many-body system: Bipartite entanglement

Product state: $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$  e.g. Mott insulator

Entangled state: $|\Psi\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$  e.g. Superfluid
Entanglement Entropy

Reduced density matrix:
\[ \rho_A = \text{tr}_B \{ \rho \} = |\Psi_A\rangle \otimes \langle \Psi_A| \]

- Product state \( \rightarrow \) Pure state
- Entangled state \( \rightarrow \) Mixed state

Quantum purity = \( \text{Tr}(\rho_A^{1/2}) \) = 1

\[ S_{1/2} (\rho_A) = - \log \text{Tr}(\rho_A^{1/2}) \] = 0

Renyi Entanglement Entropy
\[ S_n(\rho_\alpha) = \frac{1}{1-n} \log \text{Tr}\{\rho_\alpha^n\} \]
Entanglement Entropy

Reduced density matrix:
\[ \rho_A = \text{tr}_B \{ \rho \} = |\Psi_A\rangle \otimes \langle \Psi_A | \]

Quantum purity = \[ \text{Tr}(\rho \Sigma A \Sigma 2) \] = 1 < 1

Many-body Hong-Ou-Mandel interferometry

Alves and Jaksch, PRL 93, 110501 (2004)
Mintert et al., PRL 95, 260502 (2005)
Daley et al., PRL 109, 020505 (2012)
No coincidence detection for identical photons

Beam splitter operation: Rabi flopping in a double well

\[ a_L^{\dagger} \rightarrow a_L^{\dagger} - ia_R^{\dagger} \]

\[ a_R^{\dagger} \rightarrow a_L^{\dagger} + ia_R^{\dagger} \]

Also see: Kaufman A M *et al.*, Science 345, 306 (2014)
also Esslinger group
Two bosons on a beam splitter

Hong-Ou-Mandel interference

\[ a_L a_R \]

\[ a_L^\dagger \rightarrow a_L^\dagger - ia_R^\dagger \]

\[ a_R^\dagger \rightarrow a_L^\dagger + ia_R^\dagger \]

\[ a_L^\dagger a_L^\dagger \]

\[ + \]

\[ a_R^\dagger a_R^\dagger \]
Beam splitter

\[ P(1,1) \]

\[
\begin{align*}
\text{measured fidelity:} & \quad 96(4)\% \\
\text{limited by interaction} & \quad 4(4)\%
\end{align*}
\]

Quantum interference of bosonic many-body systems

How “identical” are the particles? vs. How “identical” are the states?

If $|\Psi\rangle_1 = |\Psi\rangle_2$, deterministic number parity after beam splitter

Alves and Jaksch, PRL 93 (2004)
Daley et al., PRL 109 (2012)

Also see Linke et al, arXiv:1712.08581 for experiments on two copies of a trapped ion system simulating Fermi-Hubbard model.
Quantum interference of bosonic many body systems
Making two copies of a many-body state
Measuring many-body entanglement

Mott Insulator

Locally pure

Product State

Globally pure

Superfluid

Locally mixed

Entangled

Globally pure
Measuring many-body entanglement

Mott Insulator

Superfluid

Entanglement in the ground state of a Bose-Hubbard system

\[ S_2(\rho_A) = -\log \text{Tr}\left\{ \rho_A^2 \right\} \]

Renyi entropy

Mixed

Pure

1-site

2-site

complete

Beam splitter

Entanglement in optical lattice systems:

Entanglement in the ground state of a Bose-Hubbard system

Renyi entropy versus $U/J$

Purity = $\langle \text{Parity} \rangle$

<table>
<thead>
<tr>
<th>$\Psi_1$</th>
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Mott insulator  $ightarrow$ Superfluid

Entanglement in the ground state of a Bose-Hubbard system

Renyi entropy vs. $U/J$

- **Complete**
- **2-site**
- **Pure**
- **Mixed**

$Purity = \langle \text{Parity} \rangle$

$|\Psi_1\rangle = |\Psi_2\rangle$

Entanglement in the ground state of a Bose-Hubbard system

Renyi entropy

Purity = $\langle \text{Parity} \rangle$

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Mott insulator

Superfluid

Entanglement in the ground state of a Bose-Hubbard system

Renyi entropy vs. $U/J$ for different partitioning schemes and system states.
Mutual Information $I_{AB}$

$$I_{AB} = S_2(A) + S_2(B) - S_2(AB)$$

Renyi entropy $I_{AB}$
Non equilibrium: Quench dynamics

\[ \frac{1}{\sqrt{3}} \left( -\left| \psi \right> + \left| \phi \right> + \left| \psi' \right> \right) \]

\[ \frac{1}{\sqrt{2}} \left( \left| \phi \right> + \left| \psi \right> \right) \]

Beam splitter
Probing thermalization of a pure state

Kaufman, A. et al
*Science* 353, 794 (2016)
Non equilibrium: Quench dynamics
Approximate scaling laws on six sites

Local canonical (thermal) statistics ~ local statistics from entanglement

Calabrese...Cardy, J.Stat.Mech.0504:P04010; Deutsch...Sharma, PRE 87, 042135 (2013); Santos...Rigol, PRE 86, 010102 (2013); Eisert...Plenio, 82, 277 (2010)
Thank You!