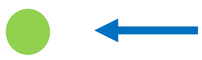
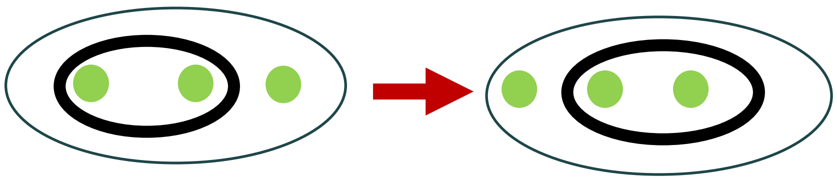
 Fibonacci anyon with topological "charge" 1

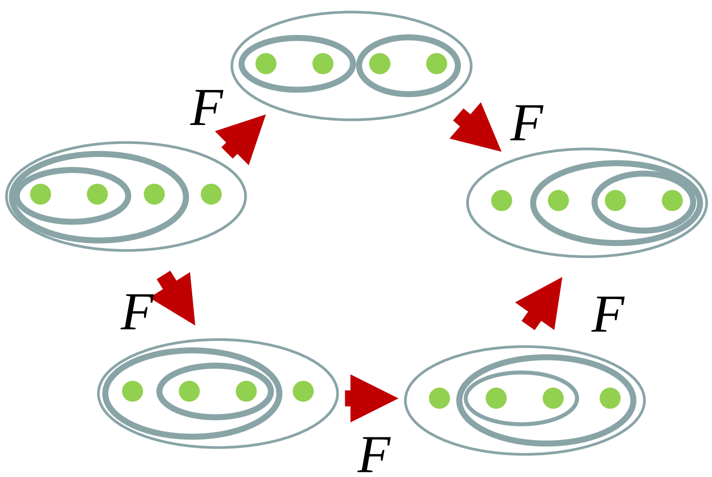
Fusion rules: 1x1 = 0+1; 1x0 = 0x1 = 1; 0x0 = 0

F Matrix:

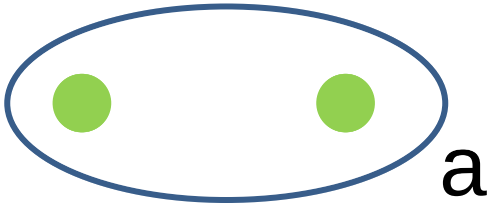


$$F = \begin{pmatrix} \varphi^{-1} & \varphi^{-1/2} \\ \varphi^{-1/2} & -\varphi^{-1} \end{pmatrix}$$

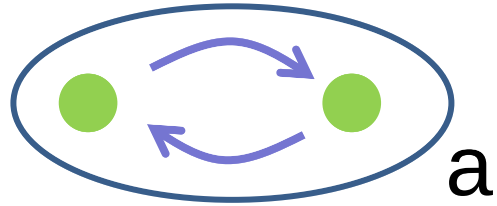
Pentagon



The R Matrix

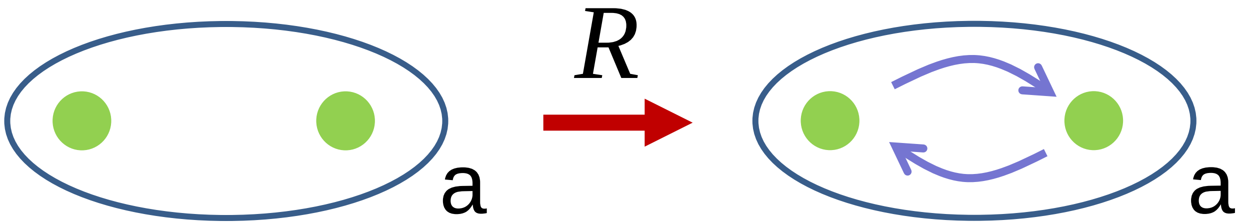


$= R_a$

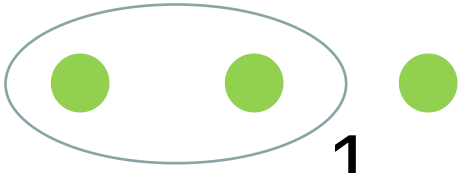


$$R = \begin{pmatrix} R_0 & 0 \\ 0 & R_1 \end{pmatrix}$$

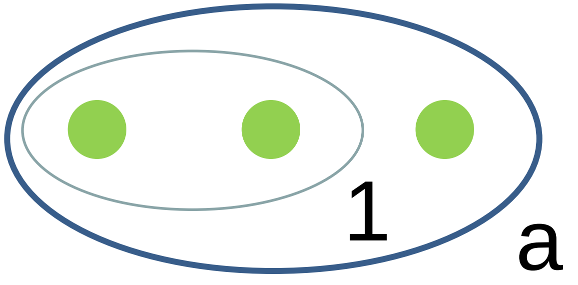
The R Matrix

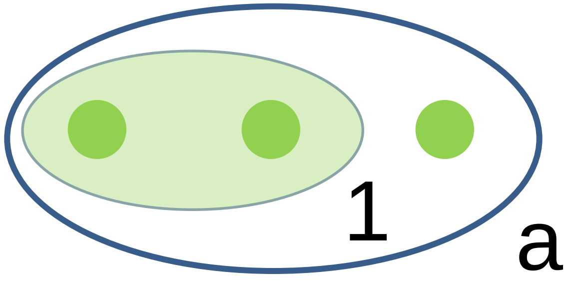


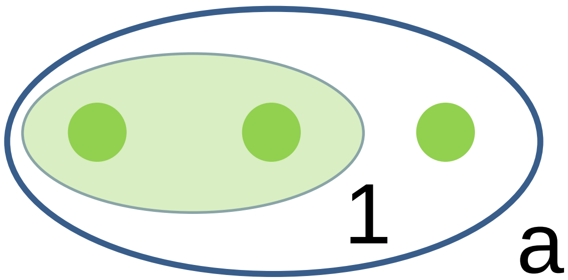
$$R = \begin{pmatrix} R_0 & 0 \\ 0 & R_1 \end{pmatrix}$$



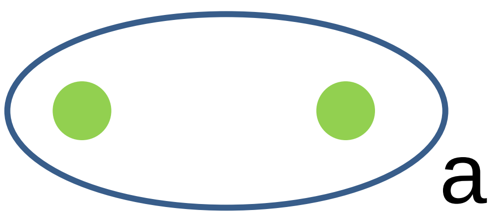
1

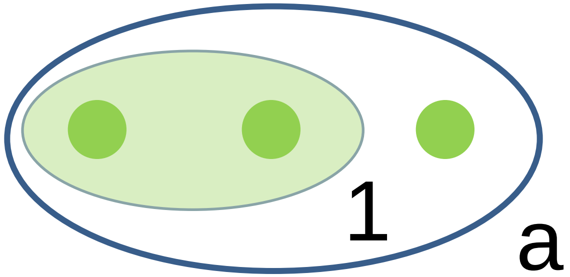

1
a


1
a

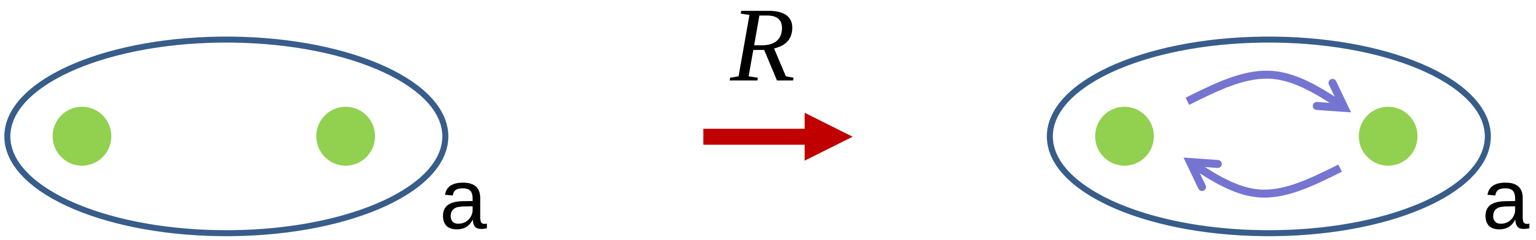


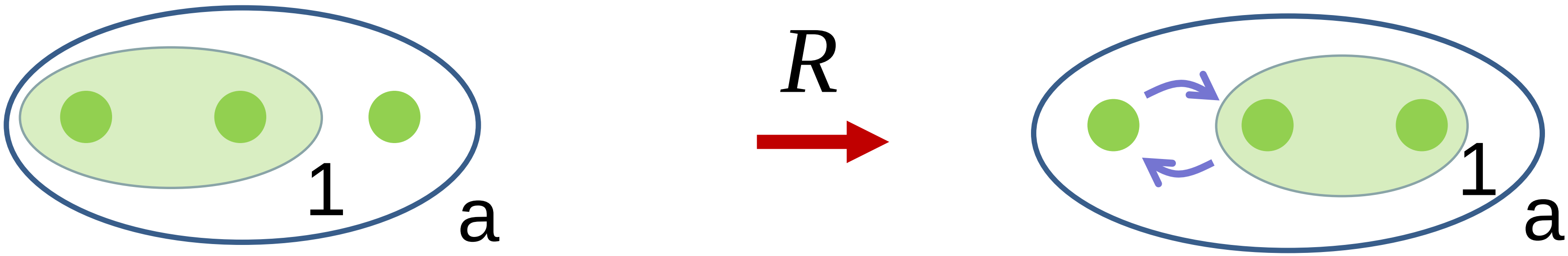
Equivalent to



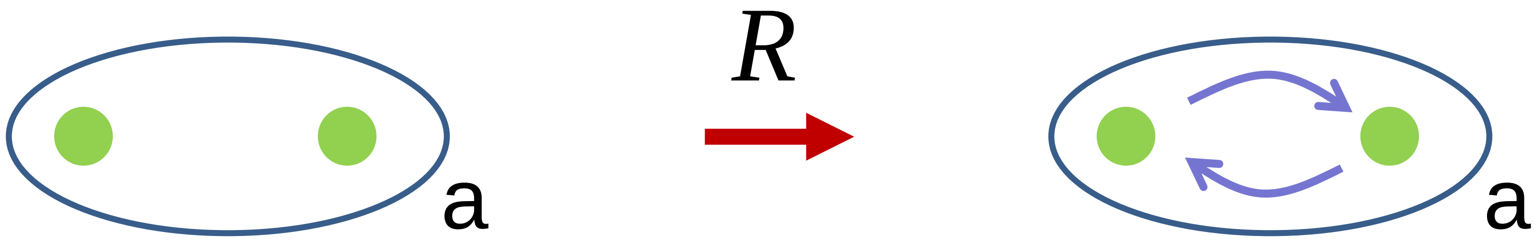


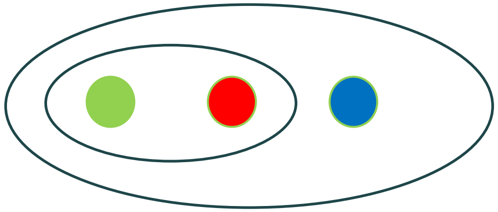
Equivalent to

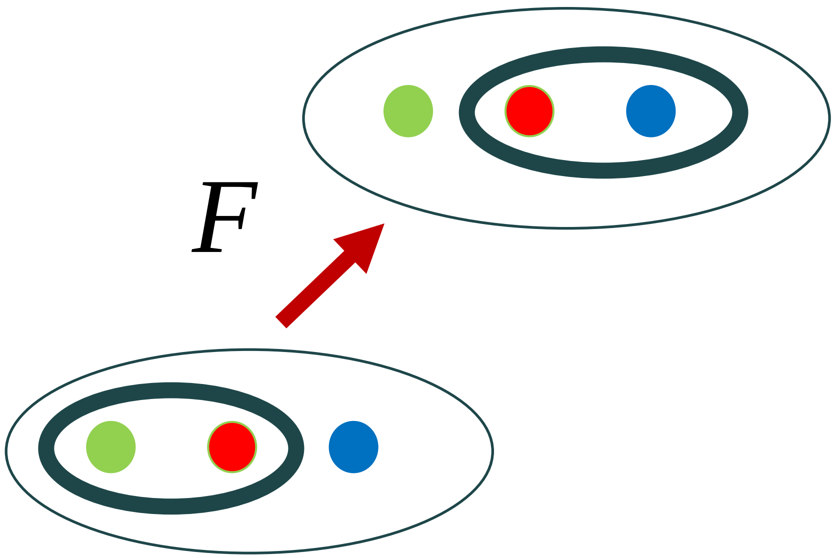




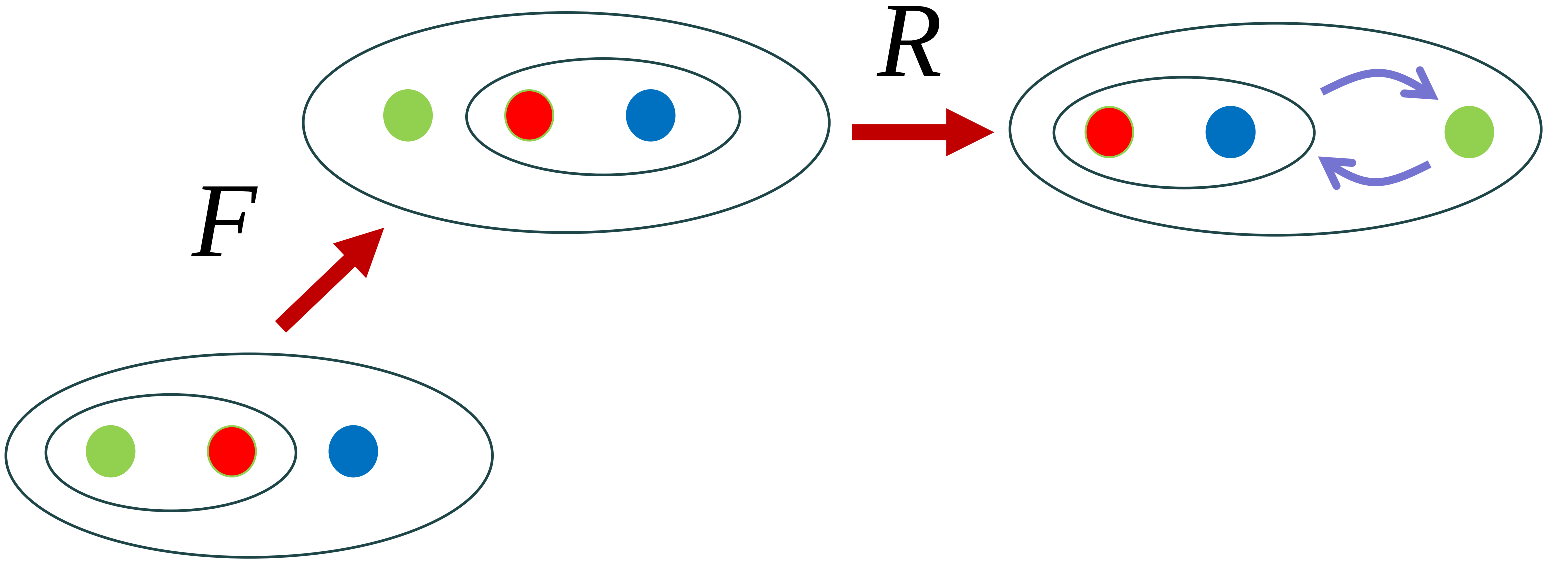
Equivalent to

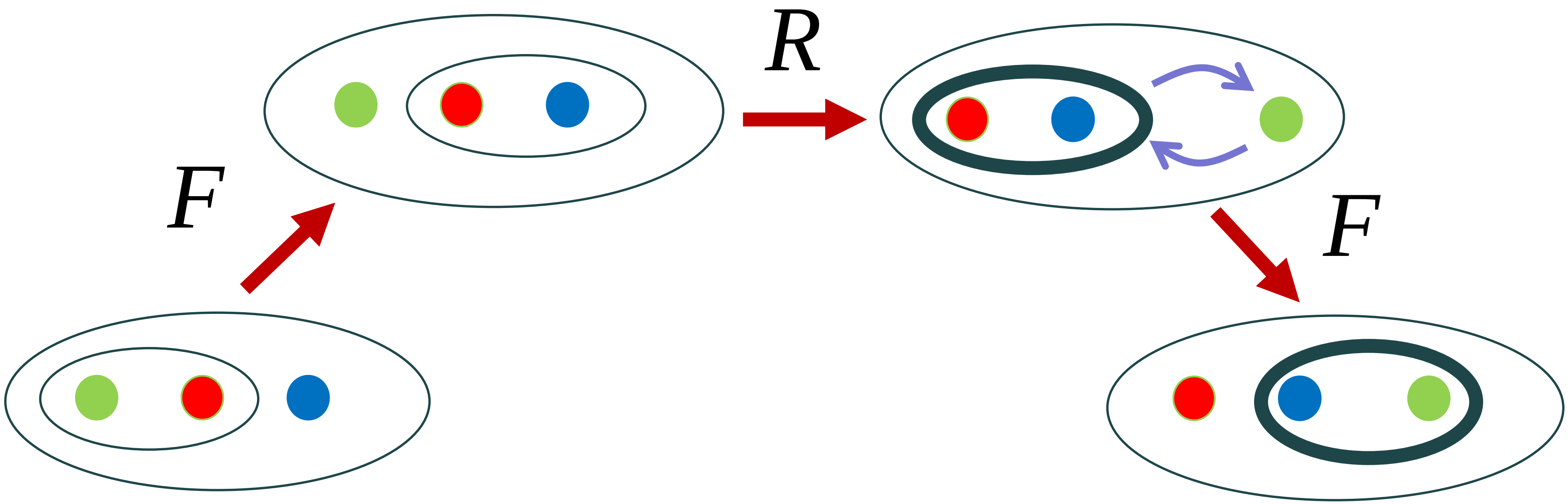


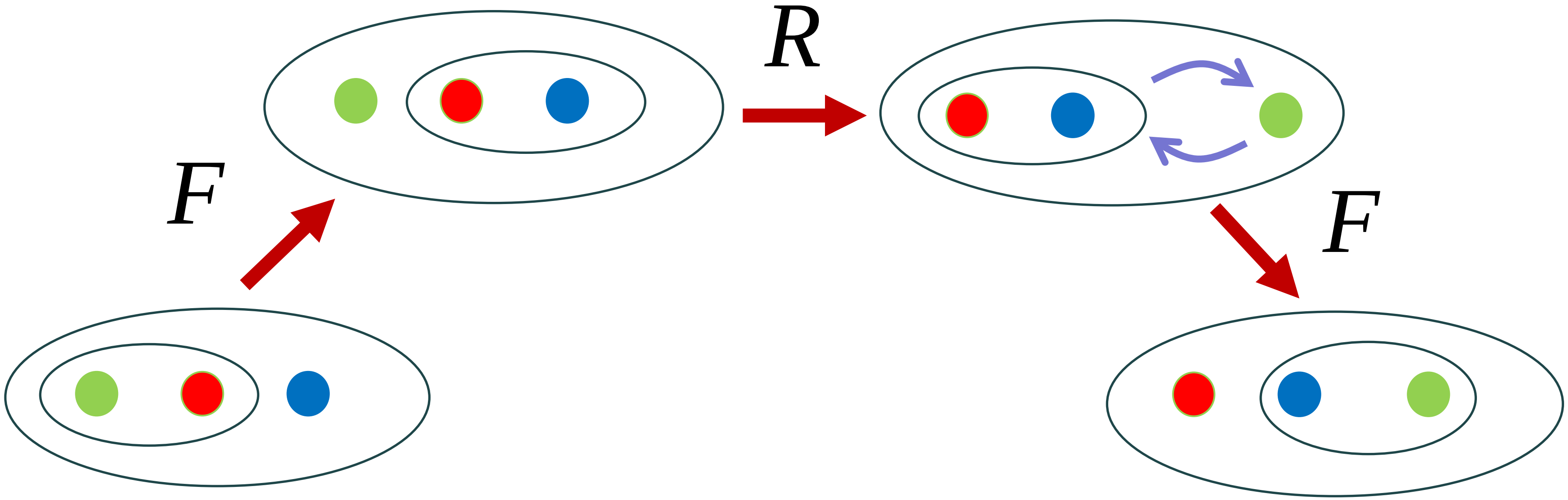




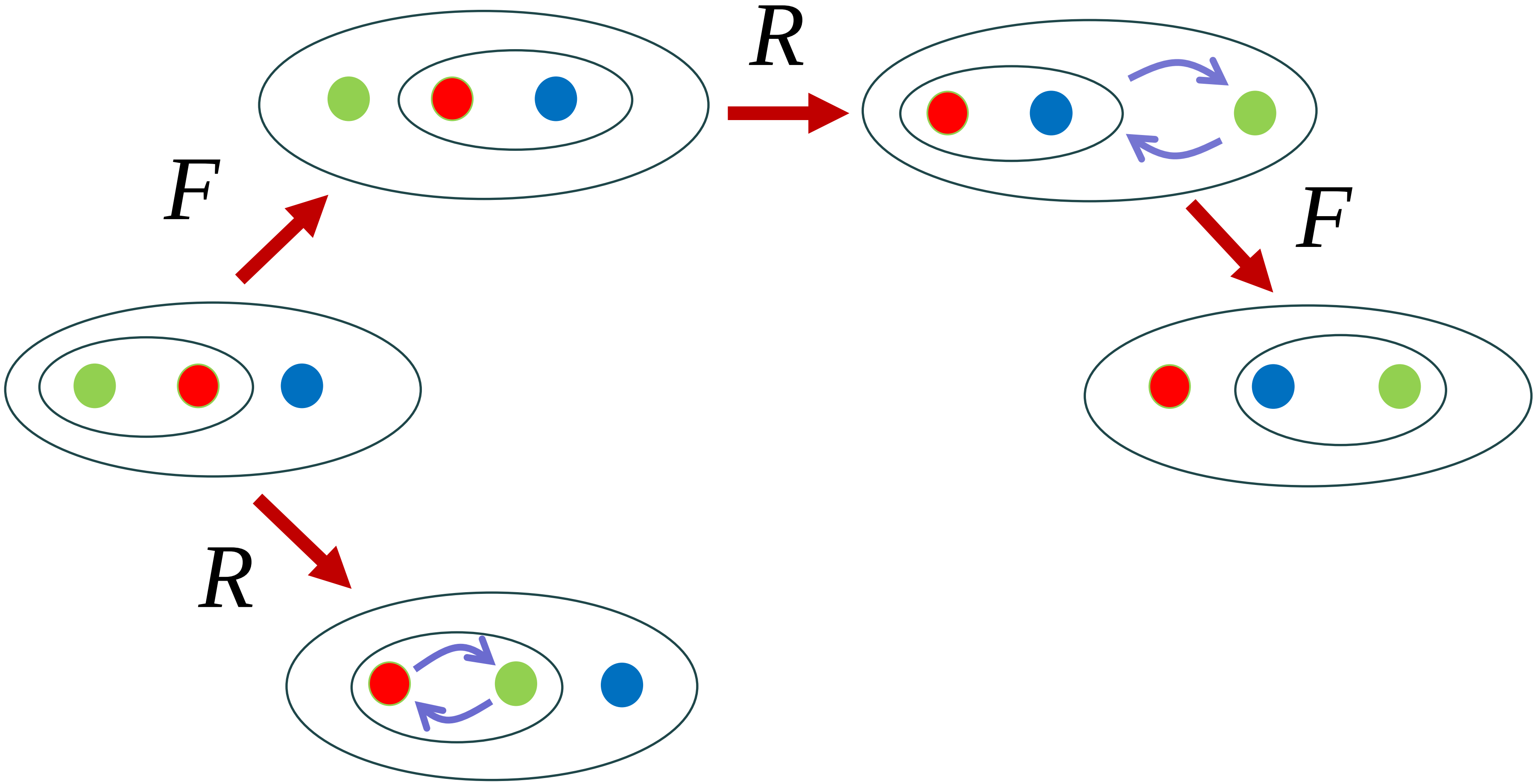
F

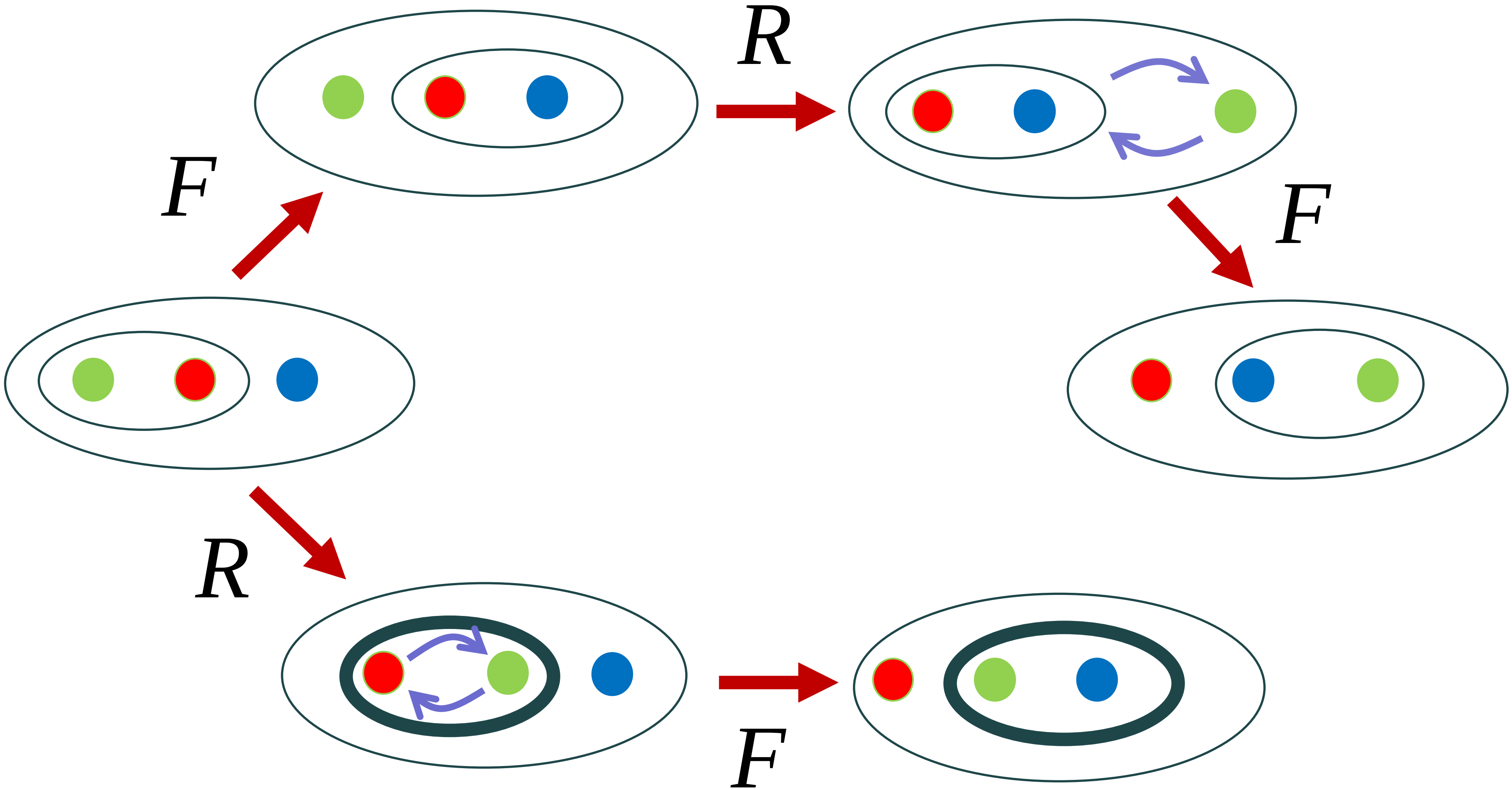

F
R


F
R
F

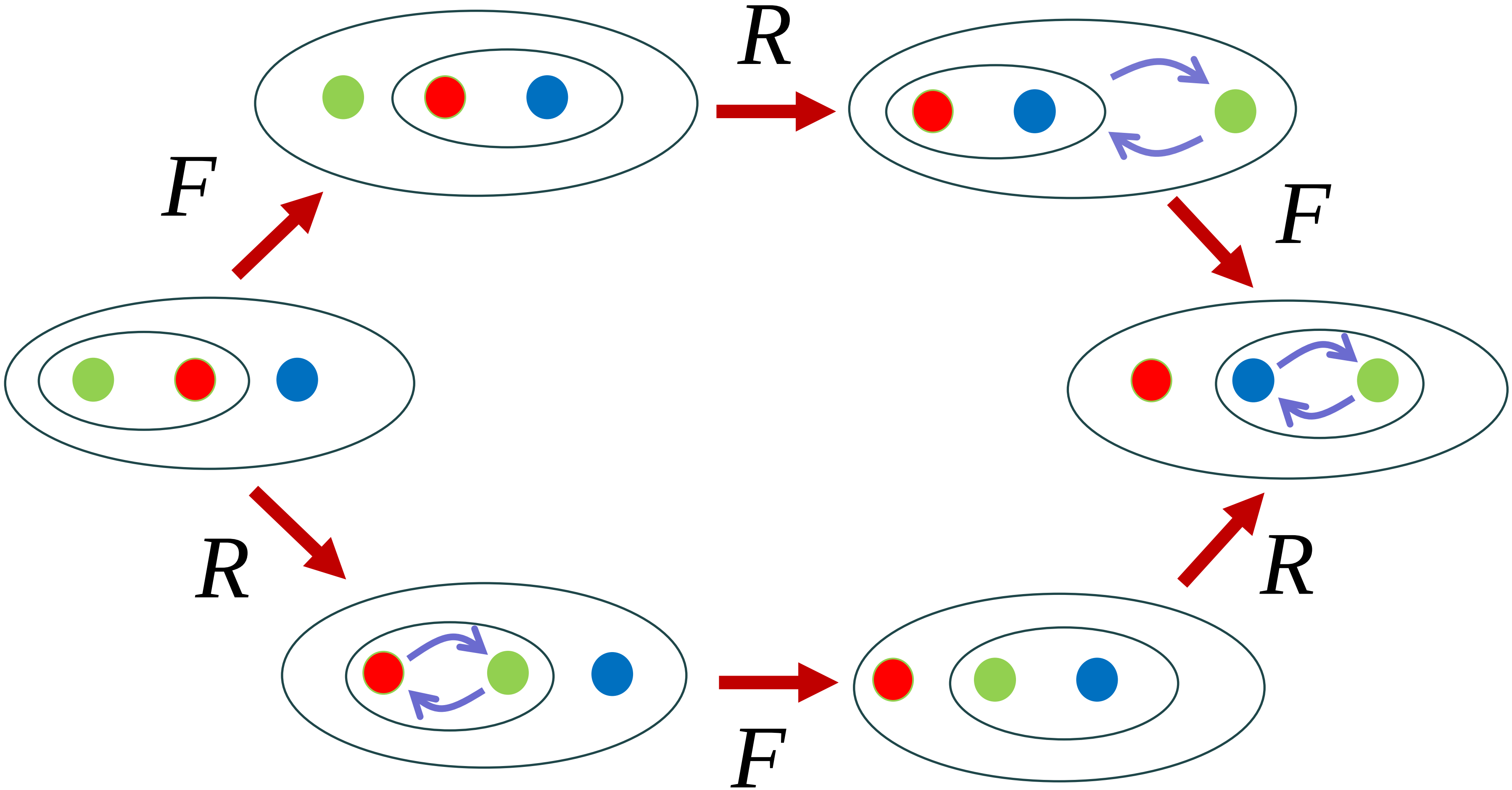


F
R
F


F
R
F
R

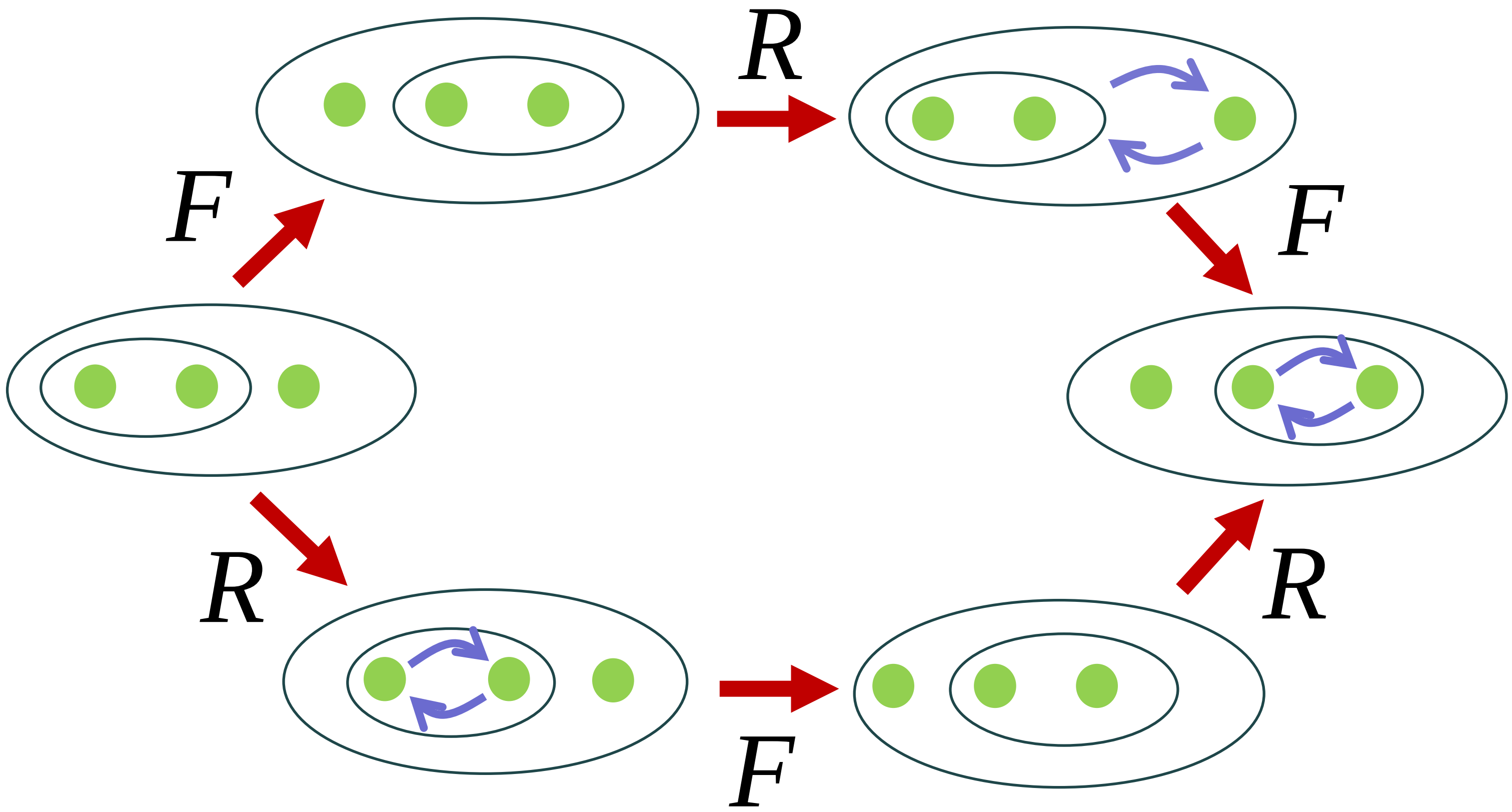


F
R
F
R
F

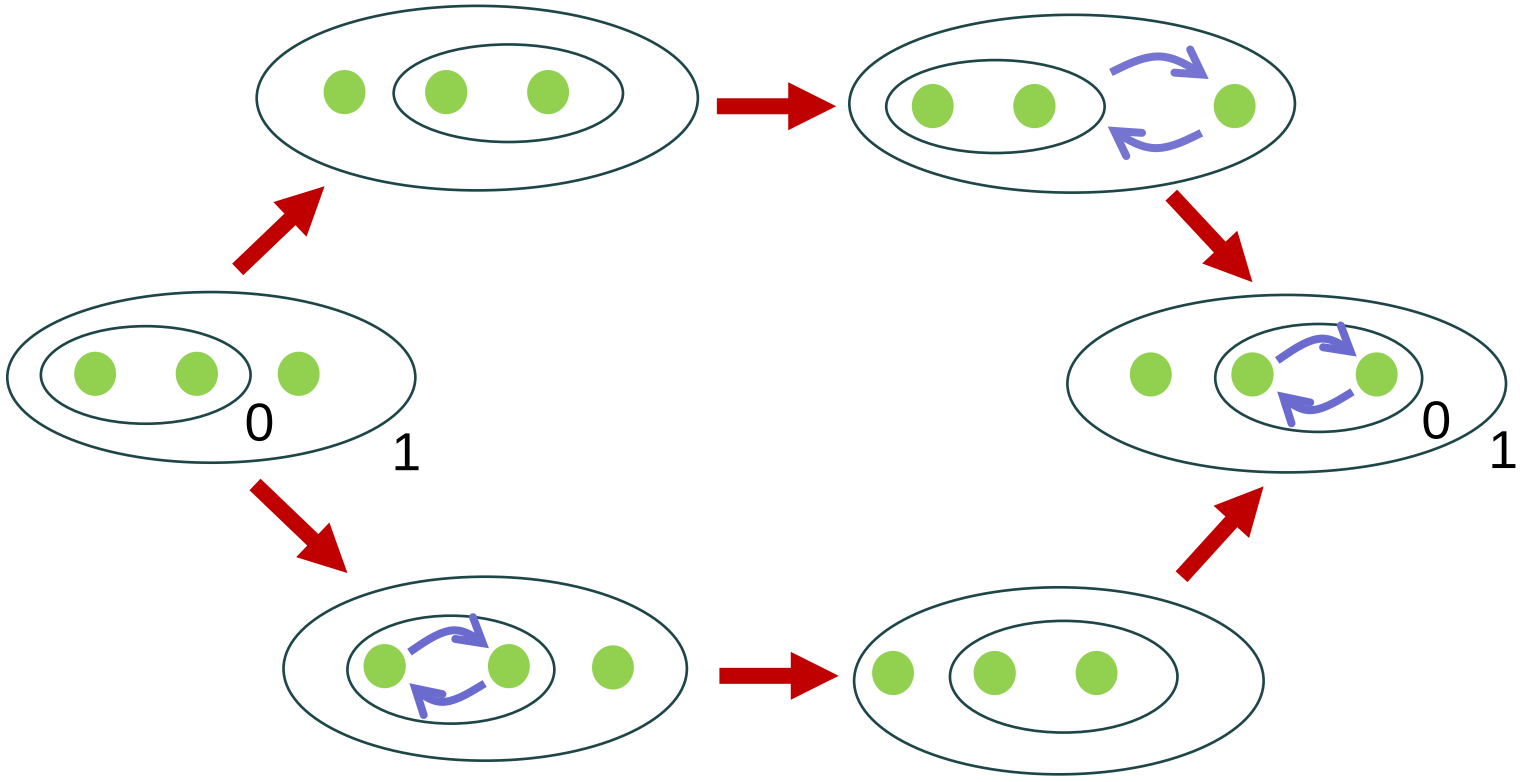


F
R
F
R
F
R

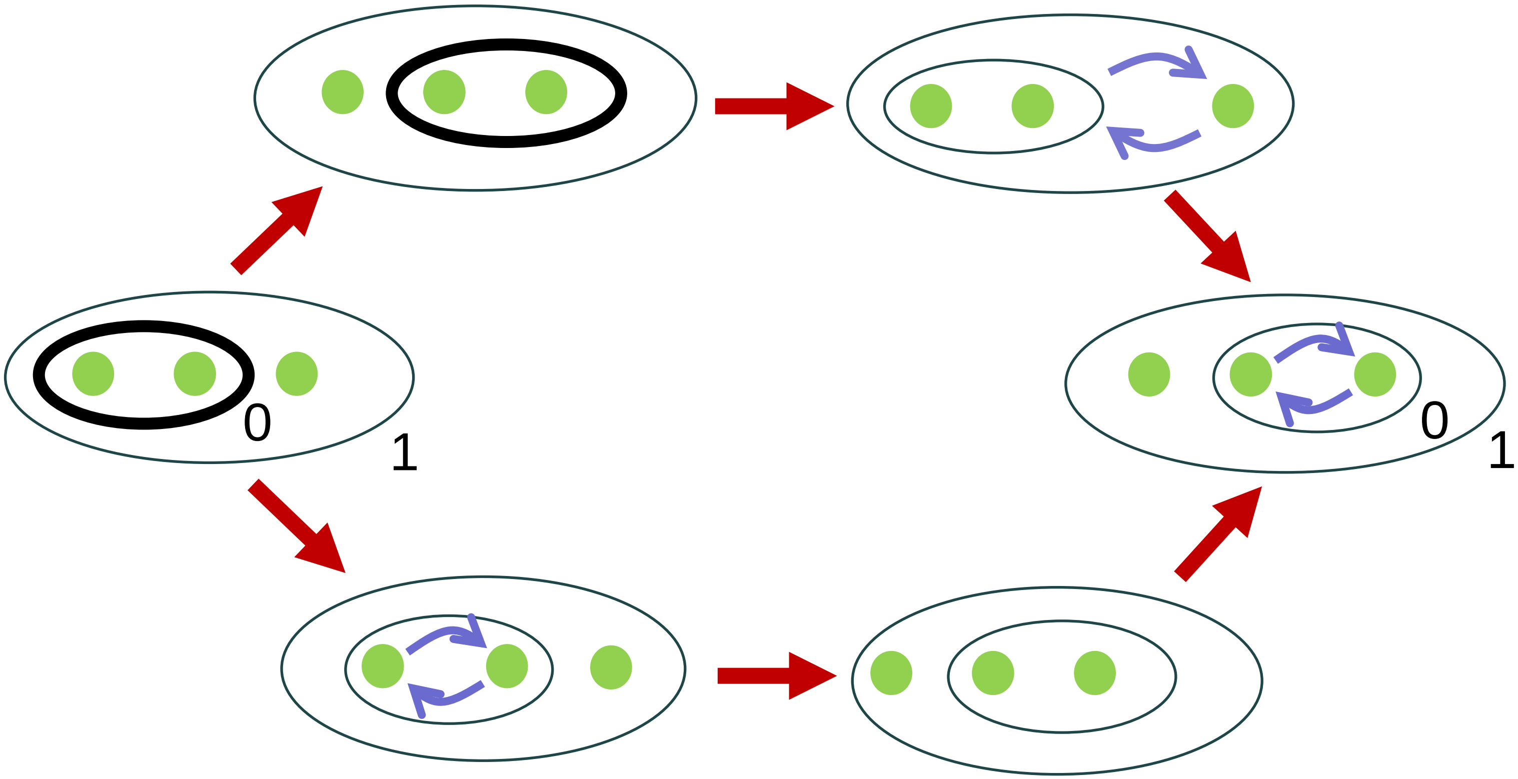
The Hexagon Equation



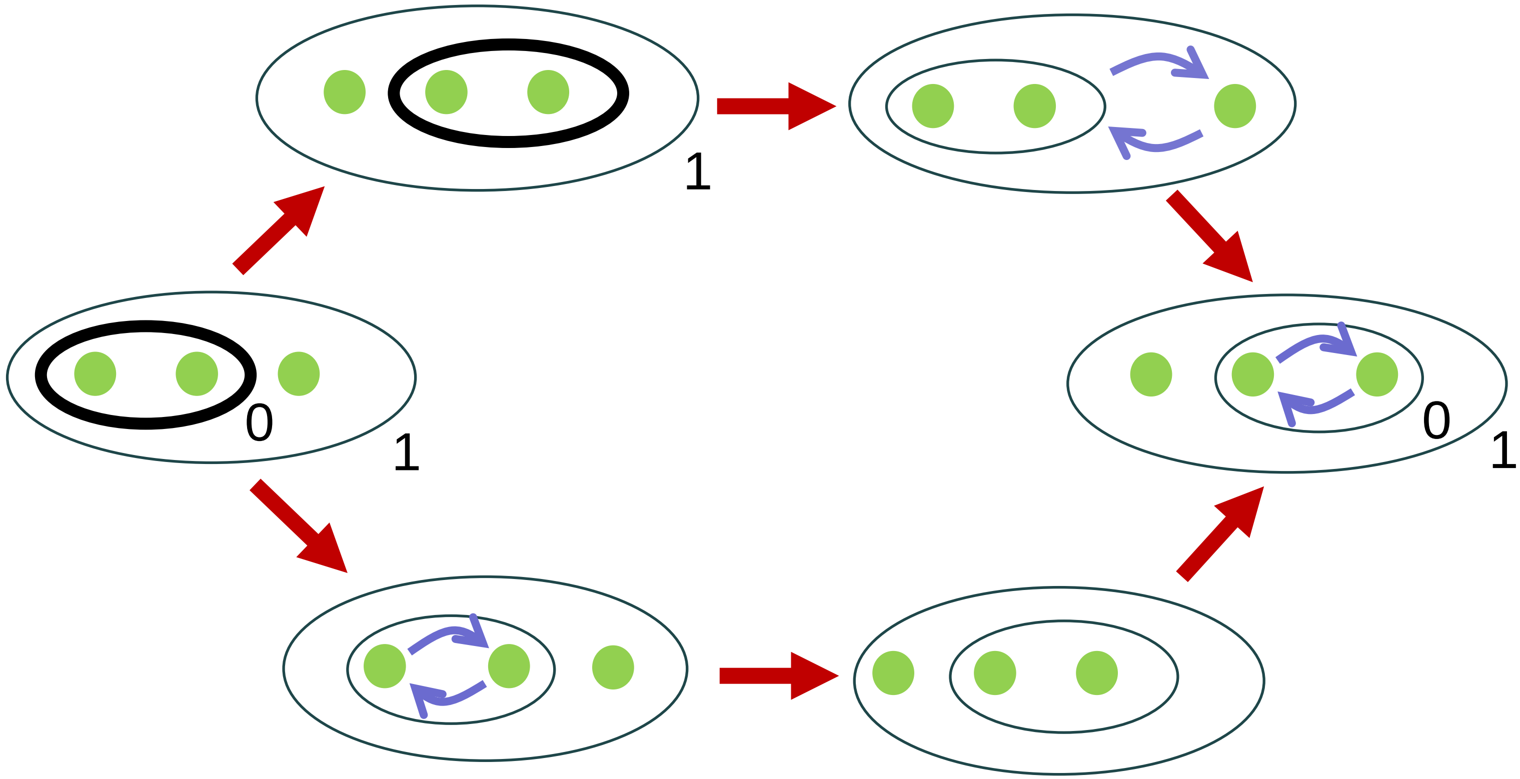
The Hexagon Equation



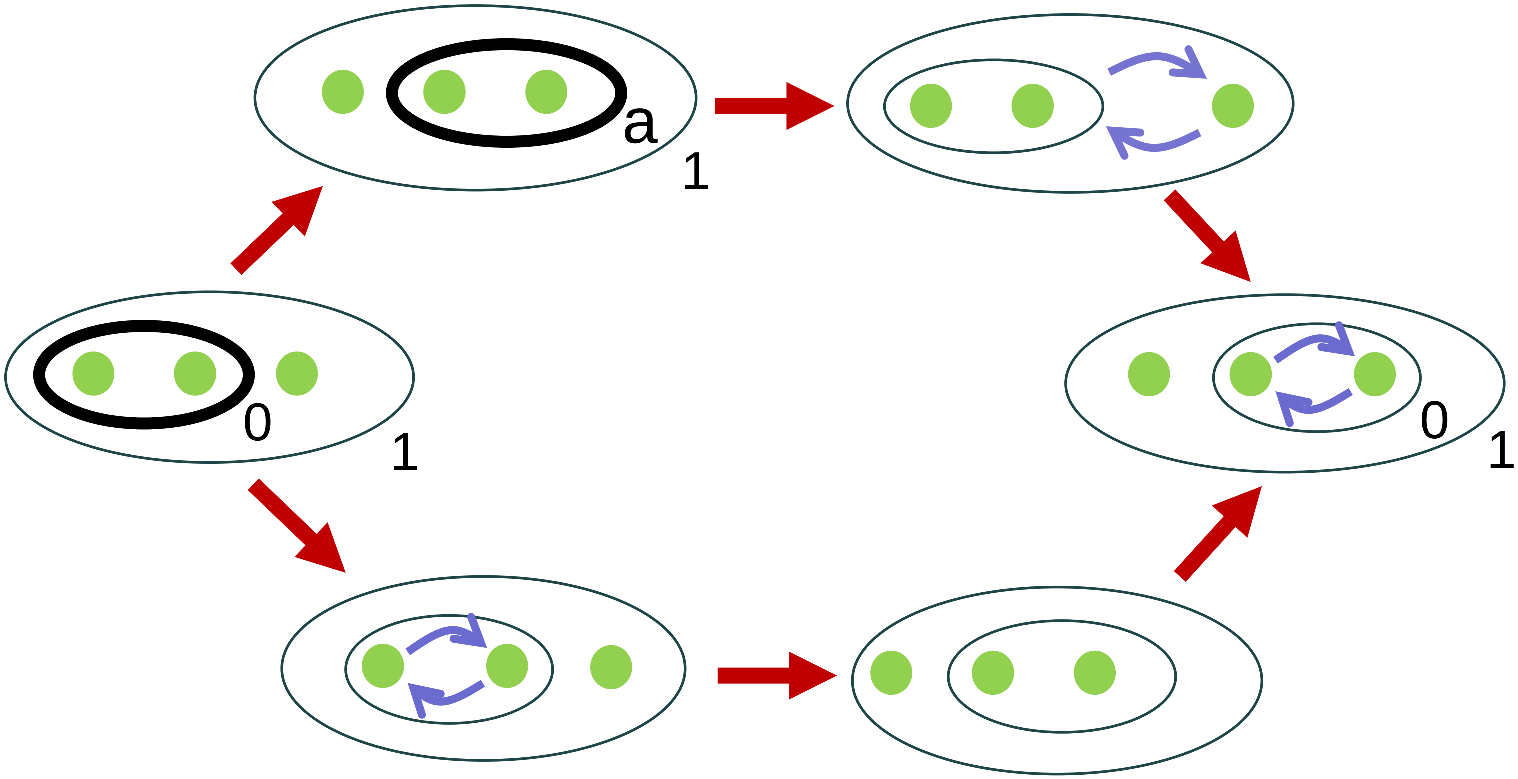
The Hexagon Equation



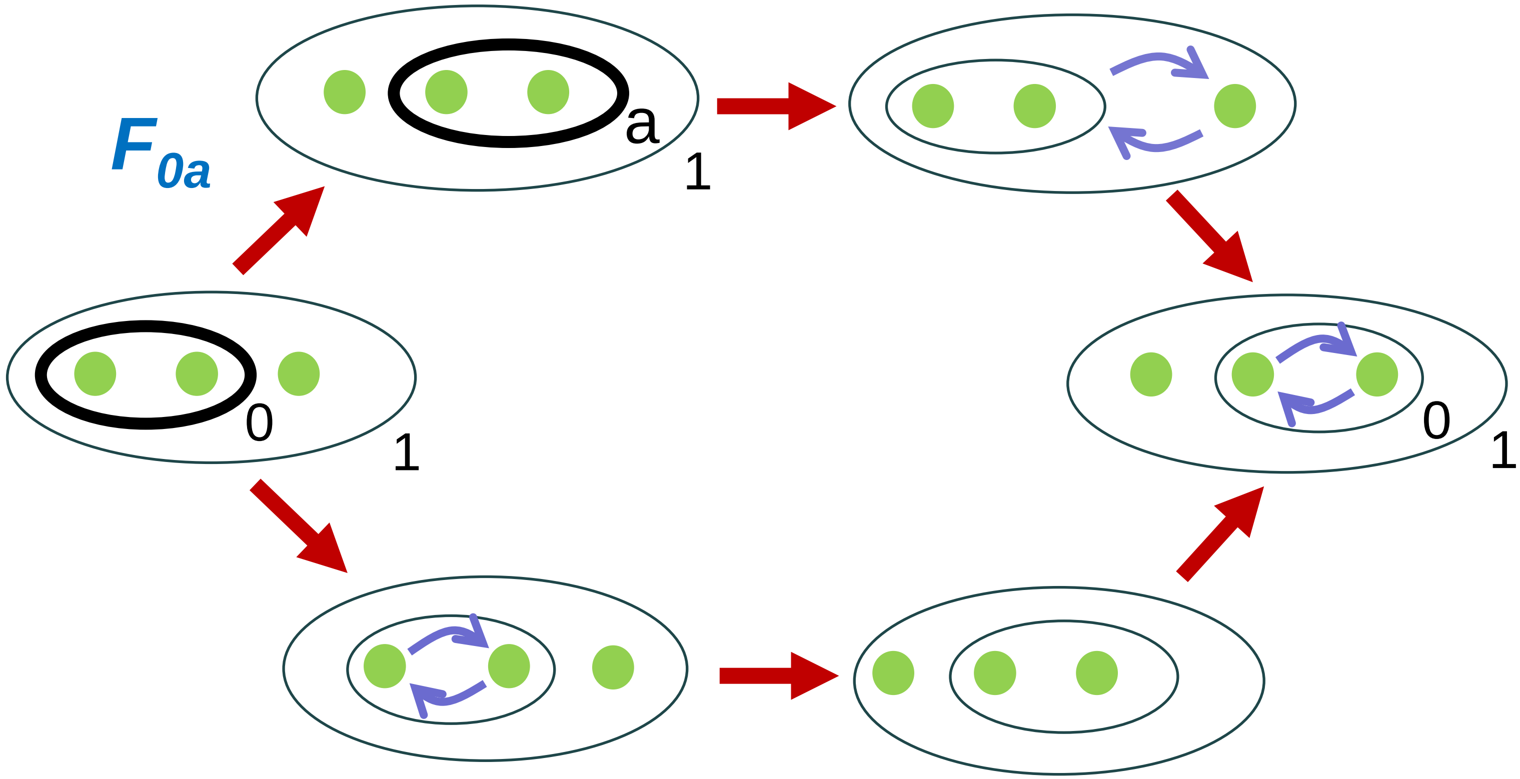
The Hexagon Equation



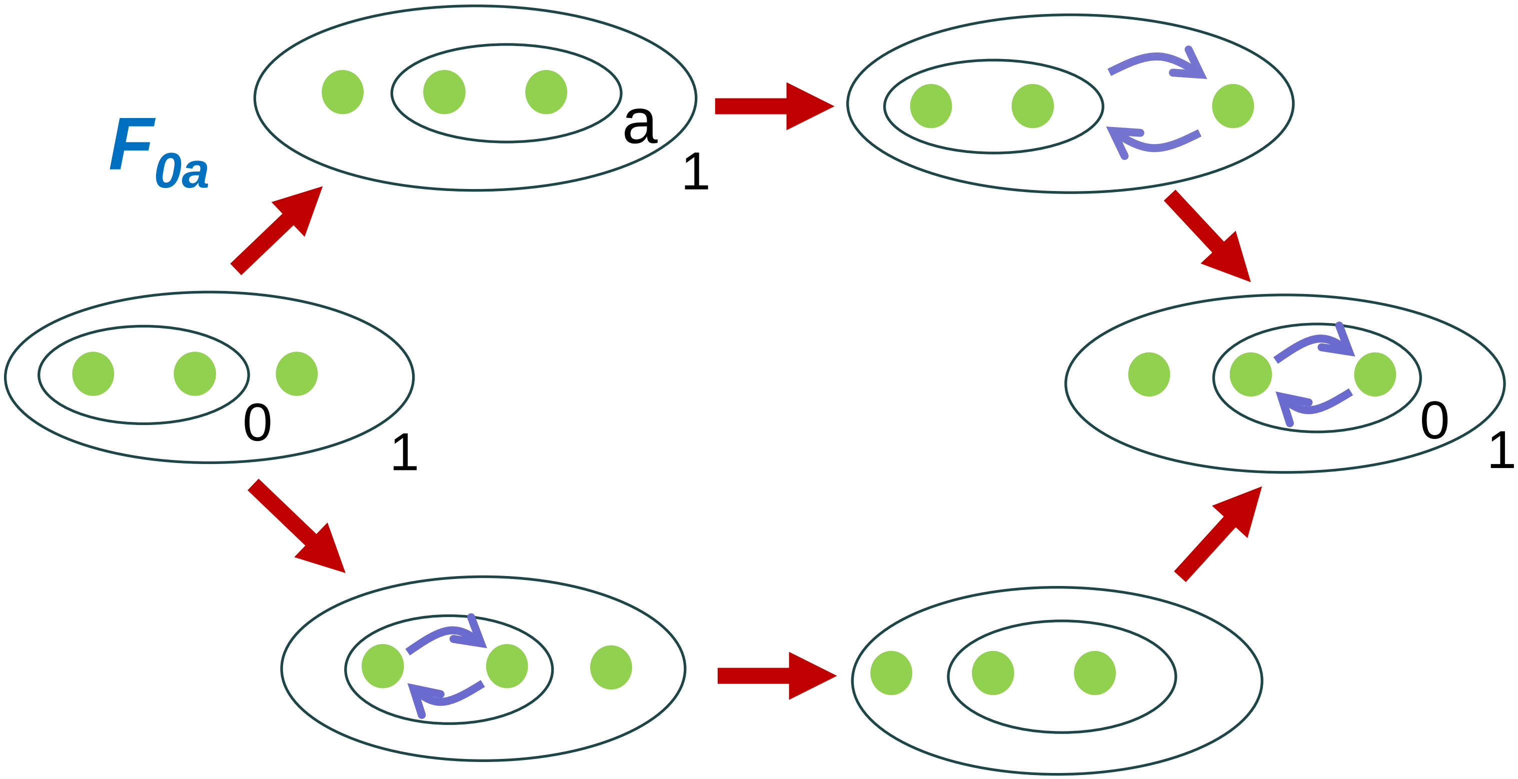
The Hexagon Equation



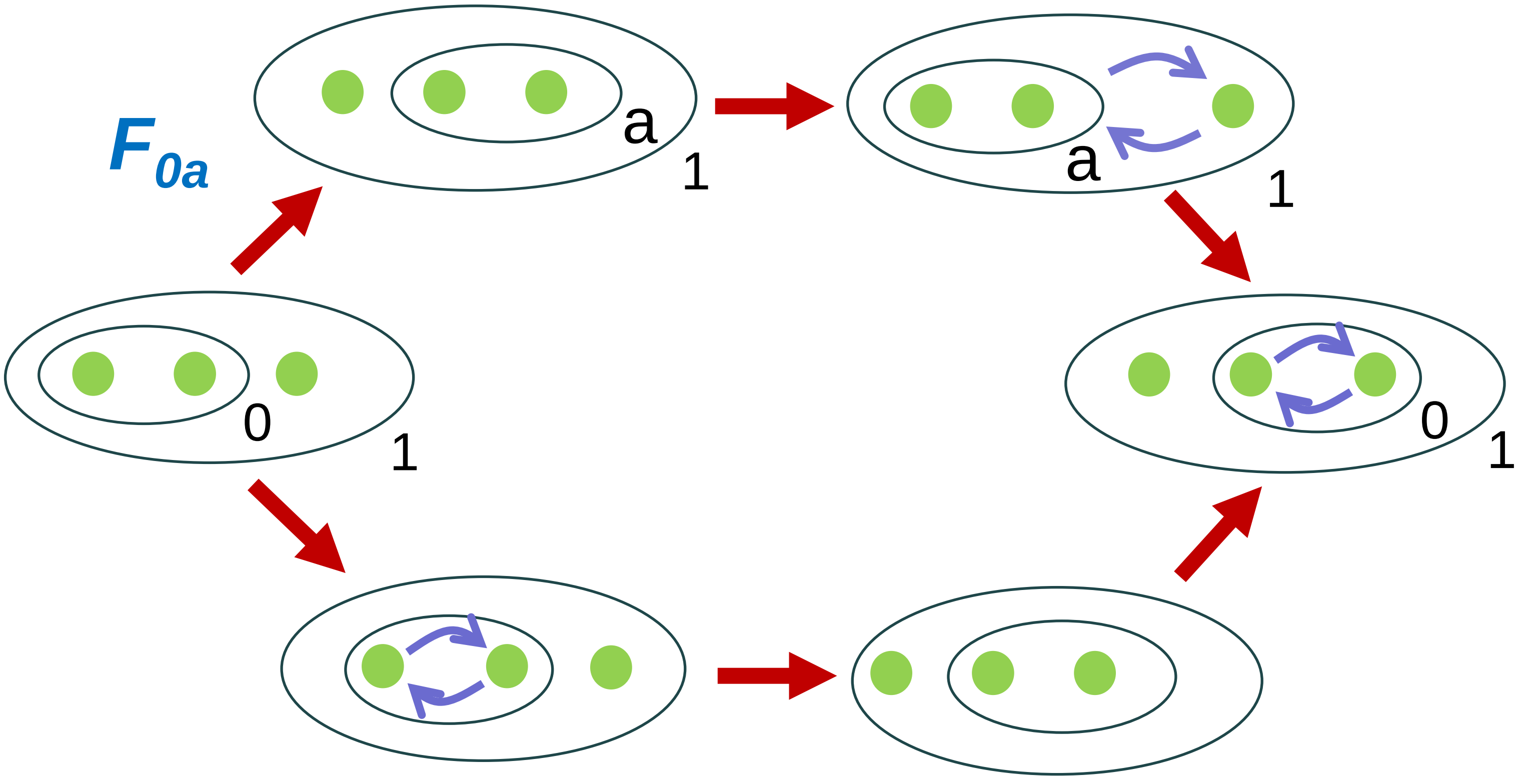
The Hexagon Equation



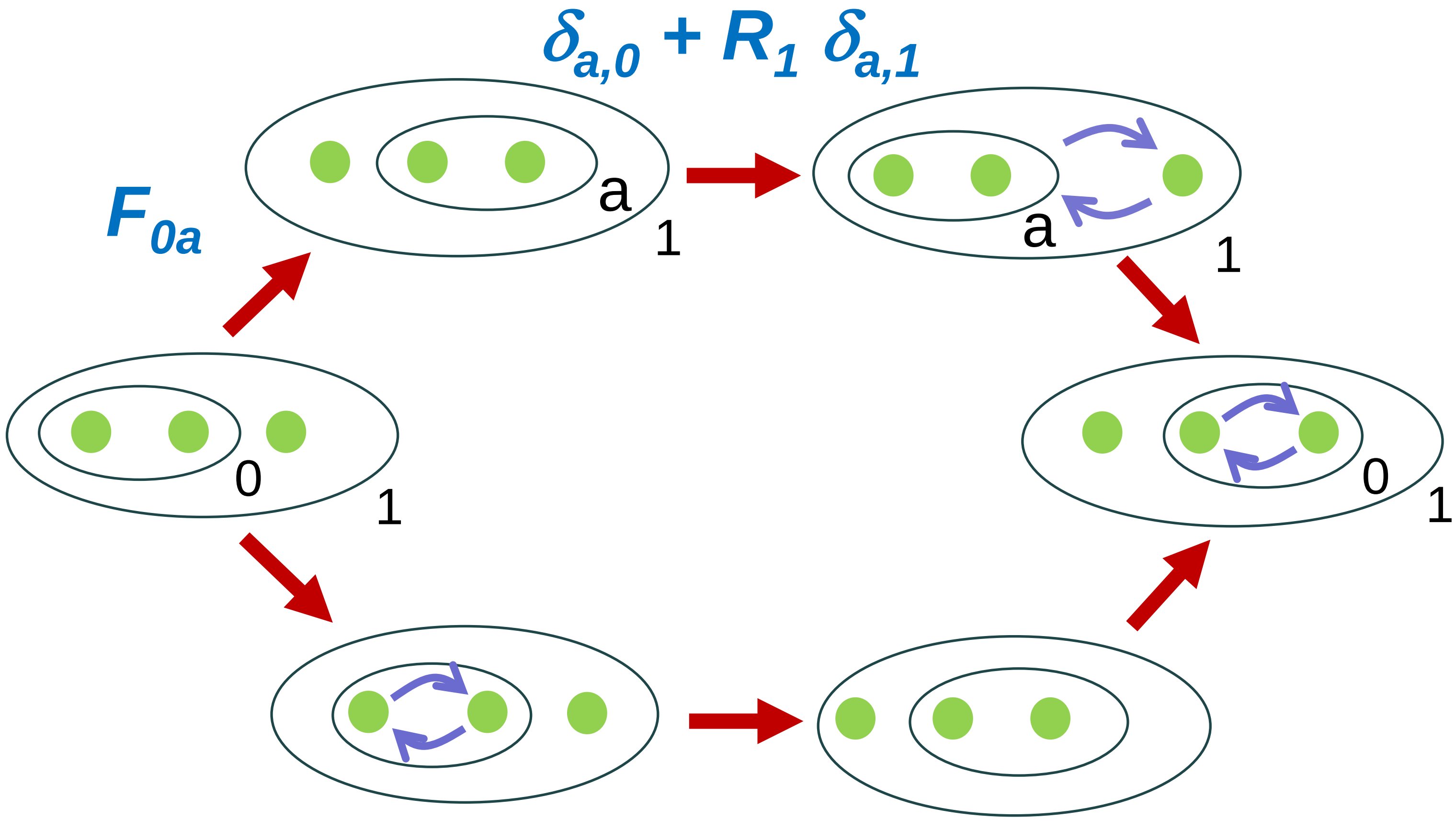
The Hexagon Equation



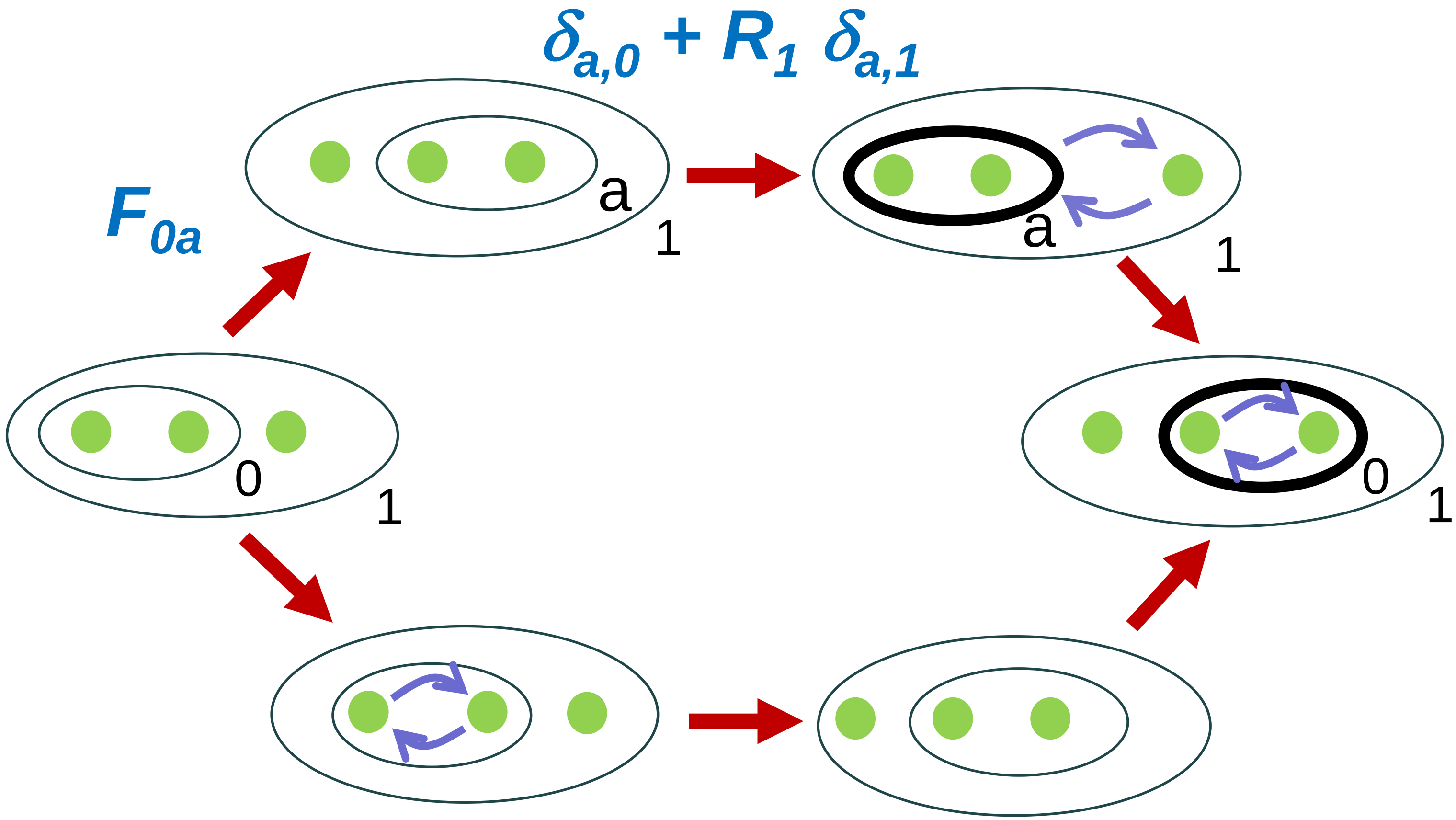
The Hexagon Equation



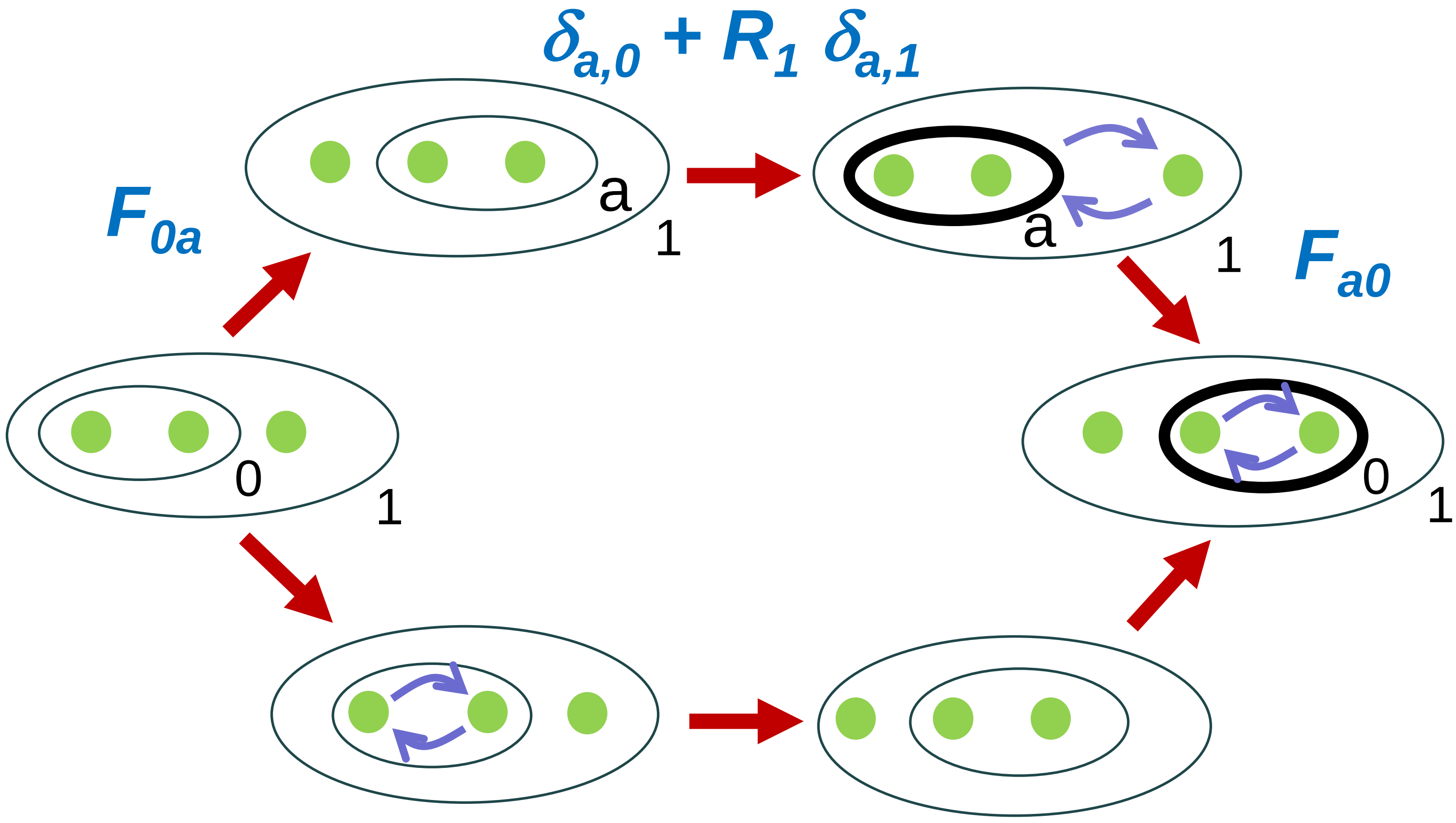
The Hexagon Equation



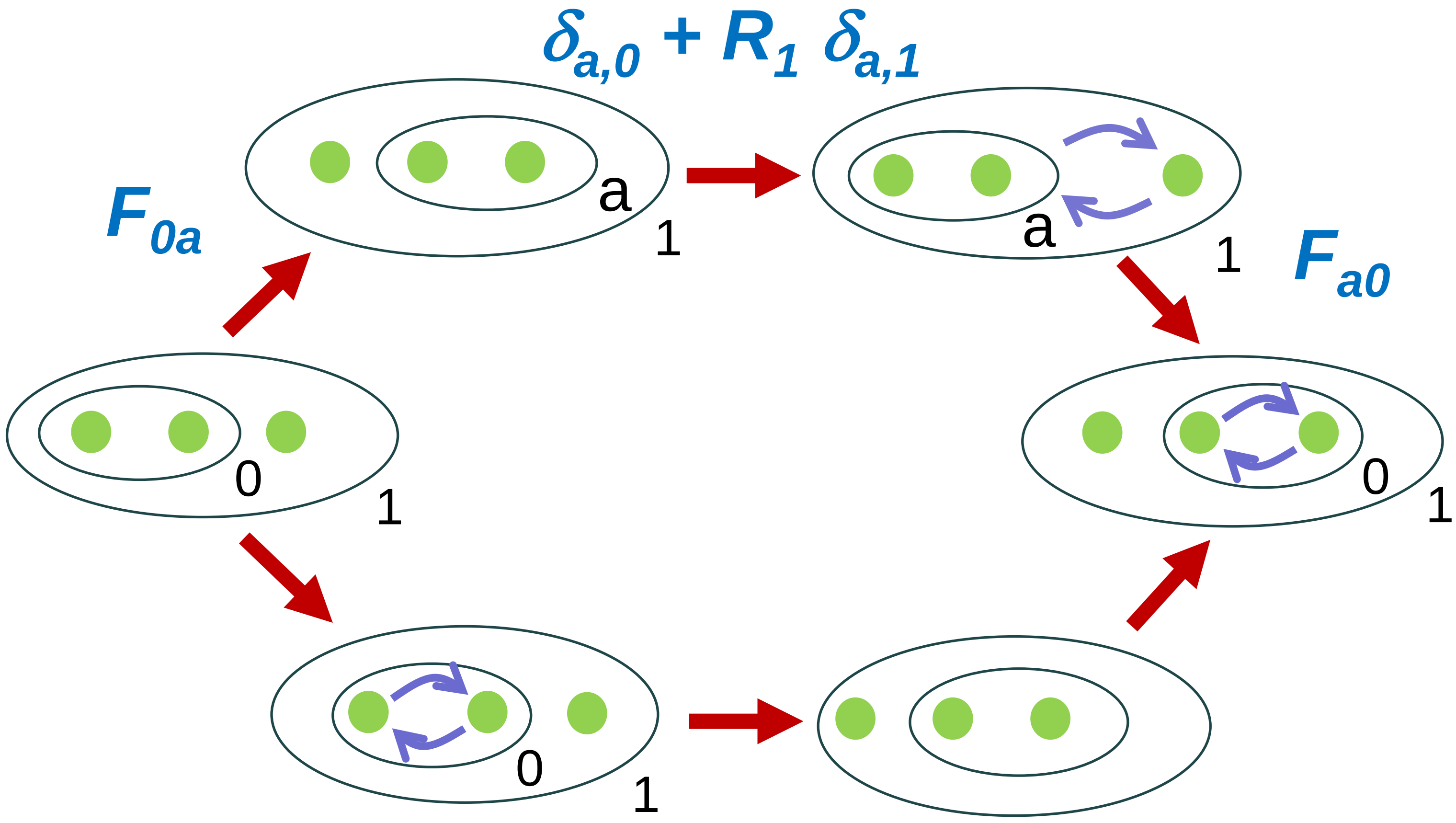
The Hexagon Equation



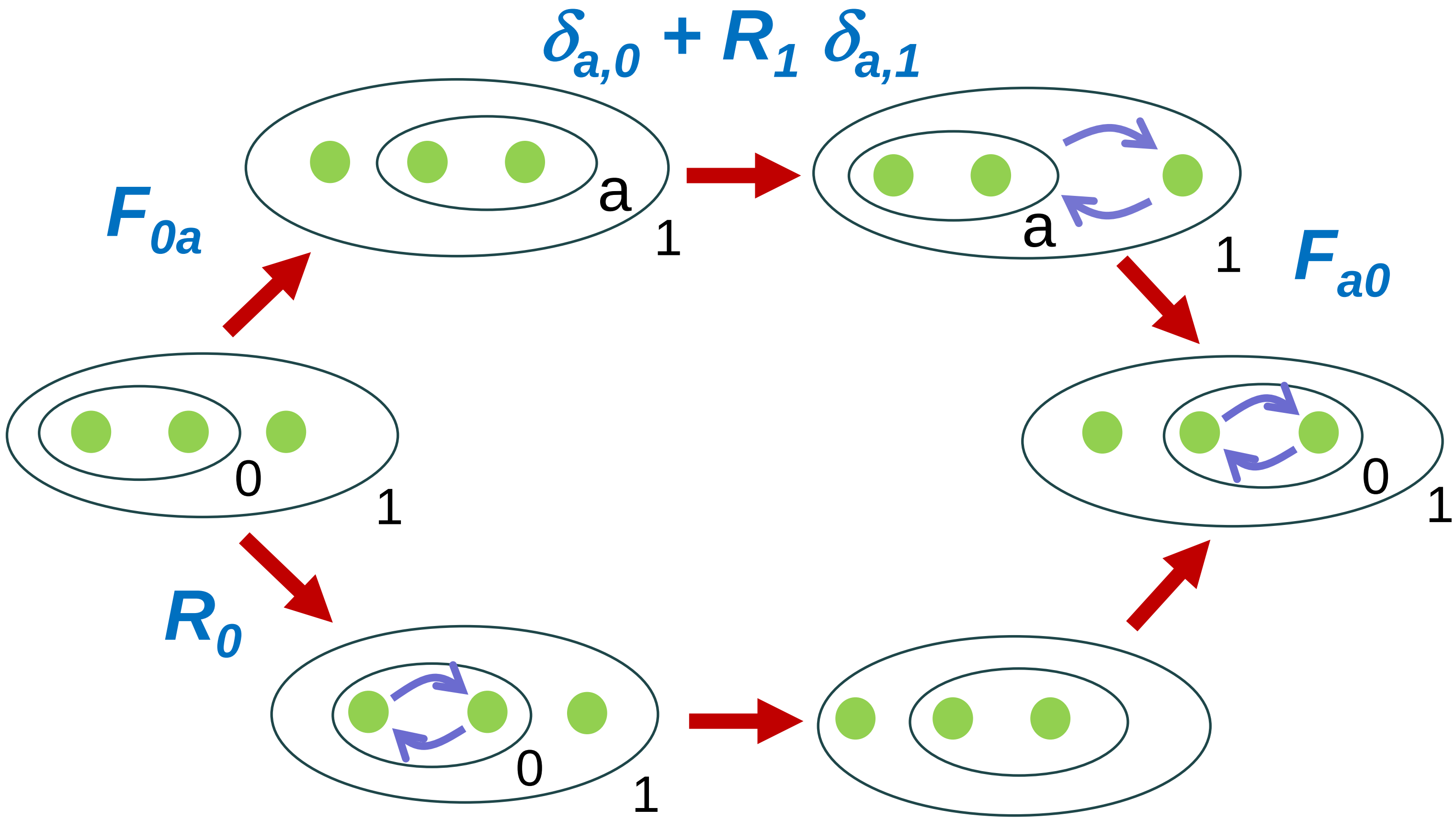
The Hexagon Equation



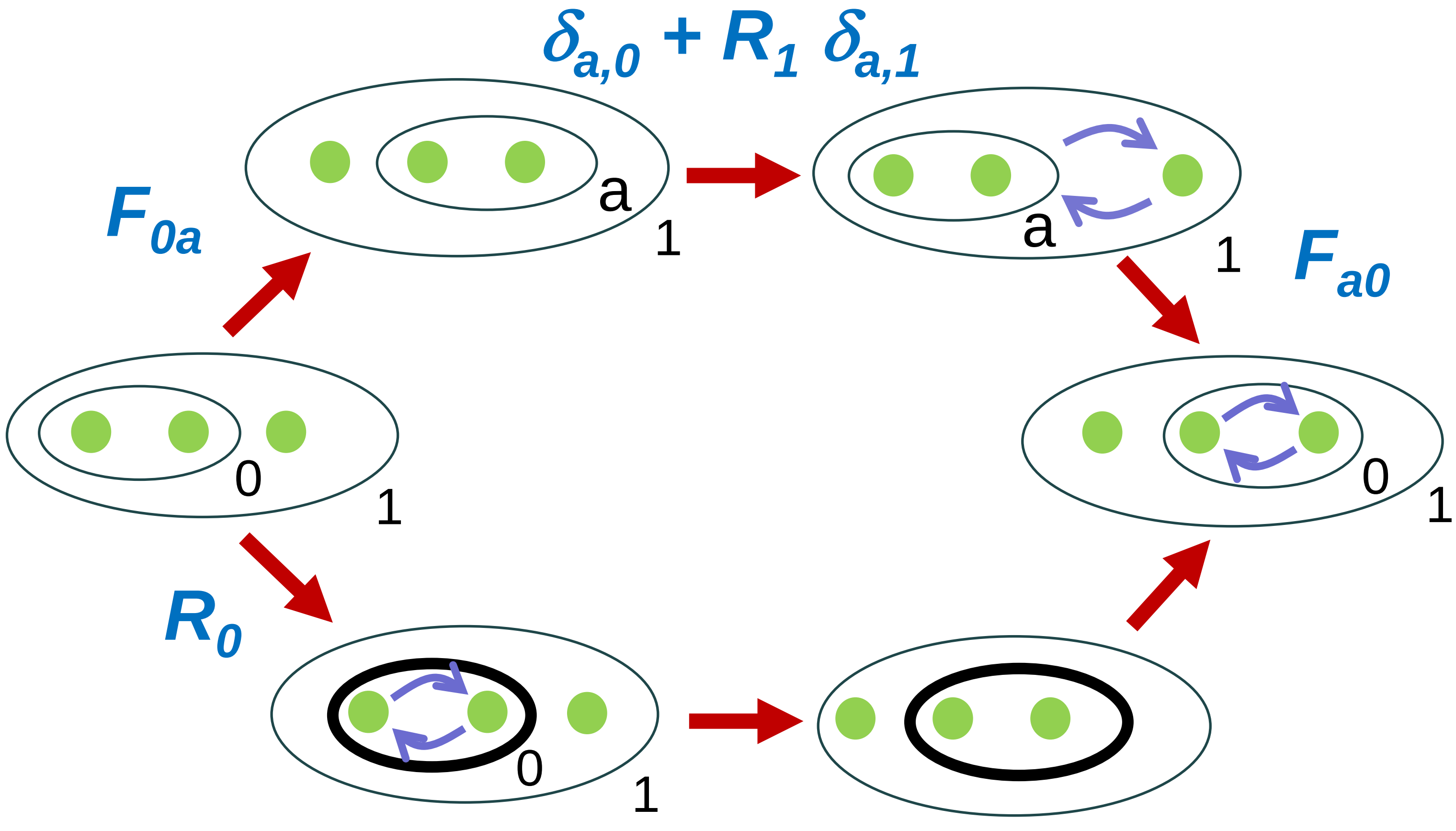
The Hexagon Equation



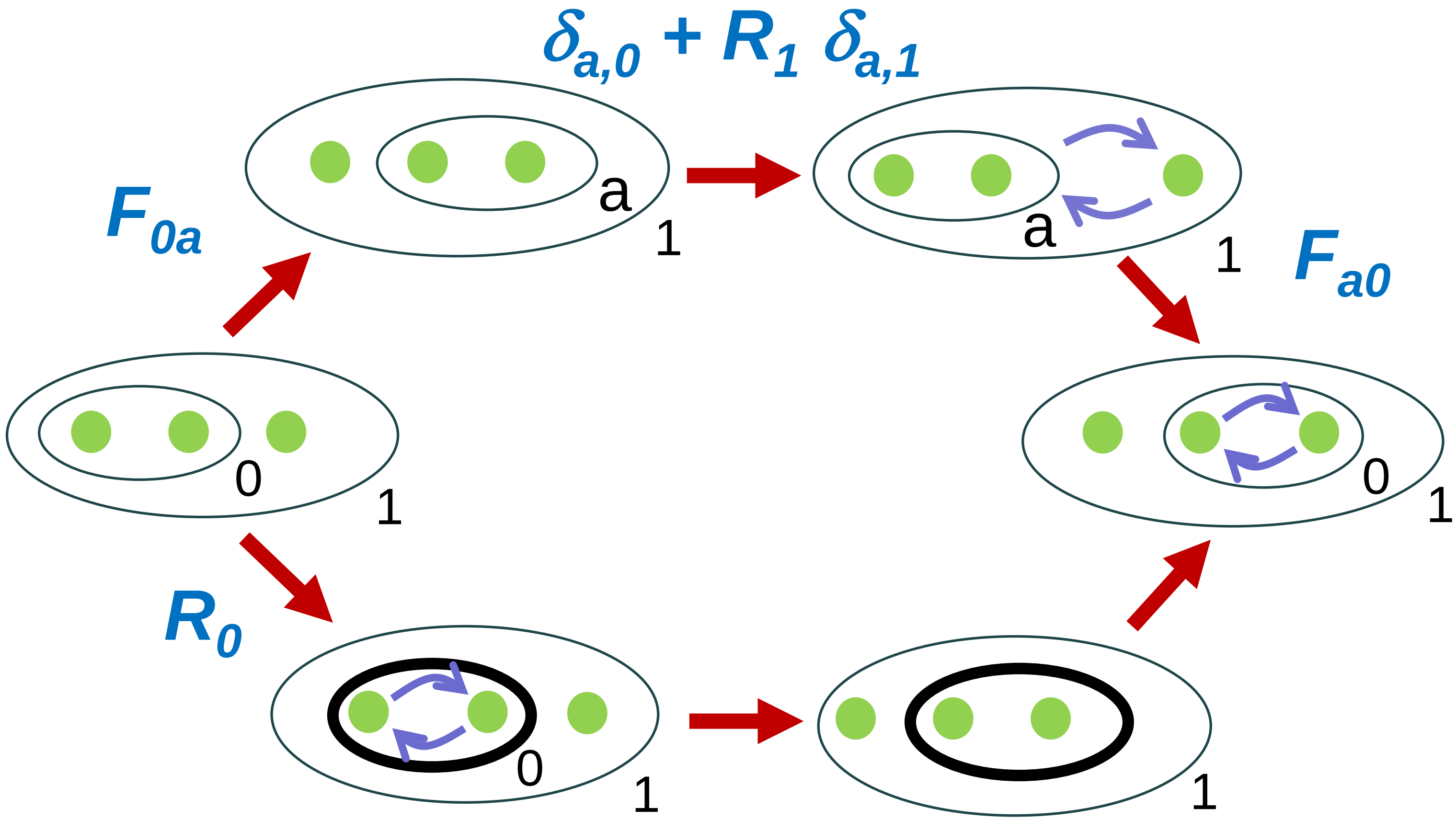
The Hexagon Equation



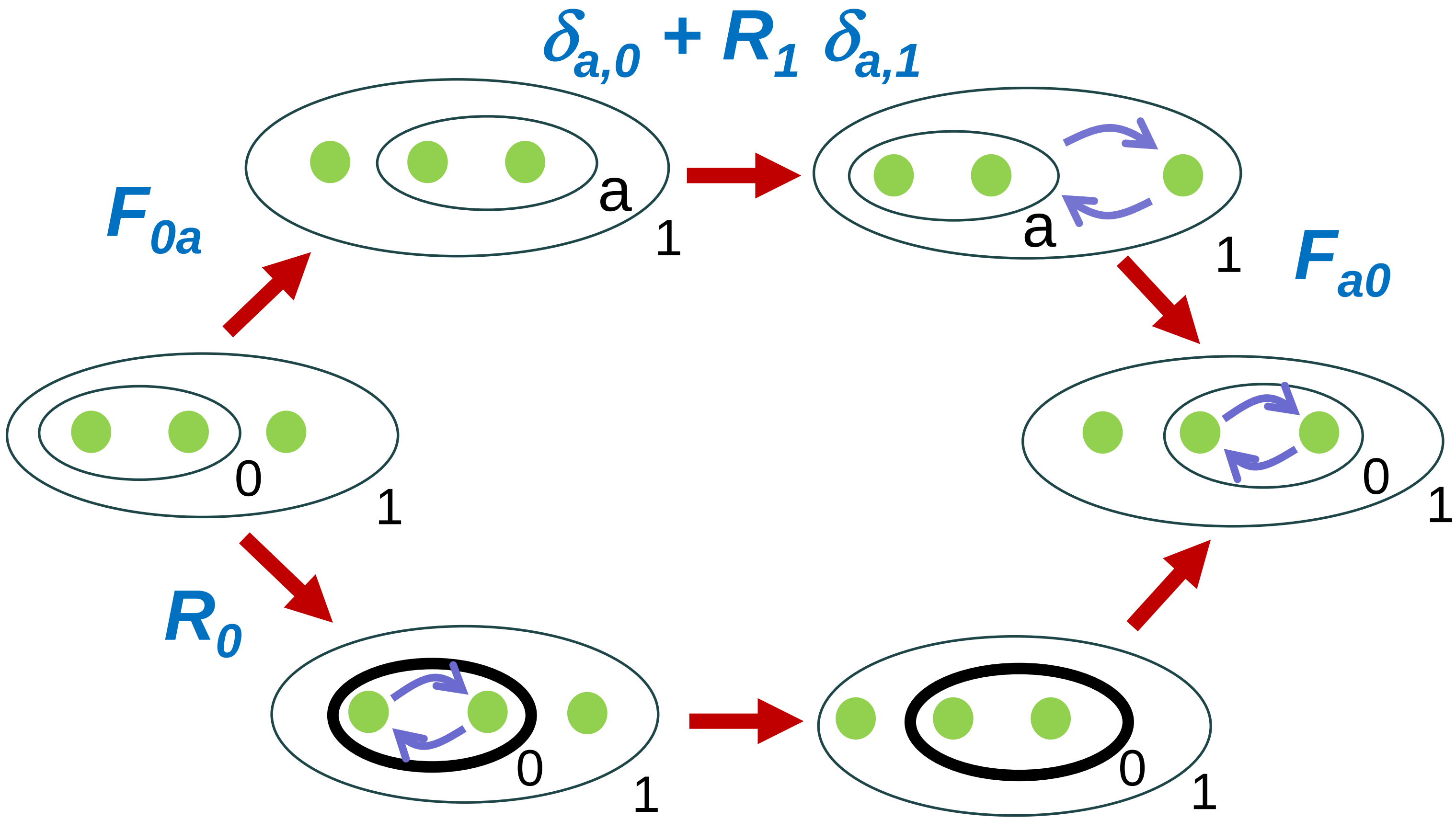
The Hexagon Equation



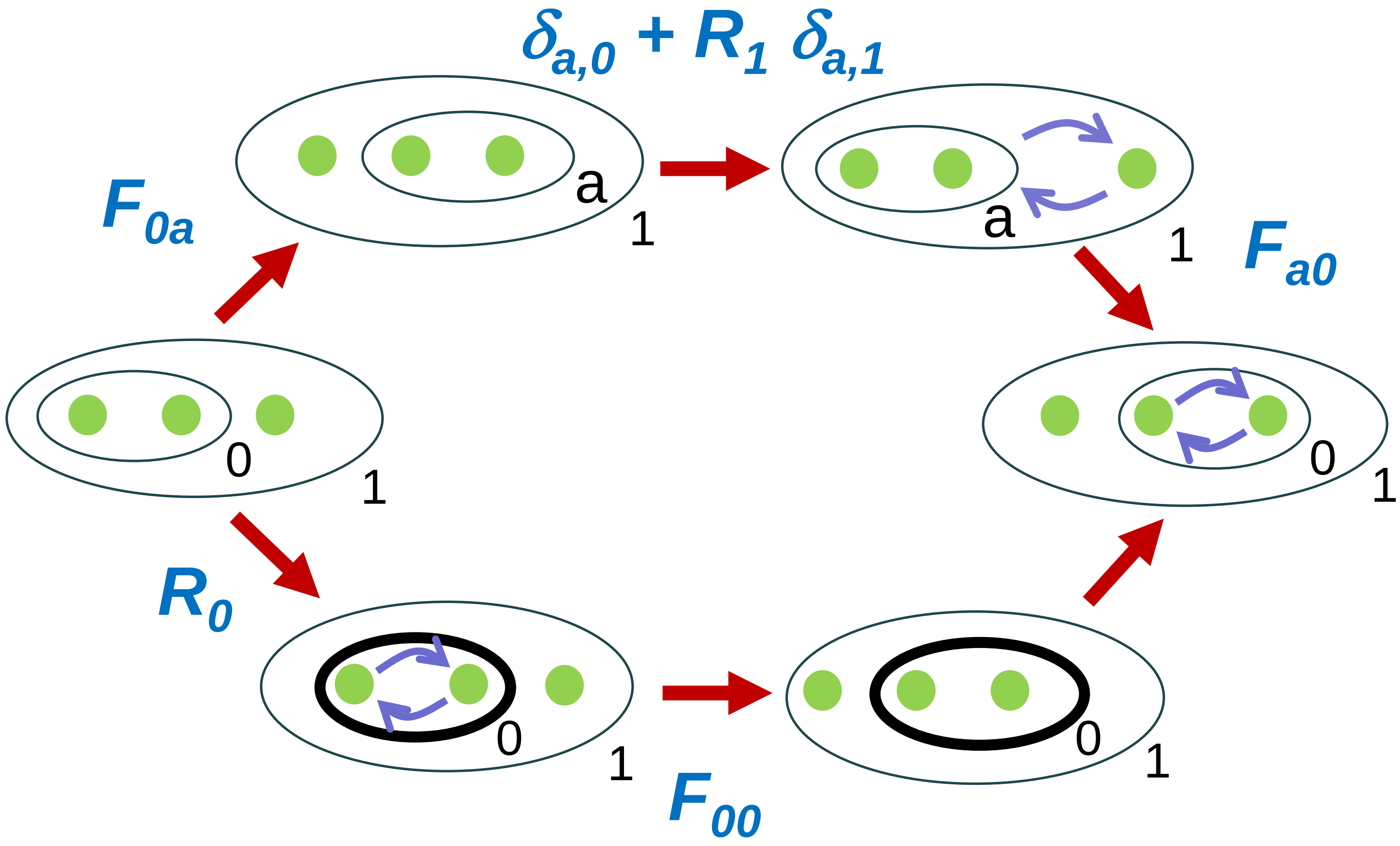
The Hexagon Equation



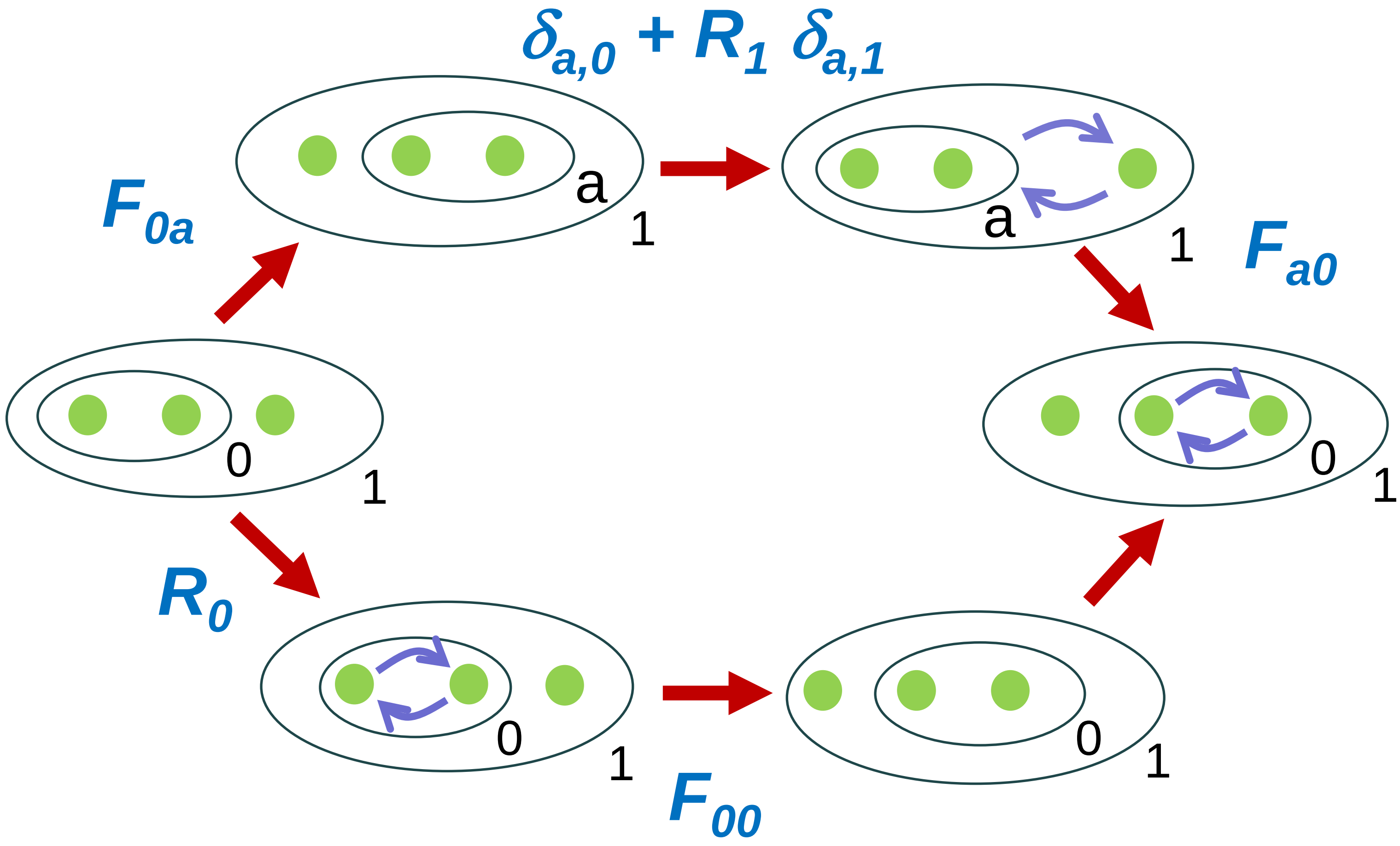
The Hexagon Equation



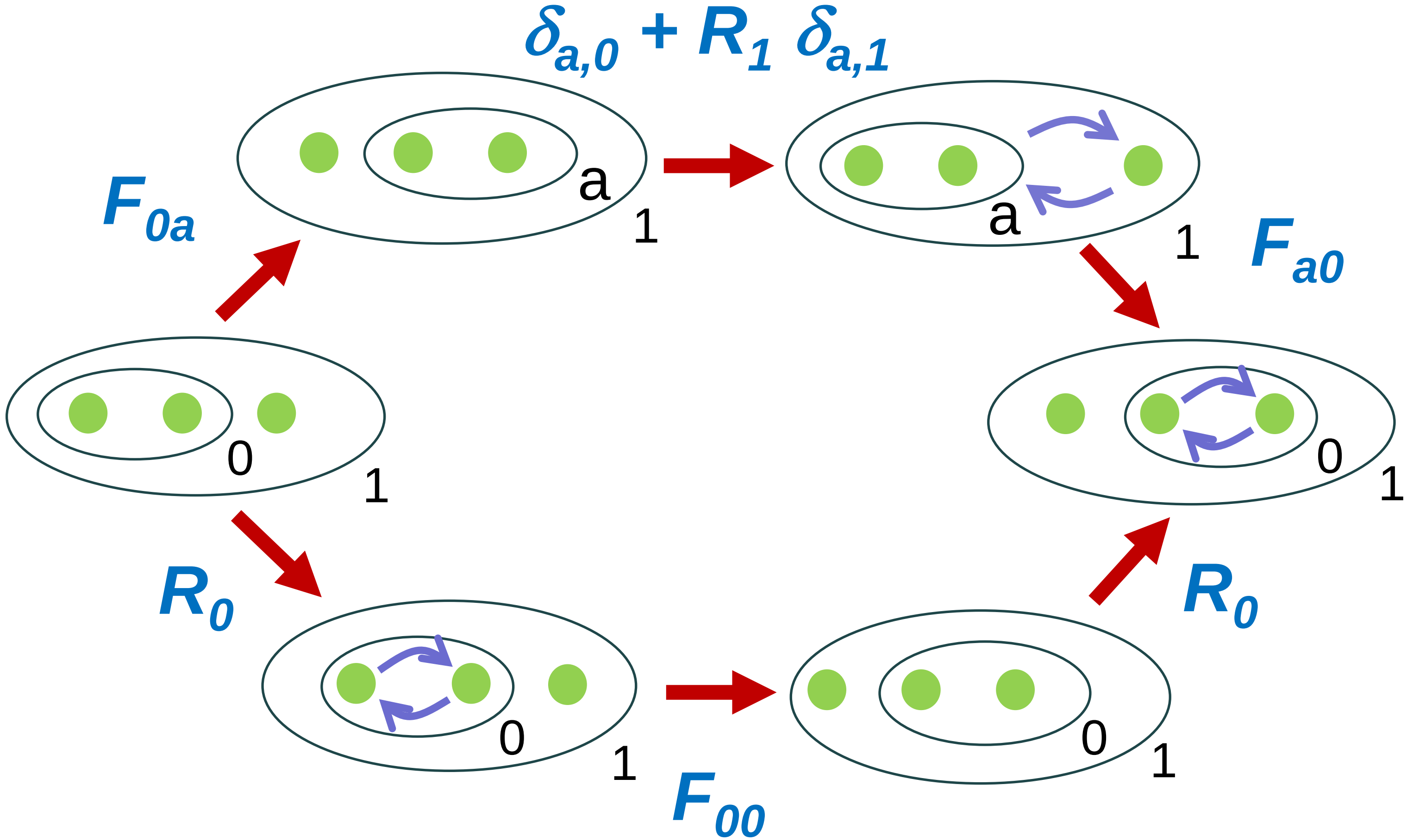
The Hexagon Equation



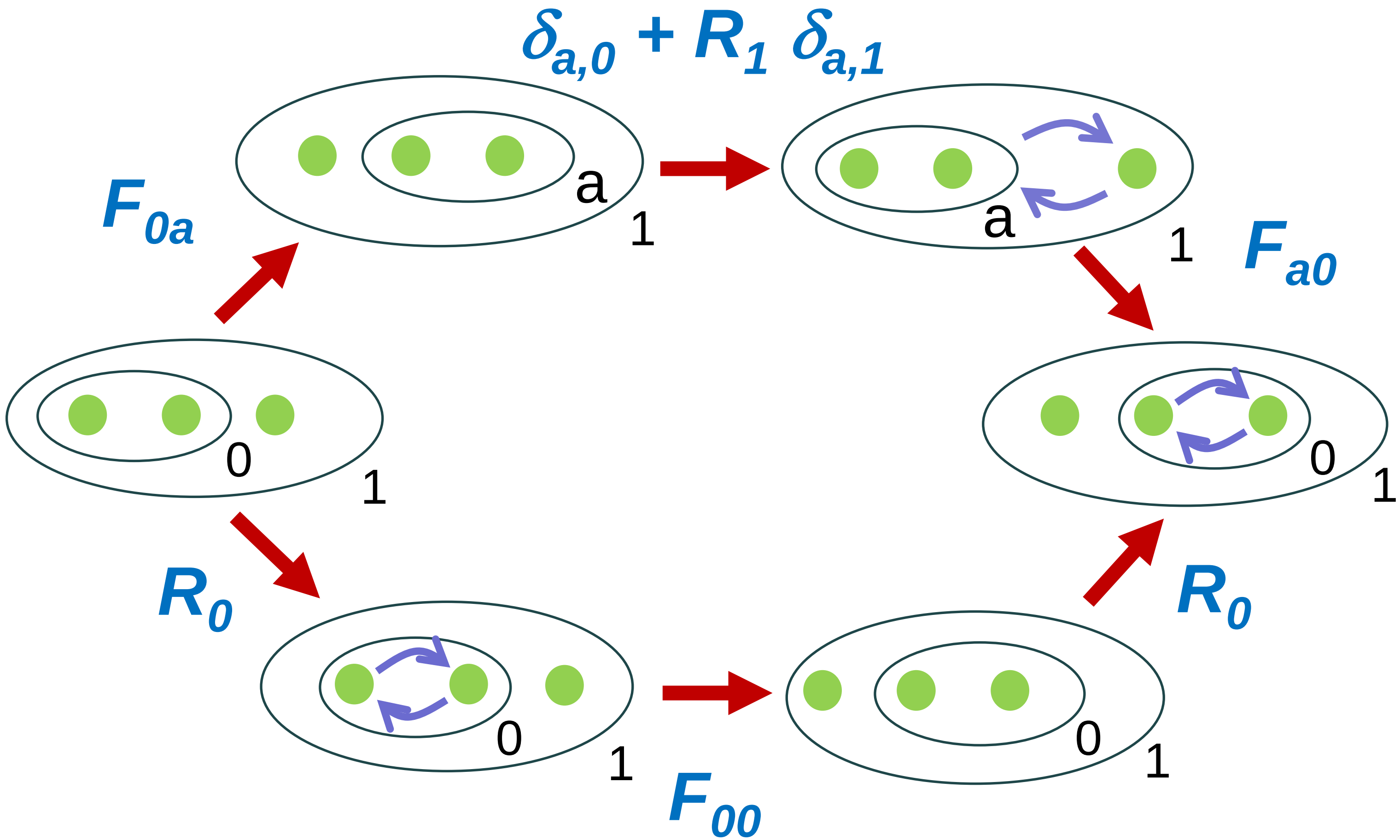
The Hexagon Equation



The Hexagon Equation



The Hexagon Equation

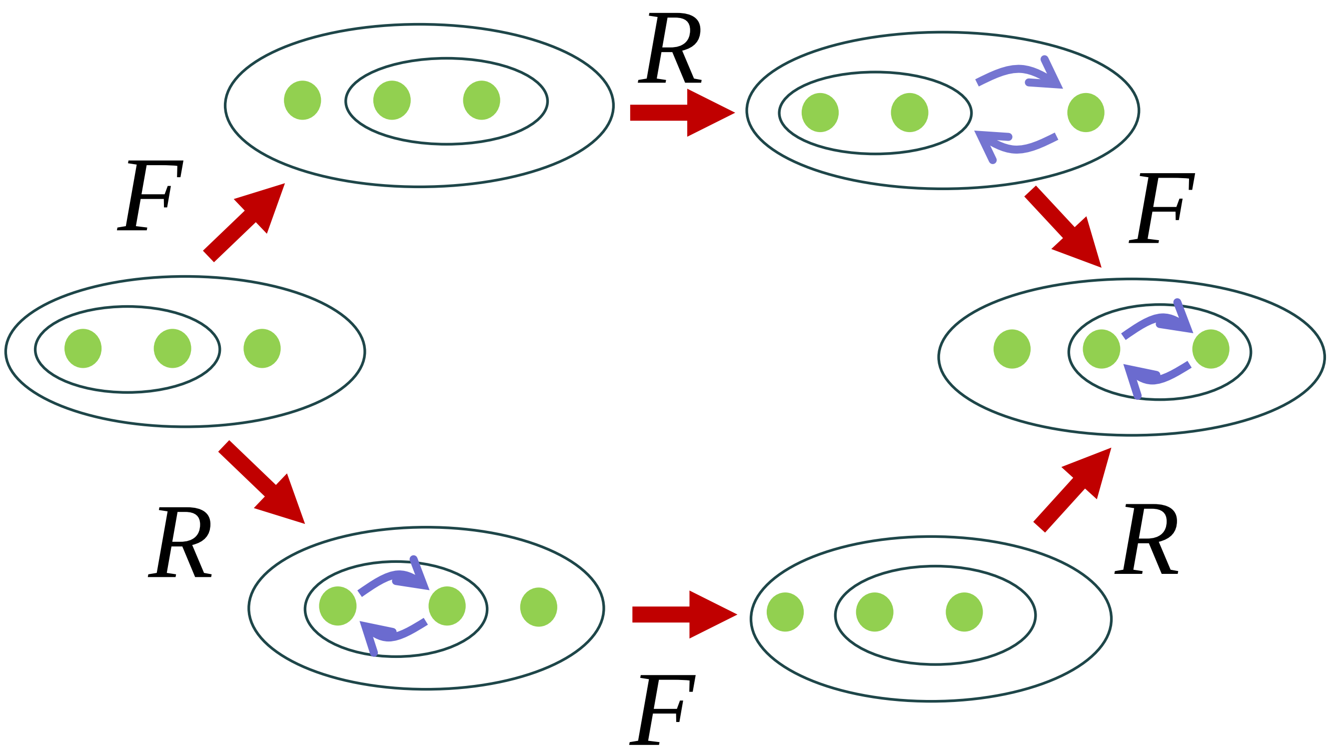


Top path

$$\sum_a F_{0a}\,(\delta_{a,0} + R_1\,\delta_{a,1})\,F_{a0} \;=\; R_0\,F_{00}\,R_0$$

Bottom path

The Hexagon Equation



Only two solutions:

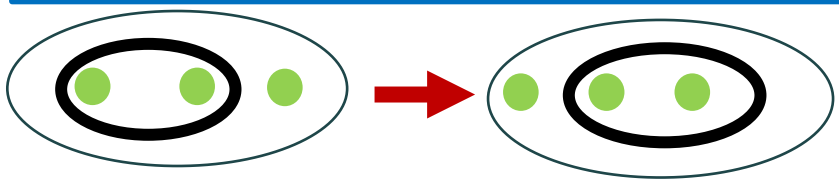
$$R = \begin{pmatrix} e^{-i4\pi/5} & 0 \\ 0 & e^{i3\pi/5} \end{pmatrix} \qquad R = \begin{pmatrix} e^{i4\pi/5} & 0 \\ 0 & e^{-i3\pi/5} \end{pmatrix}$$

Basic Content of an Anyon Theory

"Charge" values: 0, 1

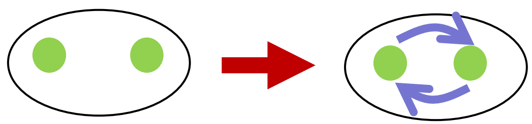
Fusion rules: 1x1 = 0+1; 1x0 = 0x1 = 1; 0x0 = 0

F Matrix:



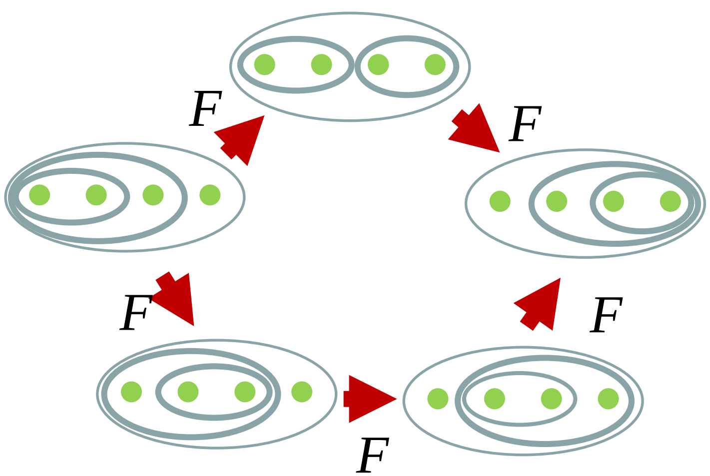
$$F = \begin{pmatrix} \varphi^{-1} & \varphi^{-1/2} \\ \varphi^{-1/2} & -\varphi^{-1} \end{pmatrix}$$

R Matrix:

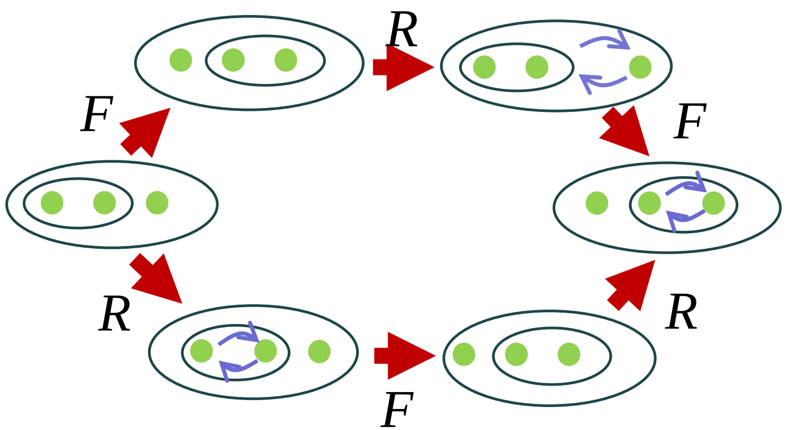


$$R = \begin{pmatrix} e^{-i4\pi/5} & 0 \\ 0 & e^{i3\pi/5} \end{pmatrix}$$

Pentagon



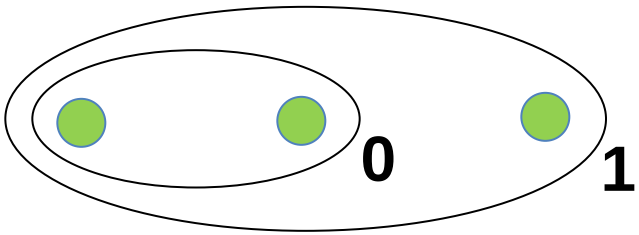
Hexagon



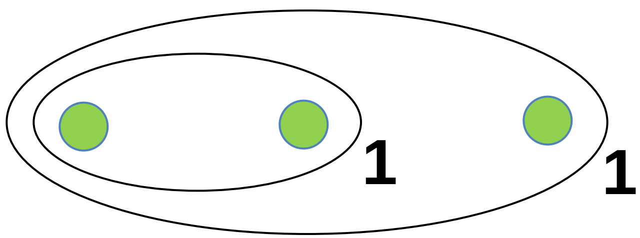
Qubit Encoding

Qubit States

$|0\rangle =$

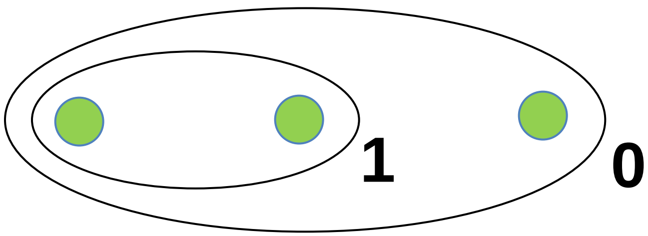


$|1\rangle =$



State of qubit is determined by "charge" of two leftmost particles

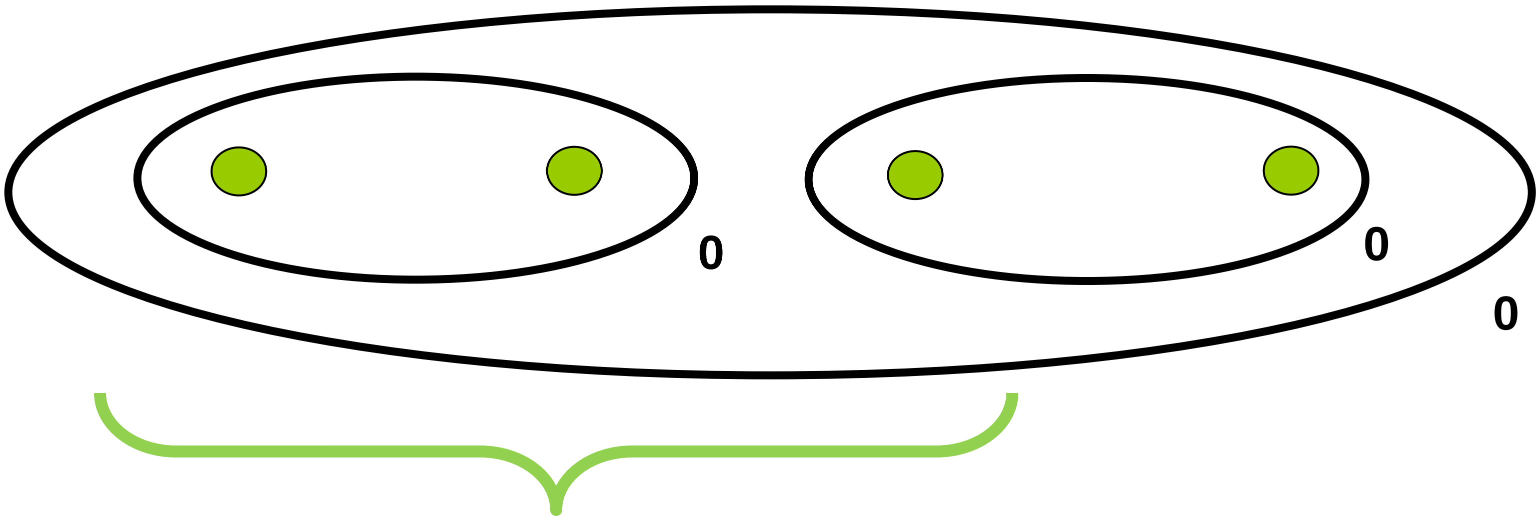
Non-Computational State



Transitions to this state are **leakage errors**

Initializing a Qubit

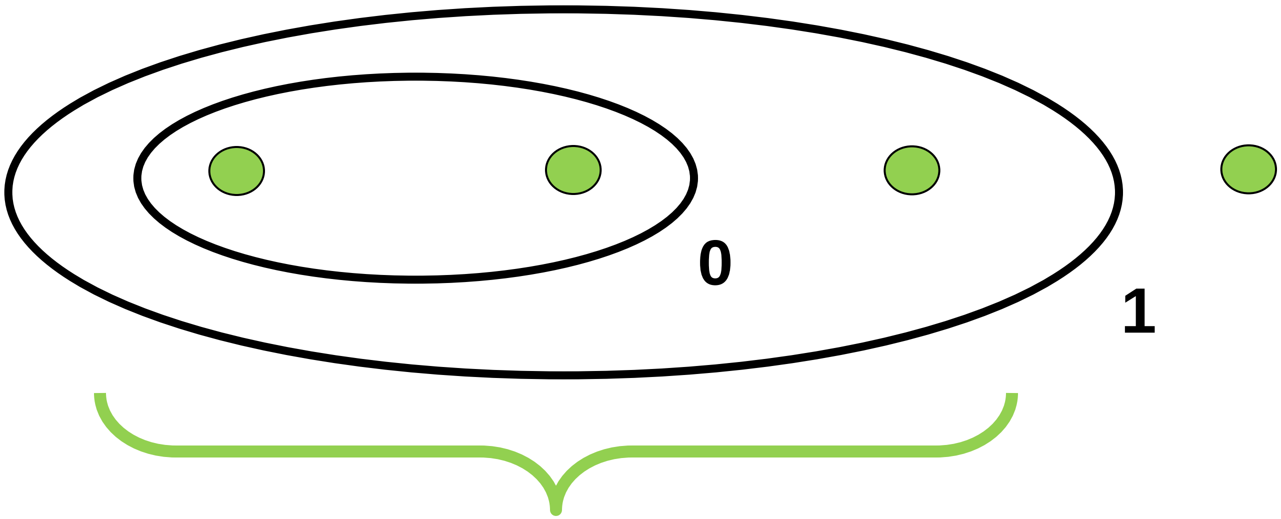
Pull two quasiparticle-quasihole pairs out of the “vacuum”.



These three particles have total q-spin 1

Initializing a Qubit

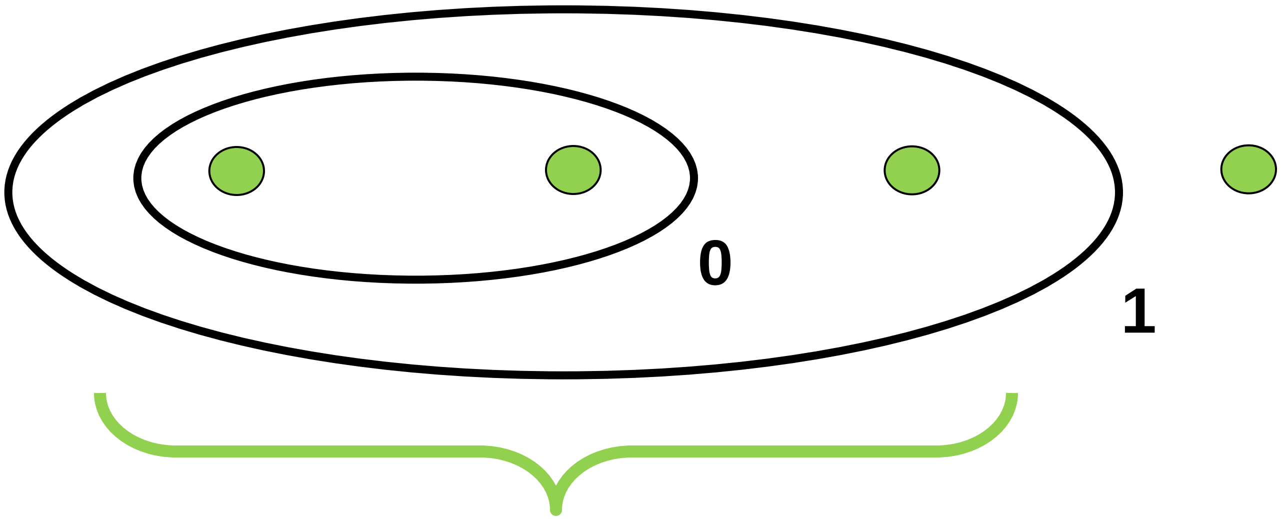
Pull two quasiparticle-quasihole pairs out of the "vacuum".



$$|0\rangle$$

Initializing a Qubit

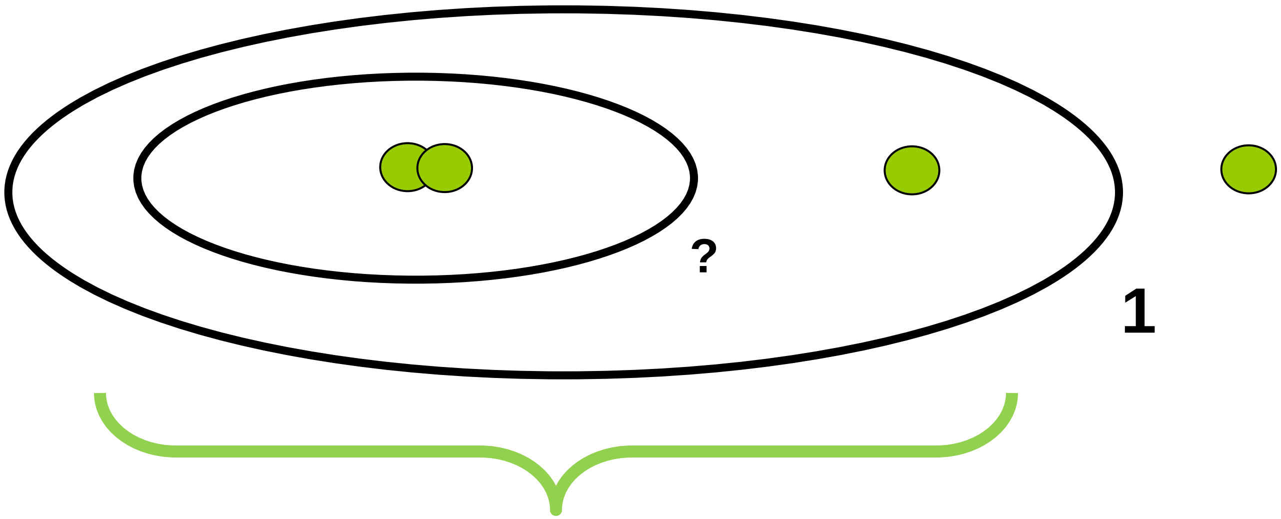
Pull two quasiparticle-quasihole pairs out of the “vacuum”.



$$|0\rangle$$

Measuring a Qubit

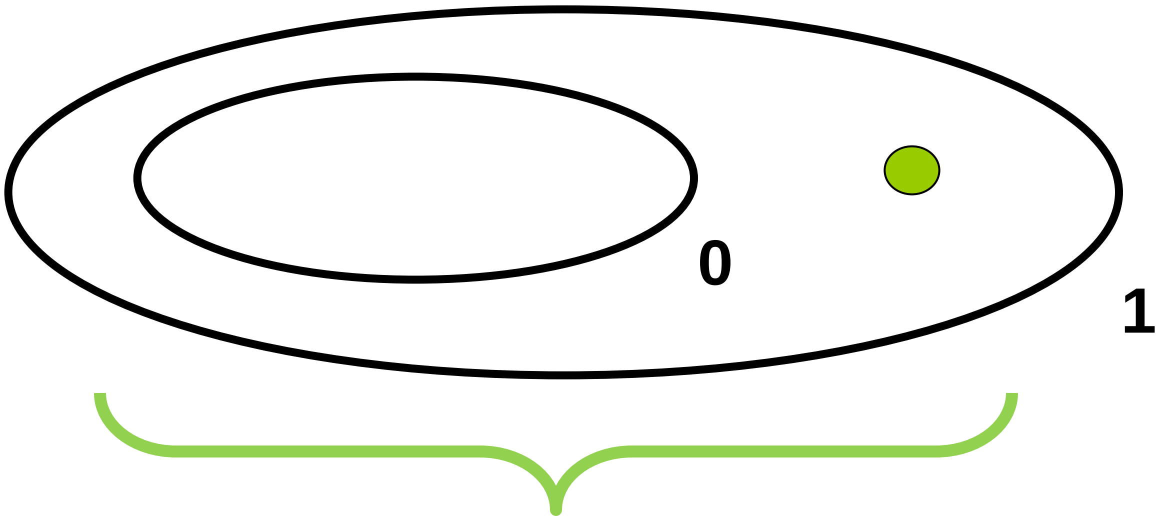
Try to fuse the leftmost quasiparticle-quasihole pair.



$$\alpha|0\rangle + \beta|1\rangle$$

Measuring a Qubit

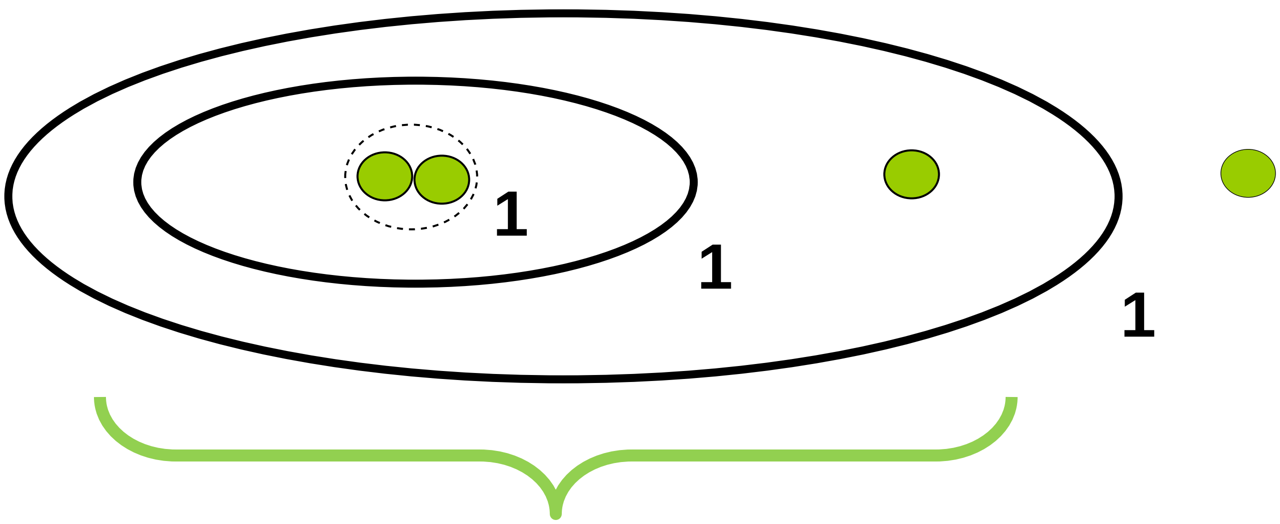
If they fuse back into the “vacuum” the result of the measurement is 0.



$$|0\rangle$$

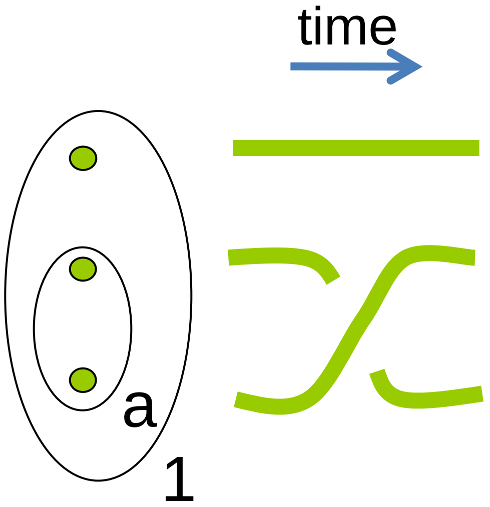
Measuring a Qubit

If they cannot fuse back into the "vacuum" the result of the measurement is 1



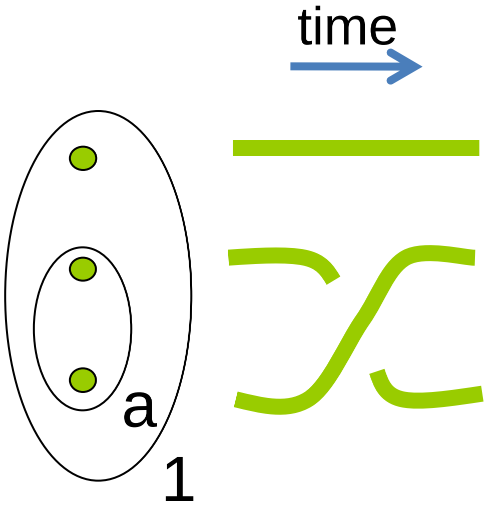
$$|1\rangle$$

Elementary Braid Matrices



$$\sigma_1 = ?$$

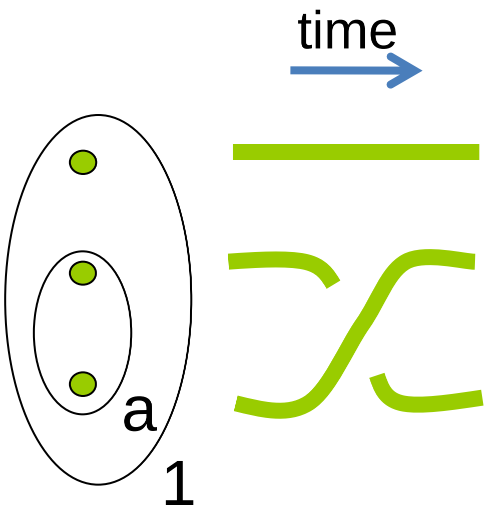
Elementary Braid Matrices



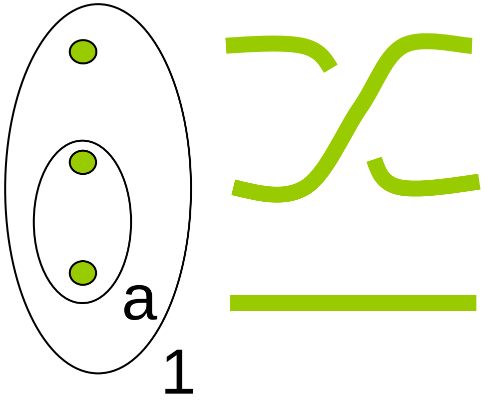
a = 0 1

$$\sigma_1 = R = \begin{pmatrix} e^{-i4\pi/5} & 0 \\ 0 & e^{i3\pi/5} \end{pmatrix}$$

Elementary Braid Matrices

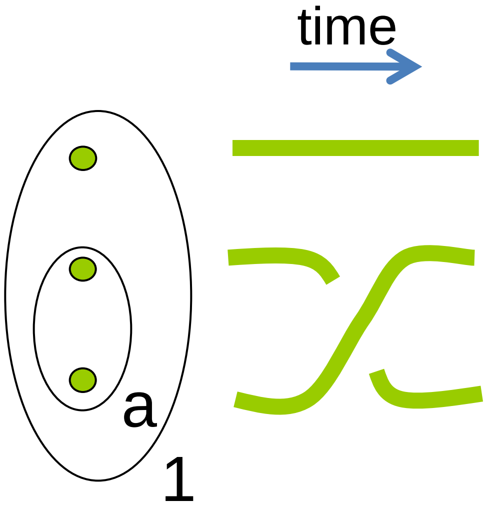


$$\sigma_1 = R = \begin{pmatrix} e^{-i4\pi/5} & 0 \\ 0 & e^{i3\pi/5} \end{pmatrix}$$

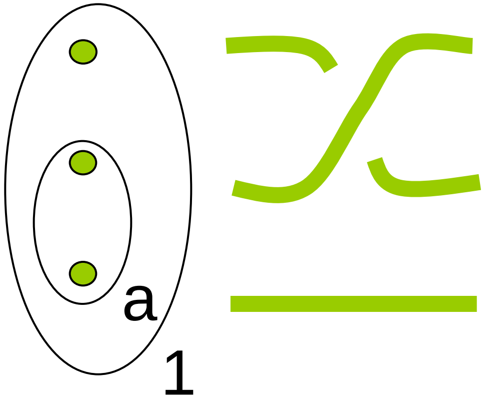


$$\sigma_2 = ?$$

Elementary Braid Matrices



$$\sigma_1 = R = \begin{pmatrix} e^{-i4\pi/5} & 0 \\ 0 & e^{i3\pi/5} \end{pmatrix}$$



$$\sigma_2 = F\,R\,F = \begin{pmatrix} -\tau\, e^{-i\pi/5} & \sqrt{\tau}\, e^{-i3\pi/5} \\ \sqrt{\tau} e^{-i3\pi/5} & -\tau \end{pmatrix}$$

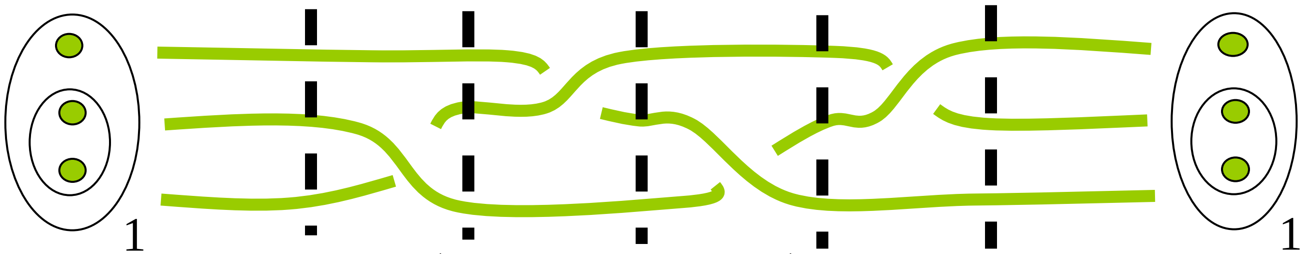
$$\tau = (\sqrt{5} - 1)/2 = \varphi^{-1}$$

Elementary Braid Matrices

$$\sigma_1 = \begin{pmatrix} e^{-i4\pi/5} & 0 \\ 0 & e^{i3\pi/5} \end{pmatrix}$$

$$\sigma_2 = F\,\sigma_1 F = \begin{pmatrix} -\tau\, e^{-i\pi/5} & \sqrt{\tau}\, e^{-i3\pi/5} \\ \sqrt{\tau} e^{-i3\pi/5} & -\tau \end{pmatrix}$$

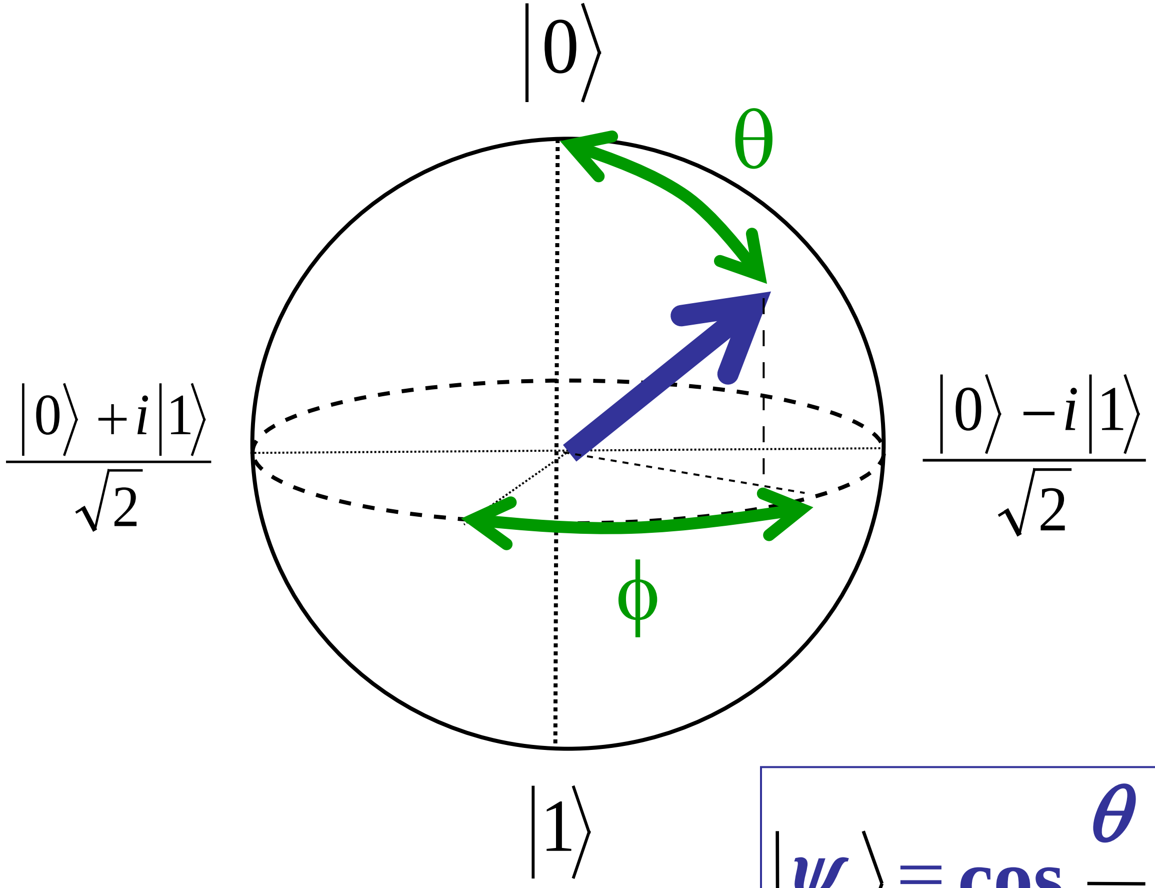
$|\Psi_i\rangle$



$$\sigma_1^{-1}\ \sigma_2\ \sigma_1^{-1}\ \sigma_2 = M$$

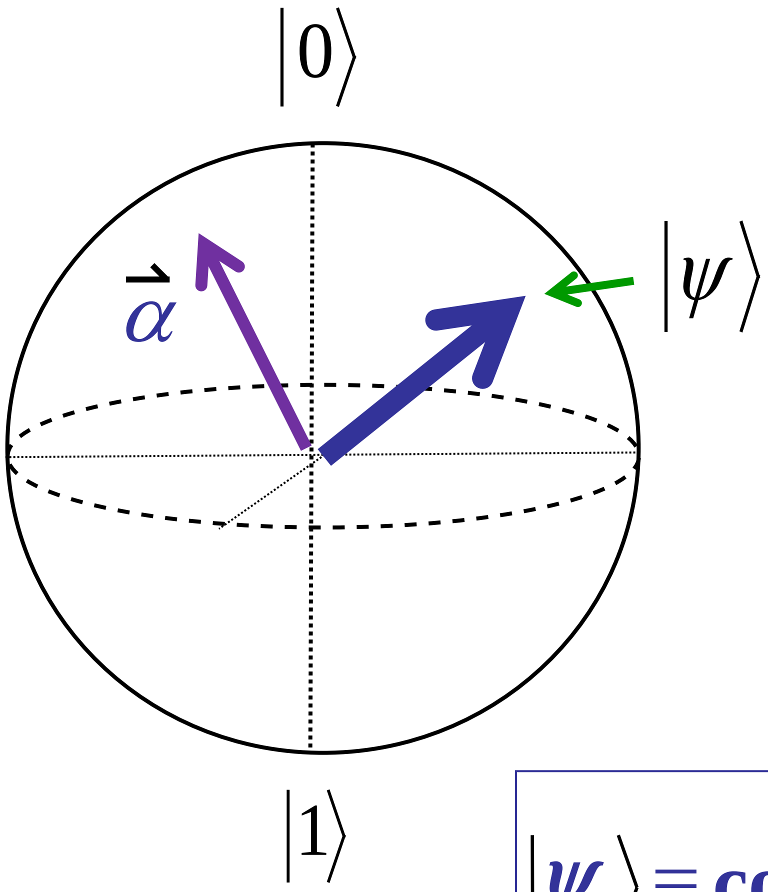
$$|\Psi_f\rangle = M^T\,|\Psi_i\rangle$$

A Quantum Bit: A Continuum of States



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{-i\phi}|1\rangle$$

Single Qubit Operations: Rotations



$\vec{\alpha}$ = rotation vector

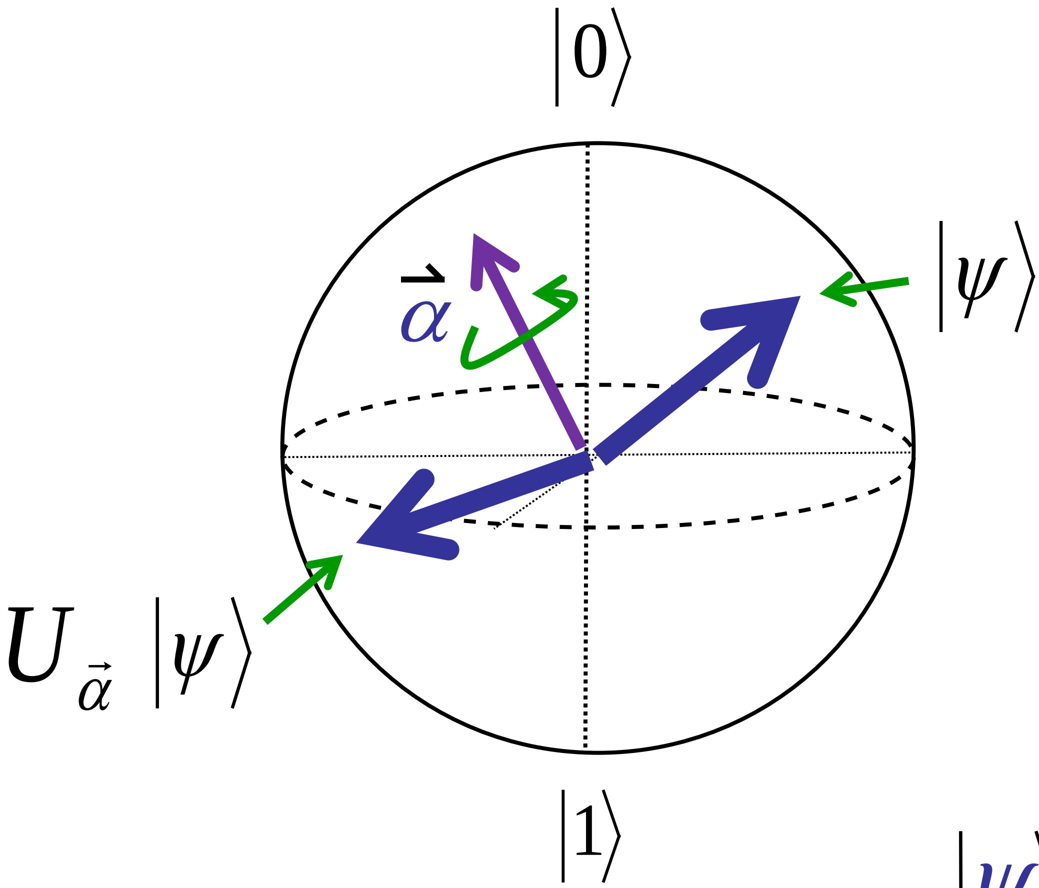
Direction of $\vec{\alpha}$ is the rotation axis

Magnitude of $\vec{\alpha}$ is the rotation angle

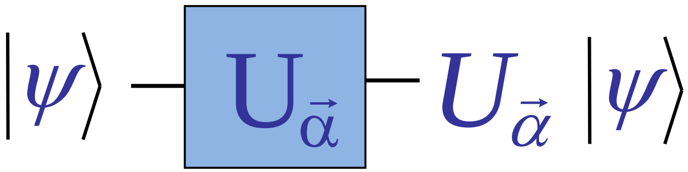
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{-i\phi}|1\rangle$$

Single Qubit Operations: Rotations

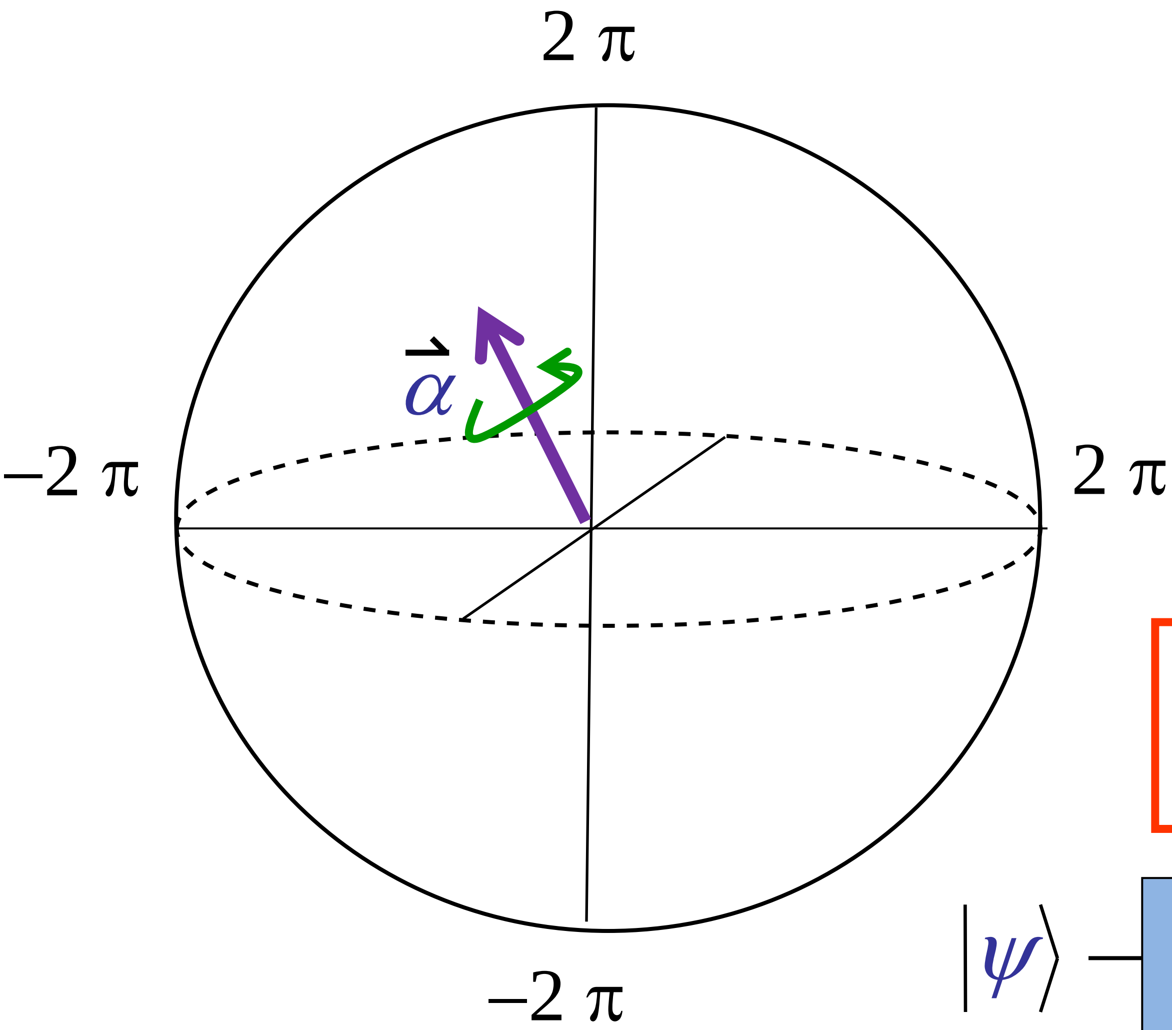
$\vec{\alpha}$ = rotation vector



$$U_{\vec{\alpha}} = \exp\frac{i\,\vec{\alpha}\cdot\vec{\sigma}}{2}$$

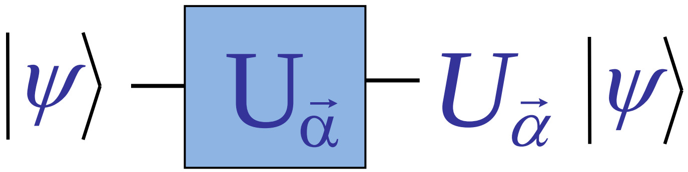


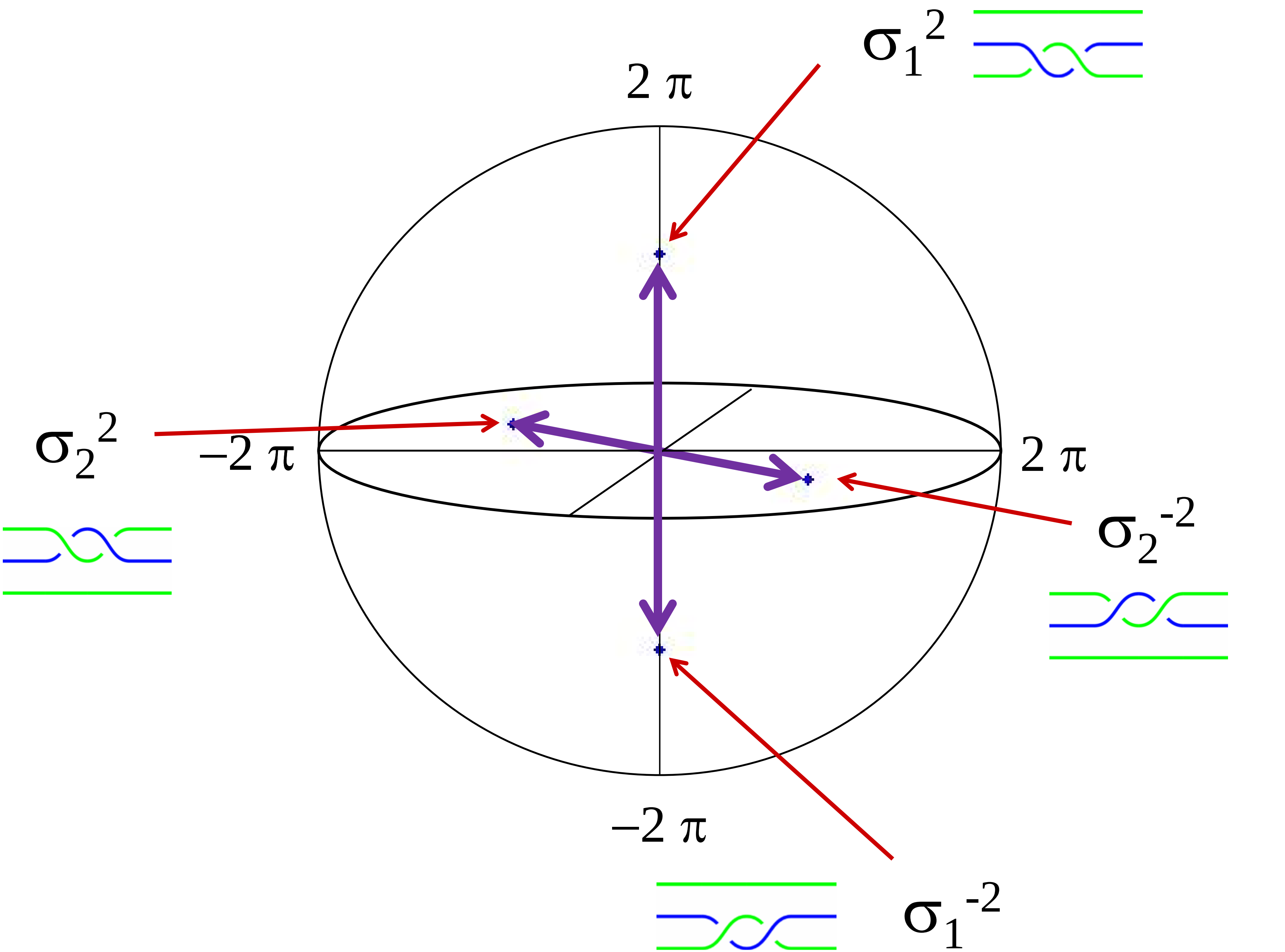
Single Qubit Operations: Rotations



The set of all single qubit rotations lives in a solid sphere of radius 2π.

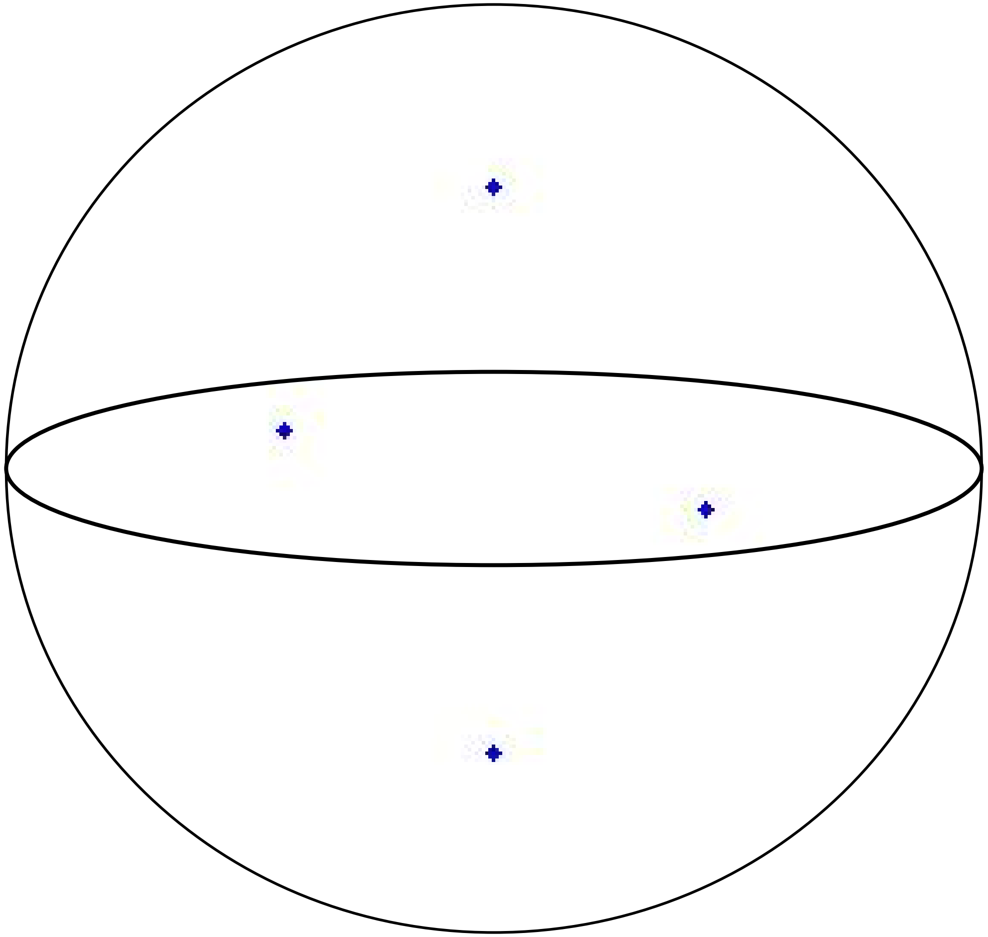
Qubits are sensitive: Easy to make errors!



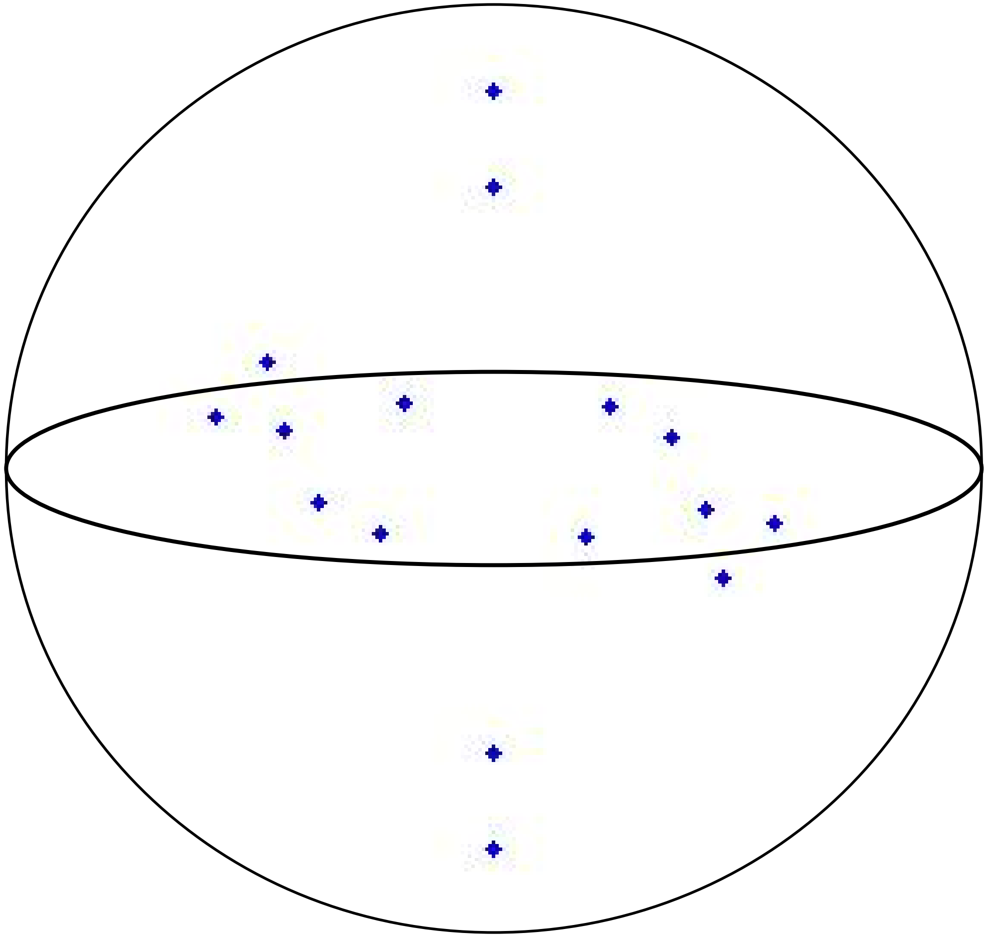


$\sigma_1{}^2$
2π
$\sigma_2{}^2$
-2π
2π
$\sigma_2{}^{-2}$
-2π
$\sigma_1{}^{-2}$

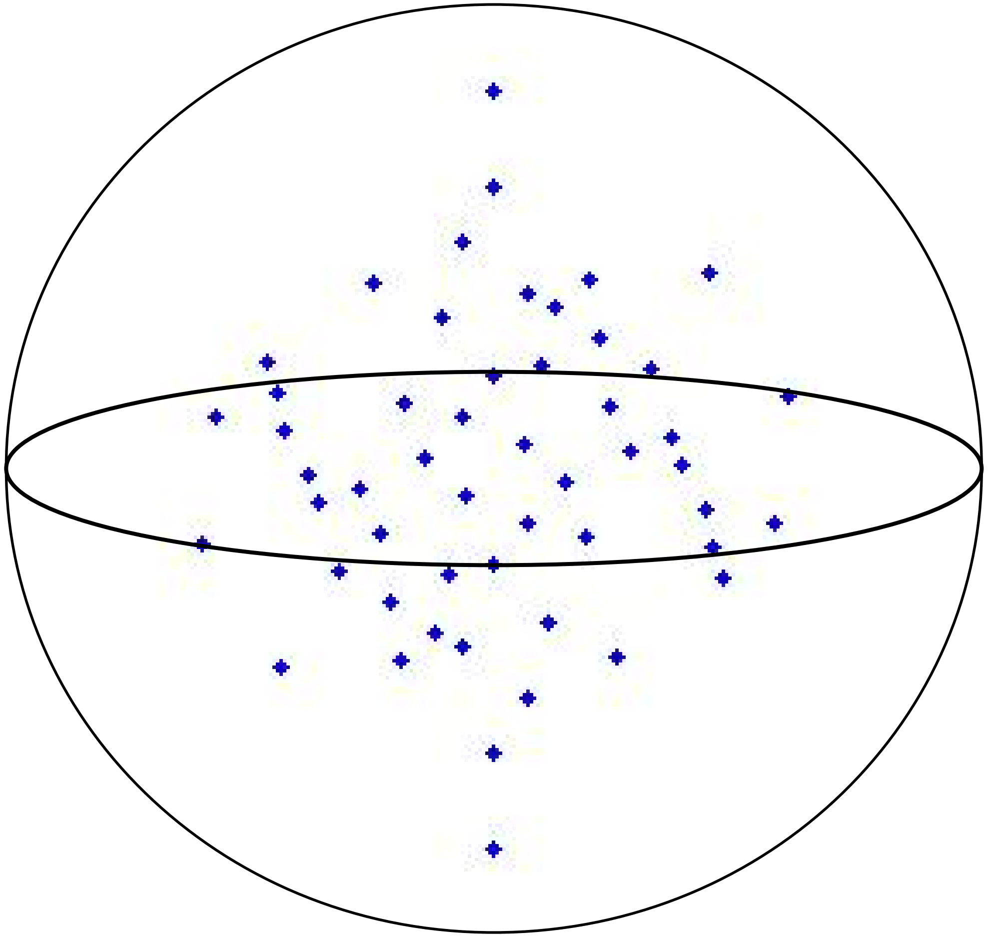
N = 1



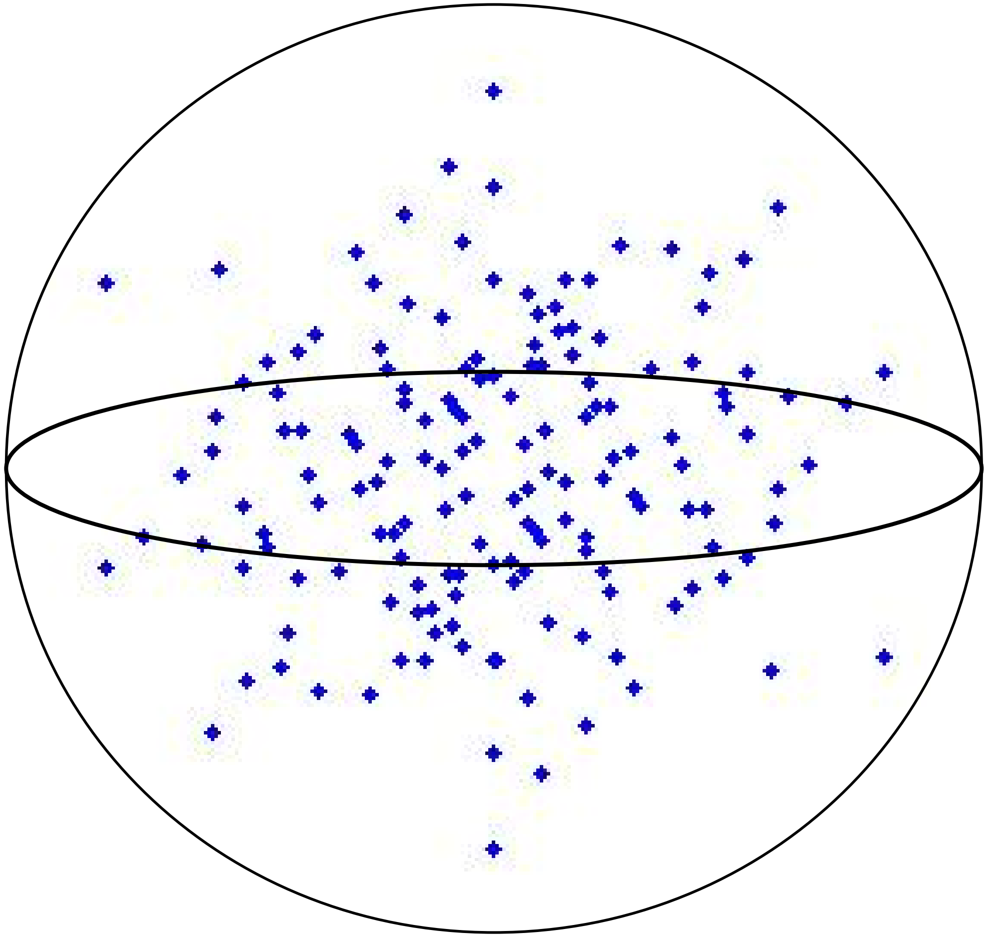
N = 2



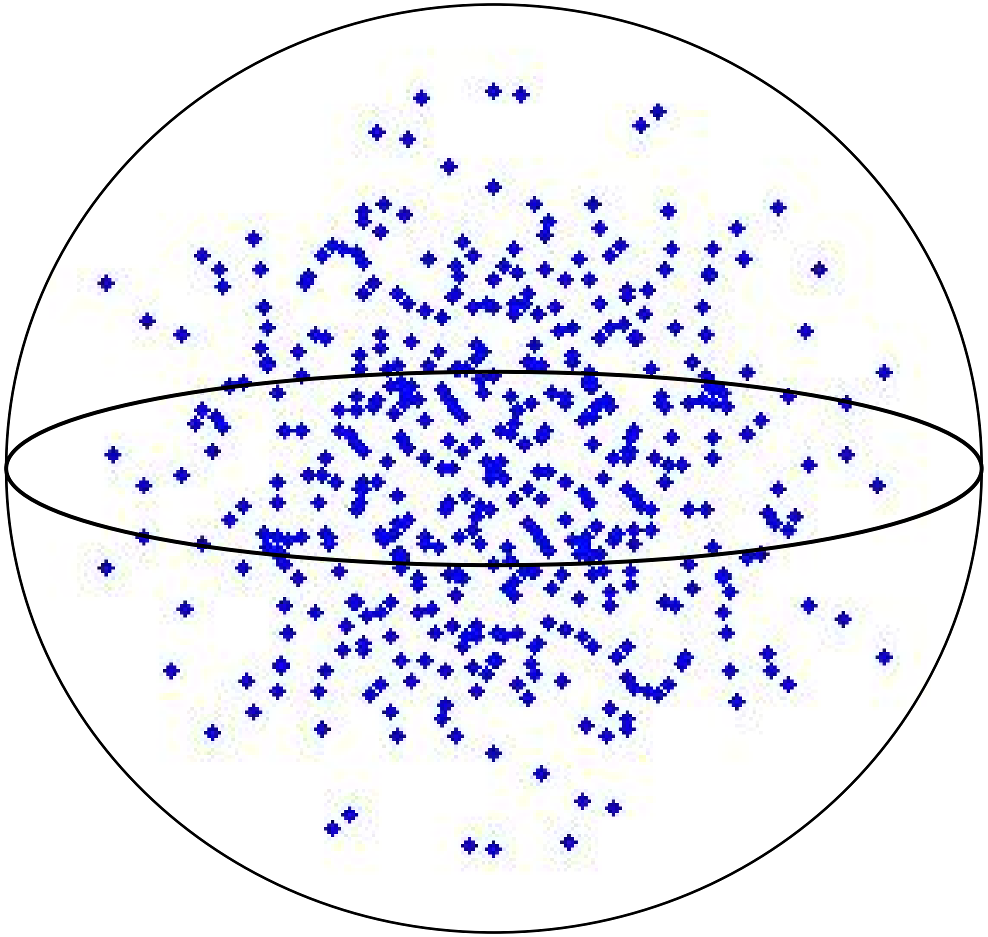
N = 3



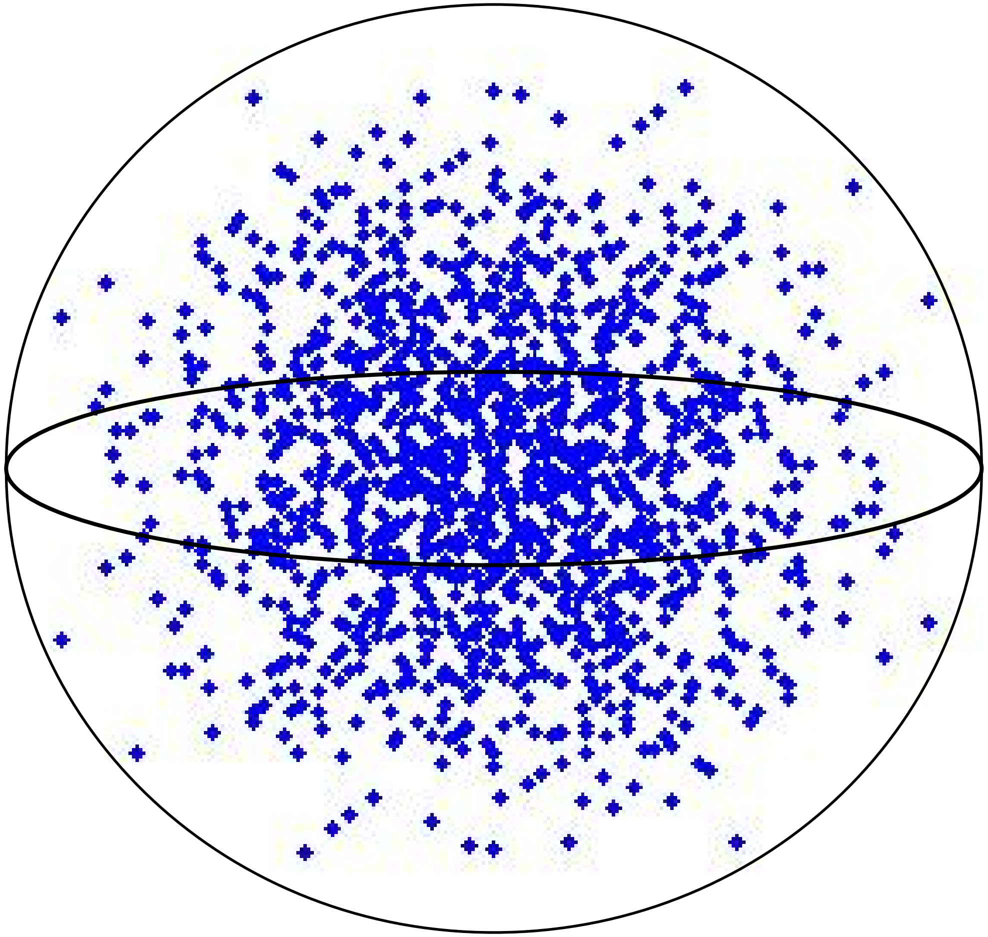
N = 4

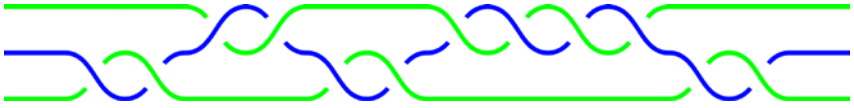


N = 5

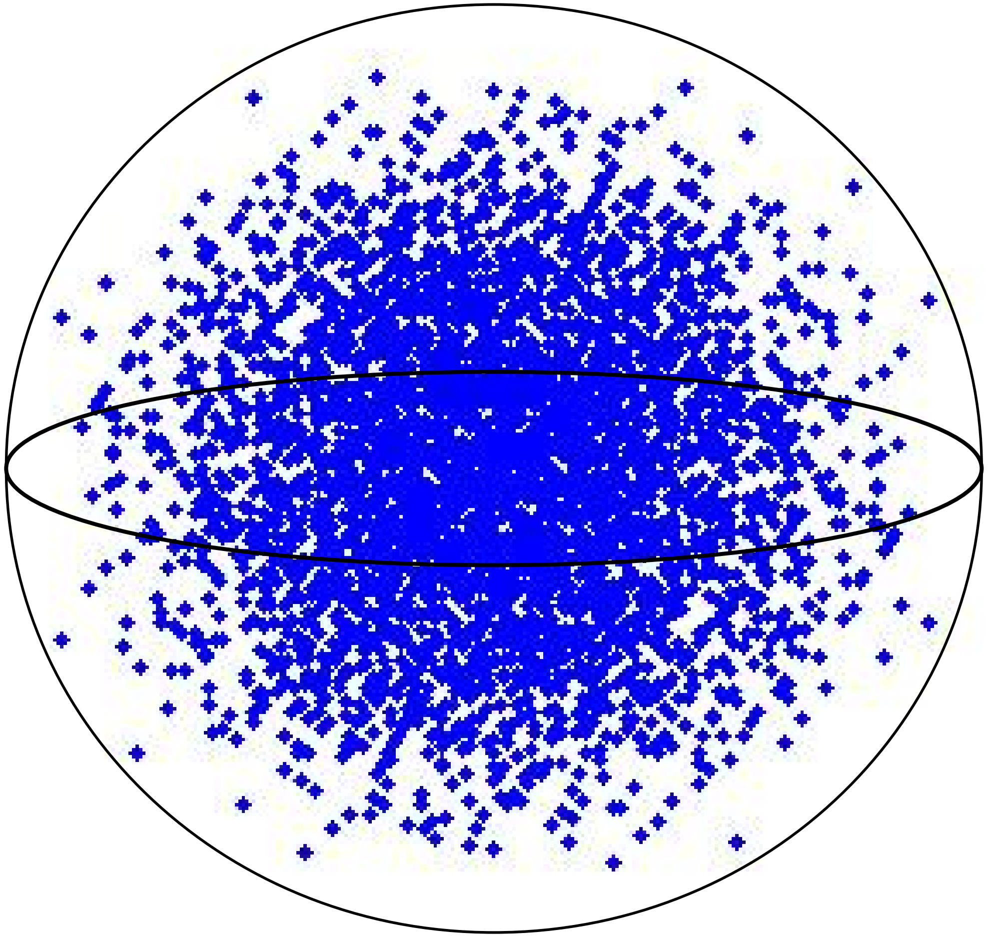


N = 6

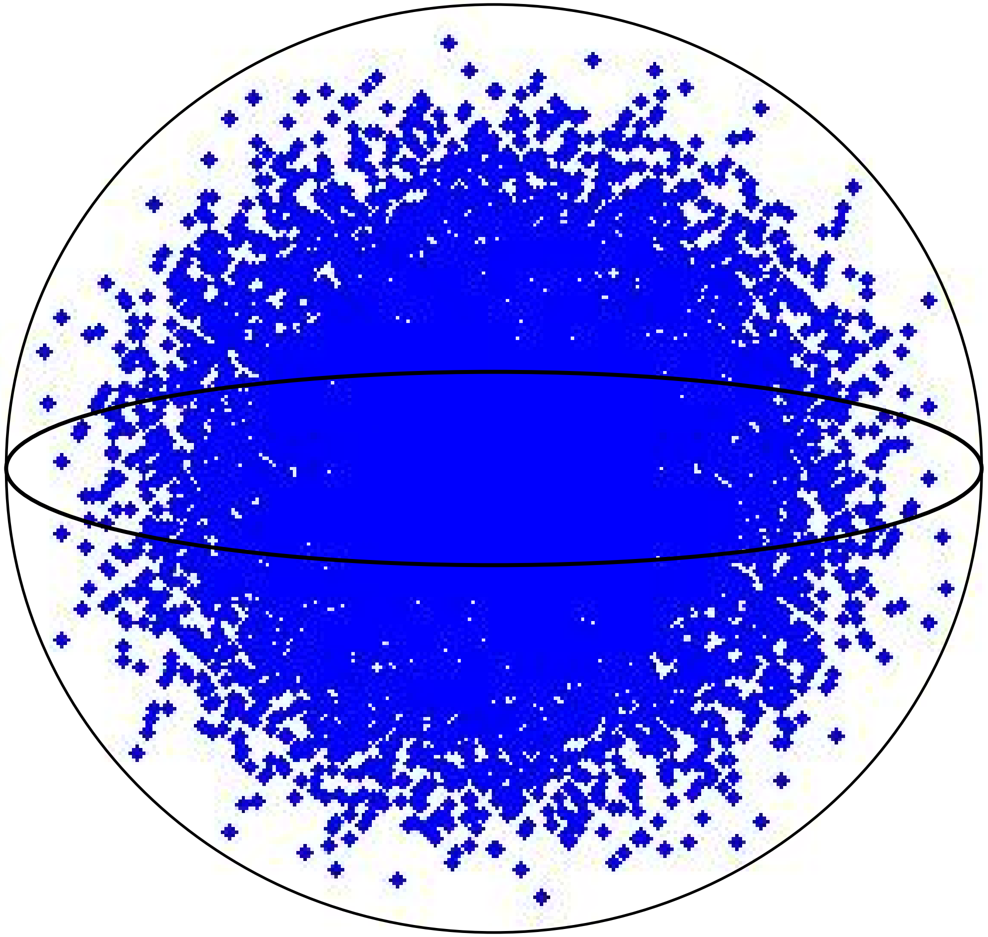




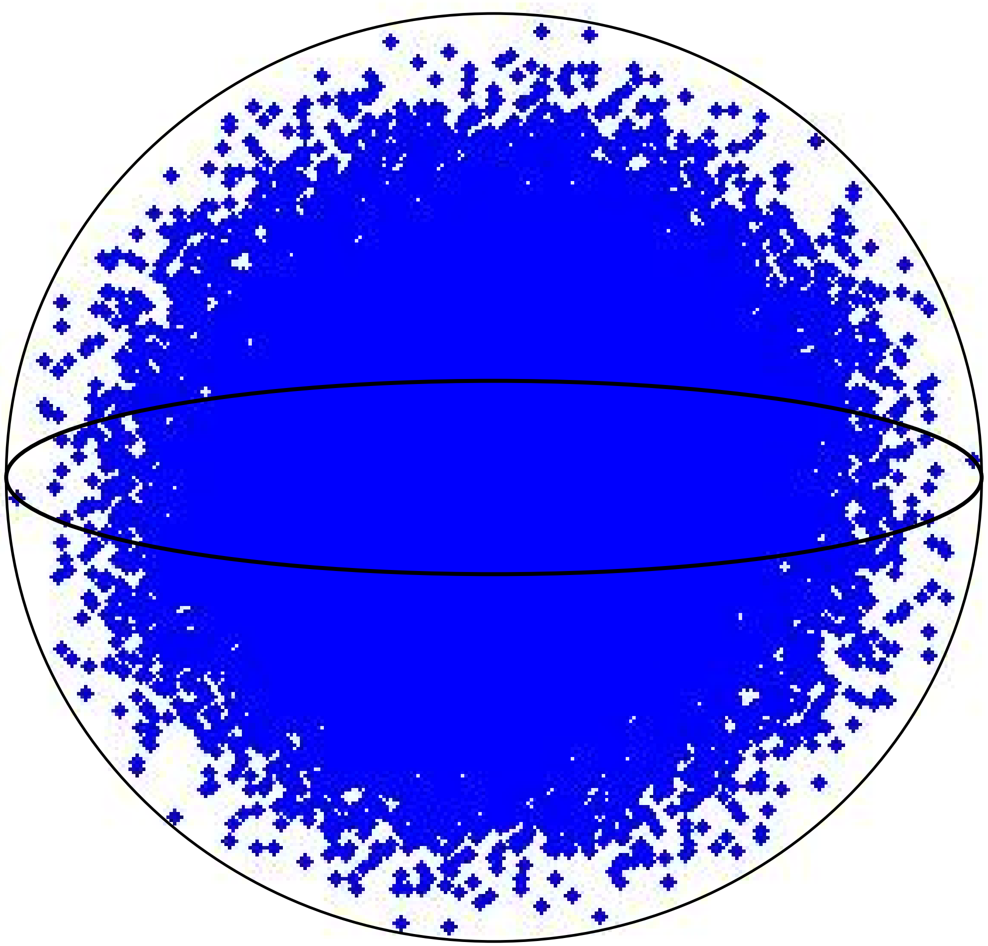
N = 7



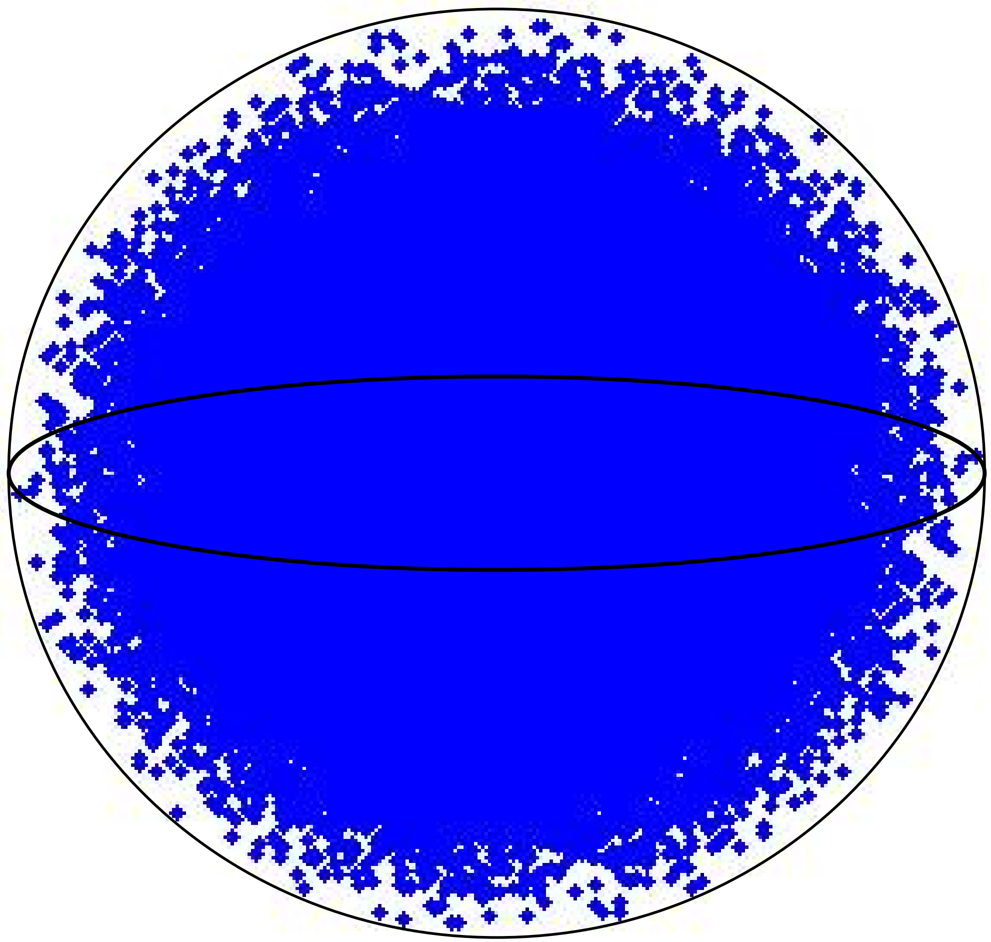
N = 8

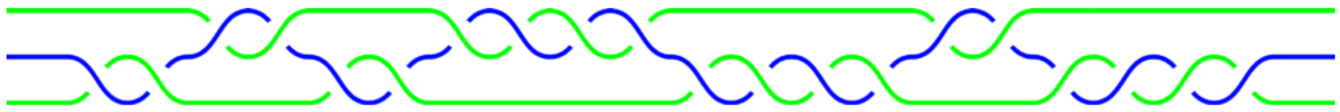


N = 9

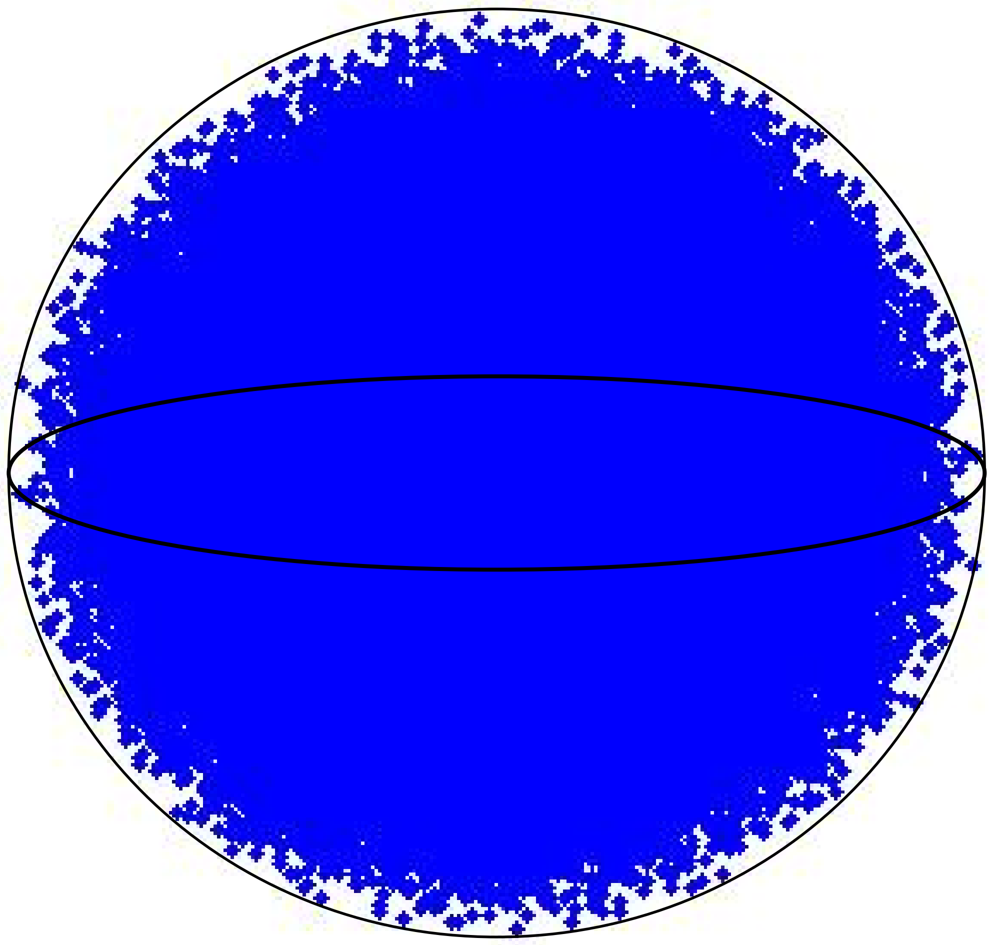


N = 10





N = 11



Brute Force Search

$$\sigma_1^{-2}\sigma_2^{-4}\sigma_1^{4}\sigma_2^{-2}\sigma_1^{2}\sigma_2^{2}\sigma_1^{-2}\sigma_2^{4}\sigma_1^{-2}\sigma_2^{4}\sigma_1^{2}\sigma_2^{-4}\sigma_1^{2}\sigma_2^{-2}\sigma_1^{2}\sigma_2^{-2}\sigma_1^{-2} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + O(10^{-3})$$

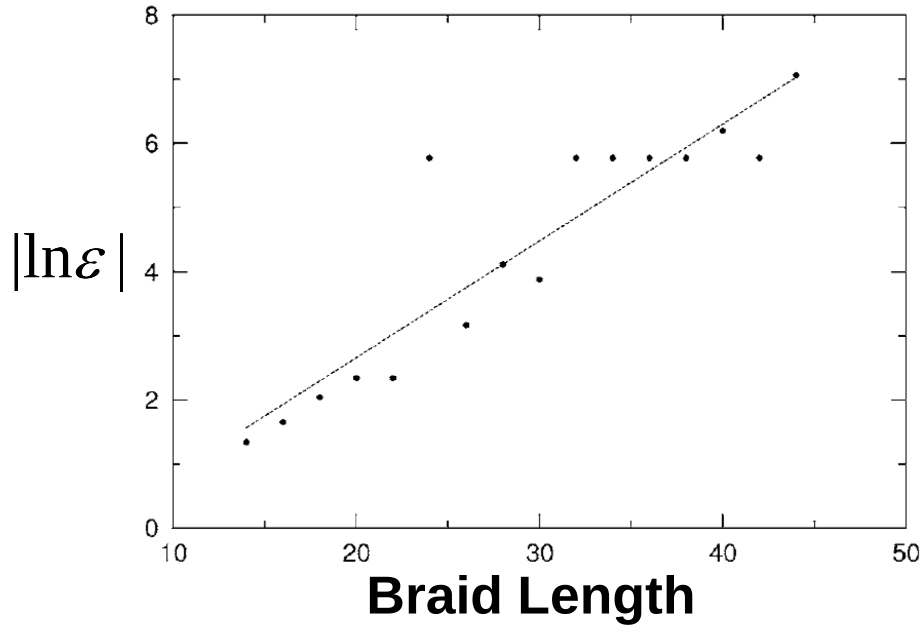
Brute Force Search

$$\sigma_1^{-2}\sigma_2^{-4}\sigma_1^{4}\sigma_2^{-2}\sigma_1^{2}\sigma_2^{2}\sigma_1^{-2}\sigma_2^{4}\sigma_1^{-2}\sigma_2^{4}\sigma_1^{2}\sigma_2^{-4}\sigma_1^{2}\sigma_2^{-2}\sigma_1^{2}\sigma_2^{-2}\sigma_1^{-2} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + O(10^{-3})$$

"error" ε

For brute force search:

Braid Length $\sim \; |\ln \varepsilon|$



L. Hormozi, G. Zikos, NEB, S.H. Simon, PRB '07

Brute Force Search

$$\sigma_1^{-2}\sigma_2^{-4}\sigma_1^{4}\sigma_2^{-2}\sigma_1^{2}\sigma_2^{2}\sigma_1^{-2}\sigma_2^{4}\sigma_1^{-2}\sigma_2^{4}\sigma_1^{2}\sigma_2^{-4}\sigma_1^{2}\sigma_2^{-2}\sigma_1^{2}\sigma_2^{-2}\sigma_1^{-2} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + O(10^{-3})$$

“error” ε

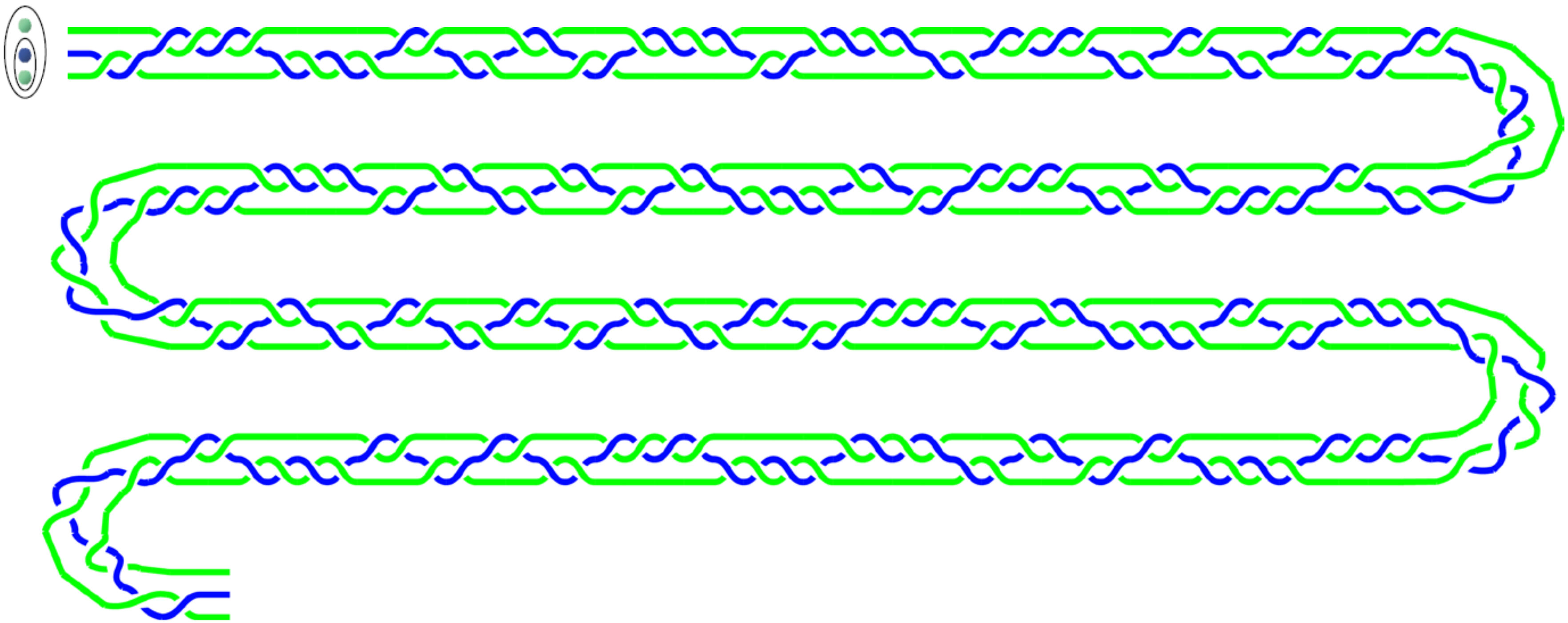
Brute force searching rapidly becomes infeasible as braids get longer.

Fortunately, a clever algorithm due to Solovay and Kitaev allows for systematic improvement of the braid given a sufficiently dense covering of *SU(2).*

Solovay-Kitaev Construction

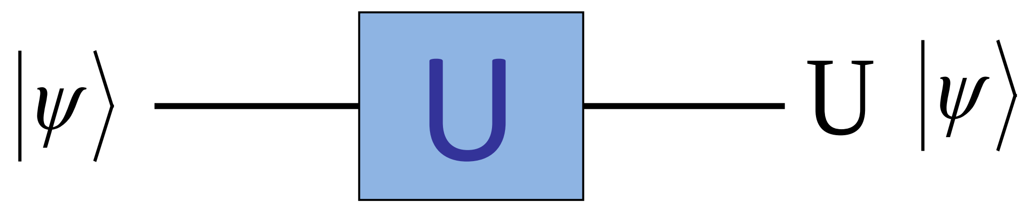
$$\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + O(10^{-5})$$

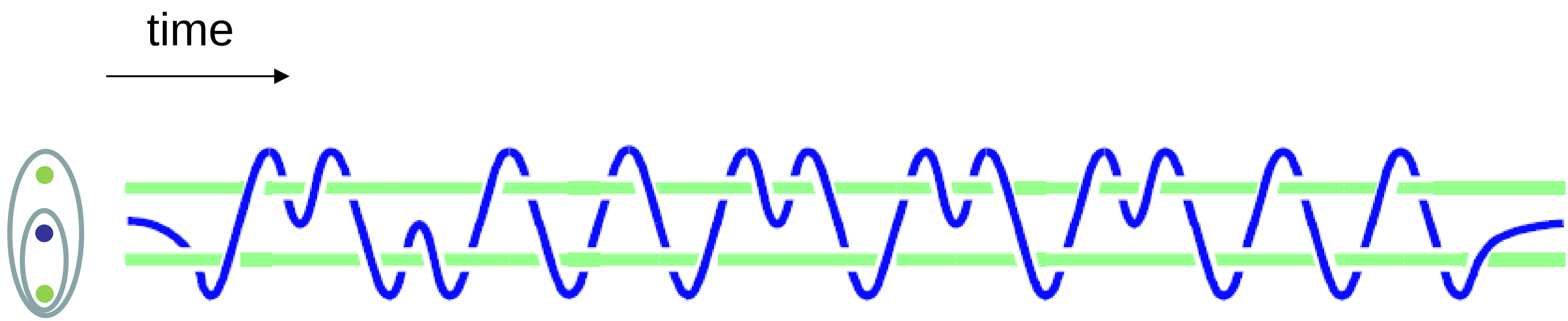
"error" ε



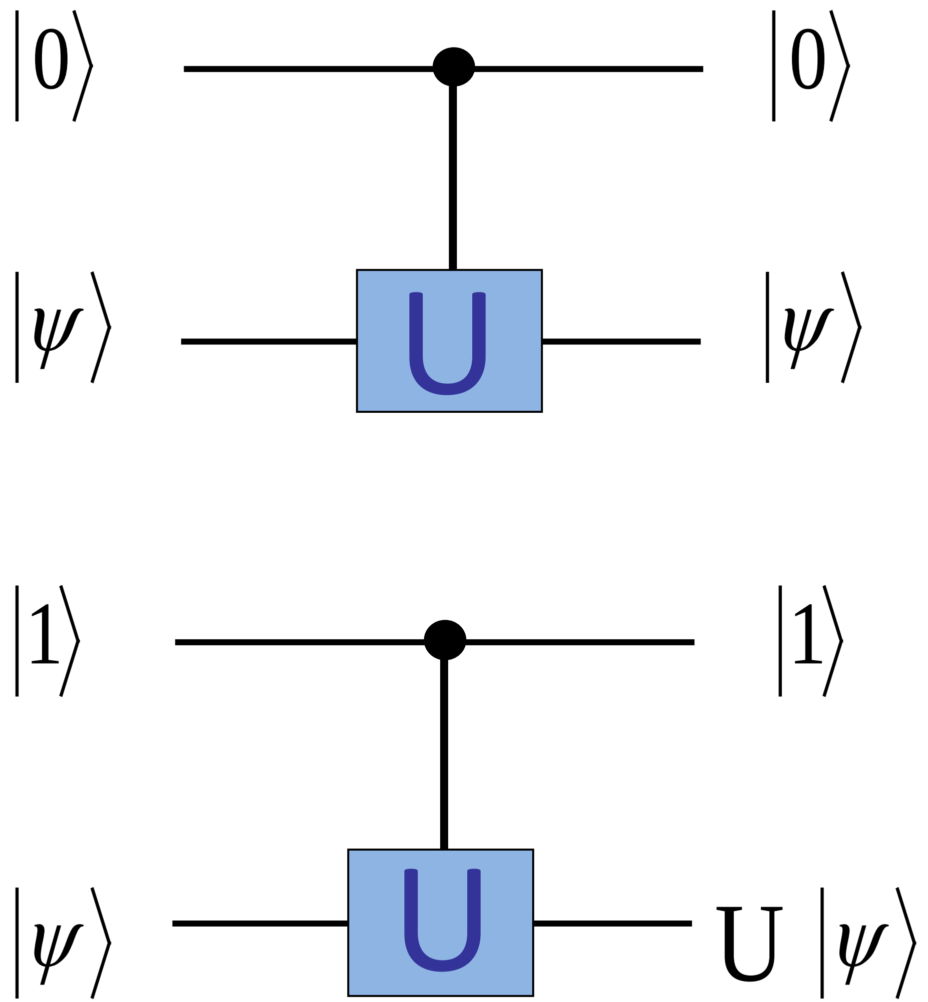
$$\text{Braid Length} \sim |\ln \varepsilon|^c, \quad c \approx 4$$

Single Qubit Operations



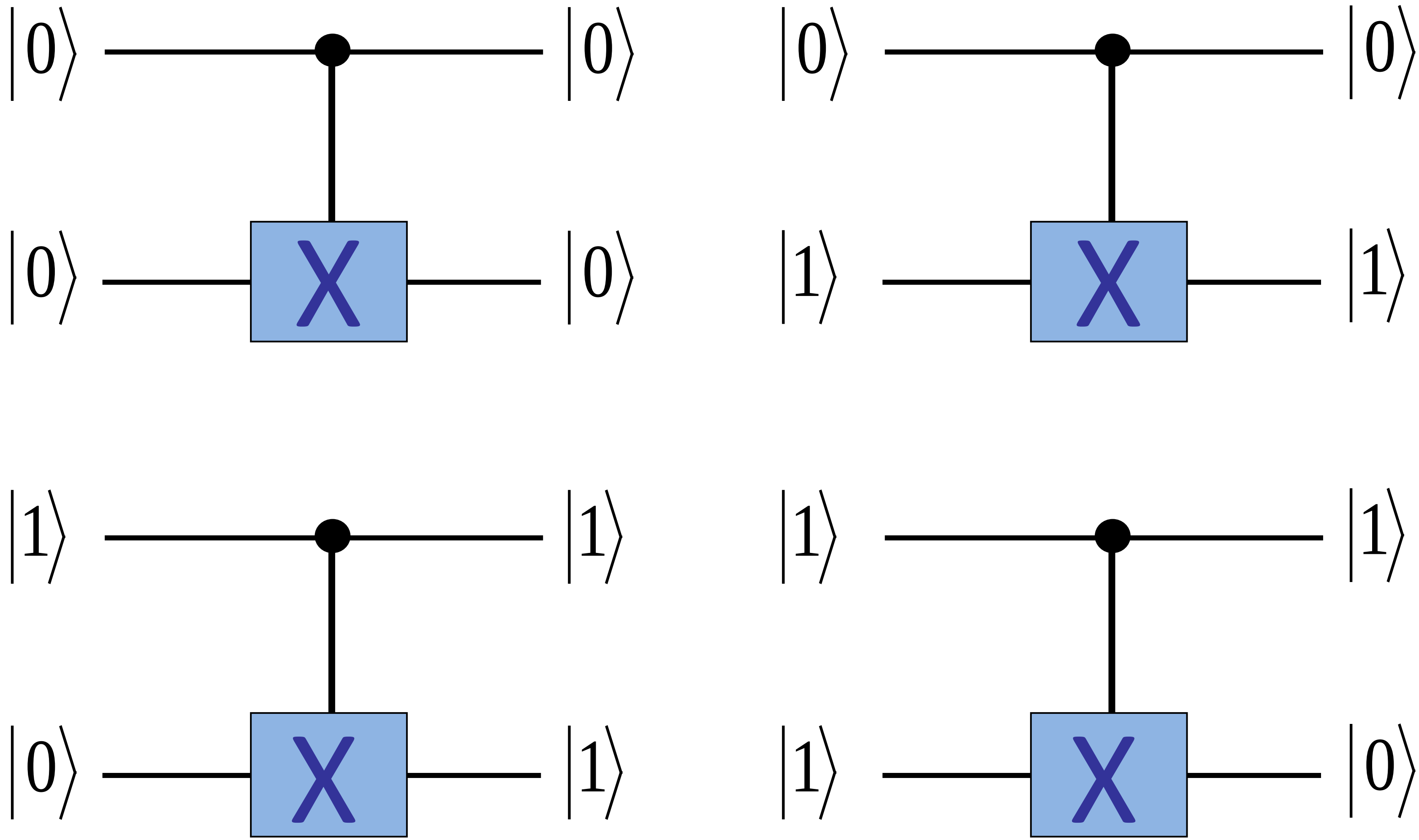


Two Qubit Gates

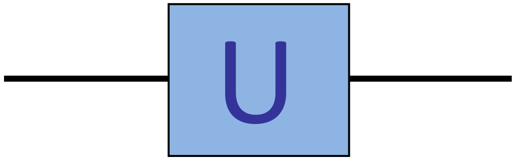


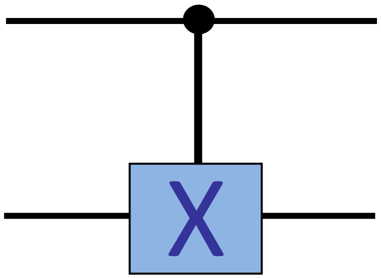
Controlled-NOT Gate

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

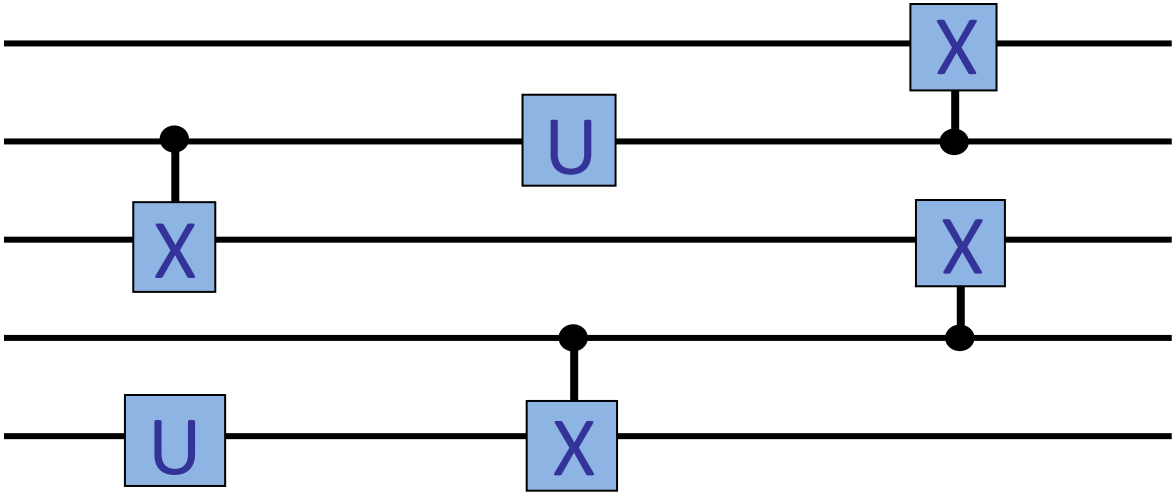


Universal Set of Gates





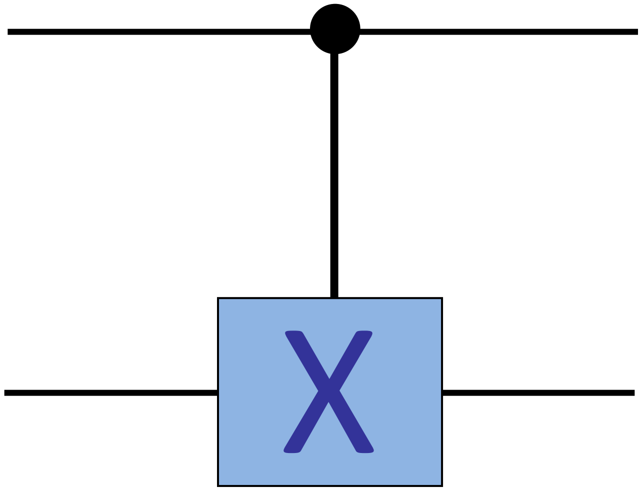
Quantum Circuit

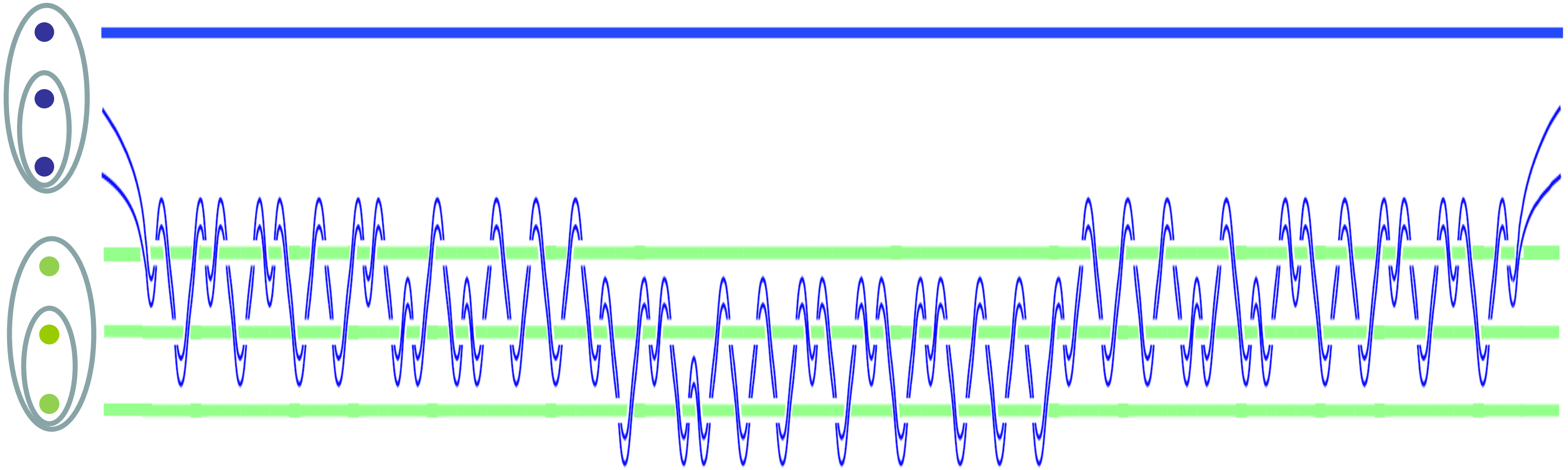


Any unitary operation can be expressed as a quantum circuit

Two Qubit Gates (CNOT)

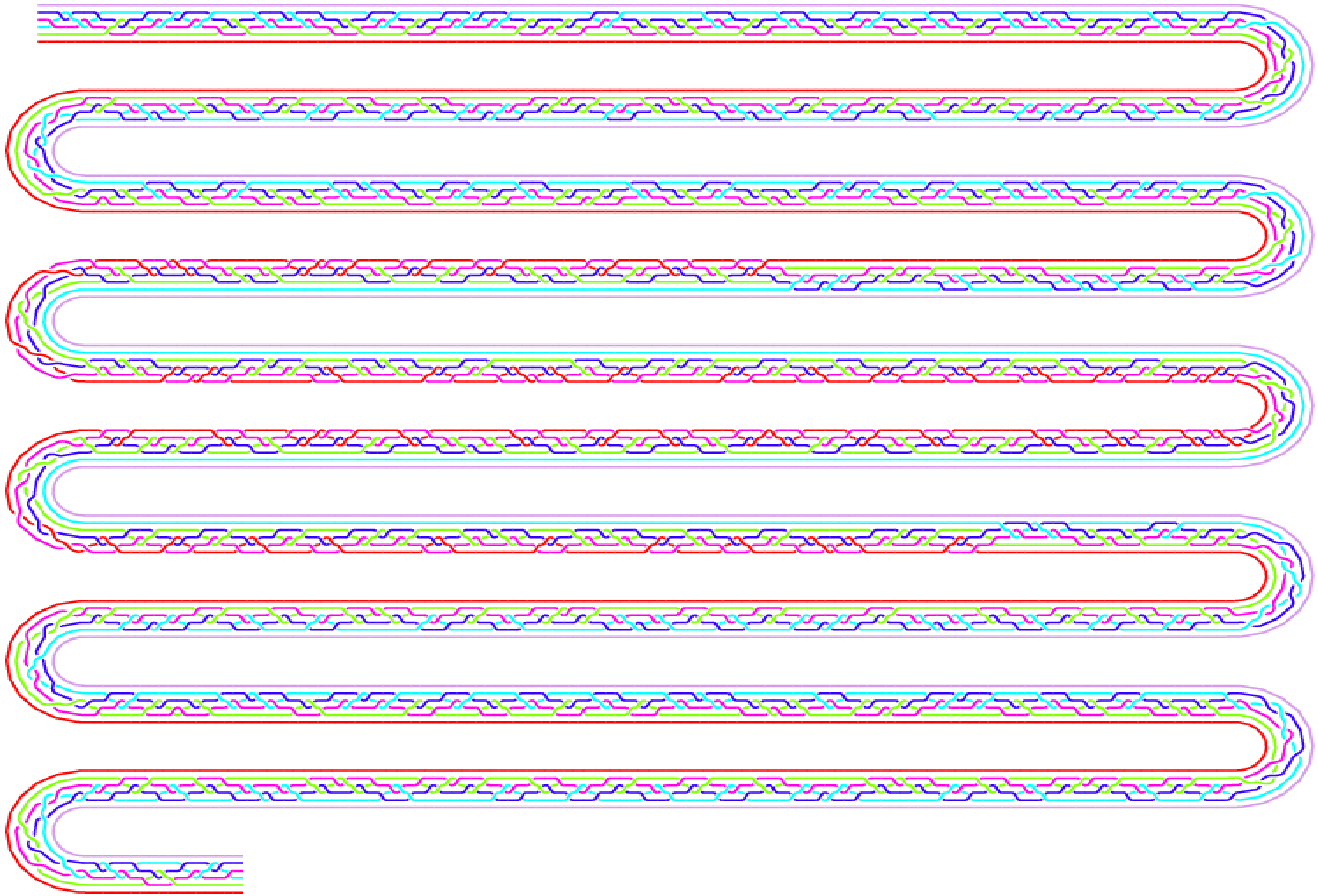
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



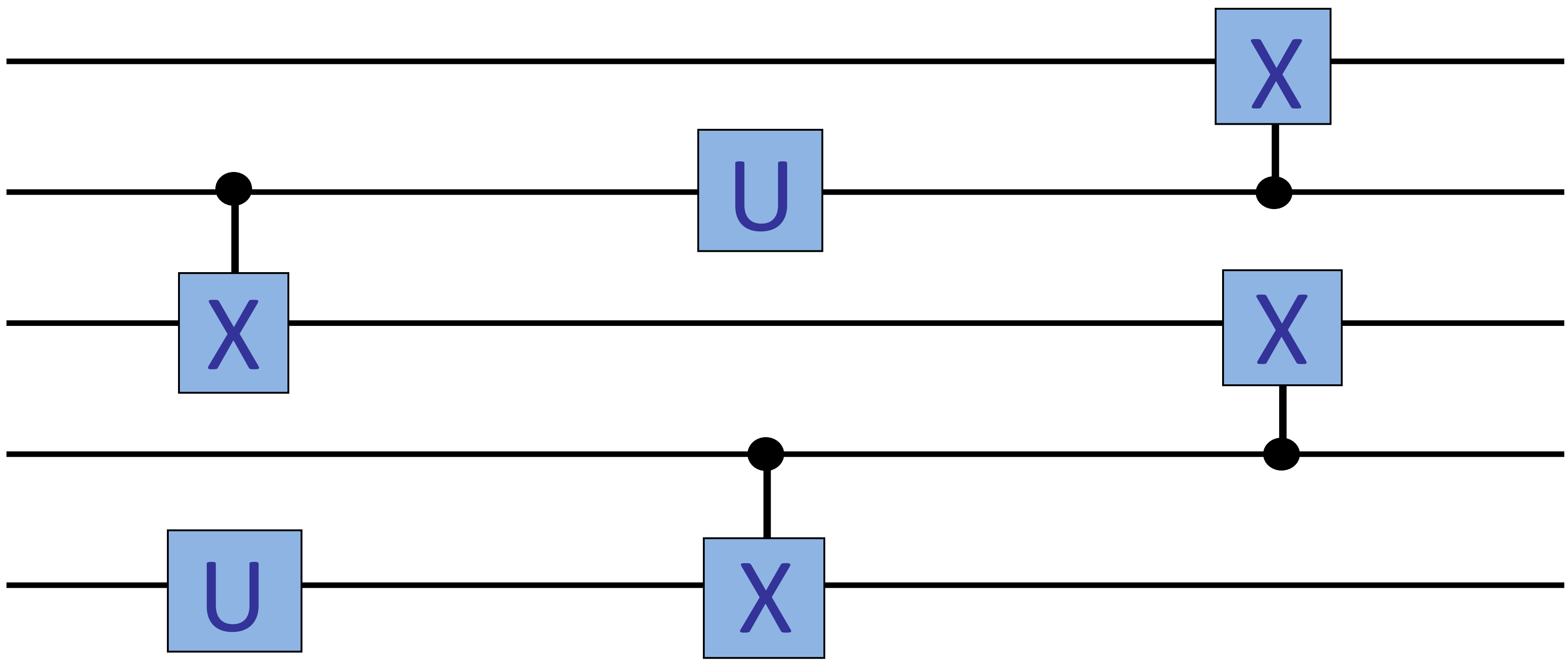


NEB, L. Hormozi, G. Zikos, & S. Simon, PRL 2005
L. Hormozi, G. Zikos, NEB, and S. H. Simon, PRB 2007

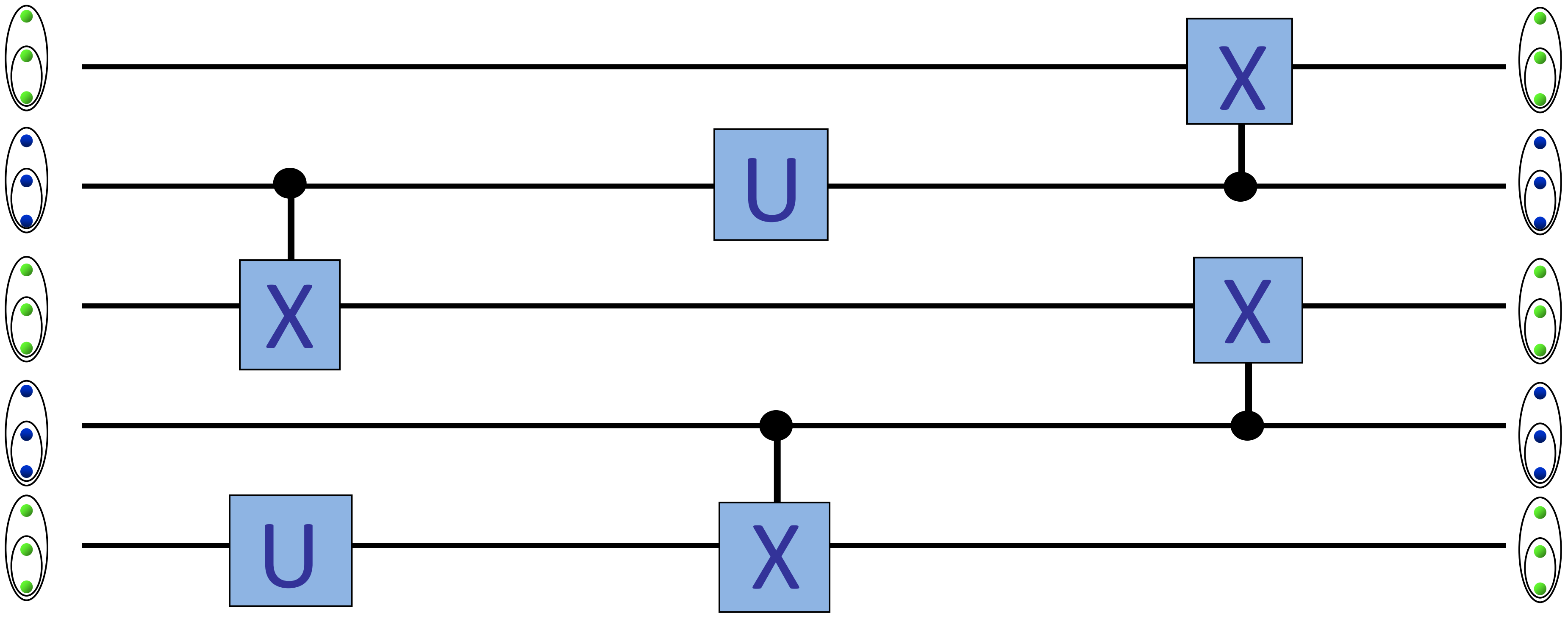
SK Improved Controlled-NOT Gate



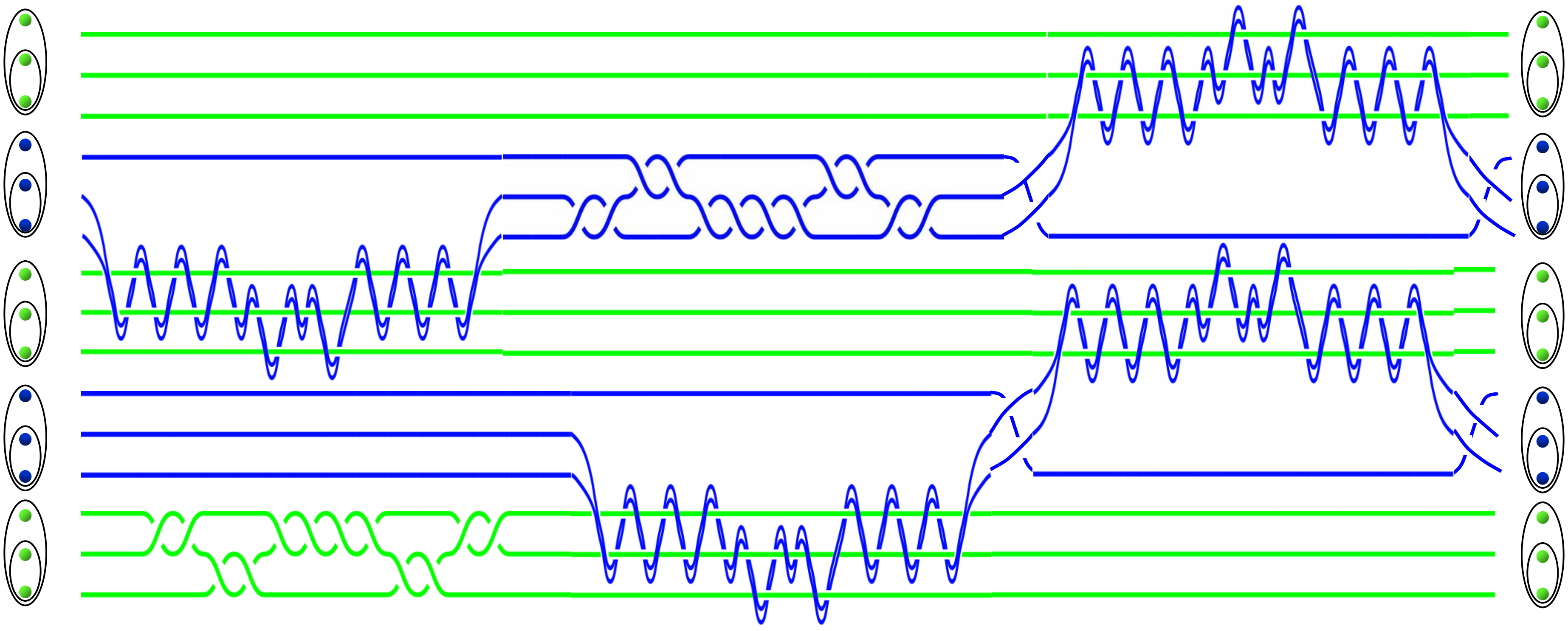
Quantum Circuit



Quantum Circuit



Braid



$SU(2)_k$ Nonabelian Particles

Quasiparticle excitations of the $\nu=12/5$ state (if it is a Read-Rezayi state) are described (up to abelian phases) by $SU(2)_3$ Chern-Simons theory.

In fact, Read and Rezayi proposed an infinite sequence of nonabelian states labeled by an index **k** with quasiparticle excitations described by $SU(2)_k$ Chern-Simons theory.

Read and Rezyai, 1999
Slingerland and Bais, 2001

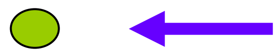
Universal for quantum computation for **k=3** and **k > 4**.

Freedman, Larsen, and Wang, 2001

But before $SU(2)_k$, there was just plain old SU(2)….

Particles with Ordinary Spin: SU(2)

1. Particles have spin $\boldsymbol{s}$ = 0, 1/2, 1, 3/2 , …



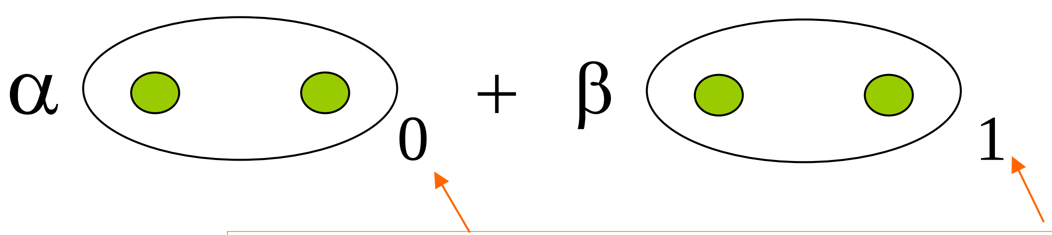
spin = ½

2. “Triangle Rule” for adding angular momentum:

$$s_1 \otimes s_2 = |s_1 - s_2| \oplus \left(|s_1 - s_2| + 1 \right) \oplus \cdots \oplus s_1 + s_2$$

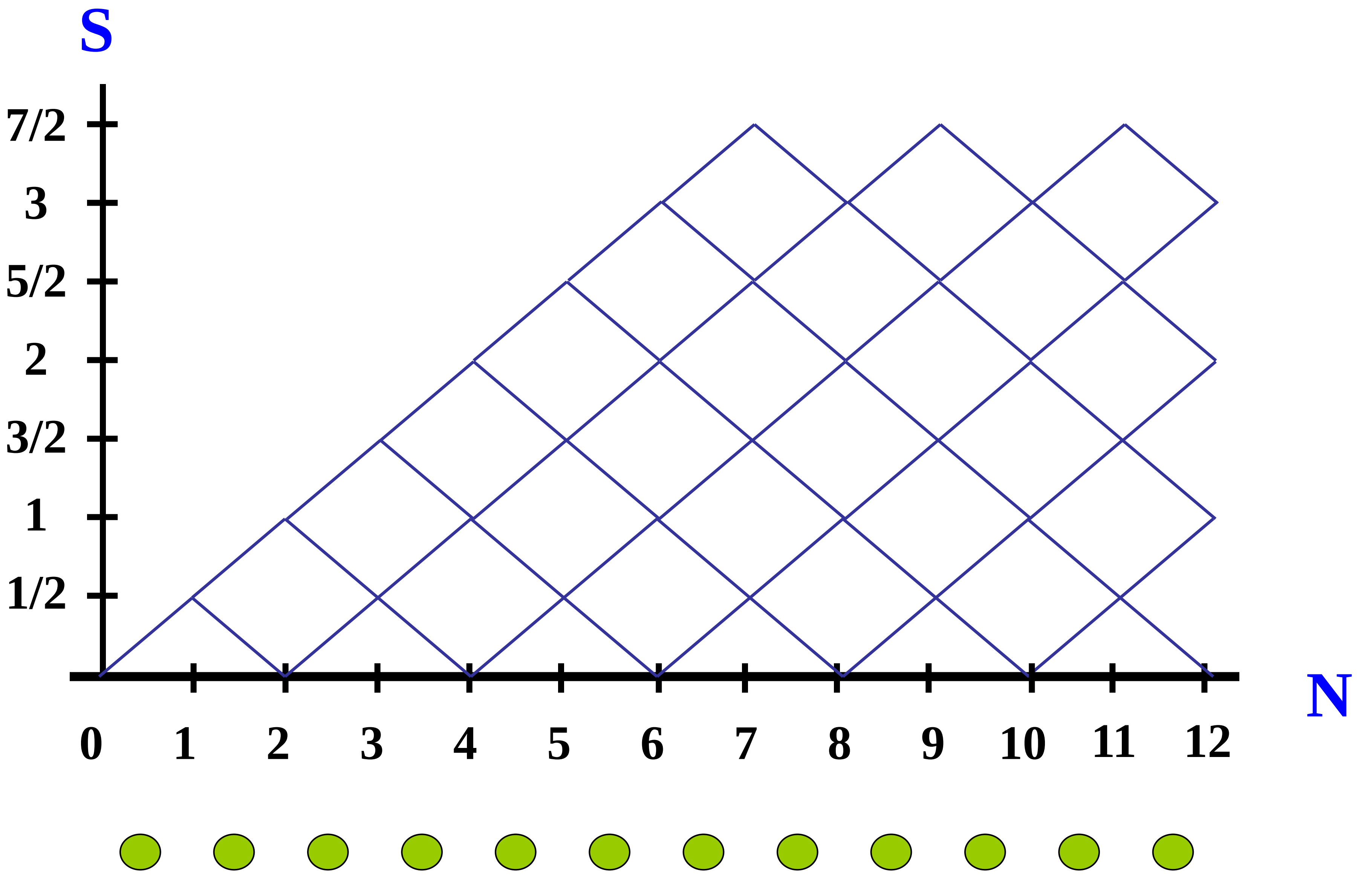
For example: $$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$

➡ Two particles can have total spin **0** or **1**.

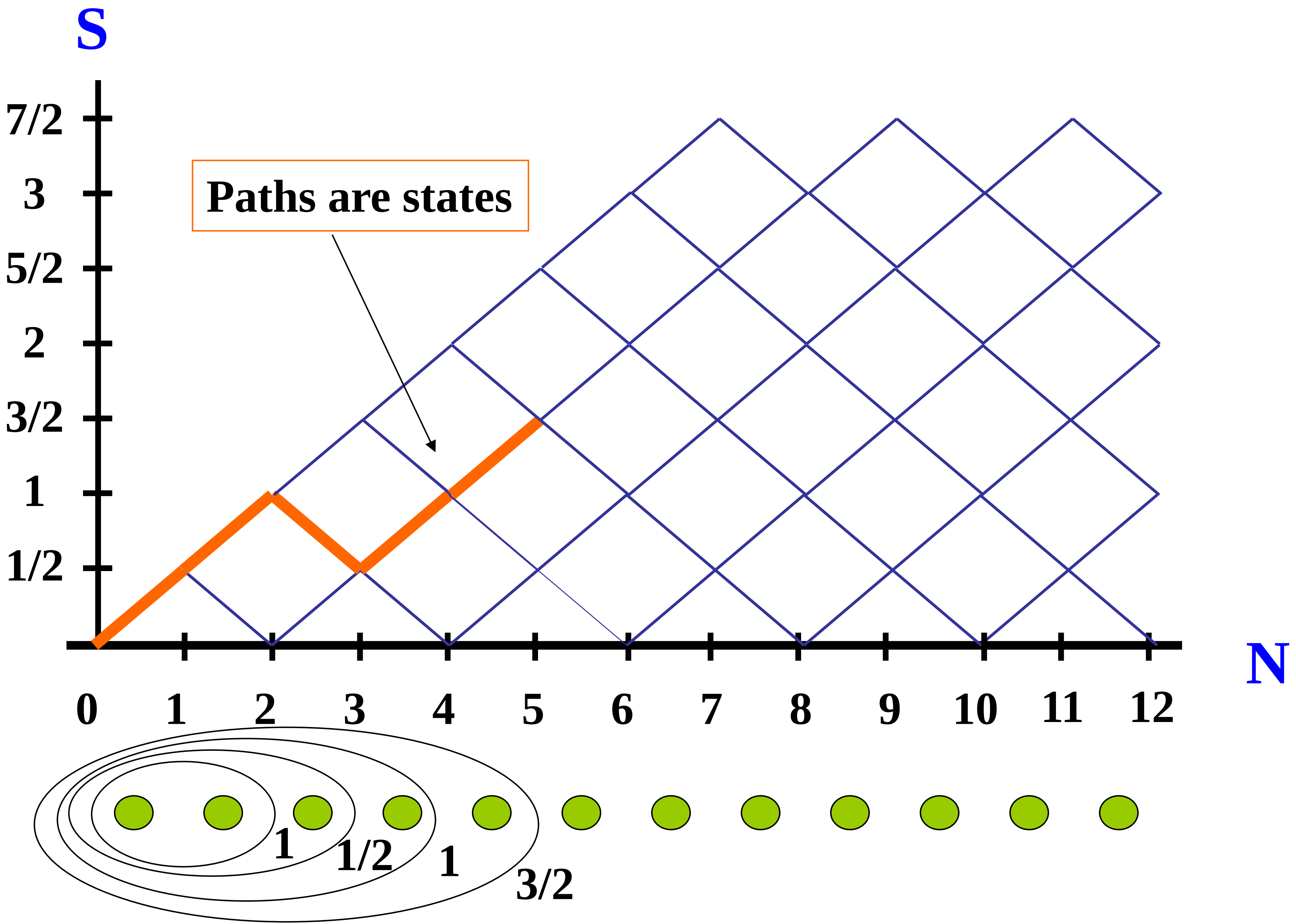


Numbers label total spin of particles inside ovals

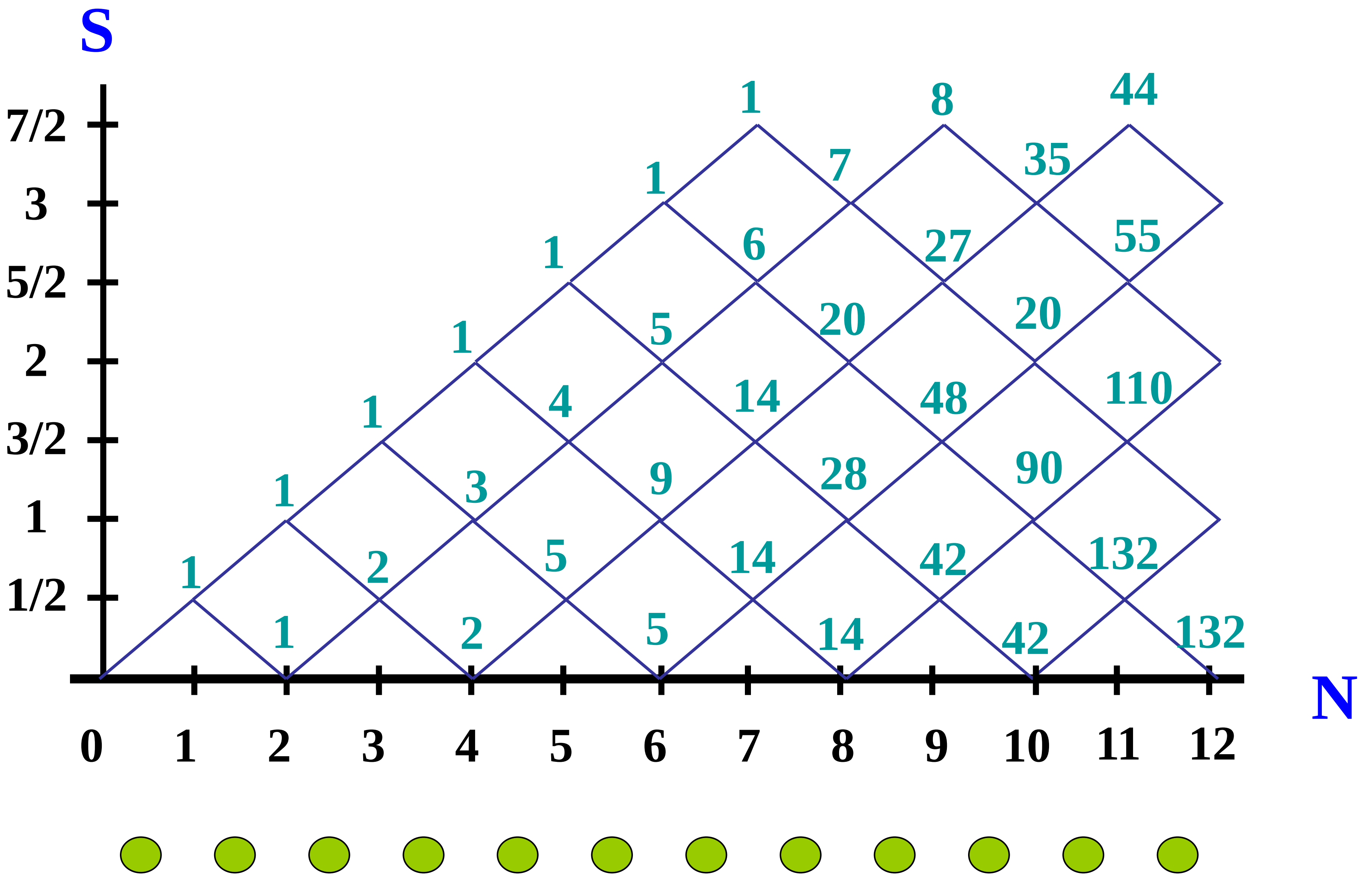
Hilbert Space



Hilbert Space

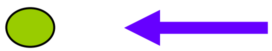


Hilbert Space



Particles with Ordinary Spin: SU(2)

1. Particles have spin ***s*** = 0, 1/2, 1, 3/2 , …



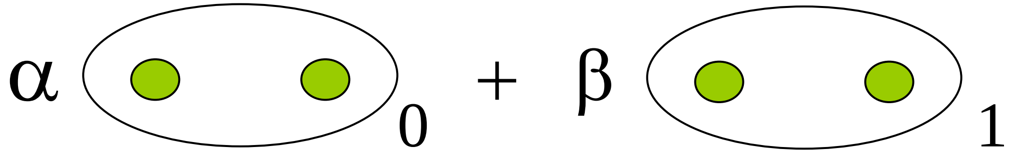
spin = ½

2. "Triangle Rule" for adding angular momentum:

$$s_1 \otimes s_2 = |s_1 - s_2| \oplus \left(|s_1 - s_2| + 1 \right) \oplus \cdots \oplus s_1 + s_2$$

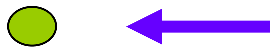
For example: $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

→ Two ● particles can have total spin **0** or **1**.



Nonabelian Particles: $SU(2)_k$

1. Particles have topological charge $\boldsymbol{s}$ = 0, 1/2, 1, 3/2 , …, k/2



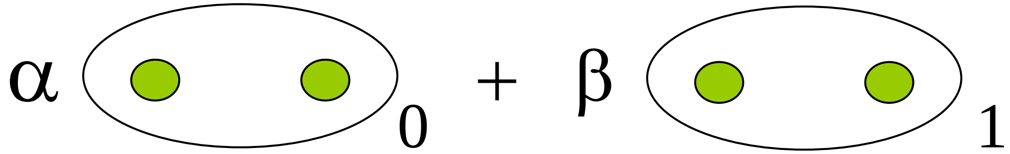
topological charge = ½

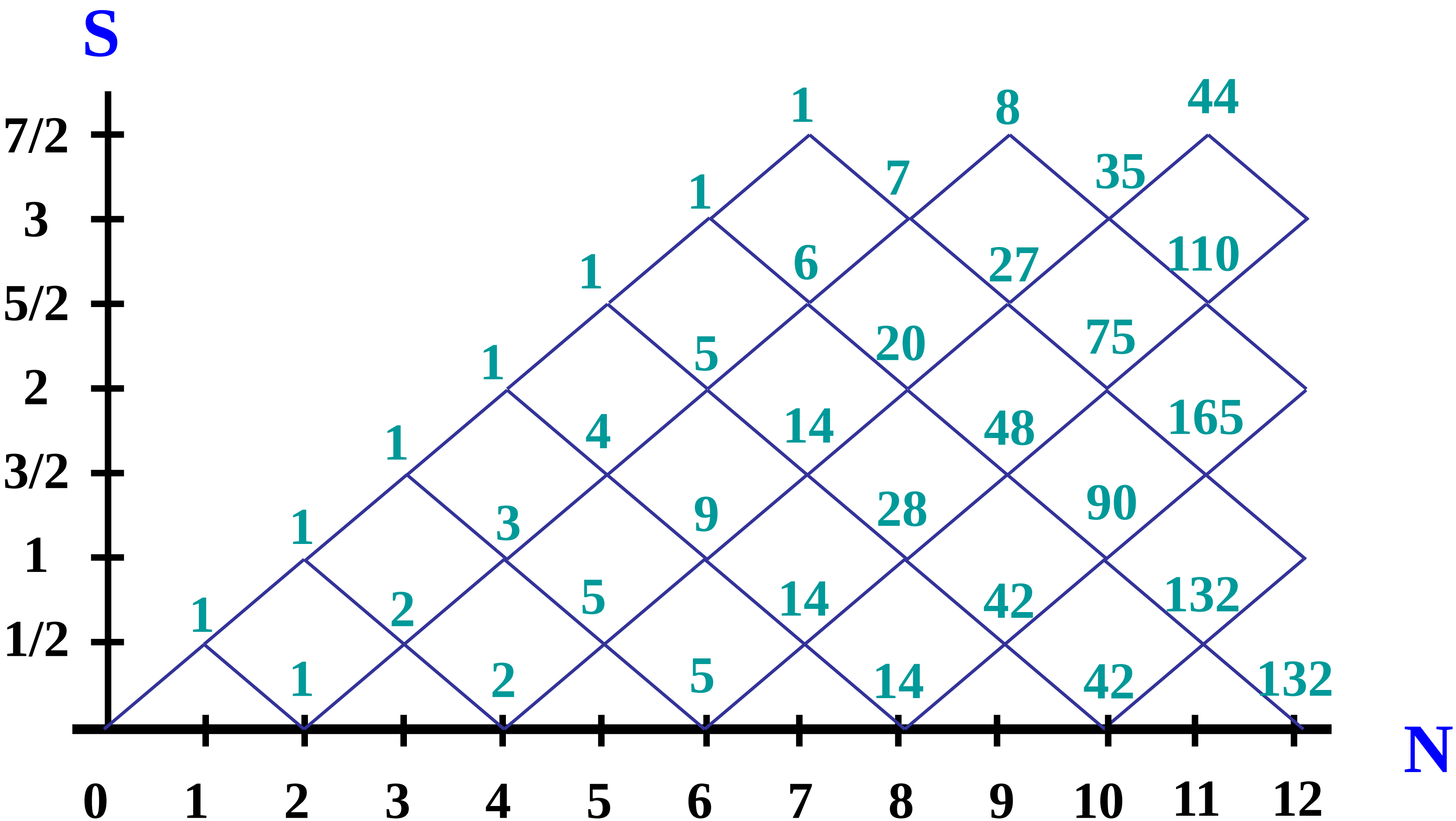
2. "Fusion Rule" for adding topological charge:

$$s_1 \otimes s_2 = |s_1 - s_2| \oplus (|s_1 - s_2| + 1) \oplus \cdots \oplus \min[s_1 + s_2, s_1 + s_2 - k/2]$$

For example: $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

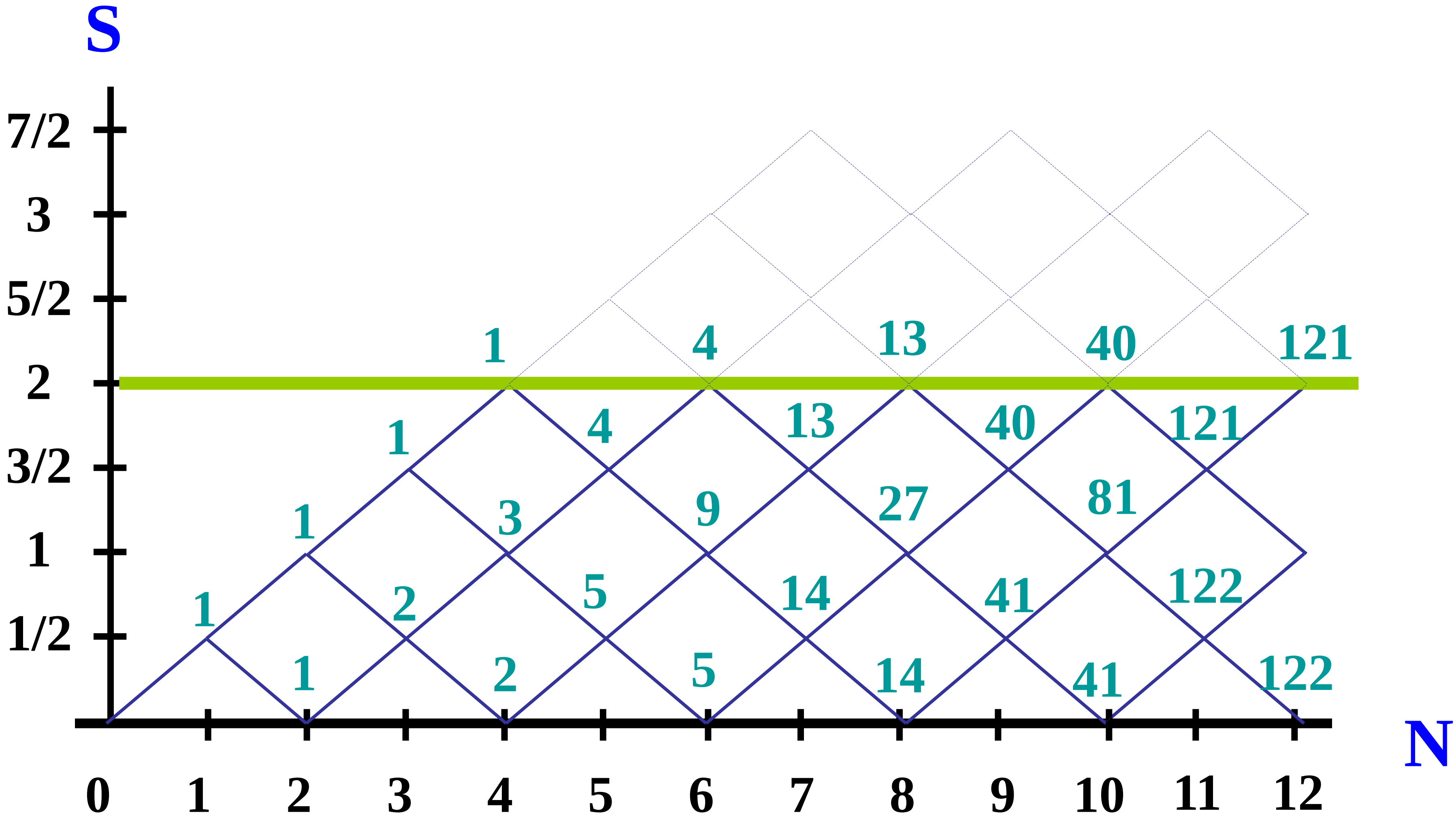
➔ Two ● particles can have total topological charge **0** or **1**.




S
7/2
3
5/2
2
3/2
1
1/2
1
8
44
1
7
35
1
6
27
110
1
5
20
75
1
4
14
48
165
1
3
9
28
90
1
2
5
14
42
132
1
2
5
14
42
132
N
0
1
2
3
4
5
6
7
8
9
10
11
12

Dim(N) ~ 2^N

k = 4


S
7/2
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5/2
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3/2
1
1/2
N
0
1
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5
6
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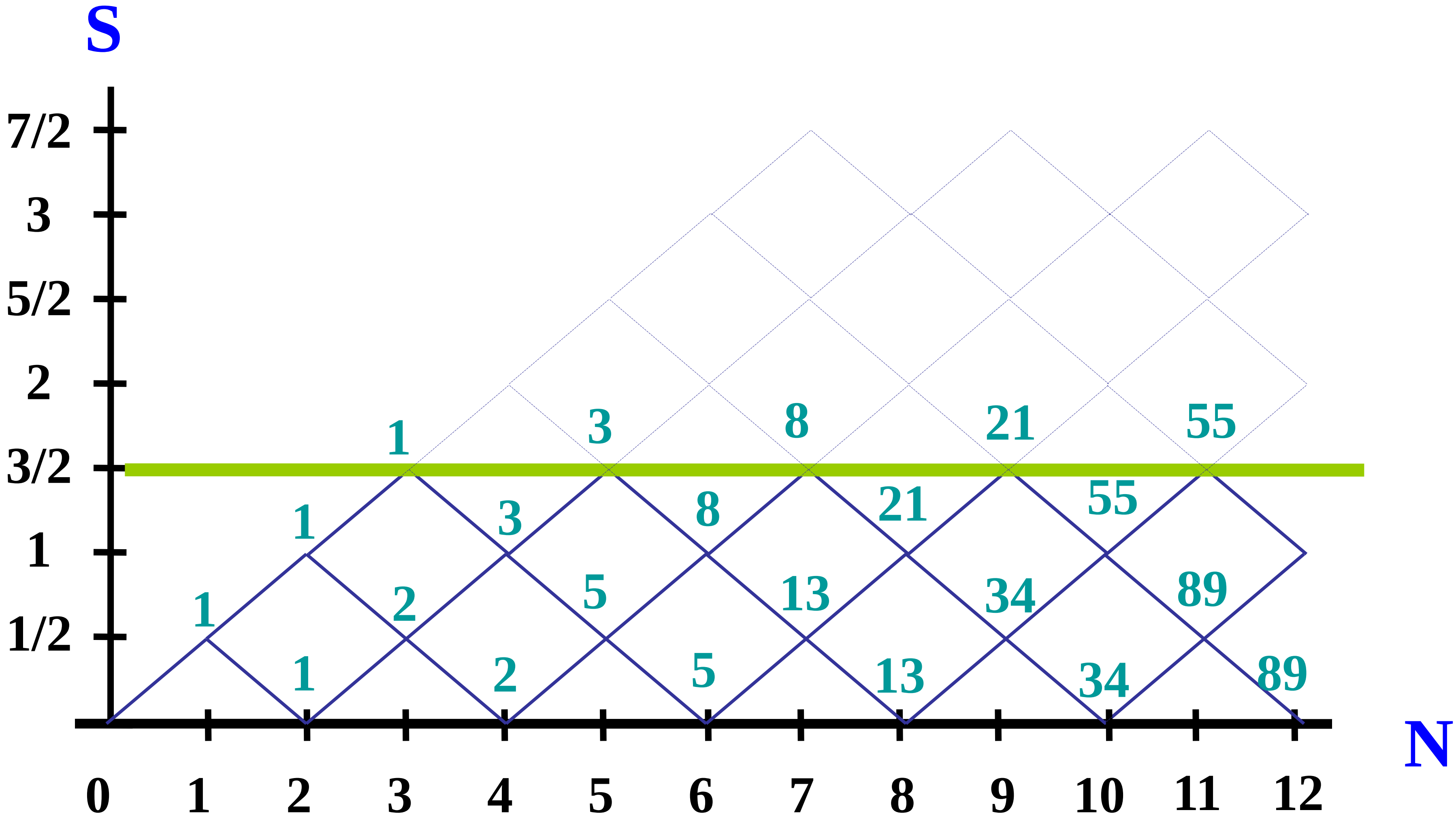
8
9
10
11
12
1
4
13
40
121
1
4
13
40
121
1
3
9
27
81
1
2
5
14
41
122
1
2
5
14
41
122

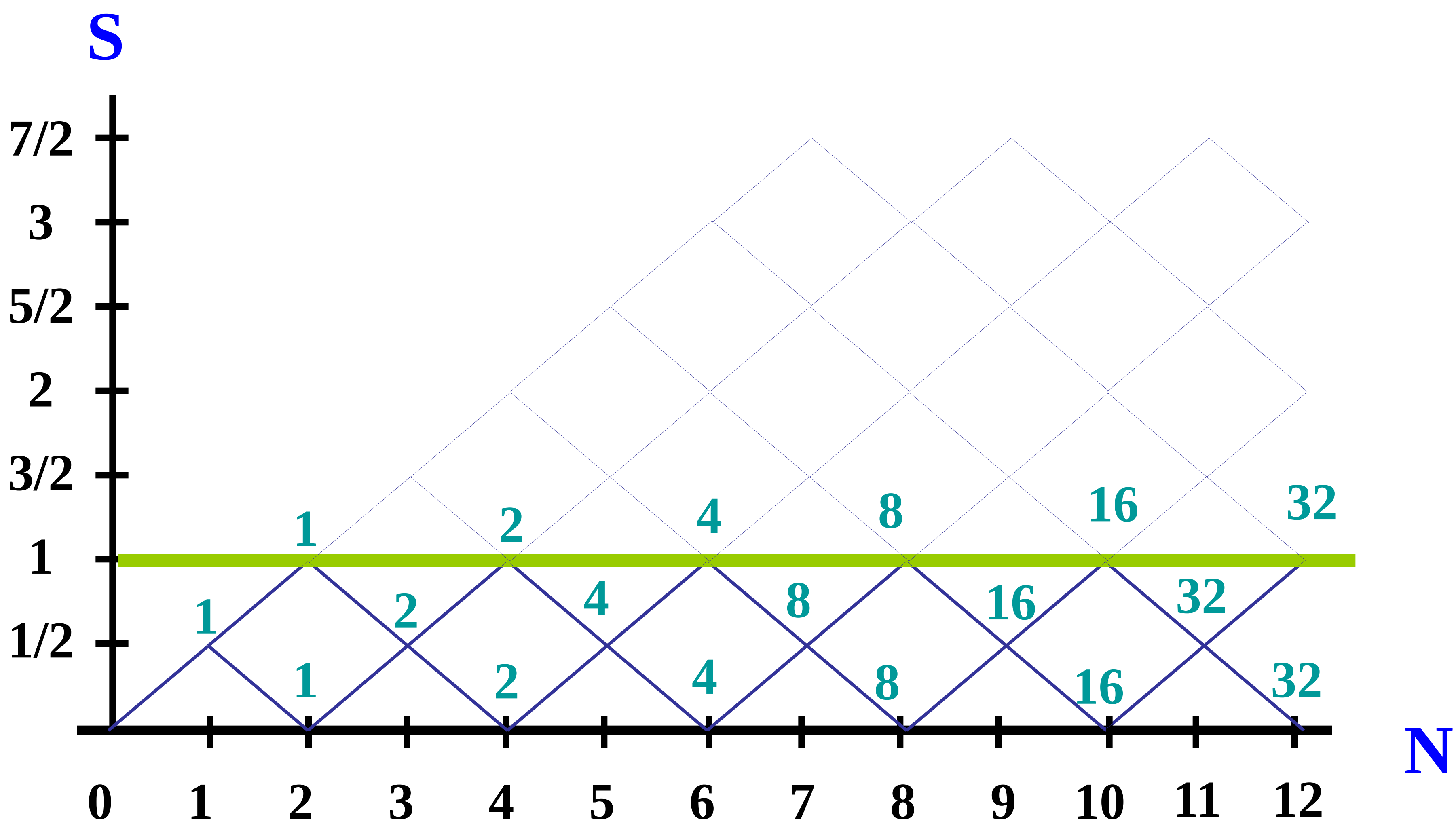
Dim(N) ~ $3^{N/2}$

k = 3



Dim(N) = Fib(N+1) ~ ϕ^N

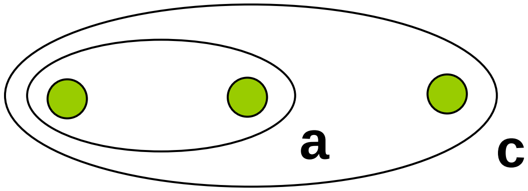
k = 2


S
7/2
3
5/2
2
3/2
1
1/2
N
0
1
2
3
4
5
6
7
8
9
10
11
12
1
2
4
8
16
32
1
2
4
8
16
32
1
2
4
8
16
32

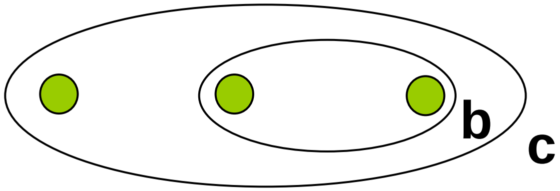
$Dim(N) = 2^{N/2}$

The F Matrix

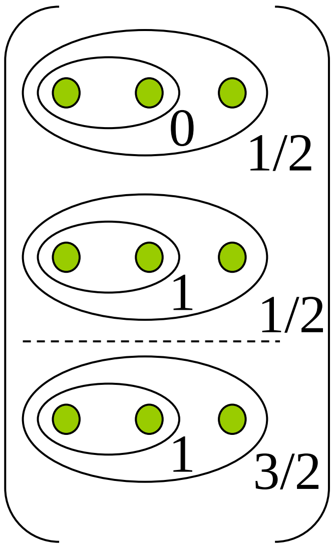
$$\sum_a F_{ab}^{c}$$



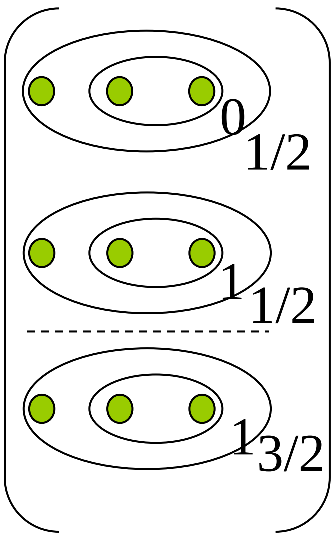
$=$



$$\left(\begin{array}{cc|c} \frac{1}{[2]_q} & \frac{\sqrt{[3]_q}}{[2]_q} & 0 \\ \frac{\sqrt{[3]_q}}{[2]_q} & -\frac{1}{[2]_q} & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$



$=$

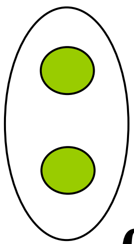


q-integers: $$[m]_q = \frac{q^{m/2} - q^{-m/2}}{q^{1/2} - q^{-1/2}} \;;\quad q = e^{i2\pi/(k+2)}$$

The R Matrix

0

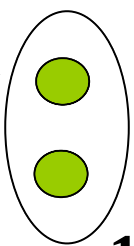
$$= e^{i\frac{2k+1}{2k+4}\pi}$$



0

1

$$= e^{i\frac{1}{2k+4}\pi}$$



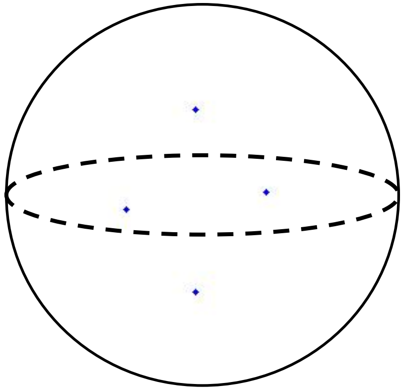
1

R matrix

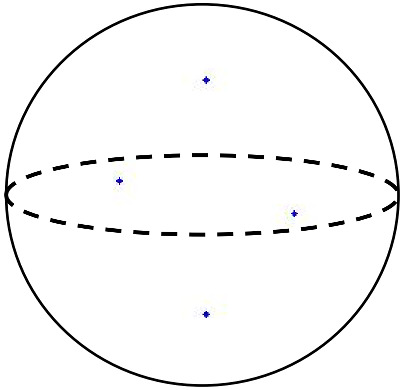
$$R = \begin{pmatrix} e^{i\frac{2k+1}{2k+4}\pi} & 0 \\ 0 & e^{i\frac{1}{2k+4}\pi} \end{pmatrix}$$

N = 1

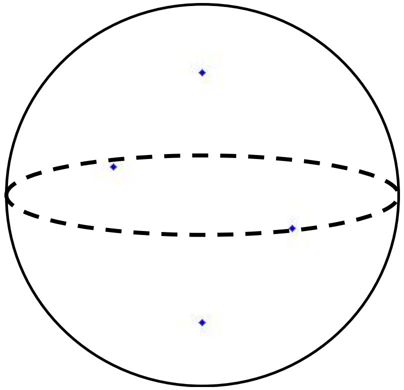
k=2



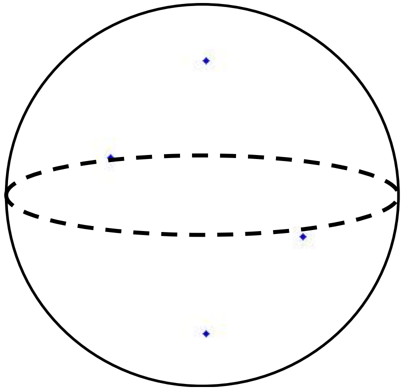
k=3



k=4

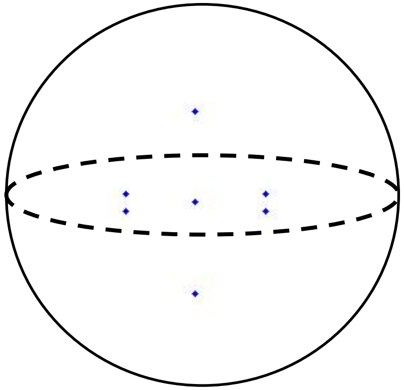


k=5

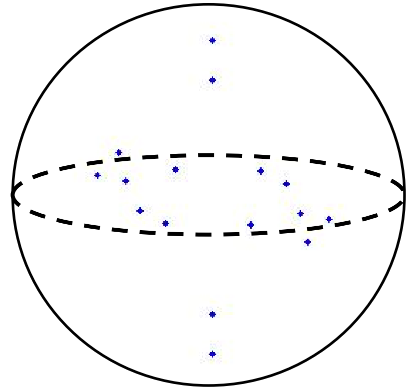


N = 2

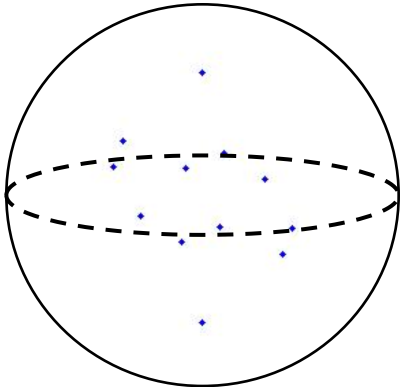
k=2



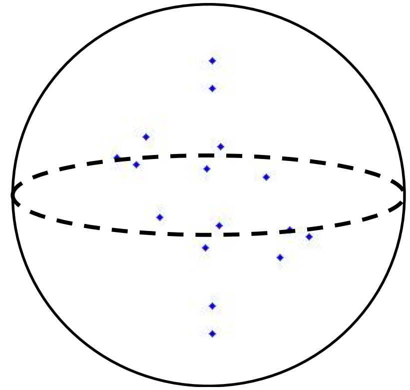
k=3



k=4

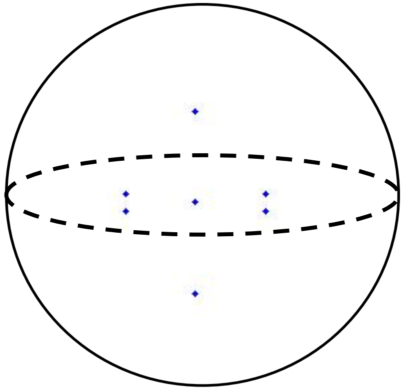


k=5

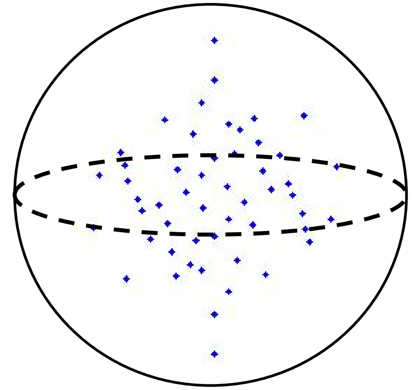


N = 3

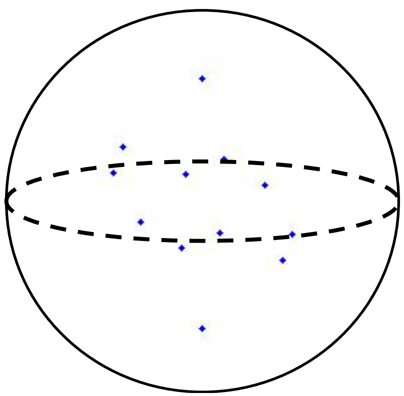
k=2



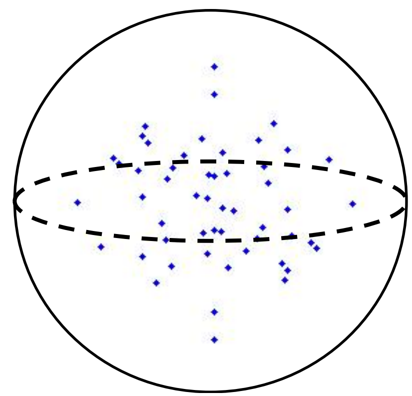
k=3



k=4

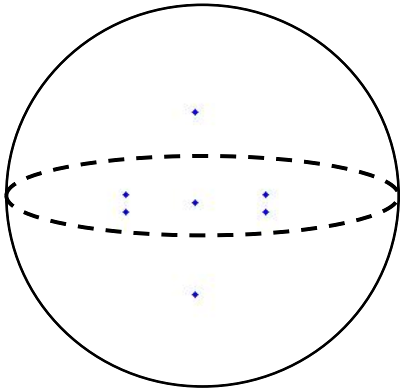


k=5

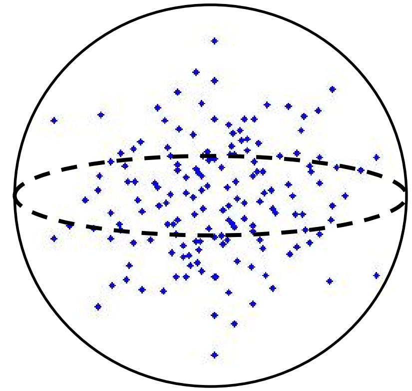


N = 4

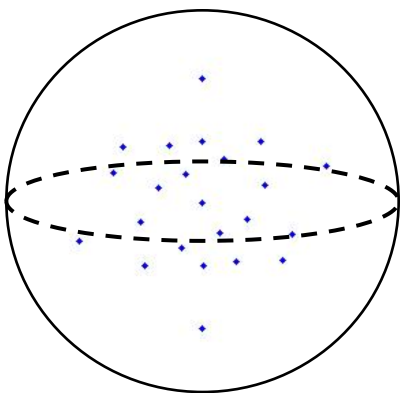
k=2



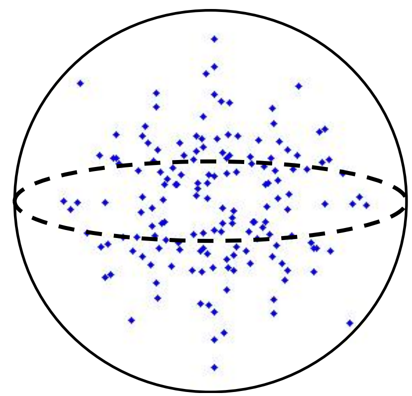
k=3

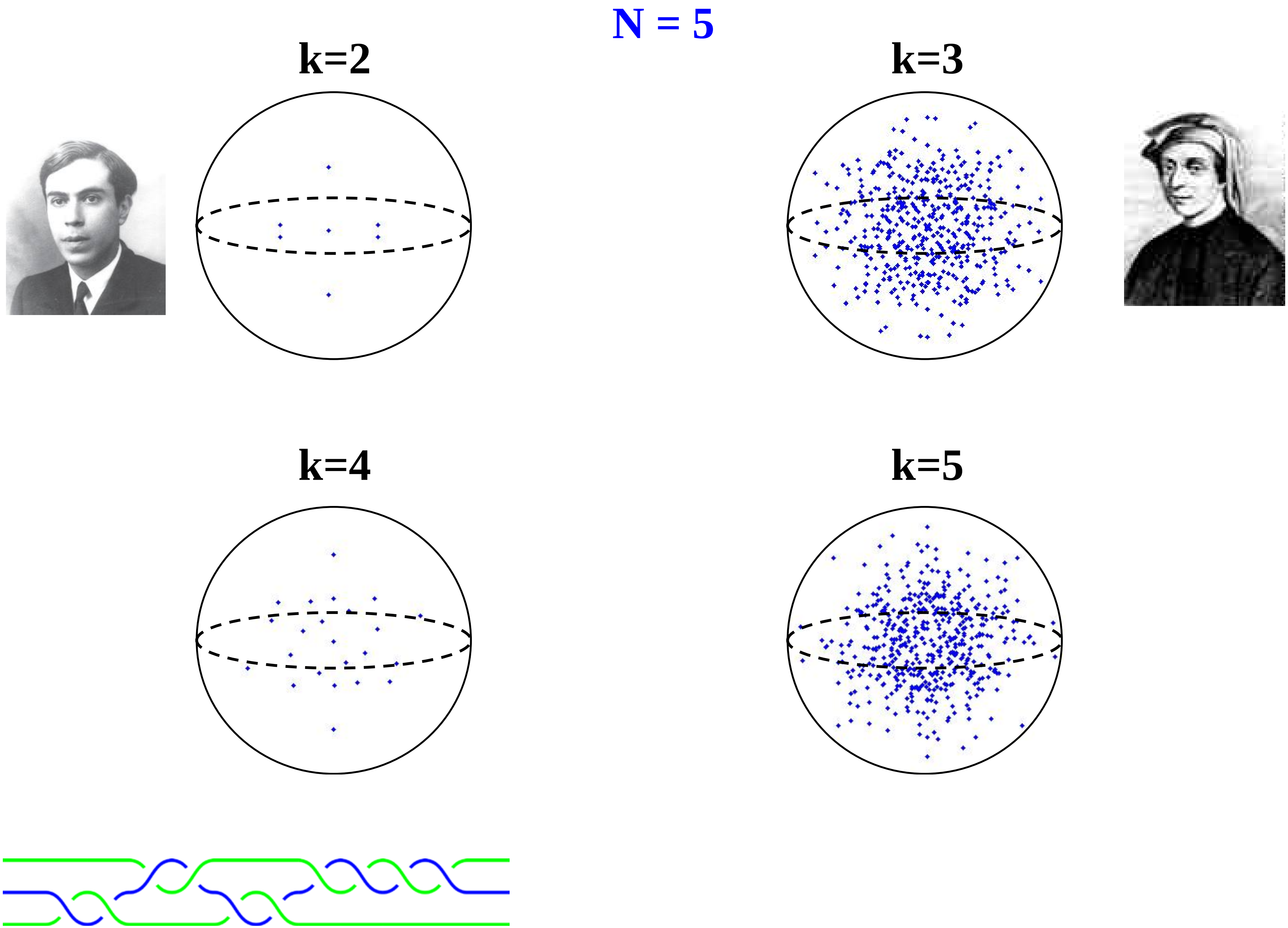


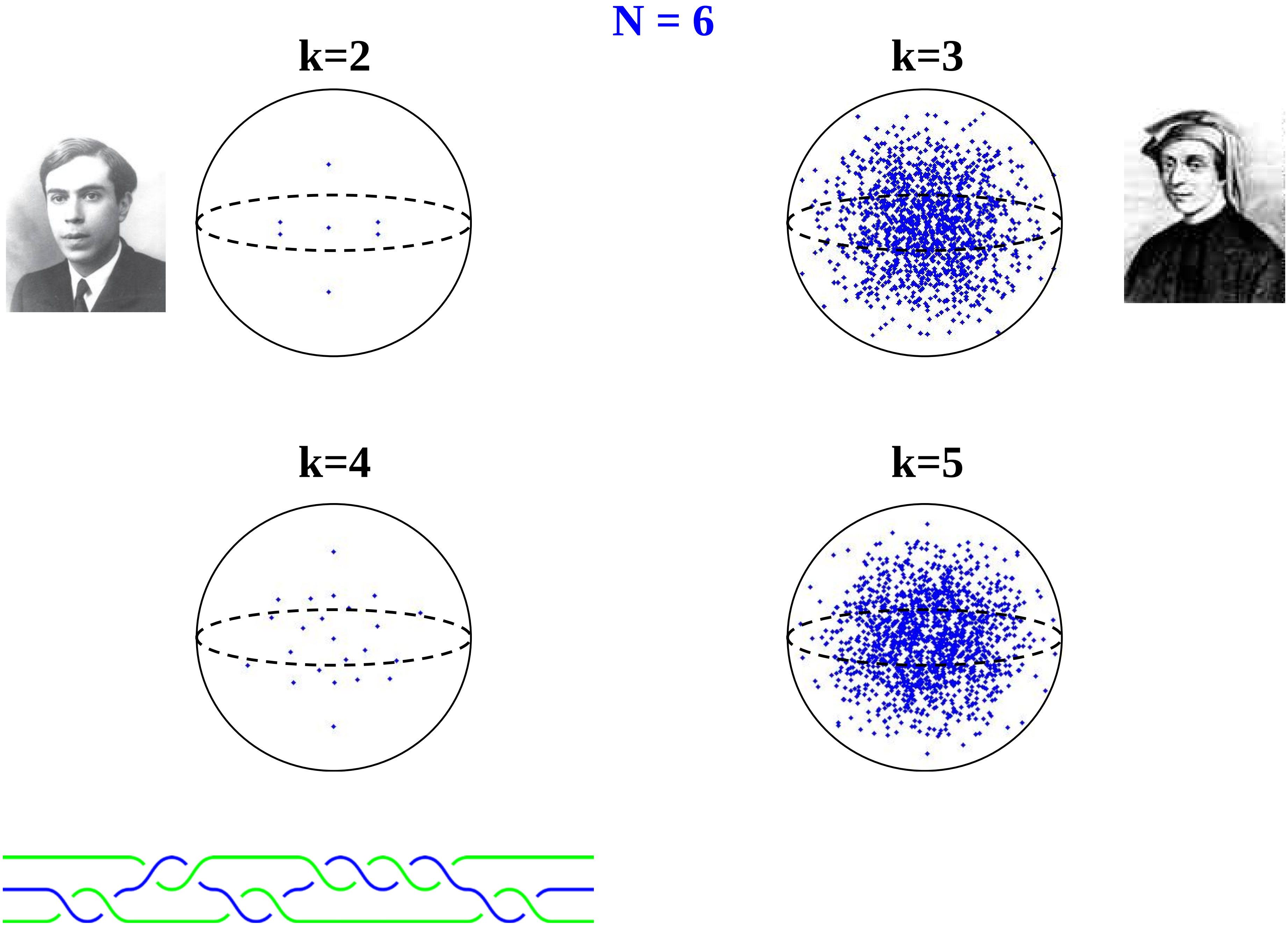
k=4



k=5

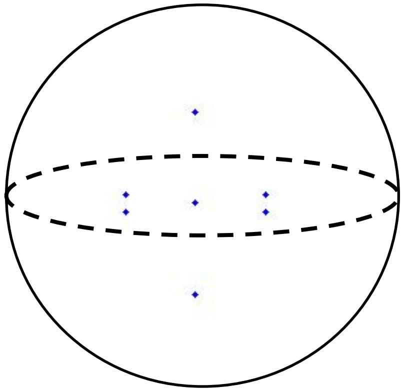



N = 5
k=2
k=3
k=4
k=5

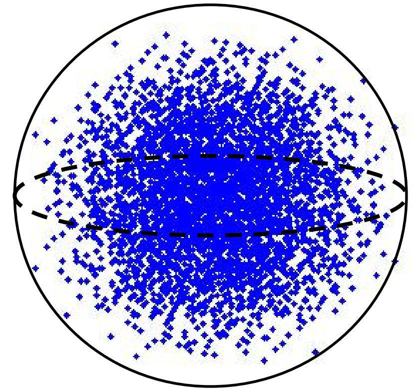

N = 6
k=2
k=3
k=4
k=5

N = 7

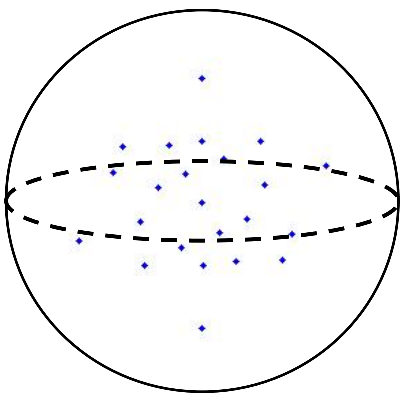
k=2



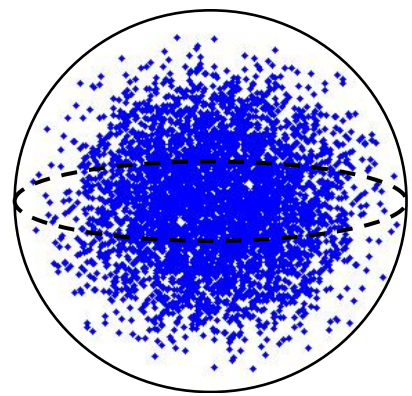
k=3



k=4

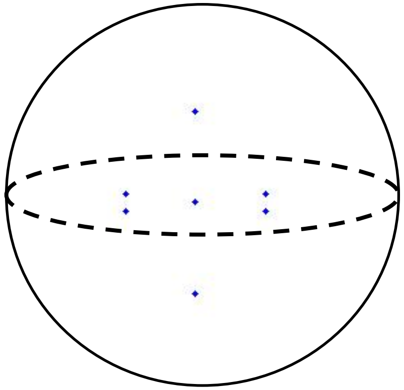


k=5

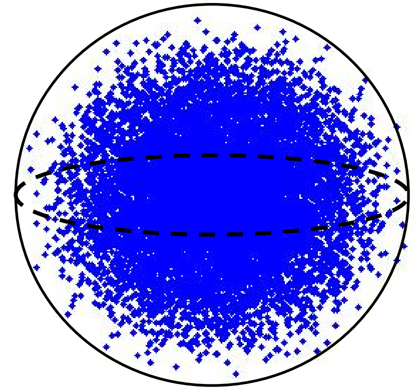


N = 8

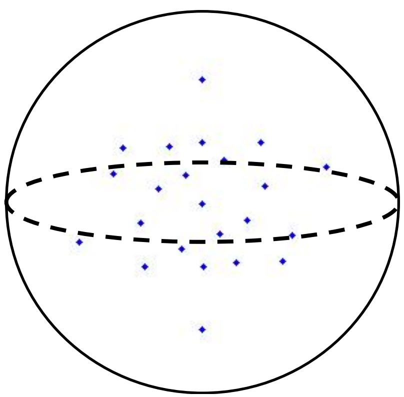
k=2



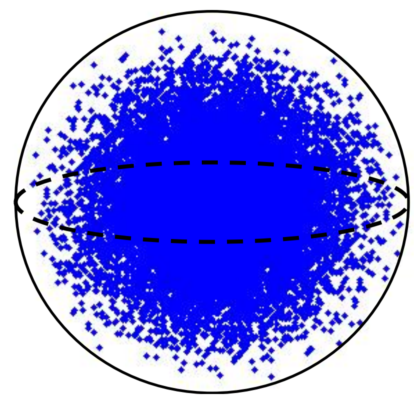
k=3

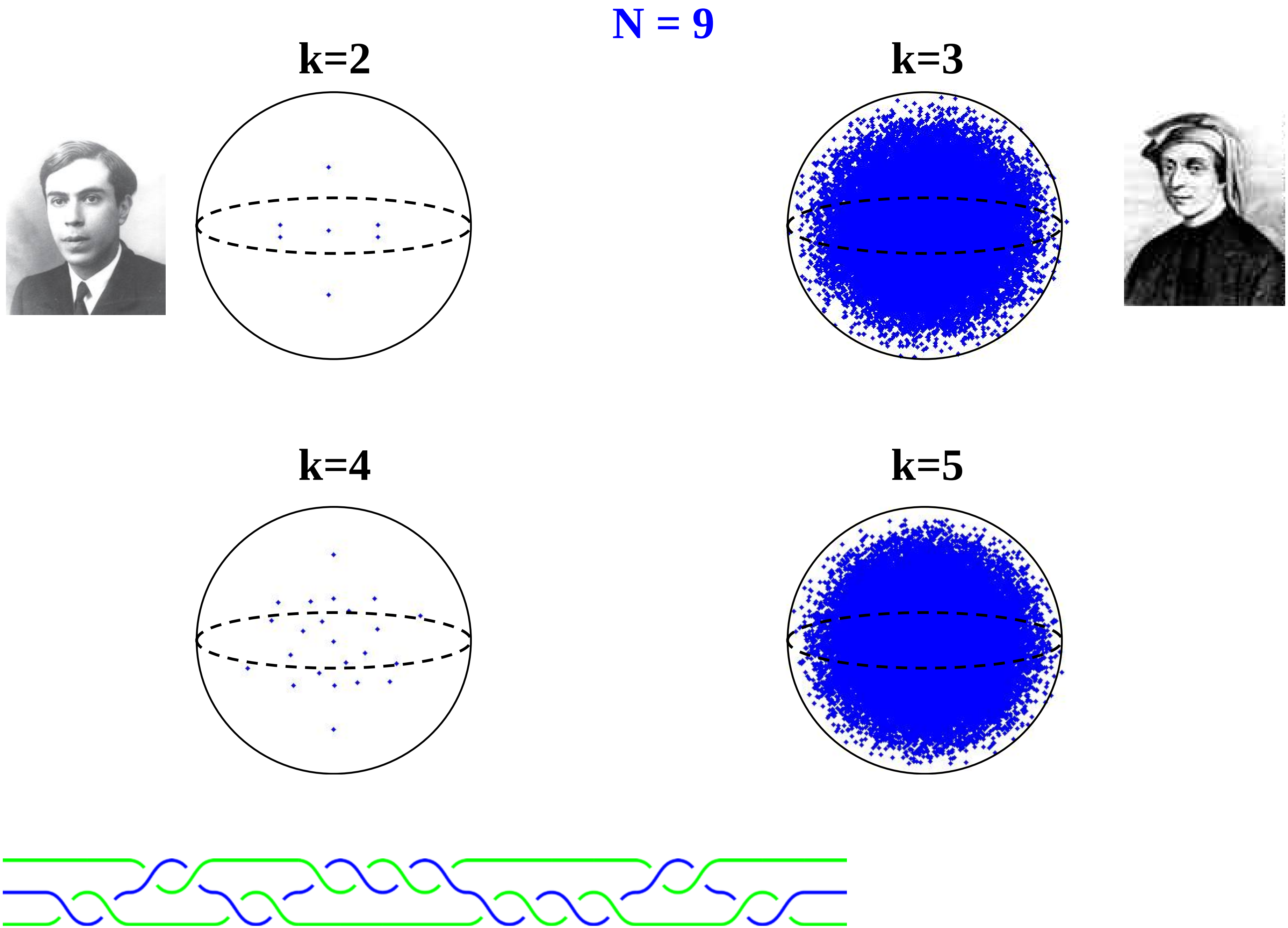


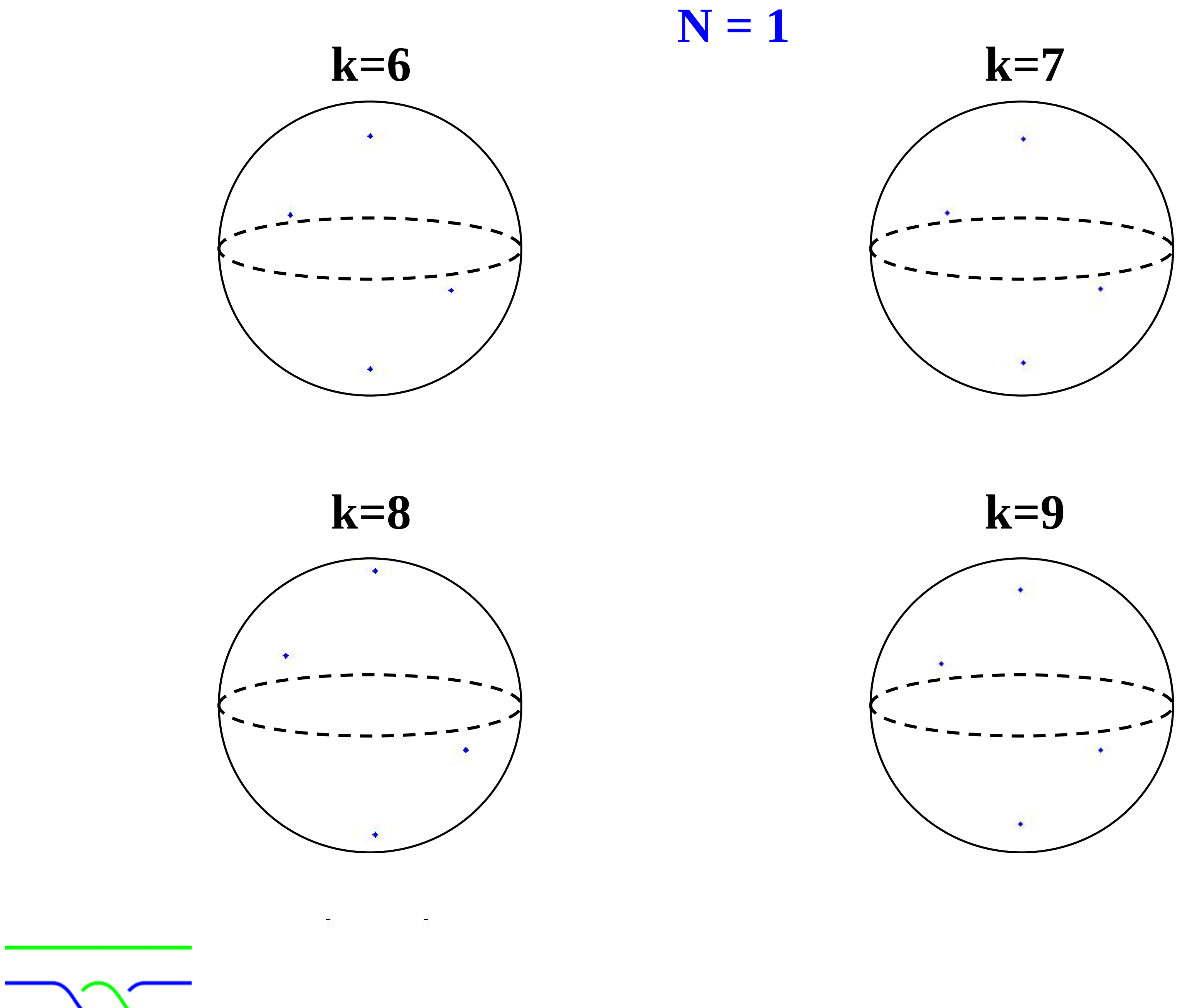
k=4



k=5

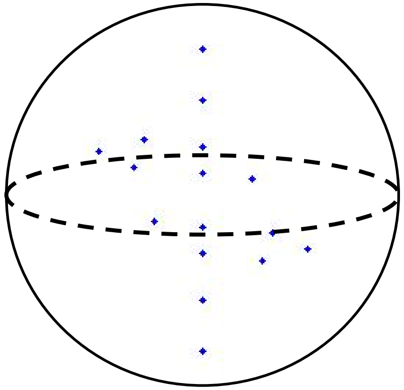



N = 9
k=2
k=3
k=4
k=5

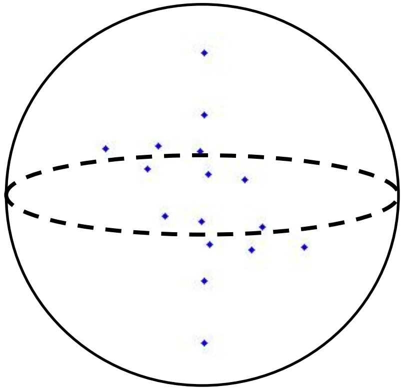

N = 1
k=6
k=7
k=8
k=9

N = 2

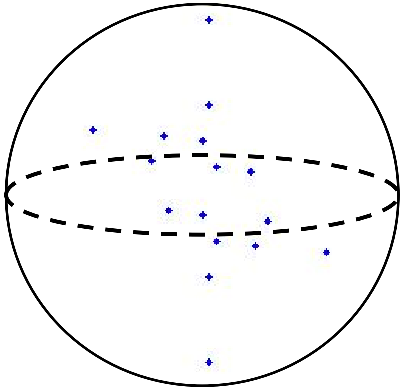
k=6



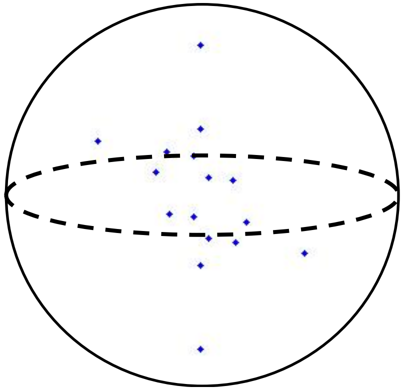
k=7



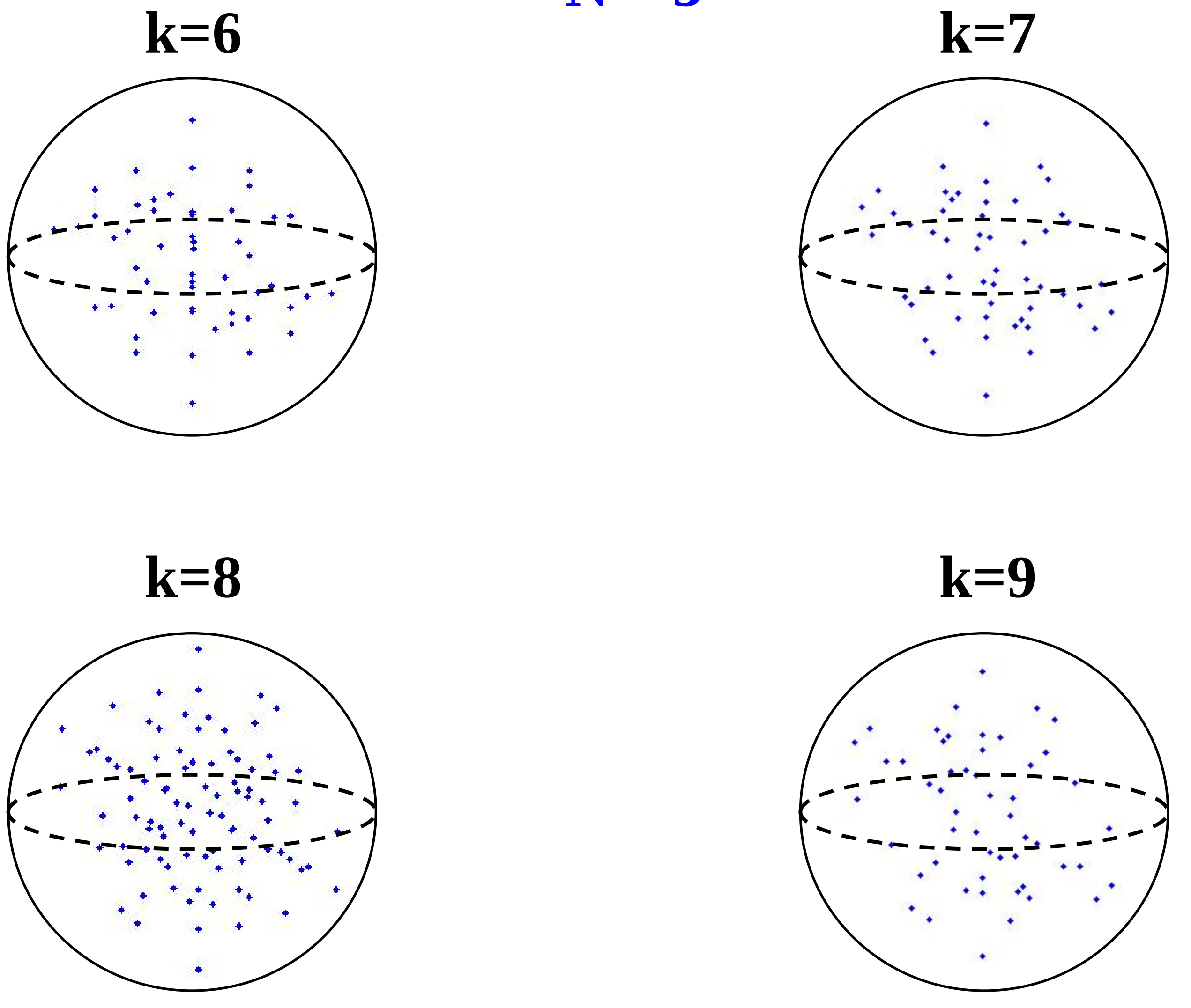
k=8

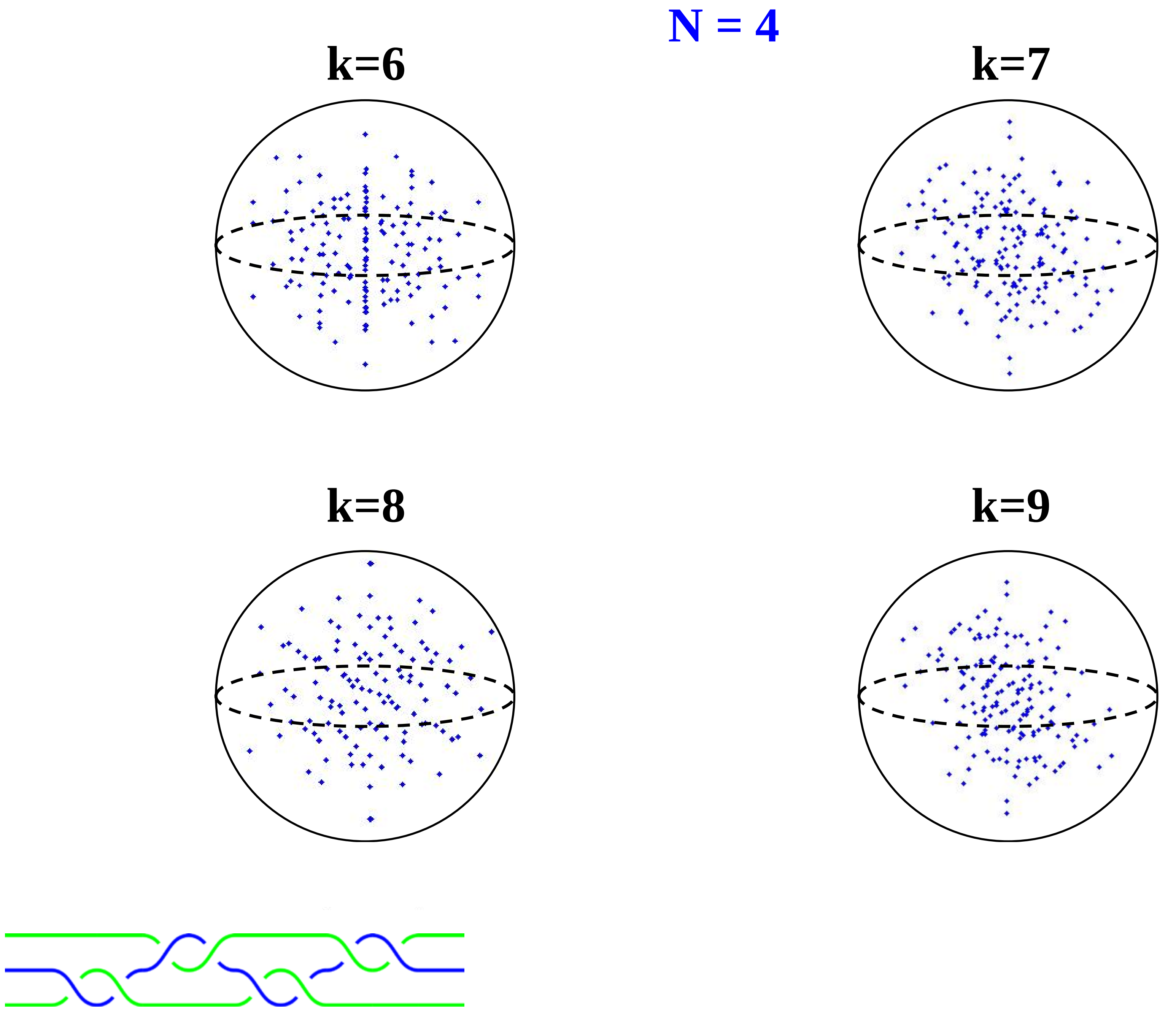


k=9

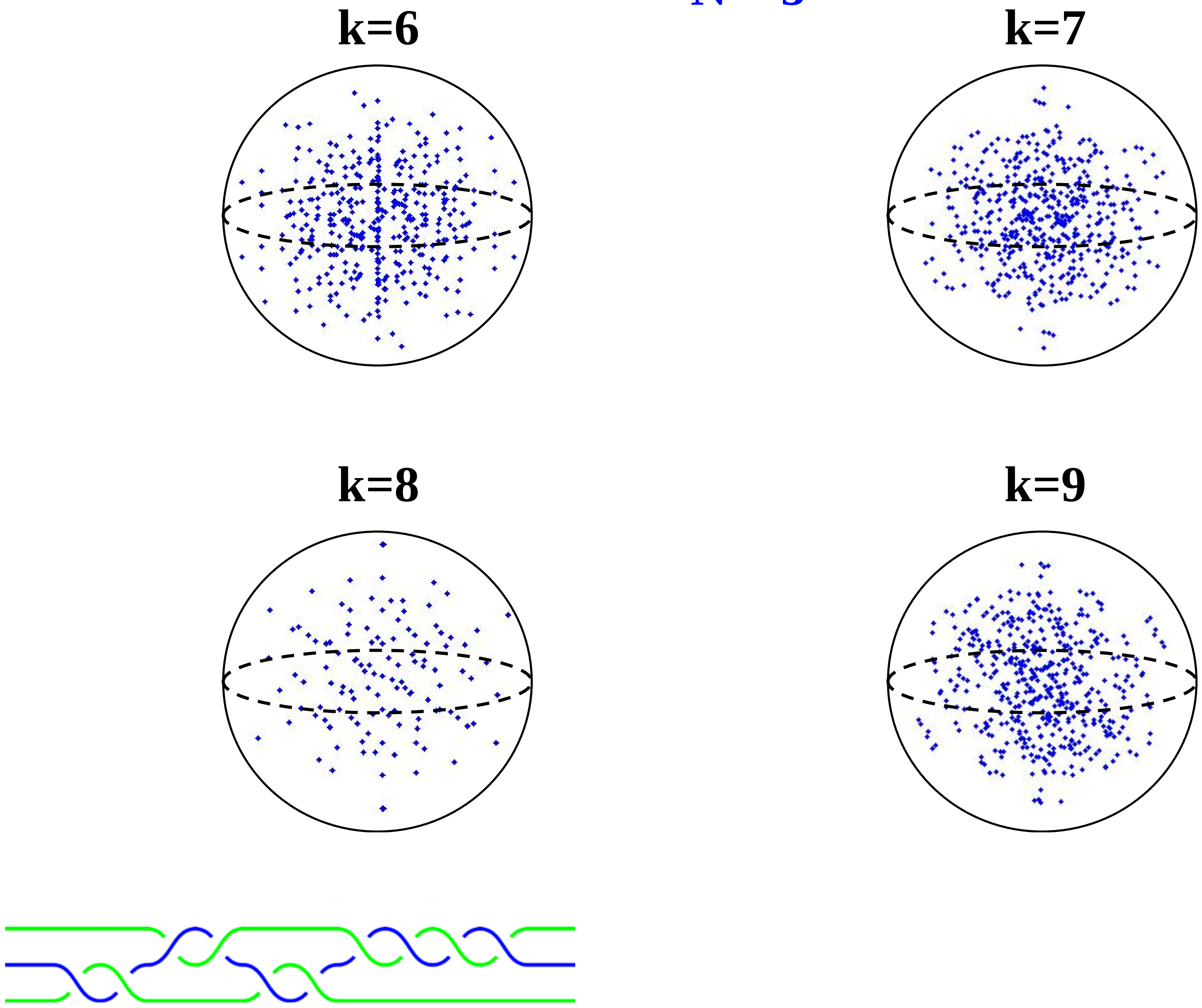


N = 3

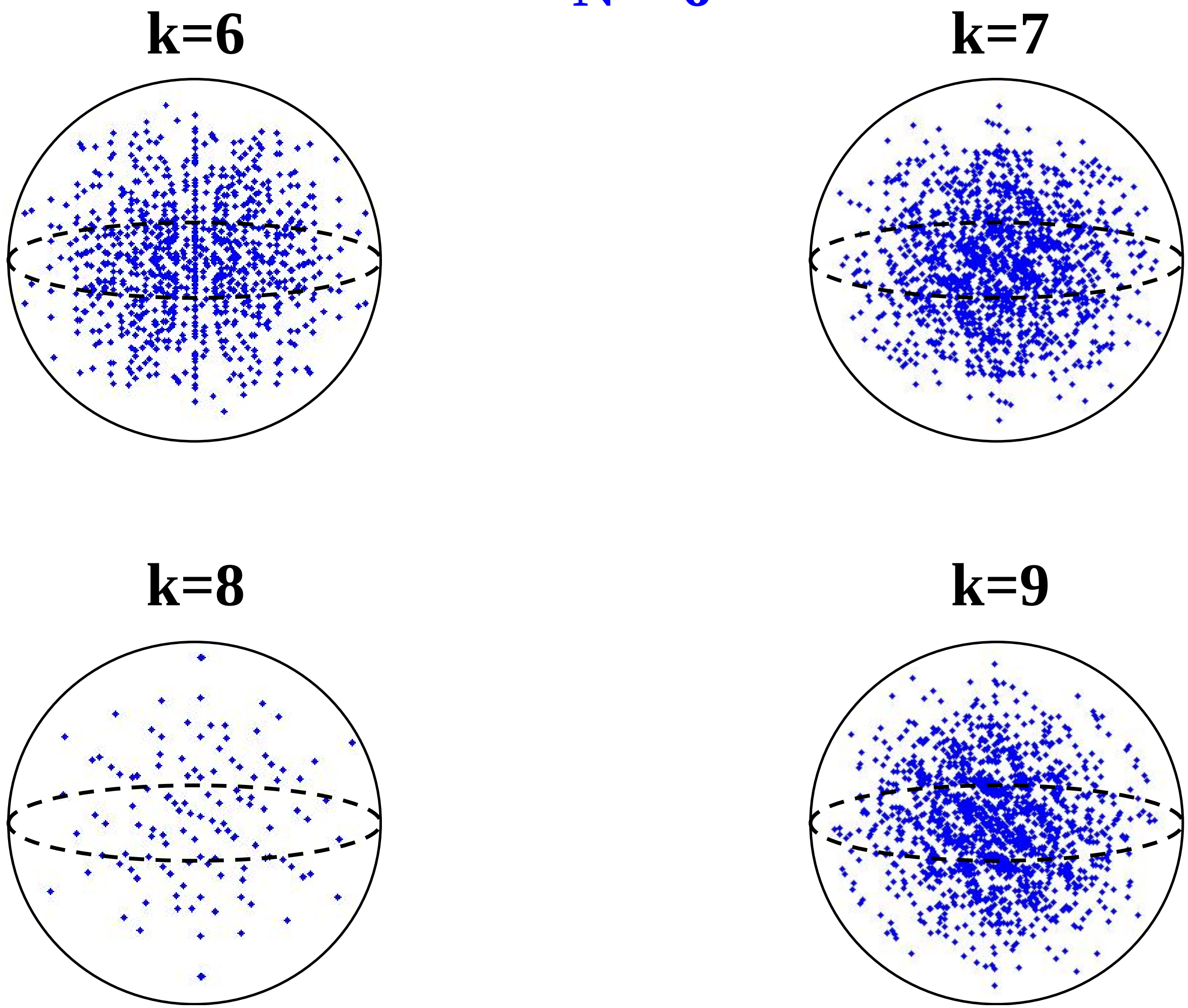


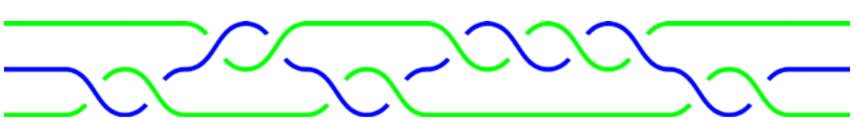

N = 4
k=6
k=7
k=8
k=9

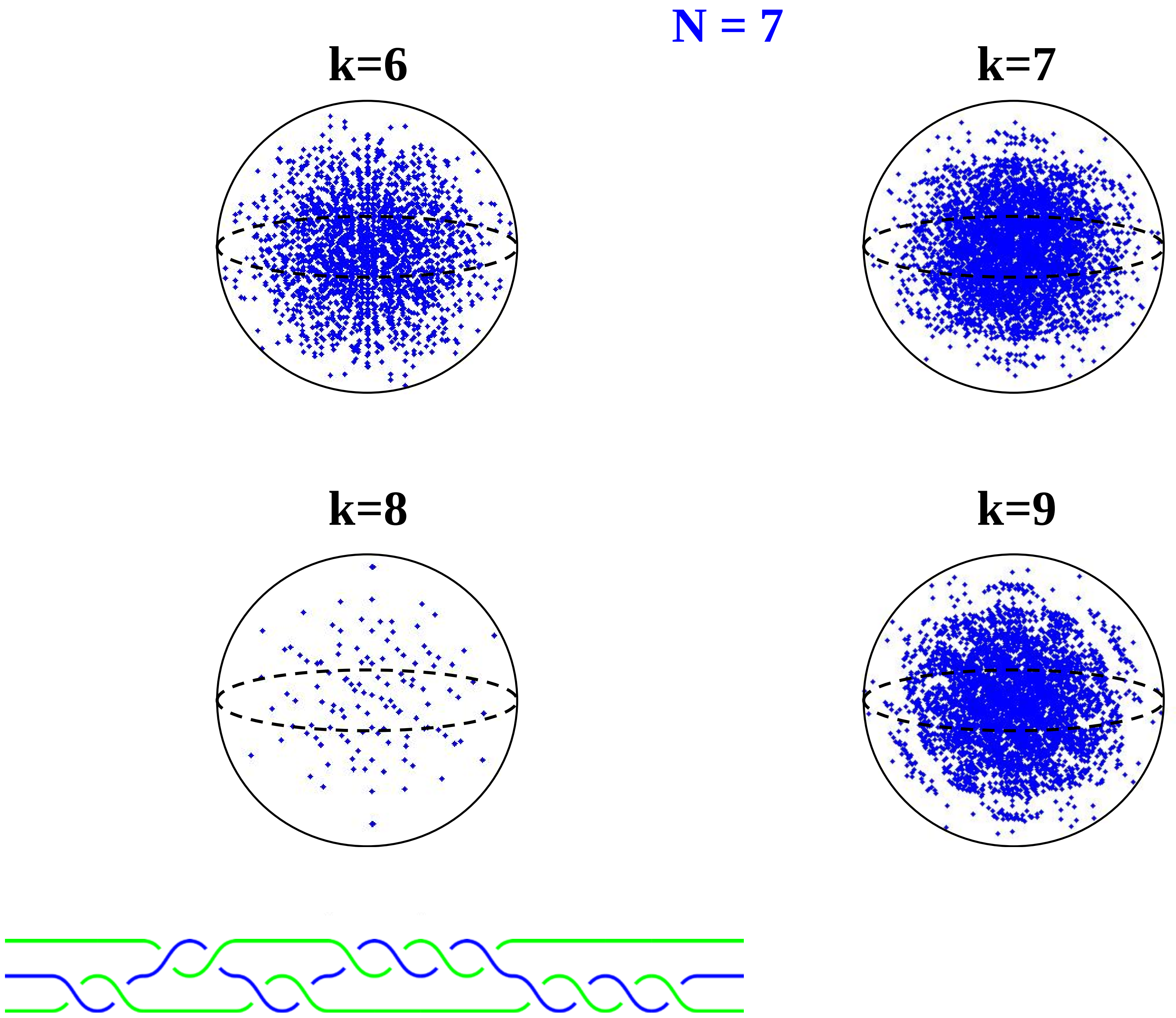
N = 5

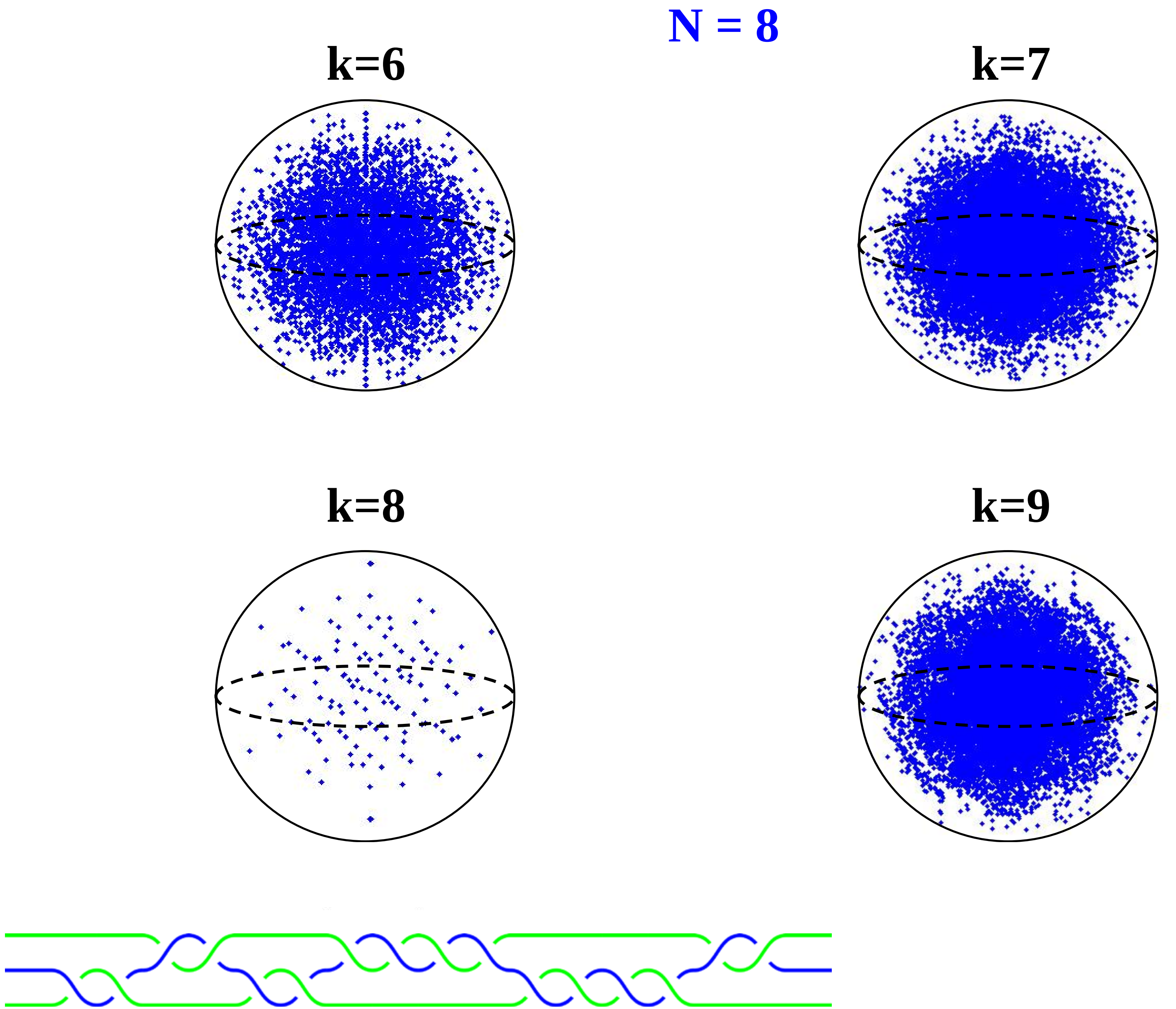


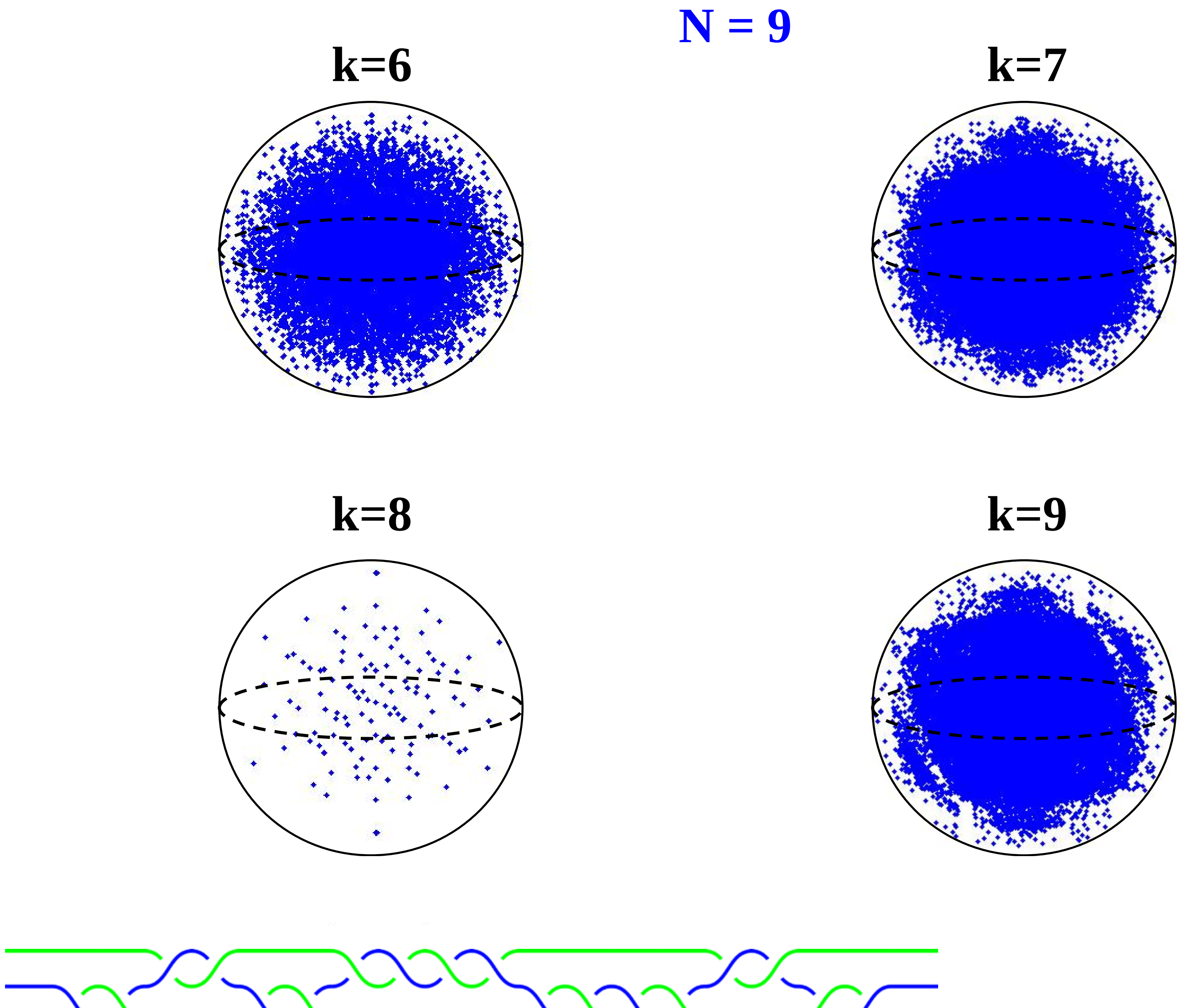
N = 6





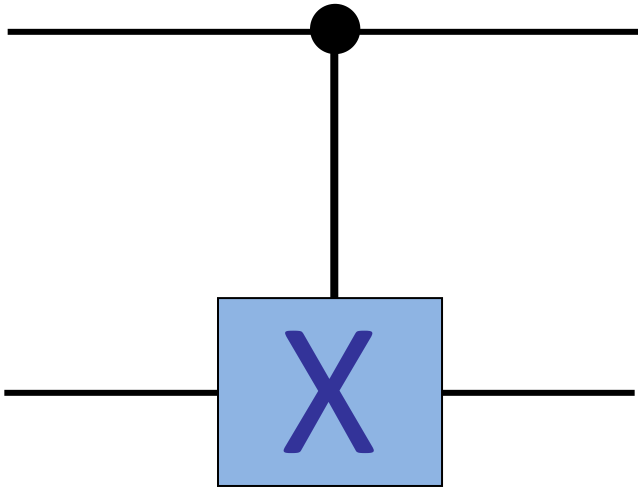

N = 7
k=6
k=7
k=8
k=9

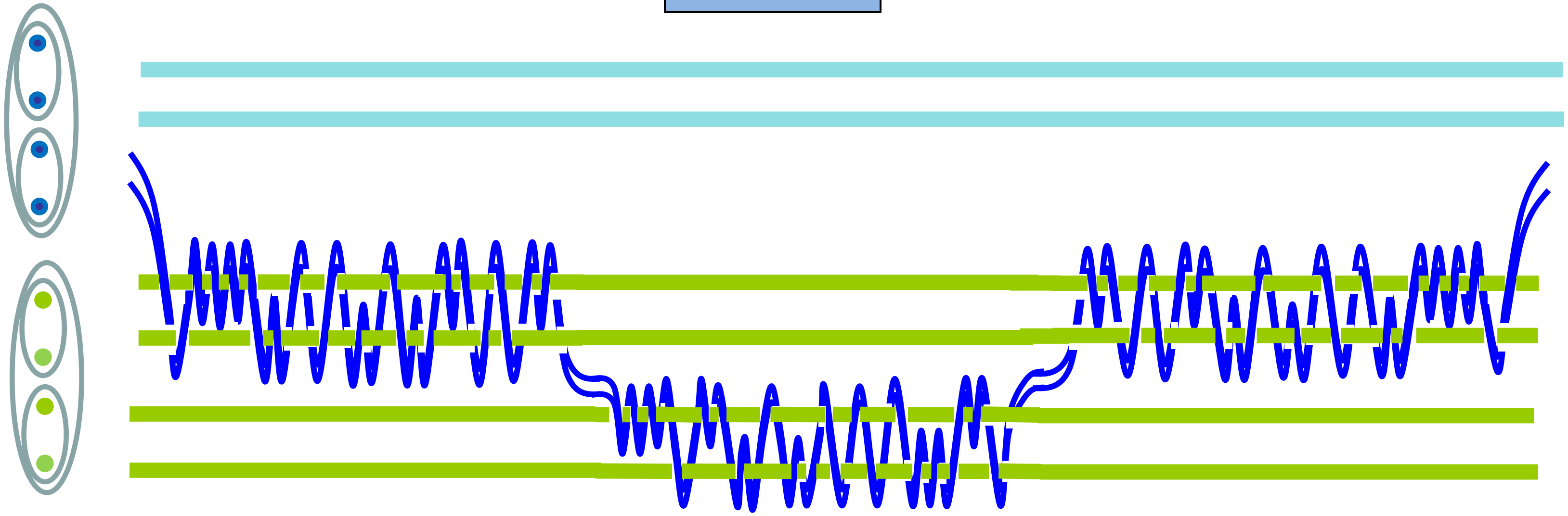

N = 8
k=6
k=7
k=8
k=9



N = 9
k=6
k=7
k=8
k=9

CNOT gate for $SU(2)_5$





L. Hormozi, NEB, & S.H. Simon, PRL 2009

“Surface Code” Approach

Key Idea: Use a quantum computer to *simulate* a theory of anyons.

The simulation “inherits” the fault-tolerance of the anyon theory.

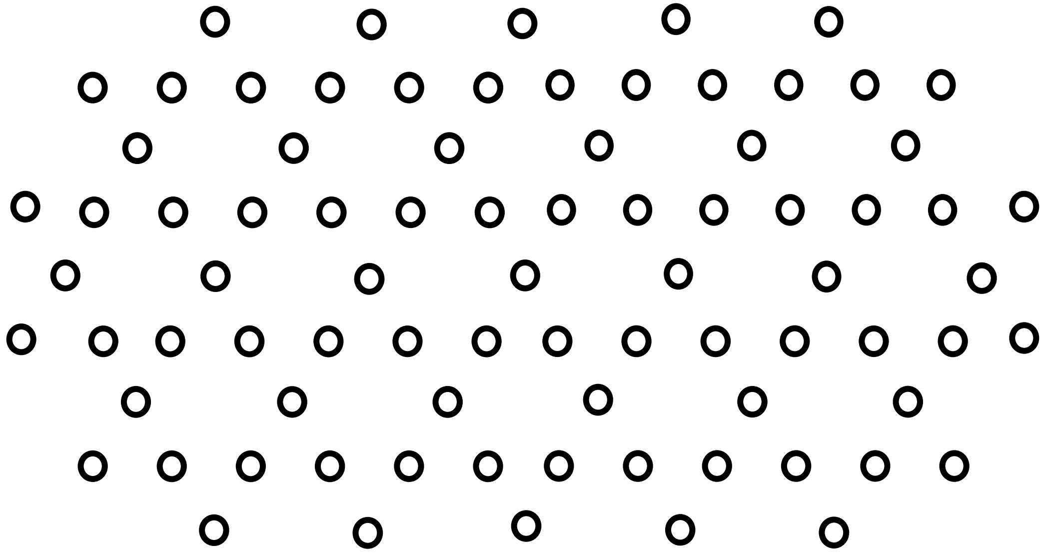
Most promising approach is based on “Abelian anyons.” High error thresholds, but can’t compute purely by braiding.

Bravyi & Kitaev, 2003
Raussendorf & Harrington, PRL 2007
Fowler, Stephens, and Groszkowski, PRA 2009

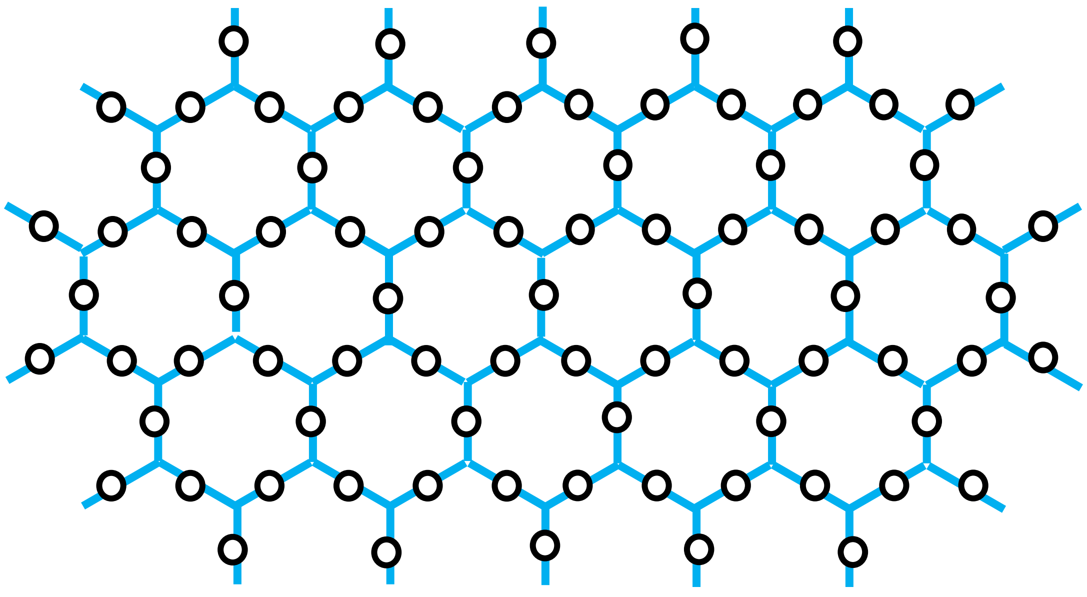
More speculative idea: simulate Fibonacci anyons.

Konig, Kuperberg, Reichardt, Ann. Phys. 2010

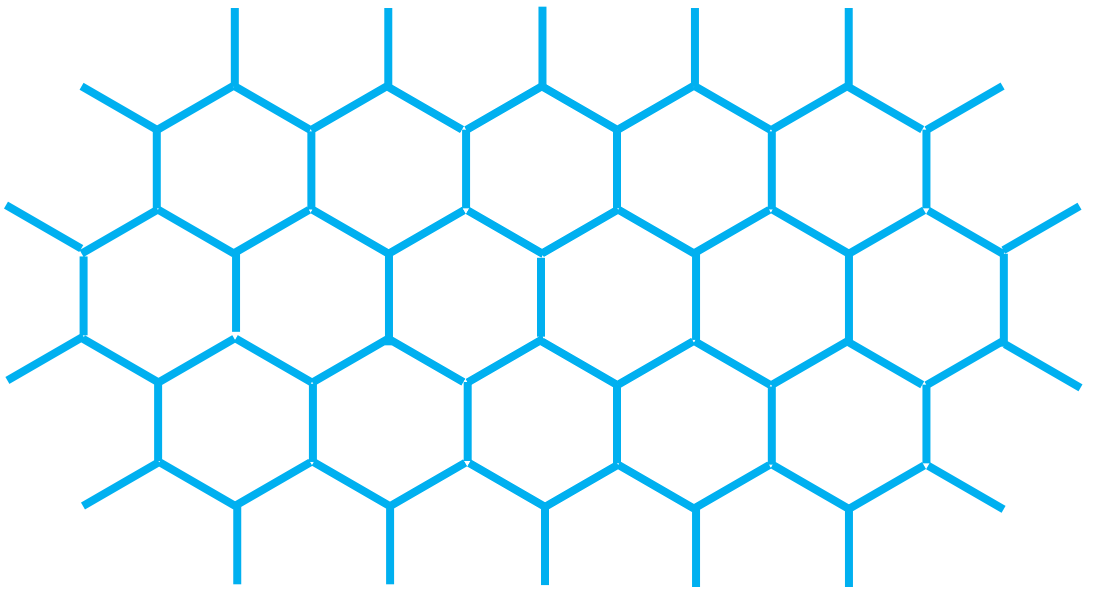
NEB, D.P. DiVincenzo, PRB 2012



2D Array of Qubits



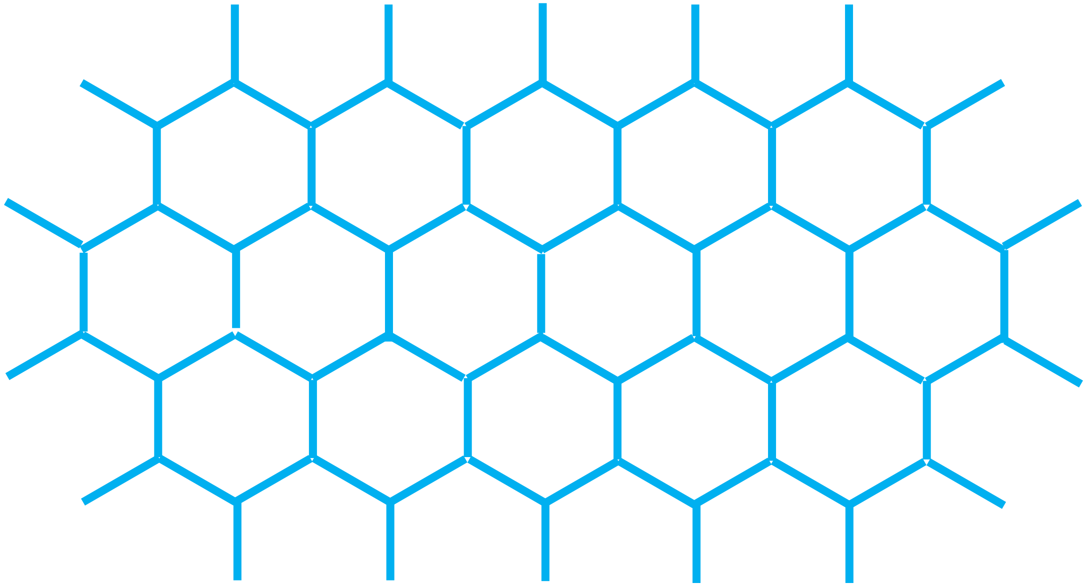
Trivalent Lattice



Trivalent Lattice

"Fibonacci" Levin-Wen Model

Levin & Wen, PRB 2005

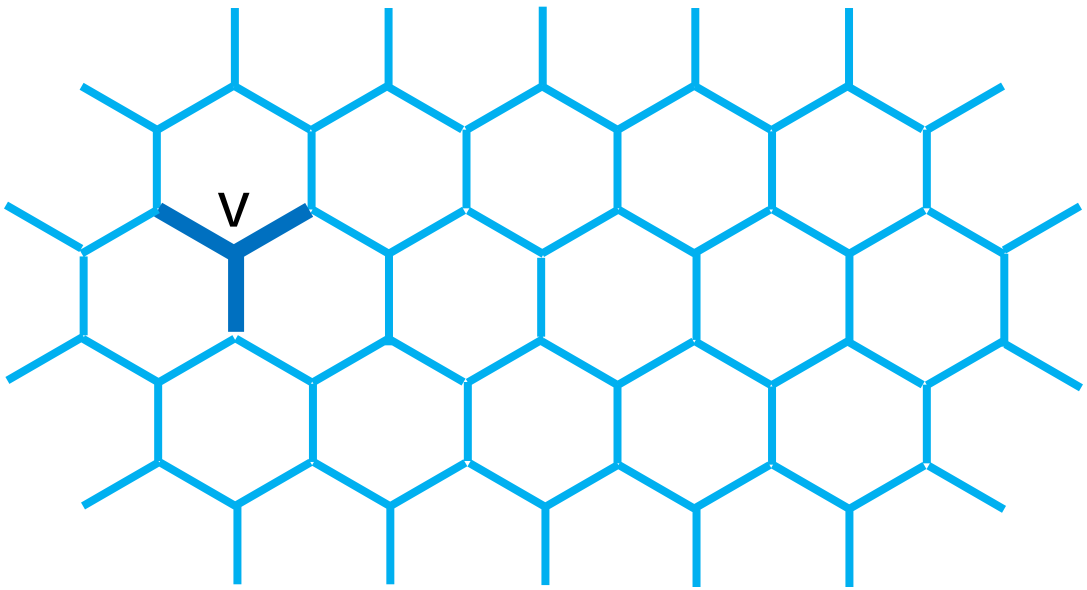


Trivalent Lattice

$$H = -\sum_{v} Q_v - \sum_{p} B_p$$

"Fibonacci" Levin-Wen Model

Levin & Wen, PRB 2005



Trivalent Lattice

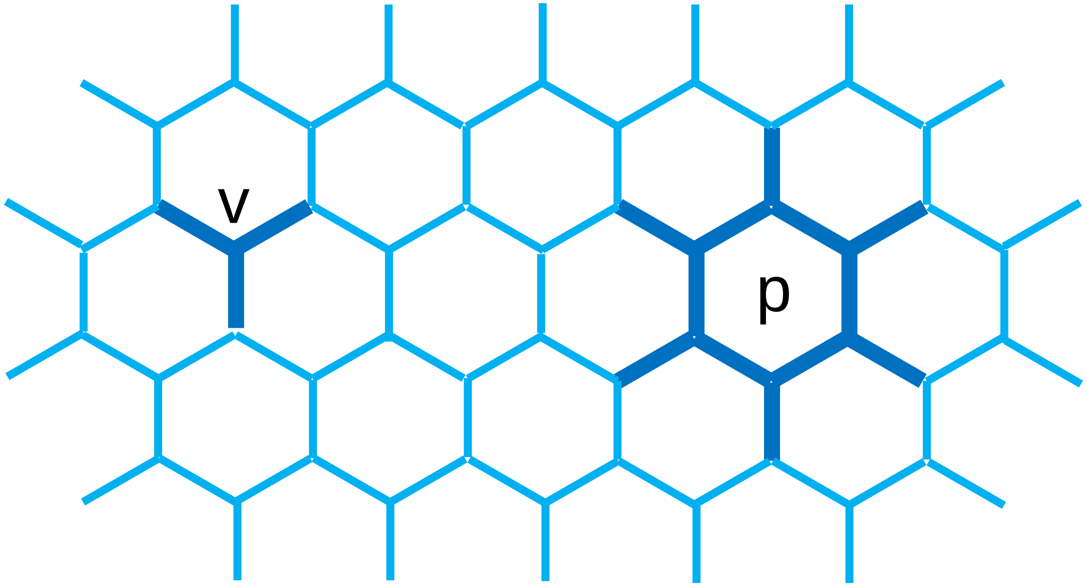
$$H = -\sum_{\mathrm{v}} Q_{\mathrm{v}} - \sum_{\mathrm{p}} B_{\mathrm{p}}$$

Vertex Operator

$Q_{\mathrm{v}} = 0,1$

“Fibonacci” Levin-Wen Model

Levin & Wen, PRB 2005



Trivalent Lattice

$$H = -\sum_{v} Q_v - \sum_{p} B_p$$

Vertex Operator

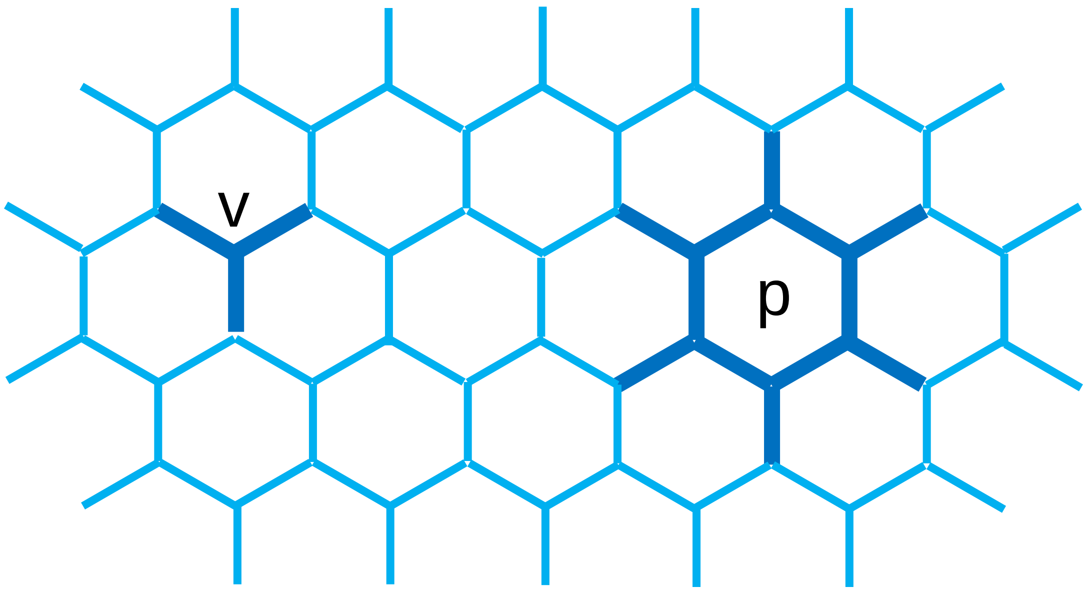
Plaquette Operator

$Q_v = 0,1$

$B_p = 0,1$

"Fibonacci" Levin-Wen Model

Levin & Wen, PRB 2005



Trivalent Lattice

$$H = -\sum_{v} Q_v - \sum_{p} B_p$$

Vertex Operator

Plaquette Operator

$Q_v = 0,1$

$B_p = 0,1$

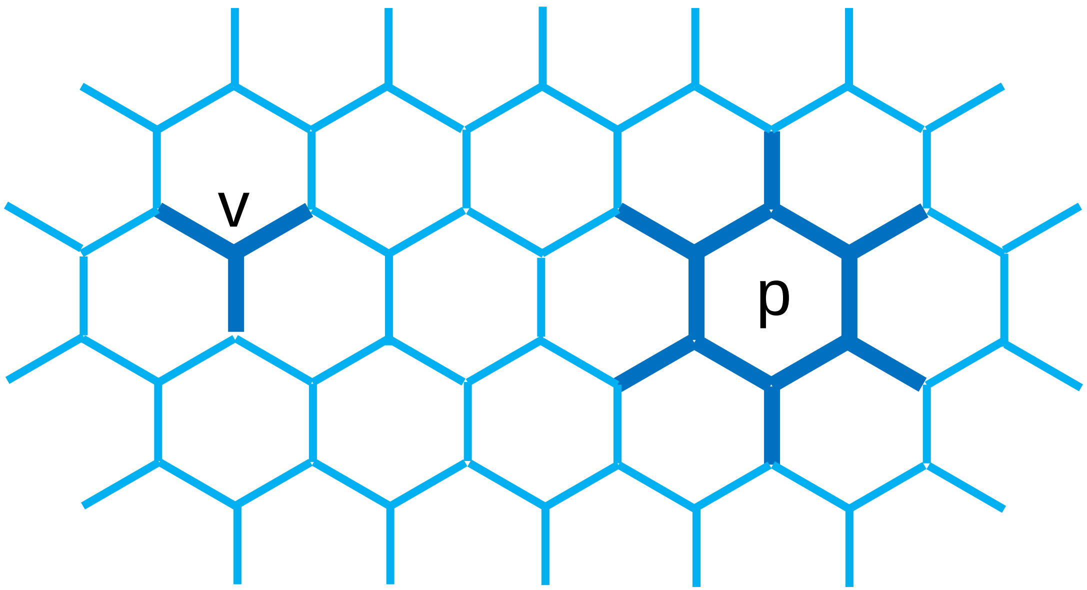
Ground State

$Q_v = 1$ on each vertex

$B_p = 1$ on each plaquette

“Fibonacci” Levin-Wen Model

Levin & Wen, PRB 2005



Trivalent Lattice

$$H = -\sum_{\mathrm{v}} Q_{\mathrm{v}} - \sum_{\mathrm{p}} B_{\mathrm{p}}$$

Vertex Operator

$Q_{\mathrm{v}} = 0,1$

Plaquette Operator

$B_{\mathrm{p}} = 0,1$

Ground State

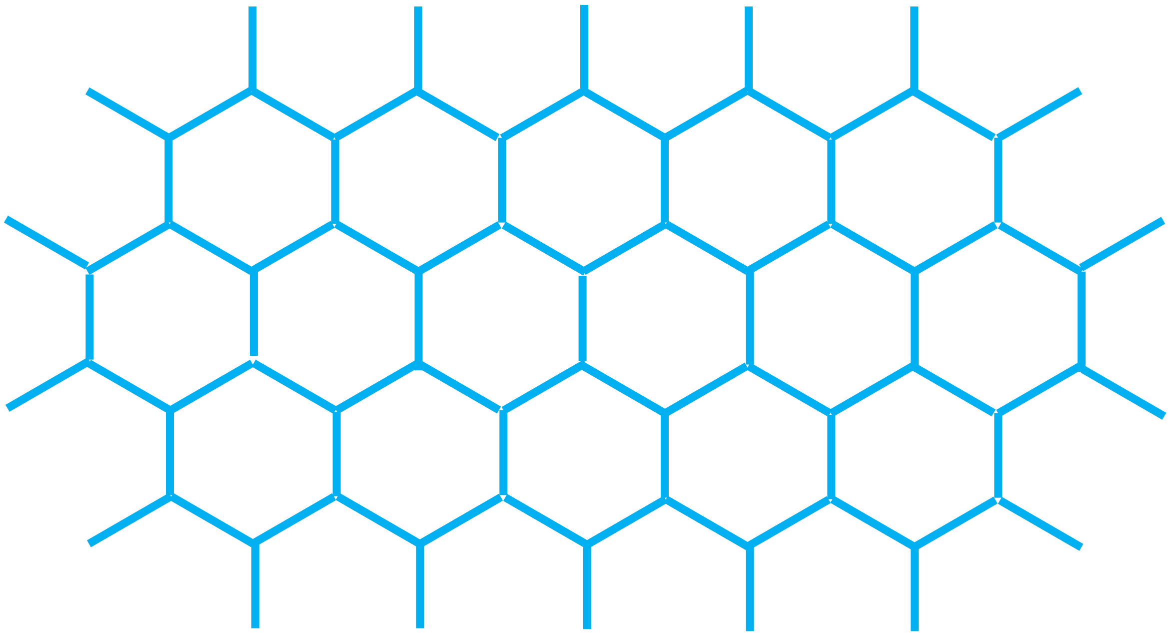
Q_{v} = 1 on each vertex

B_{p} = 1 on each plaquette

Excited States are Fibonacci Anyons

Vertex Operator: Q_v

$$H = -\sum_v Q_v - \sum_p B_p$$



$$Q_v \left| \; i \; \underset{k}{\overset{j}{\succ}} \mathbf{v} \right\rangle = \delta_{ijk} \left| \; i \; \underset{k}{\overset{j}{\succ}} \mathbf{v} \right\rangle$$

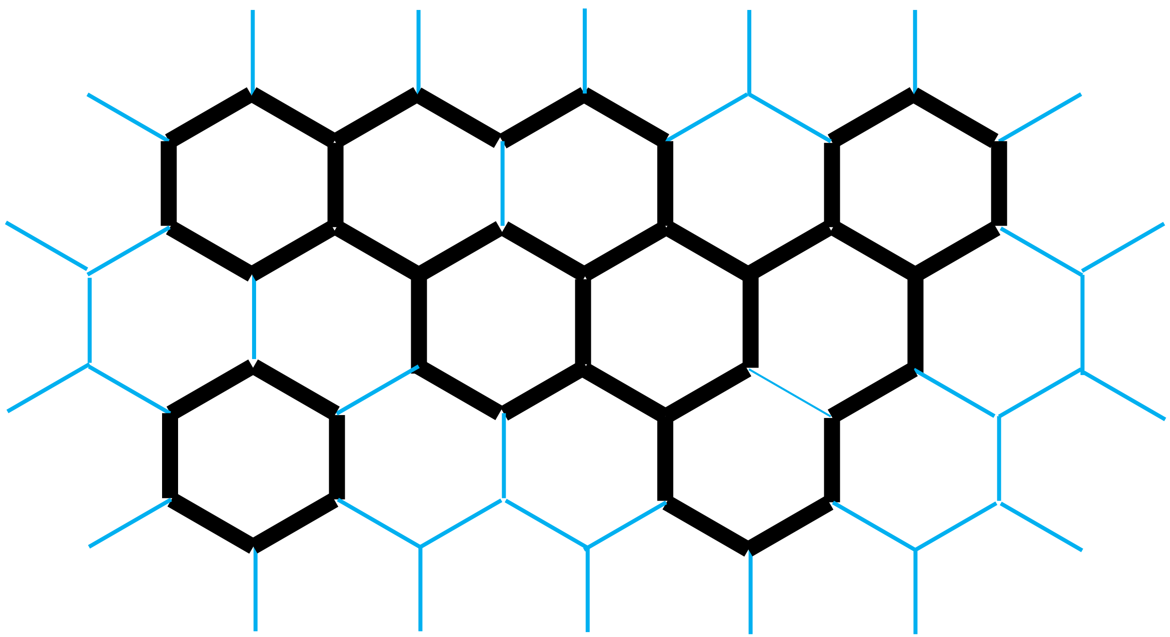
“Fibonacci” Levin-Wen Model

$$\delta_{100} = \delta_{010} = \delta_{001} = 0$$

All other $\delta_{ijk} = 1$

Vertex Operator: Q_v

$$H = -\sum_v Q_v - \sum_p B_p$$

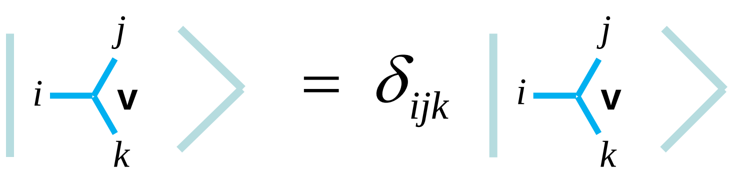


$= |0\rangle$ (thin line)

$= |1\rangle$ (thick line)

$Q_v = 1$
on each vertex
→ "branching" loop states.

$$Q_v \left| \; \right\rangle = \delta_{ijk} \left| \; \right\rangle$$



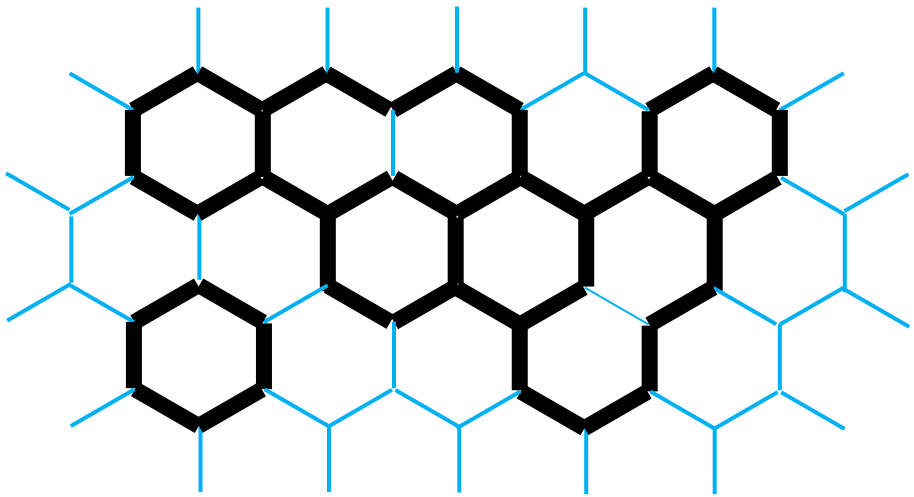
"Fibonacci" Levin-Wen Model

$$\delta_{100} = \delta_{010} = \delta_{001} = 0$$

All other $\delta_{ijk} = 1$

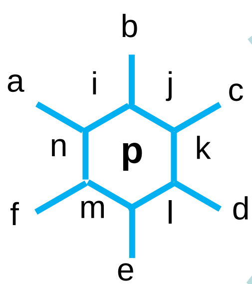
Plaquette Operator: B_{p}

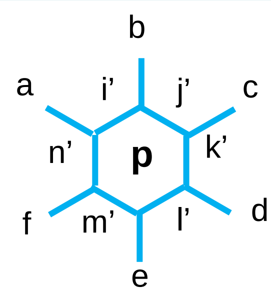
$$H = -\sum_{\mathrm{v}} Q_{\mathrm{v}} - \sum_{\mathrm{p}} B_{\mathrm{p}}$$



$B_{\mathrm{p}} = 1$
on each plaquette
→ superposition
of loop states

$$B_{\mathbf{p}}^{s}\left|\;\cdot\;\right\rangle = \sum_{i'j'k'l'm'n'} B^{s,i'j'k'l'm'n'}_{\mathbf{p},ijklmn}(abcdef)\left|\;\cdot\;\right\rangle$$





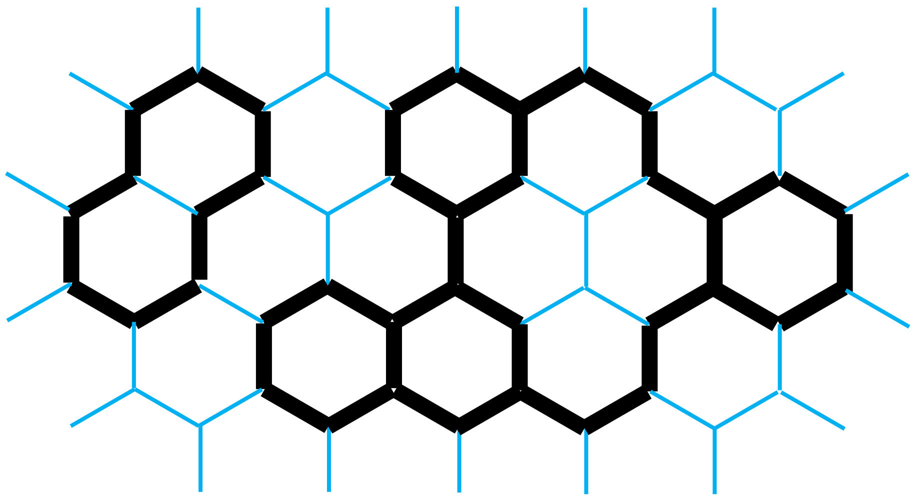
$$B_{\mathbf{p}} = \frac{B^0_{\mathbf{p}} + \varphi B^1_{\mathbf{p}}}{1+\varphi^2}$$

$$B^{s,i'j'k'l'm'n'}_{\mathbf{p},ijklmn}(abcdef) = F^{ani}_{si'n'}\, F^{bij}_{sj'i'}\, F^{cjk}_{sk'j'}\, F^{dkl}_{sl'k'}\, F^{elm}_{sm'l'}\, F^{fmn}_{sn'm'}$$

Very Complicated 12-qubit Interaction!

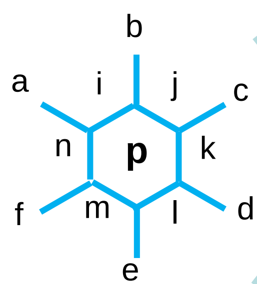
Plaquette Operator: B_p

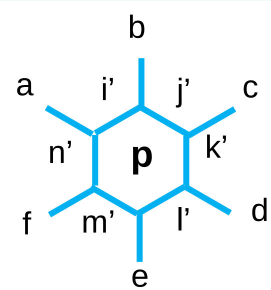
$$H = -\sum_v Q_v - \sum_p B_p$$



$B_p = 1$ on each plaquette ➡ superposition of loop states

$$B_{\mathbf{p}}^{s} \left| \vphantom{\sum} \right. \text{(hexagon diagram, edges } i,j,k,l,m,n \text{)} \left. \vphantom{\sum} \right\rangle = \sum_{i'j'k'l'm'n'} B_{\mathbf{p},ijklmn}^{s,i'j'k'l'm'n'}(abcdef) \left| \vphantom{\sum} \right. \text{(hexagon diagram, edges } i',j',k',l',m',n' \text{)} \left. \vphantom{\sum} \right\rangle$$





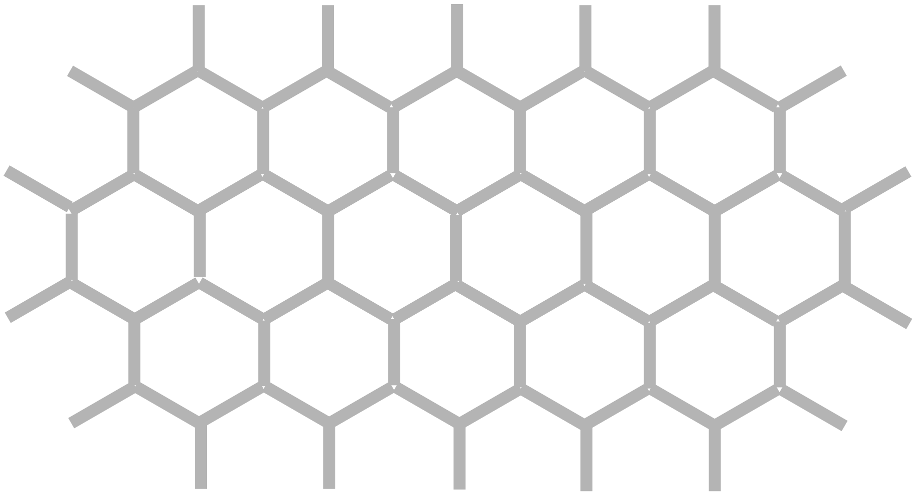
$$B_{\mathbf{p}} = \frac{B_{\mathbf{p}}^{0} + \varphi B_{\mathbf{p}}^{1}}{1+\varphi^{2}}$$

$$B_{\mathbf{p},ijklmn}^{s,i'j'k'l'm'n'}(abcdef) = F_{si'n'}^{ani} F_{sj'i'}^{bij} F_{sk'j'}^{cjk} F_{sl'k'}^{dkl} F_{sm'l'}^{elm} F_{sn'm'}^{fmn}$$

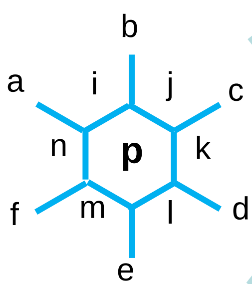
Very Complicated 12-qubit Interaction!

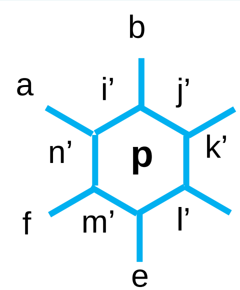
Plaquette Operator: B_{p}

$$H = -\sum_{\mathrm{v}} Q_{\mathrm{v}} - \sum_{\mathrm{p}} B_{\mathrm{p}}$$



$B_{\mathrm{p}} = 1$
on each plaquette
➡ superposition
of loop states

$B_{\mathbf{p}}^{s}$ |


⟩ $= \sum_{i'j'k'l'm'n'} B_{\mathbf{p},ijklmn}^{s,i'j'k'l'm'n'}(abcdef)$ |


⟩

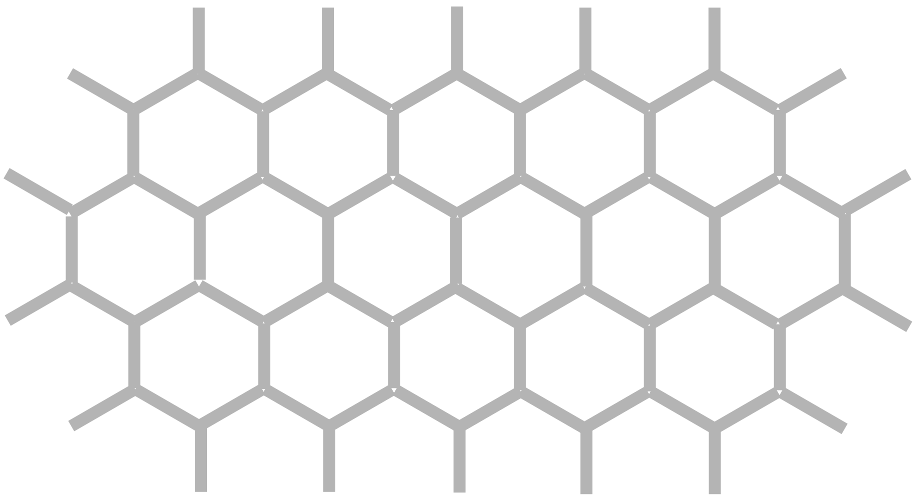
$$B_{\mathbf{p}} = \frac{B_{\mathbf{p}}^{0} + \varphi B_{\mathbf{p}}^{1}}{1+\varphi^{2}}$$

$$B_{\mathbf{p},ijklmn}^{s,i'j'k'l'm'n'}(abcdef) = F_{si'n'}^{ani} F_{sj'i'}^{bij} F_{sk'j'}^{cjk} F_{sl'k'}^{dkl} F_{sm'l'}^{elm} F_{sn'm'}^{fmn}$$

Very Complicated 12-qubit Interaction!

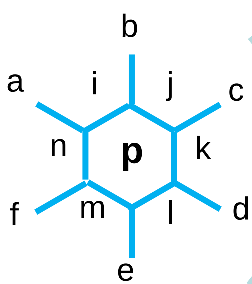
Plaquette Operator: B_p

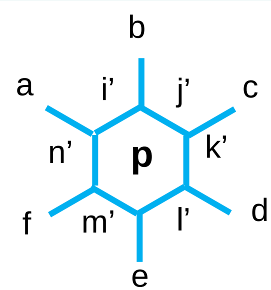
$$H = -\sum_v Q_v - \sum_p B_p$$



$B_p = 1$
on each plaquette
➡ superposition
of loop states

$$B_{\mathbf{p}}^{s}\left|\ \right\rangle = \sum_{i'j'k'l'm'n'} B_{\mathbf{p},ijklmn}^{s,i'j'k'l'm'n'}(abcdef)\left|\ \right\rangle$$





$$B_{\mathbf{p}} = \frac{B_{\mathbf{p}}^{0} + \varphi B_{\mathbf{p}}^{1}}{1+\varphi^{2}}$$

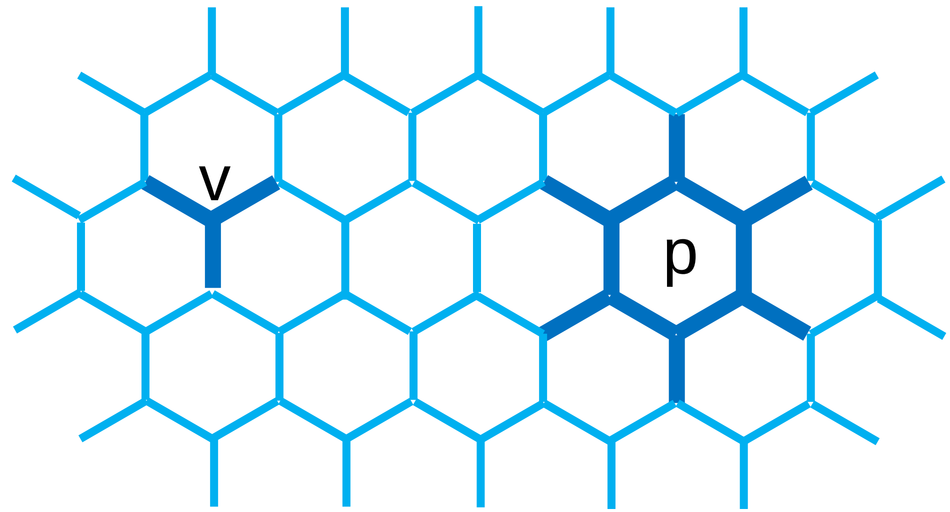
$$B_{\mathbf{p},ijklmn}^{s,i'j'k'l'm'n'}(abcdef) = F_{si'n'}^{ani} F_{sj'i'}^{bij} F_{sk'j'}^{cjk} F_{sl'k'}^{dkl} F_{sm'l'}^{elm} F_{sn'm'}^{fmn}$$

Very Complicated 12-qubit Interaction!

"Surface Code" Approach

Konig, Kuperberg, Reichardt, Ann. Phys. 2010

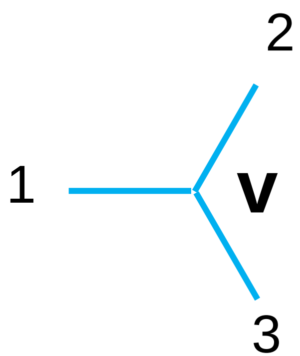
Use quantum computer to repeatedly measure Q_v and B_p.

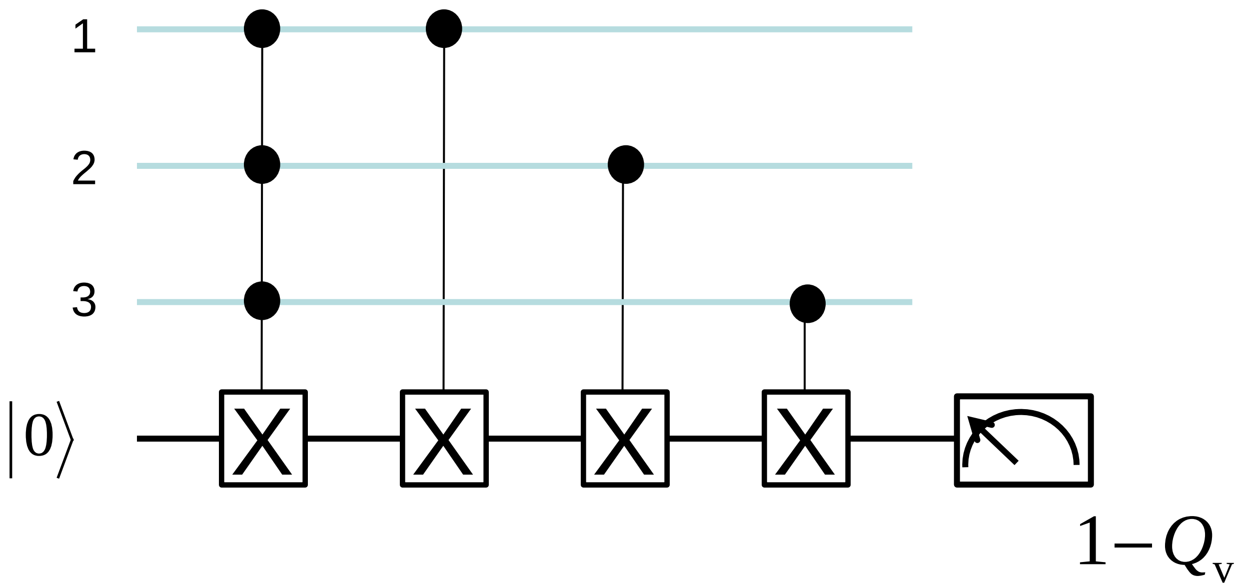


If Q_v=1 on each vertex and B_p=1 on each plaquette, good. If not, treat as an "error."

How hard is it to measure Q_v and B_p with a quantum computer?

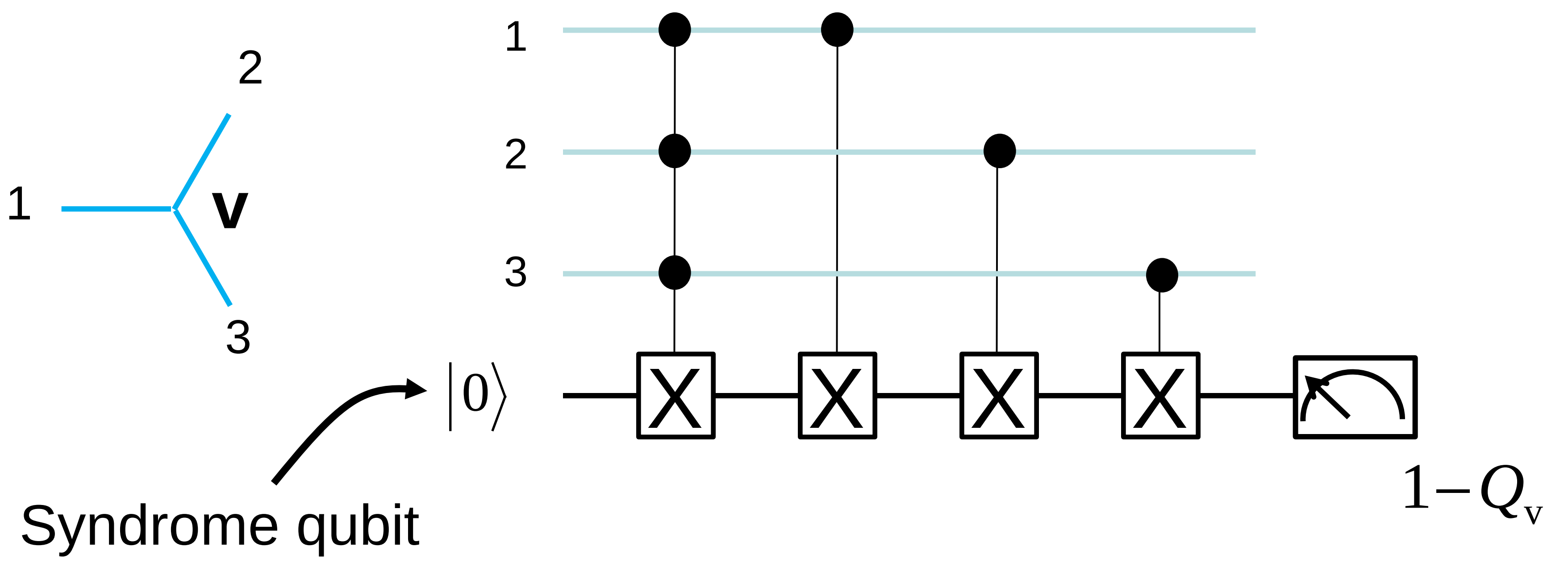
Quantum Circuit for Measuring Q_v





NEB, D.P. DiVincenzo, PRB 2012

Quantum Circuit for Measuring Q_v



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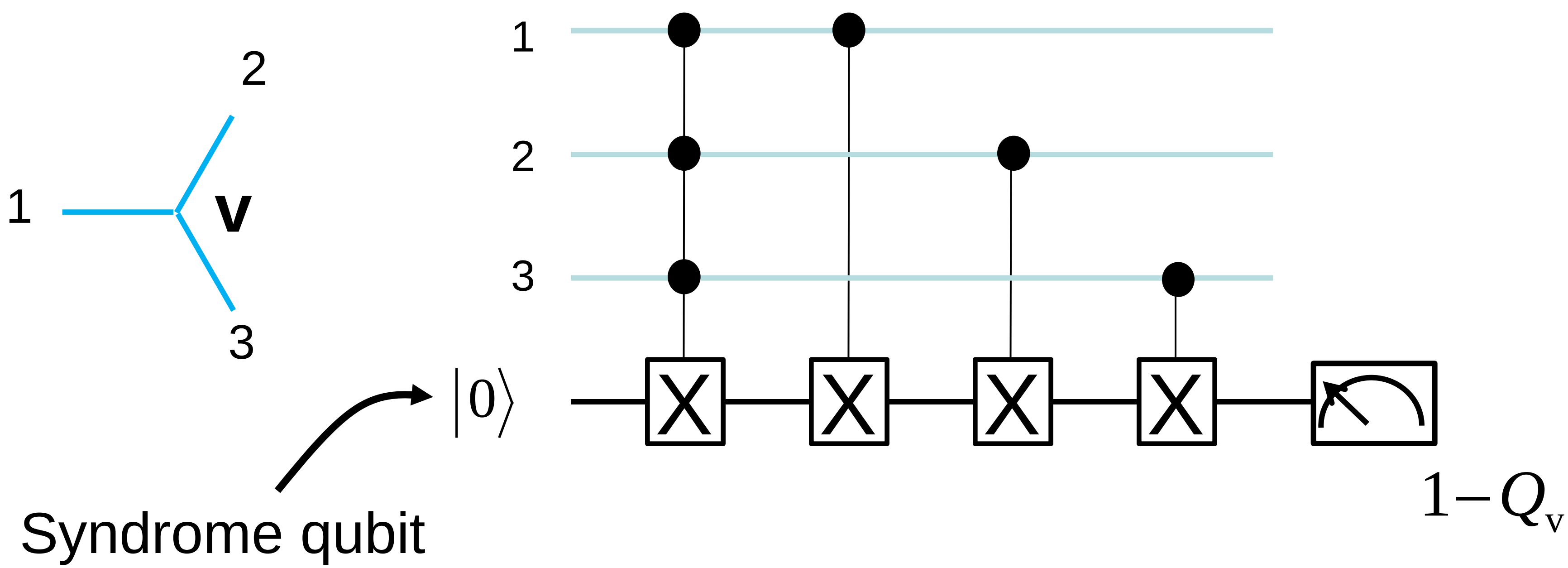
Quantum Circuit for Measuring Q_v



That was easy! What about $B_{\mathbf{p}}$?

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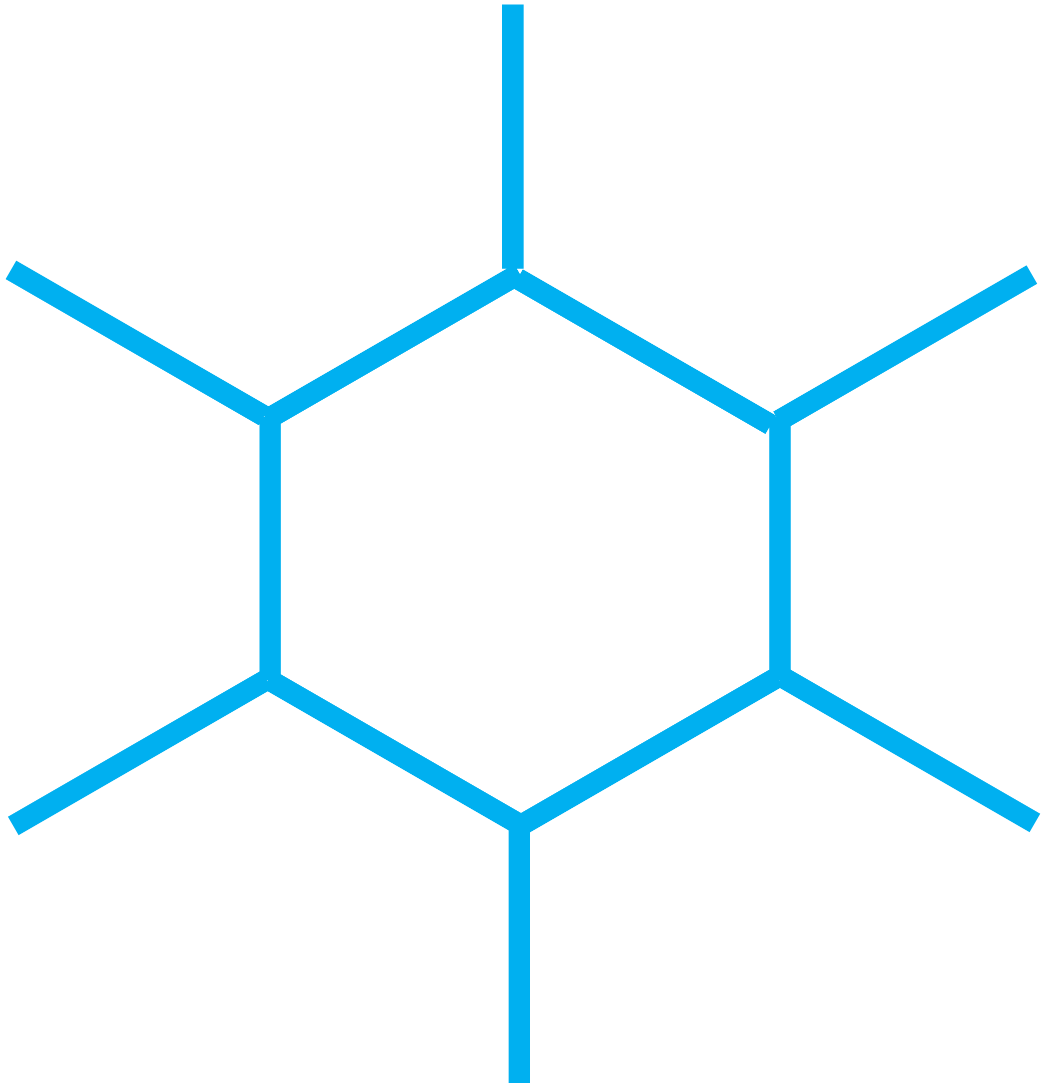
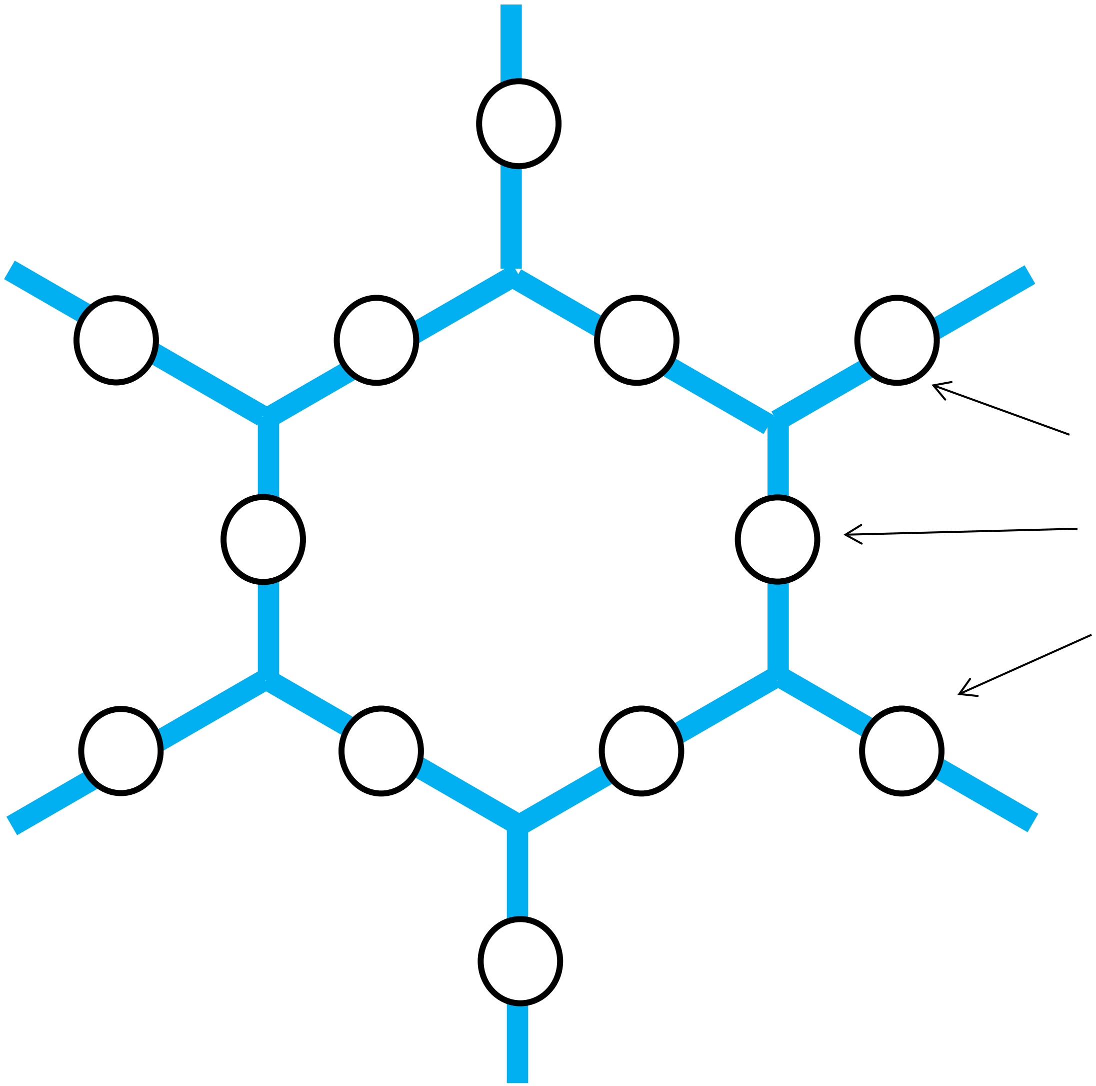
6-sided Plaquette: B_p is a 12-qubit Operator

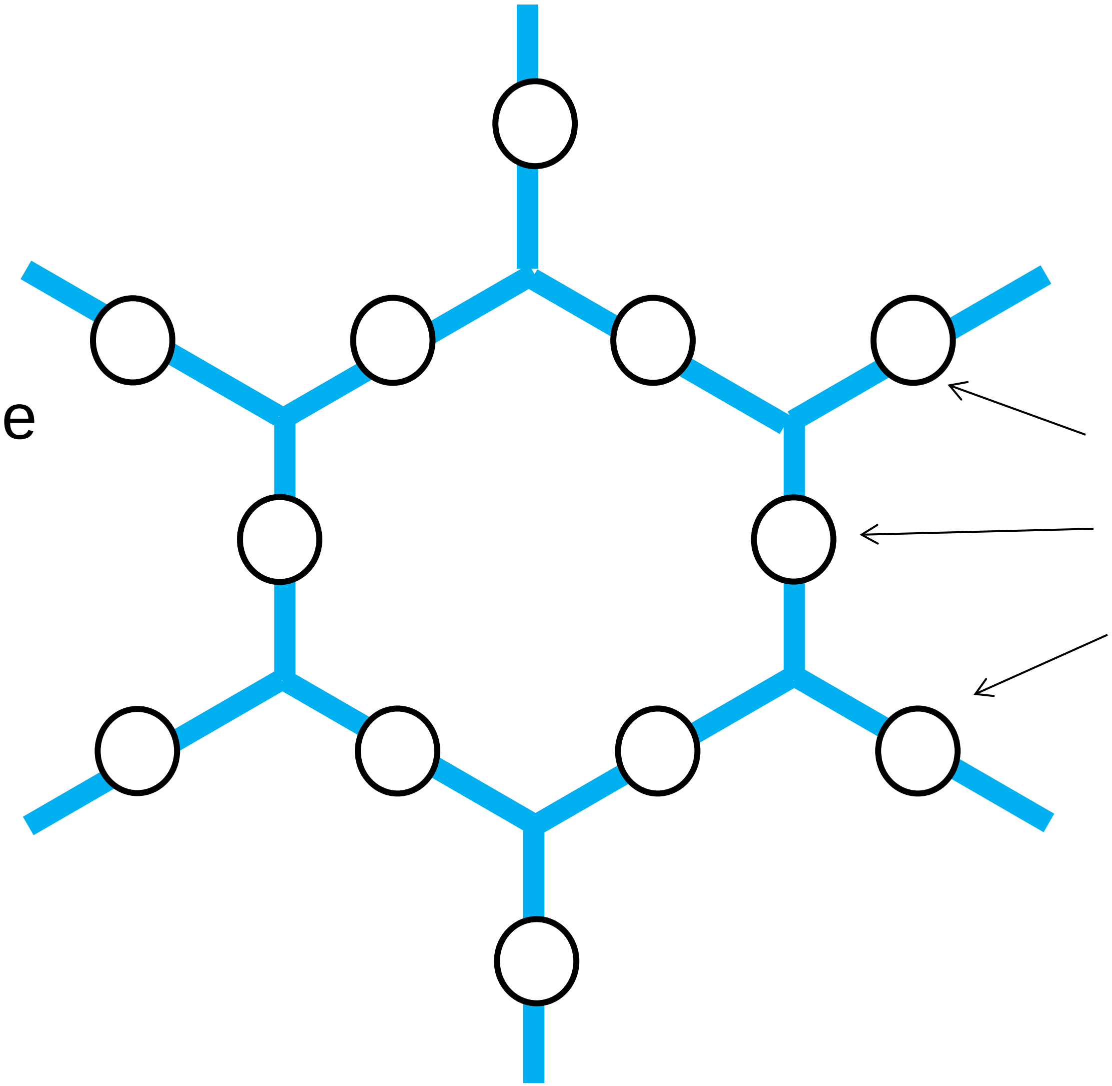


6-sided Plaquette: B_p is a 12-qubit Operator



Physical qubits are fixed in space

6-sided Plaquette: B_p is a 12-qubit Operator

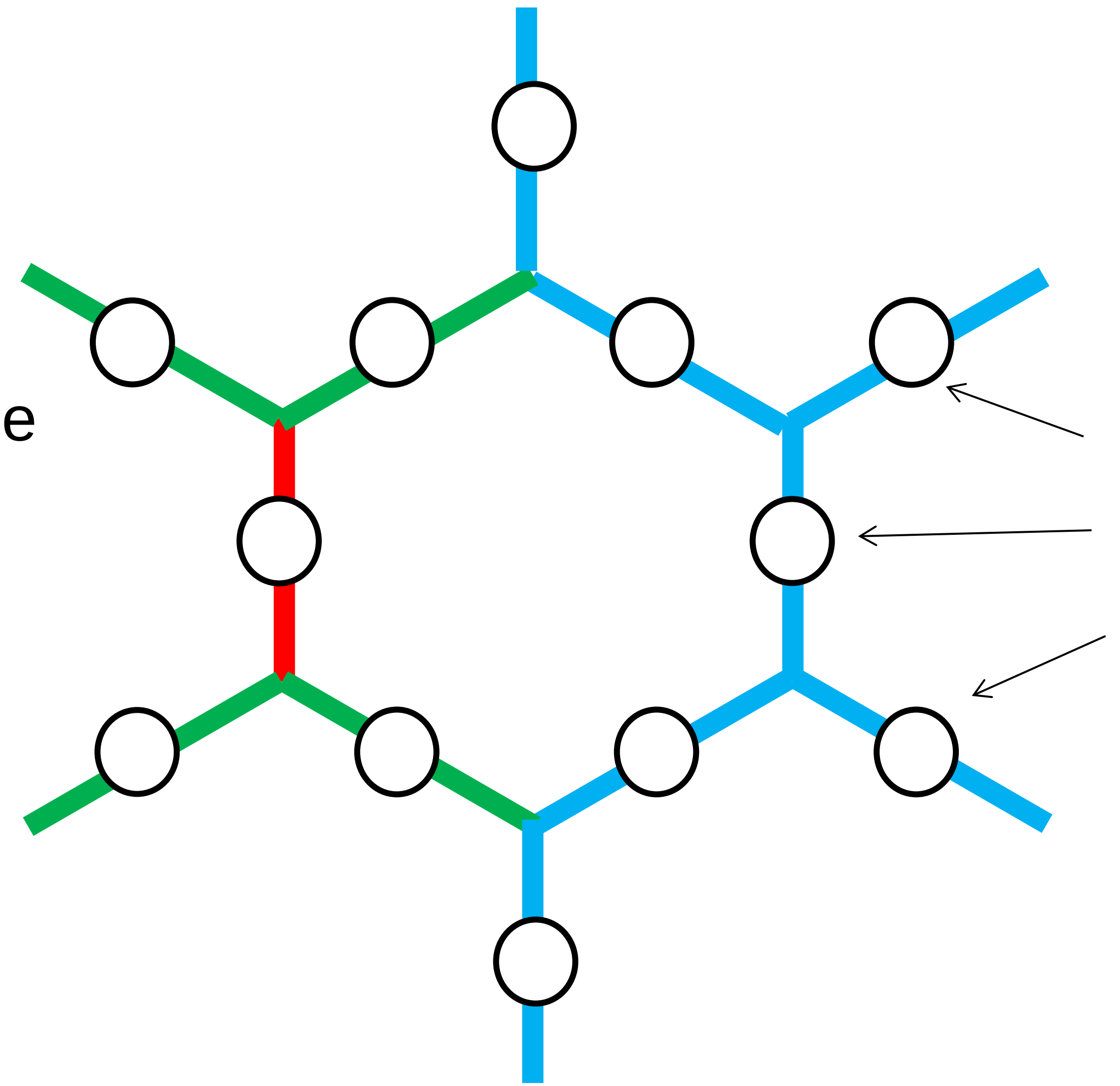


Trivalent lattice is abstract and can be “redrawn”

Physical qubits are fixed in space

6-sided Plaquette: B_p is a 12-qubit Operator

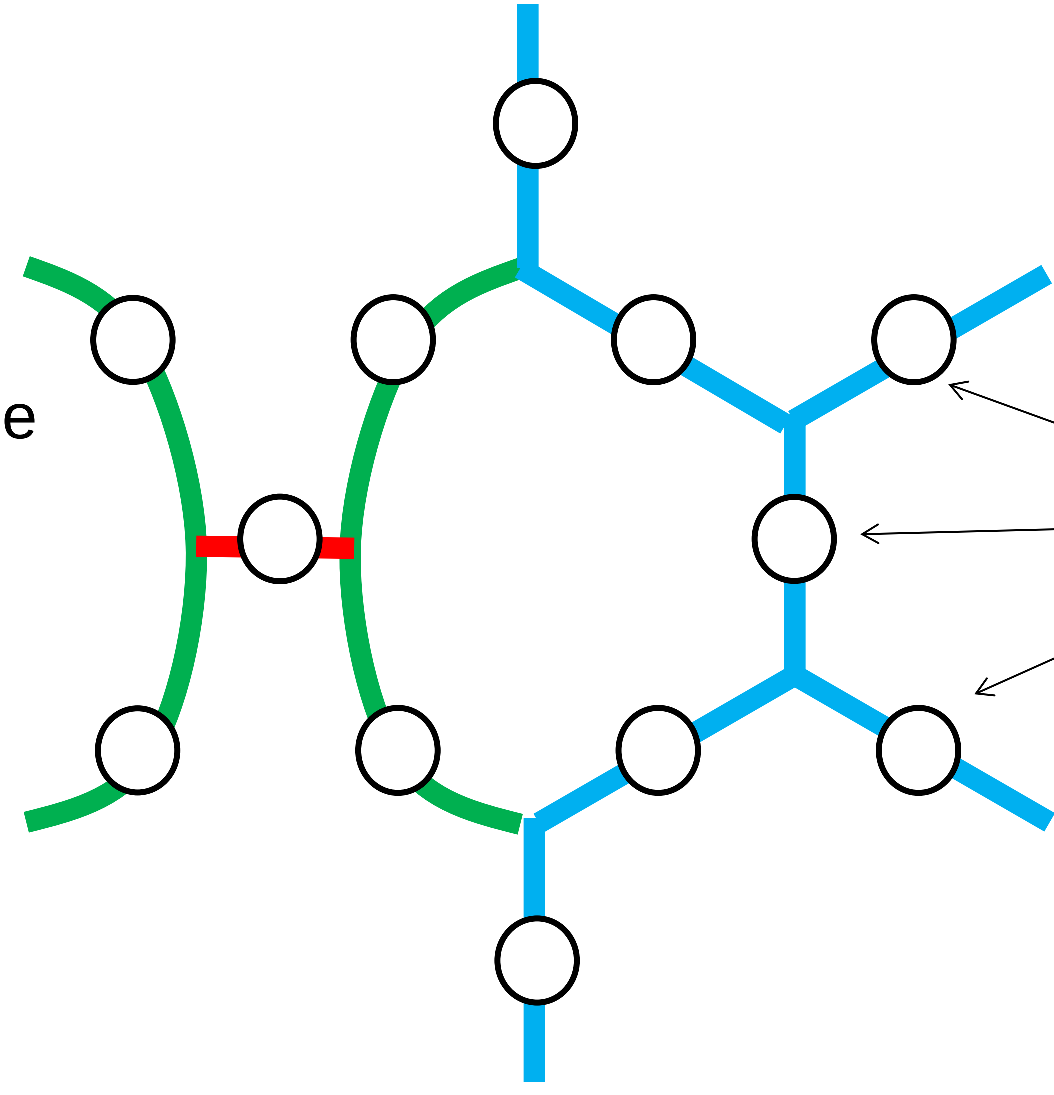
Trivalent lattice is abstract and can be "redrawn"



Physical qubits are fixed in space

6-sided Plaquette: B_p is a 12-qubit Operator

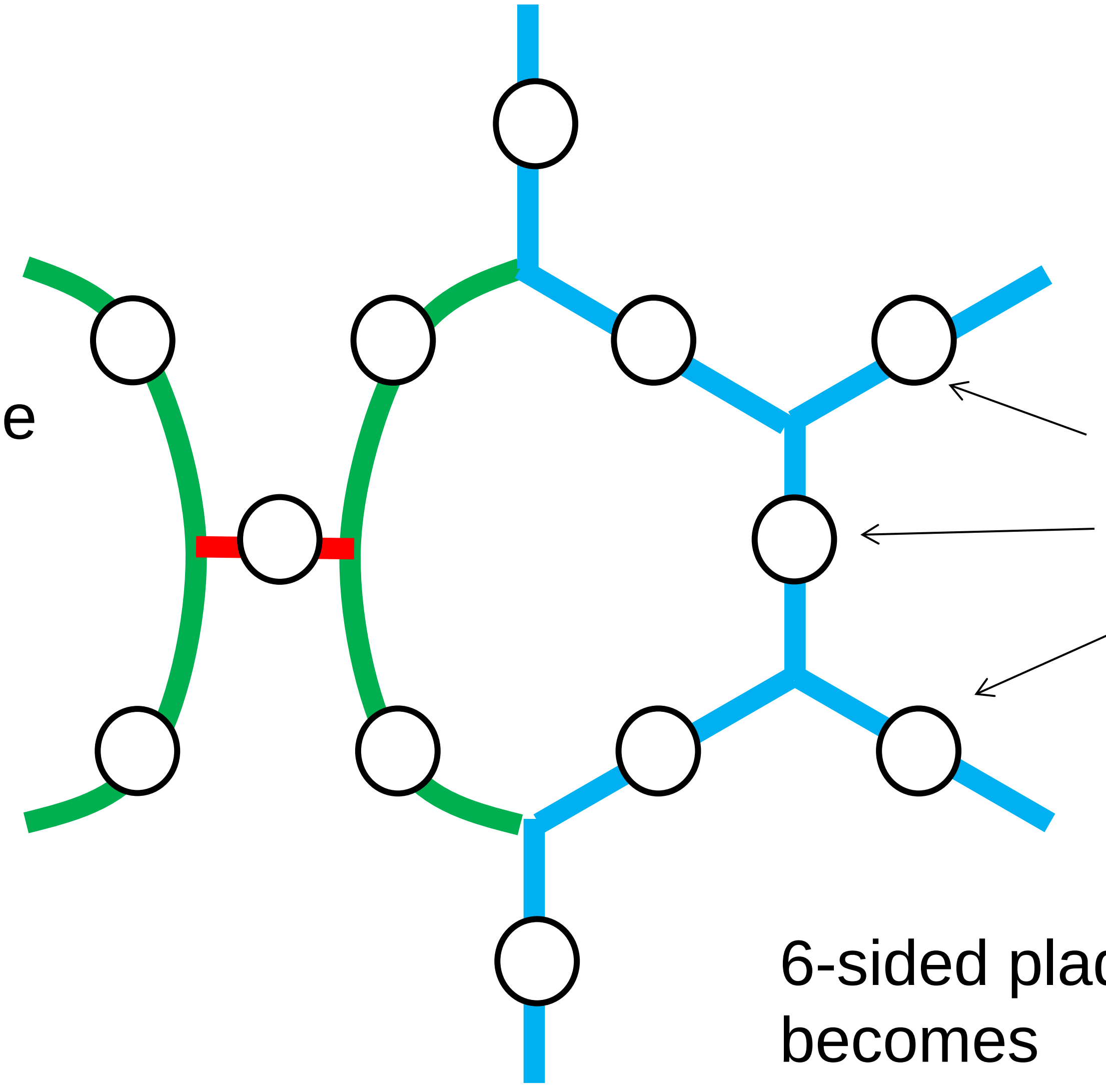
Trivalent lattice is abstract and can be "redrawn"



Physical qubits are fixed in space

6-sided Plaquette: B_p is a 12-qubit Operator

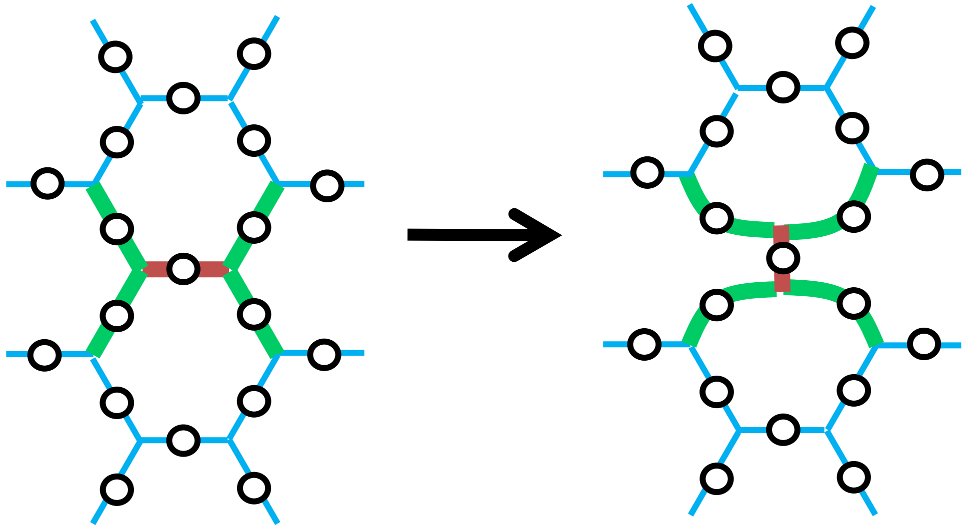
Trivalent lattice is abstract and can be "redrawn"



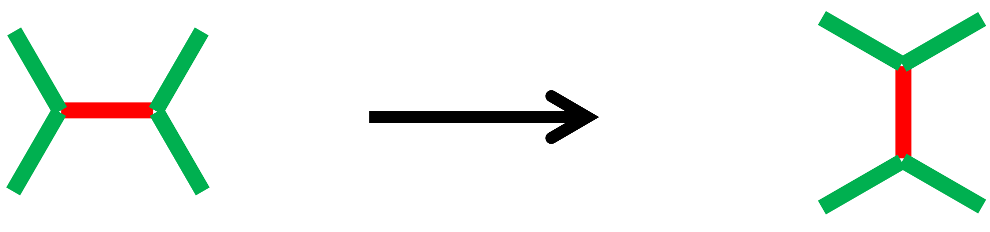
Physical qubits are fixed in space

6-sided plaquette becomes 5-sided plaquette!

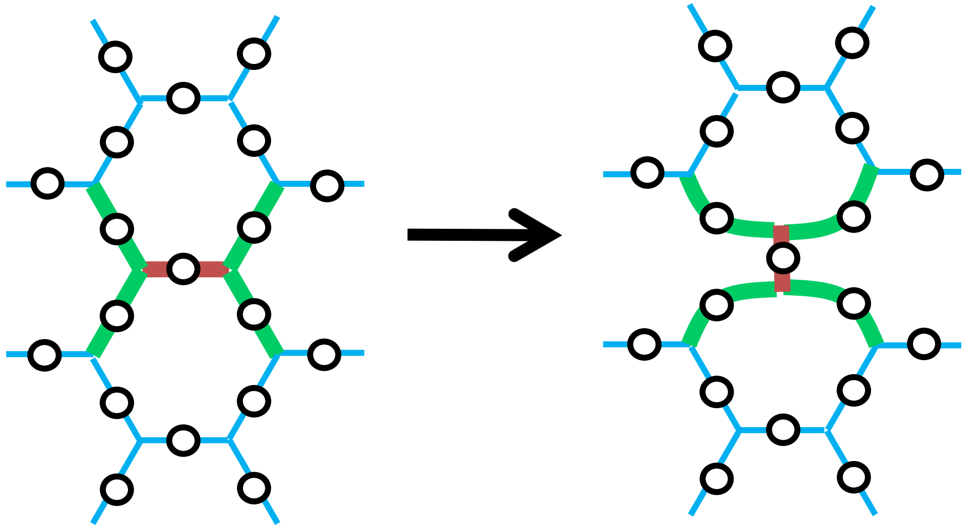
The F–Move



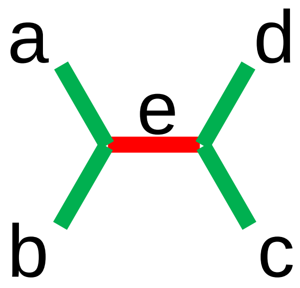
Locally redraw the lattice five qubits at a time.



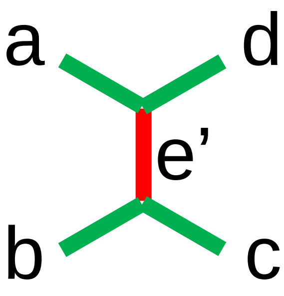
The F–Move



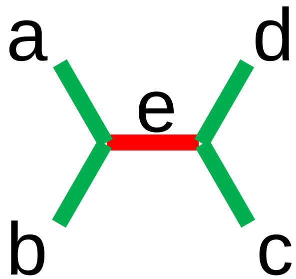
Apply a unitary operation on these five qubits to stay in the Levin-Wen ground state.



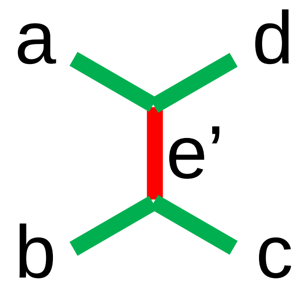
$$\sum_{e'} F^{abe}_{cde'}$$

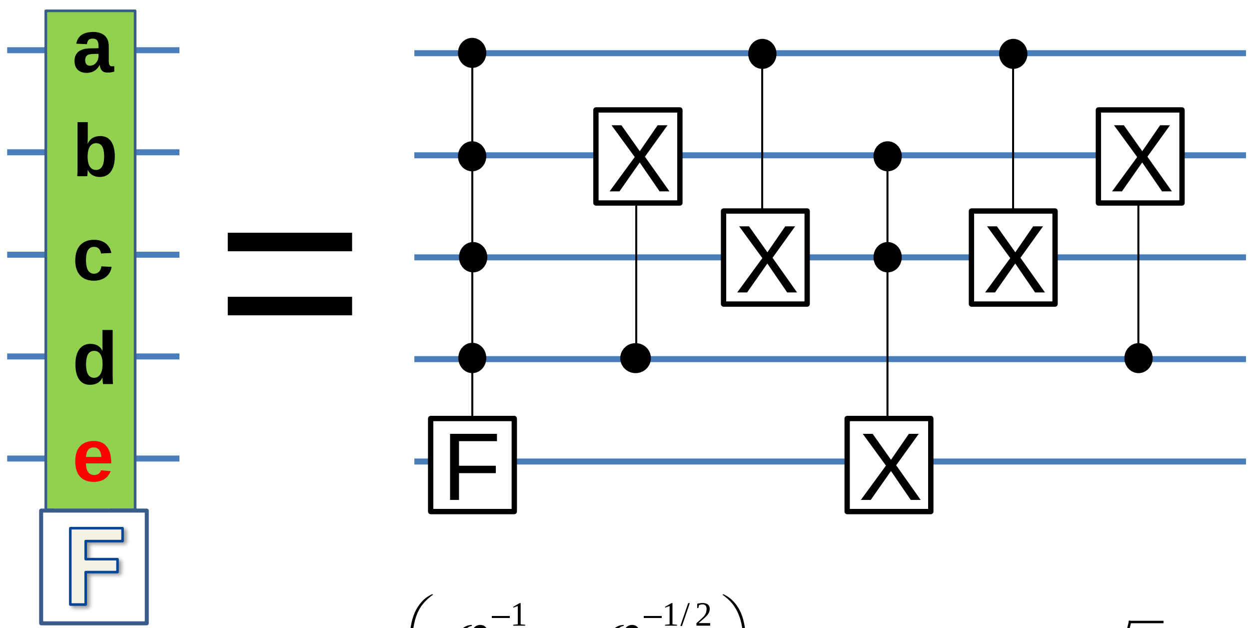


F Quantum Circuit



$$\rightarrow \sum_{e'} F^{abe}_{cde'}$$

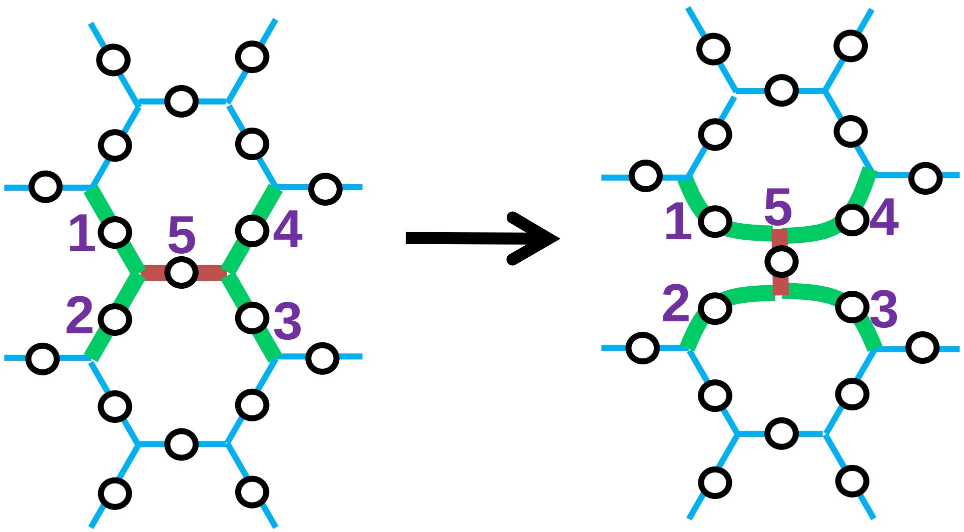


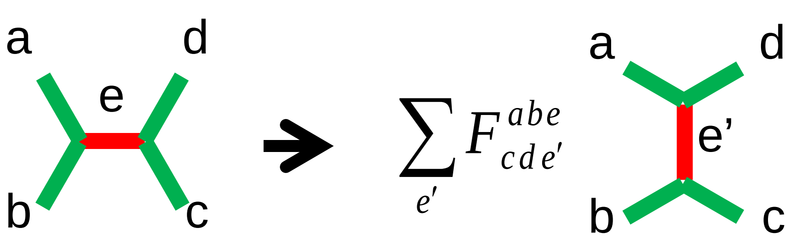


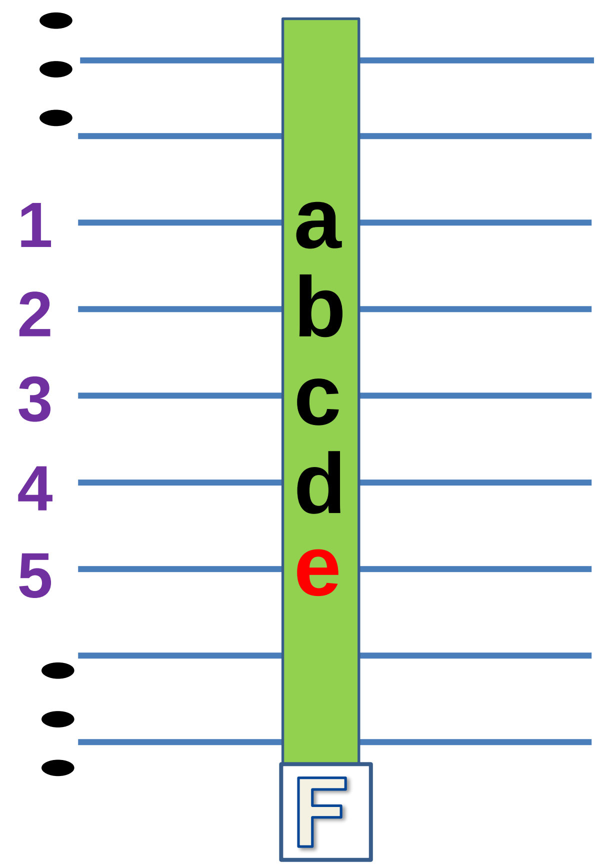
$$F = \begin{pmatrix} \varphi^{-1} & \varphi^{-1/2} \\ \varphi^{-1/2} & -\varphi^{-1} \end{pmatrix} \qquad \varphi = \frac{\sqrt{5}+1}{2}$$

NEB, D.P. DiVincenzo, PRB 2012

F Quantum Circuit

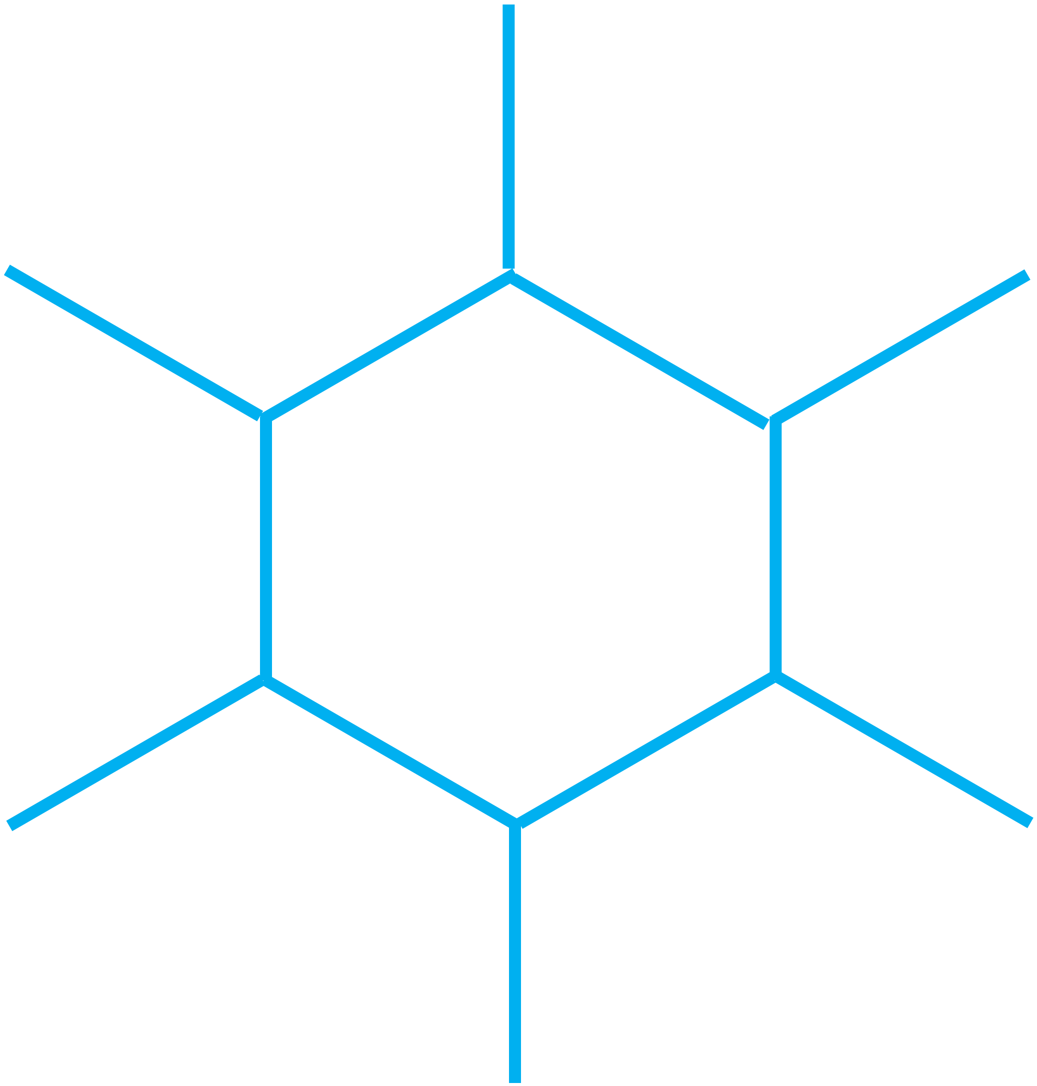




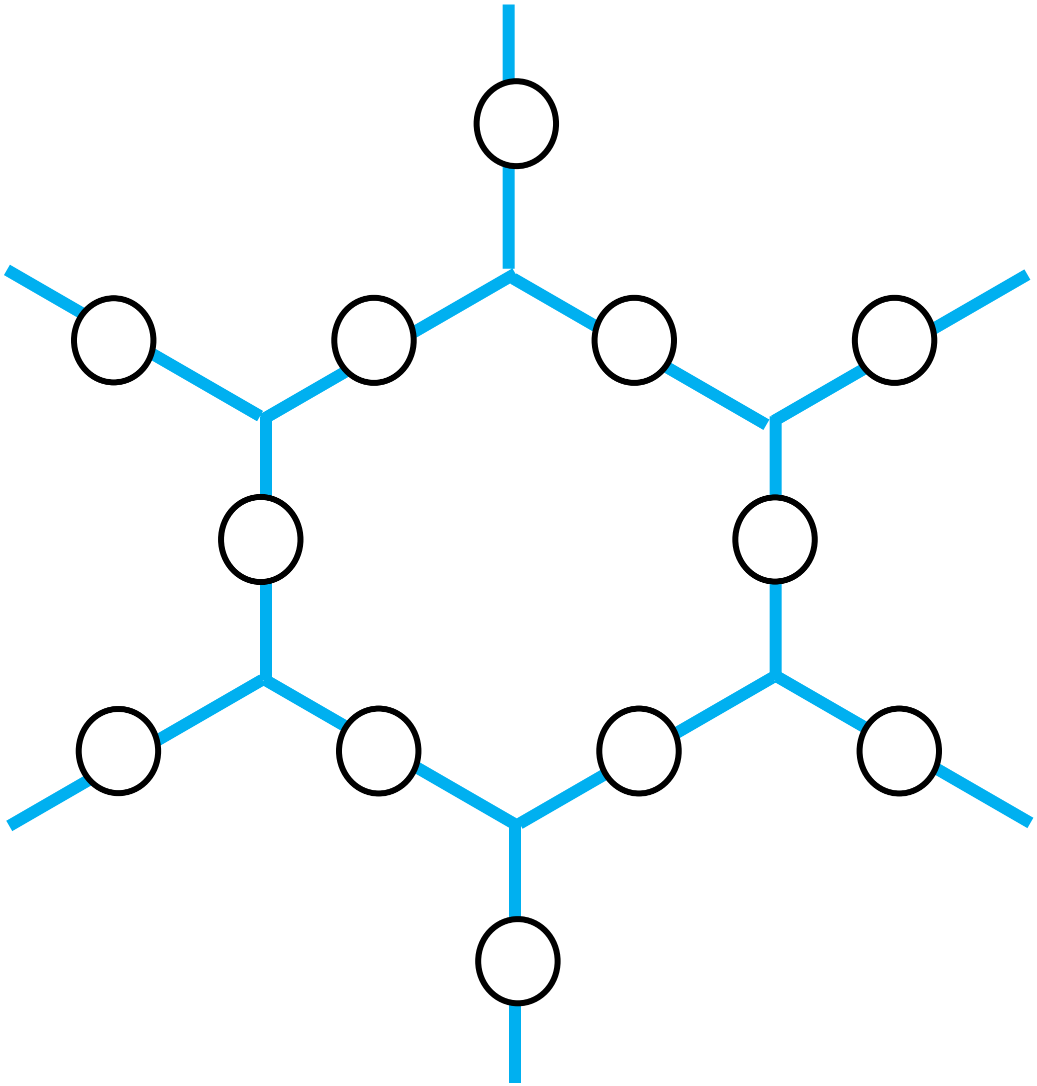


NEB, D.P. DiVincenzo, PRB 2012

Plaquette Reduction



Plaquette Reduction



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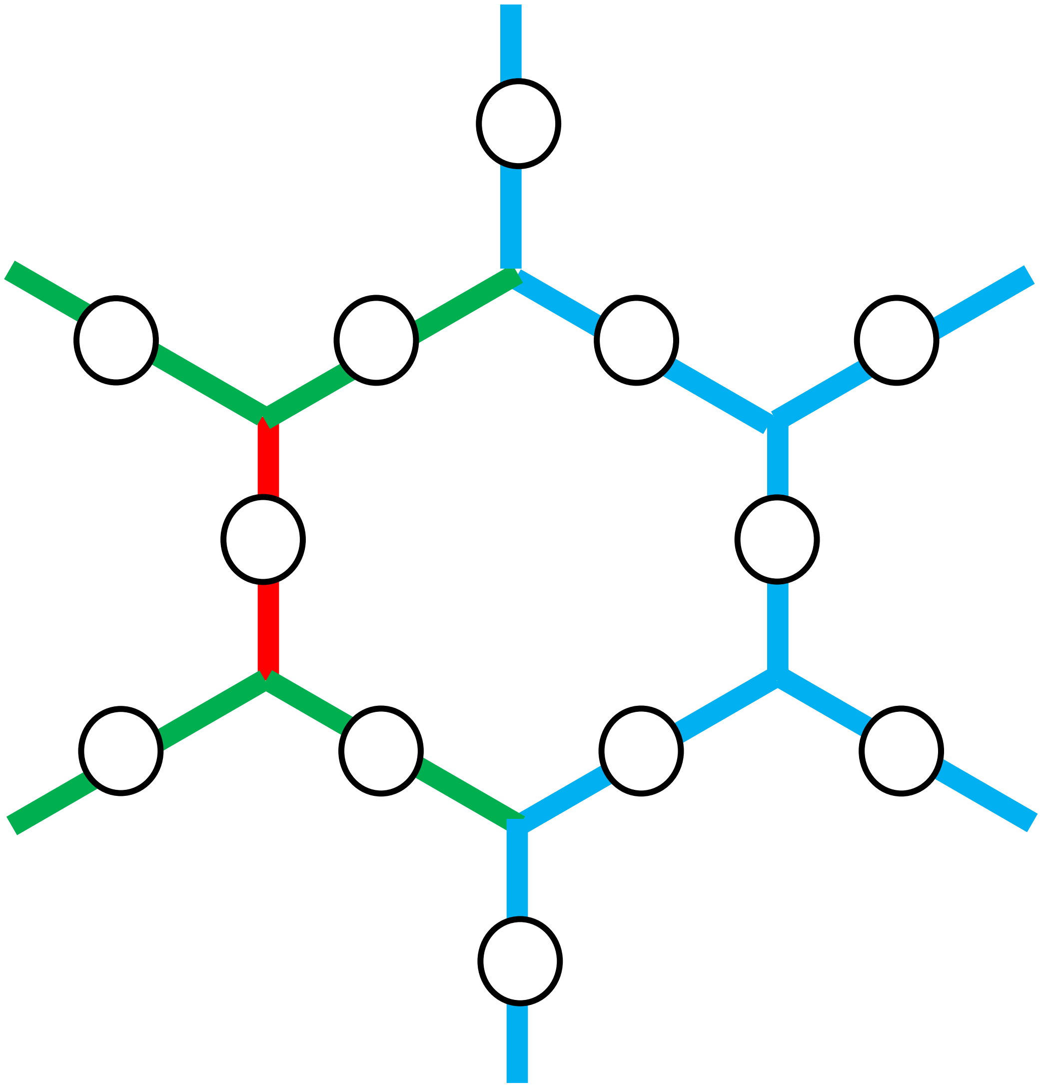

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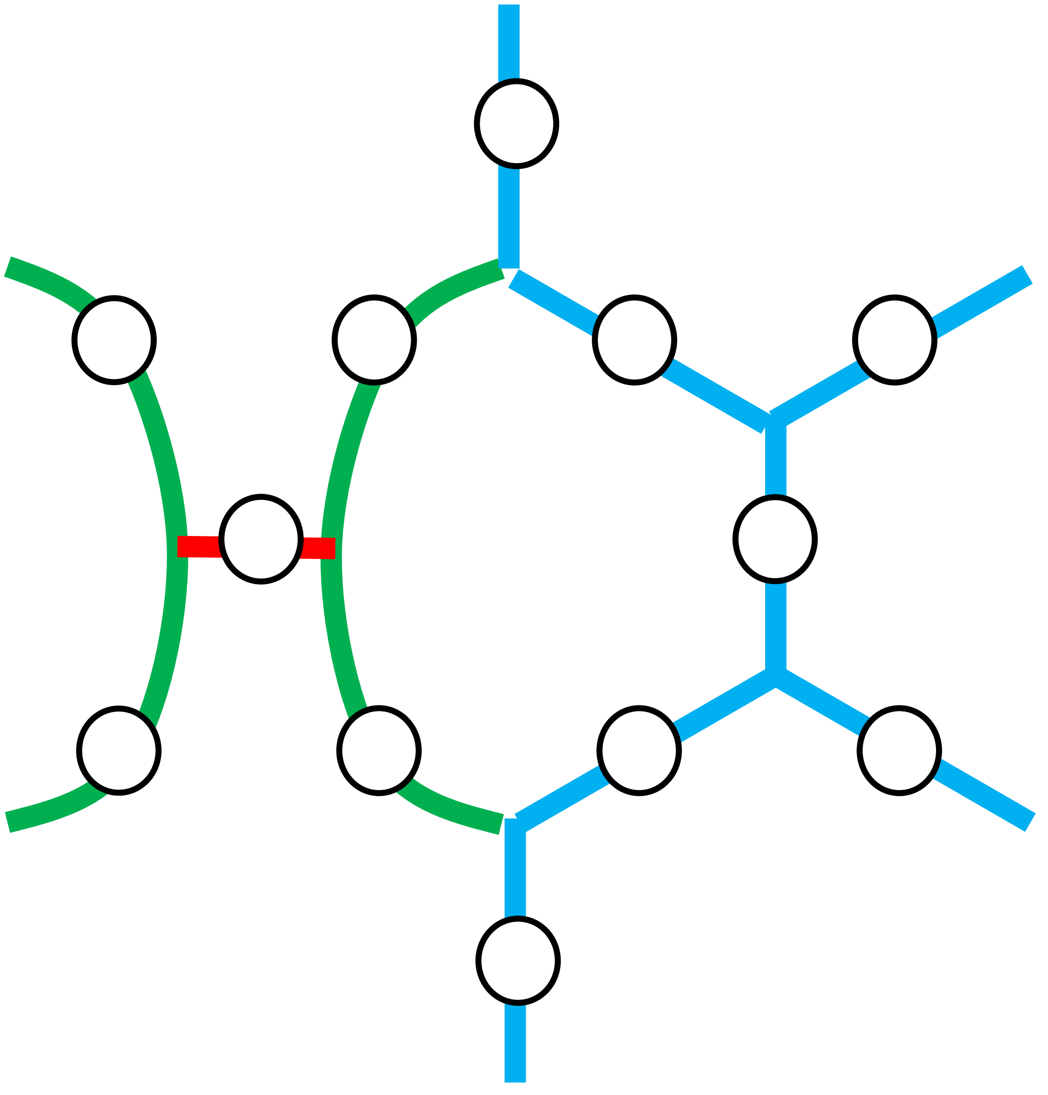

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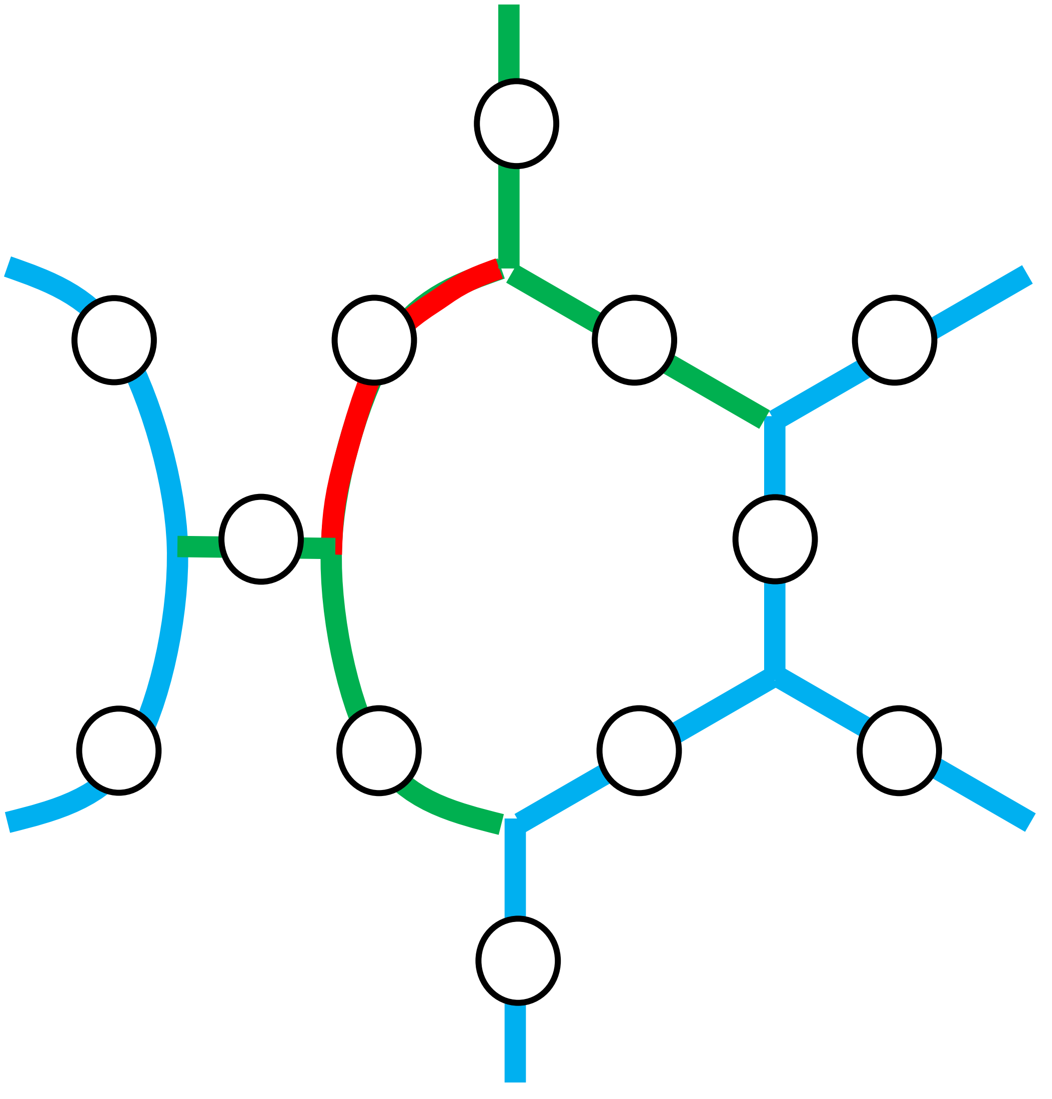

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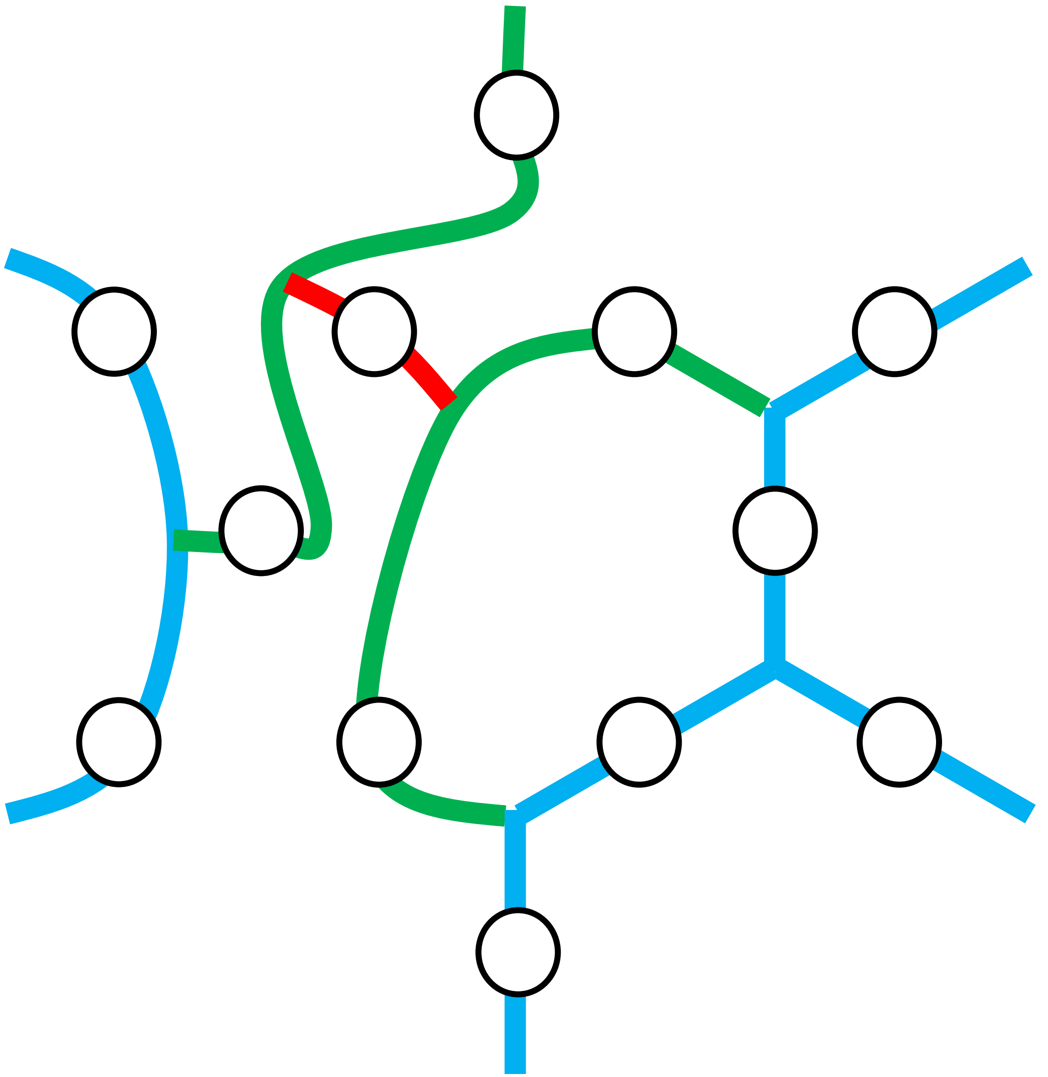

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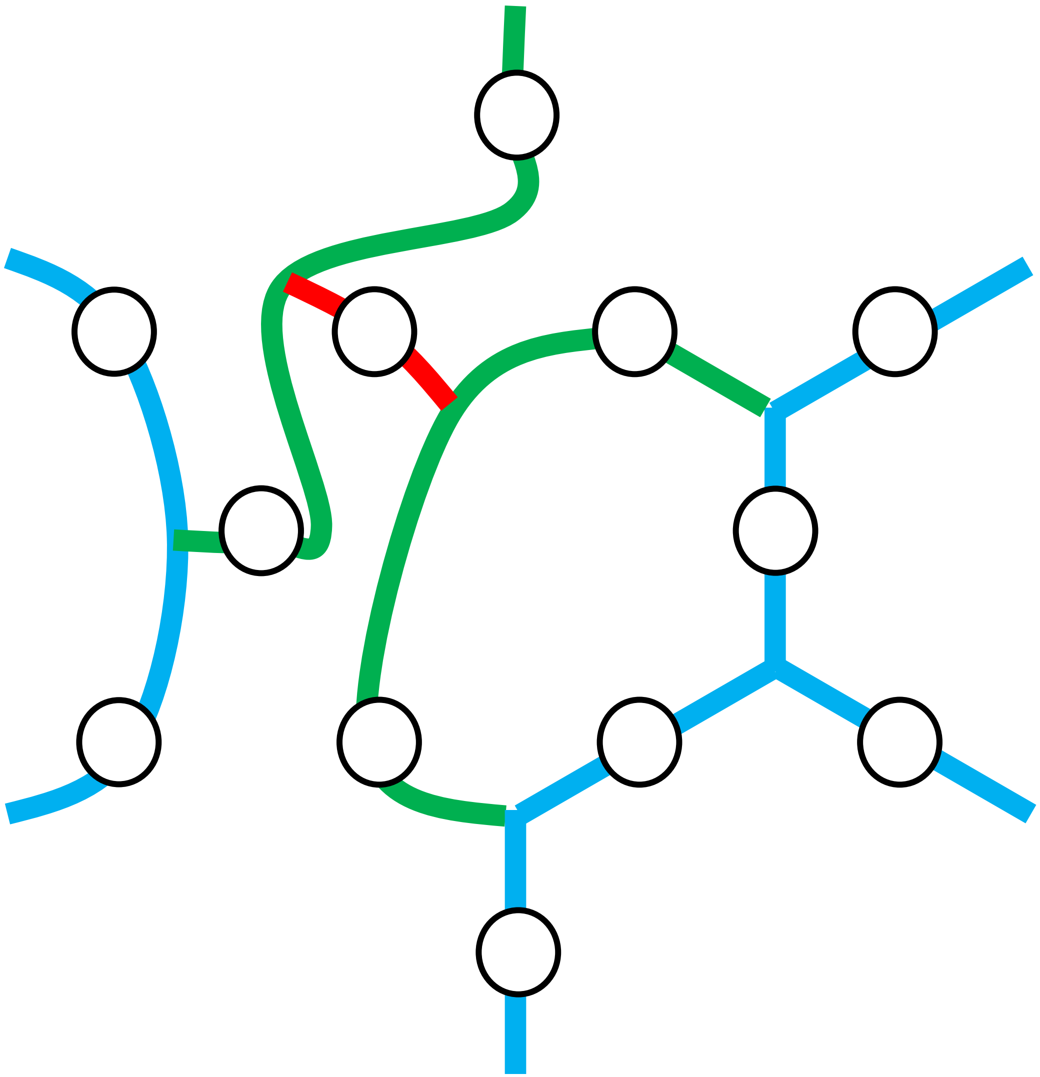

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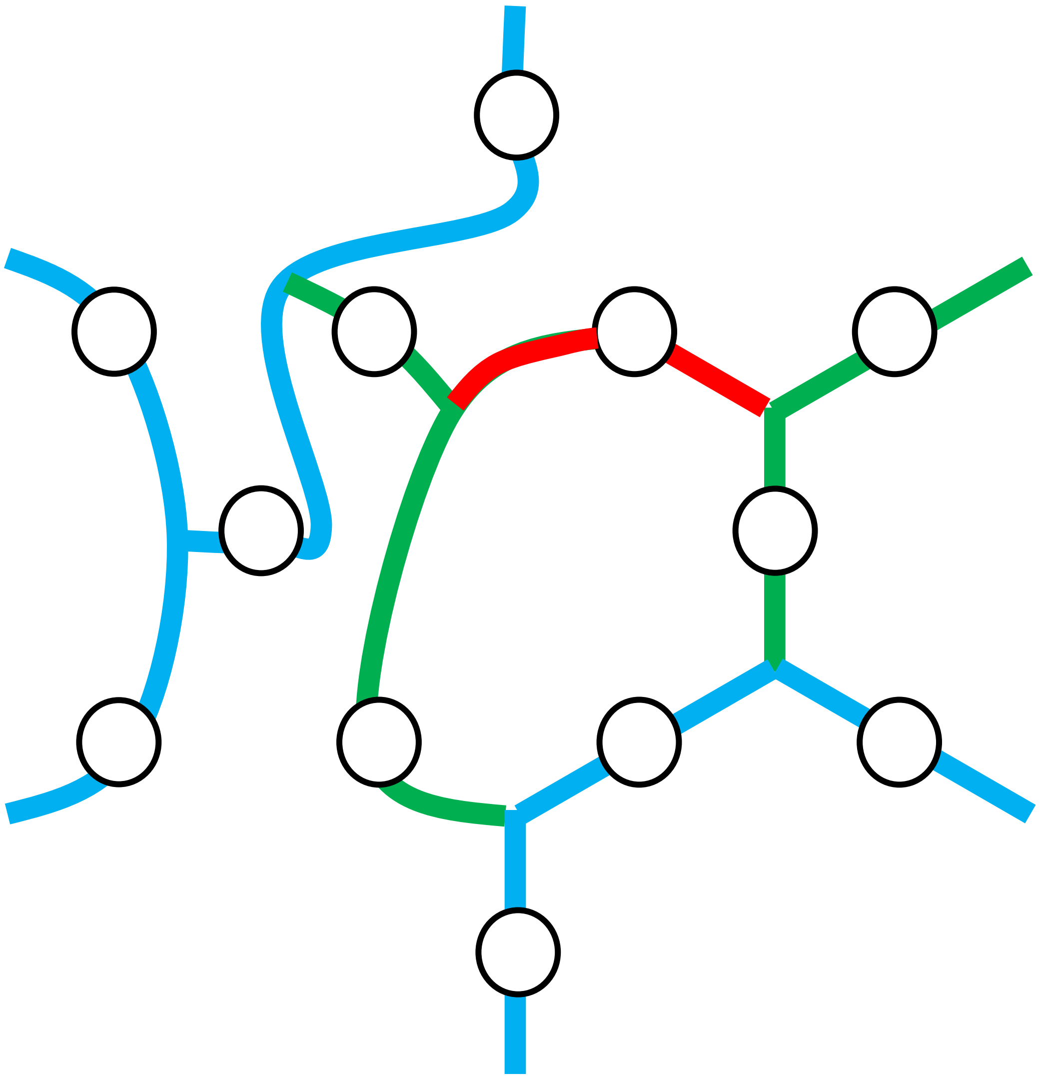

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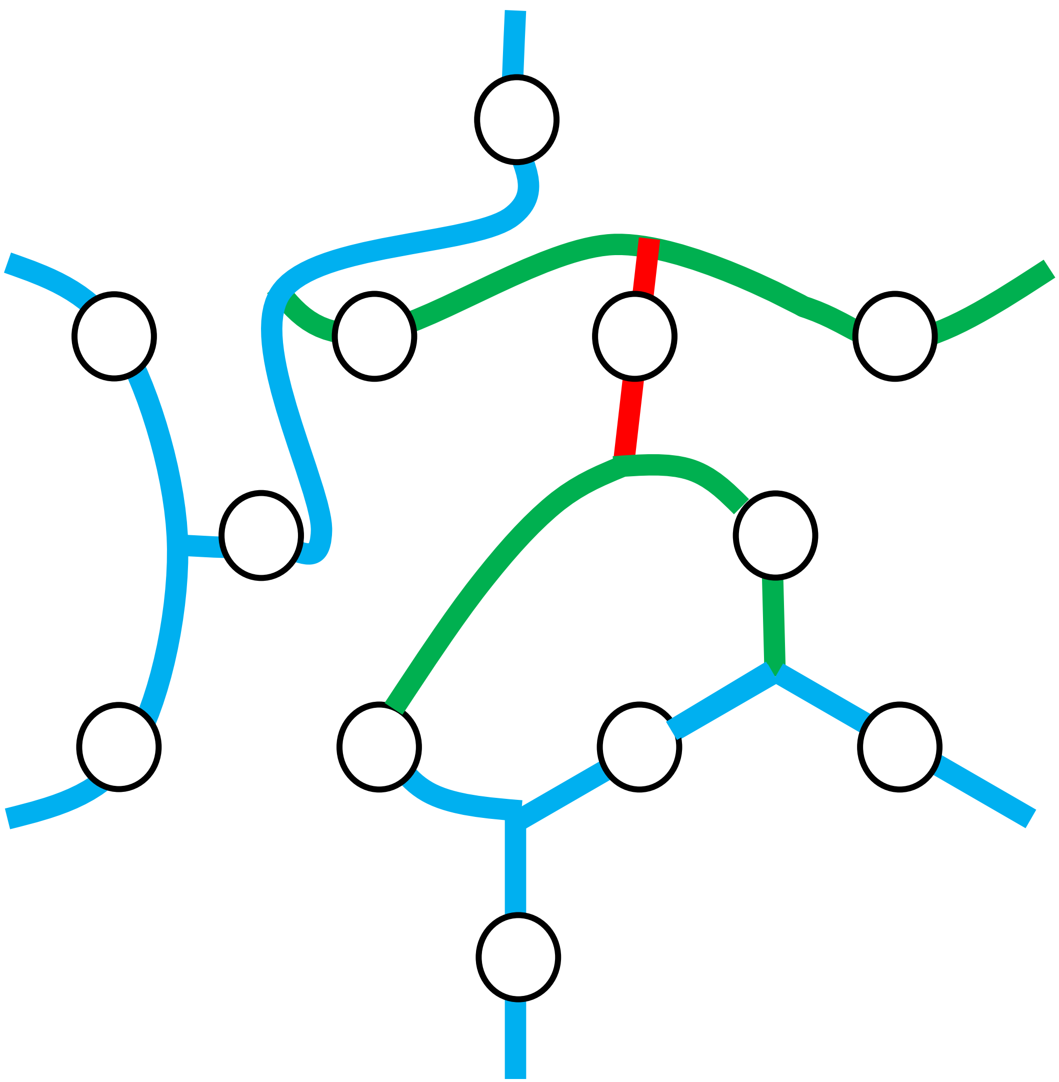

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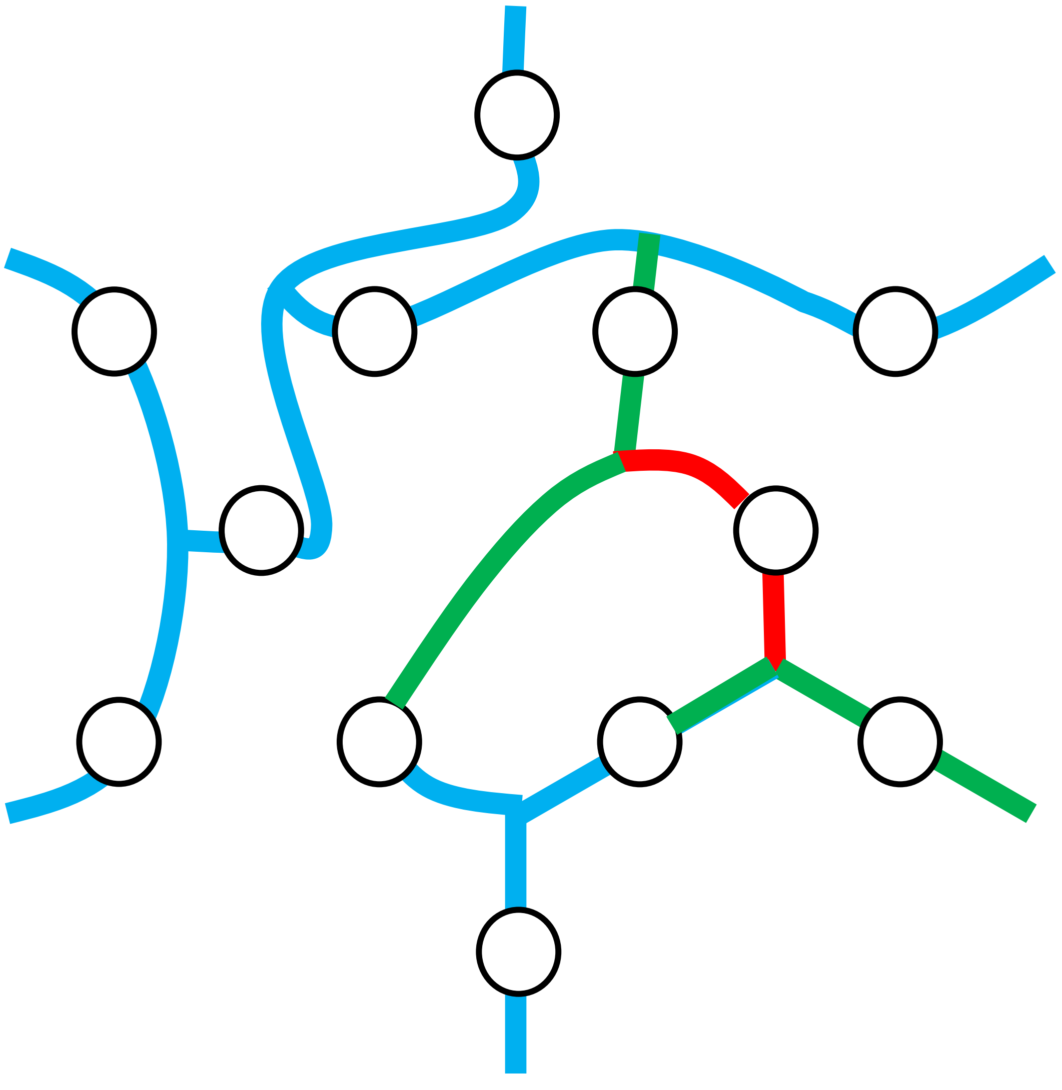

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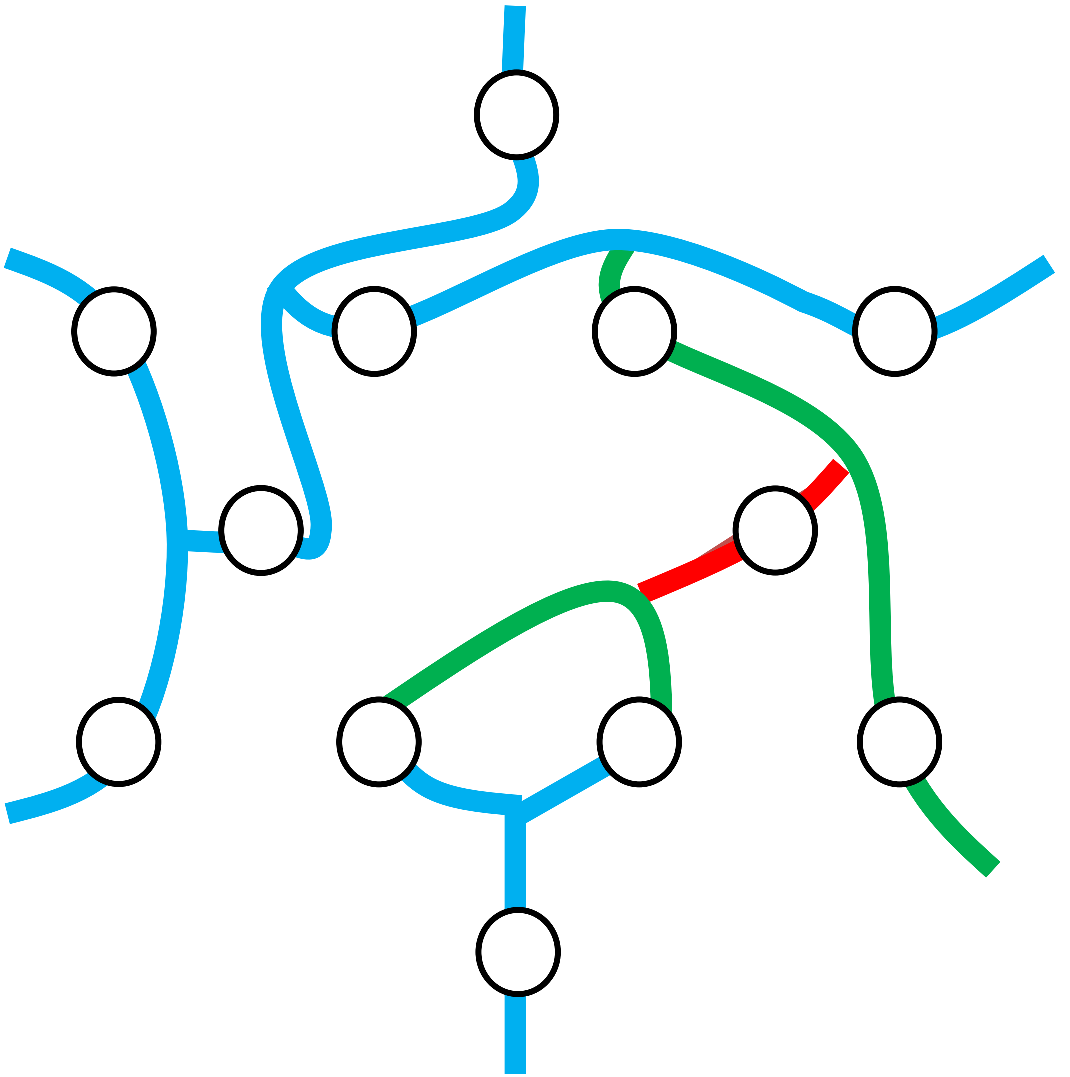

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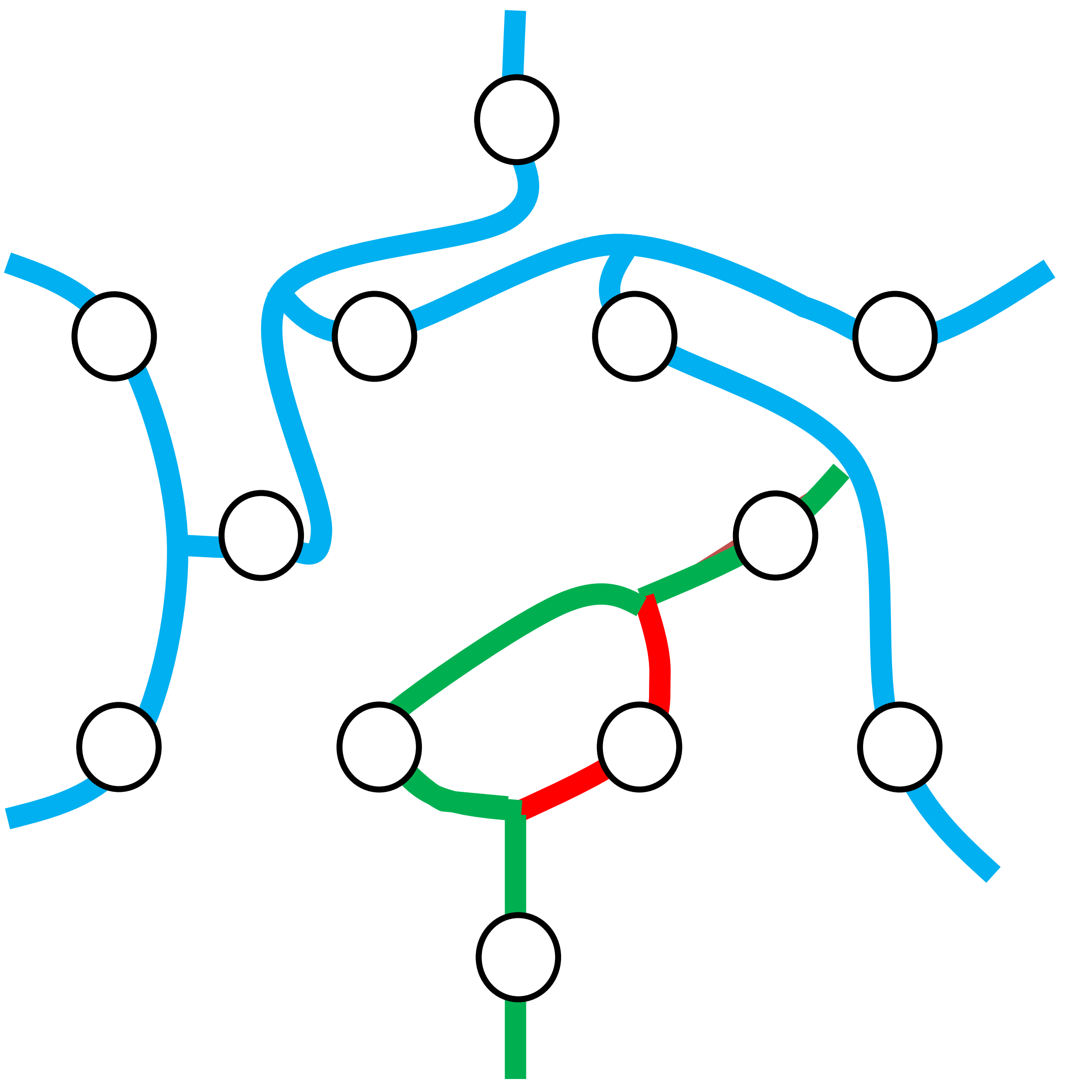

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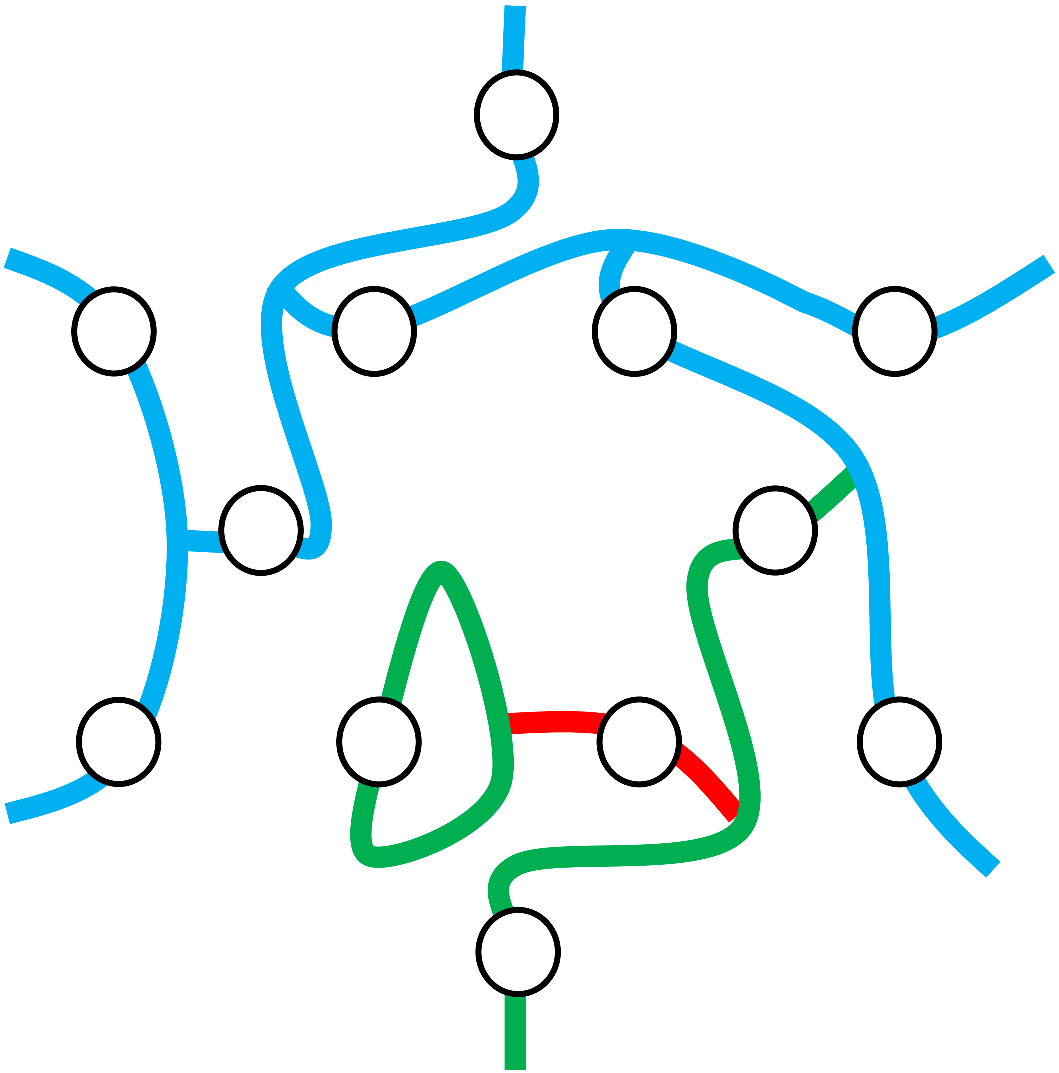

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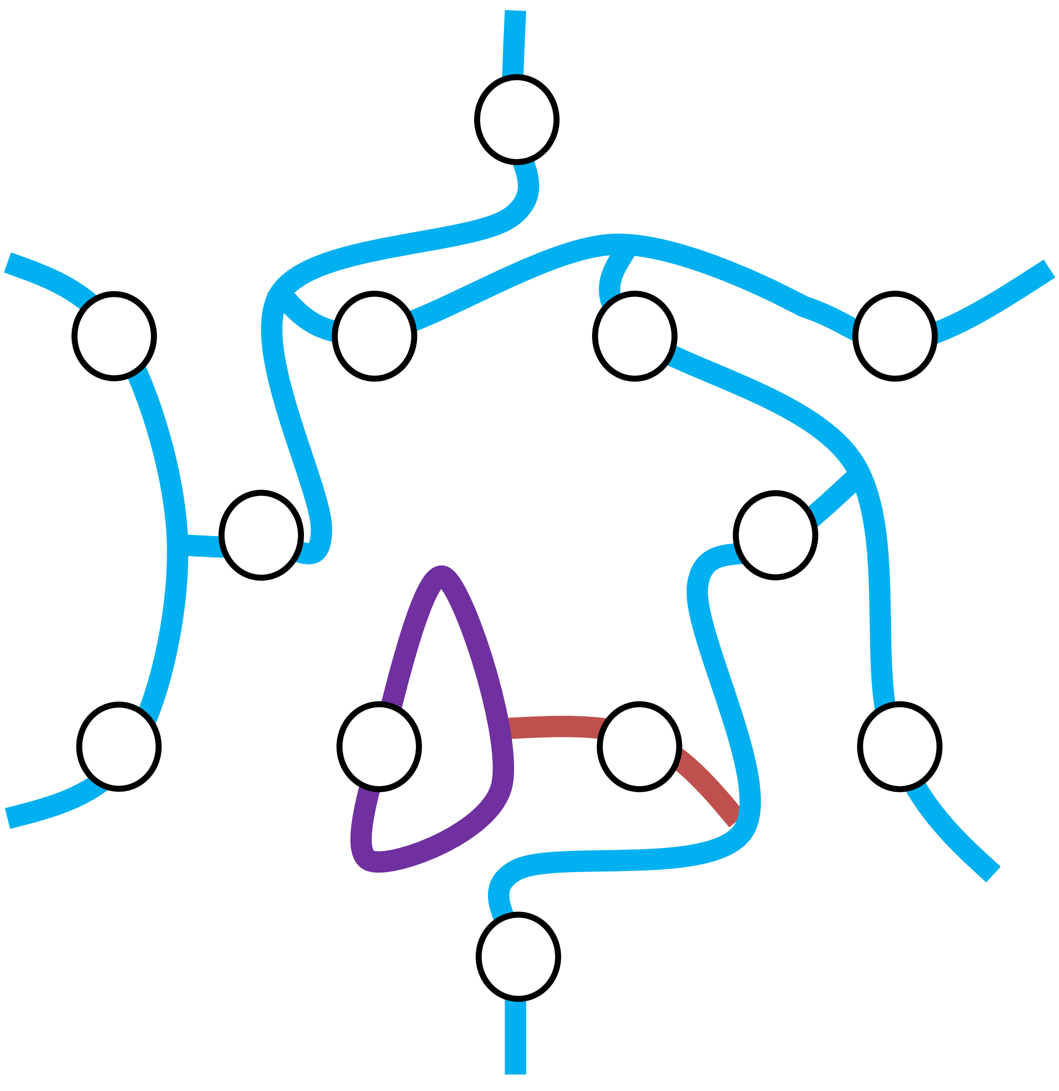

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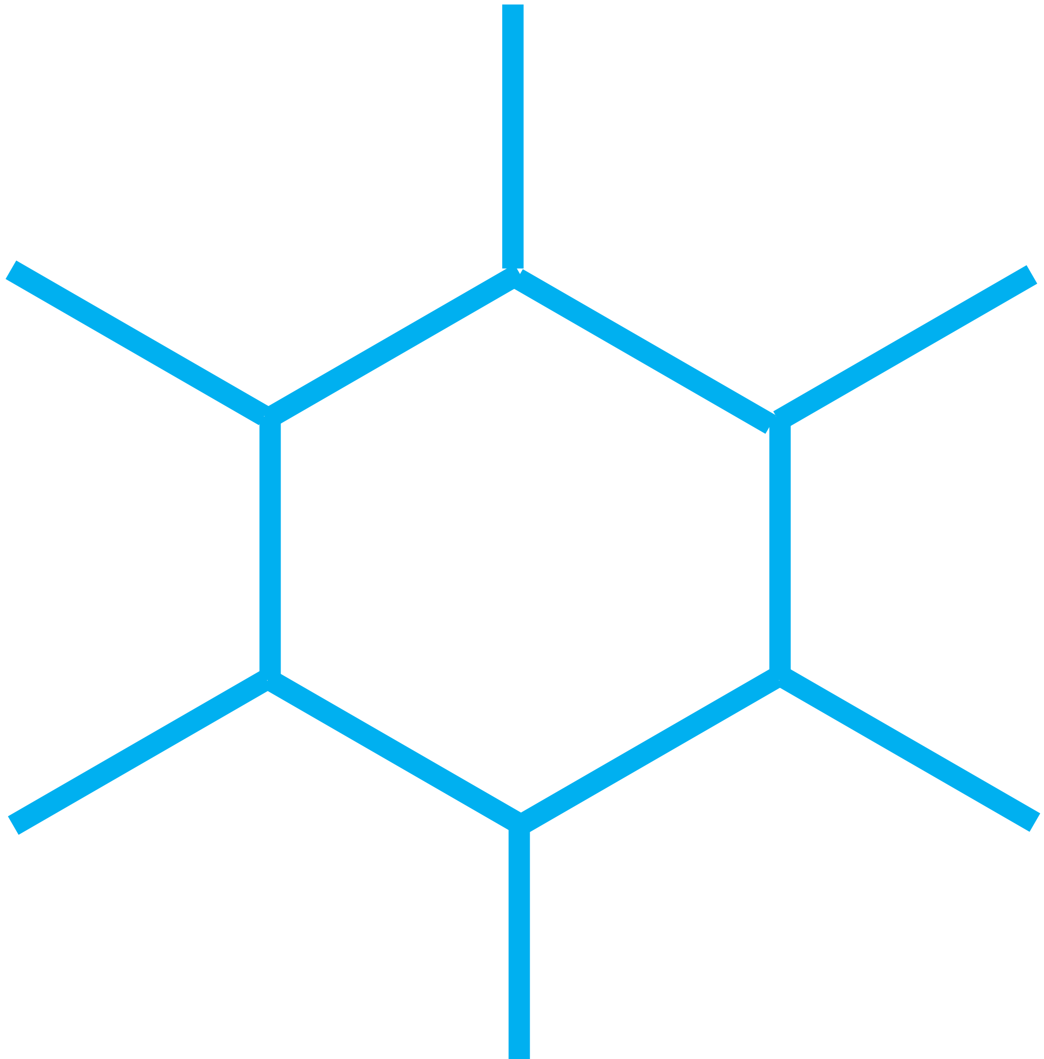

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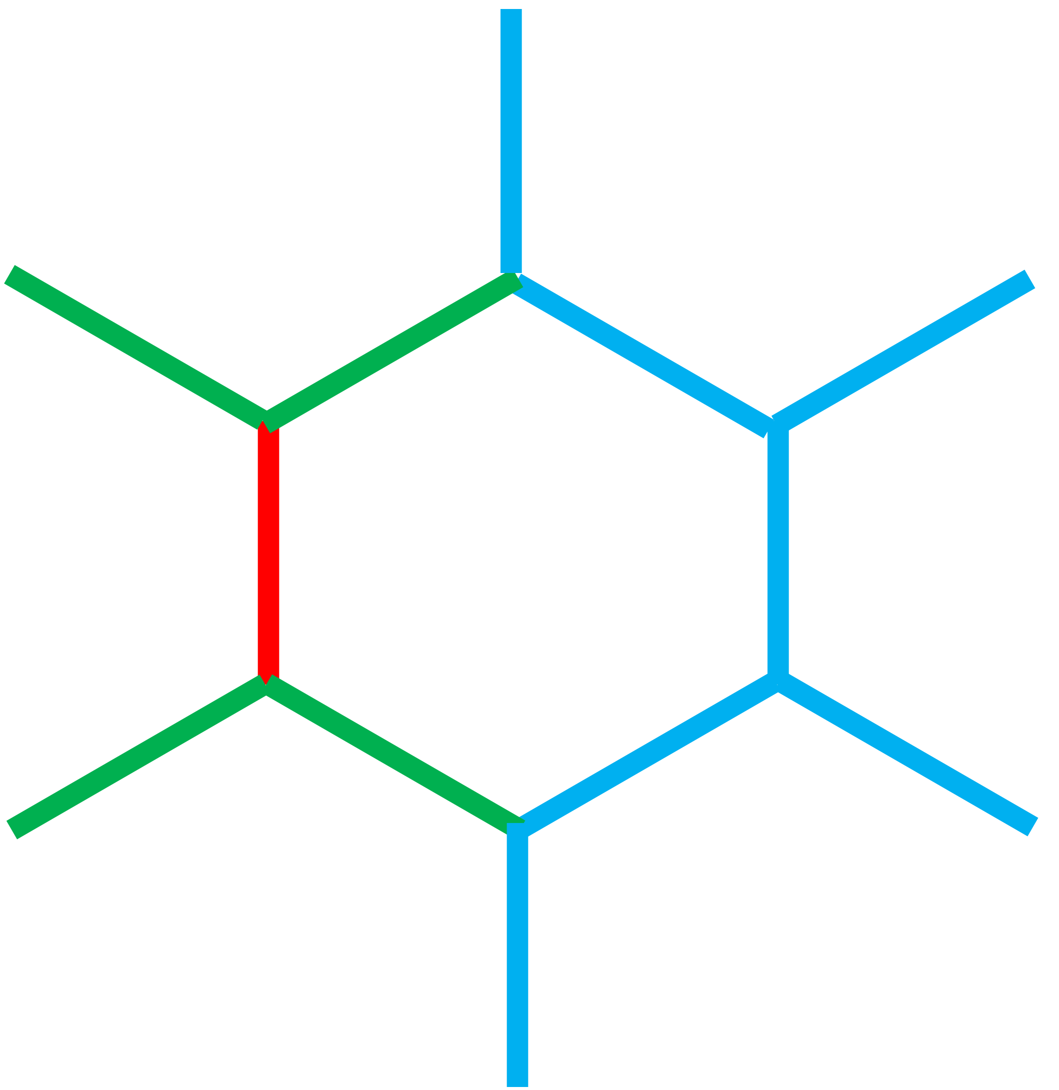

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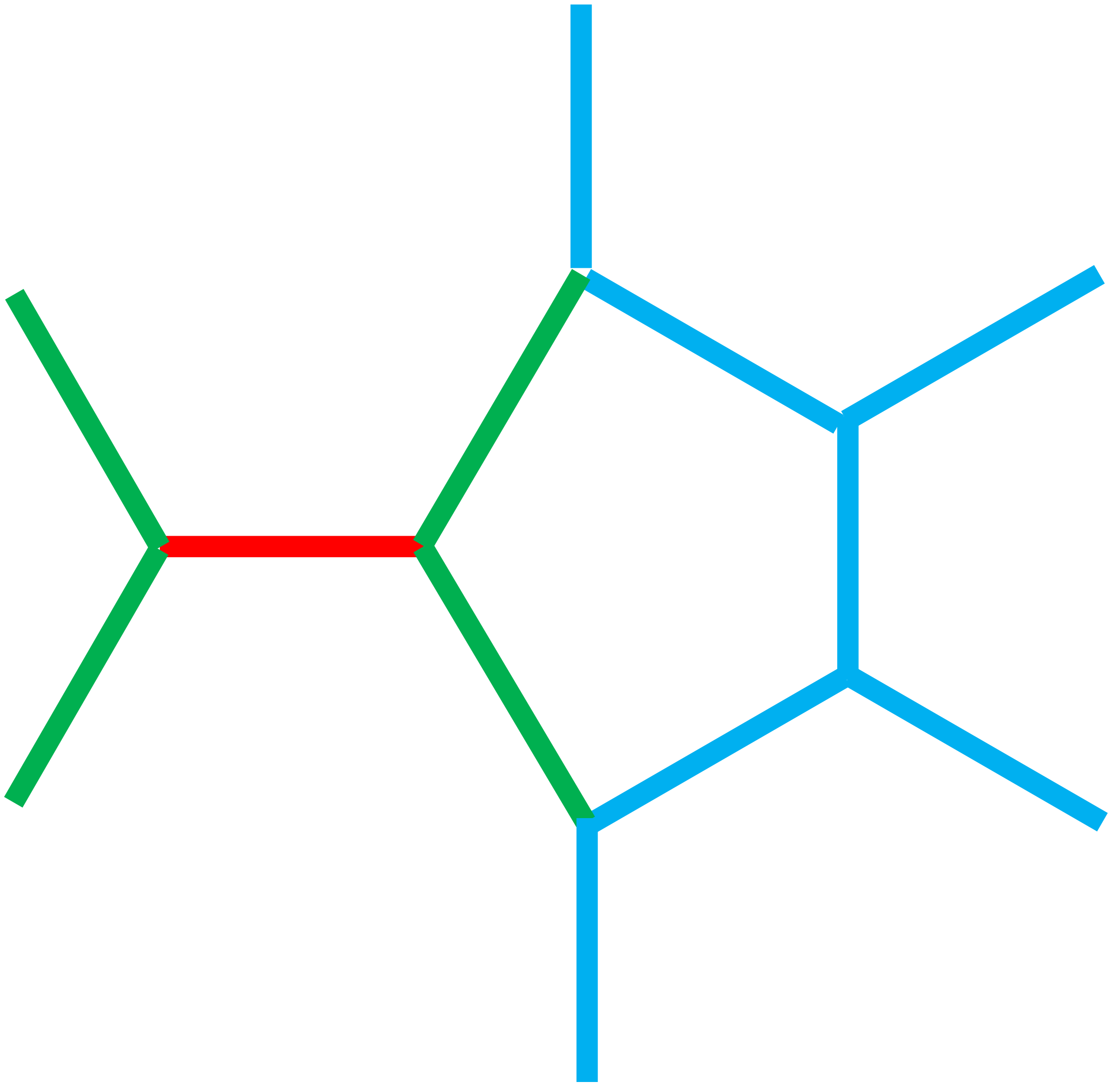

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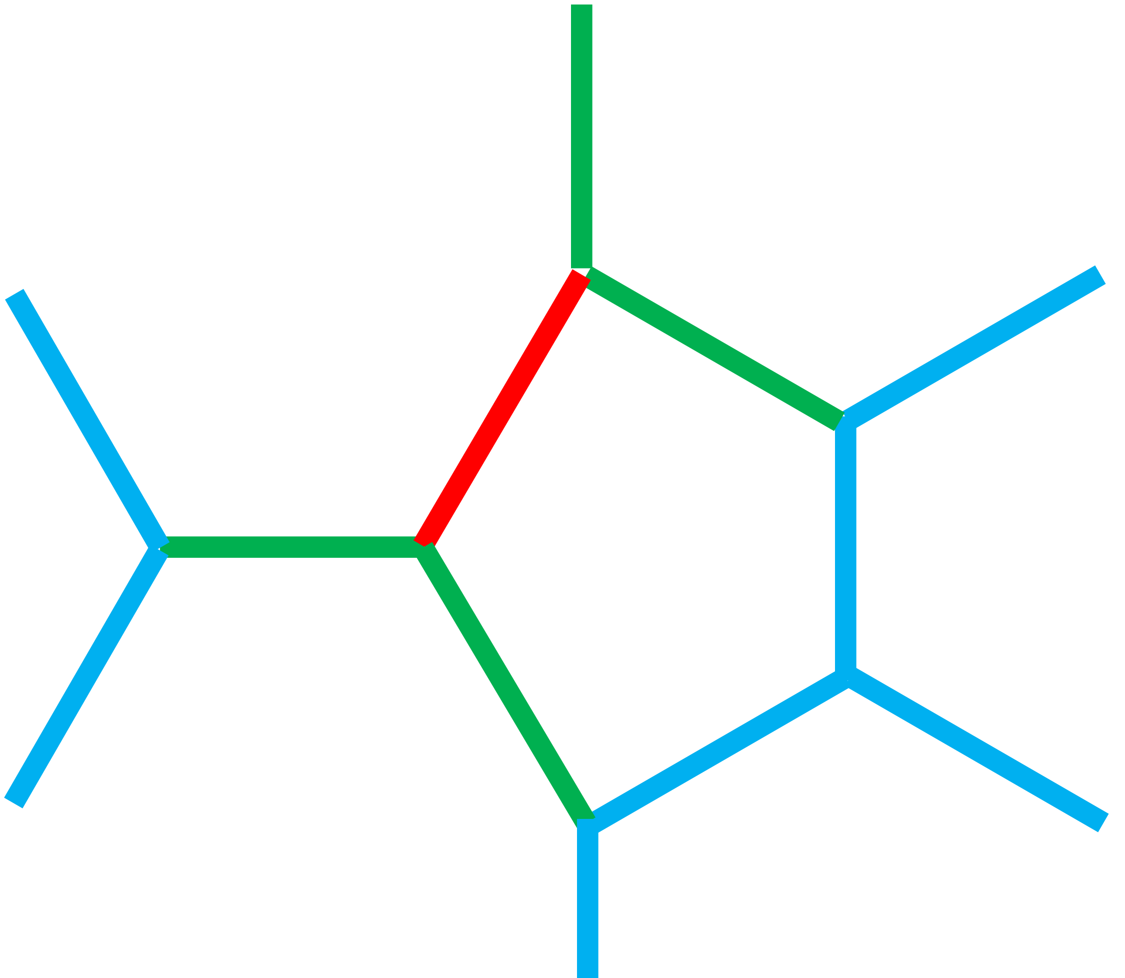

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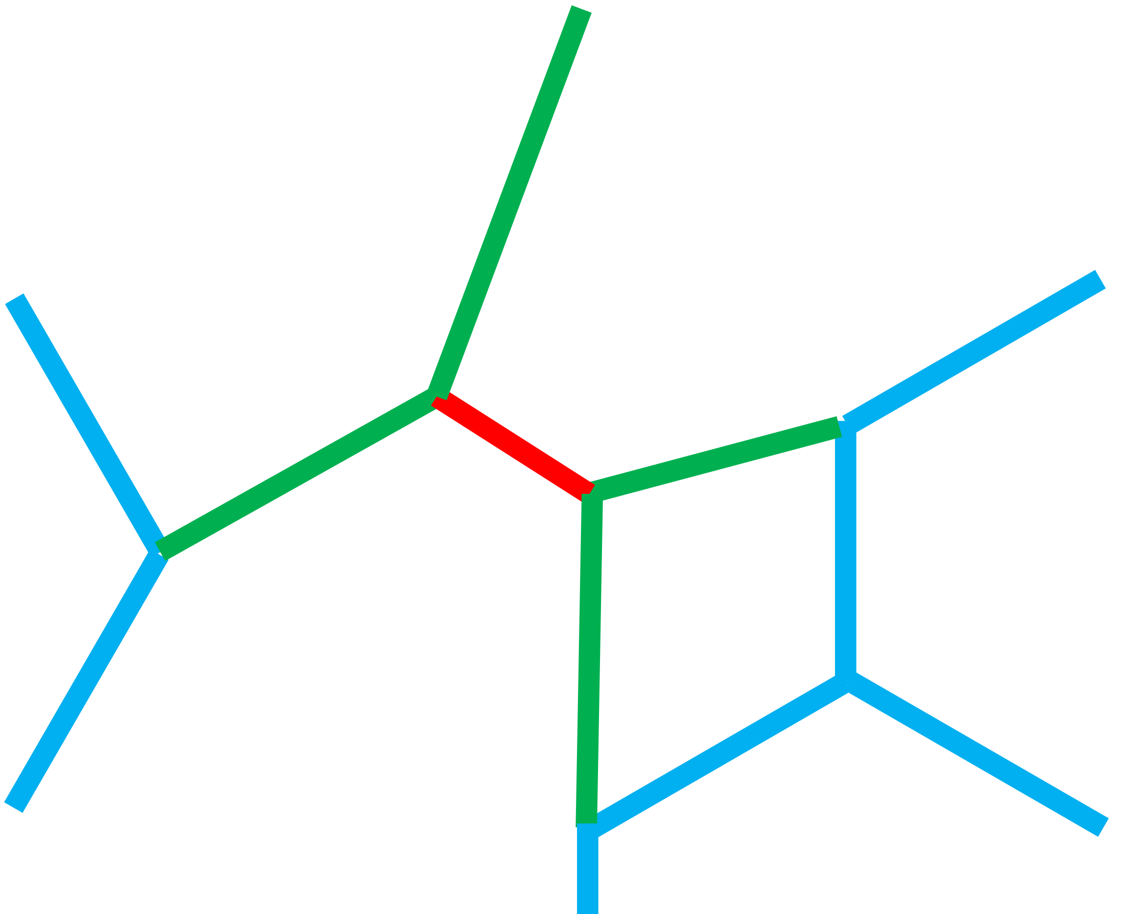

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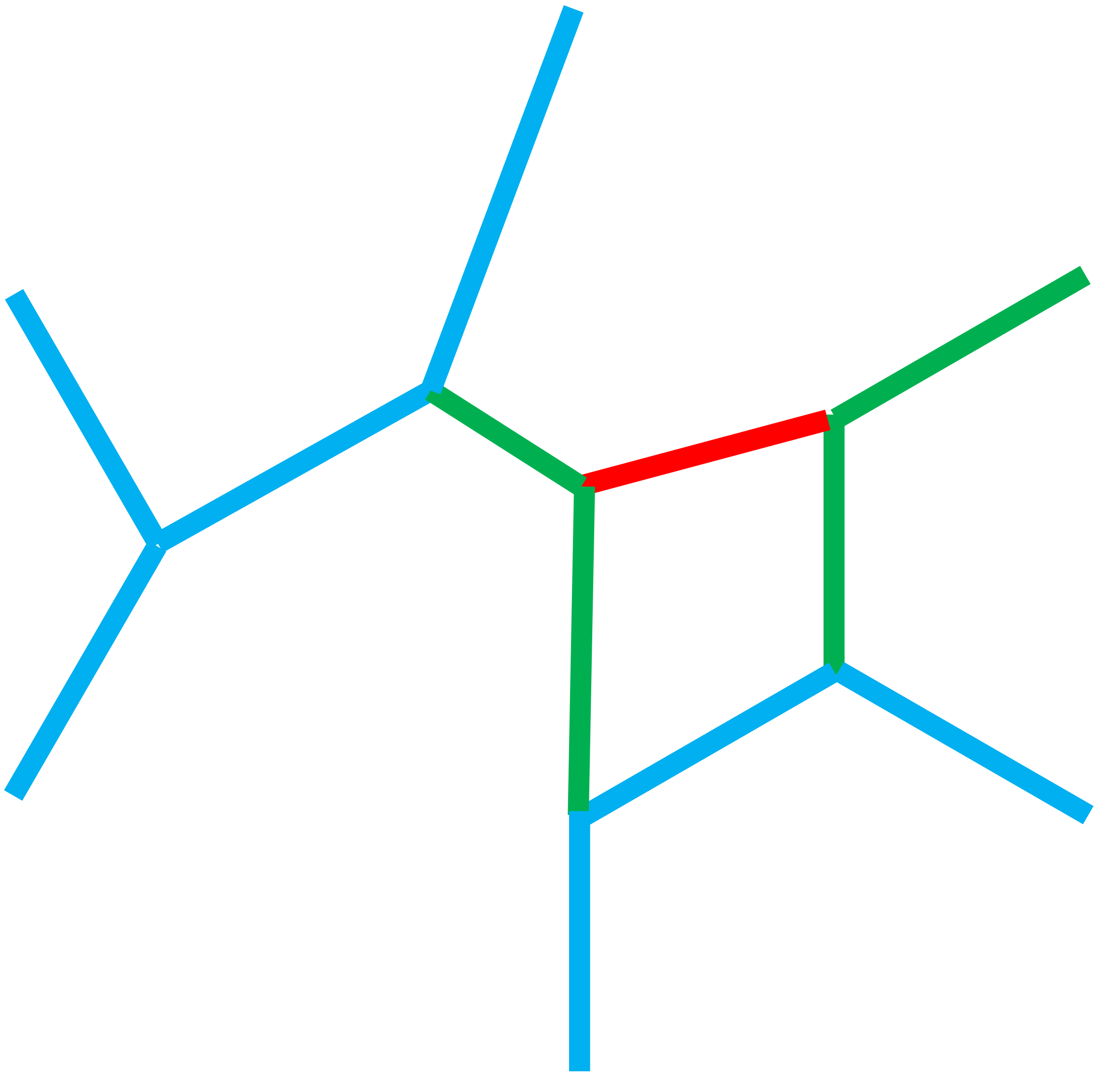

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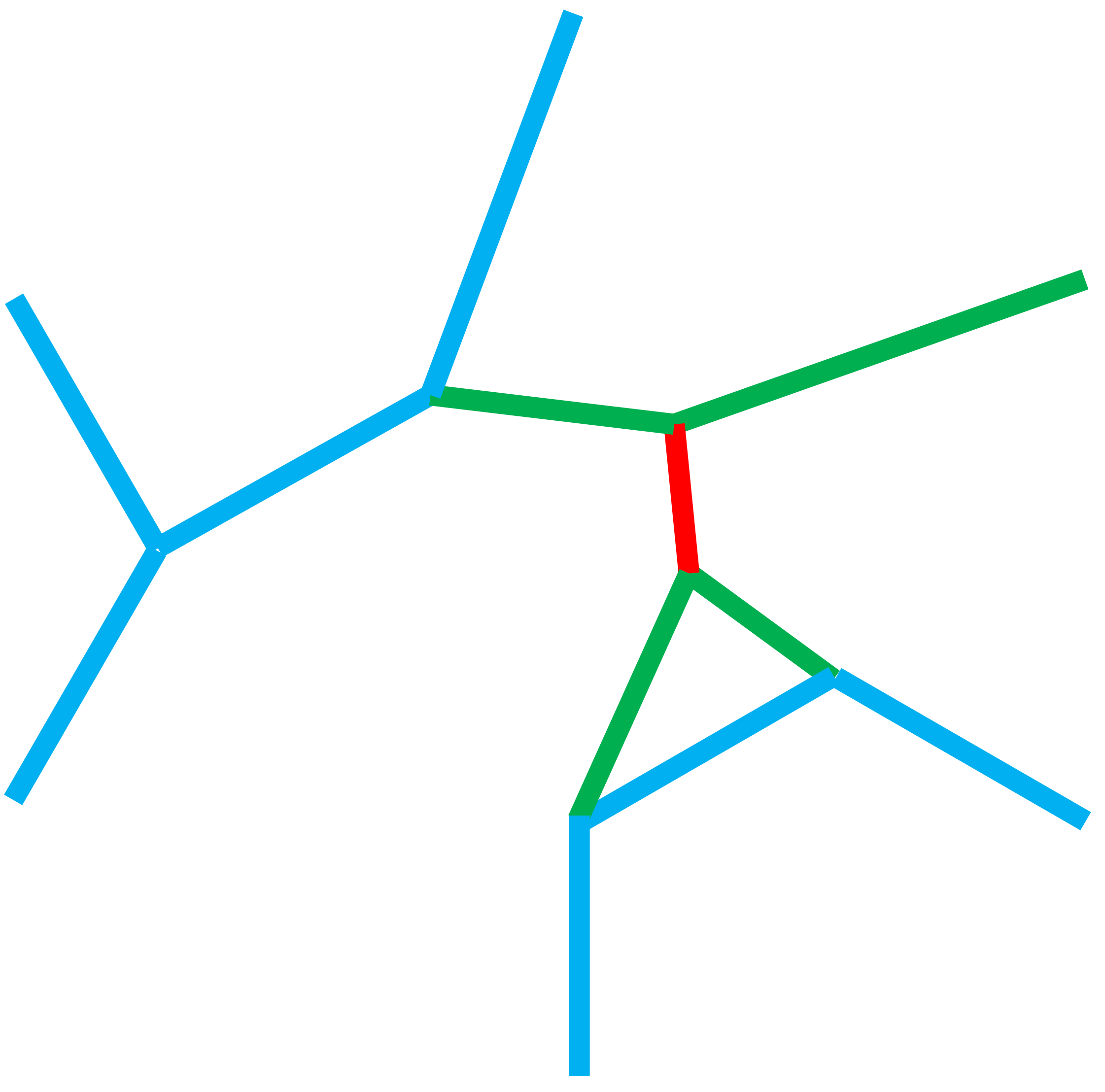

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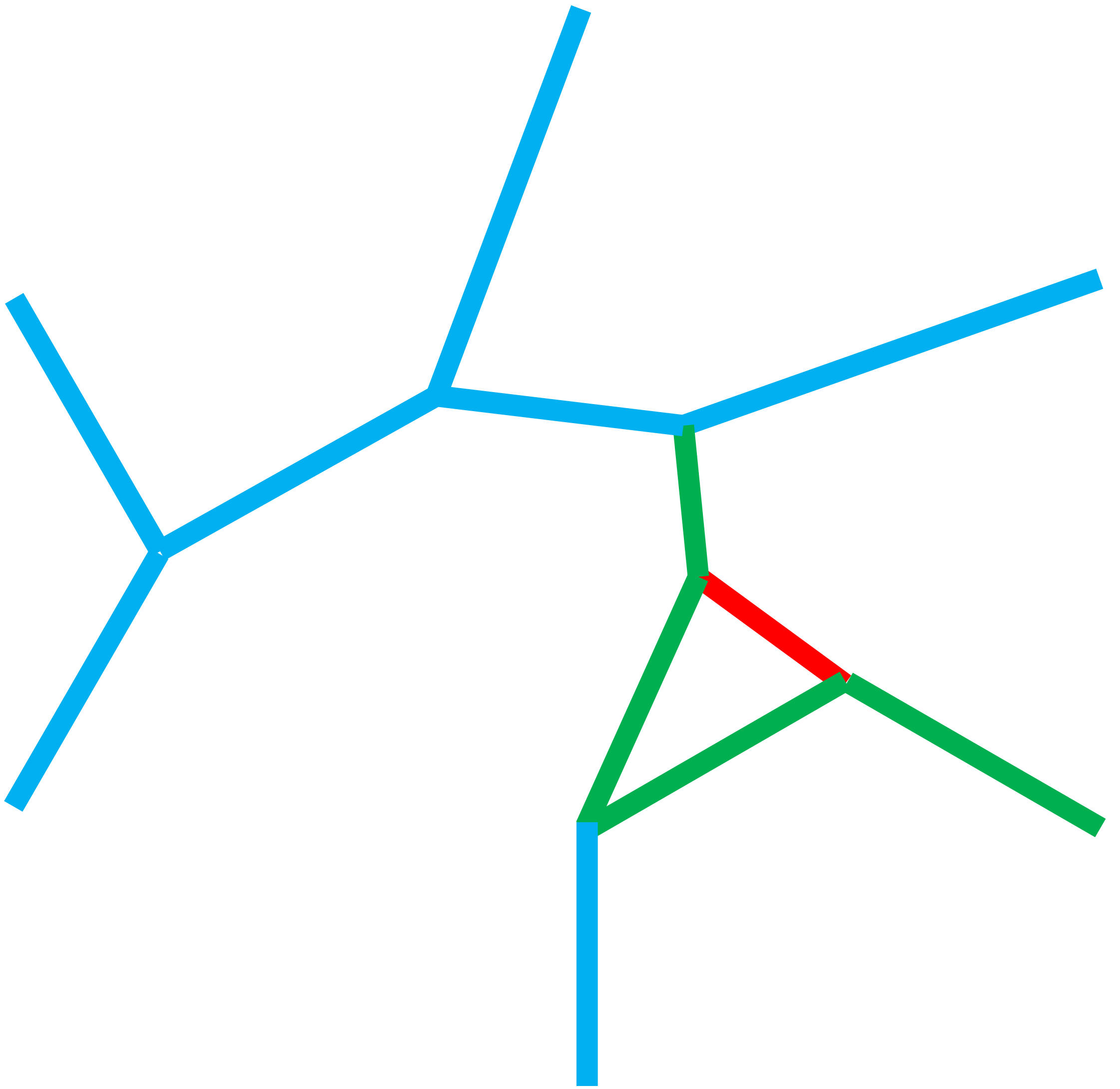

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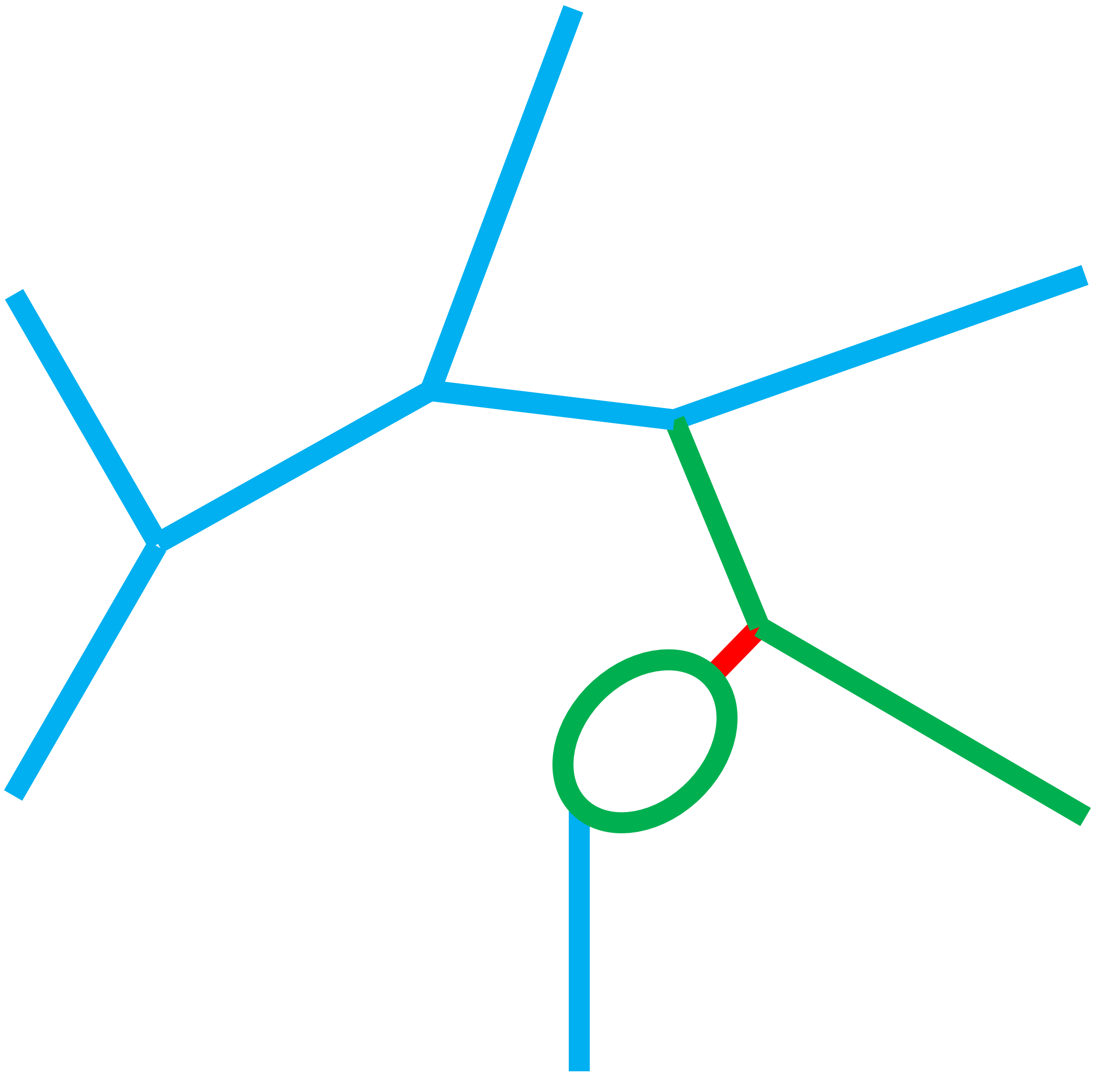

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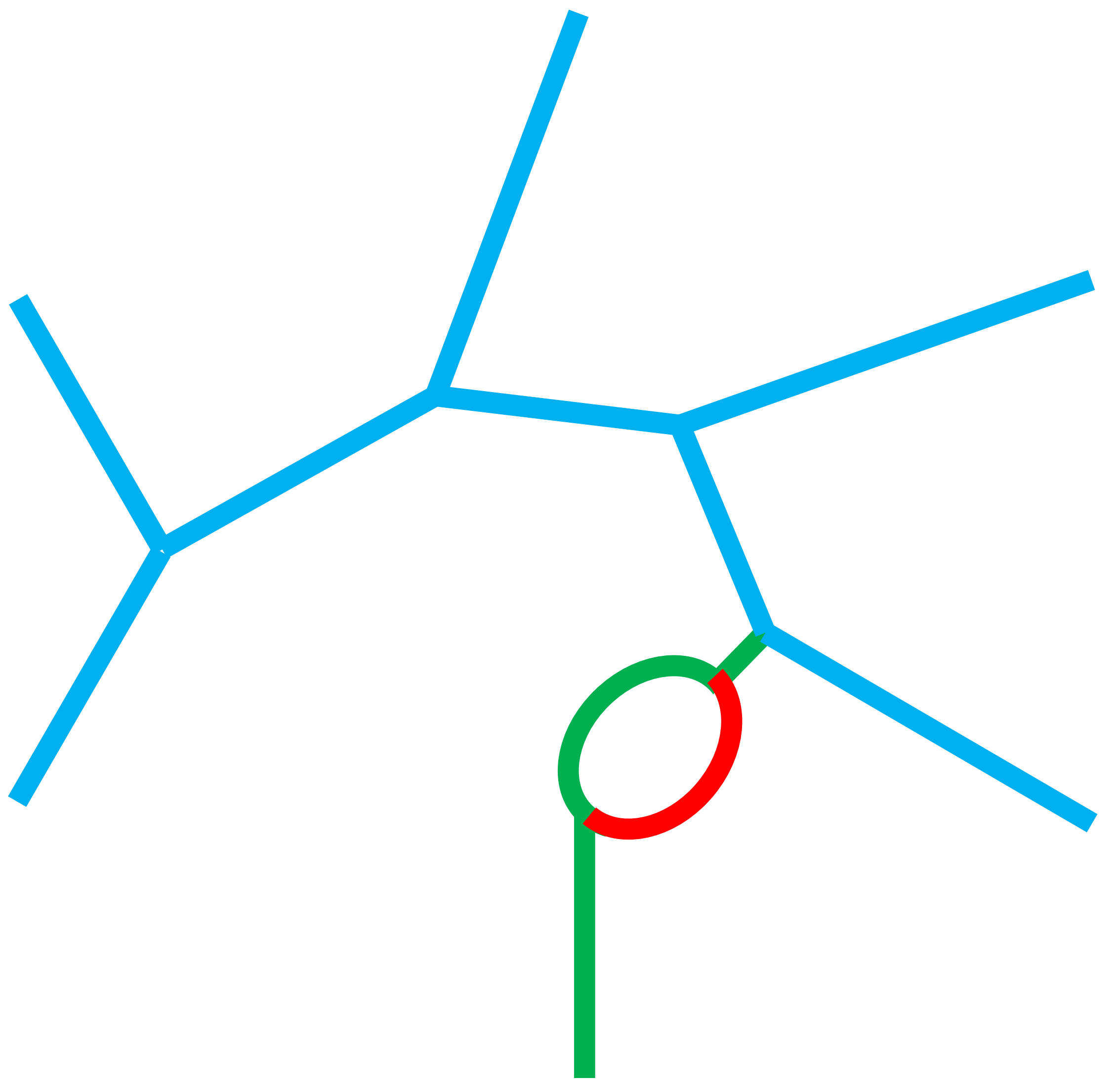

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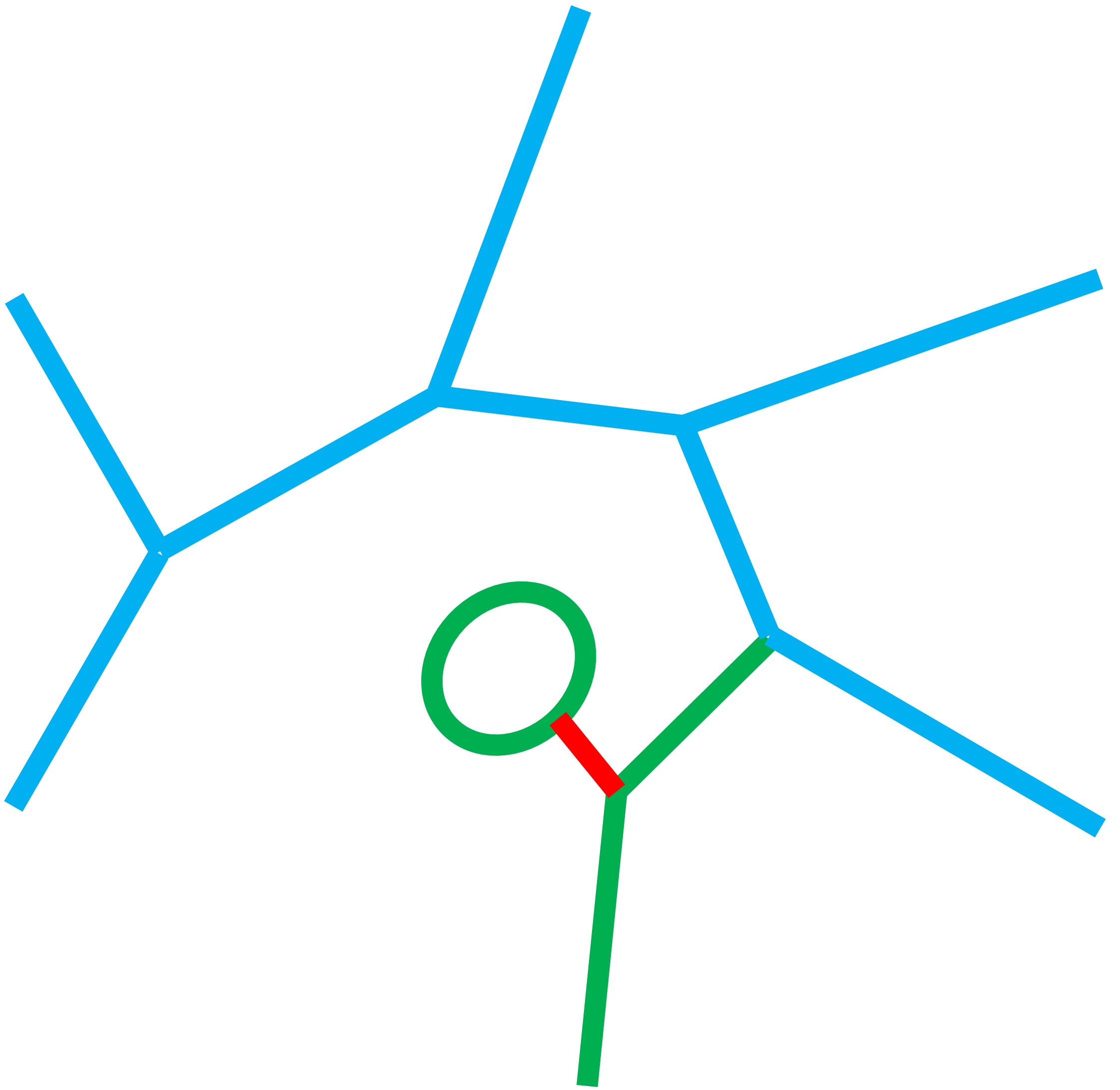

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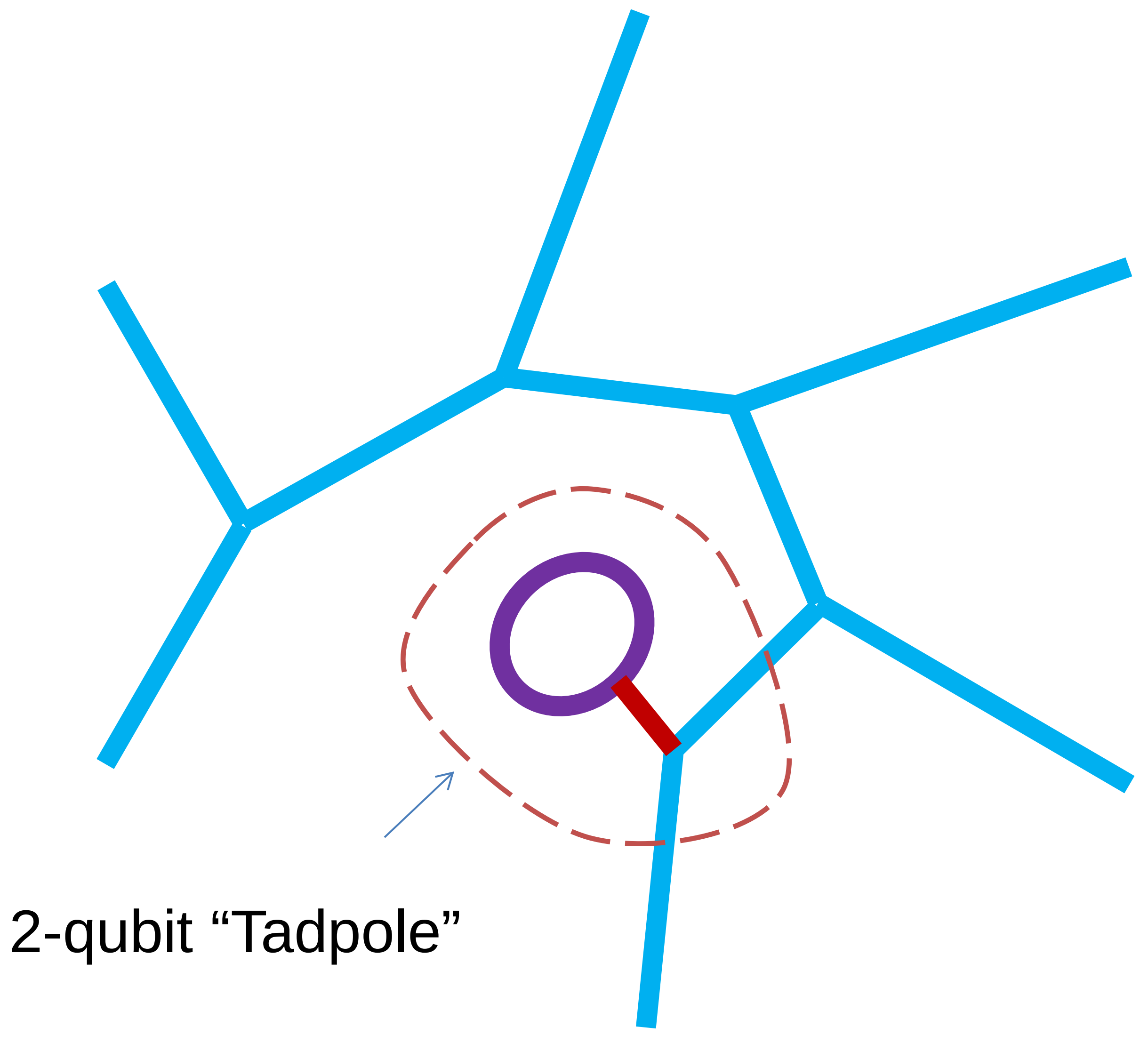
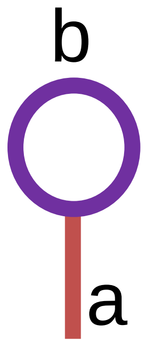
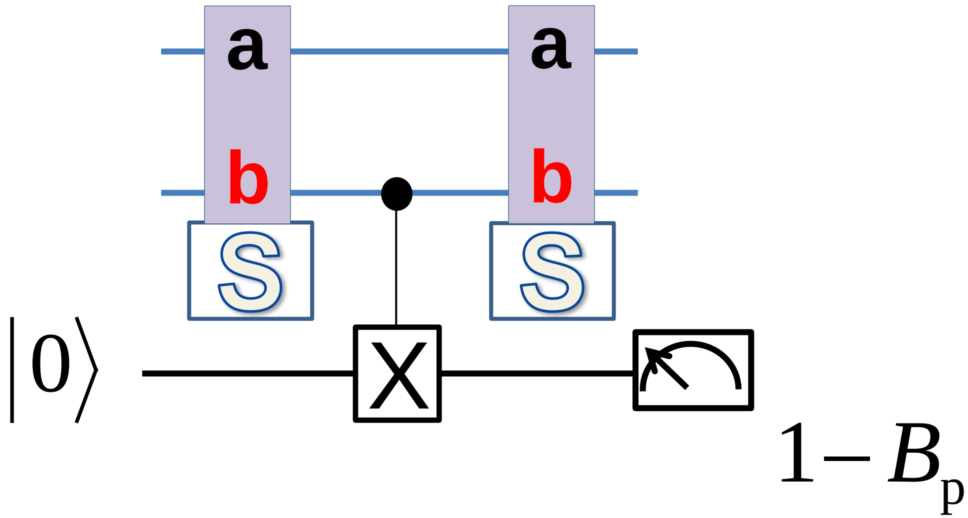
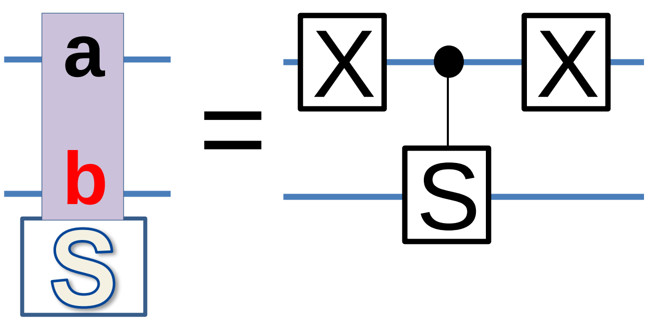


Measuring B_p for a Tadpole is Easy!



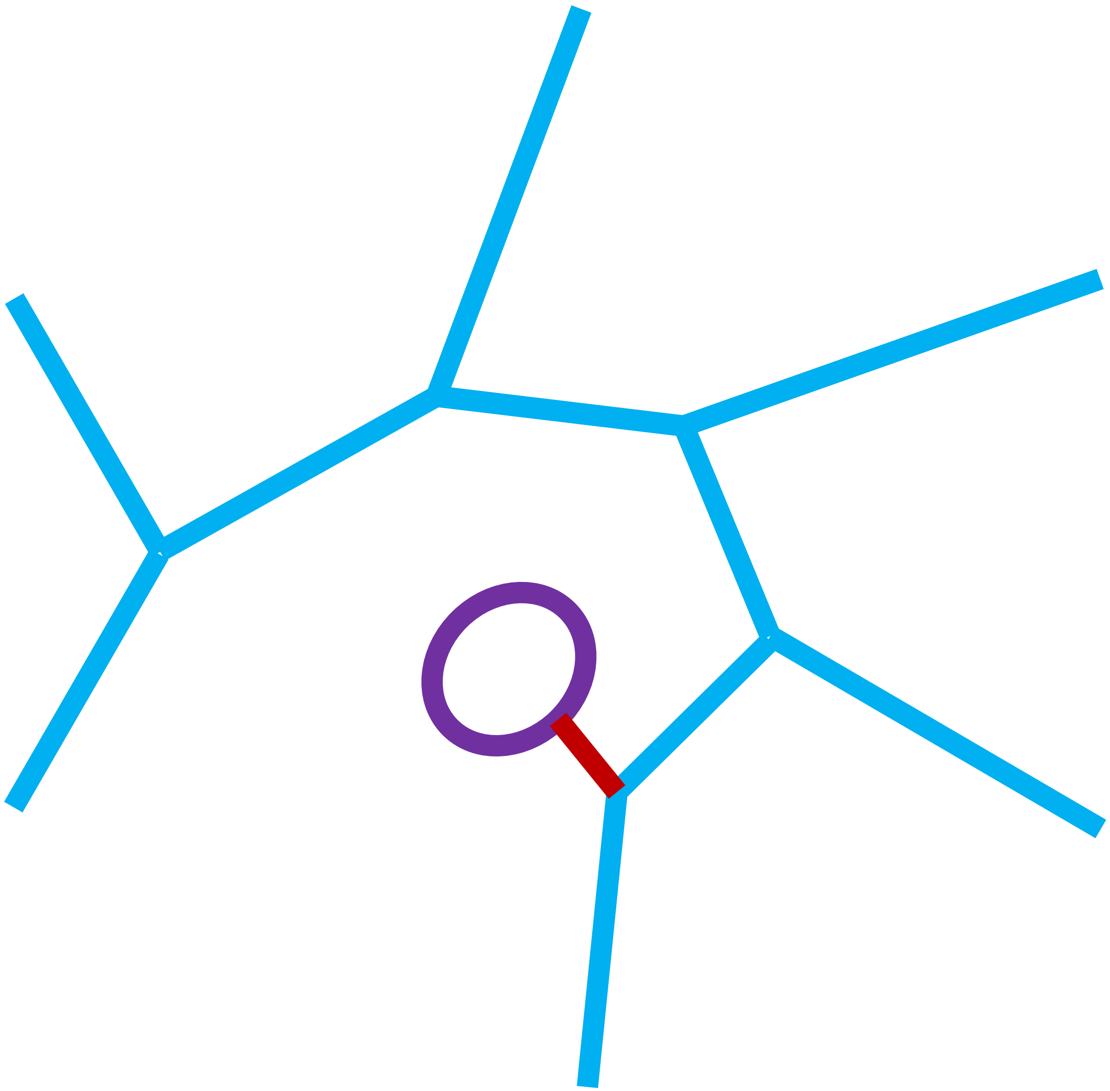


S Quantum Circuit

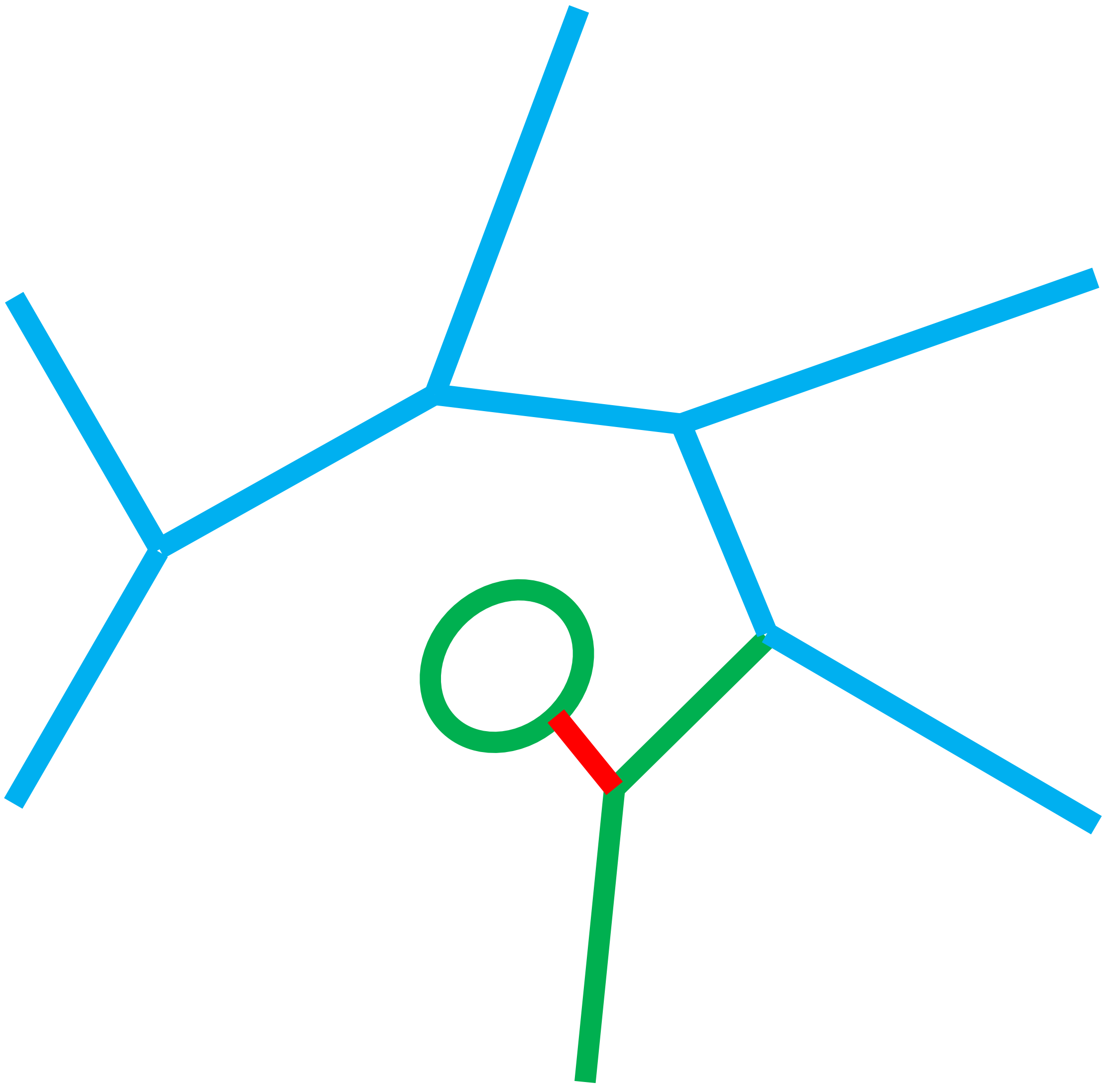


$$S = \frac{1}{\sqrt{1+\varphi^2}} \begin{pmatrix} 1 & \varphi \\ \varphi & -1 \end{pmatrix}$$

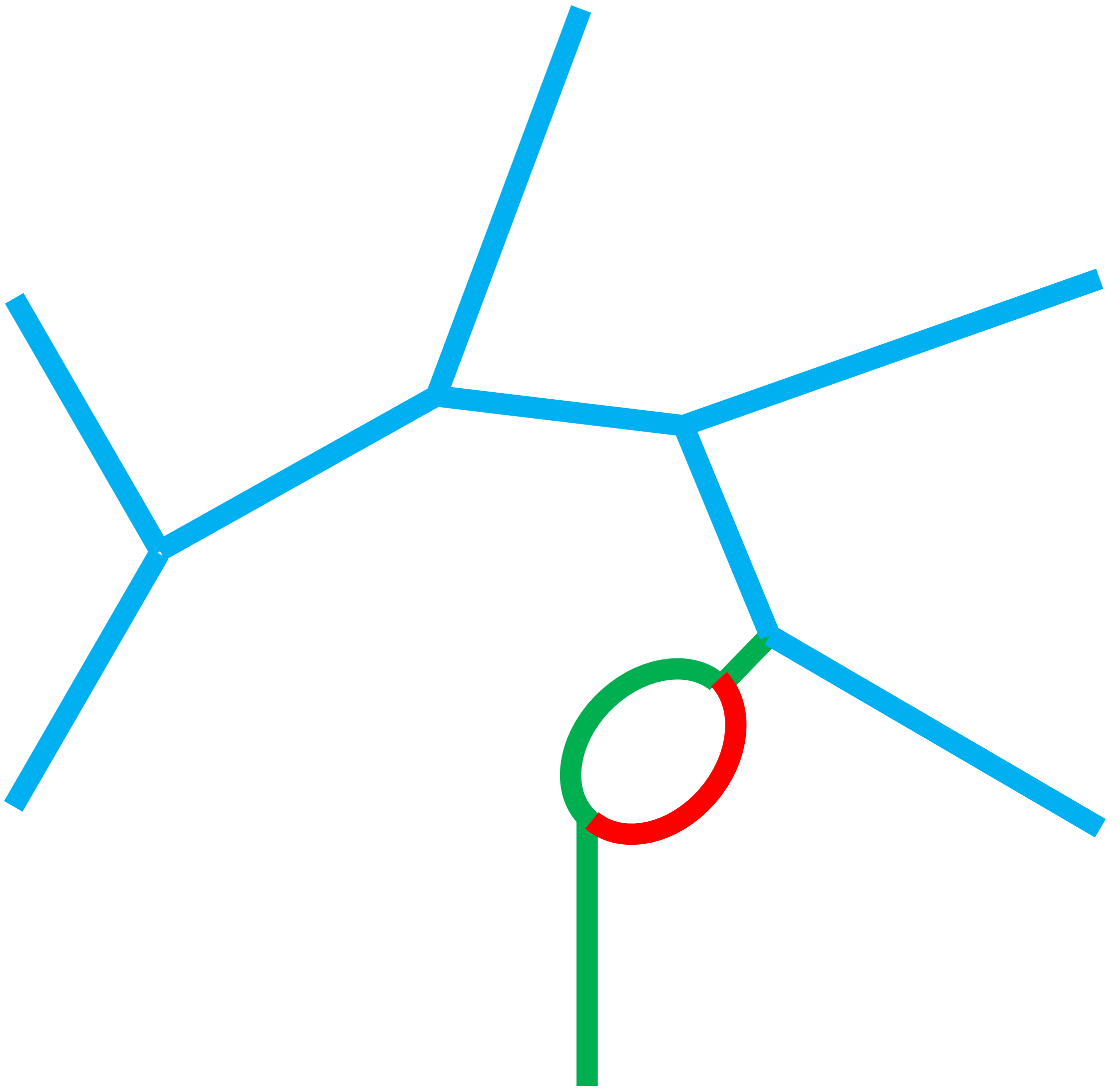
Plaquette Restoration



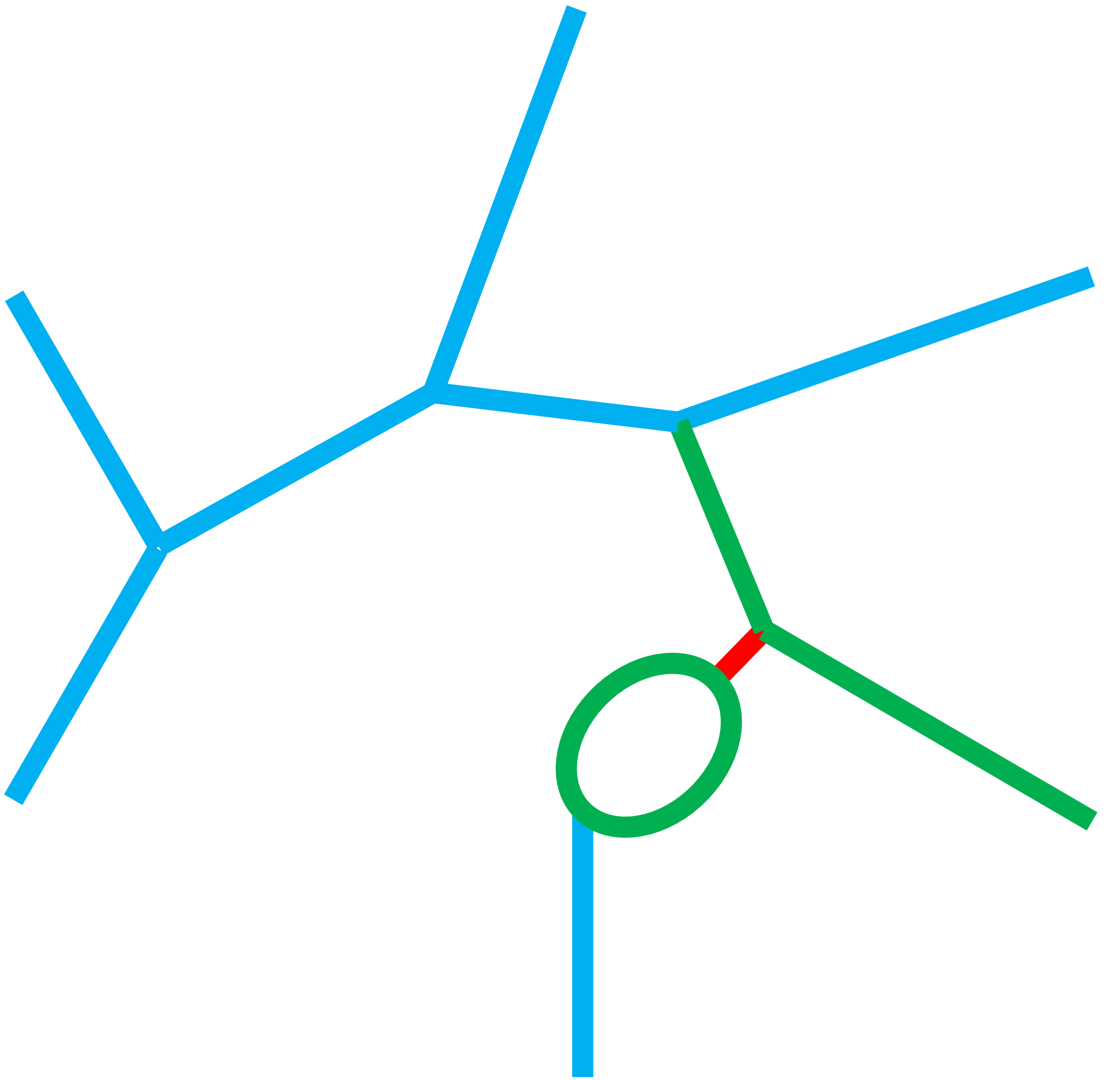
Plaquette Restoration



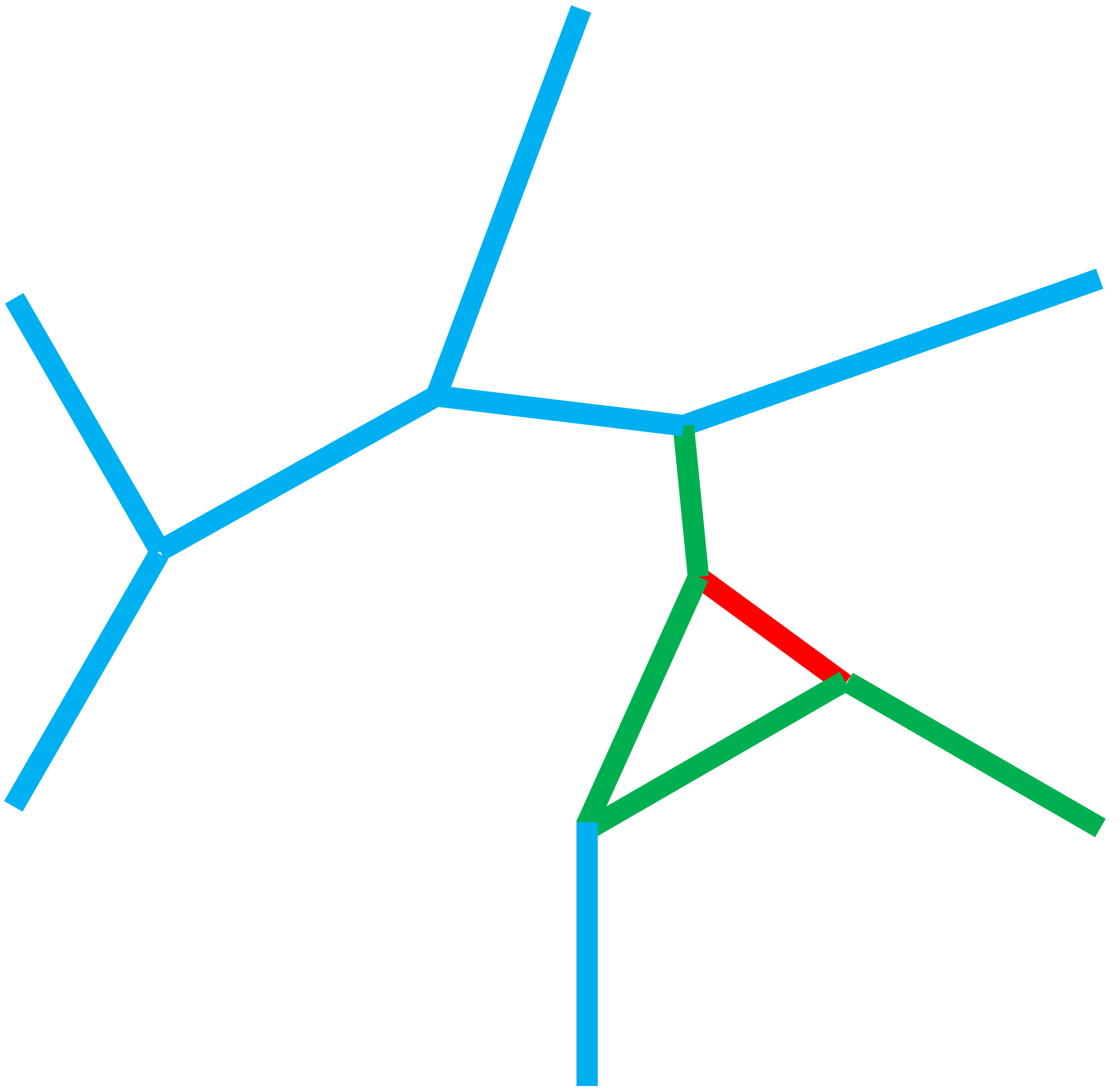
Plaquette Restoration



Plaquette Restoration



Plaquette Restoration



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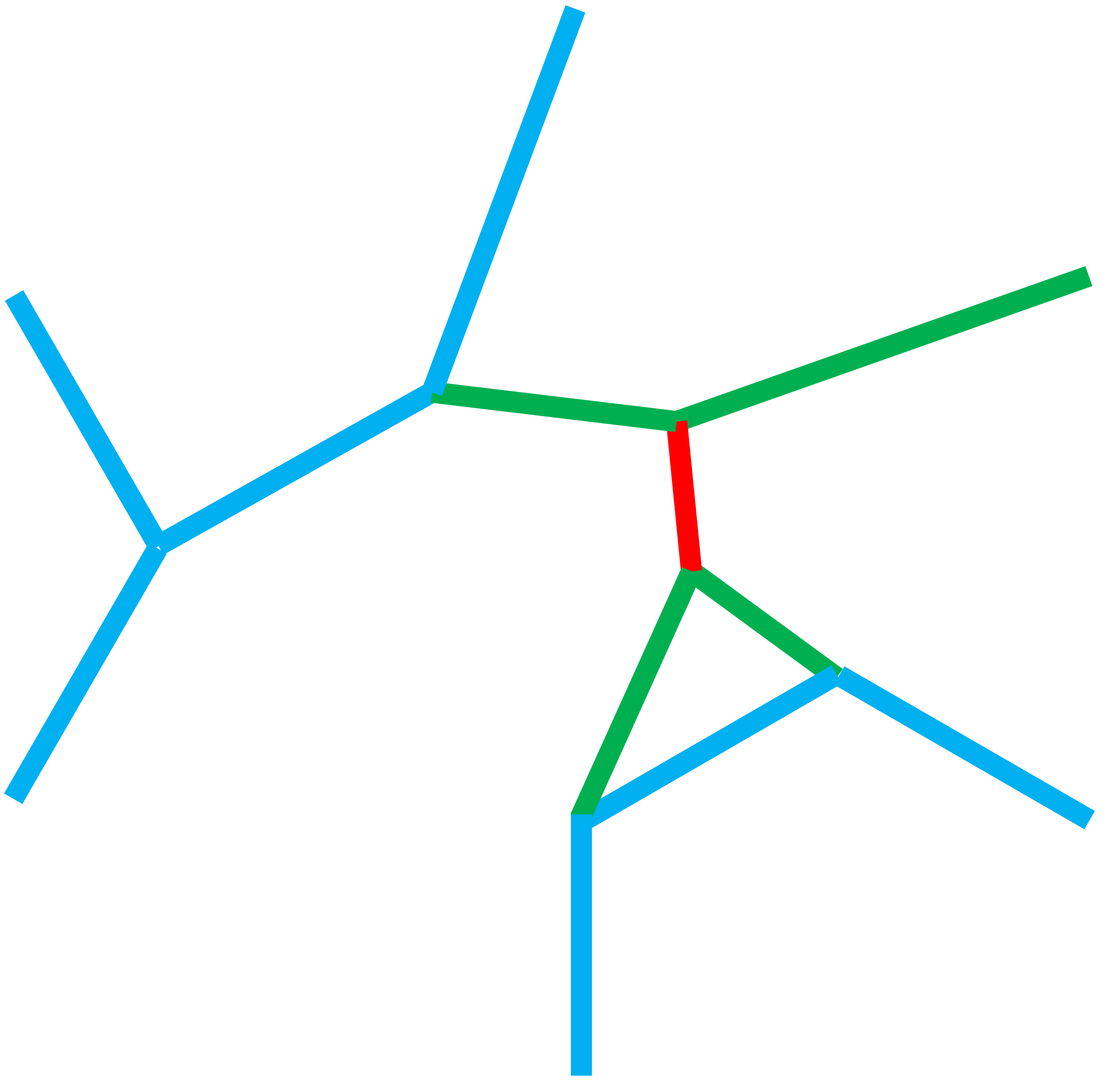

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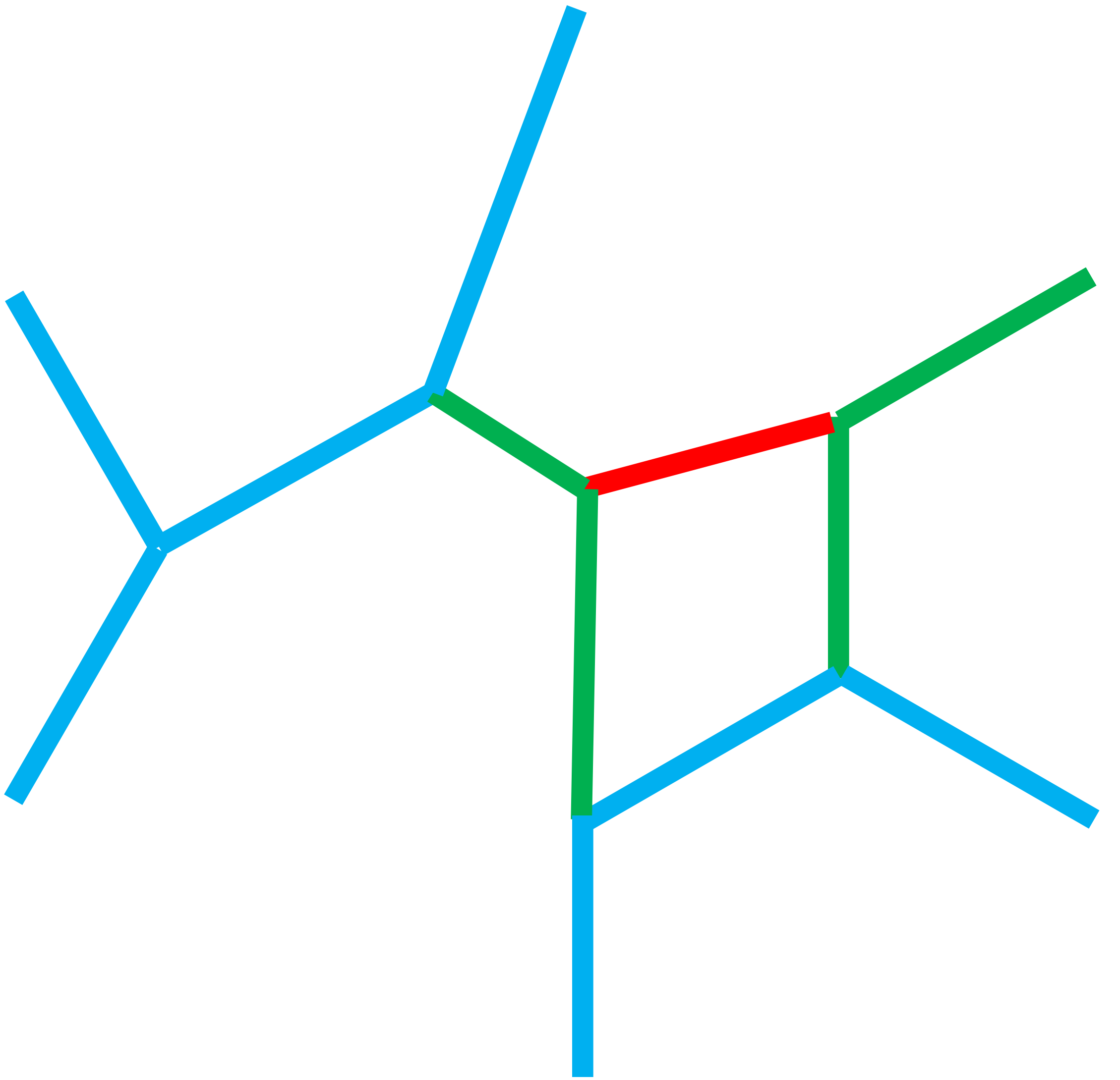

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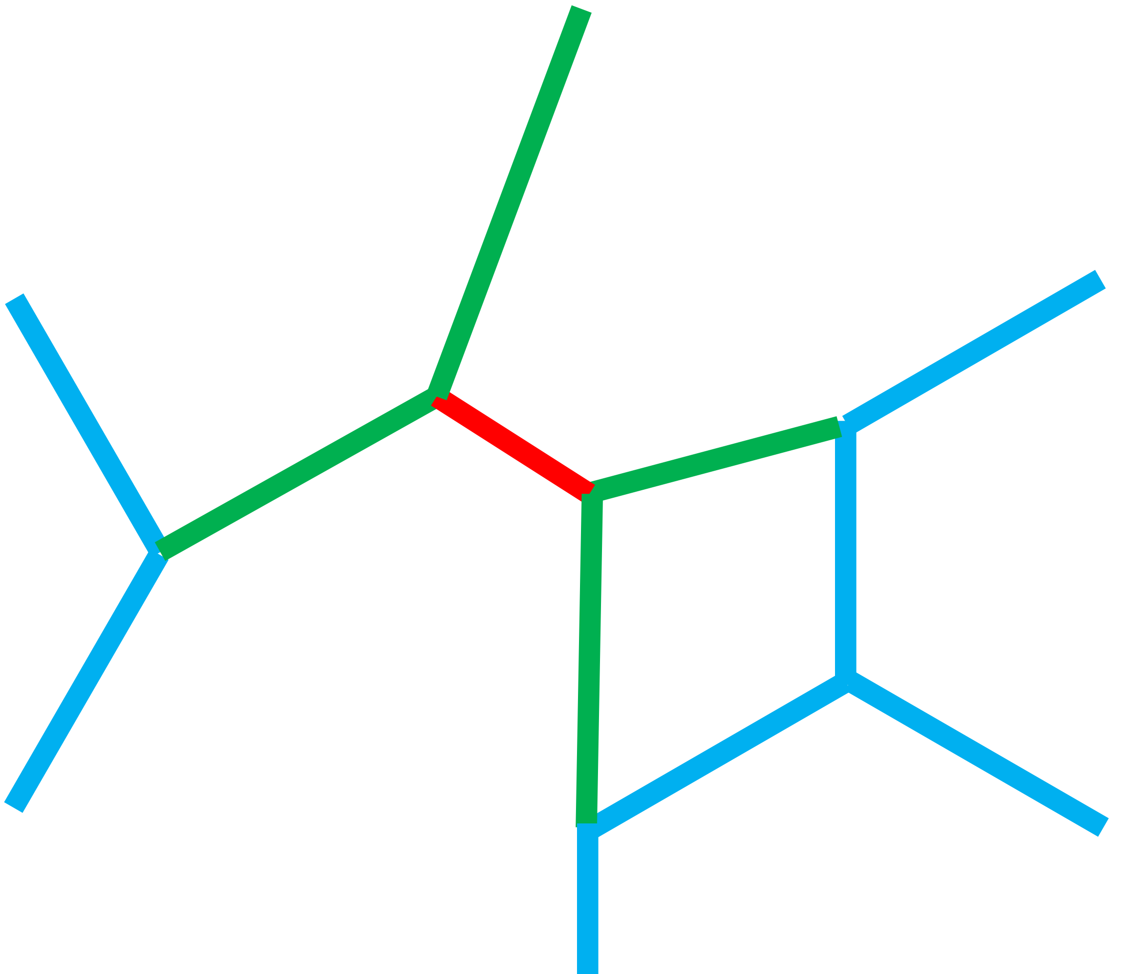

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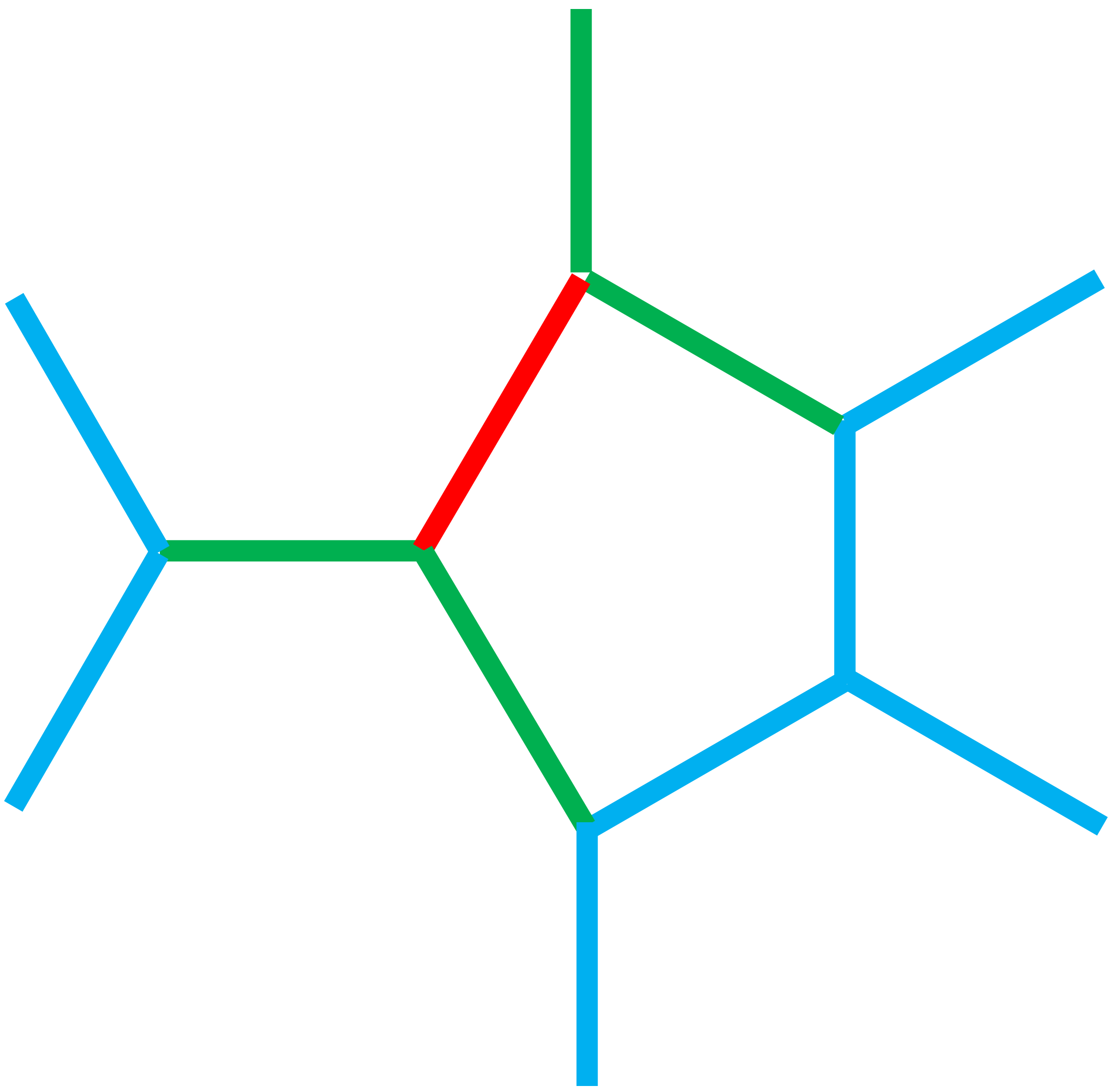

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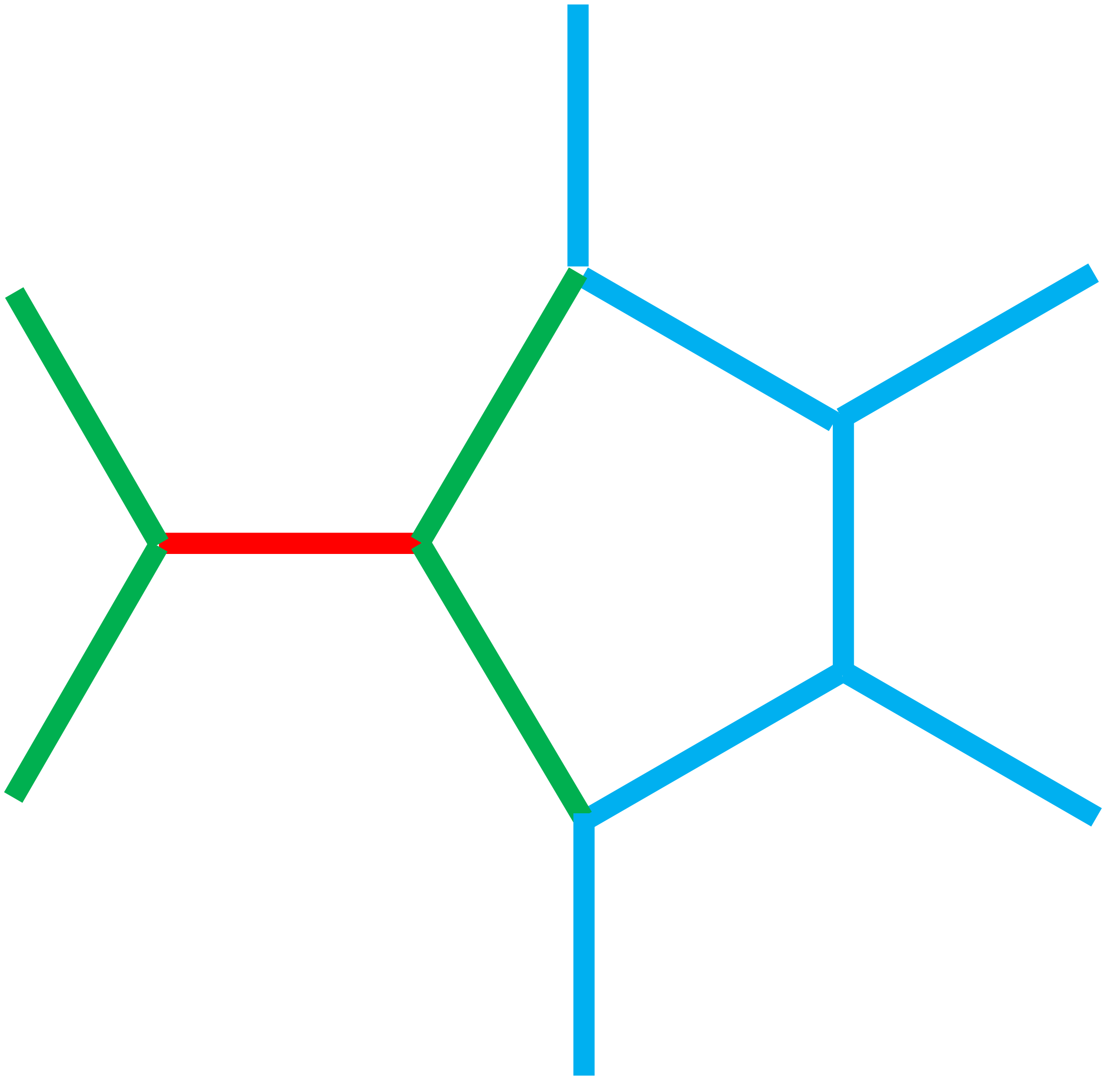

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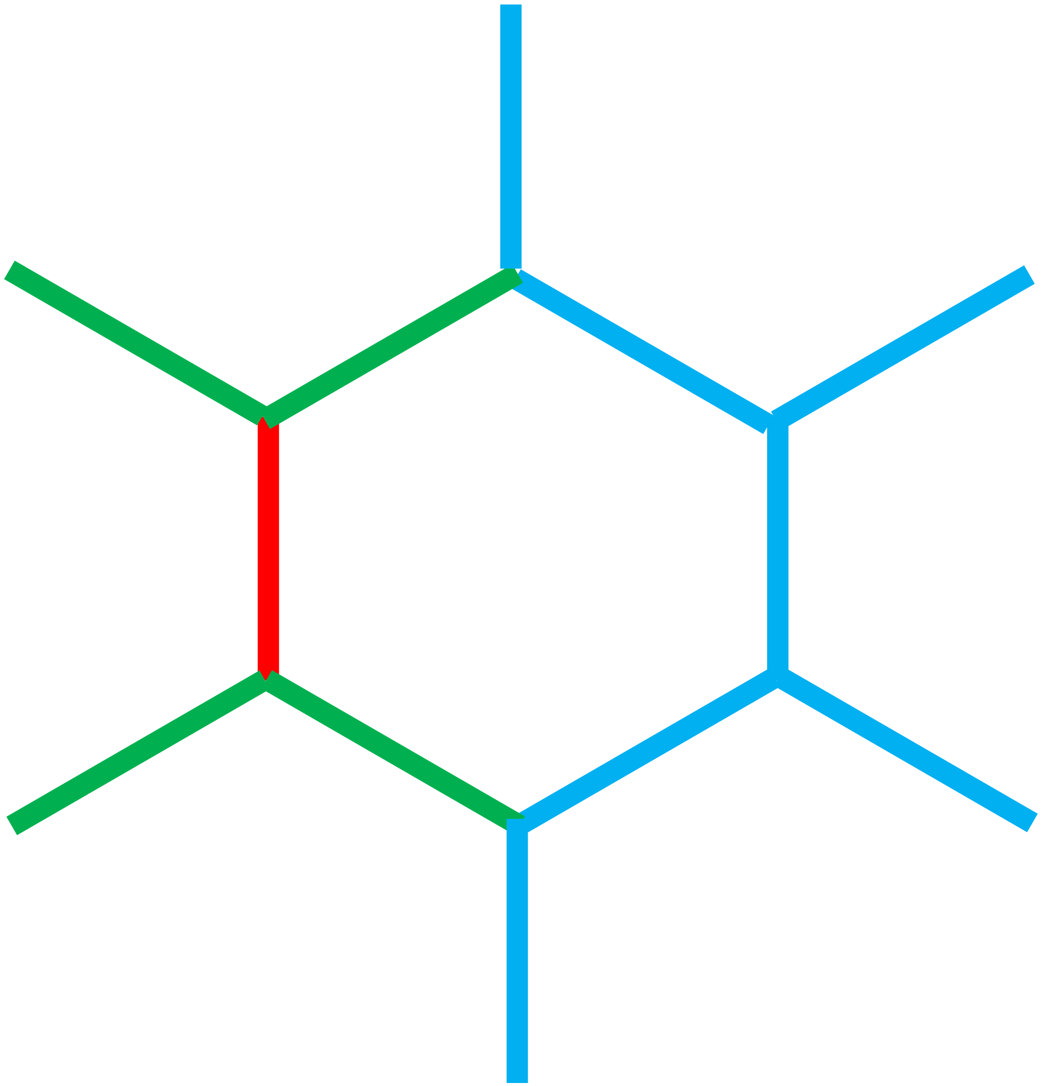

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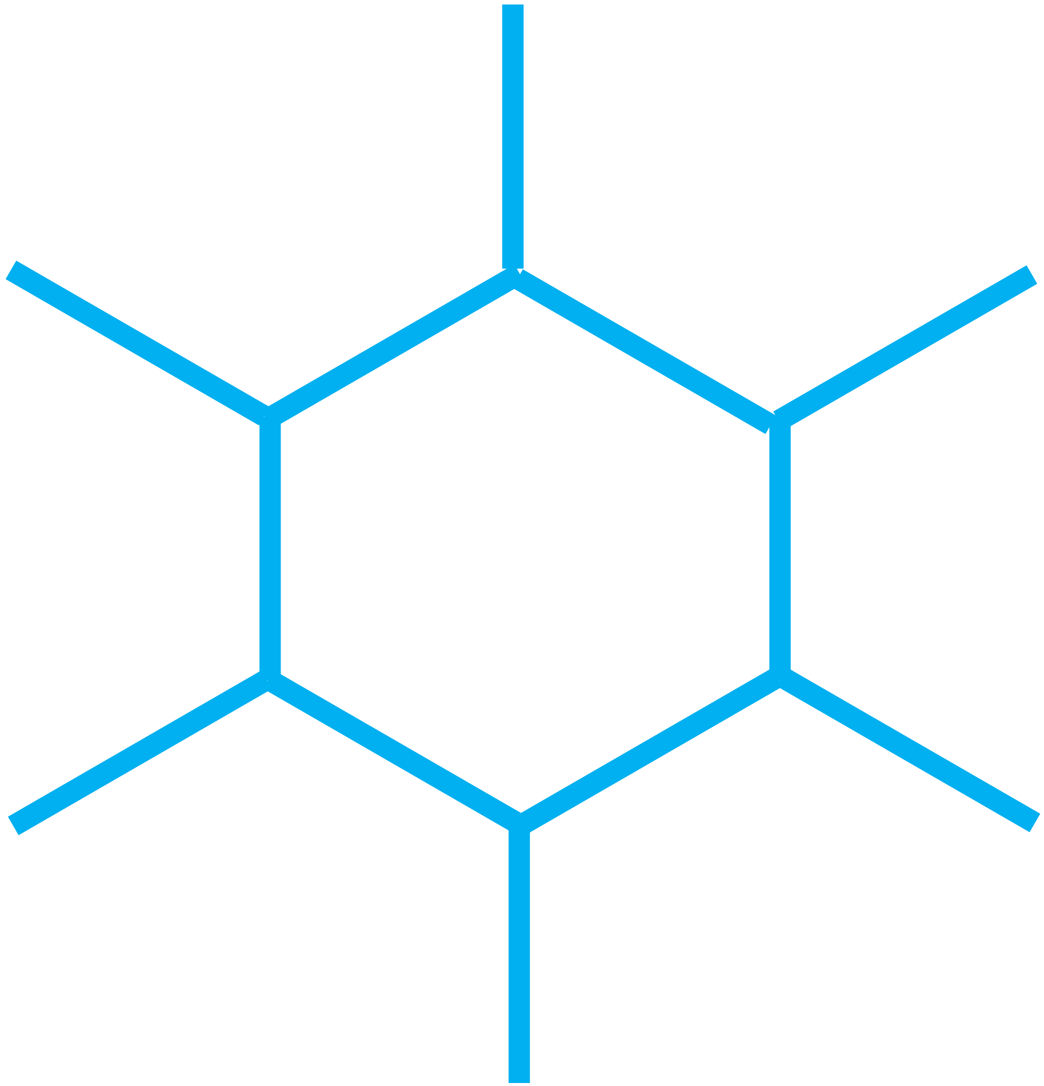
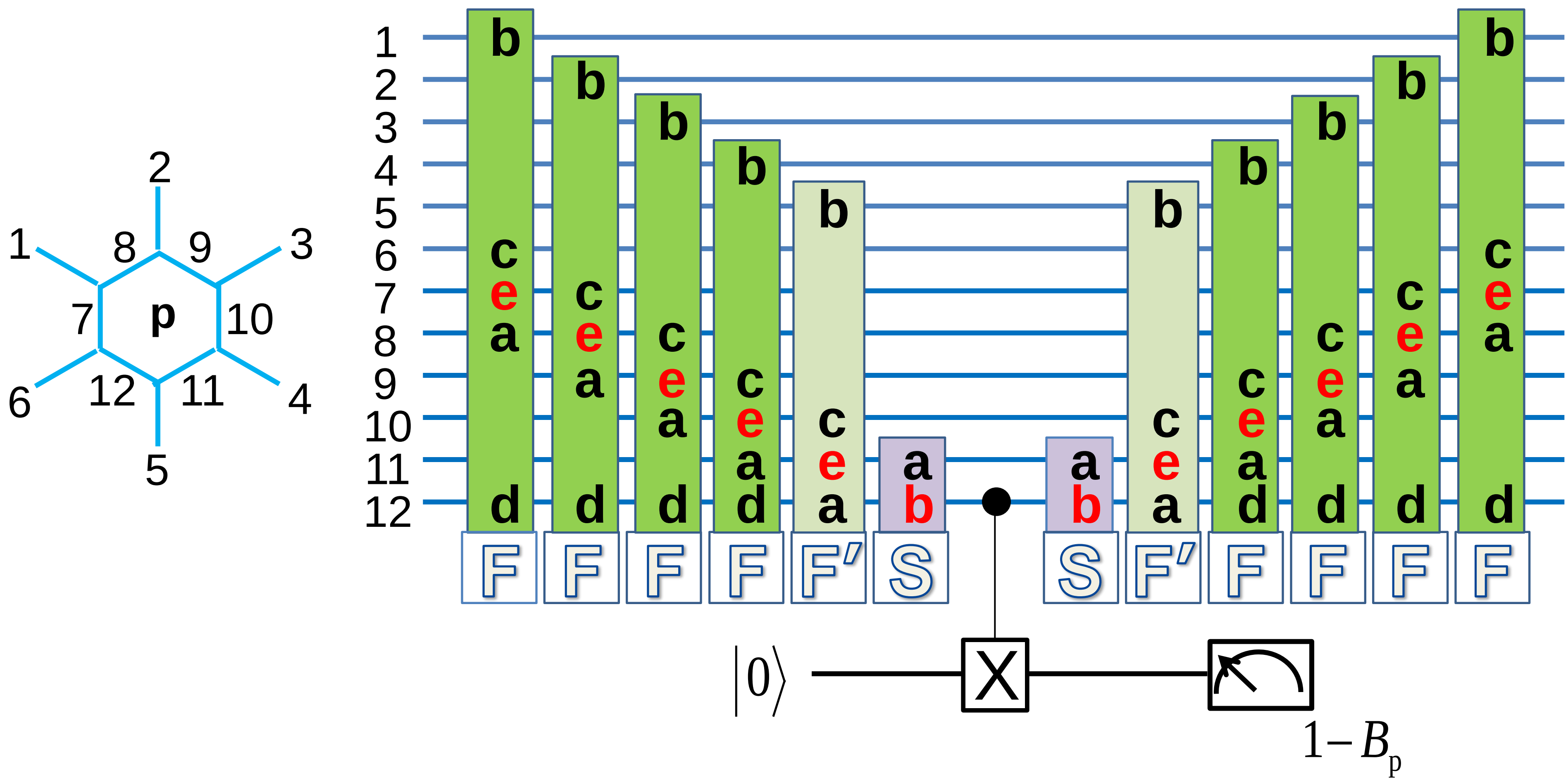


Quantum Circuit for Measuring B_p



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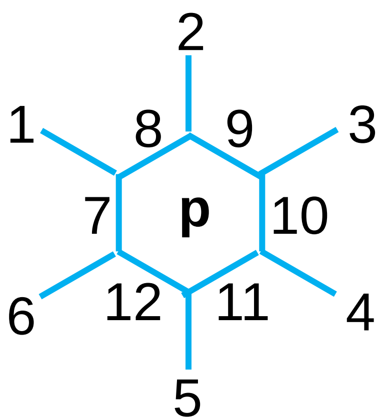
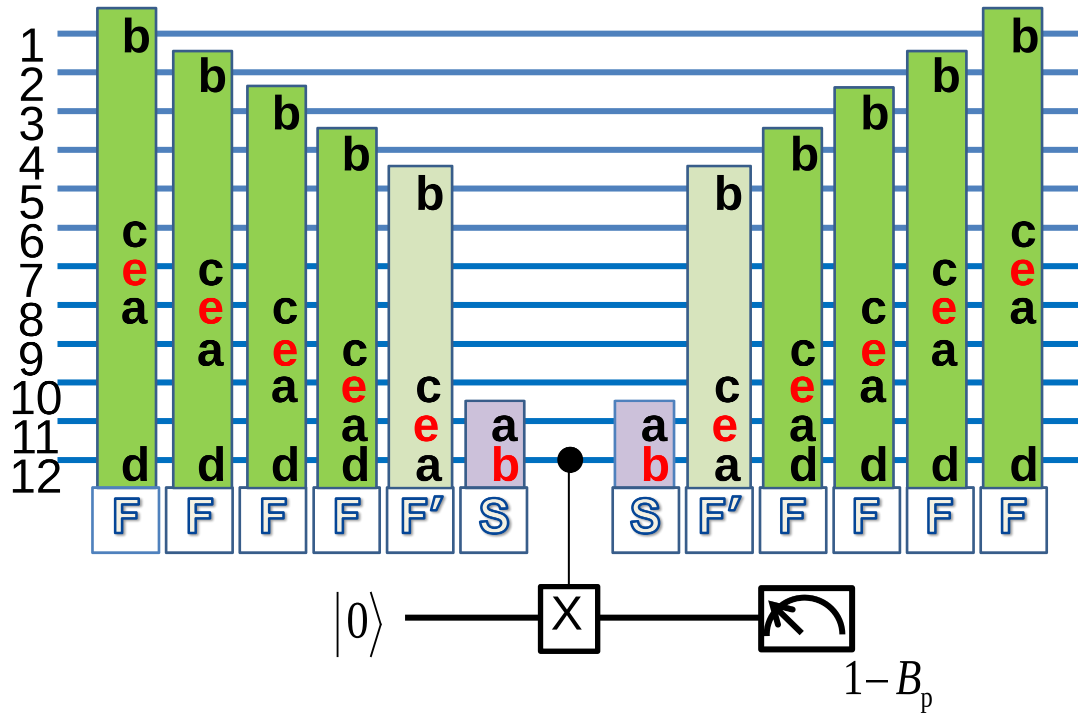
Gate Count





8	5-qubit Toffoli Gates
2	4-qubit Toffoli Gates
10	3-qubit Toffoli Gates
43	CNOT Gates
24	Single Qubit Gates

371	CNOT Gates
392	Single Qubit Rotations

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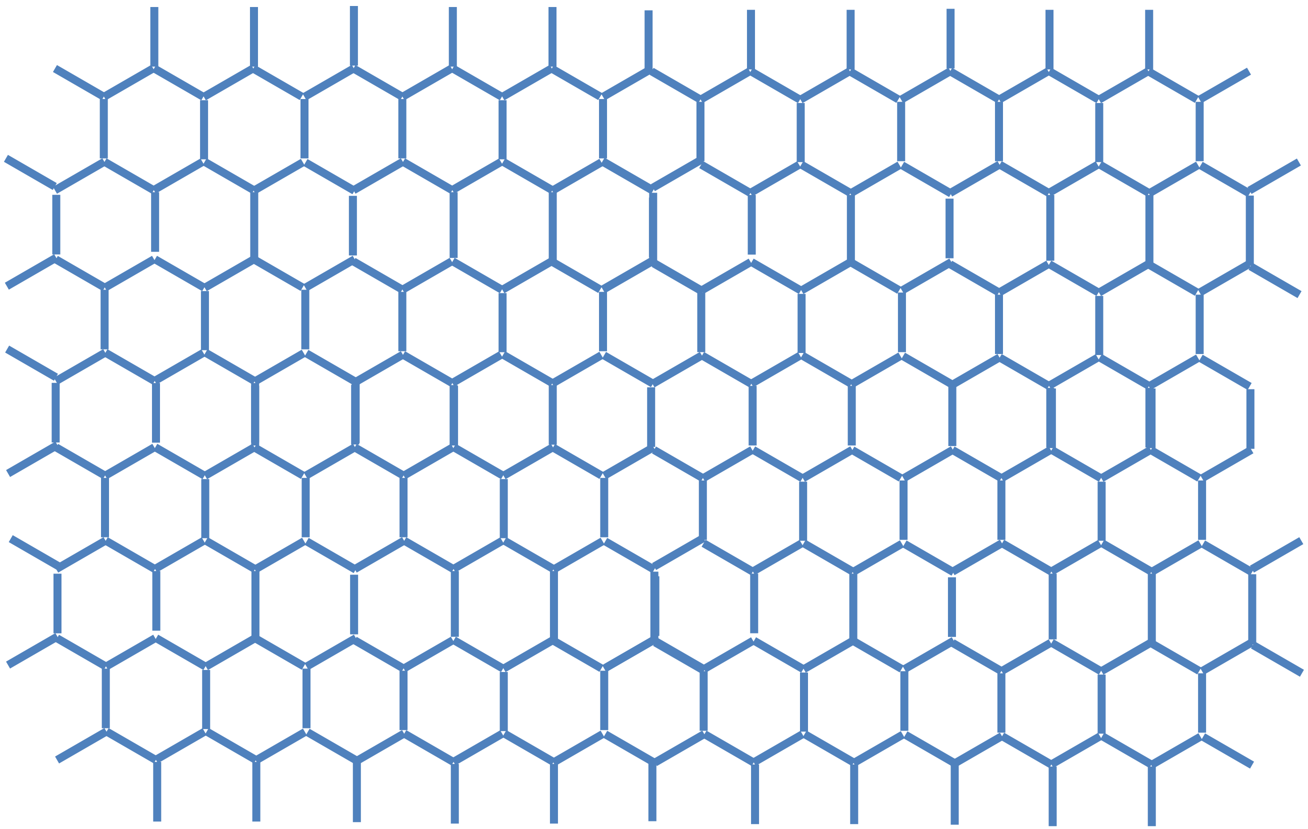
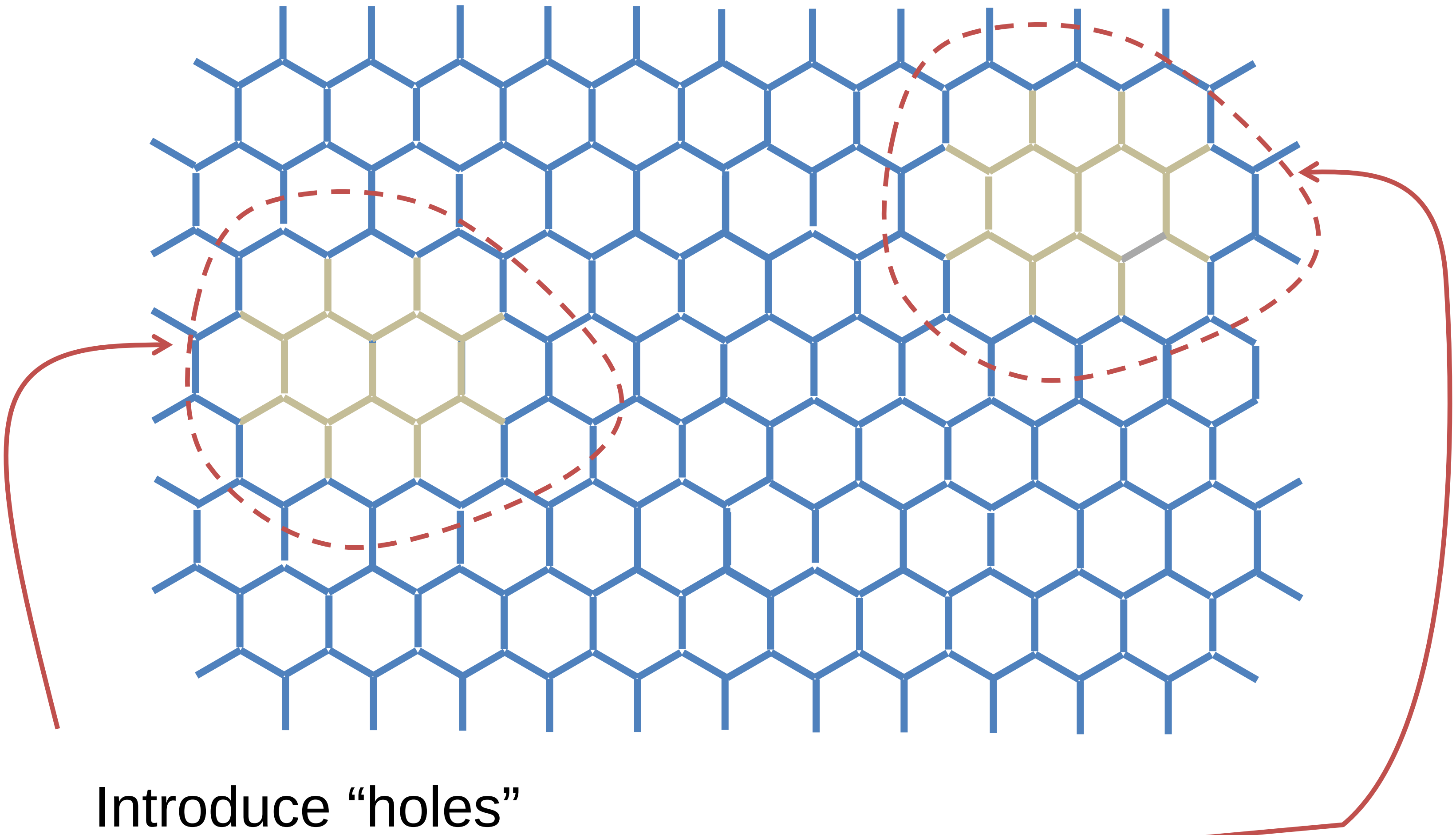
Creating Excited States: Fibonacci Anyons

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



Creating Excited States: Fibonacci Anyons

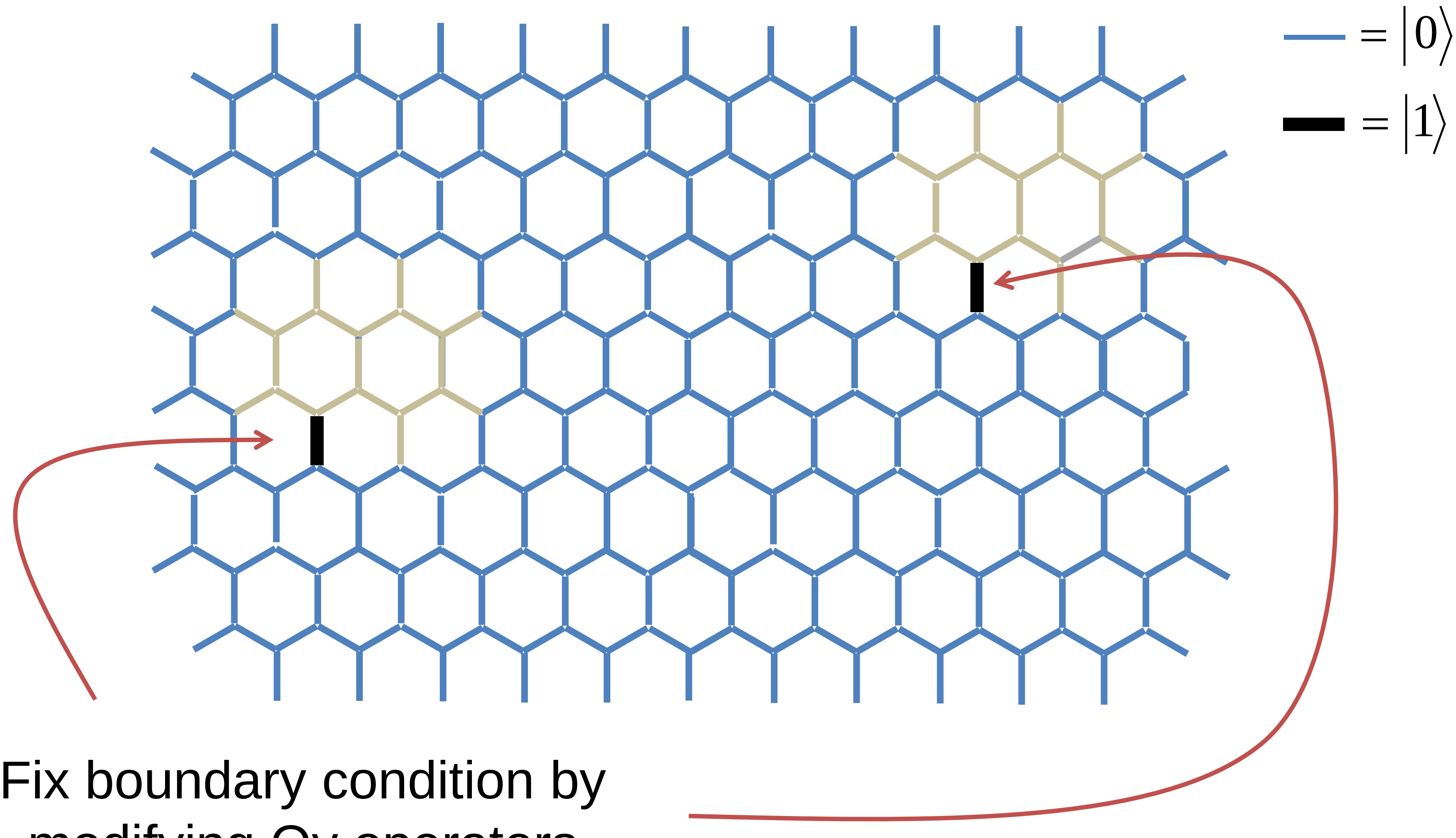
Konig, Kuperberg, Reichardt, Ann. Phys. 2010



Introduce "holes"
(Stop measuring Q_v and B_p
inside hole)

Creating Excited States: Fibonacci Anyons

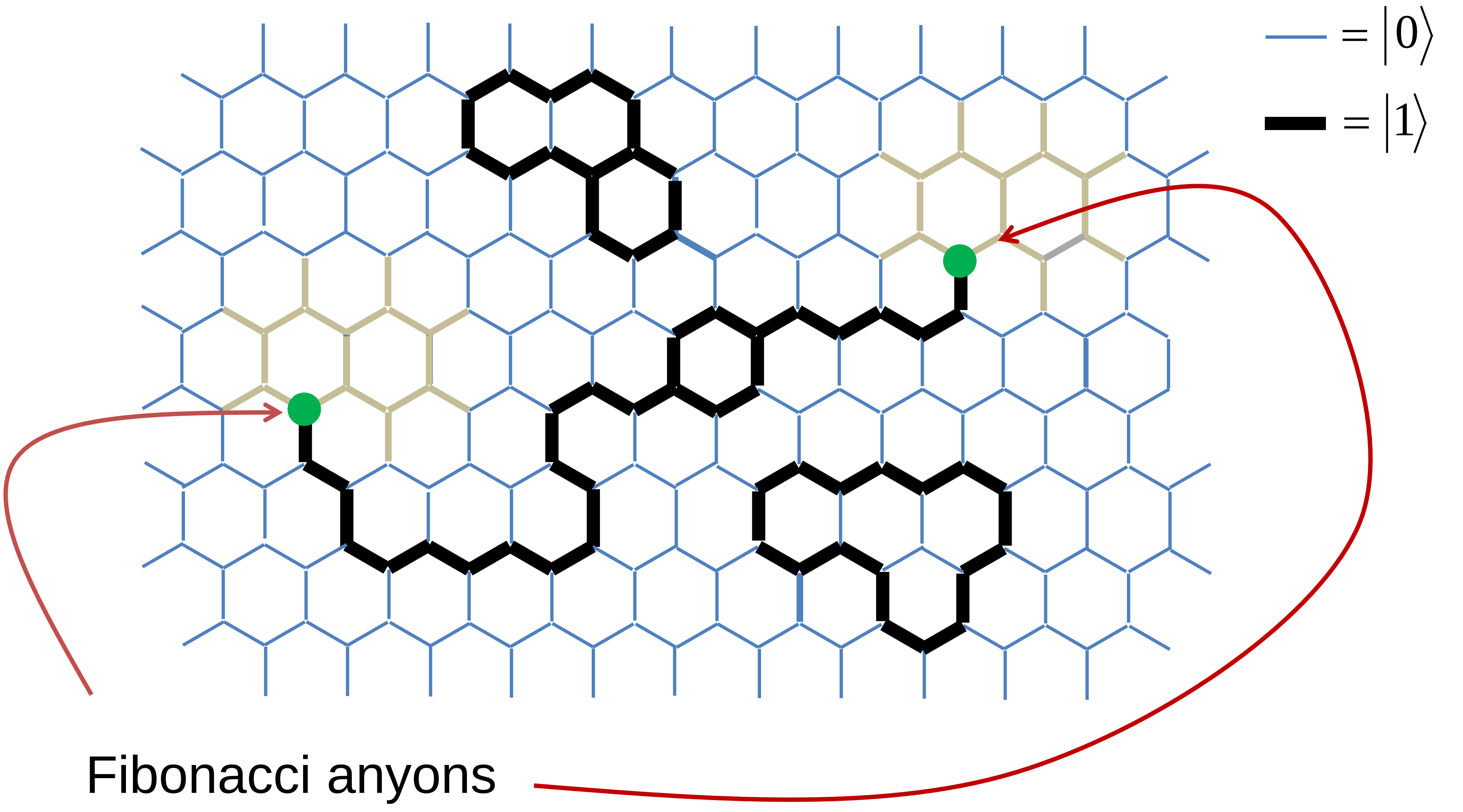
Konig, Kuperberg, Reichardt, Ann. Phys. 2010

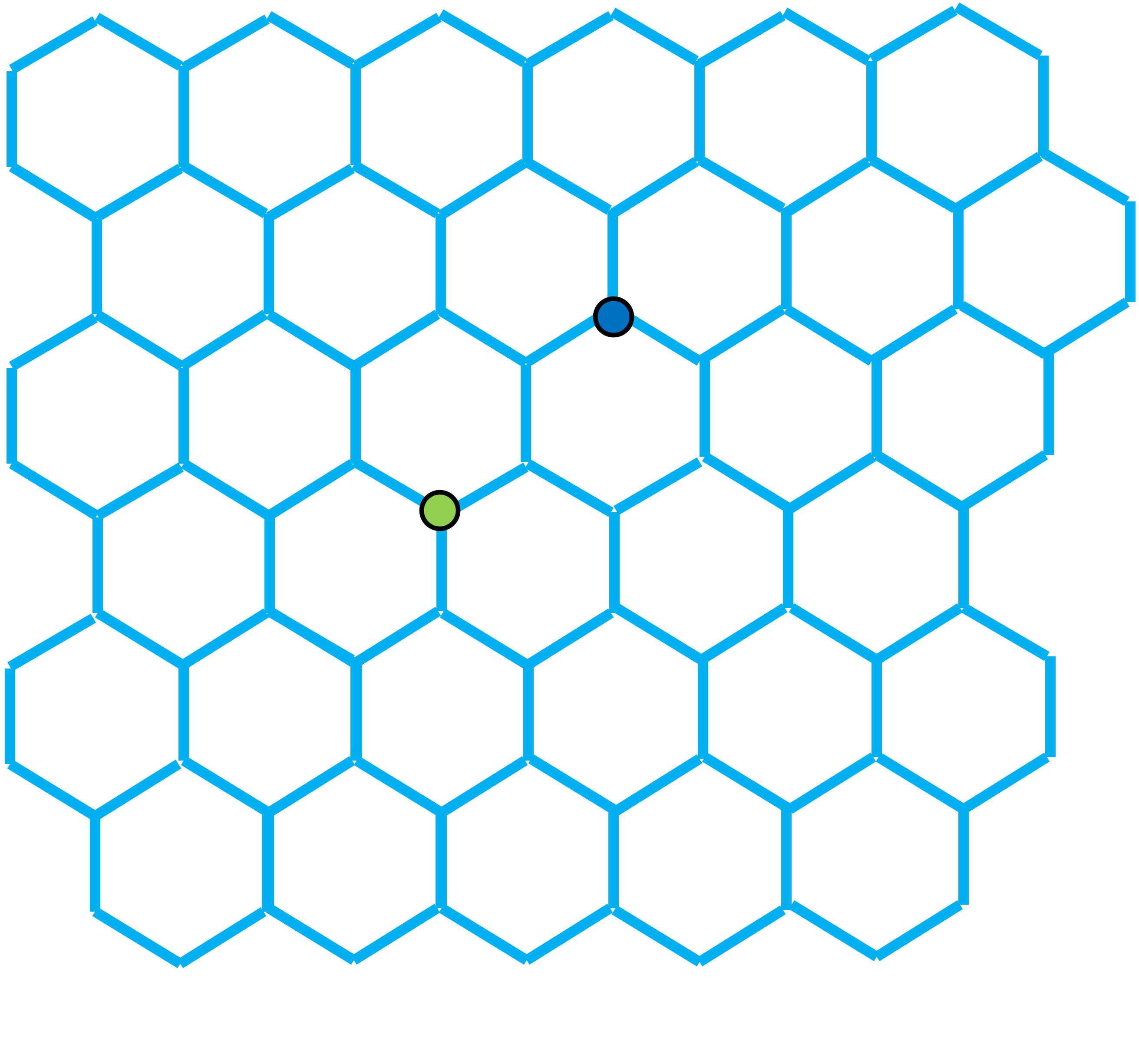


Fix boundary condition by modifying Qv operators

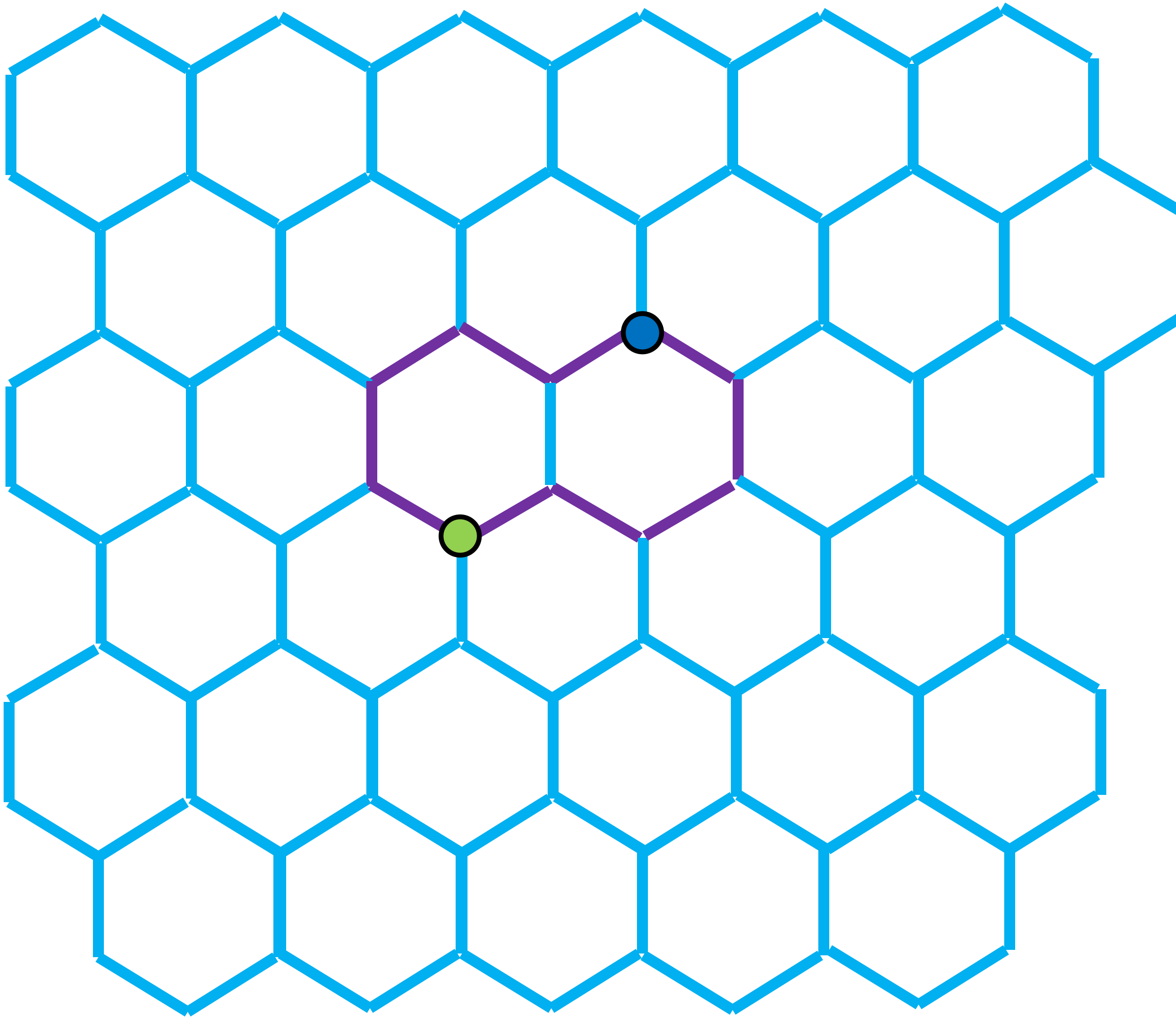
Creating Excited States: Fibonacci Anyons

Konig, Kuperberg, Reichardt, Ann. Phys. 2010

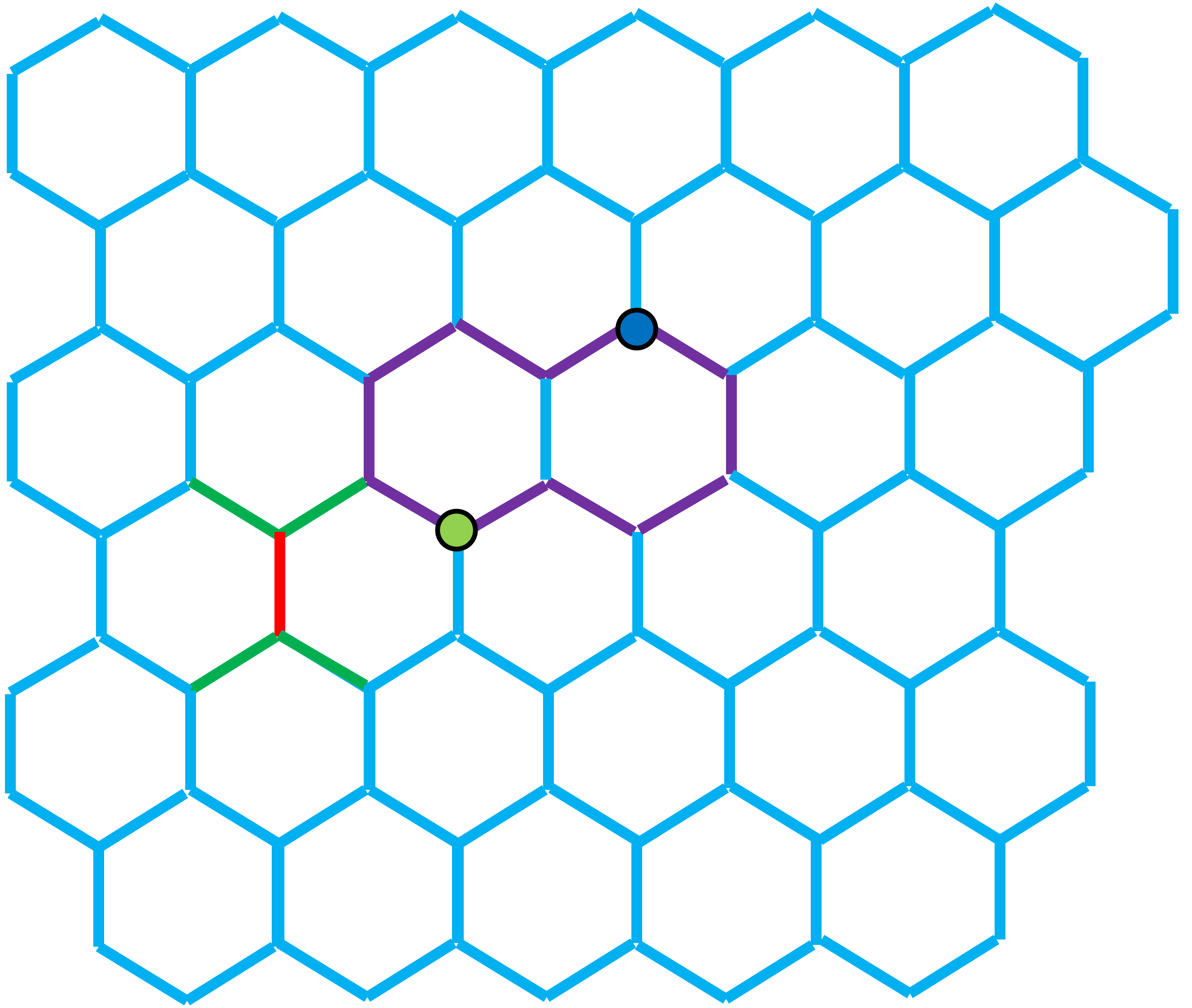




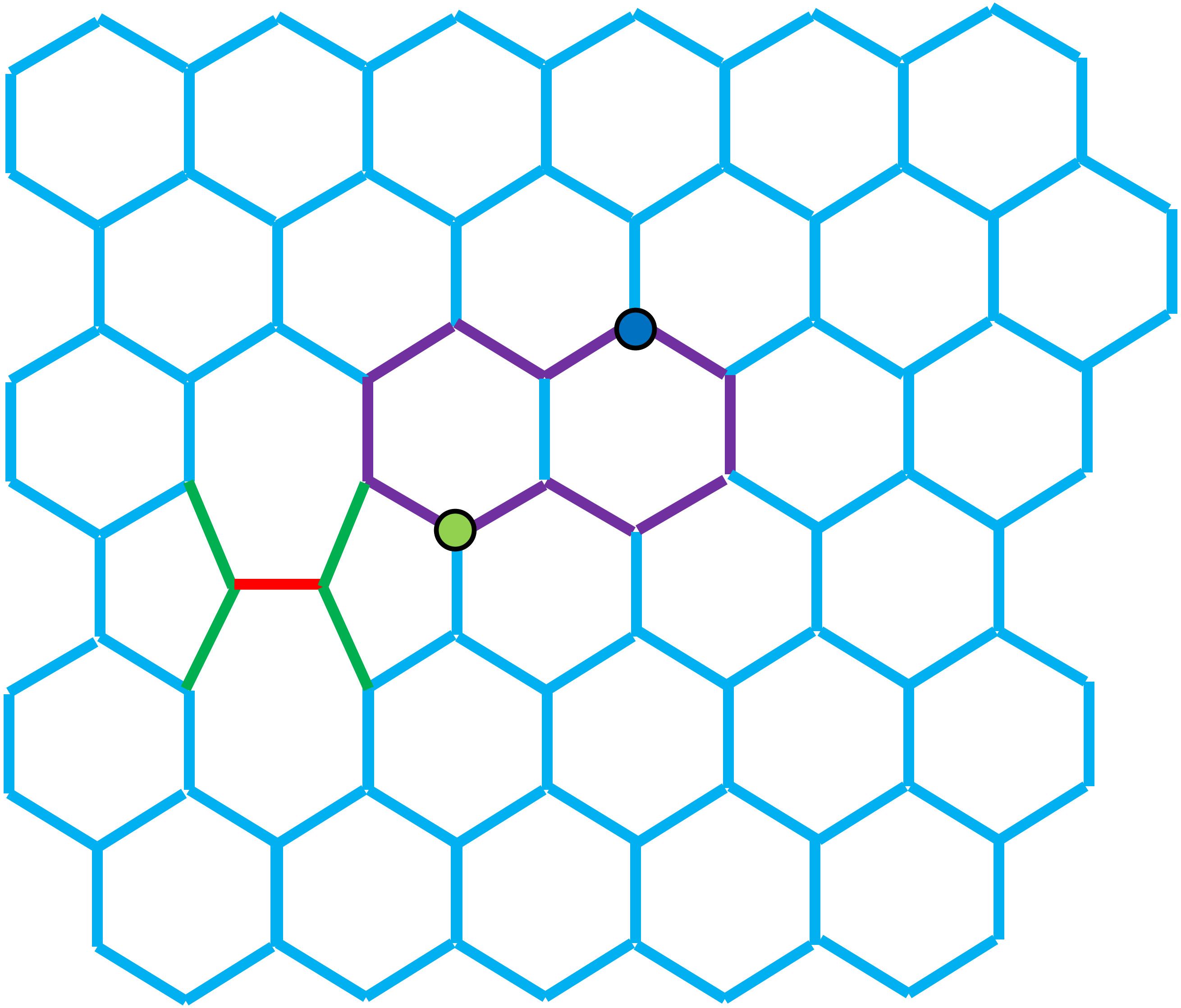
Konig, Kuperberg, Reichardt, Ann. Phys. 2010



Konig, Kuperberg, Reichardt, Ann. Phys. 2010

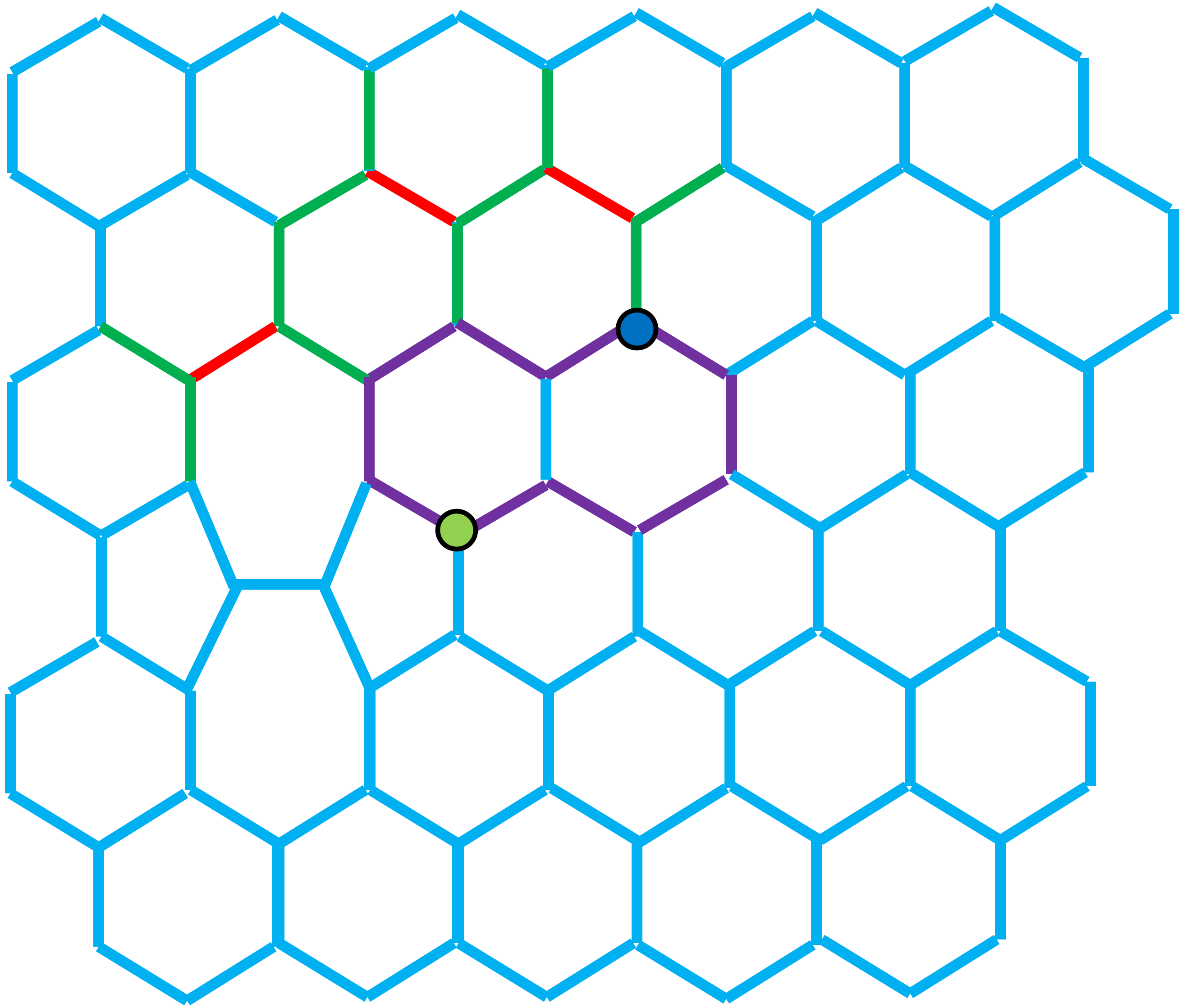


Konig, Kuperberg, Reichardt, Ann. Phys. 2010



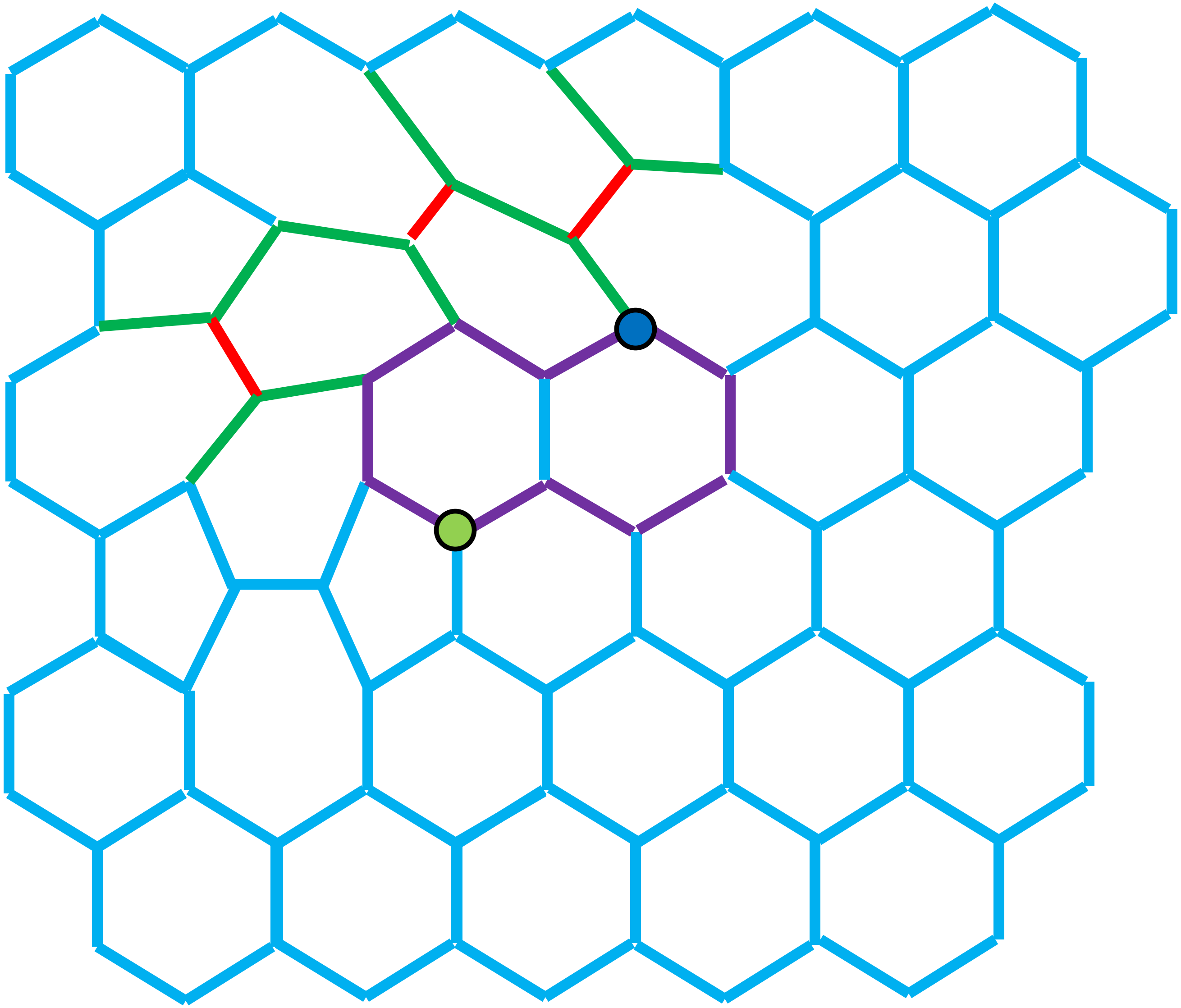
1 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



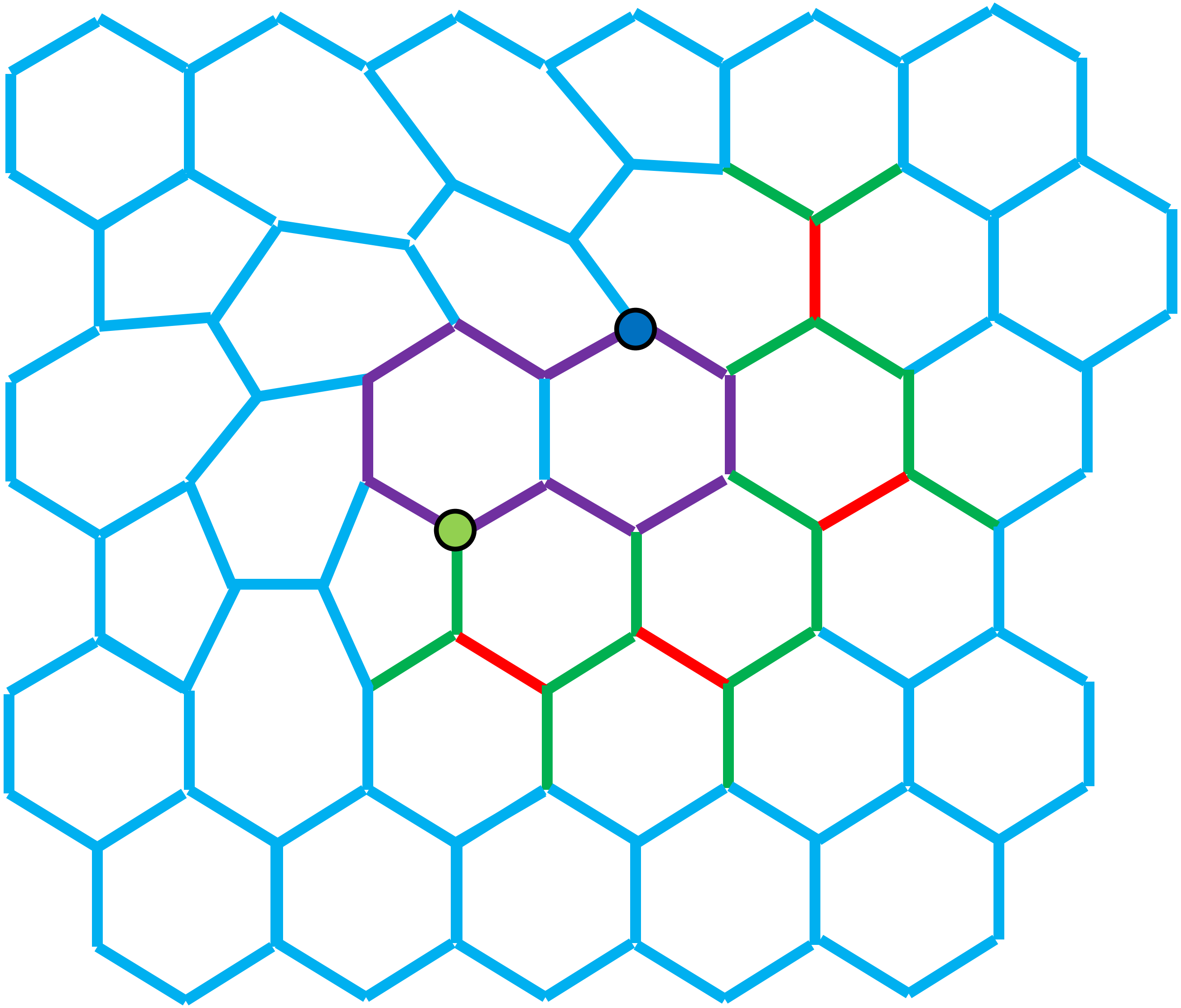
1 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



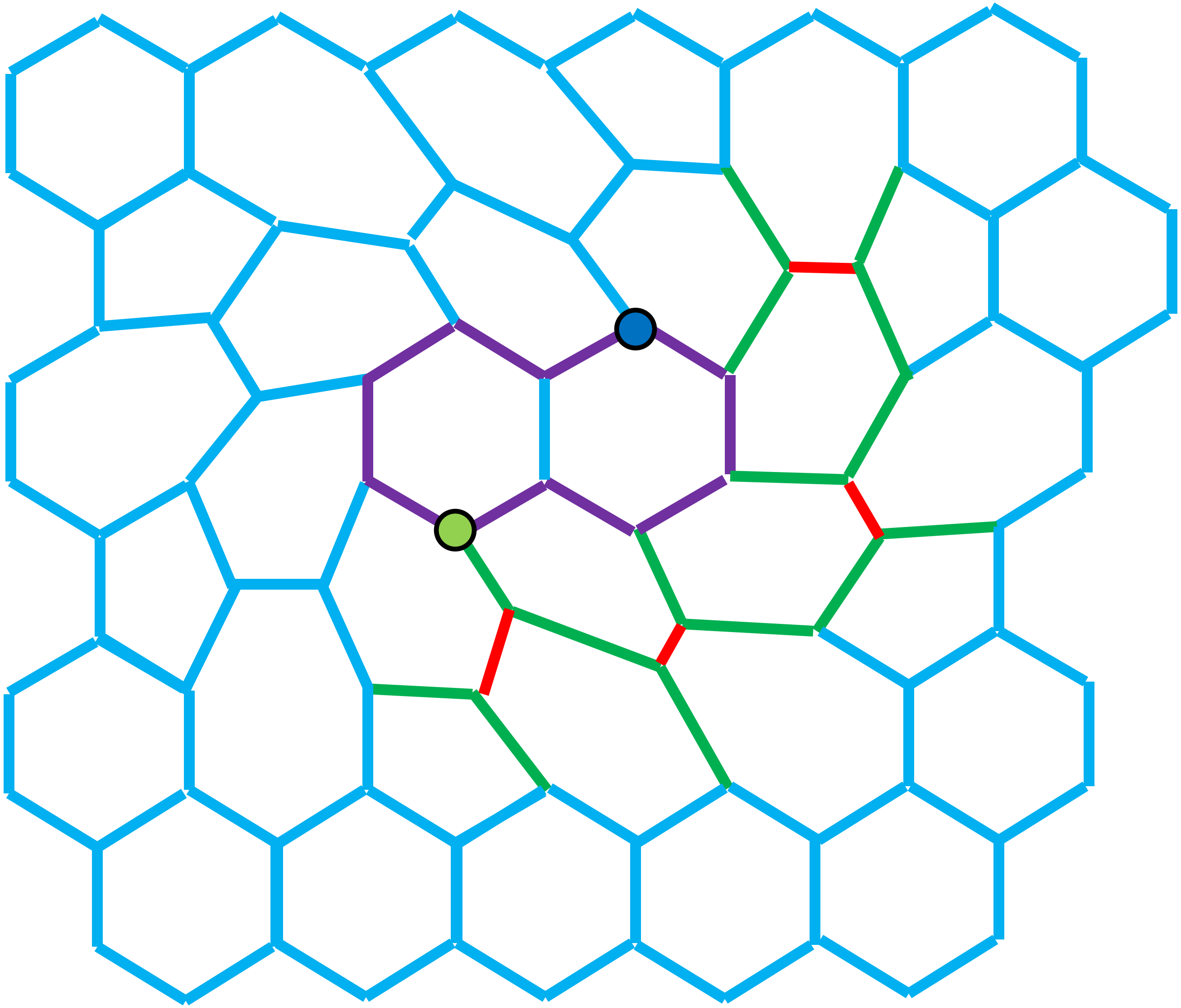
4 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



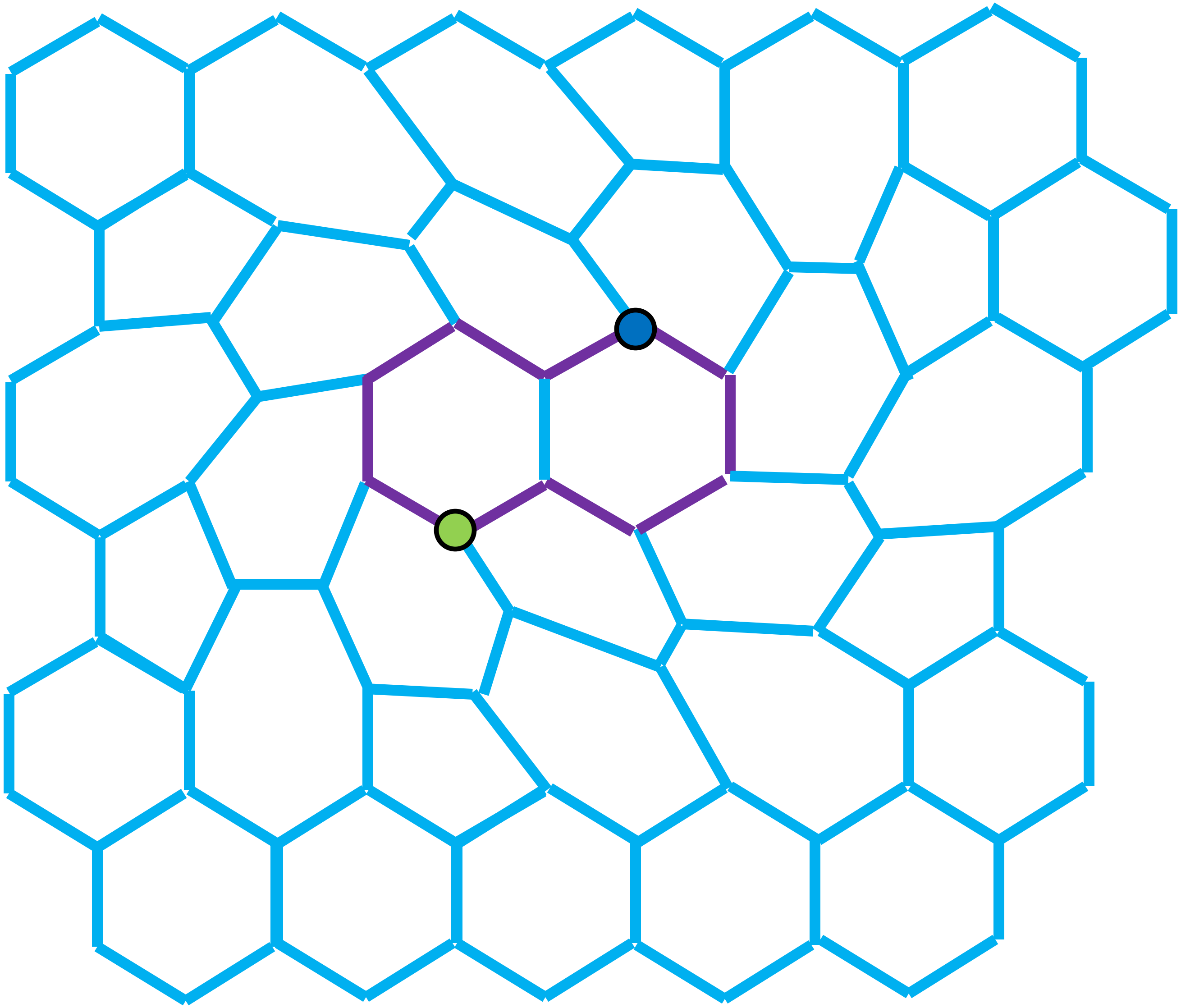
4 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



8 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



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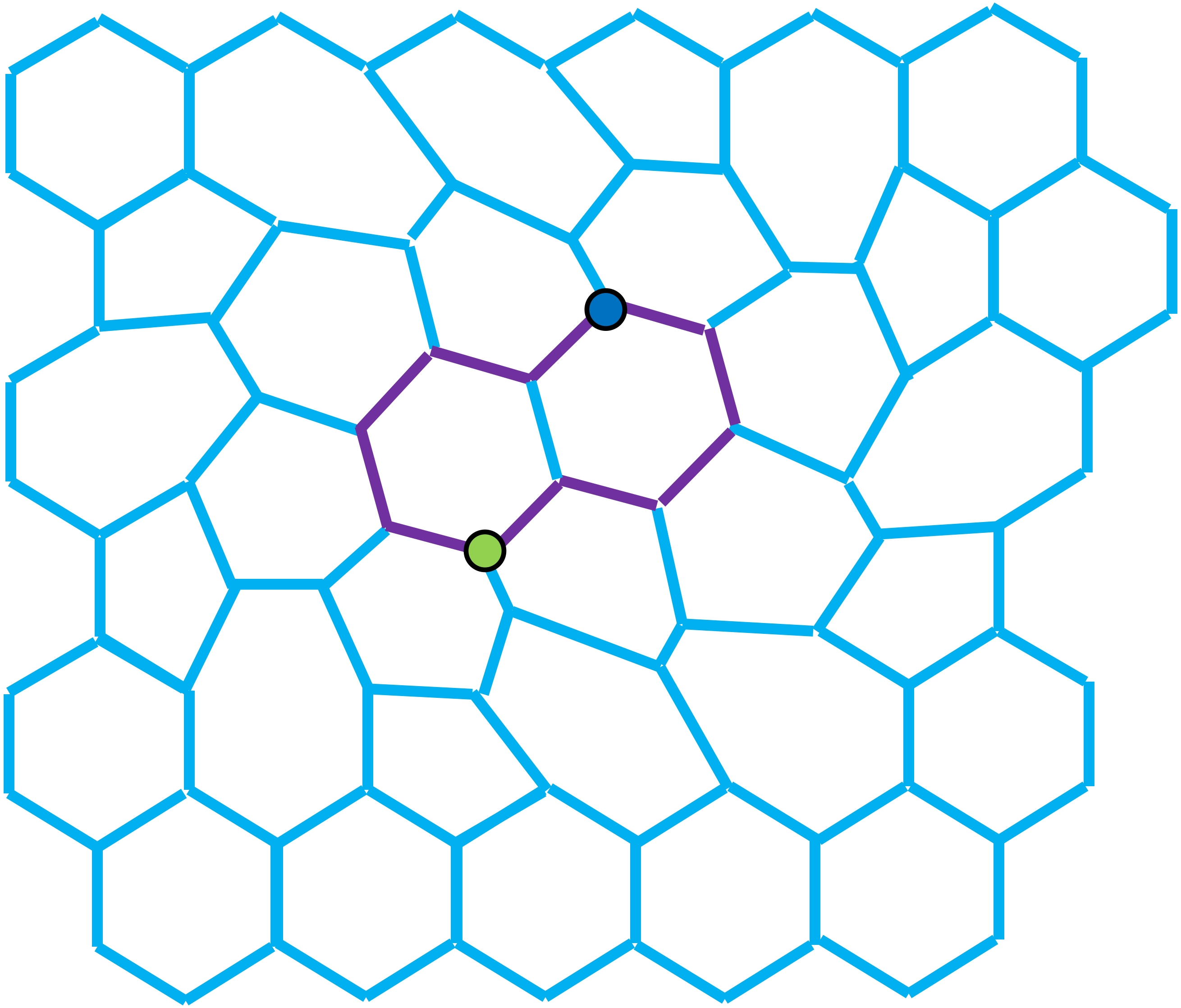
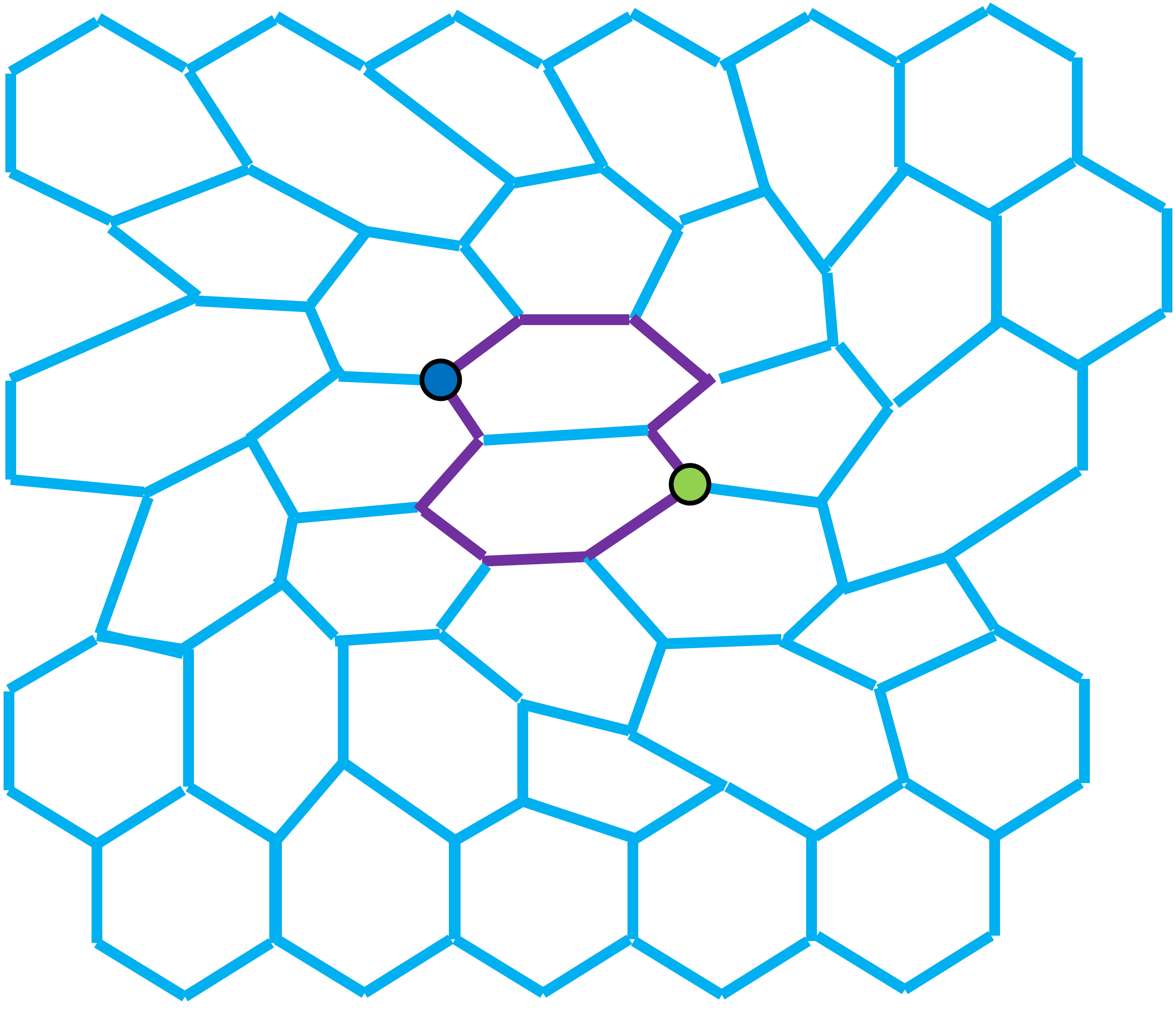
Konig, Kuperberg, Reichardt, Ann. Phys. 2010



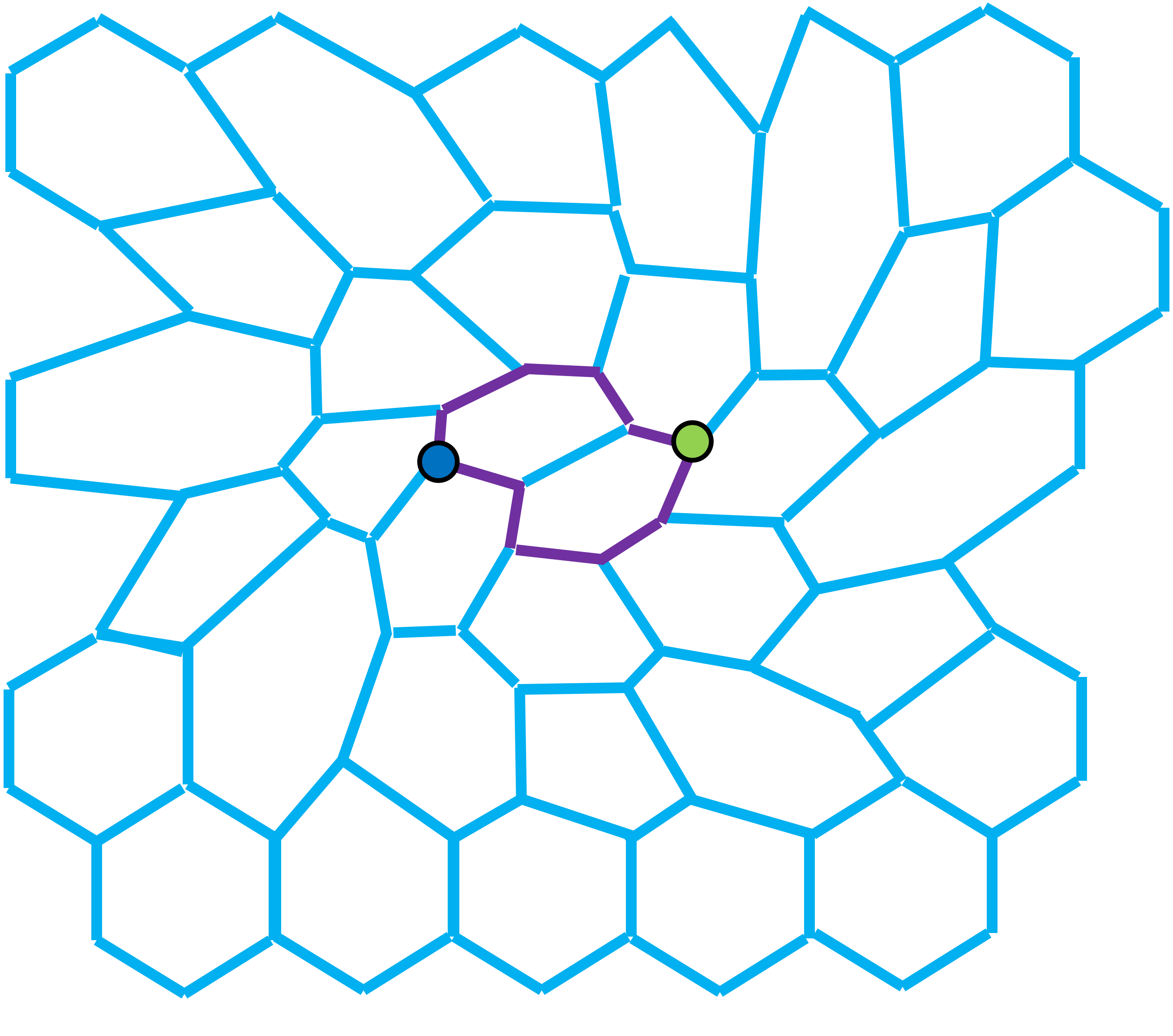
8 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



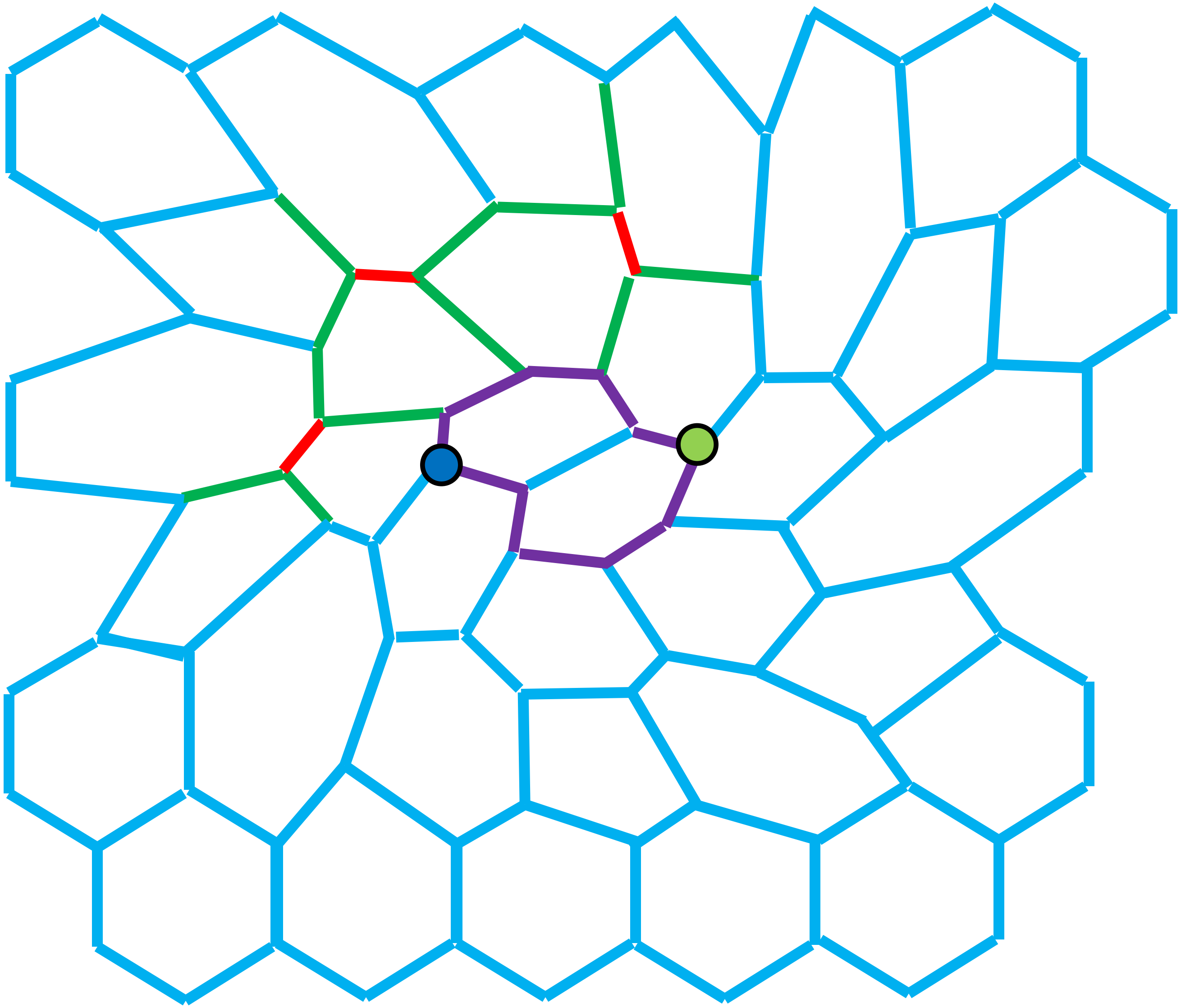
32 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



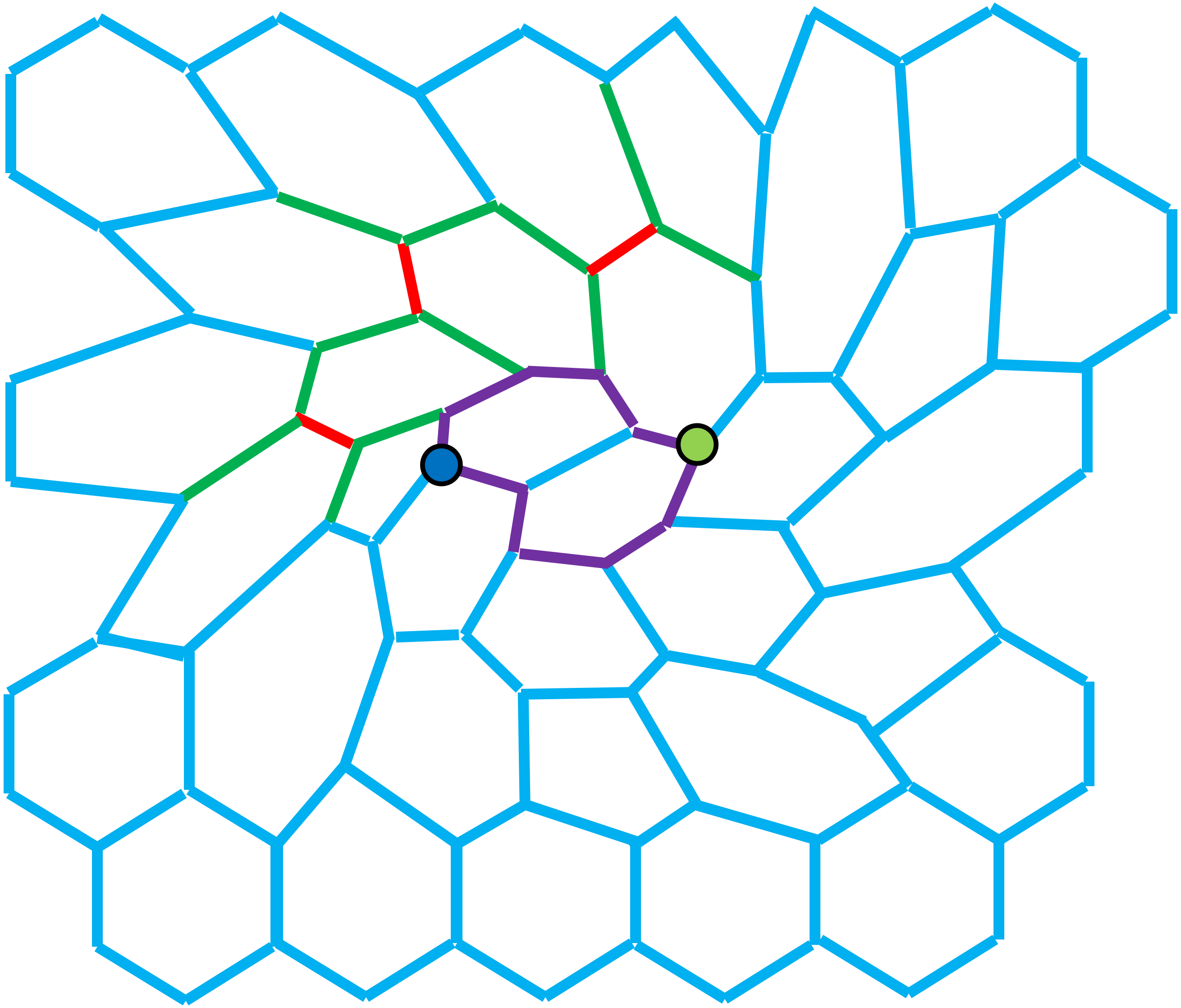
48 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



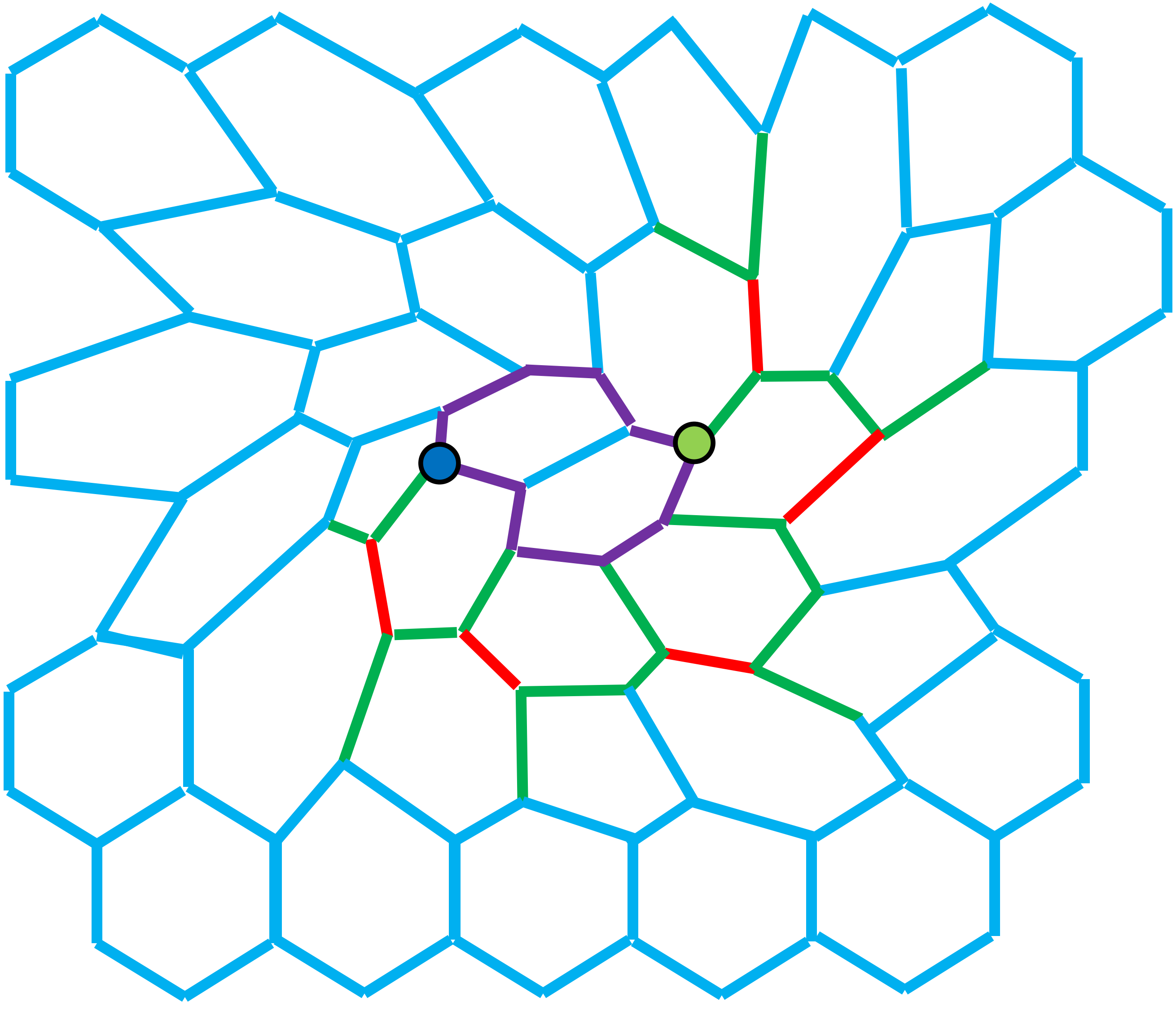
48 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



51 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



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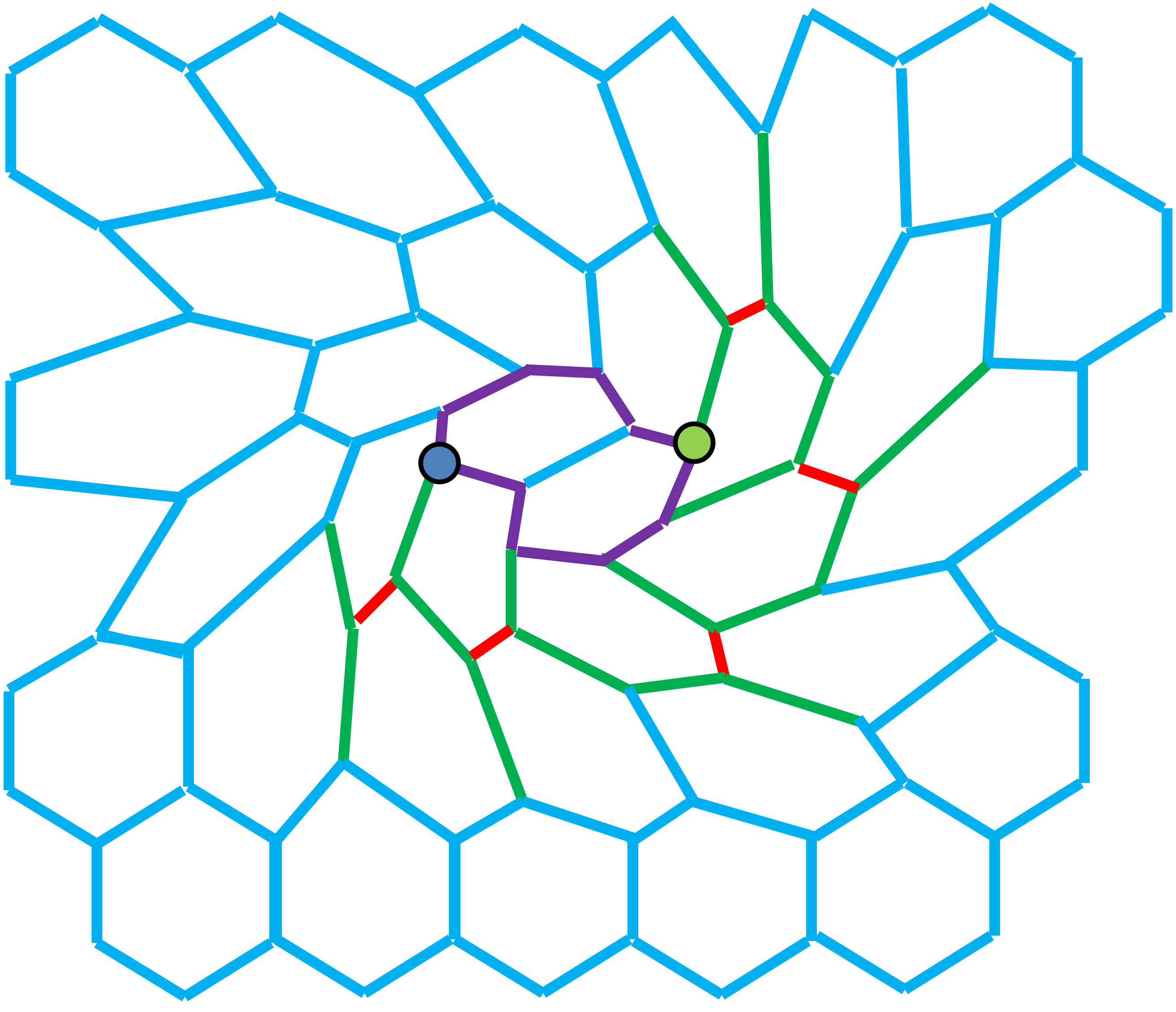
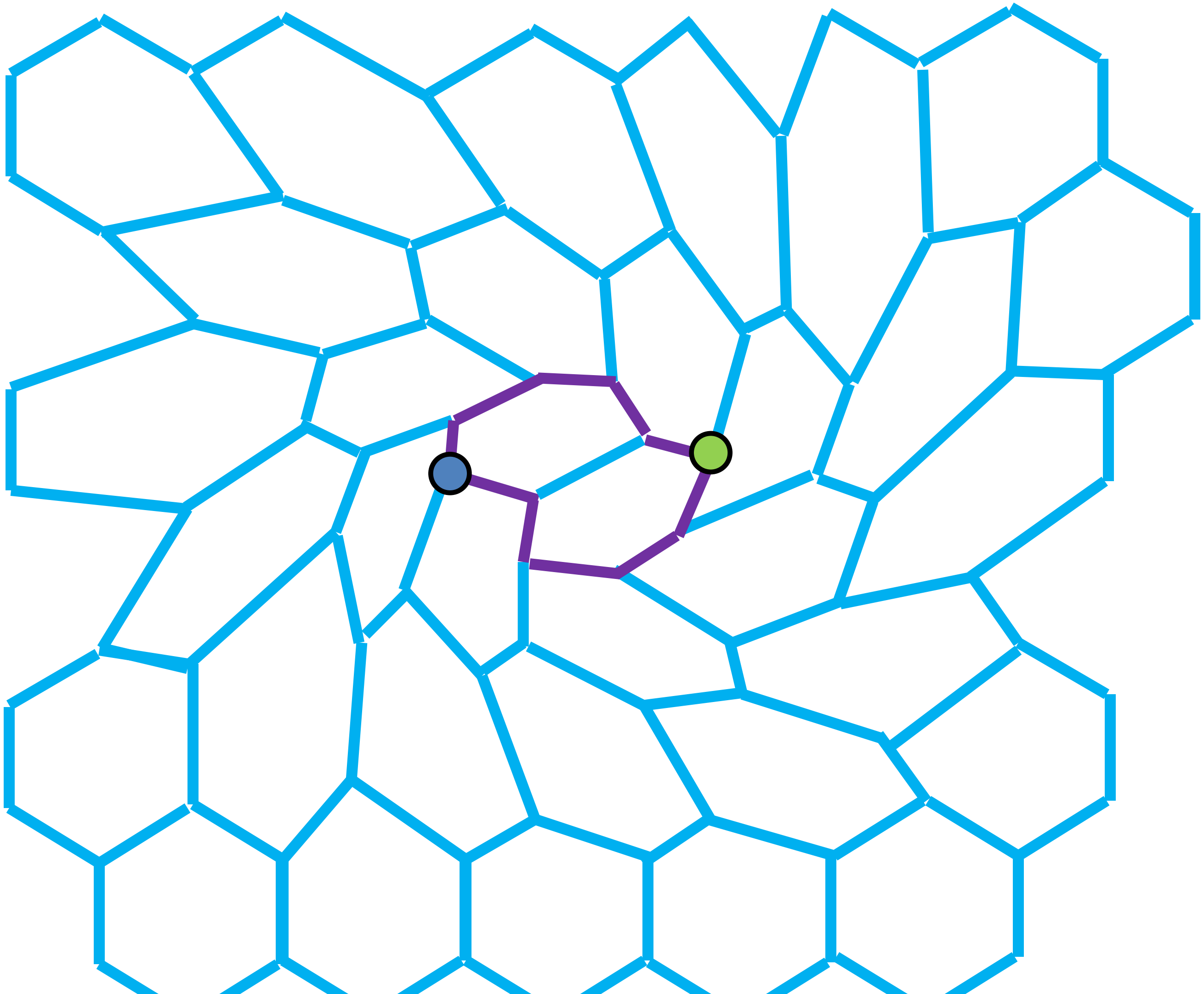
Konig, Kuperberg, Reichardt, Ann. Phys. 2010



56 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



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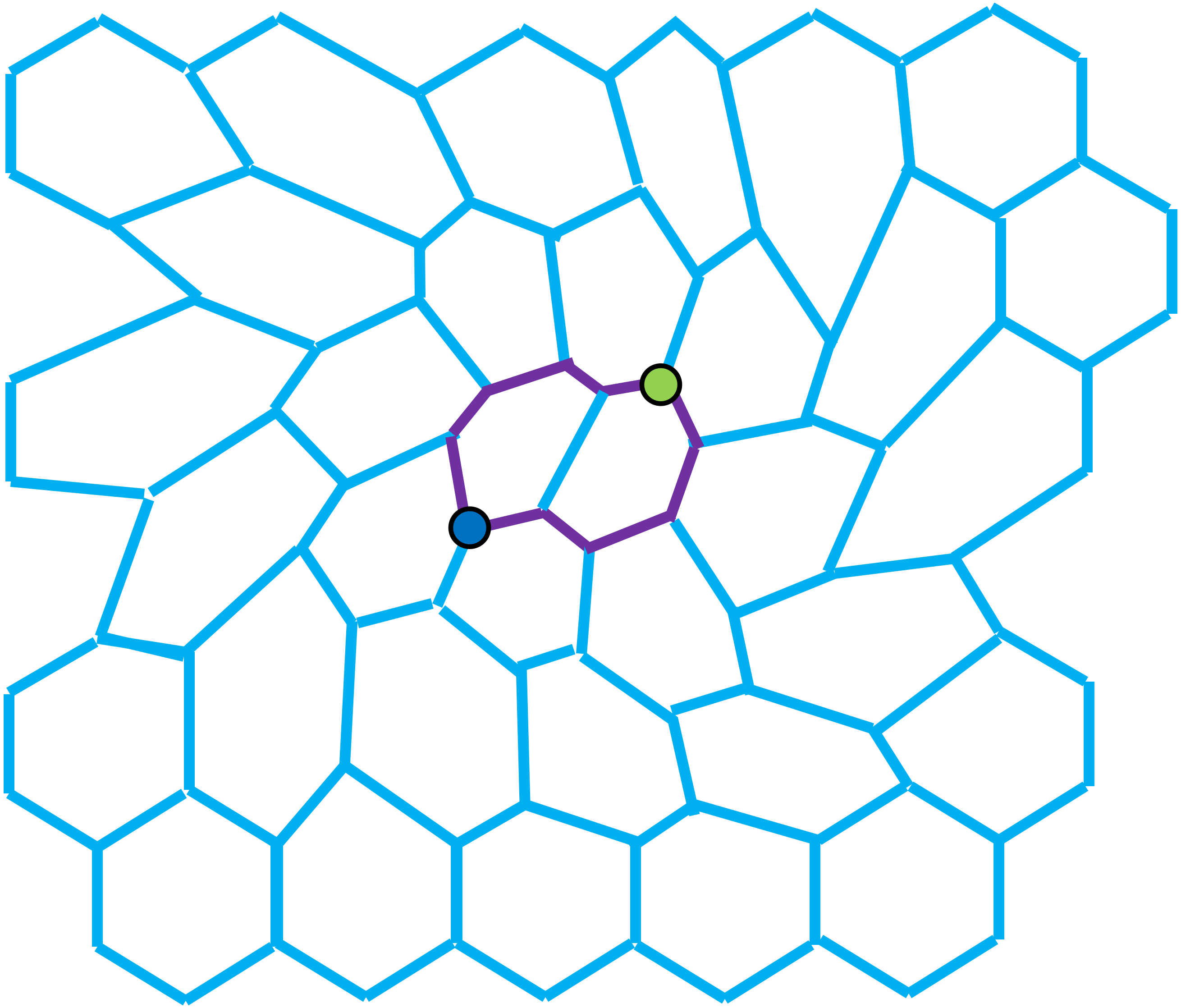
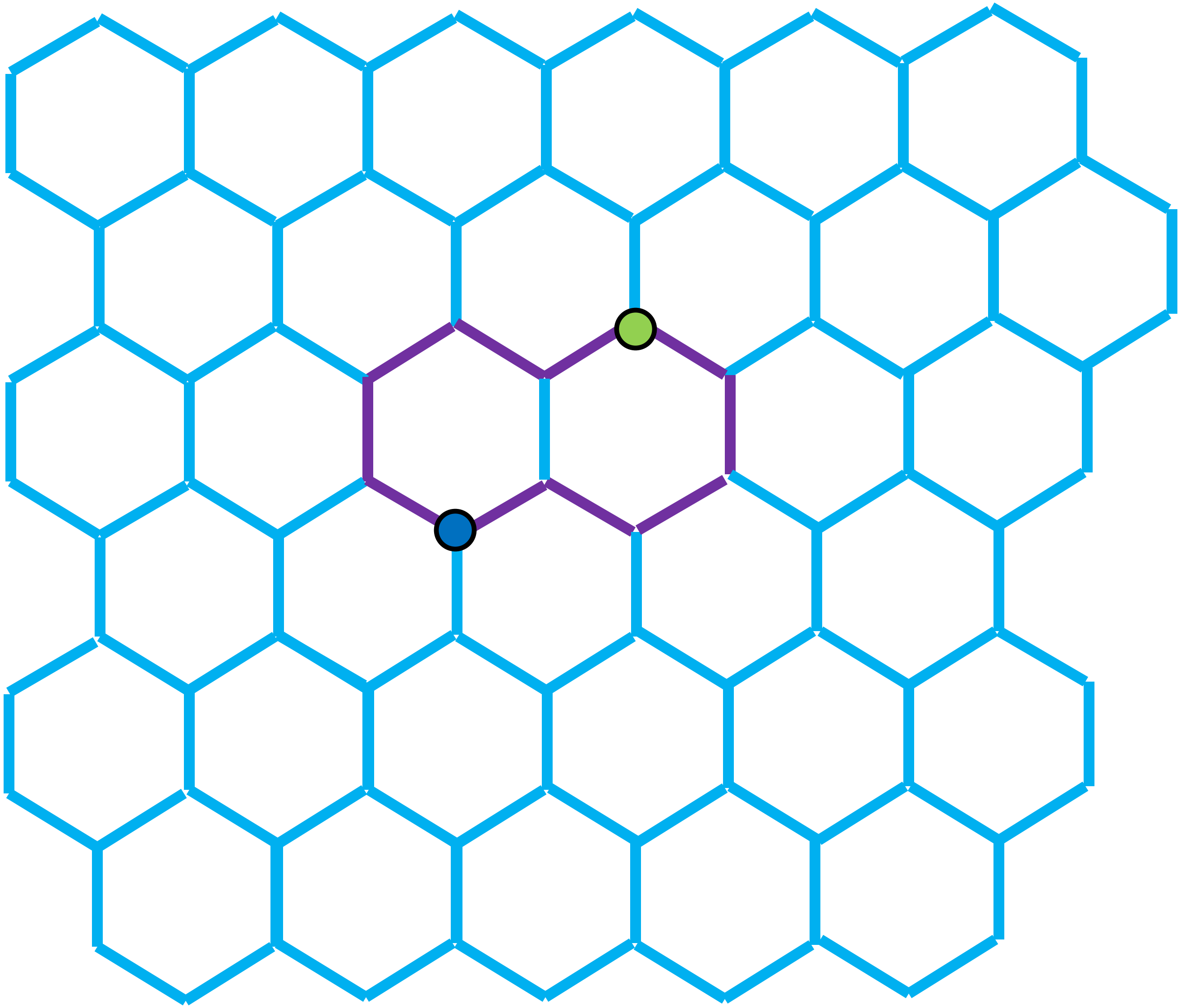


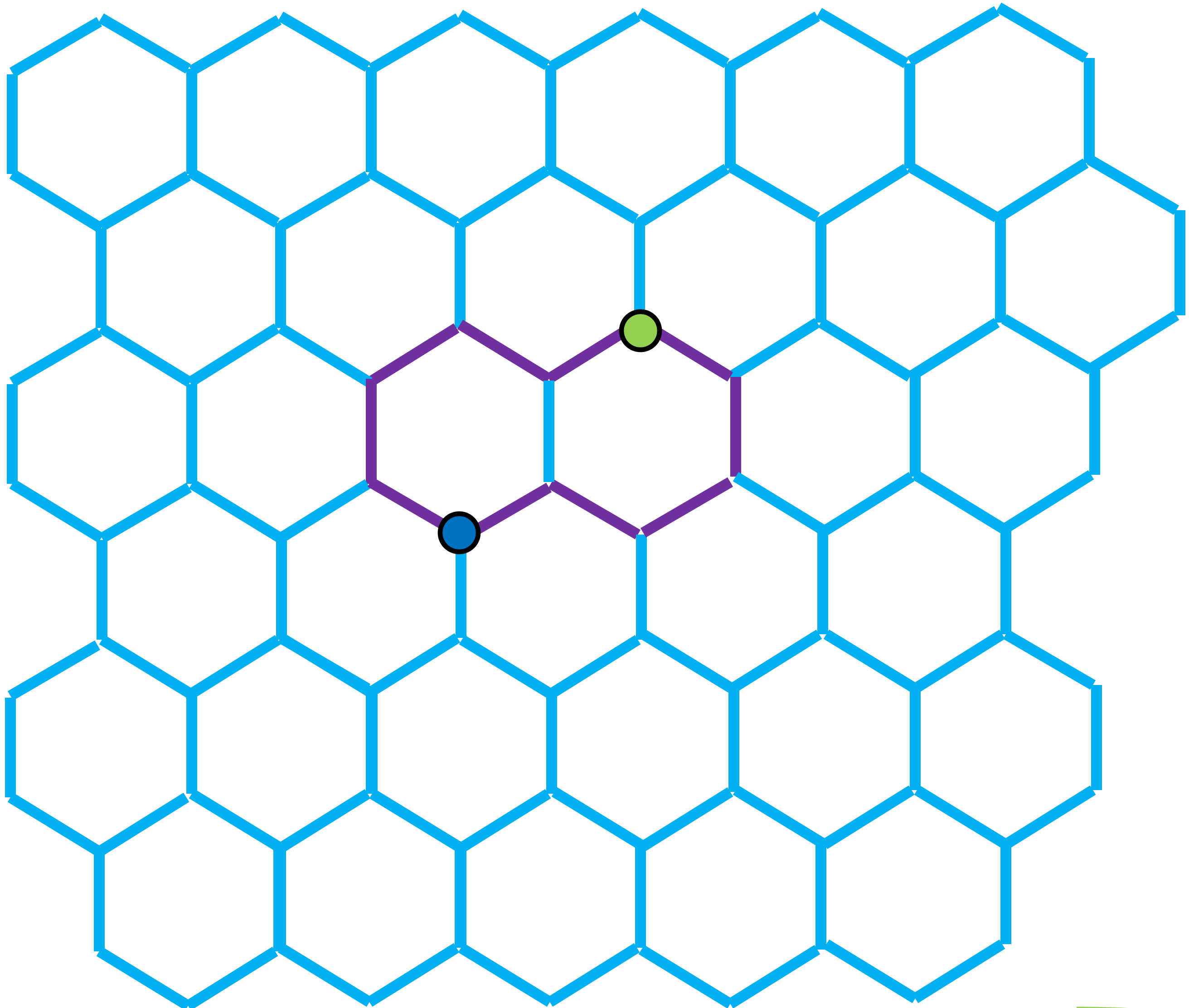
56 F-moves

Konig, Kuperberg, Reichardt, Ann. Phys. 2010



56 F-moves

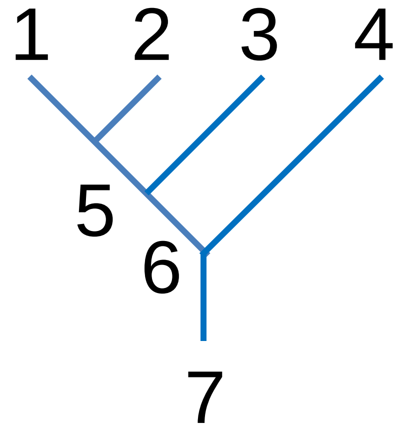
Konig, Kuperberg, Reichardt, Ann. Phys. 2010



56 F-moves

Braid

Konig, Kuperberg, Reichardt, Ann. Phys. 2010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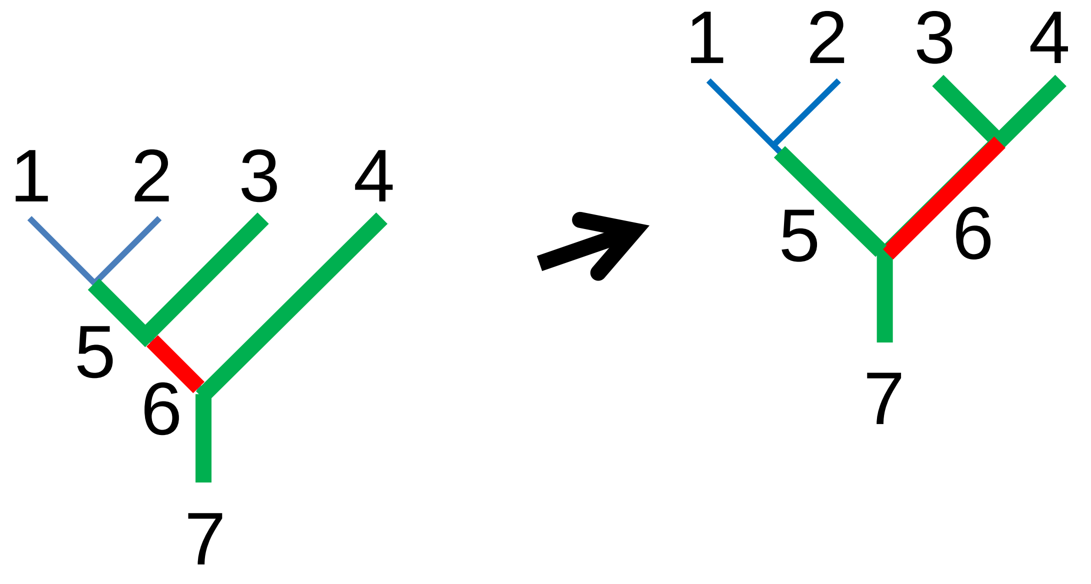
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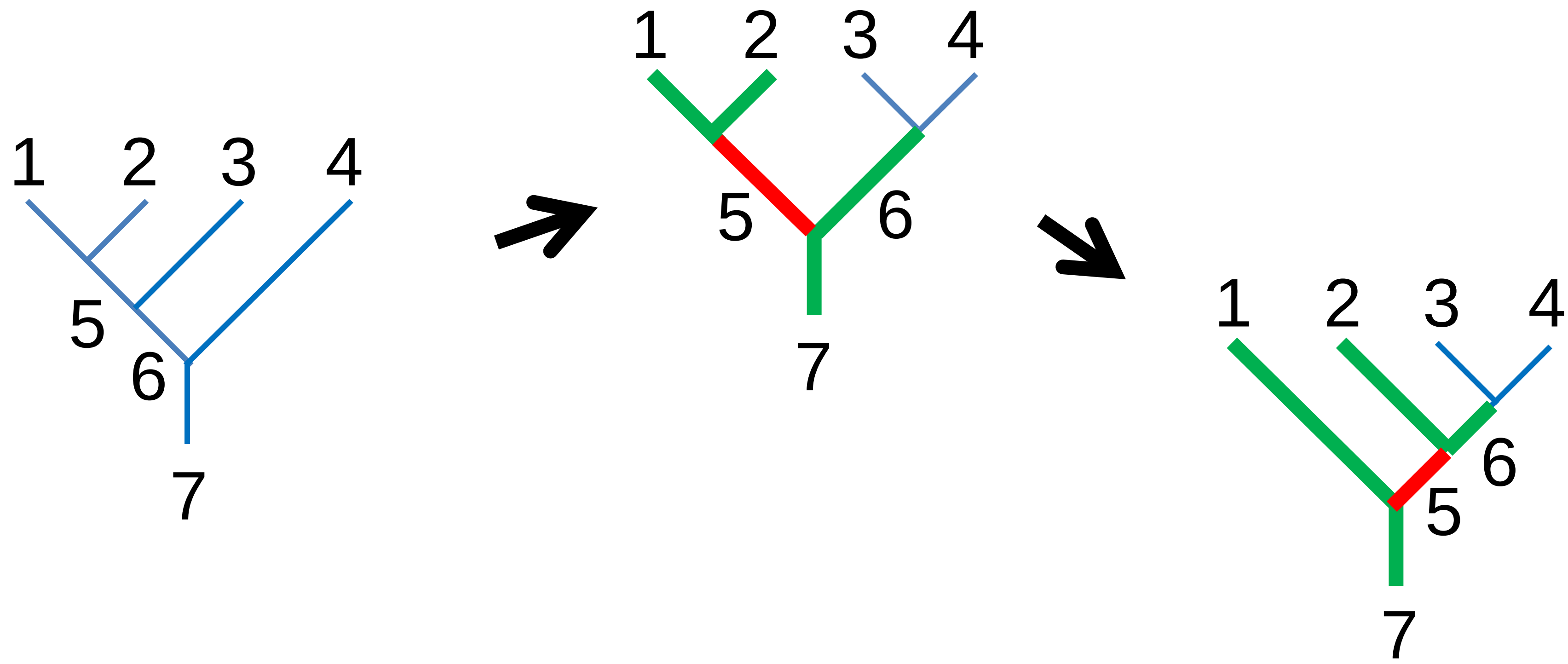

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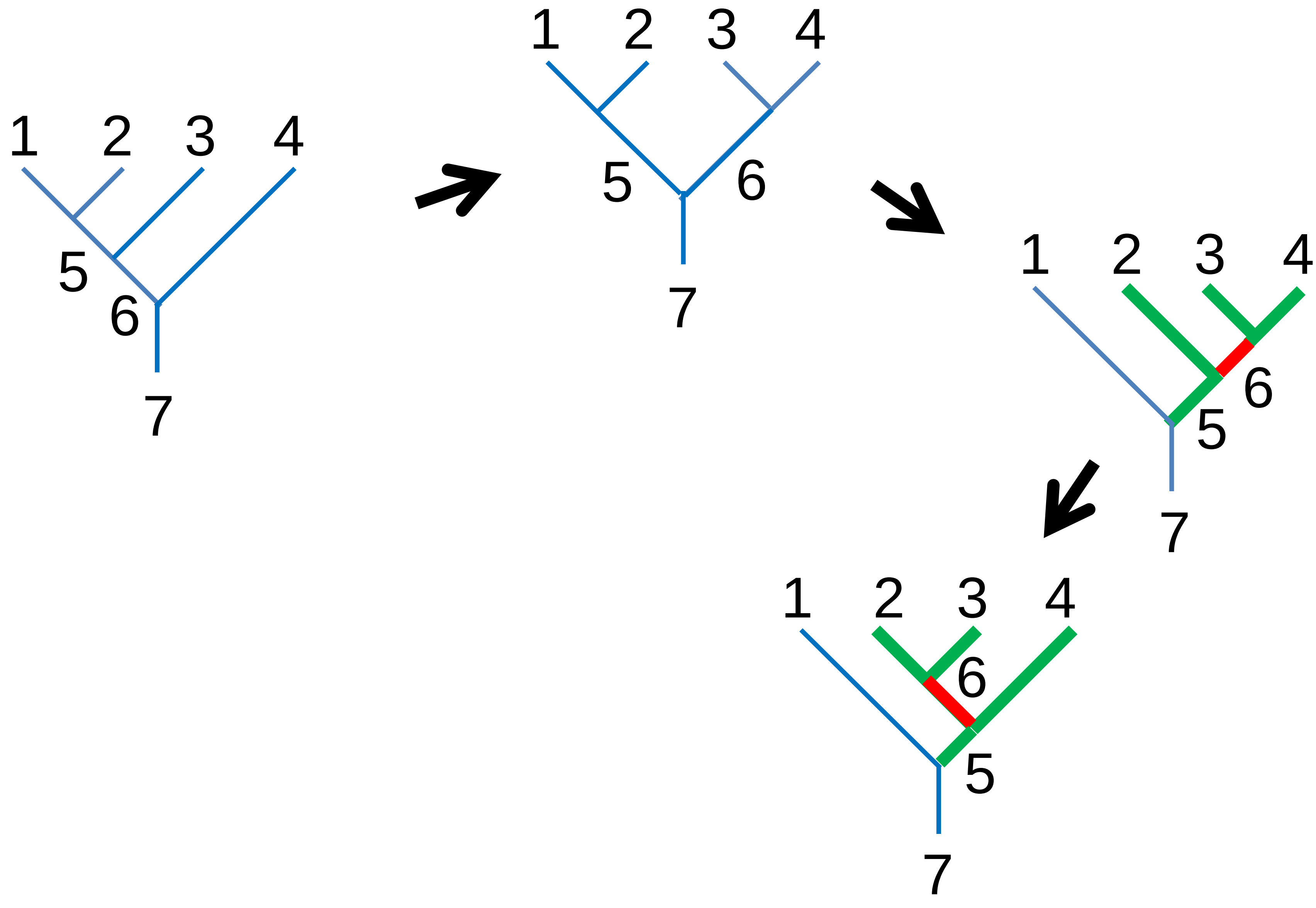

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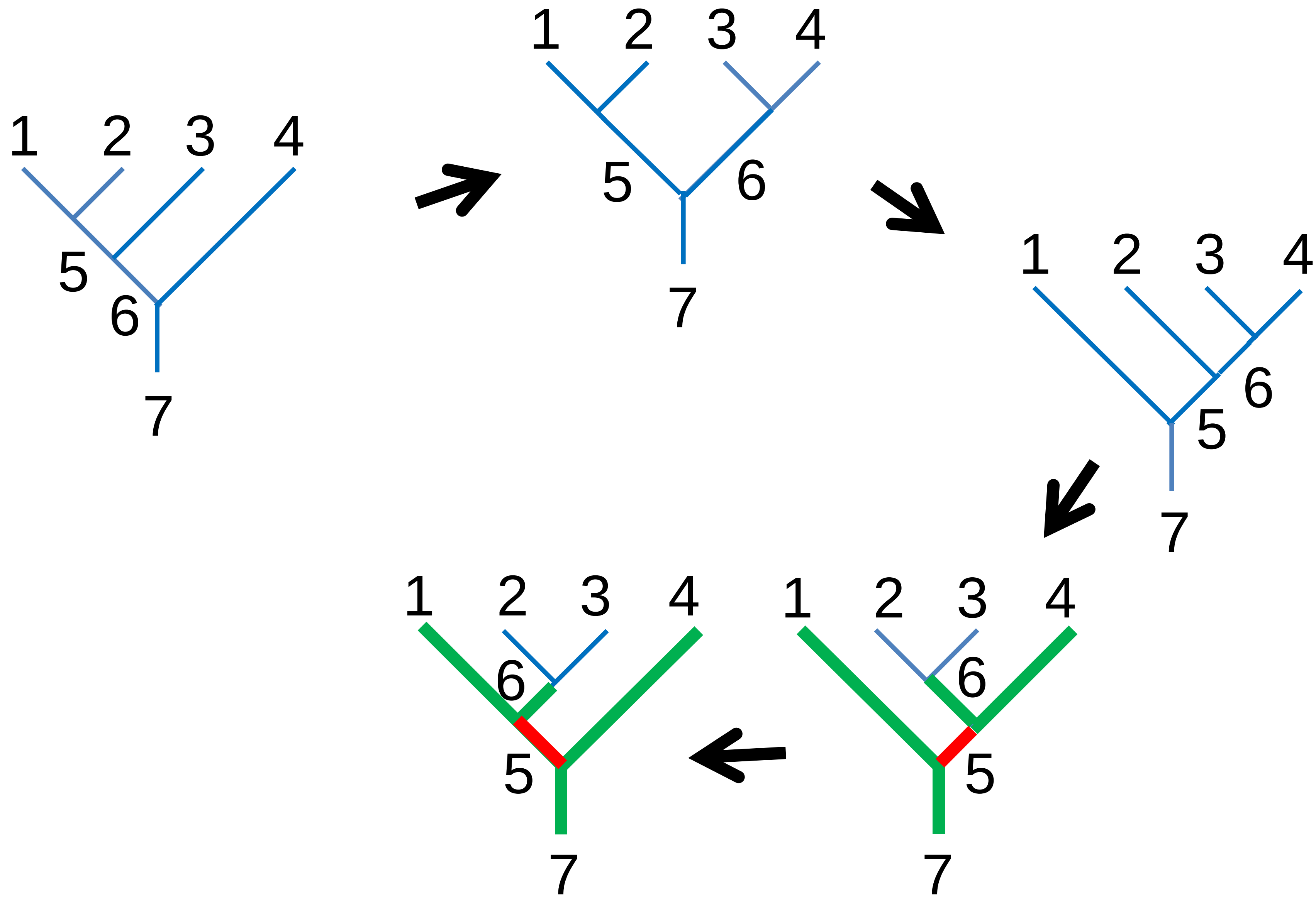

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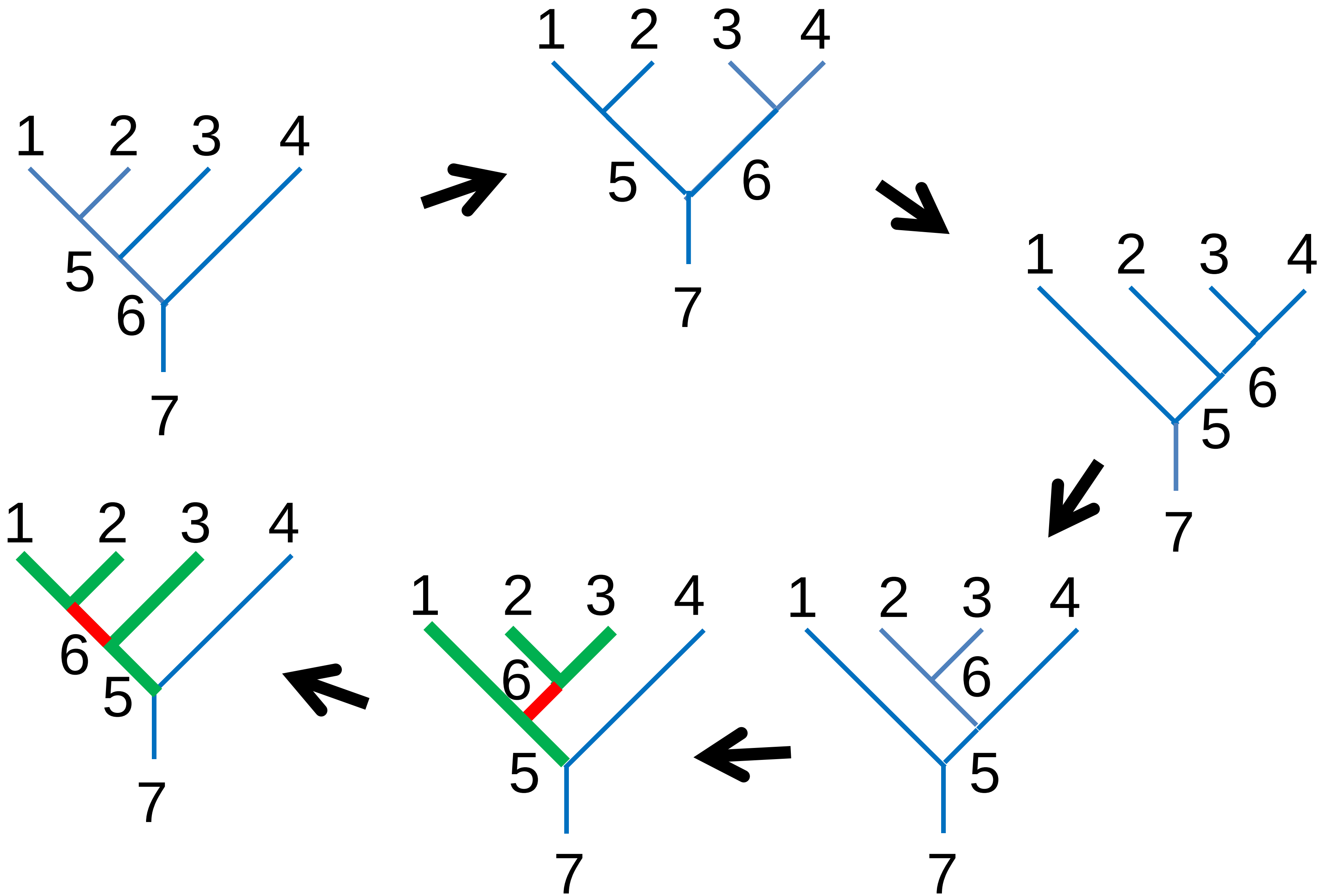
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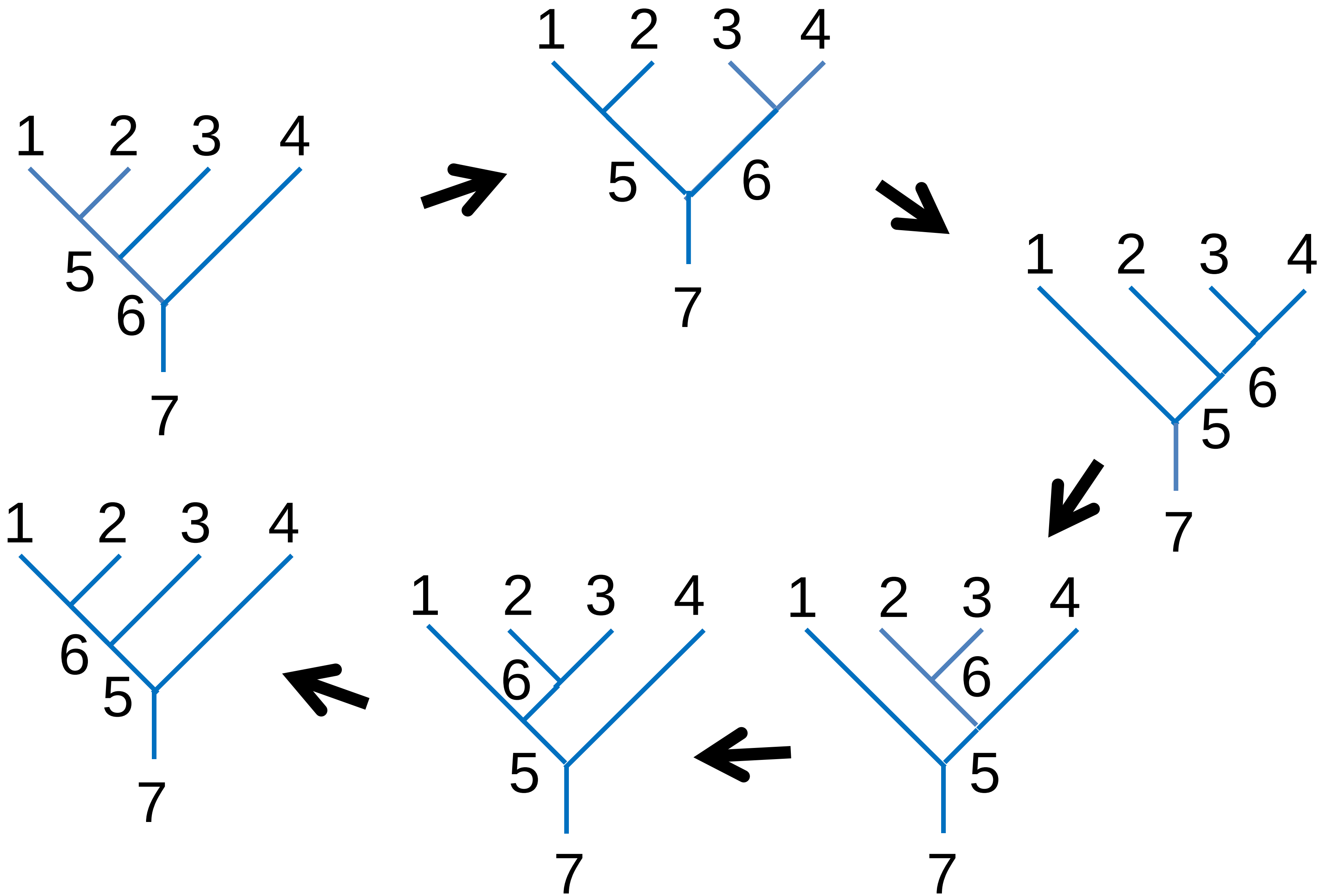
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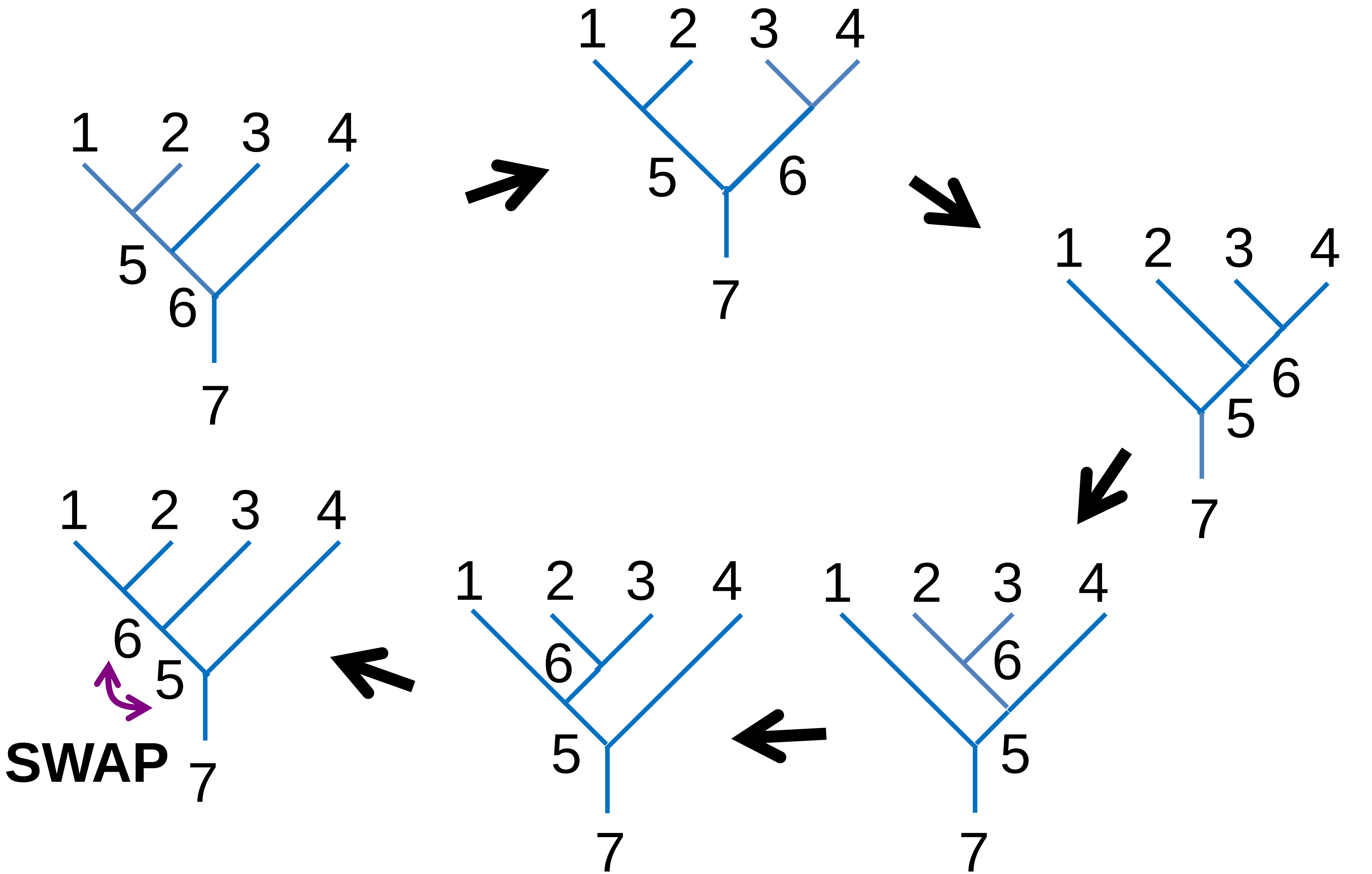

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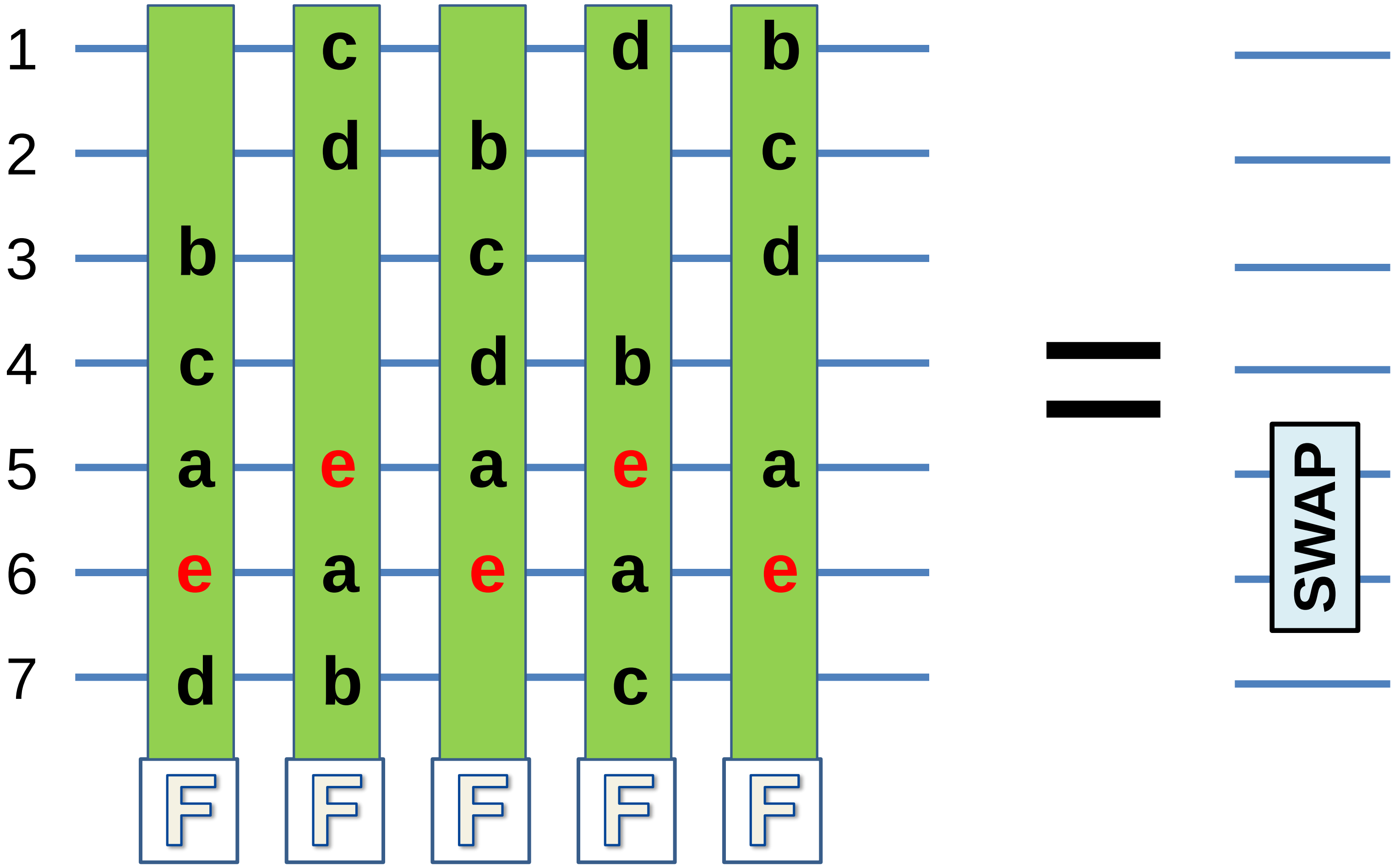
SWAP

Pentagon Equation as a Quantum Circuit



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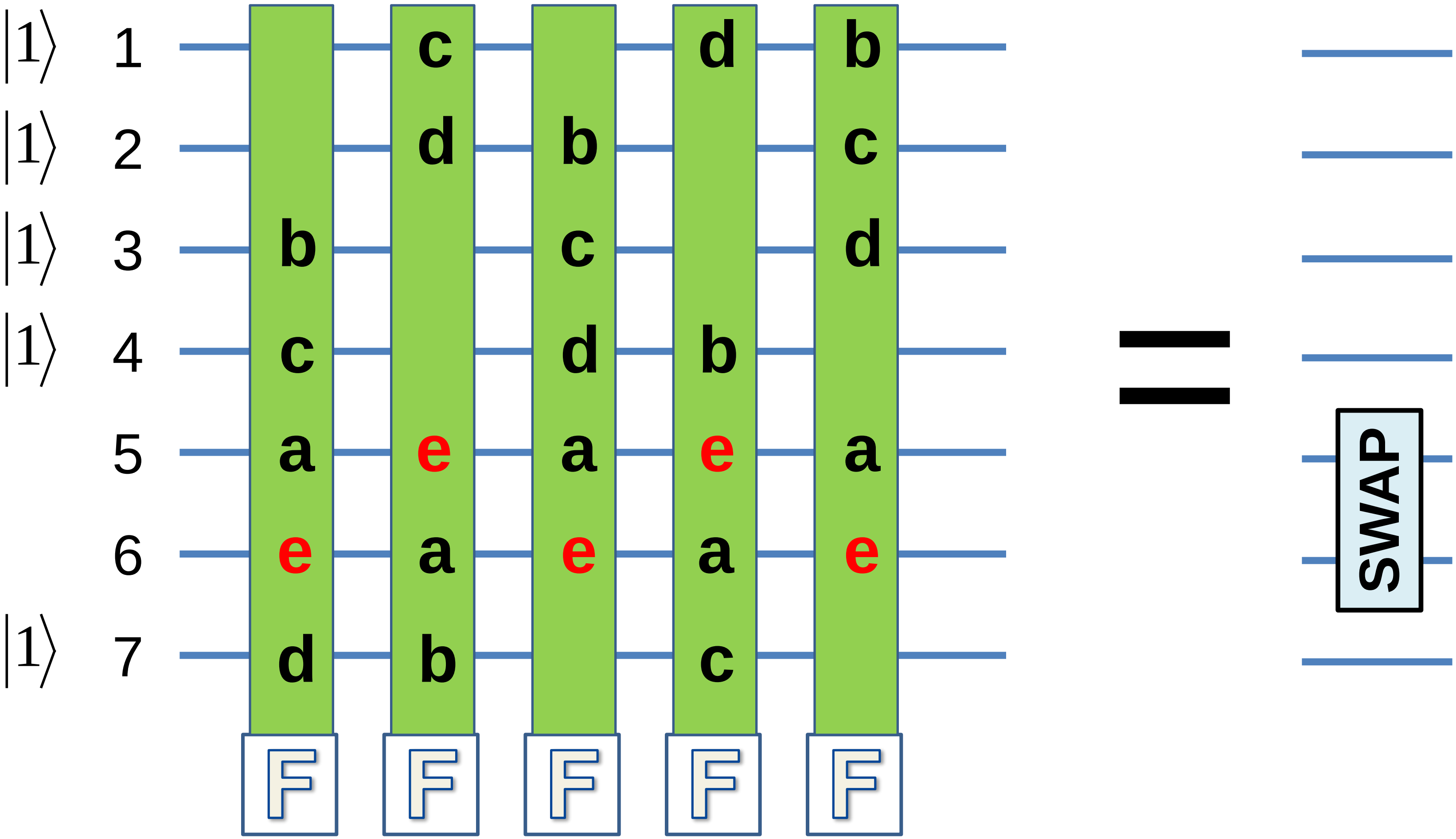
Pentagon Equation as a Quantum Circuit



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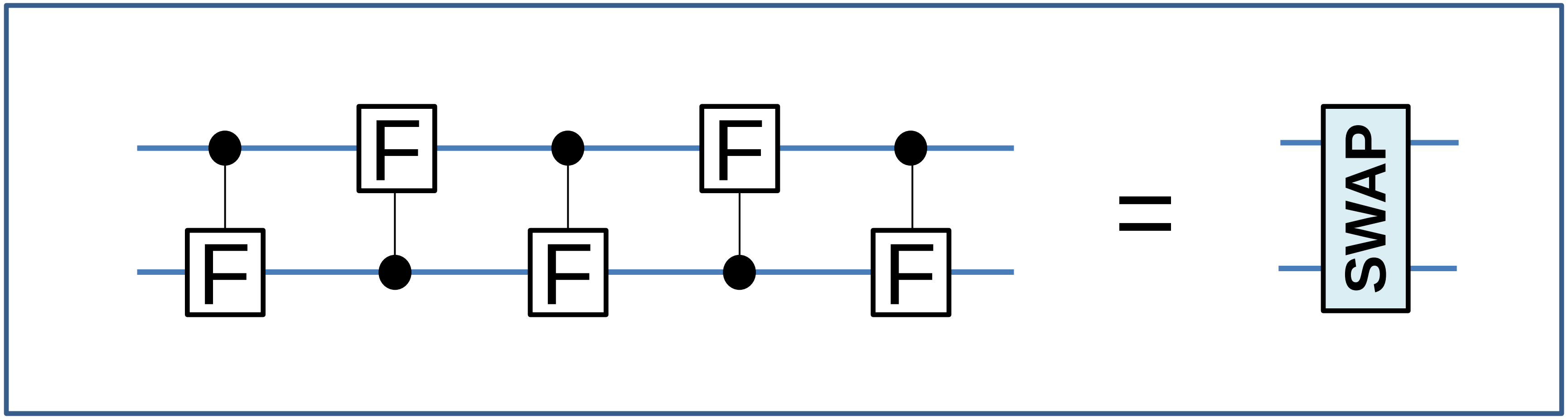
Pentagon Equation as a Quantum Circuit

$$F = \begin{pmatrix} \varphi^{-1} & \varphi^{-1/2} \\ \varphi^{-1/2} & -\varphi^{-1} \end{pmatrix} \qquad \varphi = \frac{\sqrt{5}+1}{2}$$



Simplified Pentagon Circuit

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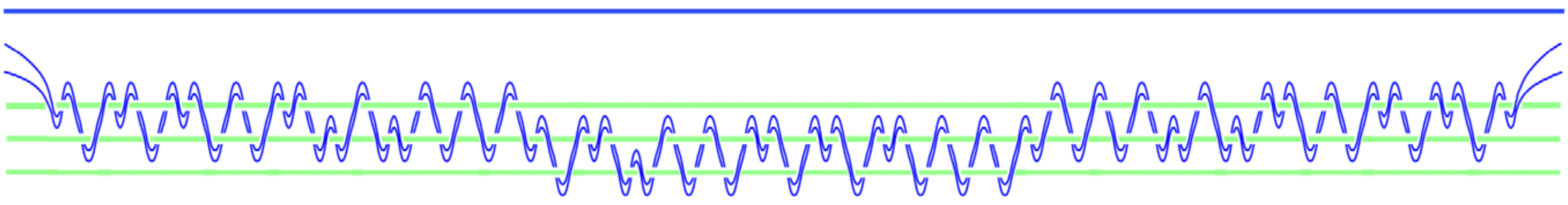
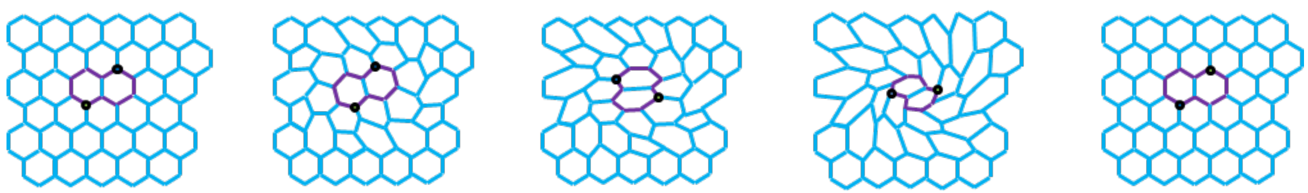
Some excellent reviews:

Non-Abelian Anyons and Topological Quantum Computation,
C. Nayak et al., Rev. Mod. Phys. 80, 1083 (2008). (arXiv:0707.1889v2)

Lectures on Topological Quantum Computation,
J. Preskill, Available online at*:* www.theory.caltech.edu/~preskill/ph219/topological.pdf





NEB, L. Hormozi, G. Zikos, S.H. Simon, Phys. Rev. Lett. 95 140503 (2005).
S.H. Simon, NEB, M.Freedman, N, Petrovic, L. Hormozi, Phys. Rev. Lett. 96, 070503 (2006).
L. Hormozi, G. Zikos, NEB, and S.H. Simon, Phys. Rev. B 75, 165310 (2007).
L. Hormozi, NEB, and S.H. Simon, Phys. Rev. Lett. 103, 160501 (2009).
M. Baraban, NEB, and S.H. Simon, Phys. Rev. A 81, 062317 (2010).
NEB and D.P. DiVincenzo, Phys. Rev. B 86, 165113 (2012).

Support: US DOE