FRACTIONAL QUANTUM ANOMALOUS HALL EFFECTS AND UNCONVENTIONAL SUPERCONDUCTORS IN RHOMBOHEDRAL GRAPHENE

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THEORY WINTER SCHOOL, 01-16-2026

Z Lu, T Han…L Ju. Nature 637, 1090, (2025)   Z Lu, T Han…L Ju. Nature 626 (8000), 759, (2024)   T. Han, Z. Lu, Z. Hadjri… LJ, Nature 643 (8072), 654 (2025)

ACKNOWLEDGEMENT

FSU group


Zhenqi Hua


Gang Shi

MIT (The Ju Group)


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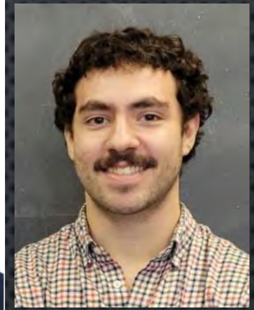
Tonghang Han


Yuxuan Yao


Jixiang Yang


Junseok Seo


Shenyong Ye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Cyprian Lewandowski

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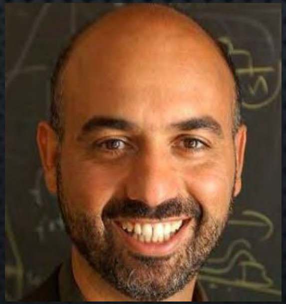
Fan Zhang

The Ashoori Group

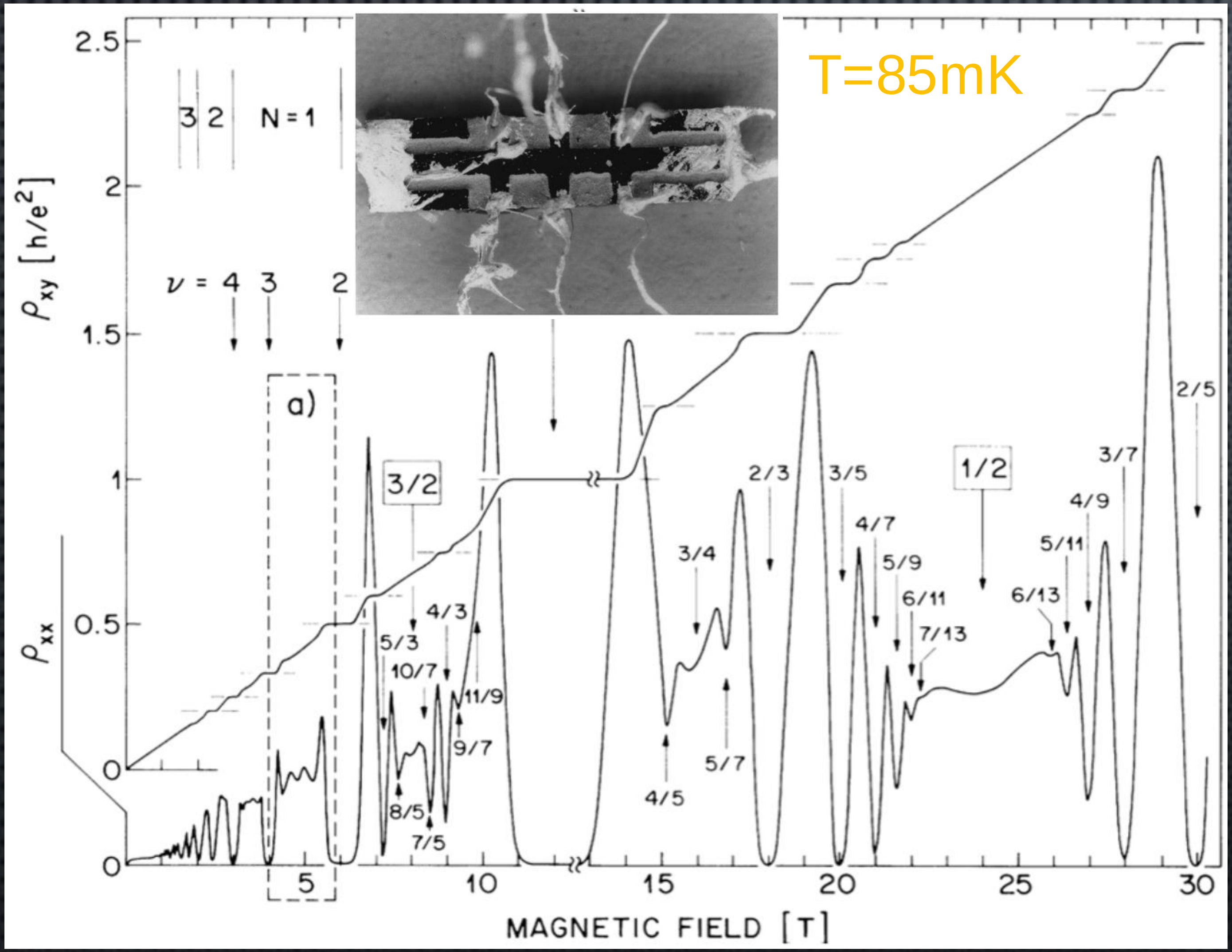

Ray Ashoori

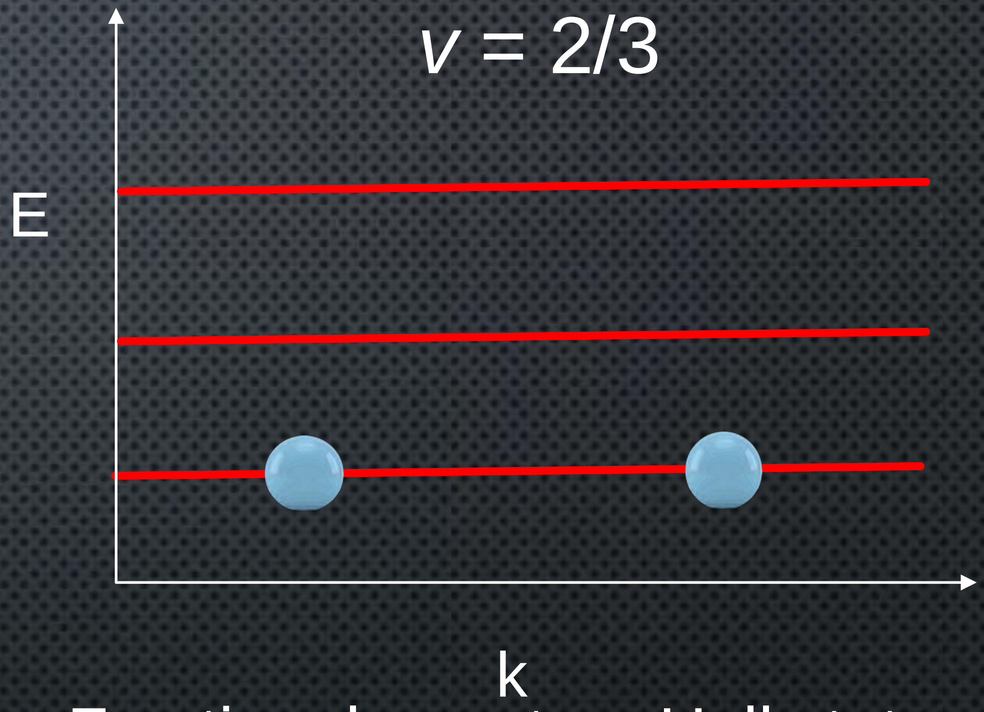
NIMS

Takashi Taniguchi

Kenji Watanabe

(Fractional) Quantum Hall Effect (1982)—Electron Topology





Fractional quantum Hall states arise from many-body gap at various fractional fillings

$$\sigma_{xy} = \frac{ne}{B} = \nu\frac{e^2}{h}$$

Stormer, Horst L.. "Nobel Lecture: The fractional quantum Hall effect." Reviews of Modern Physics 71 (1999): 875-889.

Fractional Quantum Hall Effect

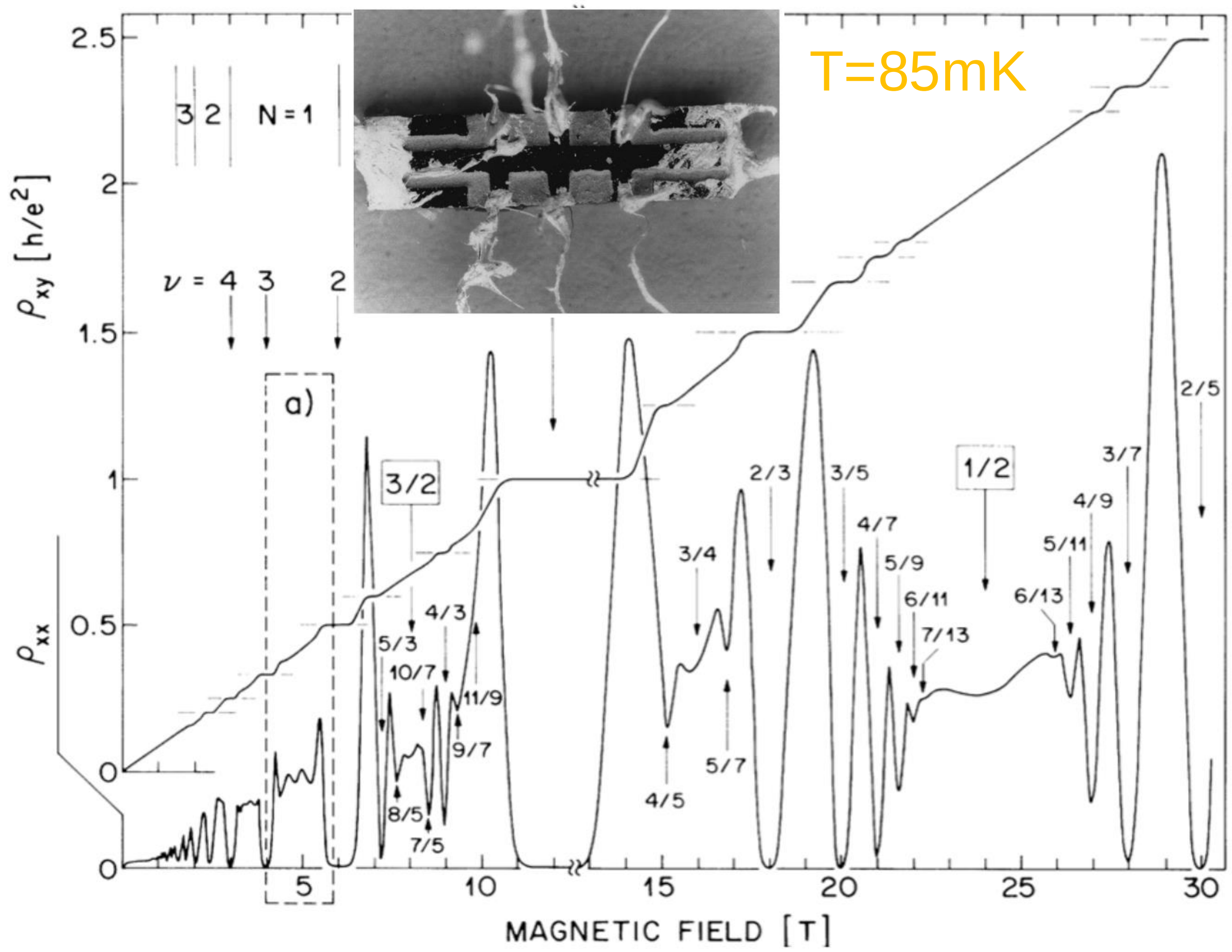


FIG. 11. First successful operation of our dilution refrigerator in high magnetic field. The sign reads: 85 mK, 280 kG, Feb. 16. 84. The proud operators are, clockwise from upper left, Albert Chang, Peter Berglund (who was largely responsible for the

Stormer, Horst L.. "Nobel Lecture: The fractional quantum Hall effect." Reviews of Modern Physics 71 (1999): 875-889.

Extremely high field needed

Fractional Quantum Hall Effect at B = 0T ?

Sun, K., Gu, Z., Katsura, H. & Das Sarma, S. Nearly flatbands with nontrivial topology. *Phys Rev Lett* (2011)

Tang, E., Mei, J. W. & Wen, X. G. High-temperature fractional quantum Hall states. *Phys Rev Lett* (2011)

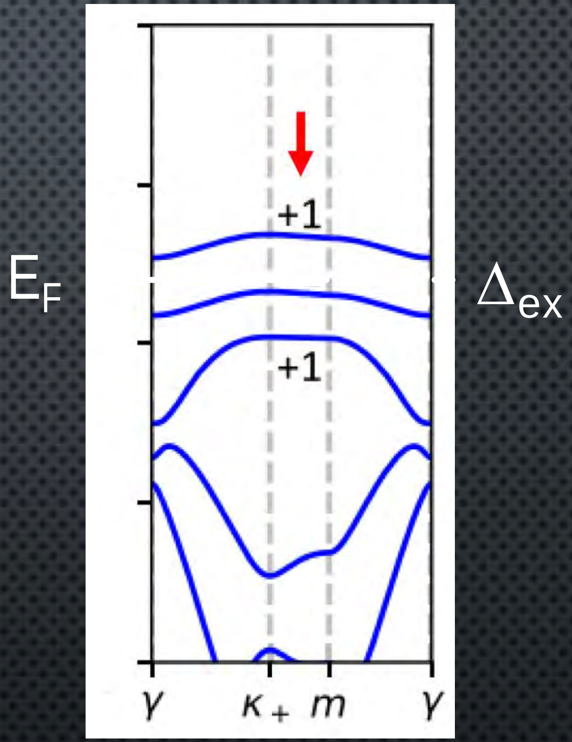
Neupert, T., Santos, L., Chamon, C. & Mudry, C. Fractional quantum Hall states at zero magnetic field. *Phys Rev Lett* (2011)

Sheng, D. N., Gu, Z. C., Sun, K. & Sheng, L. Fractional quantum Hall effect in the absence of Landau levels. *Nat Commun* (2011)

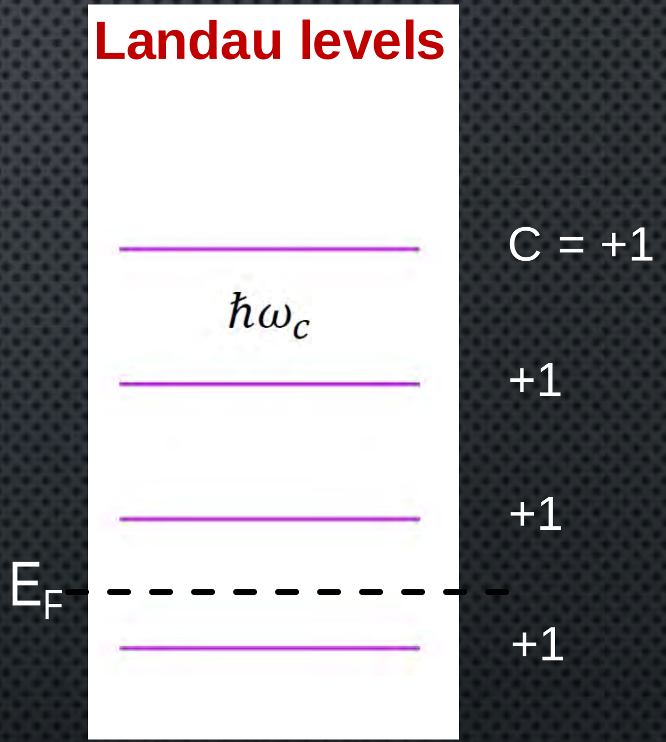
Regnault, N., Bernevig, B. Fractional Chern insulator. *Phys Rev X* (2011)

Xiao, D. et al. Interface engineering of quantum Hall effects in digital transition metal oxide heterostructures. *Nature Communications* (2011)

Topological flat band



Landau levels



Time-Reversal-Symmetry Broken by

exchange interaction spontaneously

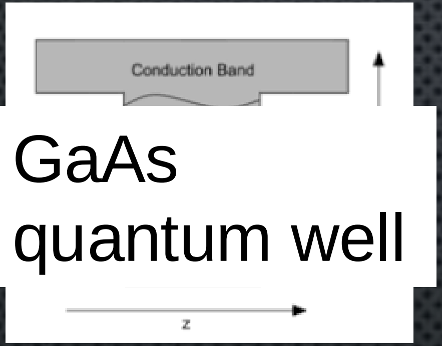
B field

Devakul, T., Crépel, V., Zhang, Y., Fu, L. *Nat Commun* **12**, 6730 (2021).

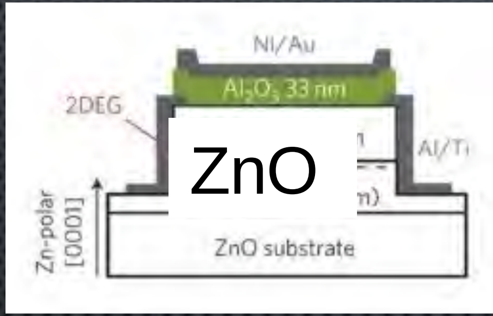
Anomalous (B = 0T) Fractional Quantum Hall Effect?

Fractional quantum Hall

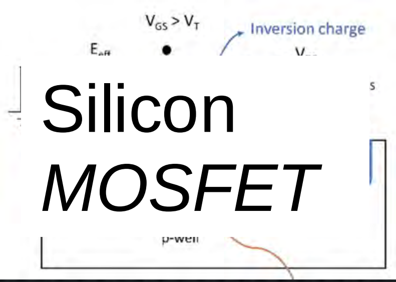
1982 – present



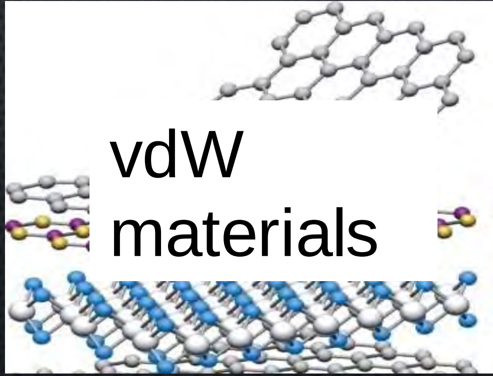
GaAs quantum well



ZnO



Silicon *MOSFET*



vdW materials

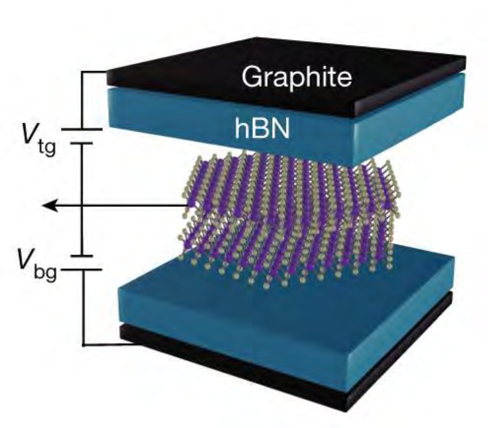
mK temperature

Strong magnetic field

Landau levels

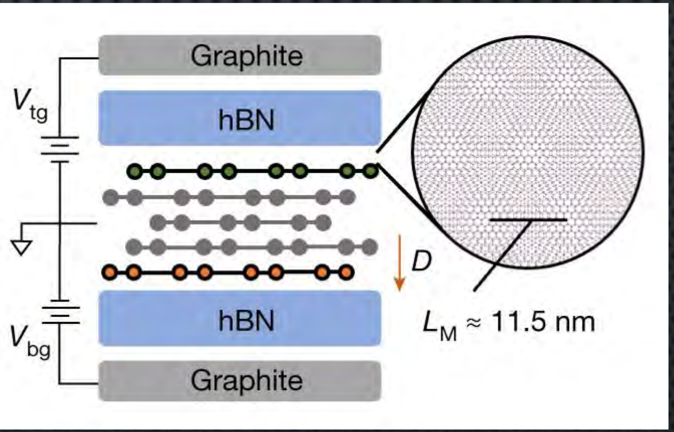
Topological material

2023 –



Park, H.et al. *Nature* (2023)

Xu, F.et al. *Phys Rev X* (2023)



Z Lu, T Han…L Ju. Nature (2023)

Isolated Topological Flat Bands

Two Pathways to Fractional Quantum Hall Effects at B=0T

Twisted TMD

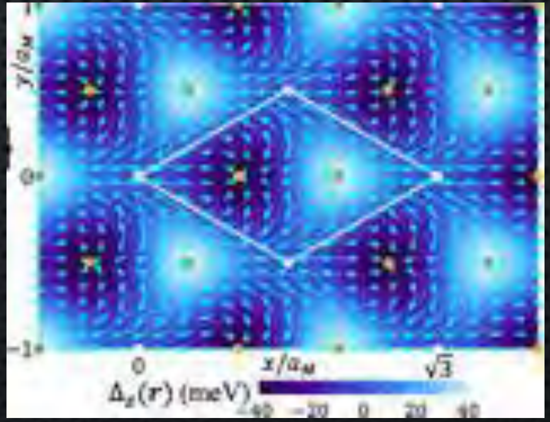
Theory

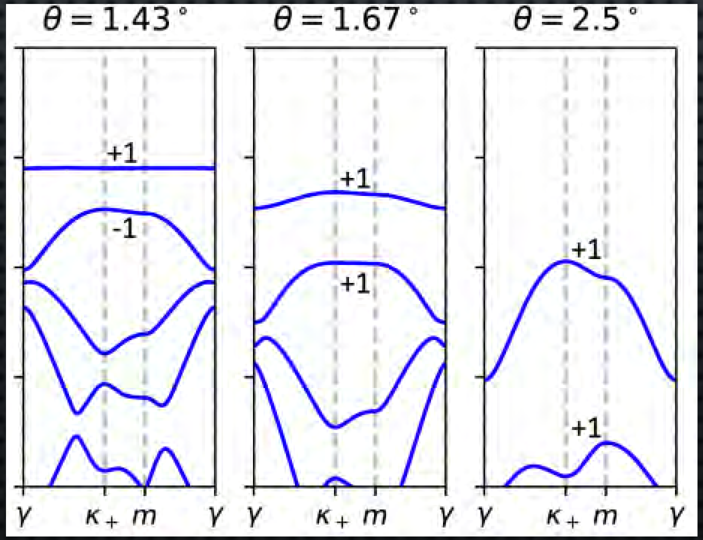
F. Wu,.. A.H. MacDonald. *PRL* (2018)

F. Wu… A.H. MacDonald. *PRL* (2019)

Li, H., Kumar, U., Sun, K. & Lin, S. Z. Phys Rev Res (2021)

Devakul, T., Crépel, V., Zhang, Y. & Fu, L. Nature Communications (2021)





Twisted MoTe2

Experiment

Observation of FQAHE in transport

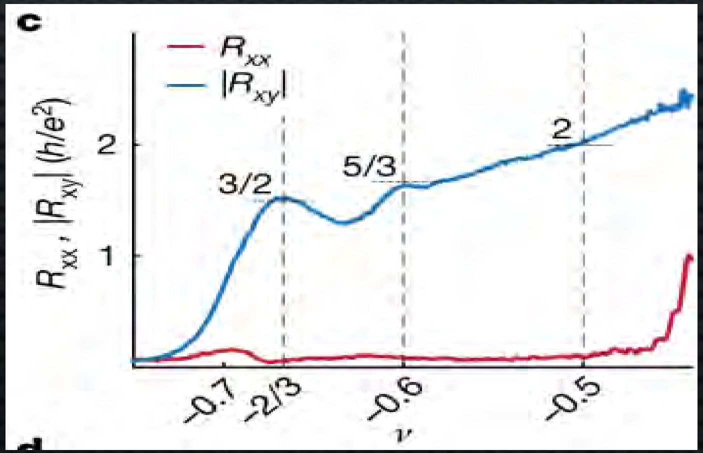
Park, H.et al. *Nature* (2023)

Xu, F.et al. *Phys Rev X* (2023)

Signatures of FCI in optics and thermodynamics

Cai, J.et al. *Nature* (2023)

Zeng, Y. et al. *Nature* (2023)



Two Pathways to Fractional Quantum Hall Effects at B=0T

Twisted/strained Graphene

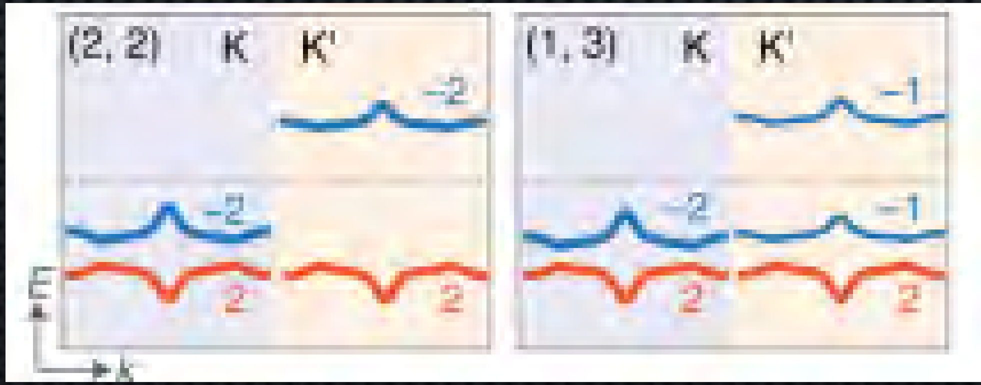
Theory

Ledwith, P. J., Tarnopolsky, G., Khalaf, E. & Vishwanath, A. Phys Rev Res (2020)

Repellin, C. & Senthil, T. *Phys Rev Res* (2020).

Abouelkomsan, A., Liu, Z. & Bergholtz, E. J. *PRL* (2020)

Gao, Q., Dong, J., Ledwith, P., Parker, D. & Khalaf, E. *PRL* (2023)



Observation … transport

Aaron L. Sharpe … 19)

Integer quan…

M. Serlin *et al.* …

Signatures … sulators > 6T

Spanton, E. et al. …

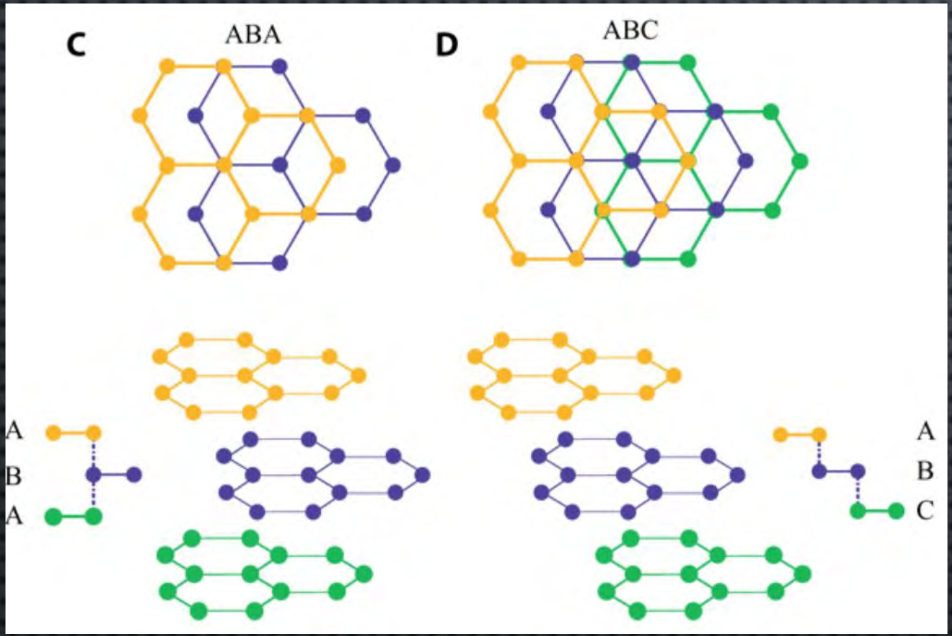
Xie, Y. et al. *Na*…

Twist angle inhomogeneity

Nature **581**, 47–52 (2020)

Graphite !?

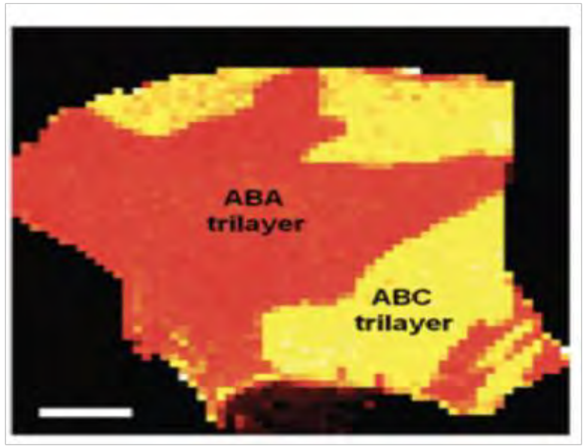
Stacking Orders in Multilayer Graphene



Bernal
(stable)

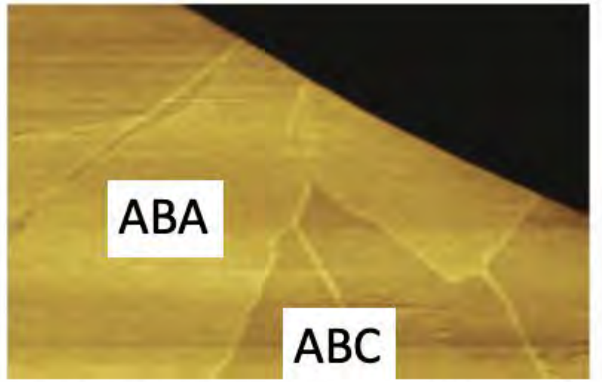
Rhombohedral
(meta-stable)

Lui, C.H., Heinz, T.F. *Nano letters*, *11*(1) (2011).



Raman mapping (hours)

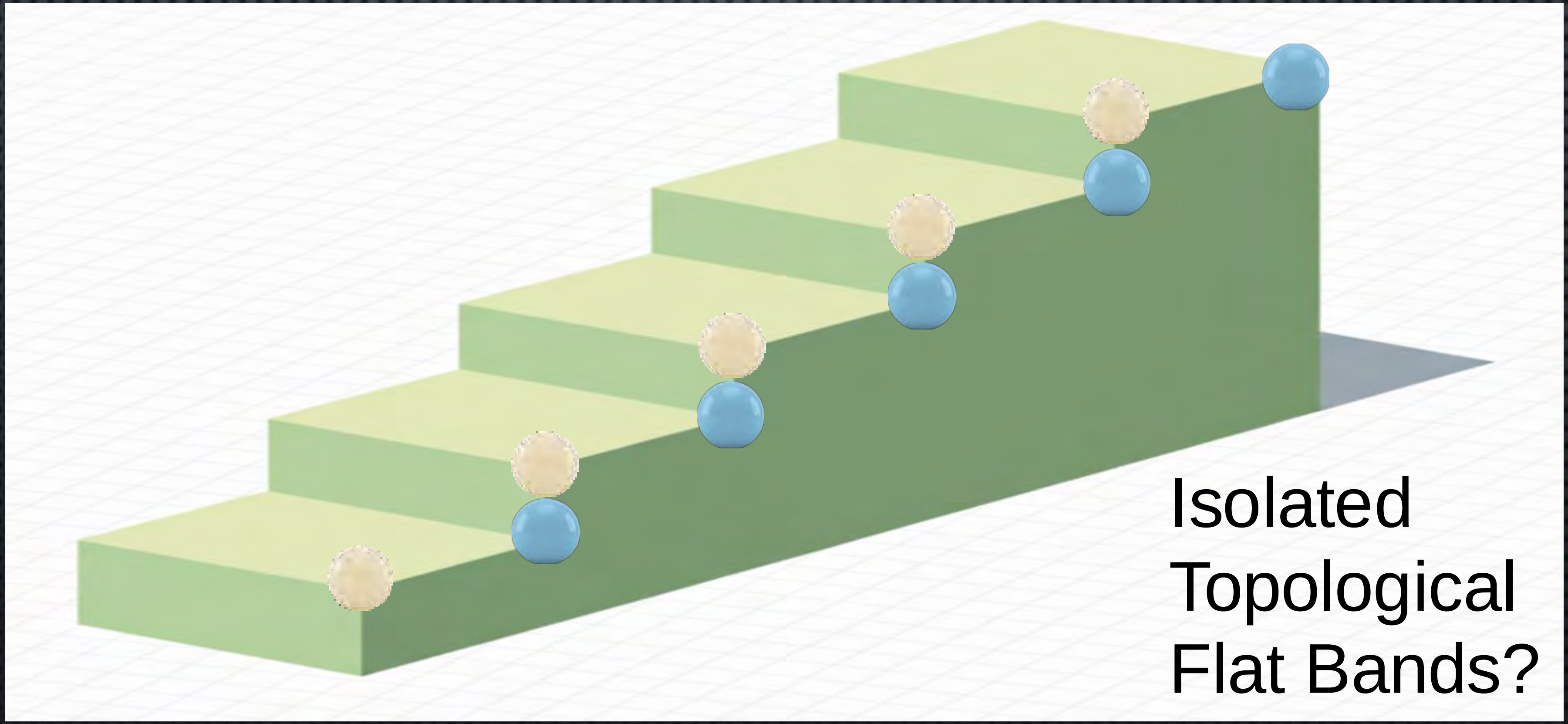
J. Lui.... T. Heinz, Nano Lett, 2010



Infrared Nanoscopy (10 mins)

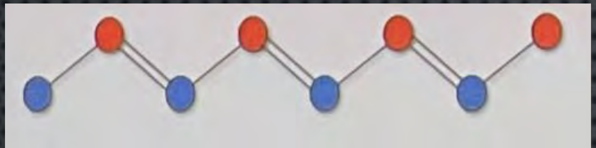
LJ... Feng Wang, Nature, 2015

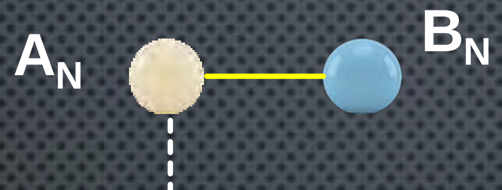
Staircase to Fractional Quantum Hall Effects at B=0T

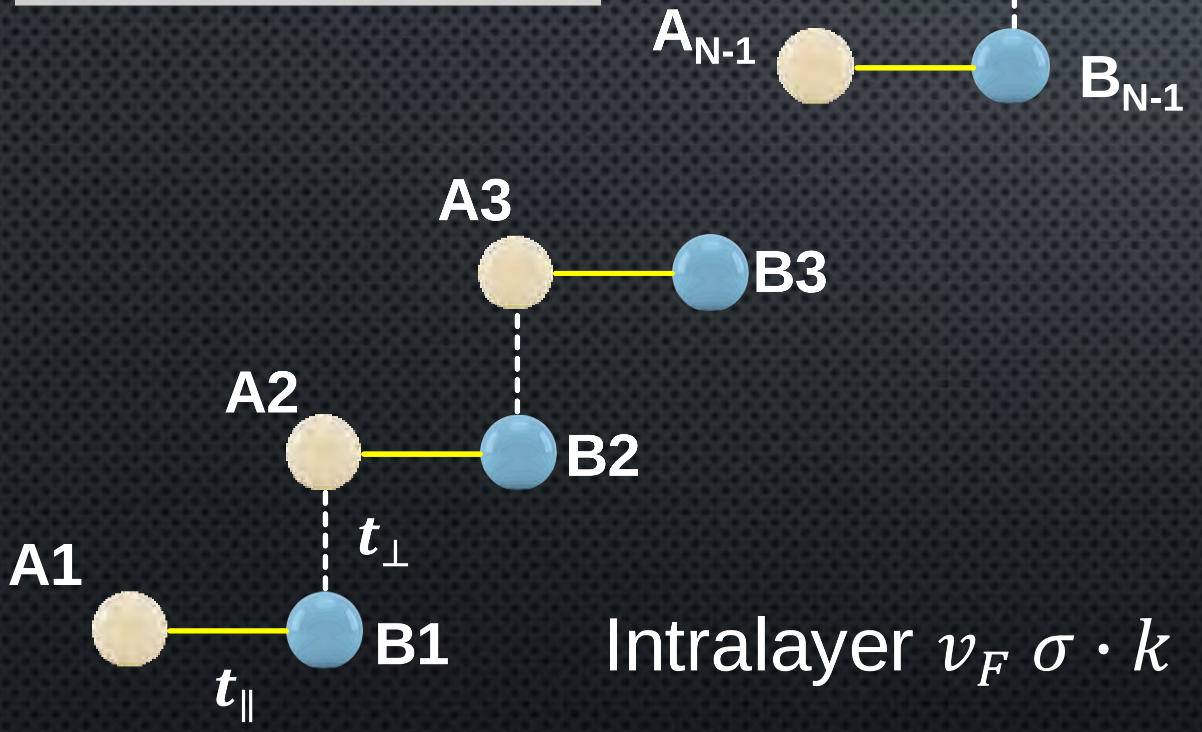


Gate-Tunable Flat Bands in Crystalline Rhombohedral Graphene

Rhombohedral (ABCABC…) N-layer

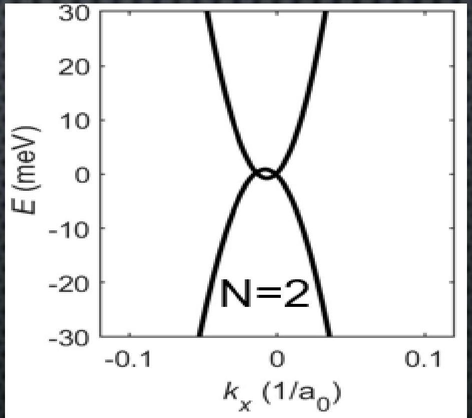


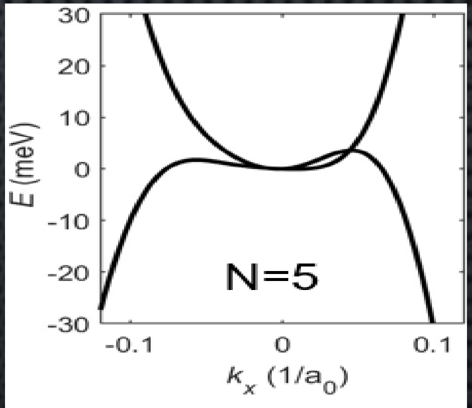




Intralayer $v_F\, \sigma \cdot k$

$$E_{eff} \approx \pm t_\perp \left(\frac{v_F}{t_\perp} k\right)^N$$





Pristine flat bands

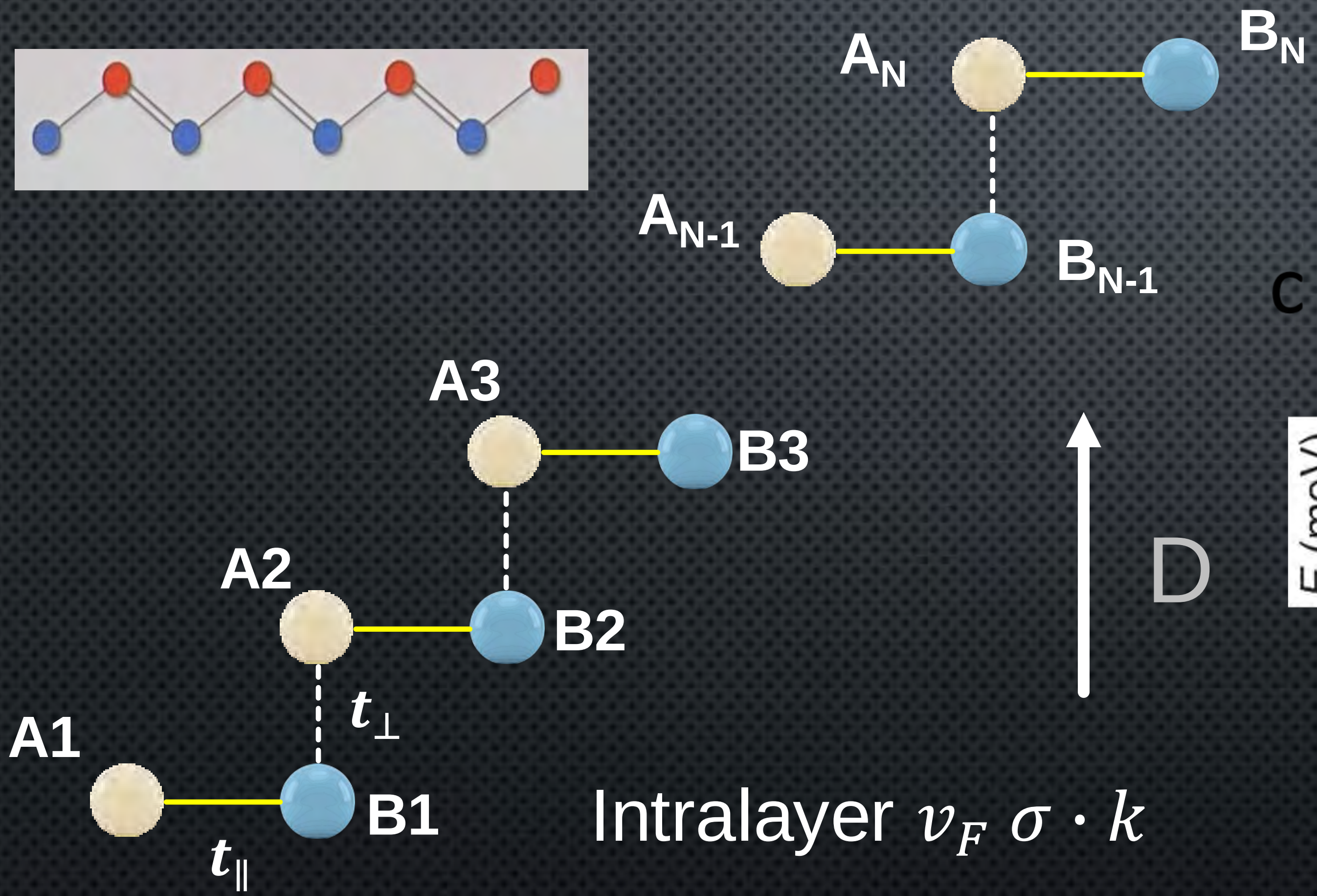
Min, H. and MacDonald, A.H. *PRB* (2008)

Zhang, F. et al. *PRB* (2010)

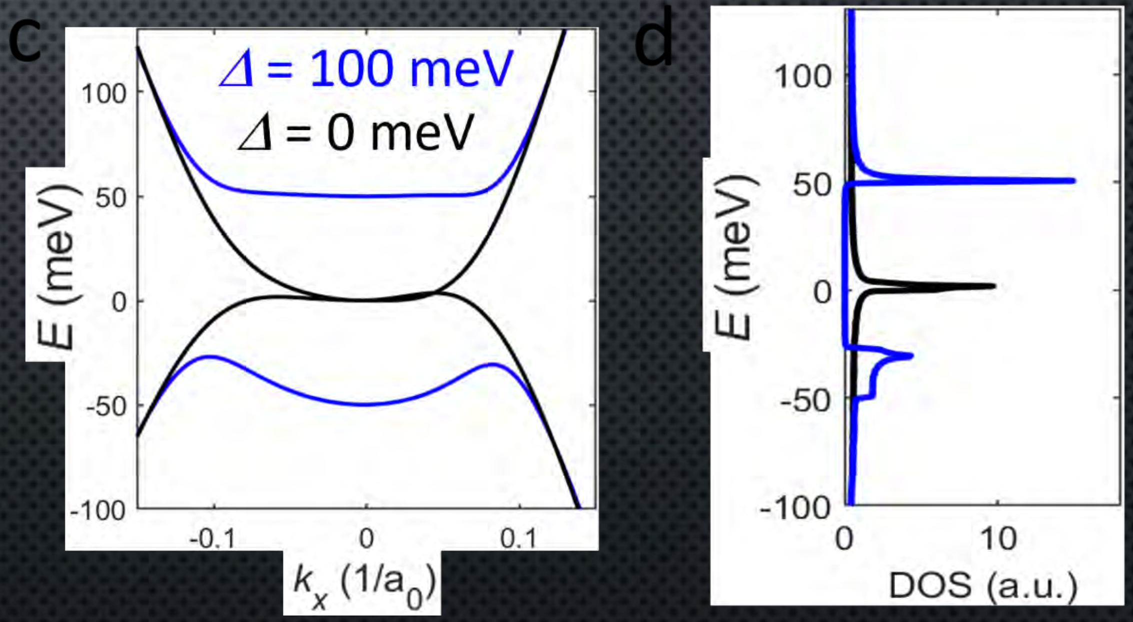
T. Han, Z. Lu…LJ, *Nat. Nanotechnol.* (2023)

Gate-Tunable Flat Bands in Crystalline Rhombohedral Graphene

Rhombohedral (ABCABC…) N-layer



$$E_{eff} \approx \pm t_\perp \left(\frac{v_F}{t_\perp} k\right)^N$$



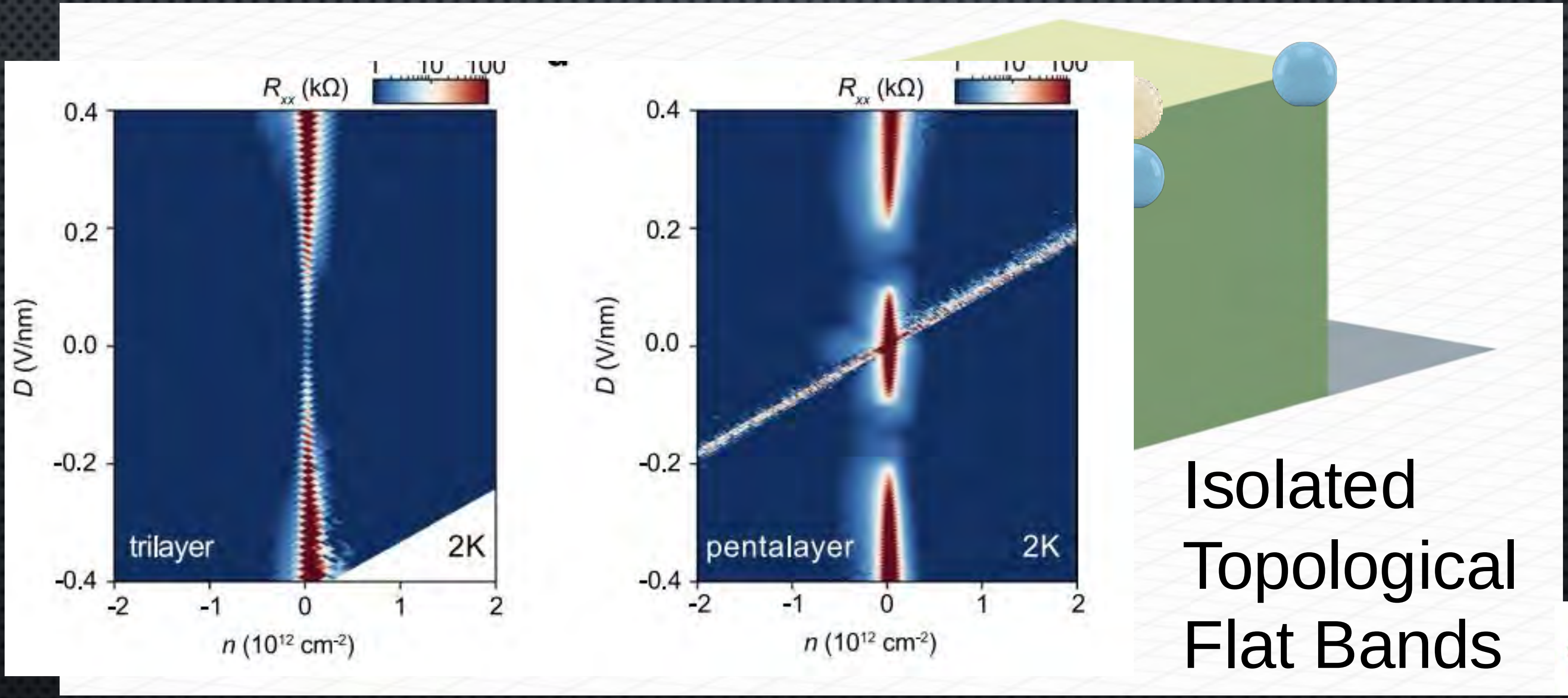
Gate tunable flat bands

Min, H. and MacDonald, A.H. *PRB* (2008)

Zhang, F. et al. *PRB* (2010)

T. Han, Z. Lu…LJ, *Nat. Nanotechnol.* (2023)

Staircase to Fractional Quantum Hall Effects at B=0T

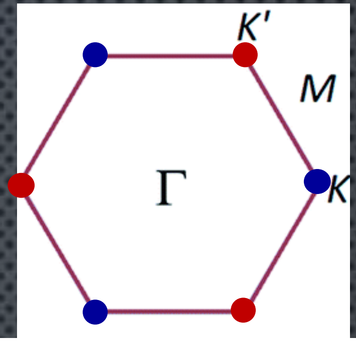


Isolated Topological Flat Bands ✓

T. Han, Z. Lu…LJ, *Nat. Nanotechnol.* (2023)

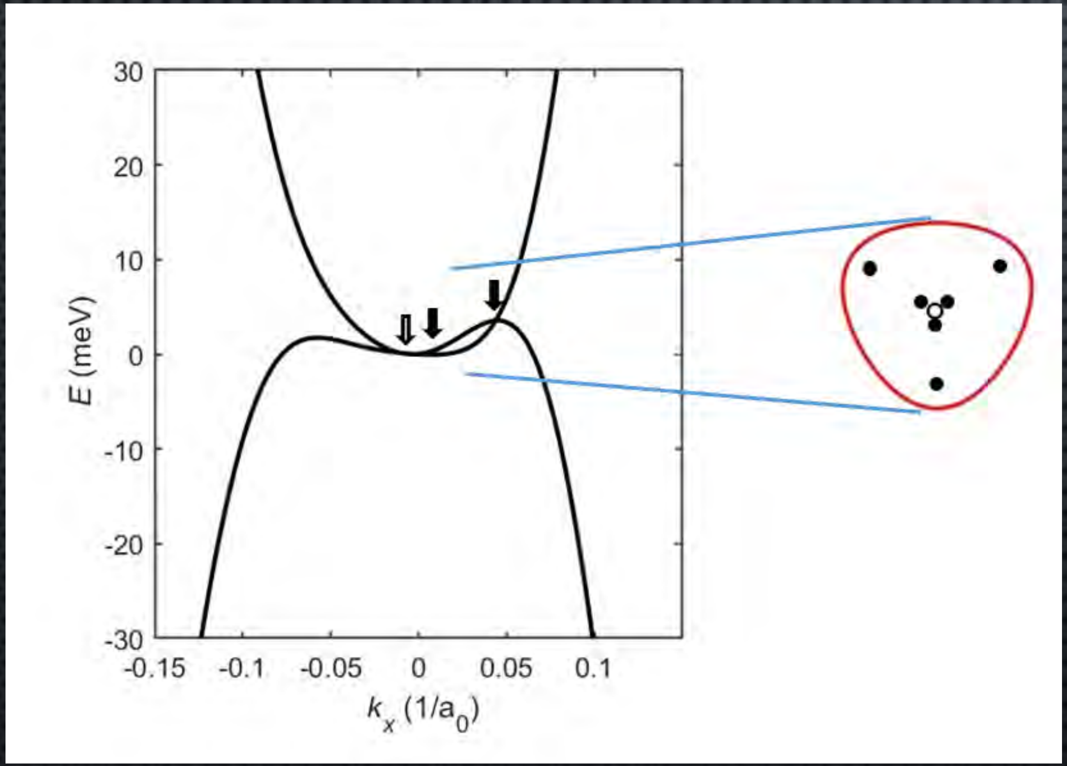
Gate-Tunable Berry Curvature

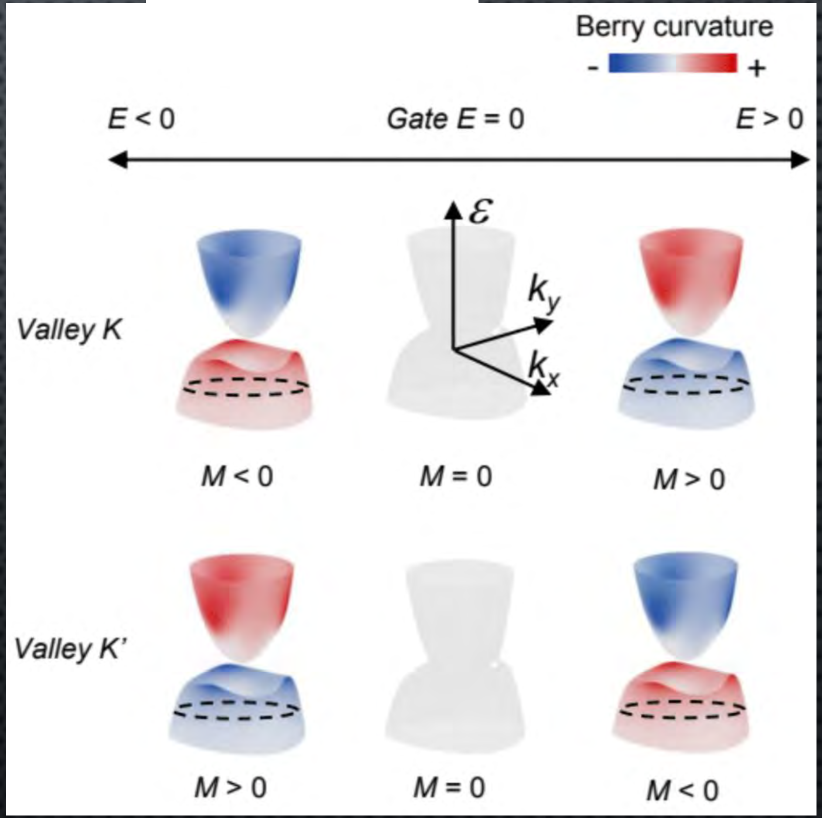
Time-reversal pair



K = -K'

Nth order Dirac cone
Singularity of nπ Berry phase at k=0

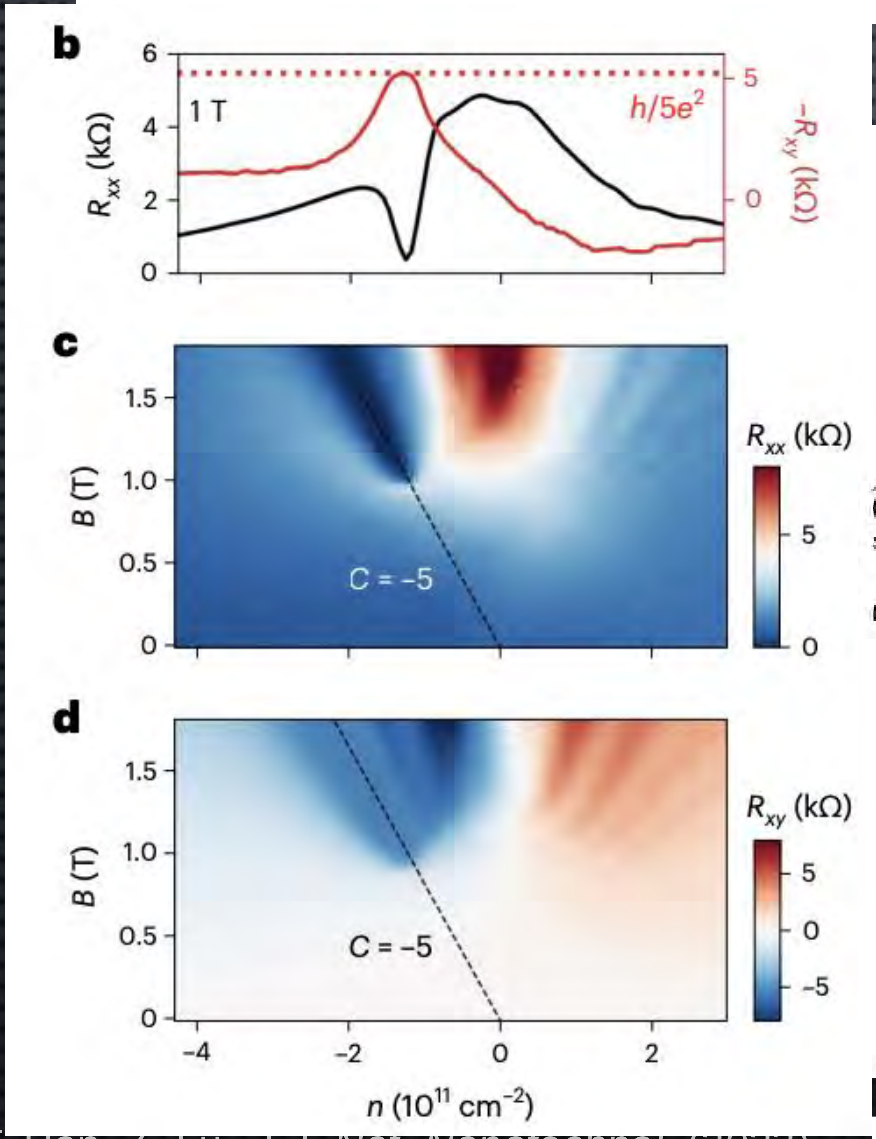




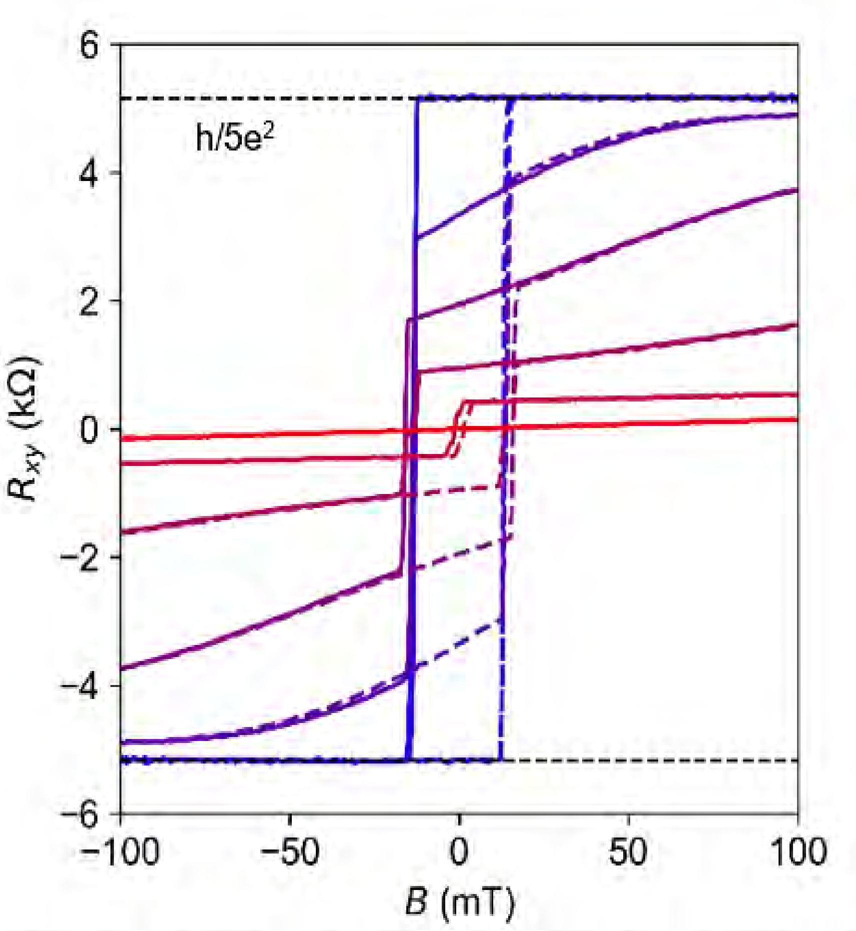
Koshino, Mikito, and Edward McCann.
Physical Review B 80.16 (2009): 165409.

T. Han, Z. Lu … LJ. *Nature* (2023)
Xiao, D., Yao, W. and Niu, Q. *PRL* (2007)

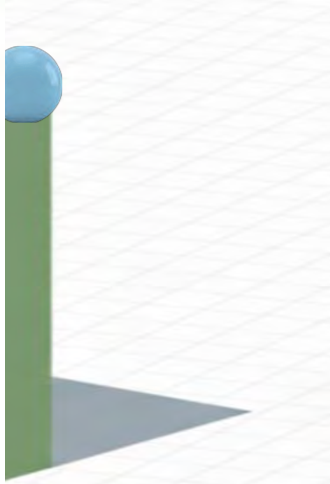
Staircase to Fractional Quantum Hall Effects at B=0T



T. Han, Z. Lu…LJ, *Nat. Nanotechnol.* (2023)

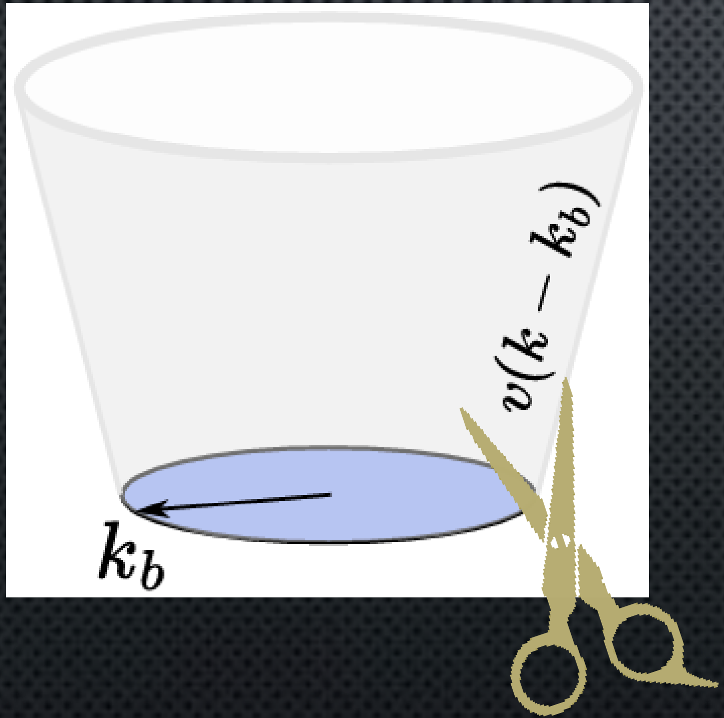


T. Han, Z. Lu… LJ, *Science 384(6696)*. (2024)

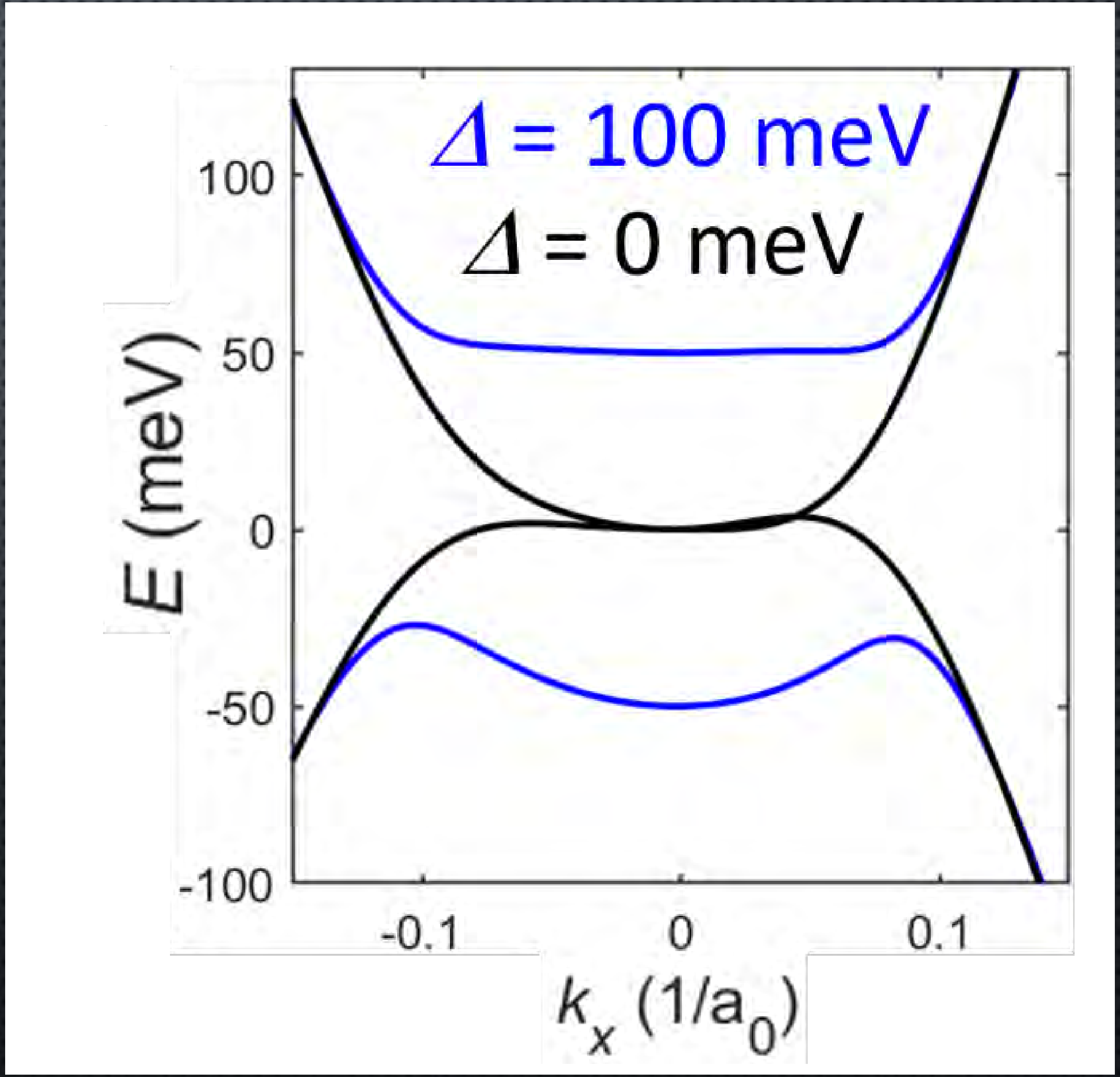


ated
ological ✓
: Bands ✓

Berry "trashcan" ?

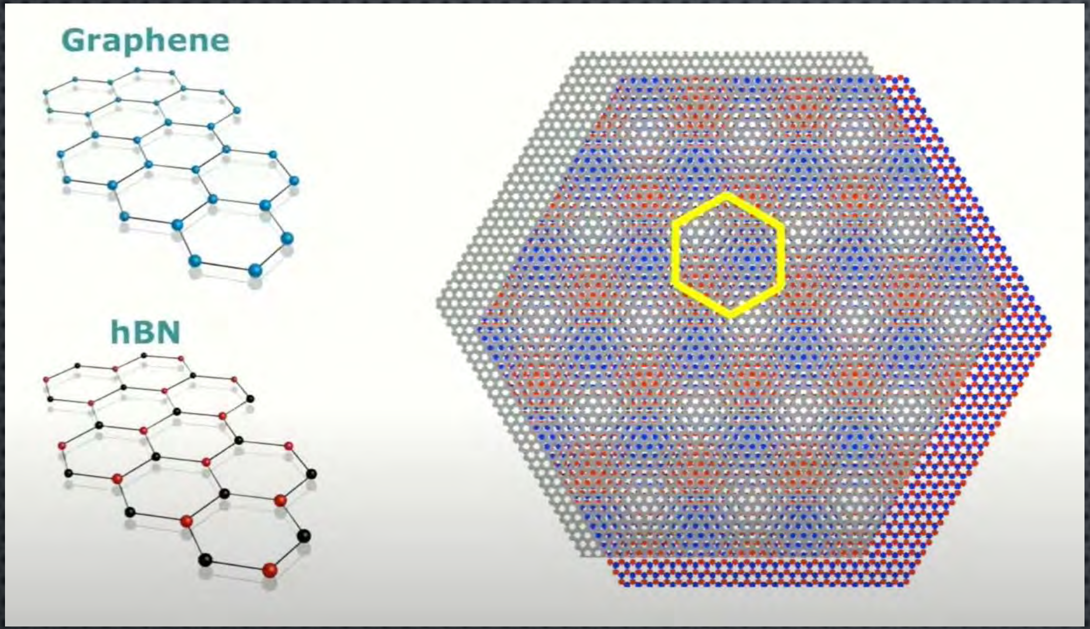


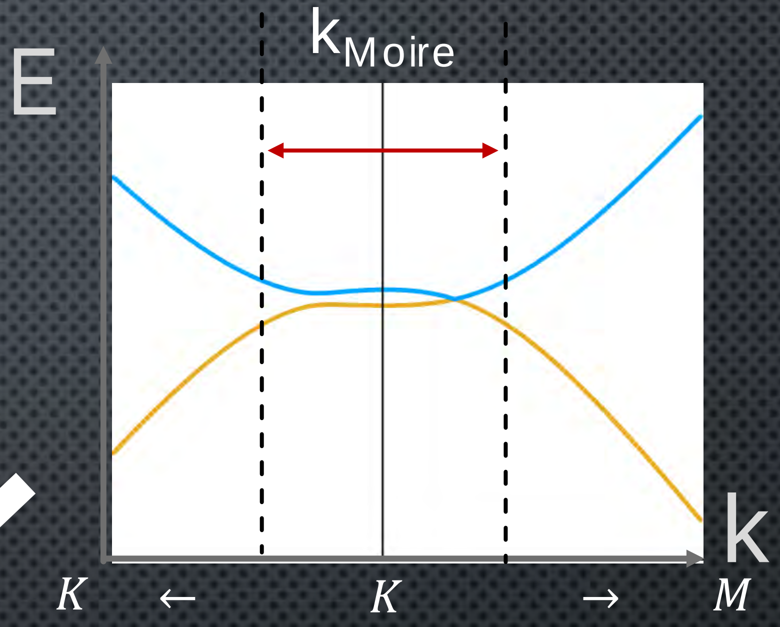
Flat band is not isolated



Andrei Bernvig, arXiv:2503.09692

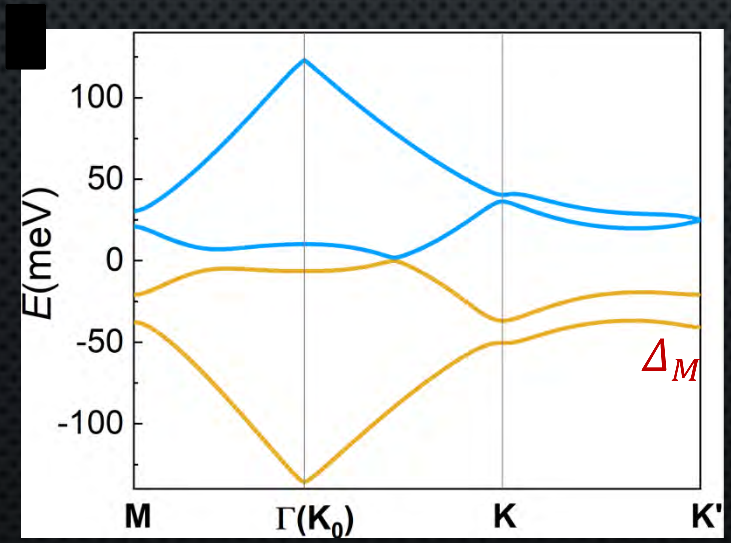
Moire Scissor



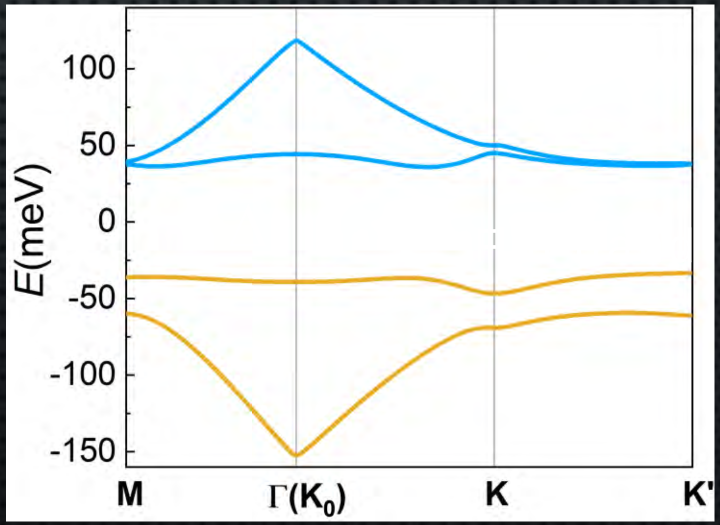


Moiré

Mini Brillouin Zone

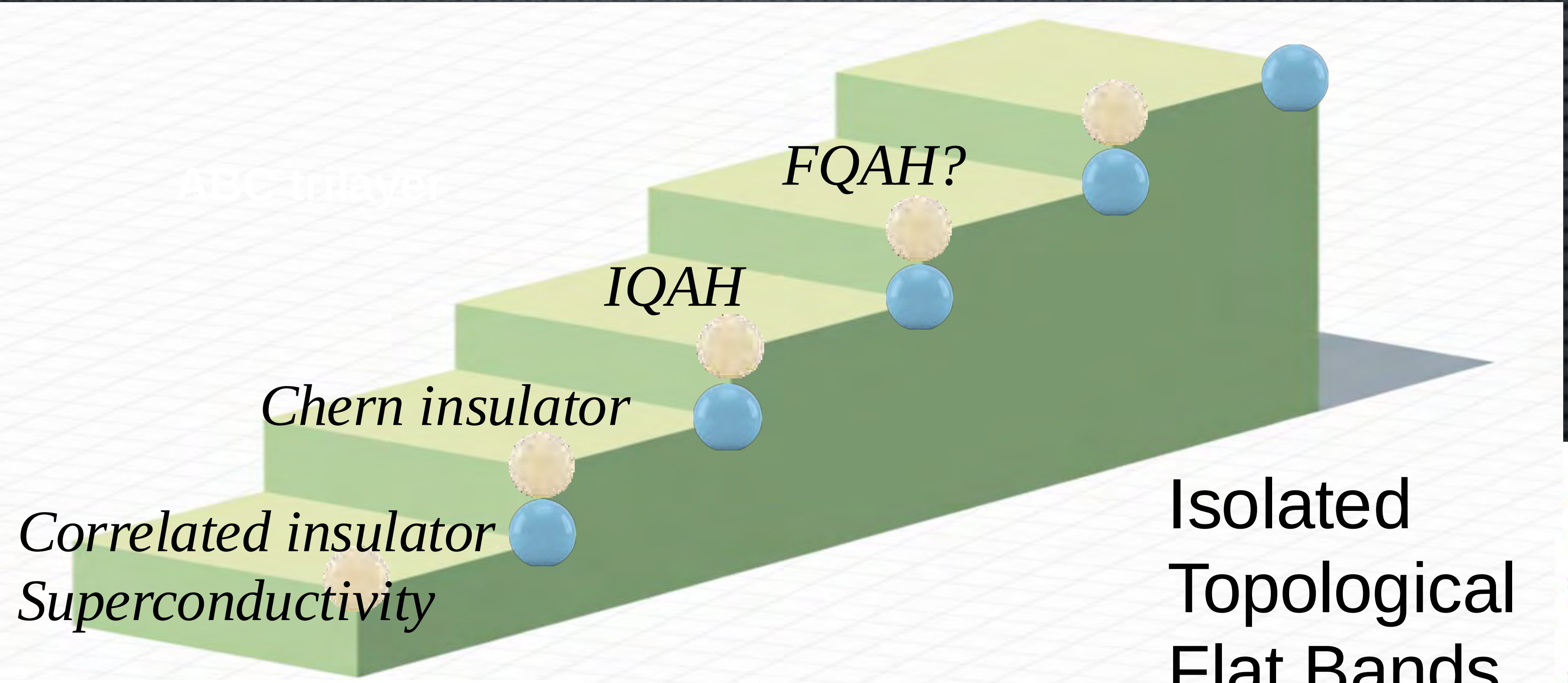


Gating



J, Yang, et al. Science 375 (6586), 1295-1299

Staircase to Fractional Quantum Hall Effects at B=0T



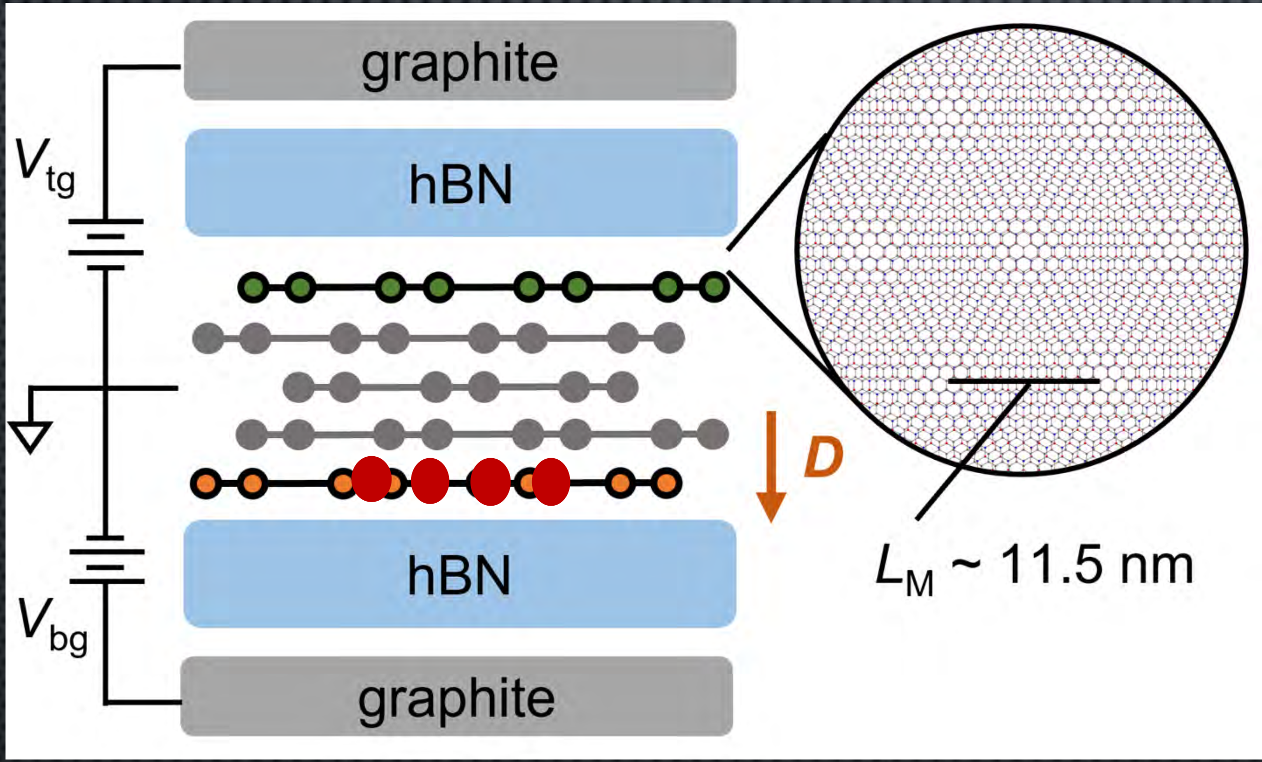
Isolated ✓
Topological ✓
Flat Bands ✓

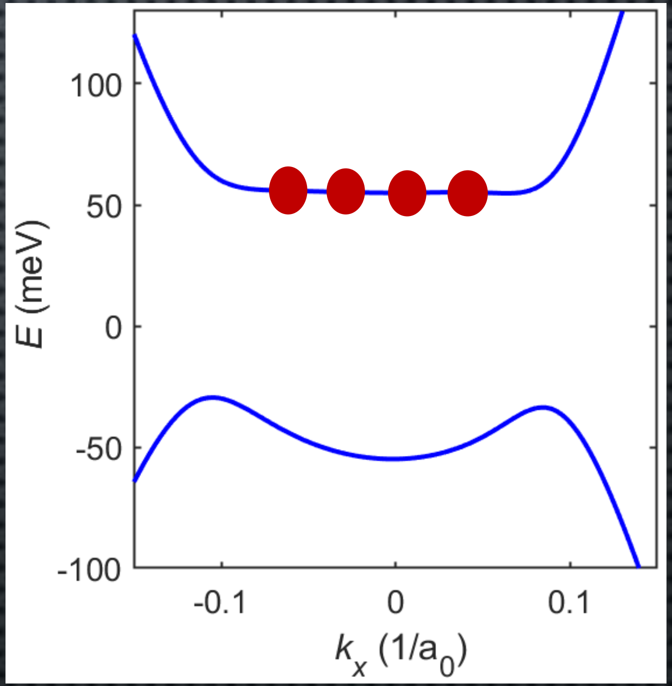
Zhou, H… Young, A. *Nature* **598**, 434–438 (2021).

T. Han, Z. Lu…LJ, *Nat. Nanotechnol.* (2023)

T. Han, Z. Lu… LJ, *Science 384(6696)*. (2024)

Rhombohedral 5-Layer Graphene/hBN Moiré





Twist angle ~0.7 degrees

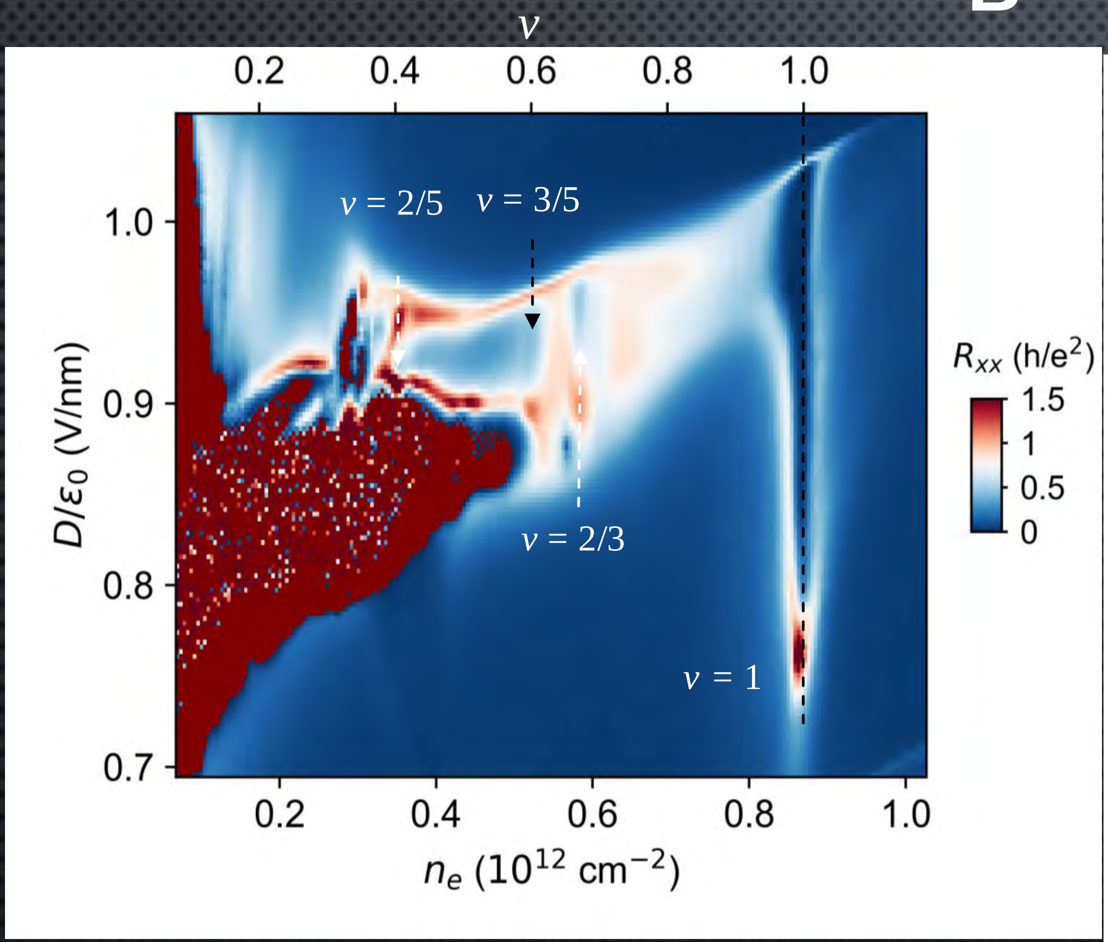
Filling the flat bottom of conduction band

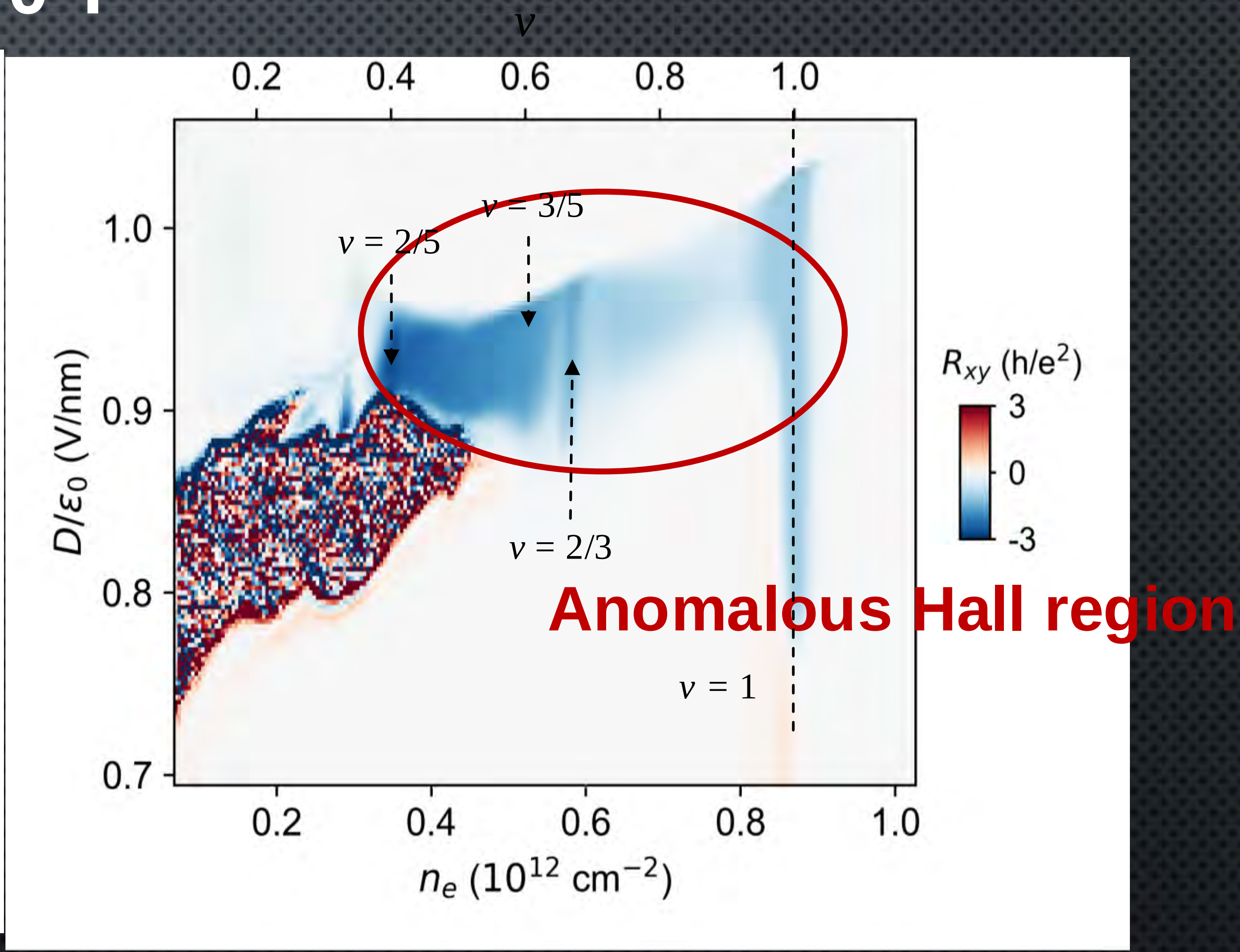
Lu Z., Han T.…LJ,

Accepted to *Nature*, Dec 19th 2023

Anomalous Hall Effect When Electrons Away from Moiré

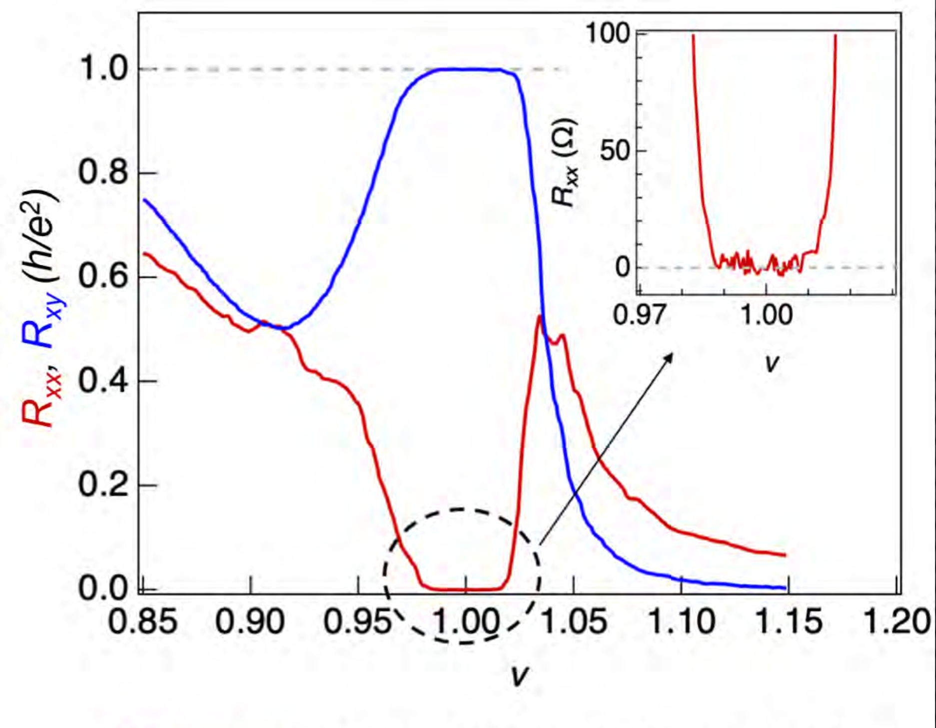
B = 0 T



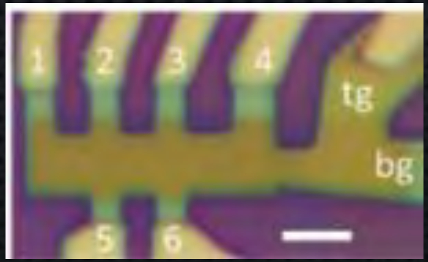


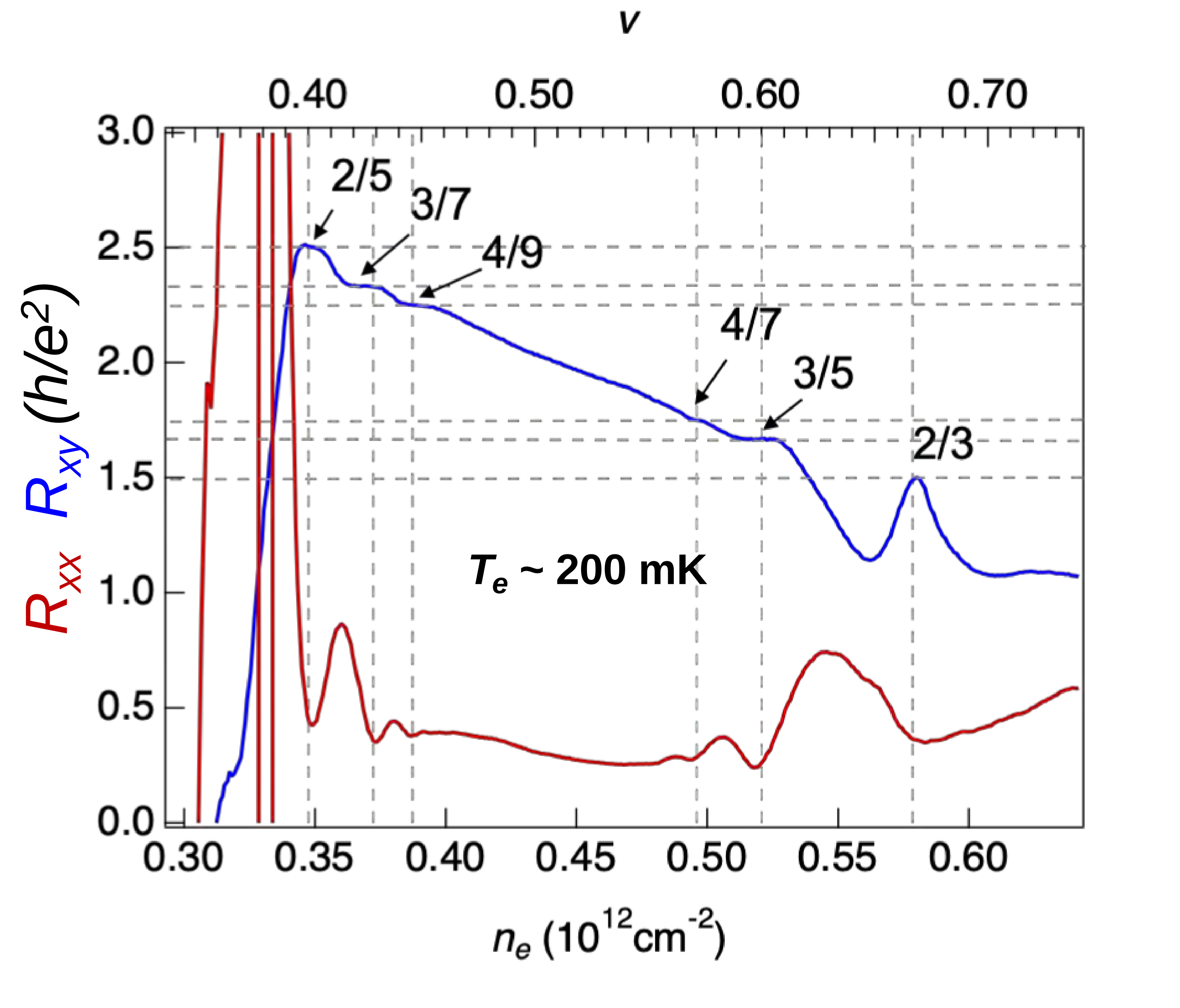
Fractional Quantum Anomalous Hall Effects

Year 2023 Device 1

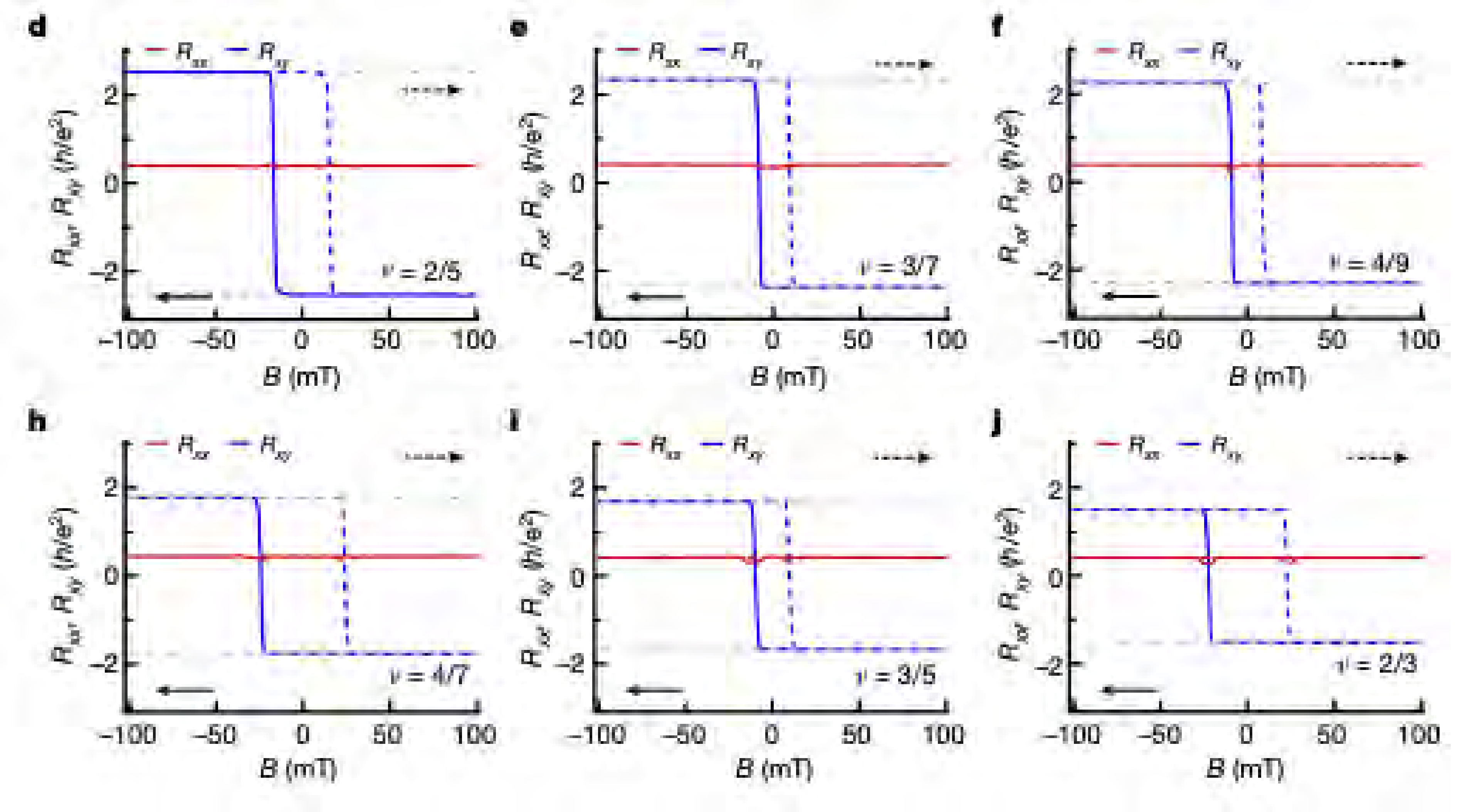


Plateau of quantized R_{xy} vs v (n_e)





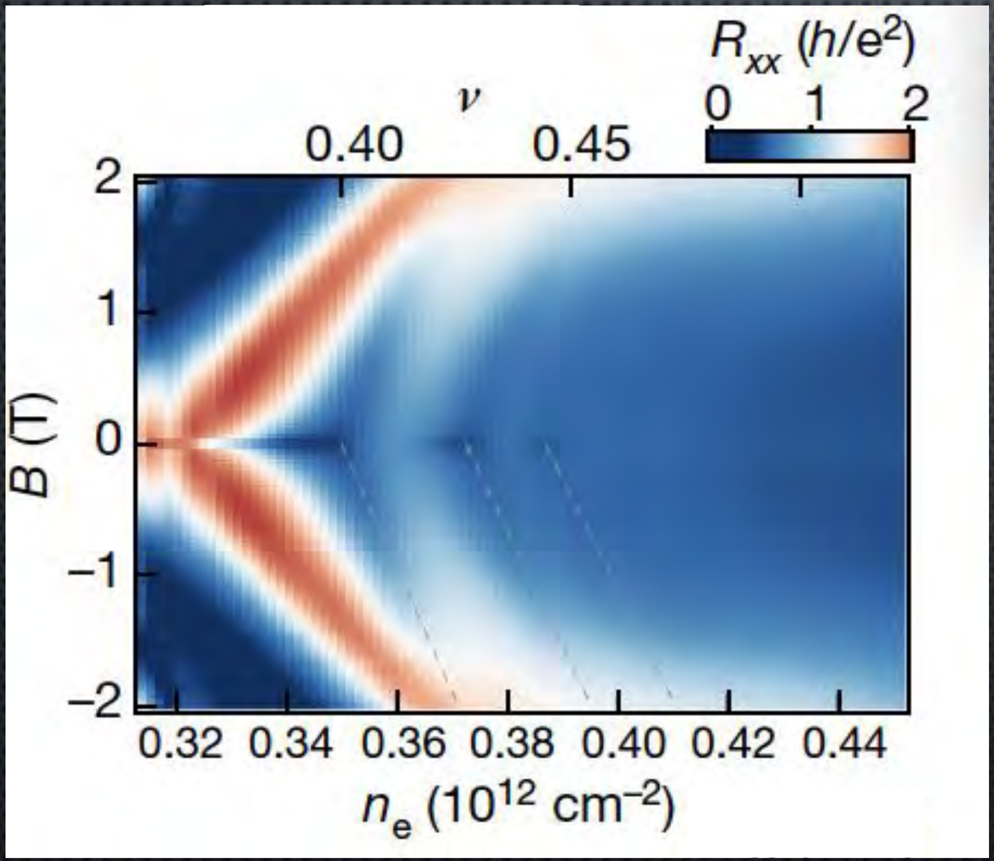
Magnetic Hysteresis

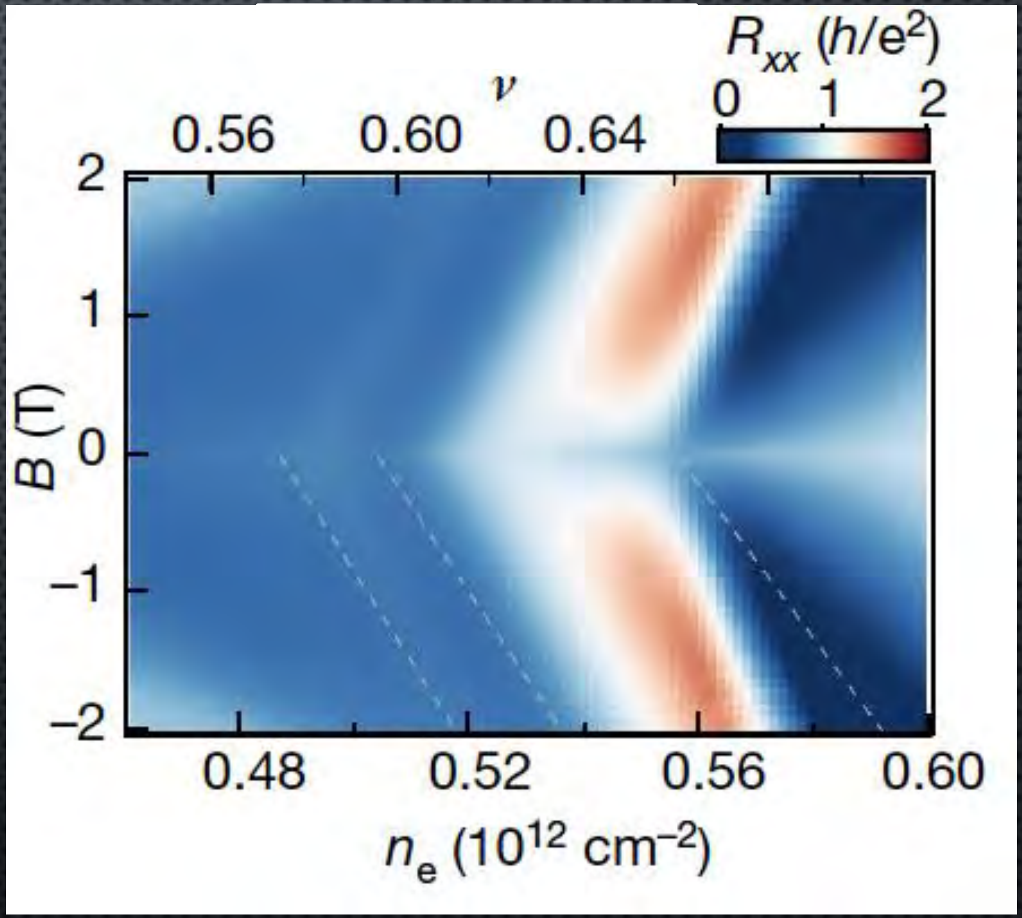


Streda Formula

$$\sigma_{xy} = e\frac{\partial \rho_e}{\partial B}$$

Dispersion of the carrier density respect to magnetic field = edge Hall conductivity

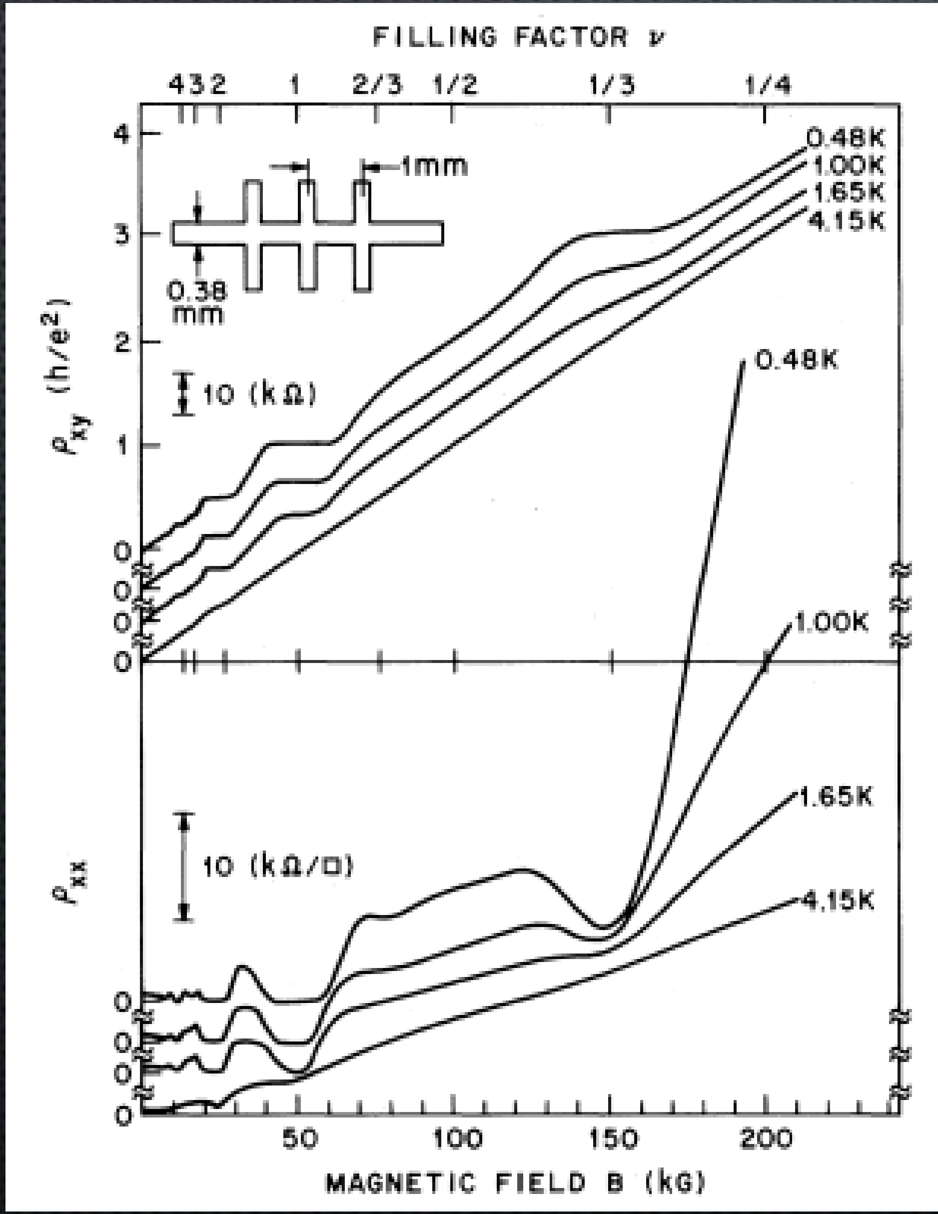


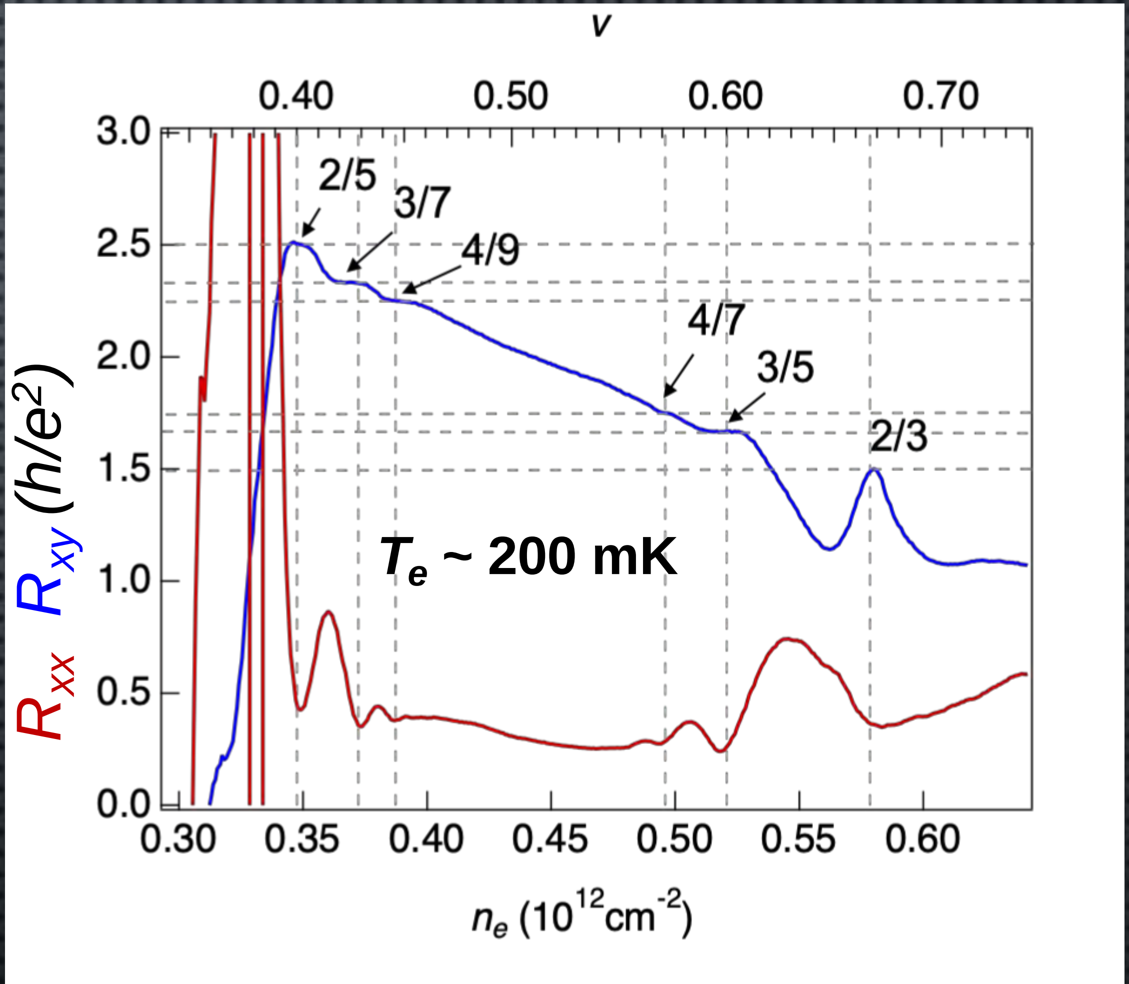


ν = 2/5, 3/7, 4/9

ν = 4/7, 3/5, 2/3

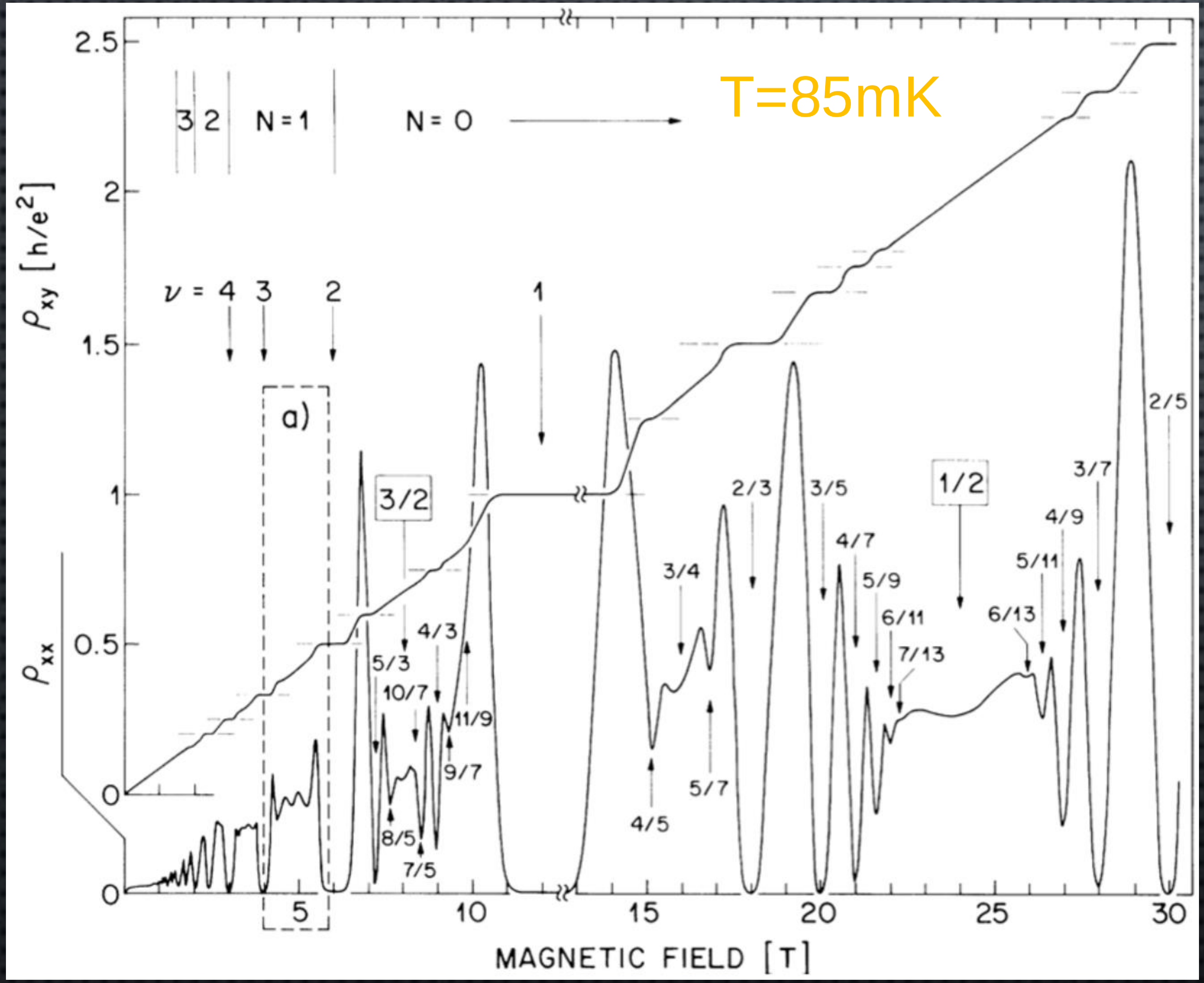
Compare with FQH



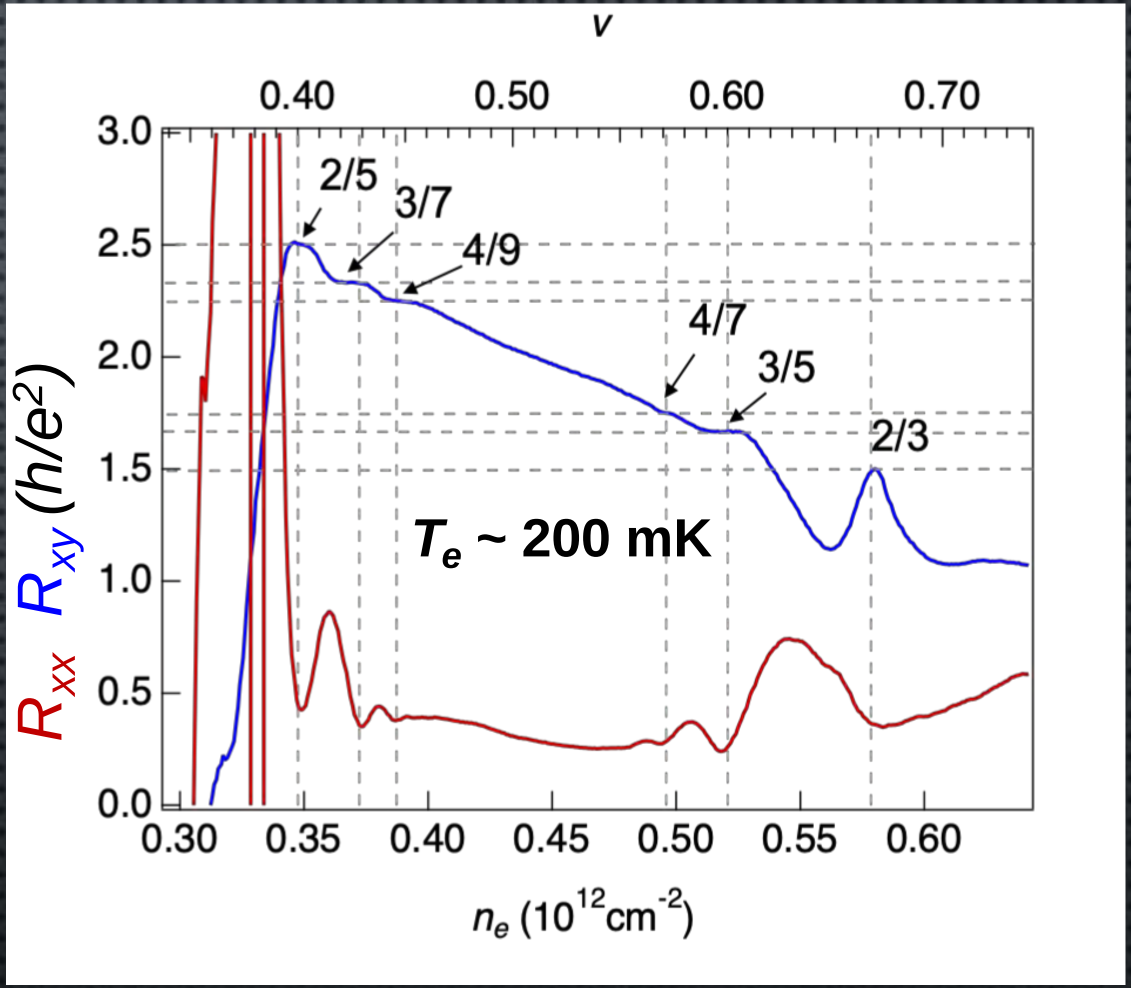


D.C. Tsui, H.L. Stormer, and A.C. Gossard, Phys. Rev. Lett. 48, 1559 (1982).

Compare with FQH



Year 2023 Device 1

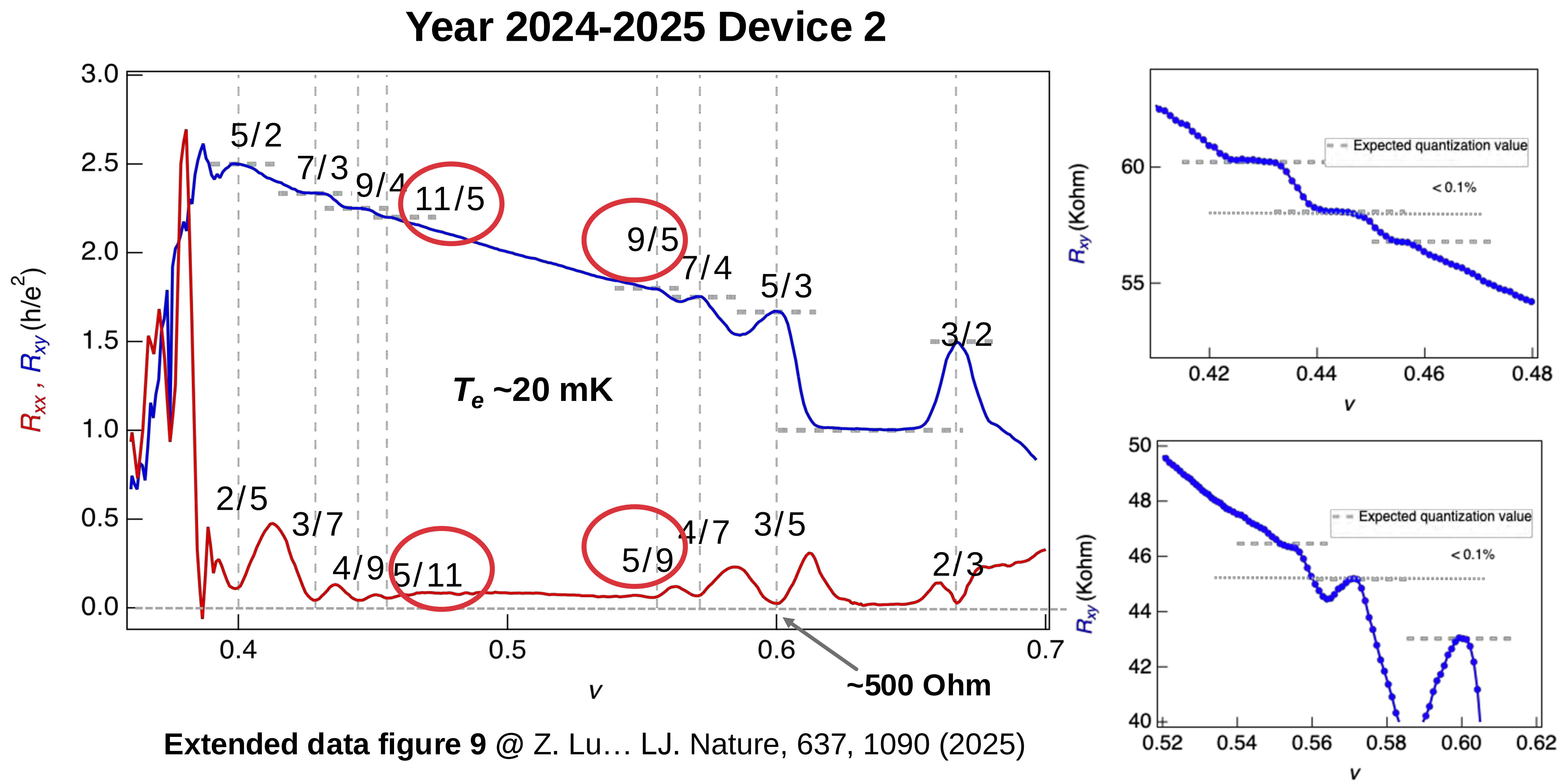


[16]This data is from R. Willett, J. P. Eisenstein, H. L. Stormer, D. C. Tsui, A. C. Gossard and H. English "*Observation of an Even-Denominator Quantum Number in the Fractional Quantum Hall Effect*", Phys. Rev. Lett. 59, 15 (1987).

Better sample quality

Lower electron temperature

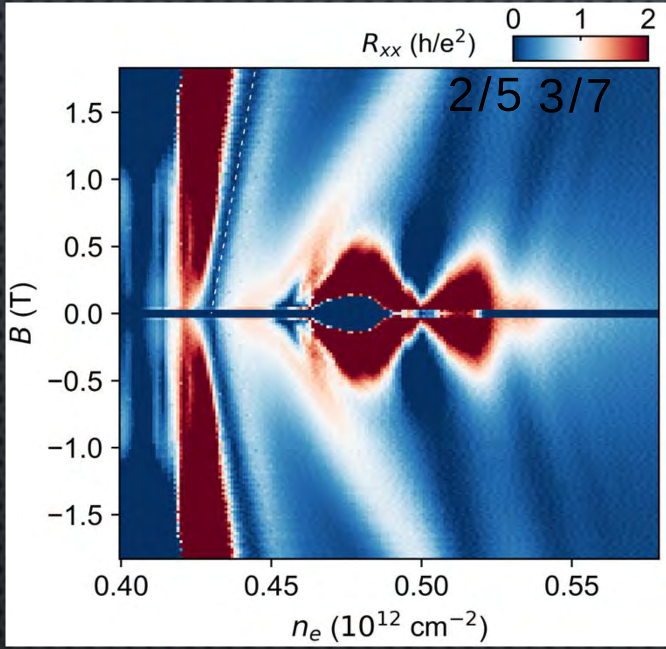
Lower Electron Temperature

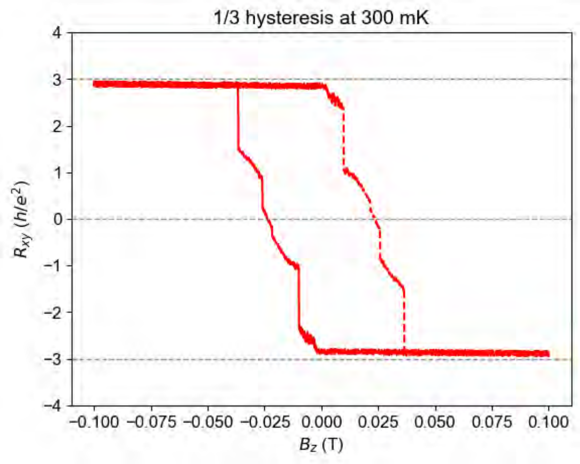


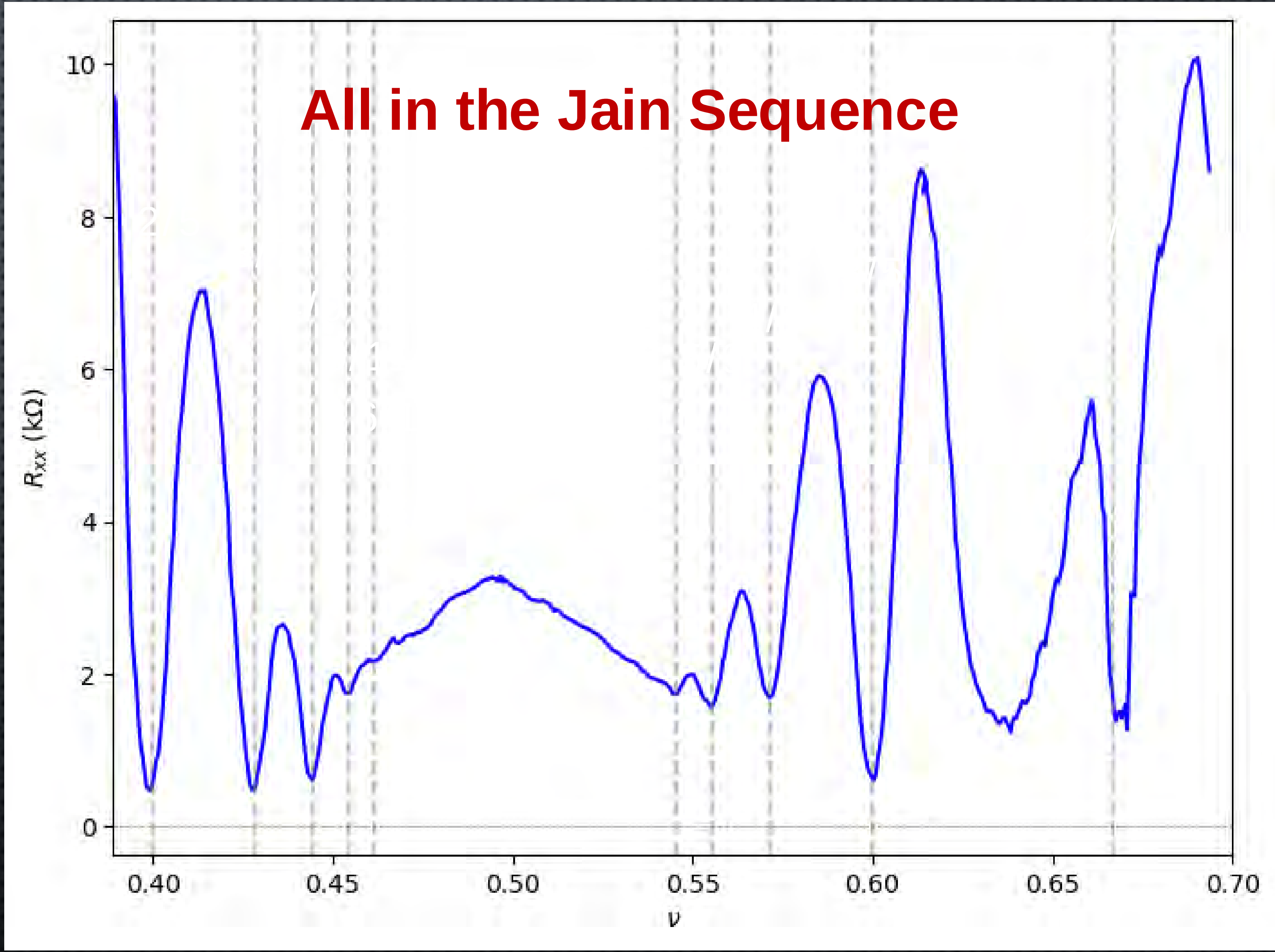
Extended data figure 9 @ Z. Lu… LJ. Nature, 637, 1090 (2025)

11 FQAHE States

Year 2025 Device 2

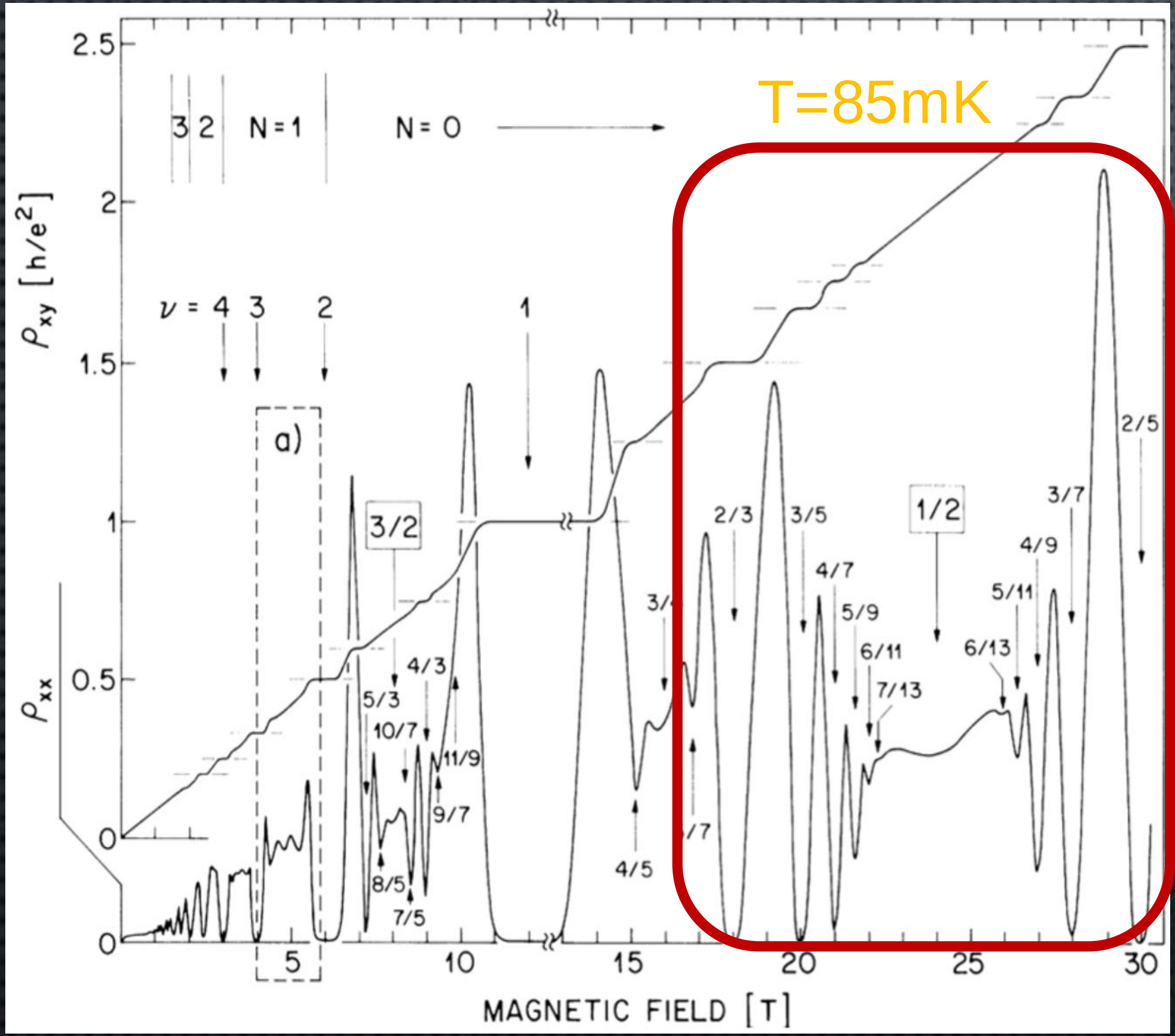




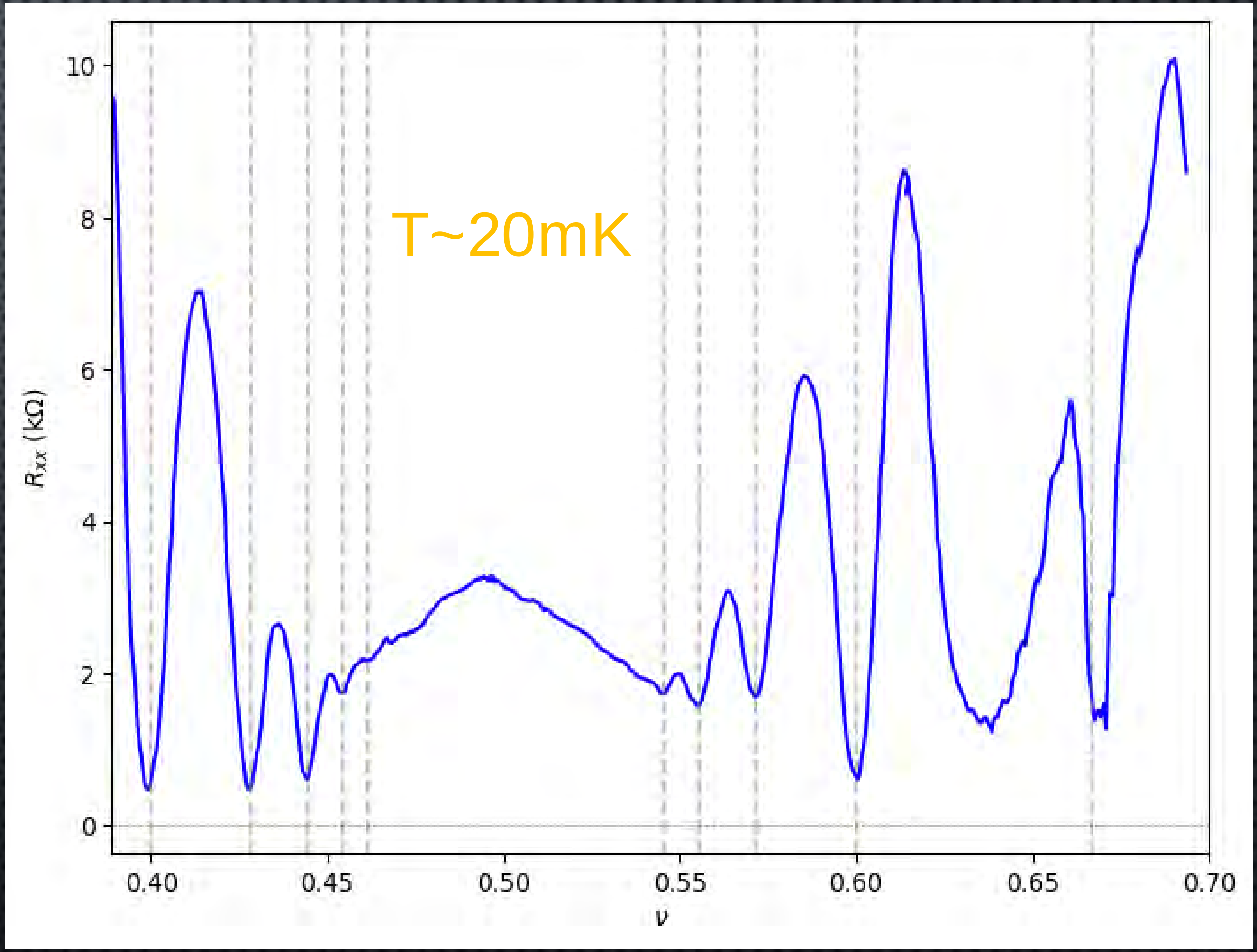


See Z. Lu… LJ. Nature, 637, 1090 (2025) and to be published

Compare with FQH

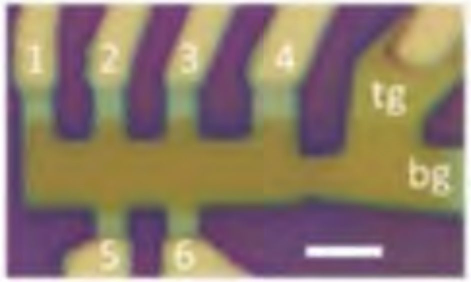
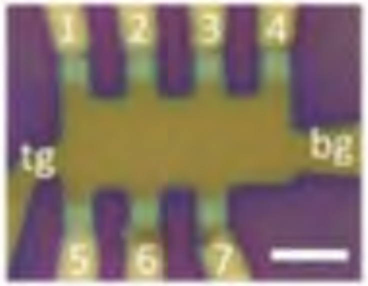
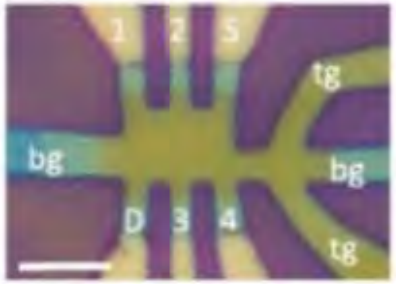


Year 2024-2025 **5L-Graphene**



[16] This data is from R. Willett, J. P. Eisenstein, H. L. Stormer, D. C. Tsui, A. C. Gossard and H. English "*Observation of an Even-Denominator Quantum Number in the Fractional Quantum Hall Effect*", Phys. Rev. Lett. 59, 15 (1987).

Reproducibility

	Device #1	Device #2	Device #3
Optical Image			
Layer number	Pentalayer	Pentalayer	Tetralayer
Angle with hBN	~0.77 deg	~0.77 deg	**~0.3 deg**

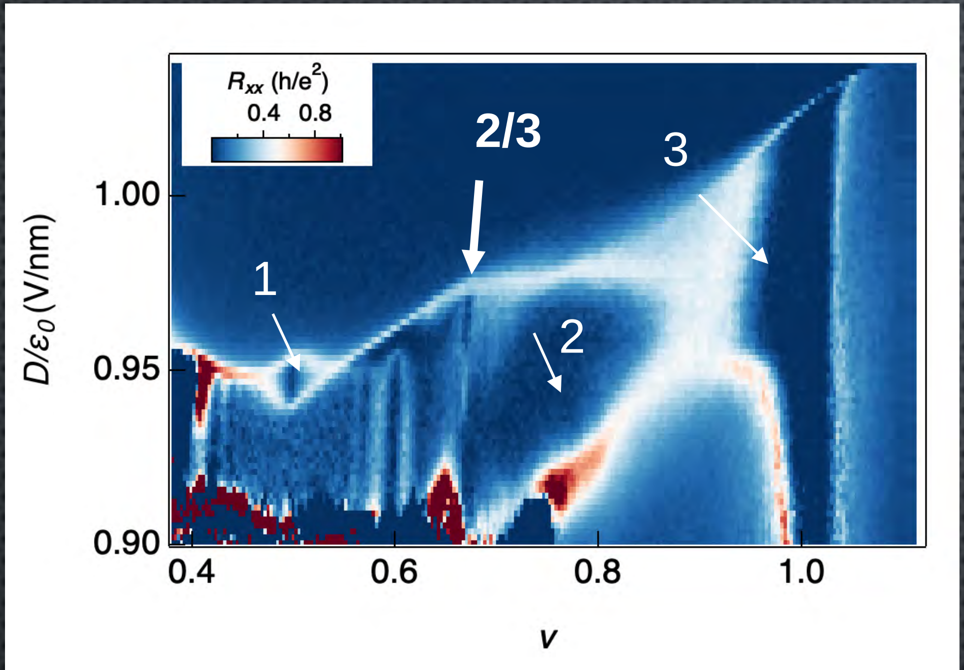
v= 2/3, 3/5, 4/7, 4/9, 3/7,2/5… **v= 2/3, 3/5**

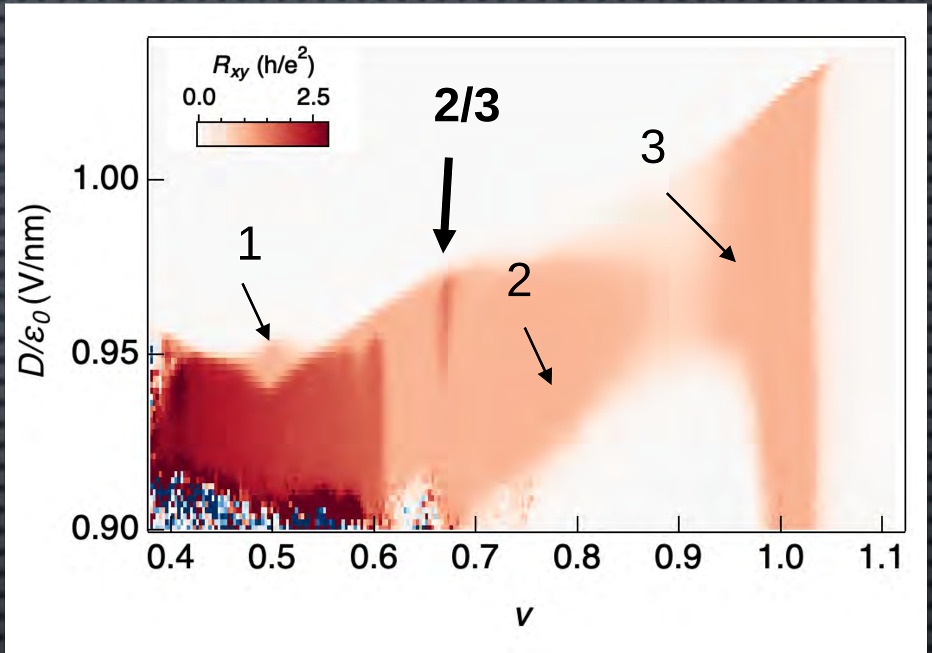
v= 2/3 and 3/5 at 5L rG/hBN observed by Zhengguang Lu (FSU)

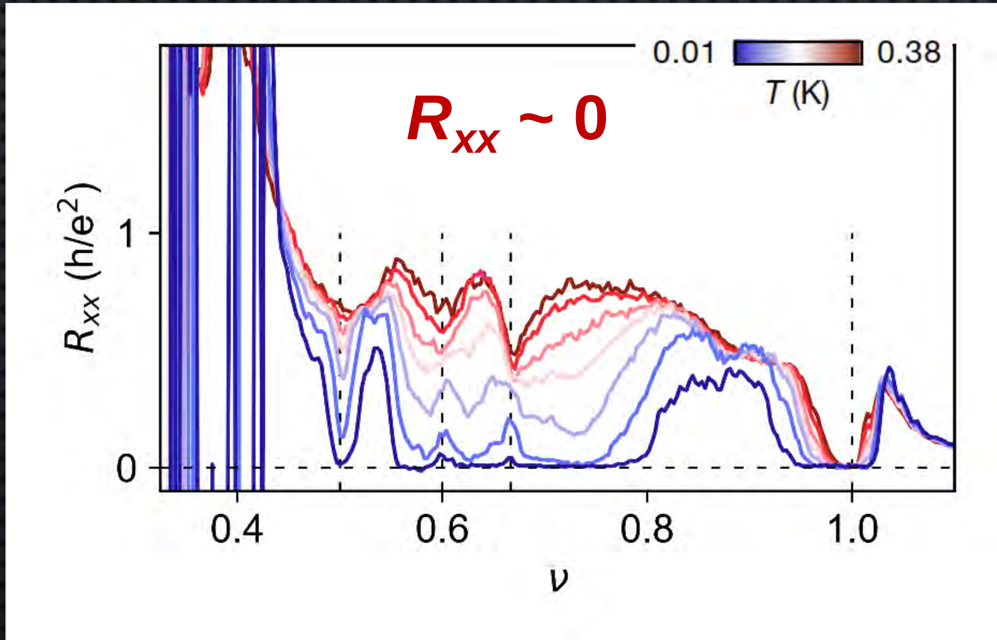
v= 2/3 at 6L rG/hBN observed by Xiaobo Lu (Peking U): Nat. Mater. 24, 1042–1048 (2025).

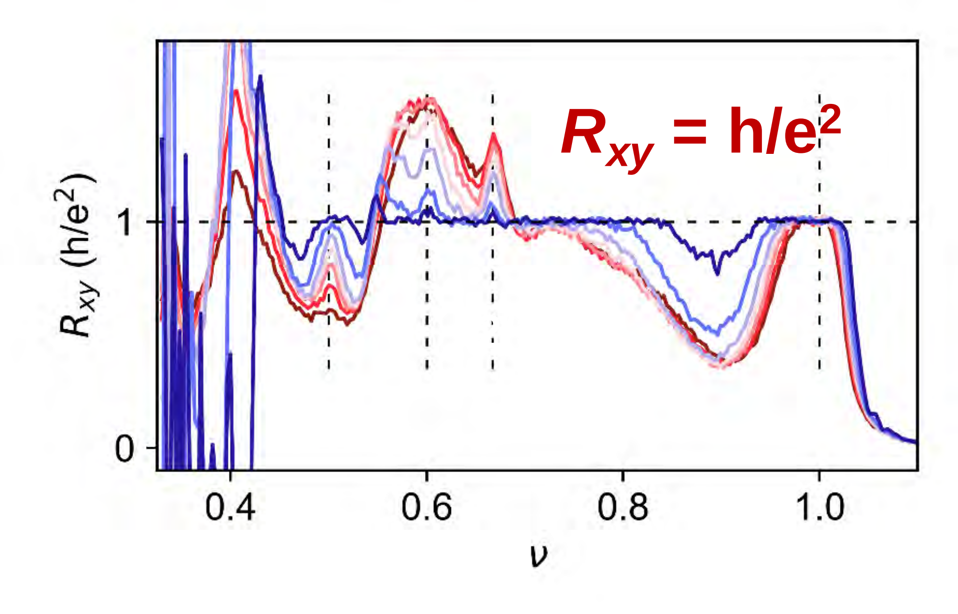
v= 2/3 at 4L rG/hBN observed by Andrea Young (UCSB): Nature 639, 342–347 (2025)

Extended Quantum Anomalous Hall states









Three regions:
Rxx~0
Rxy=h/e^2

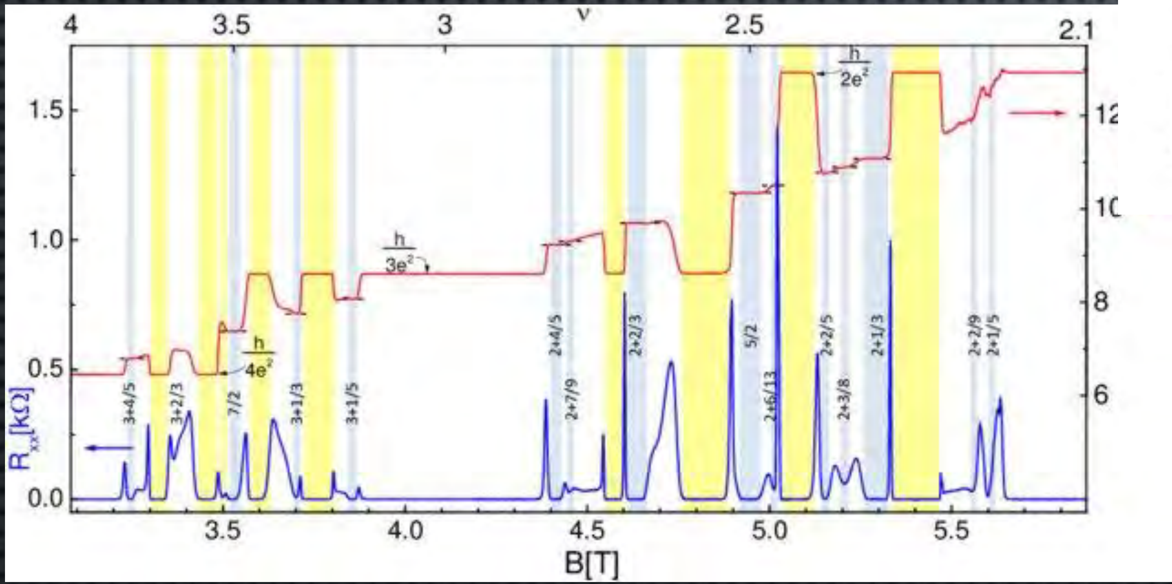
Extended over Large range of Fillings

Only exist at Very low T_e

Underlying Mechanism ?

Anomalous version of RIQHE ?

Reentrant integer quantum Hall



Gábor Csáthy, 2010

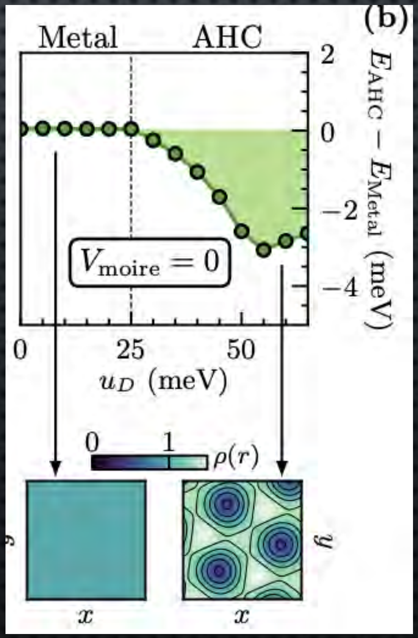
IQH liquid + Wigner crystal

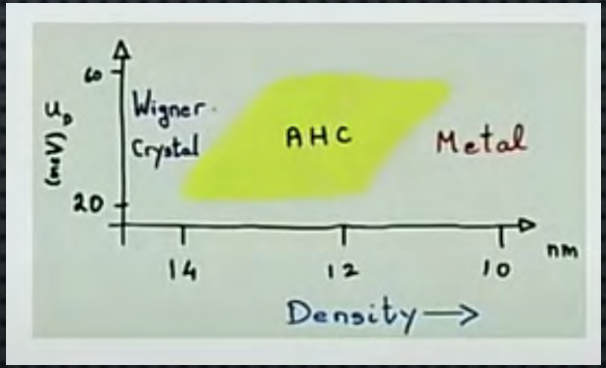
Hall crystal B. I. Halperin, et al. (1986)

VOLUME 57, NUMBER 7 PHYSICAL REVI

Compatibility of Crystalline Order and the Quantized Hall Effect

Anomalous Hall crystal ?



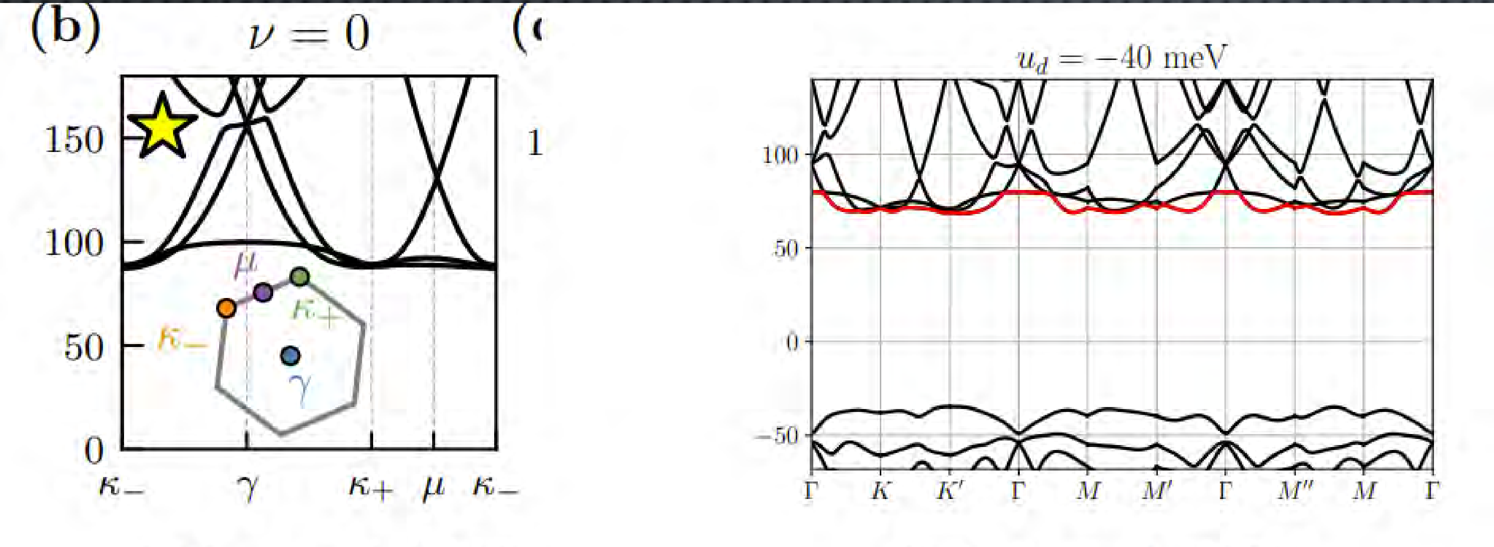


Breaks time reversal and continuous translational symmetry spontaneously

@ A. Vishwanath, T. Senthil, Y. Zhang, F. Zhang

Puzzles

1. How do electrons feel the moiré?
2. Multiple moiré bands entangled



J. Dong et al. (2023).   Z. Dong et al. (2023).

No isolated flat band in moiré-distant band structure

Anomalous Hall Crystals or Moiré Chern Insulators? Spontaneous versus explicit translational symmetry breaking in graphene pentalayers

1. **Theory of fractional quantum anomalous Hall phases in pentalayer rhombohedral graphene moiré structures**
Authors: Zhihuan Dong, Adarsh S. Patri, and T. Senthil
arXiv:2311.03445; DOI: 10.48550/arXiv.2311.03445

2. **Fractional quantum anomalous Hall effects in rhombohedral multilayer graphene in the moiréless limit and in Coulomb imprinted superlattice**
Authors: Boran Zhou, Hui Yang, and Ya-Hui Zhang
arXiv:2311.04217; DOI: 10.48550/arXiv.2311.04217

3. **Anomalous Hall Crystals in Rhombohedral Multilayer Graphene I: Interaction-Driven Chern Bands and Fractional Quantum Hall States at Zero Magnetic Field**
Authors: Junkai Dong, Taige Wang, Tianle Wang, Tomohiro Soejima, Michael P. Zaletel, Ashvin Vishwanath, and Daniel E. Parker
arXiv:2311.05568; DOI: 10.48550/arXiv.2311.05568

4. **Theory of fractional Chern insulator states in pentalayer graphene moiré superlattice**
Authors: Zhongqing Guo, Xin Lu, Bo Xie, Jianpeng Liu
arXiv:2311.14368; DOI: 10.48550/arXiv.2311.14368

5. **Moiré Fractional Chern Insulators III: Hartree-Fock Phase Diagram, Magic Angle Regime for Chern Insulator States, the Role of the Moiré Potential and Goldstone Gaps in Rhombohedral Graphene Superlattices**
Authors: Yves H. Kwan, Jiabin Yu, Jonah Herzog-Arbeitman, Dmitri K. Efetov, Nicolas Regnault, and B. Andrei Bernevig
arXiv:2312.11617; DOI: 10.48550/arXiv.2312.11617

Recommended with a commentary by S.A. Parameswaran, University of Oxford
|View Commentary (pdf)|

This commentary may be cited as:
DOI: 10.36471/JCCM_January_2024_02
https://doi.org/10.36471/JCCM_January_2024_02

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Theoretical Mystery on Rhombohedral Graphene FCI

Yves H. Kwan, Jiabin Yu, Jonah Herzog-Arbeitman, Dmitri K. Efetov, Nicolas Regnault, B. Andrei Bernevig. arXiv:2312.11617

Jiabin Yu, Jonah Herzog-Arbeitman, Yves H. Kwan, Nicolas Regnault, B. Andrei Bernevig. arXiv:2407.13770 (2024)

Zhongqing Guo, Xin Lu, Bo Xie, and Jianpeng Liu. Phys. Rev. B 110, 075109 (2023)

Koji Kudo, Ryota Nakai, and Kentaro Nomura. Phys. Rev. B 110, 245135 (2024)

Zhihuan Dong, Adarsh S. Patri, and T. Senthil. Phys. Rev. B 110, 205130 (2024)

Ming Xie and Sankar Das Sarma. Phys. Rev. B 109, L241115 (2024)

Sheng, D. N., Reddy, A. P., Abouelkomsan, A., Bergholtz, E. J. & Fu, L. Phys. Rev. Lett. 133, 66601 (2024).

Dong, Z., Patri, A. S. & Senthil, T. *Phys. Rev. Lett.* 133, 206502 (2024).

Dong, J. et al. *Phys. Rev. Lett.* 133, 206503 (2024).

Hongyu Lu, Han-Qing Wu, Bin-Bin Chen, Kai Sun, Zi Yang Meng. arXiv:2403.03258

Ke Huang, Xiao Li, Sankar Das Sarma, and Fan Zhang. Phys. Rev. B 110, 115146 (2024)

Boran Zhou, Hui Yang, and Ya-Hui Zhang. Phys. Rev. Lett. 133, 206504 (2024)

Ke Huang, Sankar Das Sarma, Xiao Li. arXiv:2408.05139 (2024)

Tixuan Tan and Trithep Devakul. Phys. Rev. X 14, 041040 (2024)

Zhaochen Liu, Jing Wang. arXiv:2401.13413 (2024)

Tixuan Tan, Julian May-Mann, Trithep Devakul. arXiv:2409.06775 (2024)

Sankar Das Sarma and Ming Xie. Phys. Rev. B 110, 155148 (2024)

Soejima, T. et al. Phys. Rev. B 110, 205124 (2024).

Dong, Z., Patri, A. S. & Senthil, T. Phys. Rev. B 110 ,205130 (2024).

Zezhu Wei, Ang-Kun Wu, Miguel Gonçalves, Shi-Zeng Lin. arXiv:2409.05043 (2024)

H Li, BA Bernevig, N Regnault, Physical Review B 112 (7), 075130, (2025)

No consensus yet!!!

Flat band + Berry curvature + ~~moiré~~ = FQAHE

No QAHE, but Superconductor

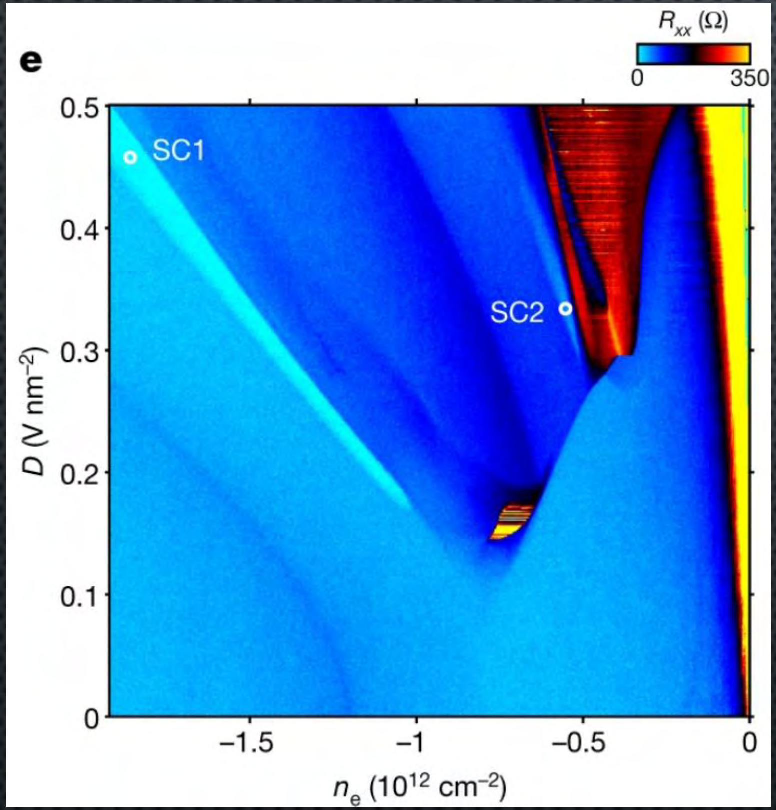
Different

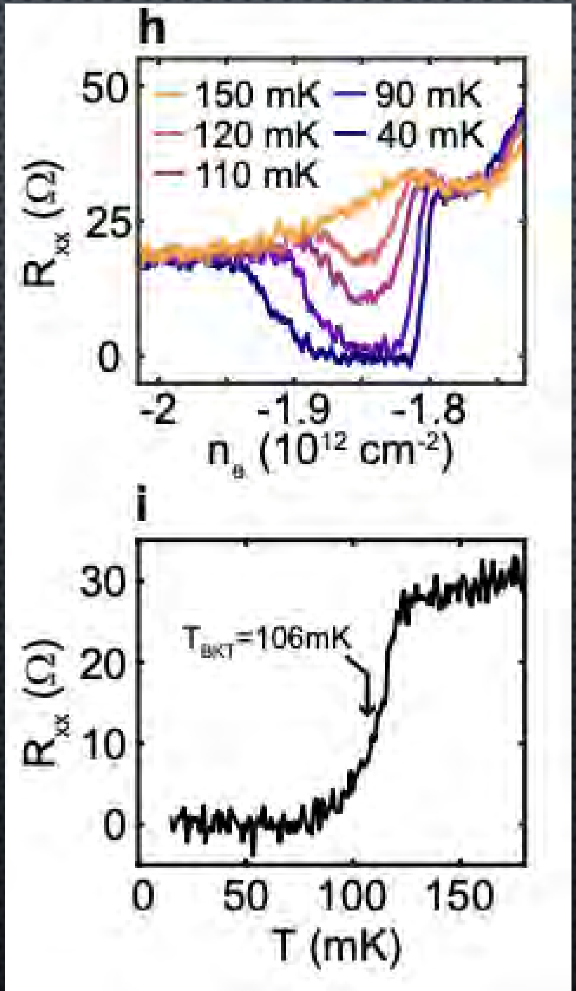
From all known superconductors

T. Han, Z. Lu, Z. Hadjri… LJ, Nature (2025)

Superconductivity in Crystalline Rhombohedral Graphene

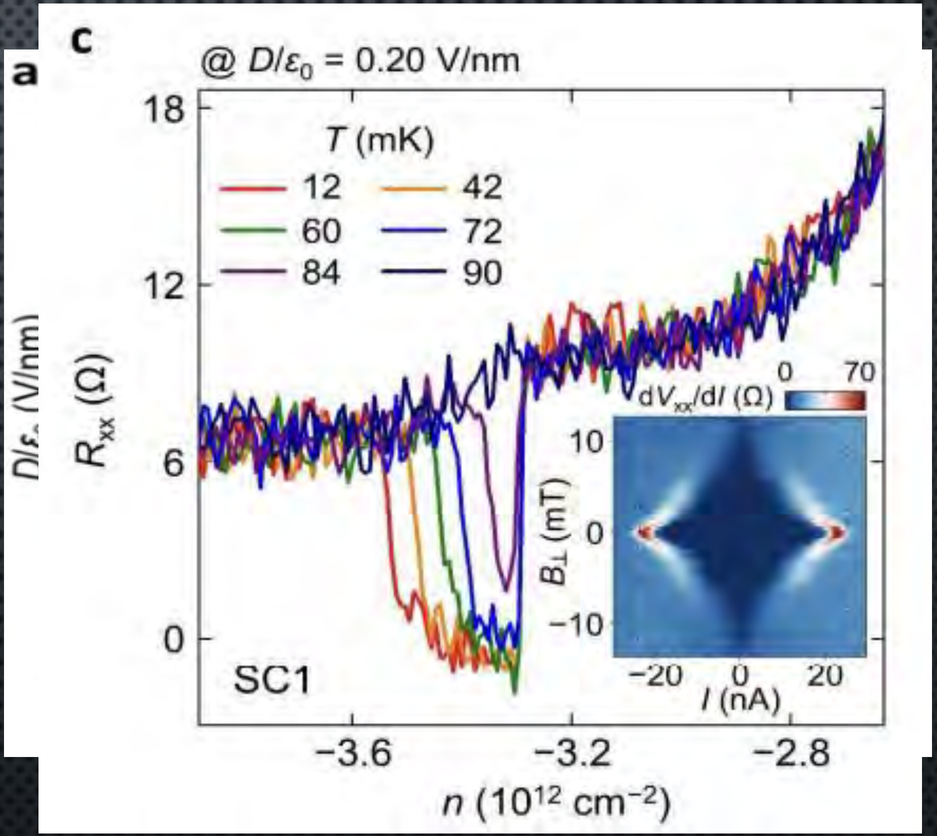
ABC trilayer





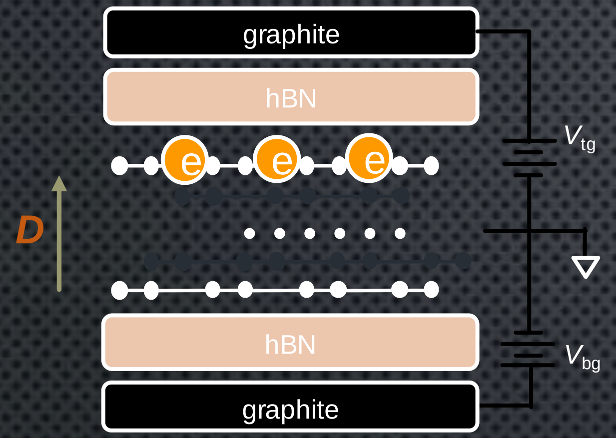
Zhou, H… Young,
A. *Nature* **598**, 434–438 (2021).

Bare Pentalayer RG

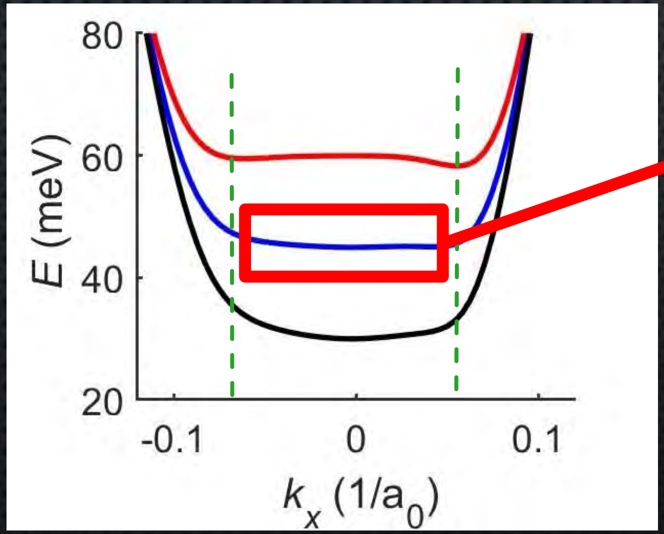


Zero resistance within few Ohms

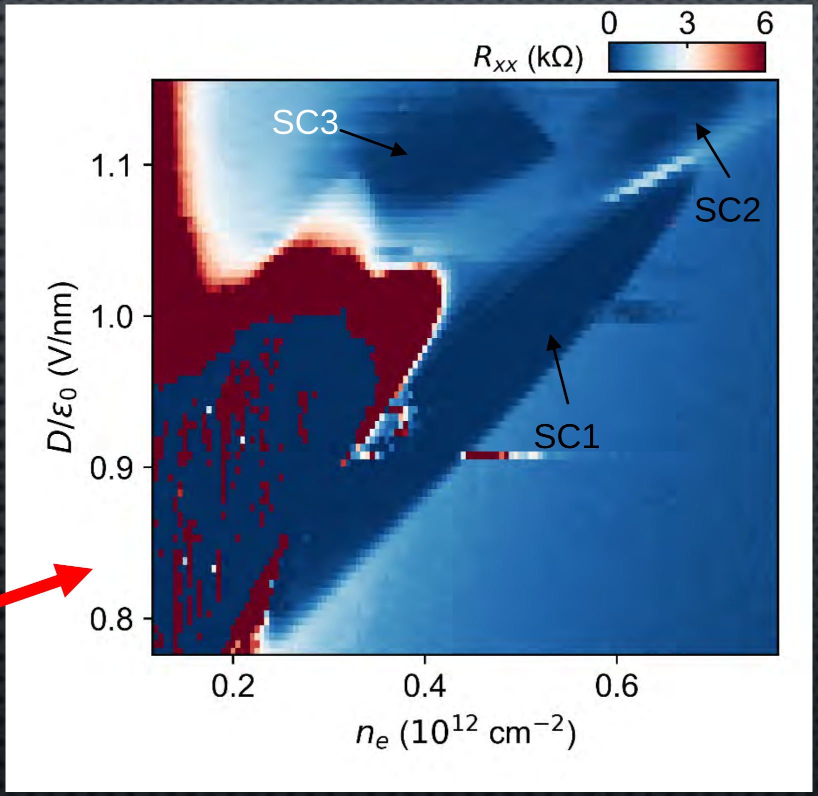
Superconductivities in Bare Rhombohedral Graphene



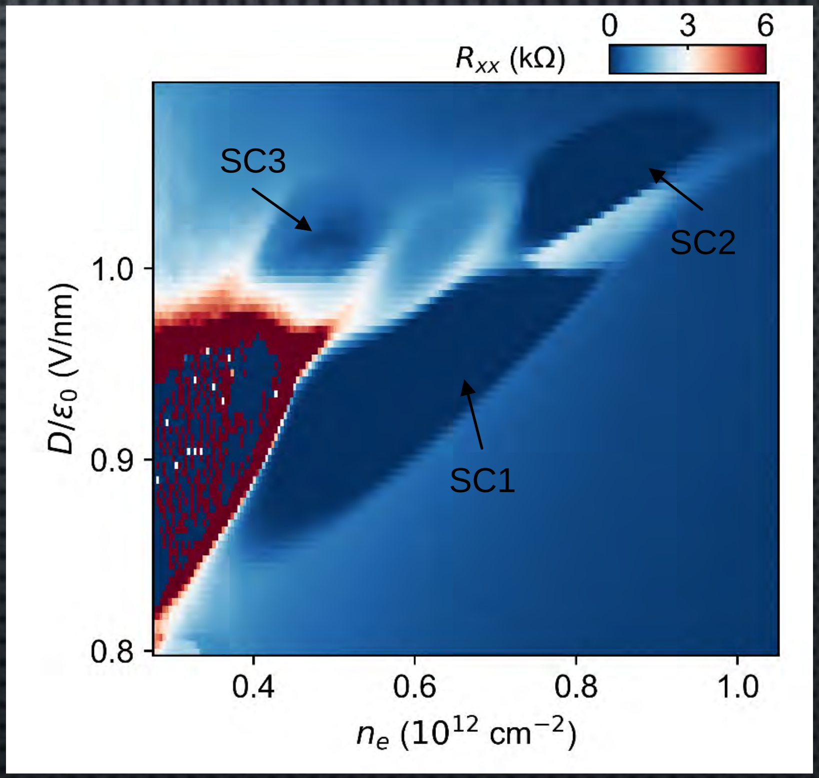
No moiré



4LG - ABCA

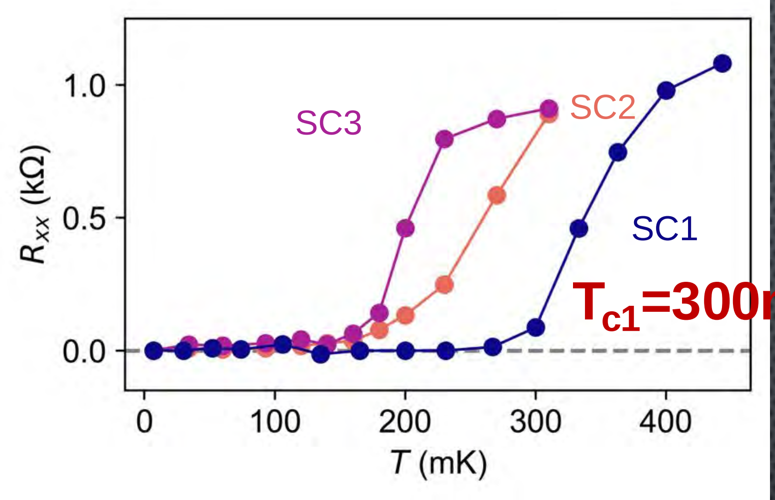


5LG - ABCAB

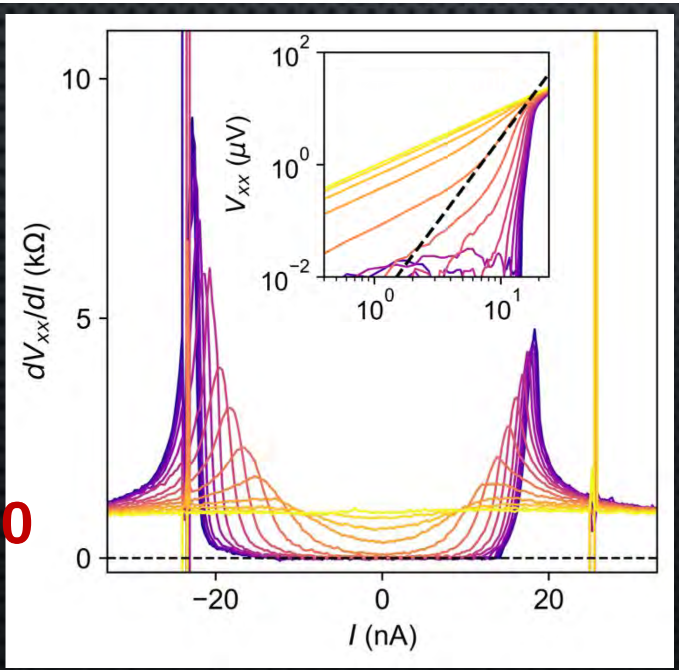


Superconductivities in Bare Rhombohedral Graphite

4LG

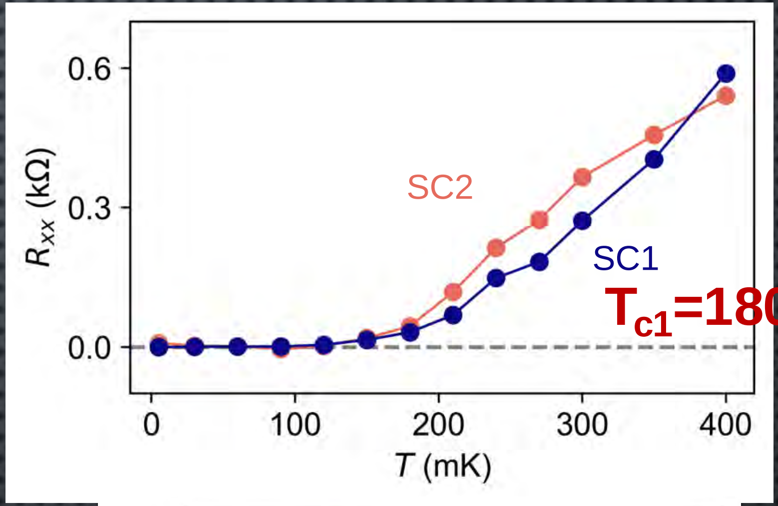


T_{c1}=300mK

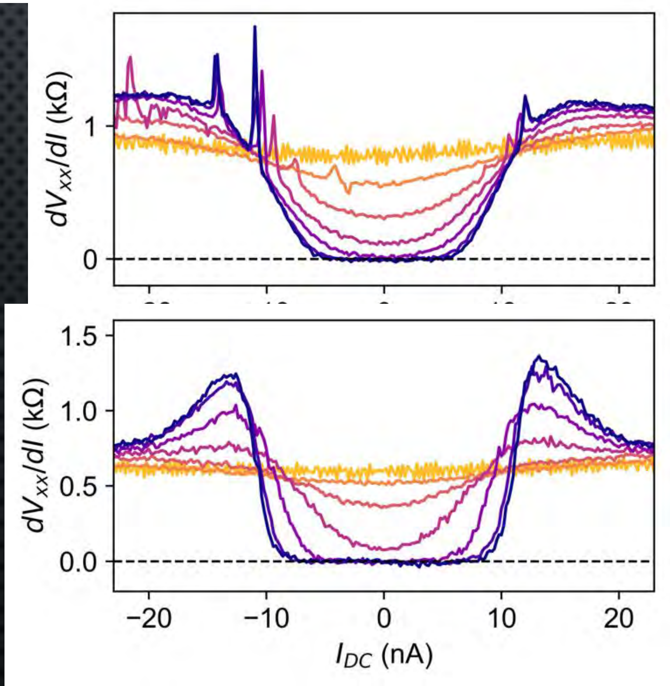


Superconducting diode effect at B=0

5LG

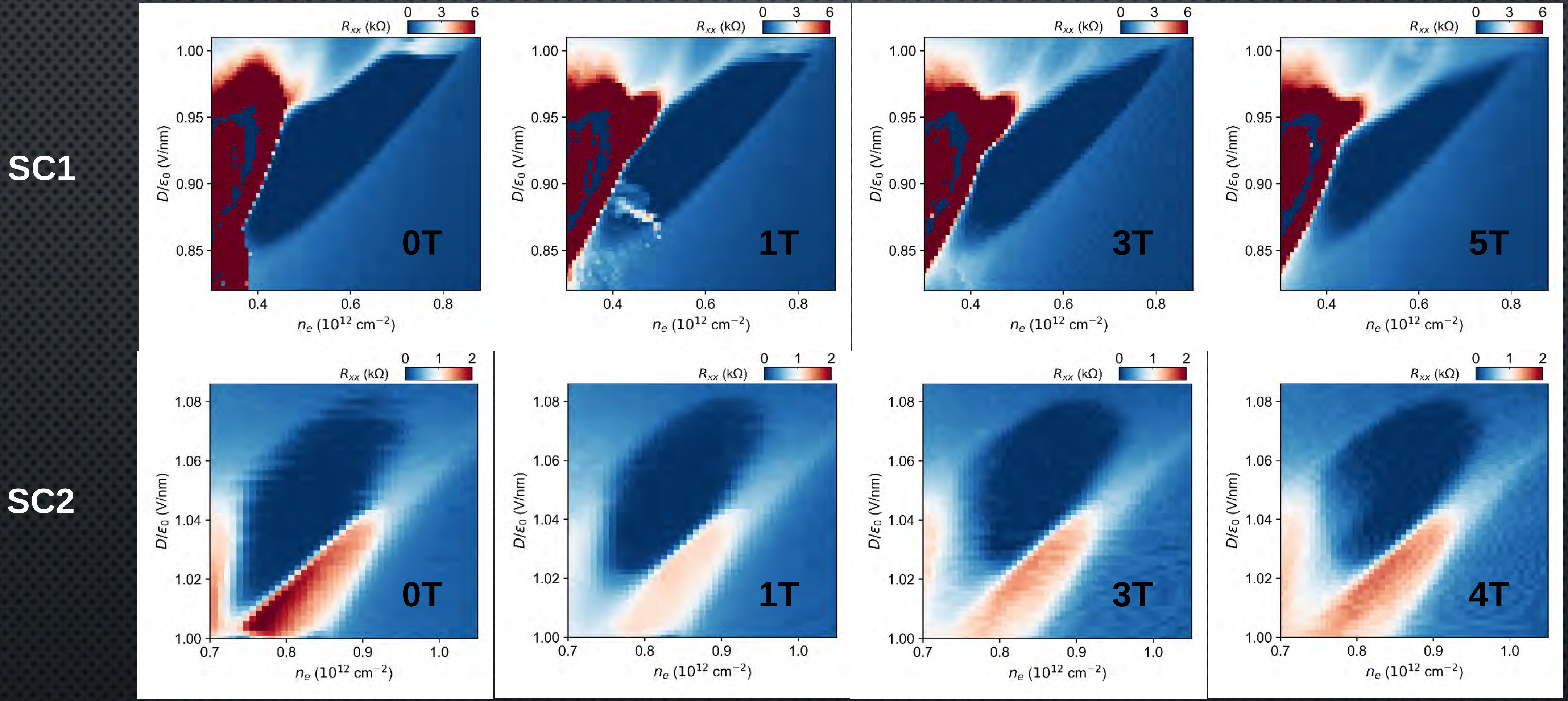


T_{c1}=180mK



In-Plane Magnetic Field Dependence

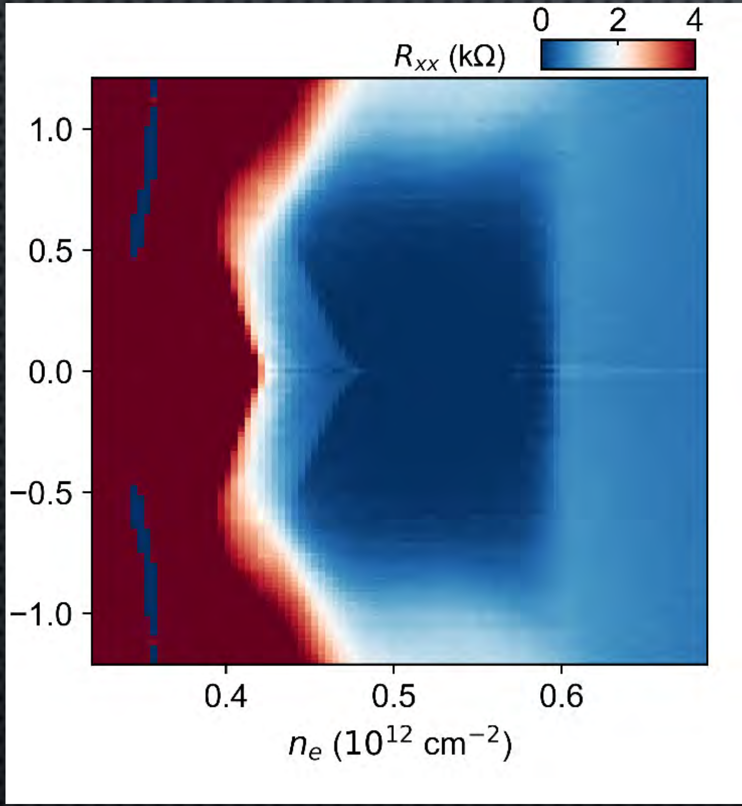
5L Device P1: T_c = 180mK, B_{Pauli} ~0.34 T



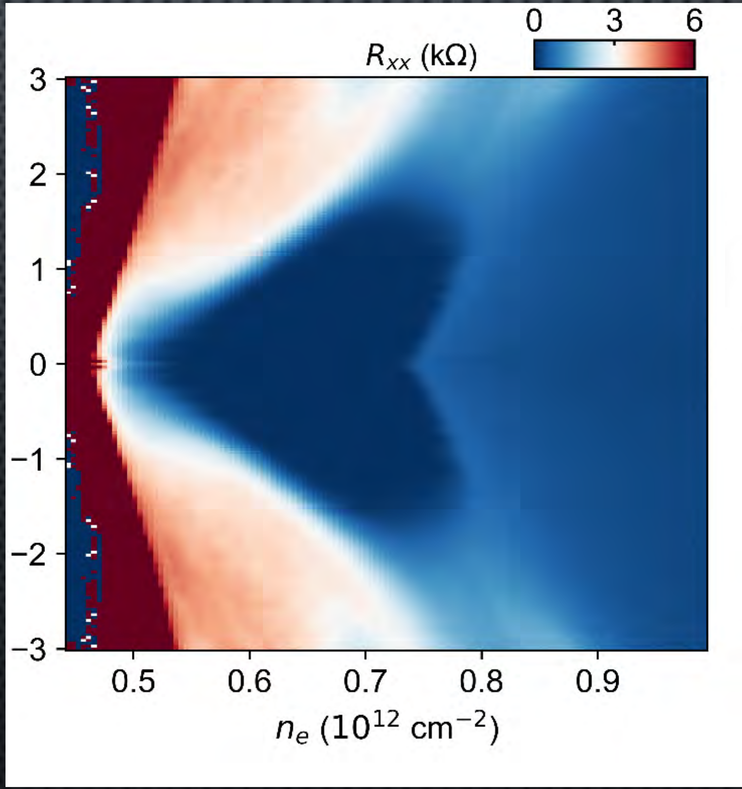
SCs survive largely from $B_{//}$ up to 5 T, exceeding the Pauli limit by at least 15 times

Out-of-Plane Magnetic Field Dependence

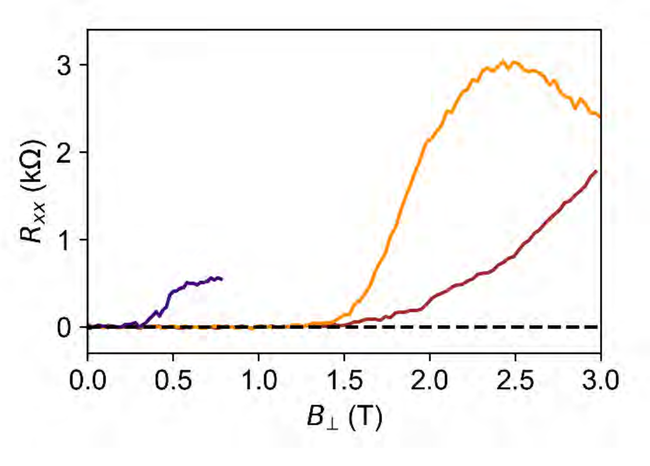
4LG



5LG

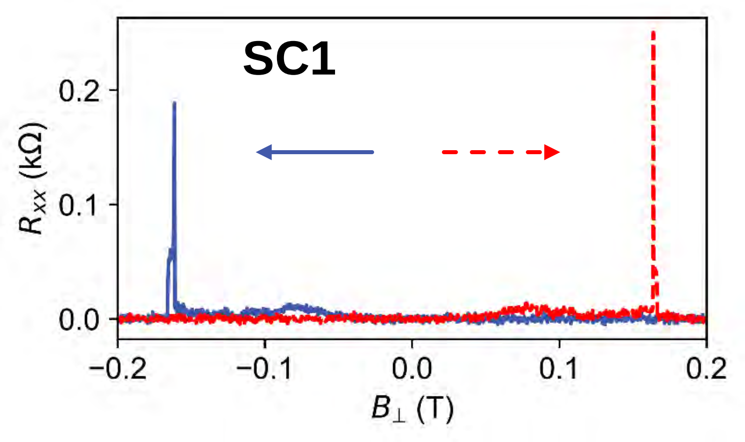


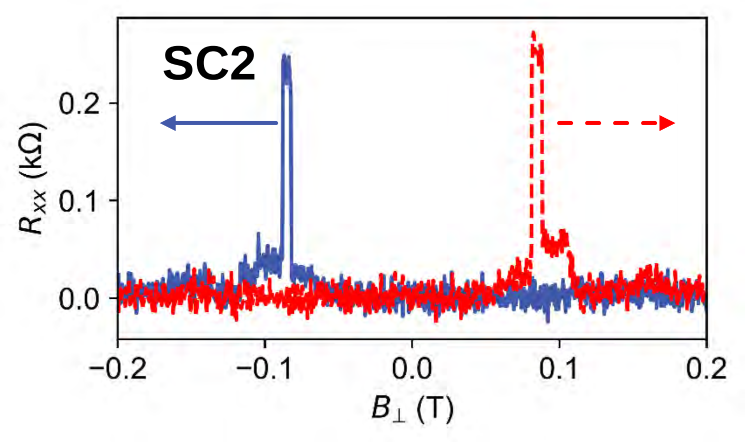
5LG



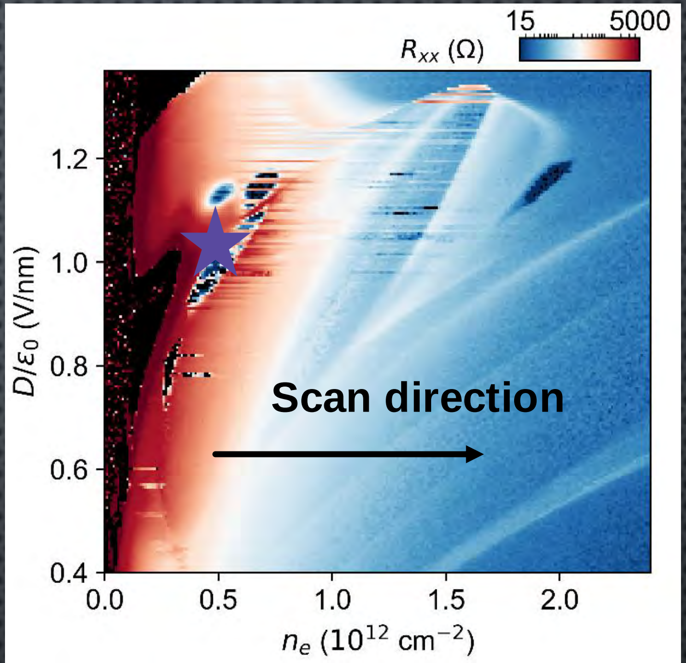
$R_{xx} = 0$ up to 1.4T

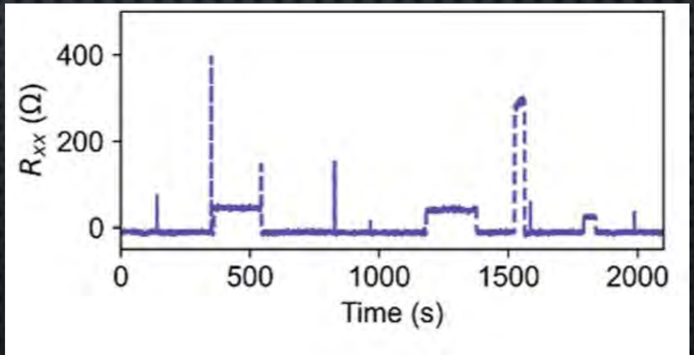
Magnetic Hysteresis in SC vs Out-of-Plane Magnetic Field





Hysteresis



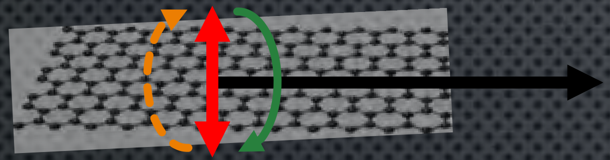


Fluctuations

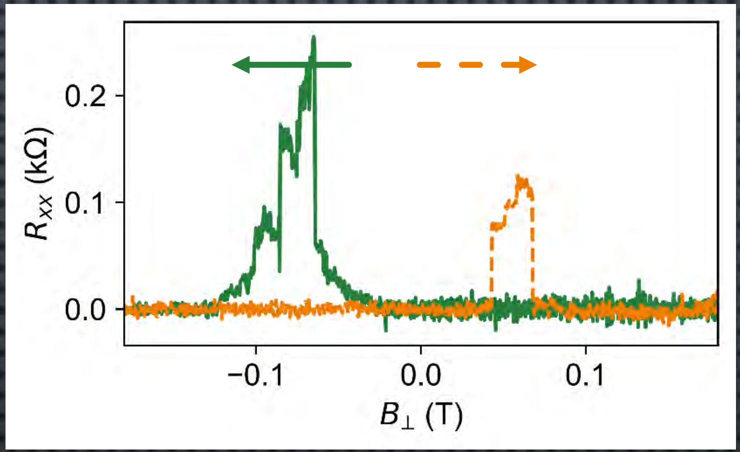
Behaves like a magnet!

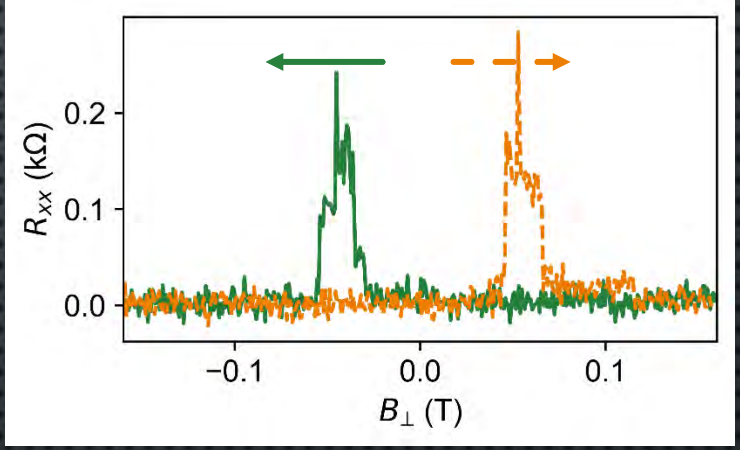
Origin of Magnetic Hysteresis: Spin or Valley?

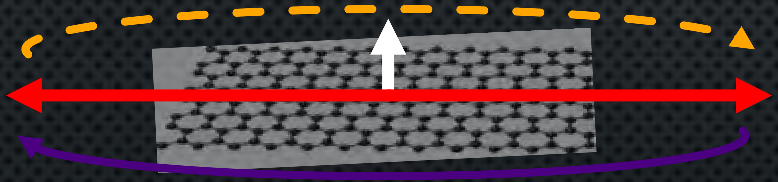
Orbital (valley) magnetization only couples to the out-of-plane B field



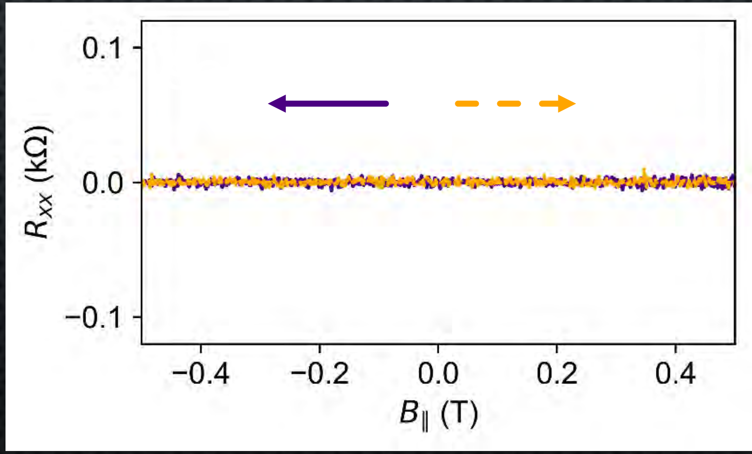
Spin is polarized
Only valley flipping

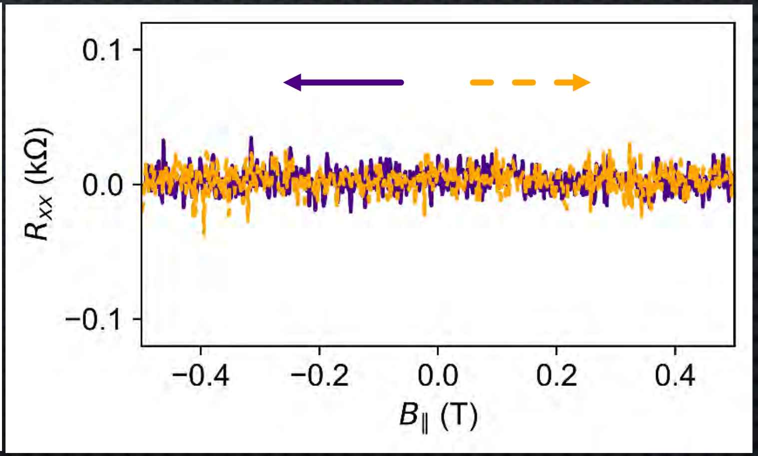




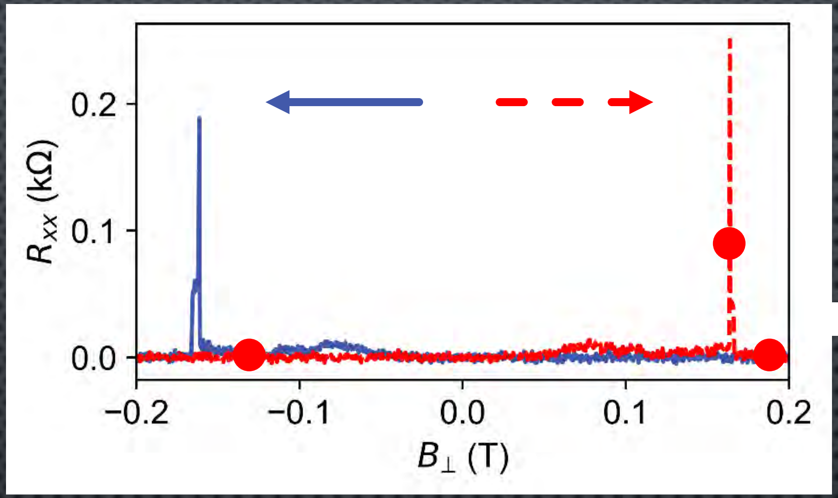


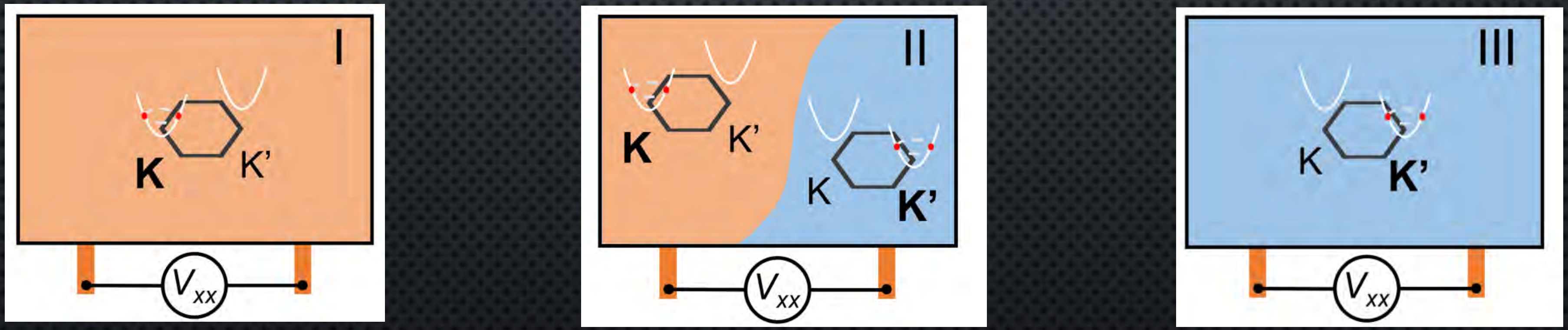
Valley is polarized
Only Spin flipping





Magnetic Hysteresis is from Polarized Valley





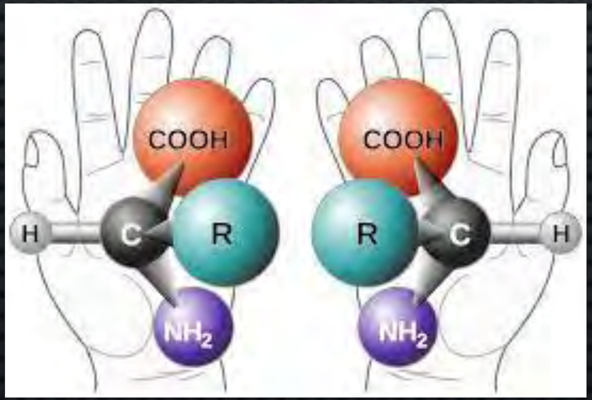
Superconducting state is valley polarized!

Spin- and valley-polarized superconductivity?!

Implications:

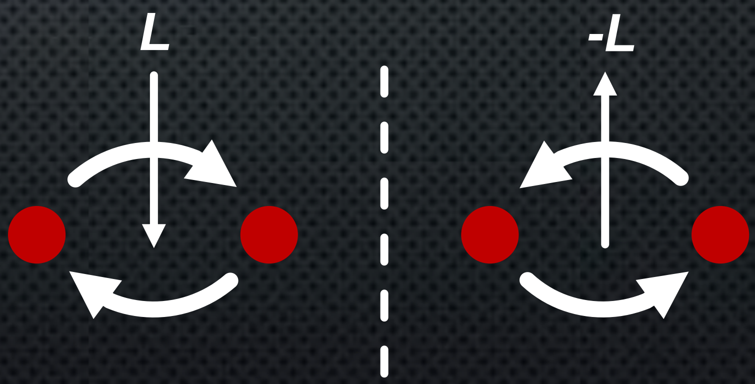
1. Spin triplet
2. Finite momentum pairing (FFLO)
3. Chiral superconductor (time-reversal-symmetry breaking at the orbital level)

Chiral molecules



Break mirror symmetry

Chiral superconductor



Break mirror symmetry (P)

Break time-reversal symmetry (T)

Theory Developments Suggest Topological SC

arXiv:2409.06701 [pdf, other] cond-mat.supr-con cond-mat.mes-hall cond-mat.mtrl-sci cond-mat.str-el

Intravalley spin-polarized superconductivity in rhombohedral tetralayer graphene

Authors: Yang-Zhi Chou, Jihang Zhu, Sankar Das Sarma

arXiv:2409.13829 [pdf, other] cond-mat.supr-con cond-mat.mes-hall

Chiral and topological superconductivity in isospin polarized multilayer graphene

Authors: Max Geier, Margarita Davydova, Liang Fu

arXiv:2409.18067 [pdf, other] cond-mat.str-el cond-mat.supr-con doi 10.1103/PhysRevB.111.014508

Topological chiral superconductivity beyond pairing in a Fermi liquid

Authors: Minho Kim, Abigail Timmel, Long Ju, Xiao-Gang Wen

arXiv:2411.02503 [pdf, other] cond-mat.supr-con cond-mat.str-el

Topological incommensurate Fulde-Ferrell-Larkin-Ovchinnikov superconductor and Bogoliubov Fermi surface in rhombohedral tetra-layer graphene

Authors: Hui Yang, Ya-Hui Zhang

arXiv:2412.07145 [pdf, ps, other] cond-mat.supr-con cond-mat.str-el

Chiral finite-momentum superconductivity in the tetralayer graphene

Authors: Qiong Qin, Congjun Wu

arXiv:2502.17555 [pdf, other] cond-mat.mes-hall cond-mat.str-el cond-mat.supr-con

Quarter Metal Superconductivity

Authors: Chiho Yoon, Tianyi Xu, Yafis Barlas, Fan Zhang

arXiv:2502.19474 [pdf, other] cond-mat.str-el

Band Renormalization, Quarter Metals, and Chiral Superconductivity in Rhombohedral Te Graphene

Authors: Guillermo Parra-Martinez, Alejandro Jimeno-Pozo, Vo Tien Phong, Hector Sainz-Cruz, Daniel Kaplan, Peleg Yuval Oreg, Pierre A. Pantaleon, Jose Angel Silva-Guillen, Francisco Guinea

arXiv:2503.15471 [pdf, other] cond-mat.str-el cond-mat.supr-con

Finite-momentum pairing and superlattice superconductivity in valley-imbalanced rhombohedral graphene

Authors: Maine Christos, Pietro M. Bonetti, Mathias S. Scheurer

Spontaneous vortex lattice due to orbital magnetization in valley polarized superconductors

Ammar Jahin, Shi-Zeng Lin

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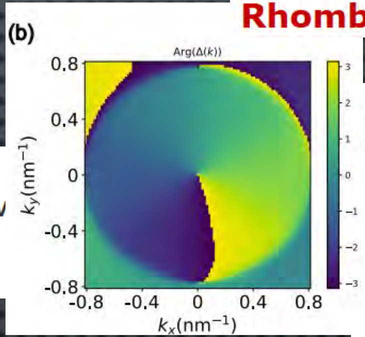
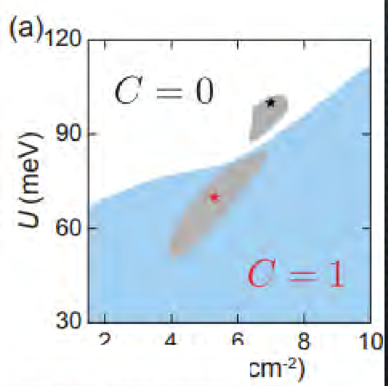
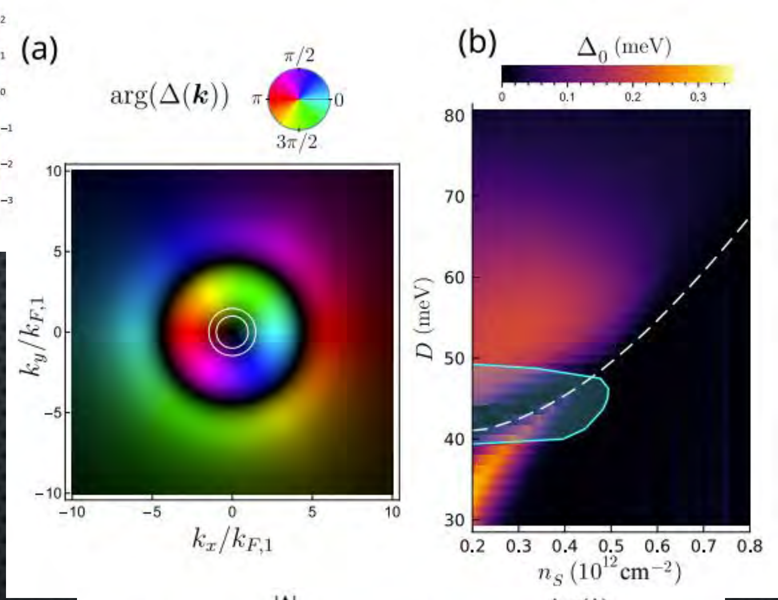
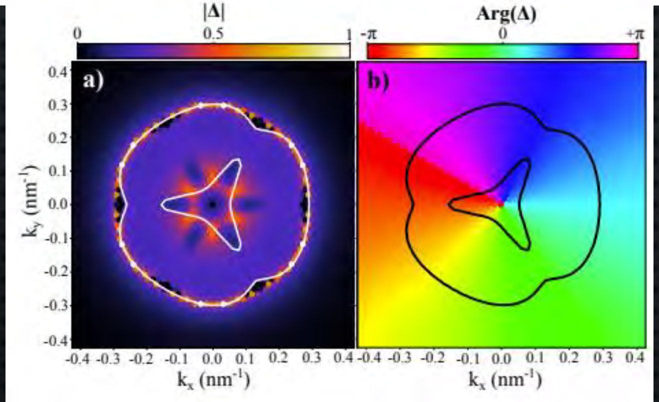
Chiral Superconductivity in the Flat bands of Rhombohedral Graphene





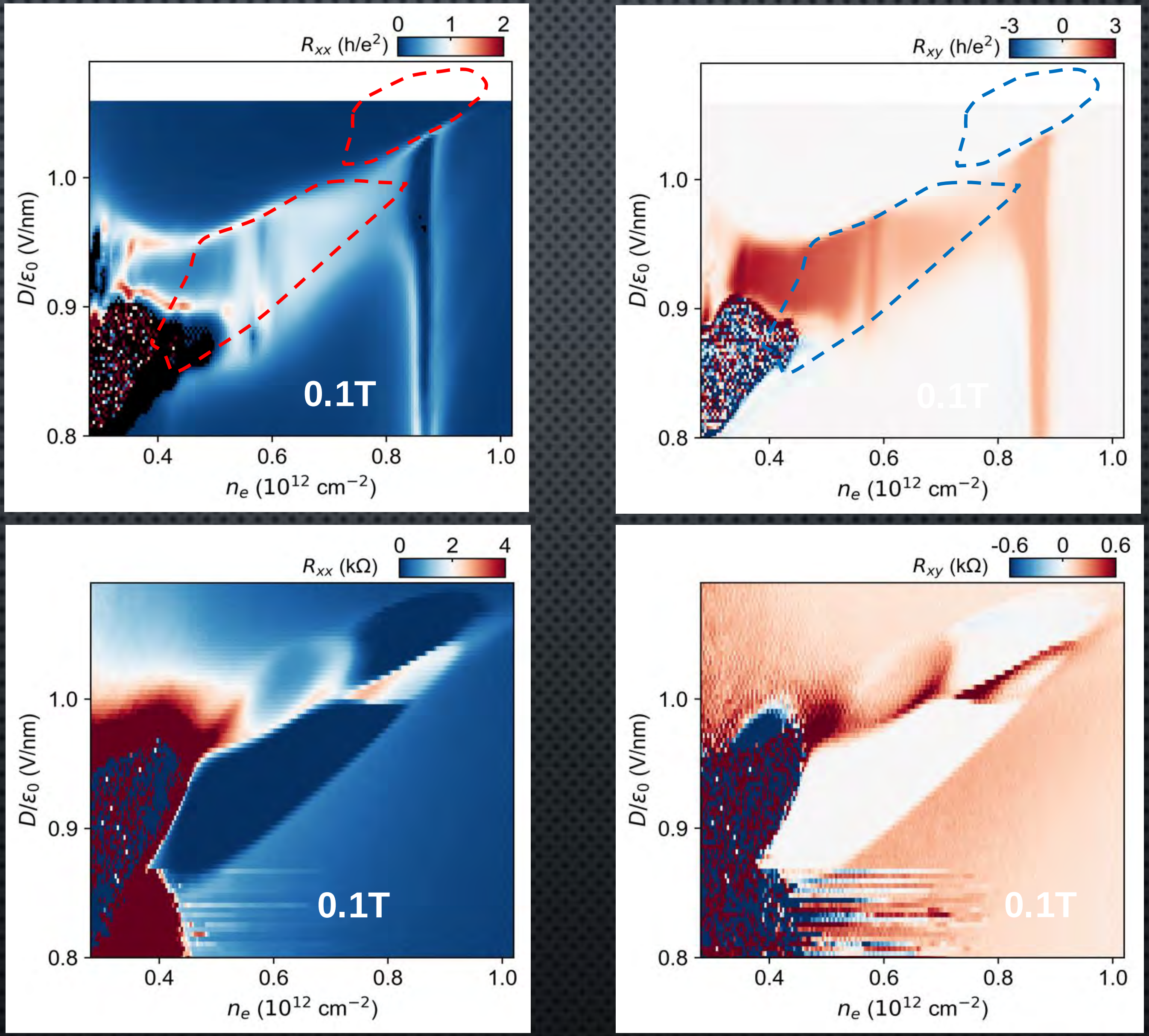




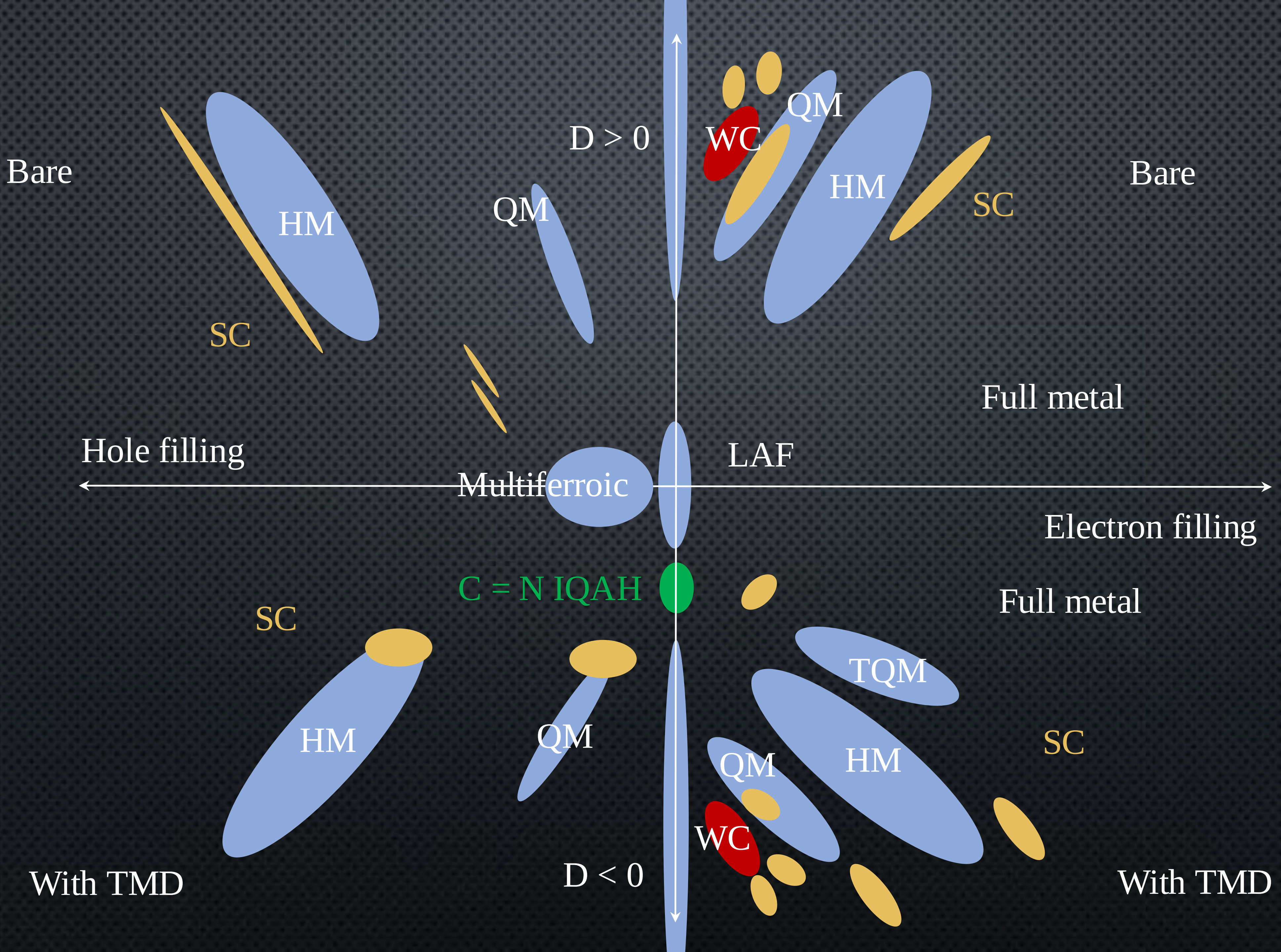
Removing moire

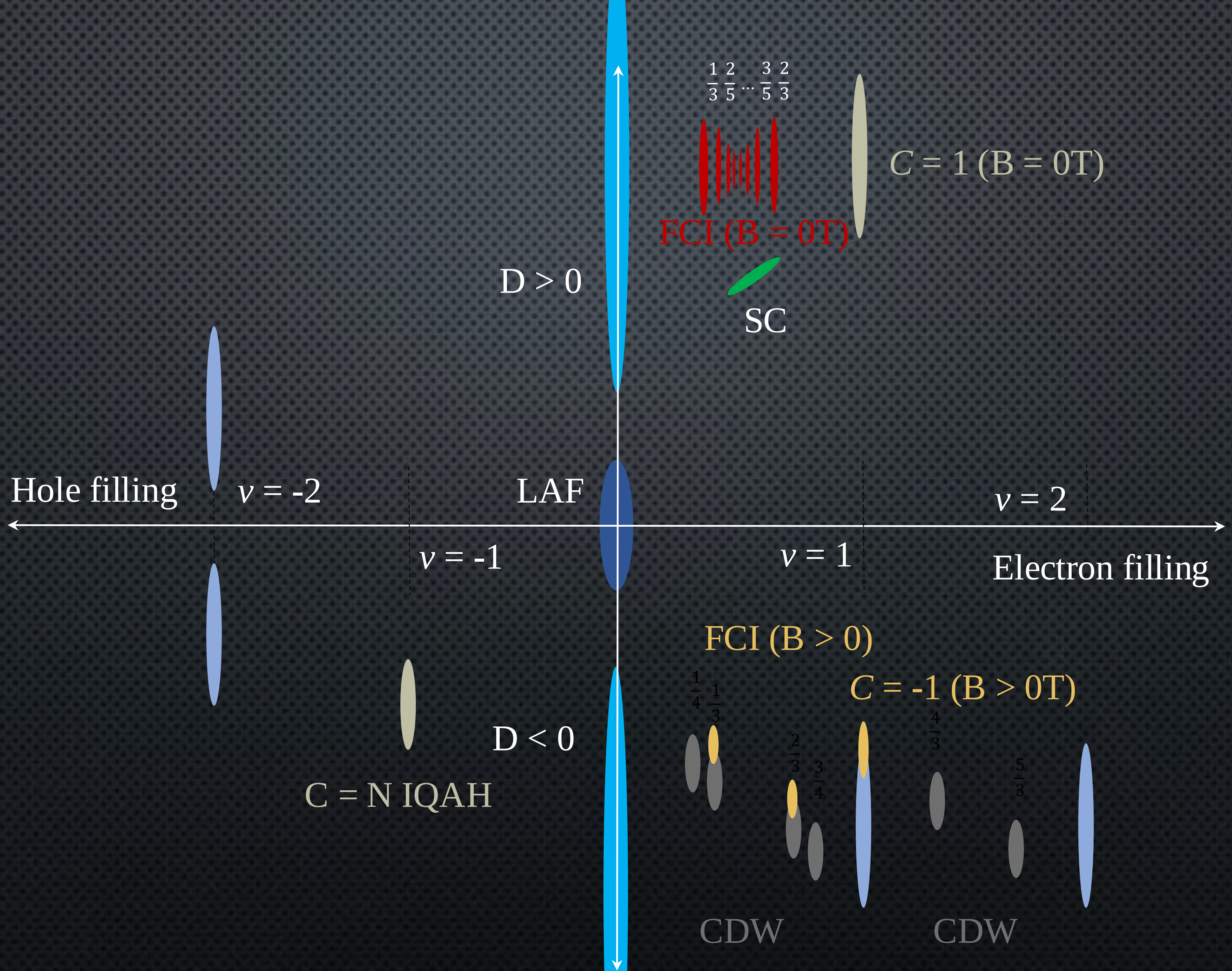
5L RGr with weak moiré

5L RGr with no moiré



Superconductor!


Bare
HM
SC
QM
D > 0
WC
QM
HM
SC
Bare
Full metal
Hole filling
LAF
Multiferroic
Electron filling
C = N IQAH
Full metal
SC
TQM
HM
QM
SC
QM
HM
WC
D < 0
With TMD
With TMD



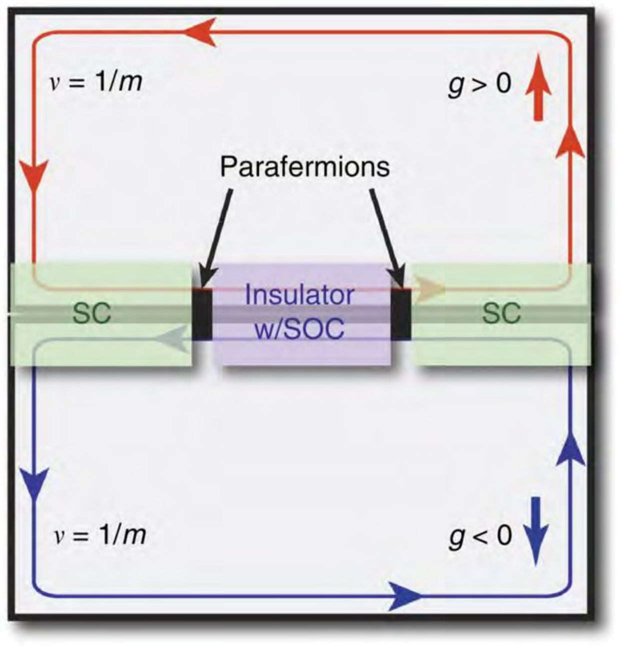
$\frac{1}{3}\ \frac{2}{5} \cdots \frac{3}{5}\ \frac{2}{3}$
C = 1 (B = 0T)
FCI (B = 0T)
D > 0
SC
Hole filling
v = -2
LAF
v = 2
v = -1
v = 1
Electron filling
FCI (B > 0)
C = -1 (B > 0T)
$\frac{1}{4}\ \frac{1}{3}$
$\frac{2}{3}\ \frac{3}{4}$
$\frac{4}{3}$
$\frac{5}{3}$
D < 0
C = N IQAH
CDW
CDW

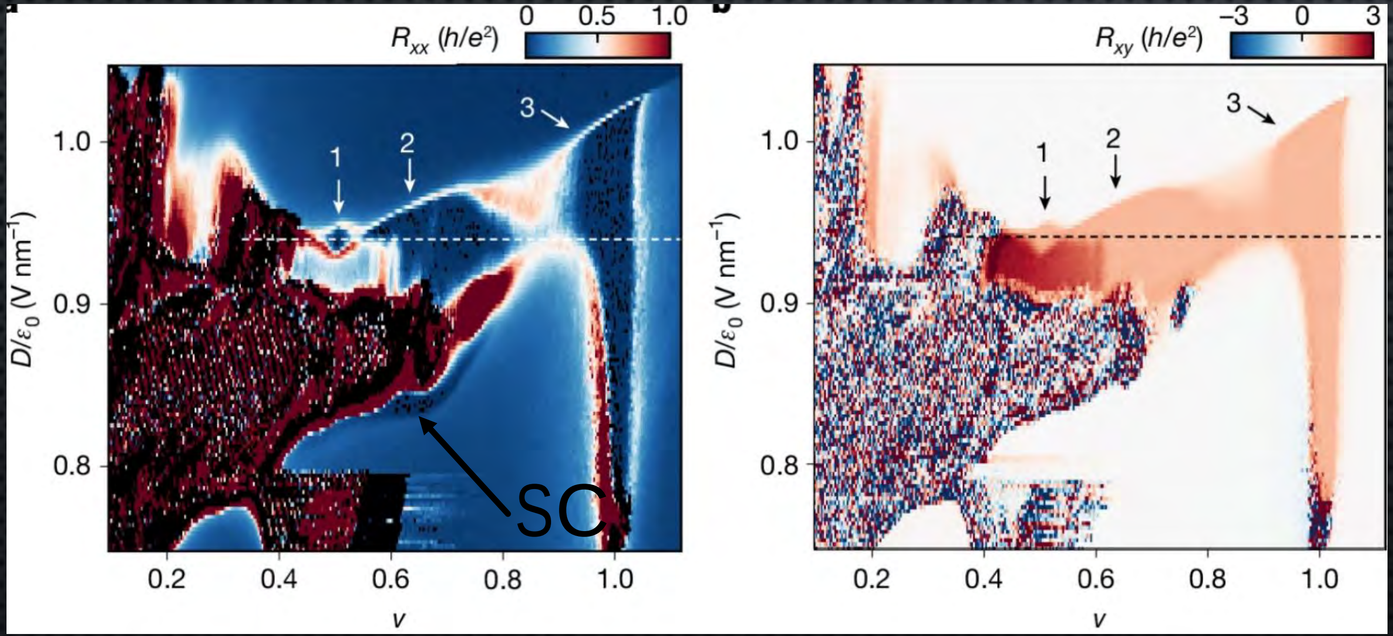
Topological Quantum Computation—Parafermions at Gate-Defined Junctions

Parafermion at the SC/FQAHE interface

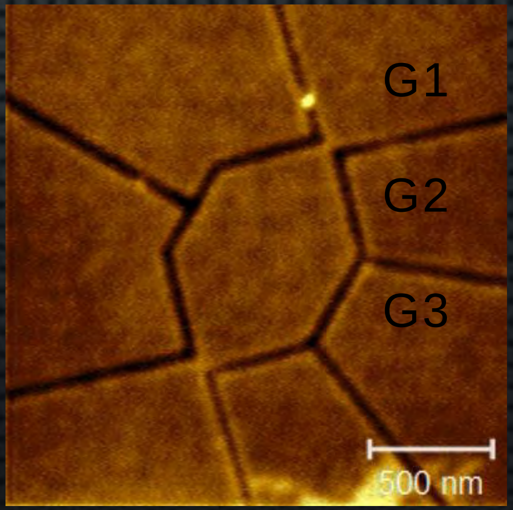
Lindner, N. H., Berg, E., Refael, G. & Stern, A.
Phys. Rev. X **2**, 041002 (2012).

Clarke, D. J., Alicea, J. & Shtengel, K.
Nature Communications (2013)





Z. Lu… LJ. Nature, 637, 1090 (2025)



Split gates cut by AFM in graphite

THANKS!