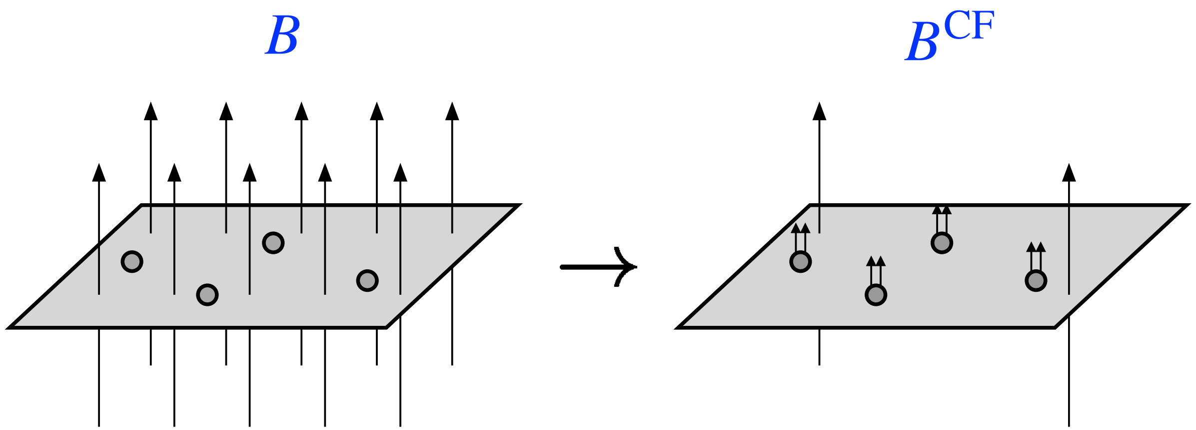
Quantitative Successes of the Composite Fermion Theory

The composite fermion: pictorial view



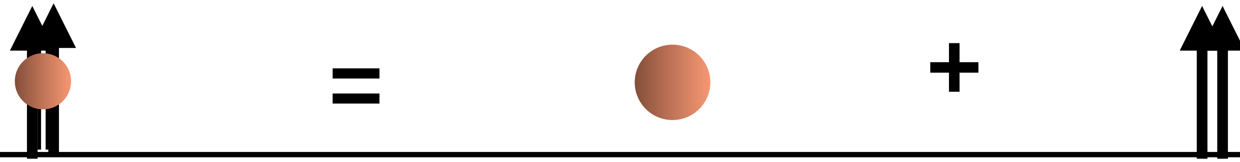
$$B^{\mathrm{CF}} = B - 2m\rho\phi_0 \qquad \nu^{\mathrm{CF}} = \frac{\nu}{1-2m\nu} \qquad \nu = \frac{\nu^{\mathrm{CF}}}{2m\nu^{\mathrm{CF}} \pm 1}$$

$\phi_0 = hc/e$, ρ = density $\quad \nu^{\mathrm{CF}} = \rho\phi_0/|B^{\mathrm{CF}}|$ = CF filling factor

- Postulate: Strongly interacting electrons at B transform into weakly interacting composite fermions at B^{CF}. The CFs form their own Landau-like levels called "Λ levels," and have a filling factor ν^{CF}.

In particular: $\nu^{\mathrm{CF}} = p \;\Leftrightarrow\; \nu = \dfrac{p}{2mp \pm 1}$

Microscopic theory: composite-fermionization



$$\Psi^{\alpha}_{\nu=\frac{\nu^{\rm CF}}{2m\nu^{\rm CF}\pm 1}} = \mathscr{P}_{\rm LLL}\Phi^{\alpha}_{\pm\nu^{\rm CF}}(\{z_i^{\rm CF}, z_i\})\prod_{j<k}(z_j - z_k)^{2m}$$

$z_j = x_j - iy_j$

wave function of interacting electrons in the lowest Landau level

projects into the lowest Landau level

wave function of noninteracting electrons

$\Phi_{-\nu^*} = [\Phi_{\nu^*}]^*$

the Laughlin-Jastrow factor attaches 2m quantized vortices to each electron

The minimal model: 2D electrons in the lowest Landau level

$$H\Psi = E\Psi$$

$$H = \sum_j \frac{1}{2m}(\vec{p}_j + e\overrightarrow{A}(\vec{r}))^2 + \sum_{j<k} \frac{e^2}{|\vec{r}_j - \vec{r}_k|} + E_{\text{Zeeman}} + \sum_j V_{\text{disorder}}(\vec{r}_j)$$

$B \to \infty$ no disorder

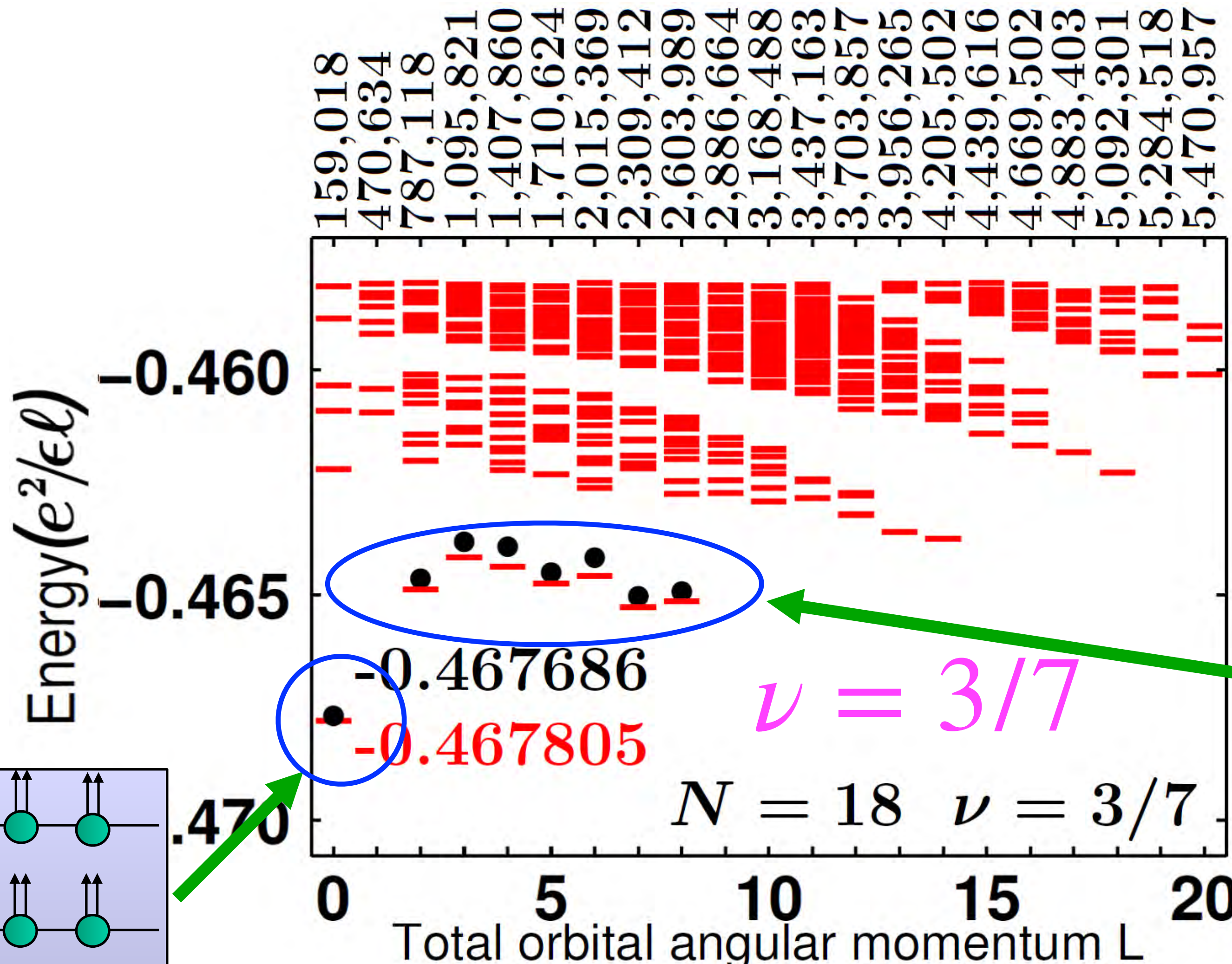
$$H_{\text{minimal}} = \sum_{j<k} \frac{1}{|\vec{r}_j - \vec{r}_k|} \quad \text{(lowest Landau level)}$$

No parameters. No mass. No kinetic energy.

Objective:

- solve this problem as a function of the filling factor
- identify the underlying physics
- predict, calculate

$\nu = 3/7$: ground state + neutral excitations



Ajit Balram
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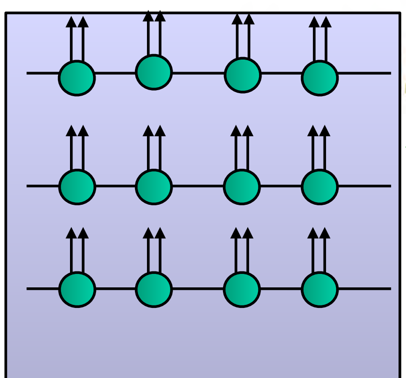
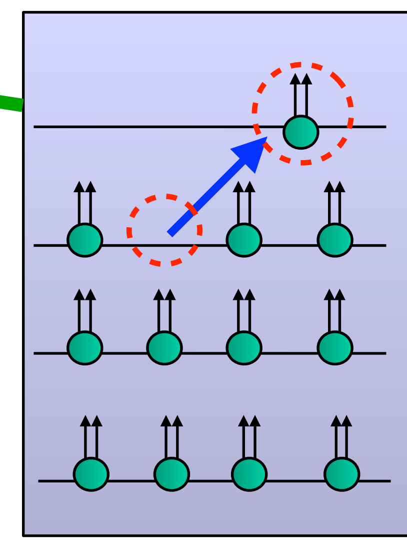




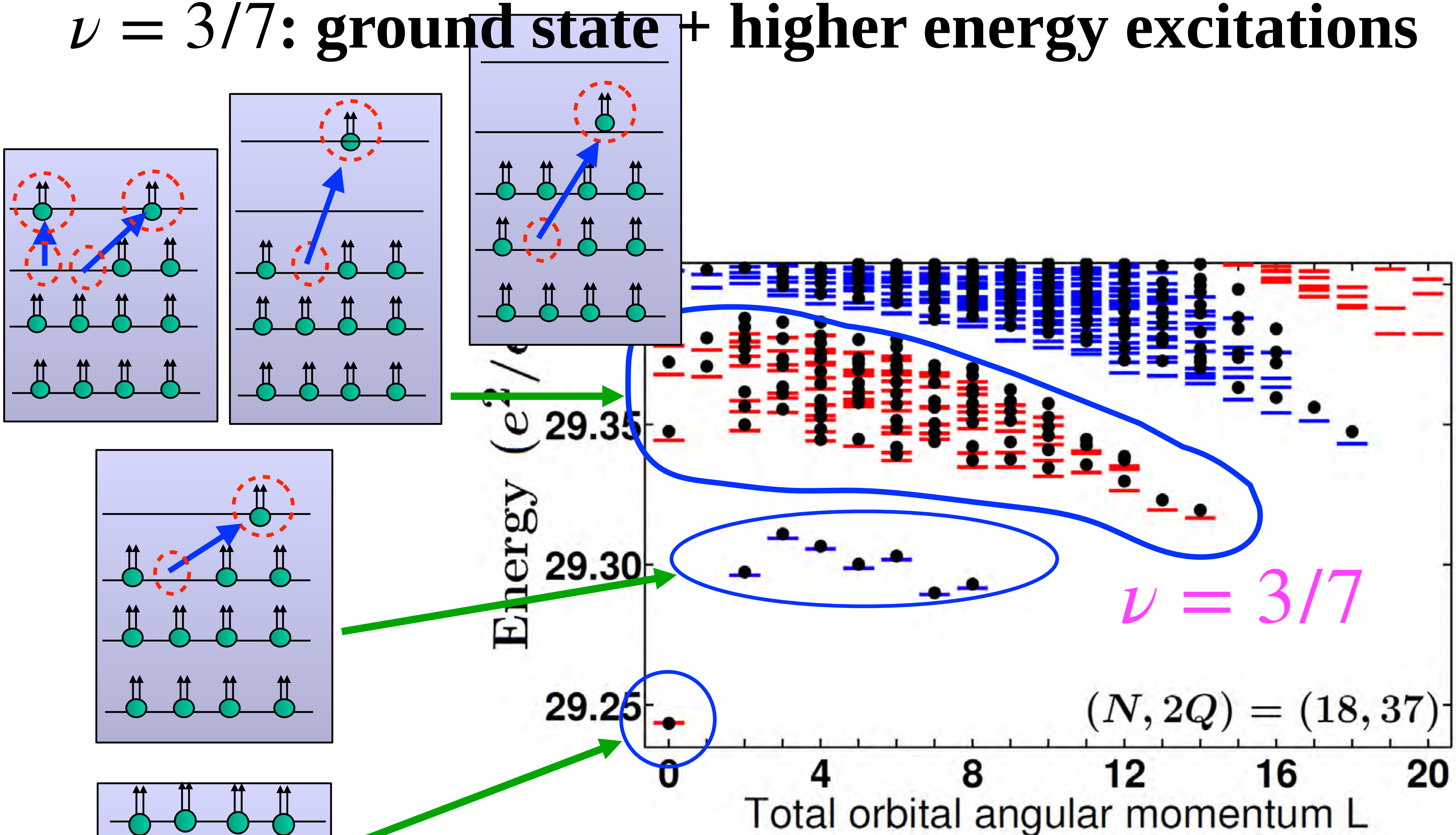
Almost exact agreement with no parameters!

$$\Psi_{3/7} = \mathcal{P}_{\rm LLL} \Phi_3 \prod_{j<k} (z_j - z_k)^2$$

$$\Psi^{\rm ex}_{3/7} = \mathcal{P}_{\rm LLL} \Phi^{\rm ex}_3 \prod_{j<k} (z_j - z_k)^2$$

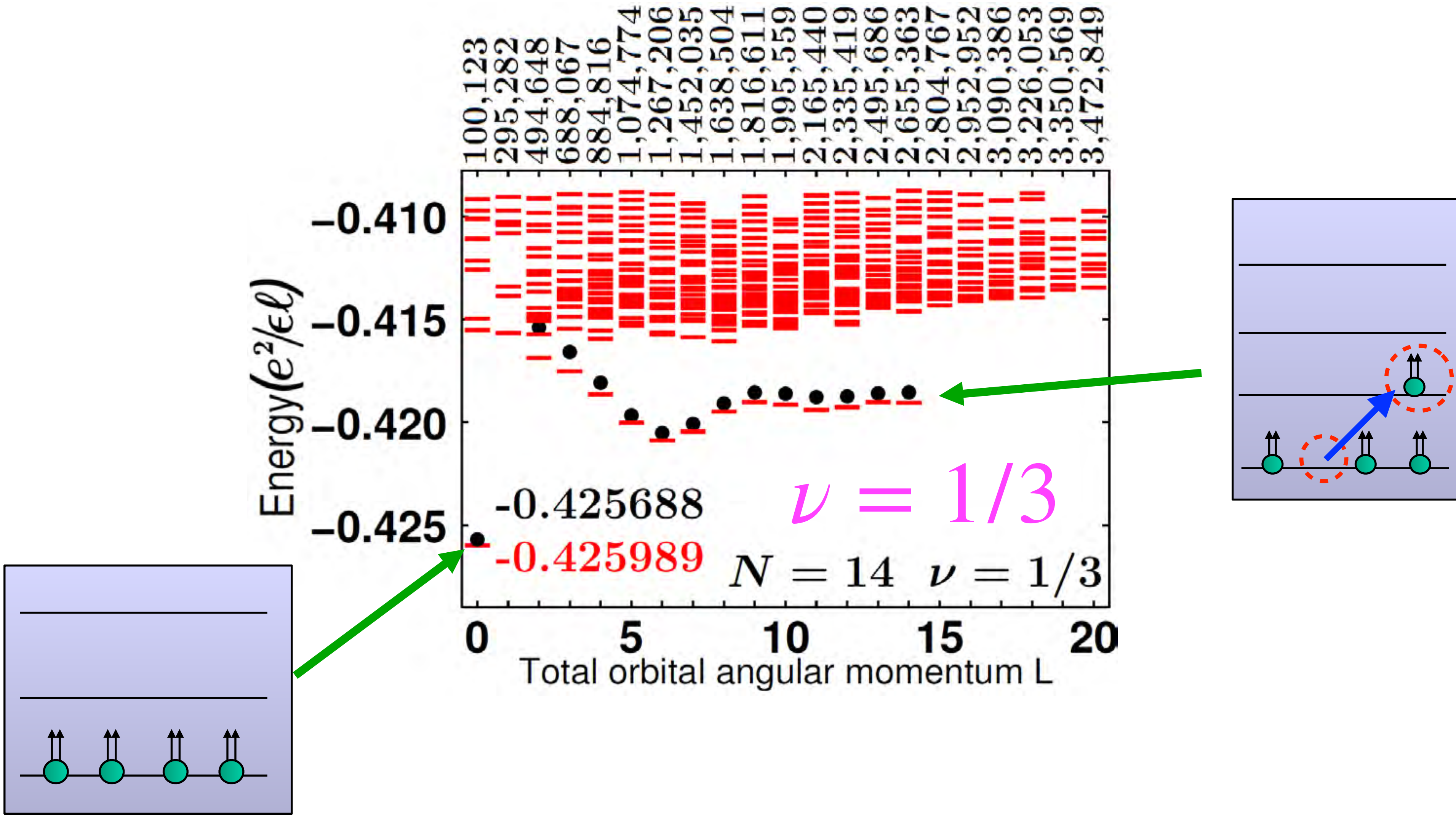
Balram, Wójs, Jain 2013

$\nu = 3/7$: ground state + higher energy excitations



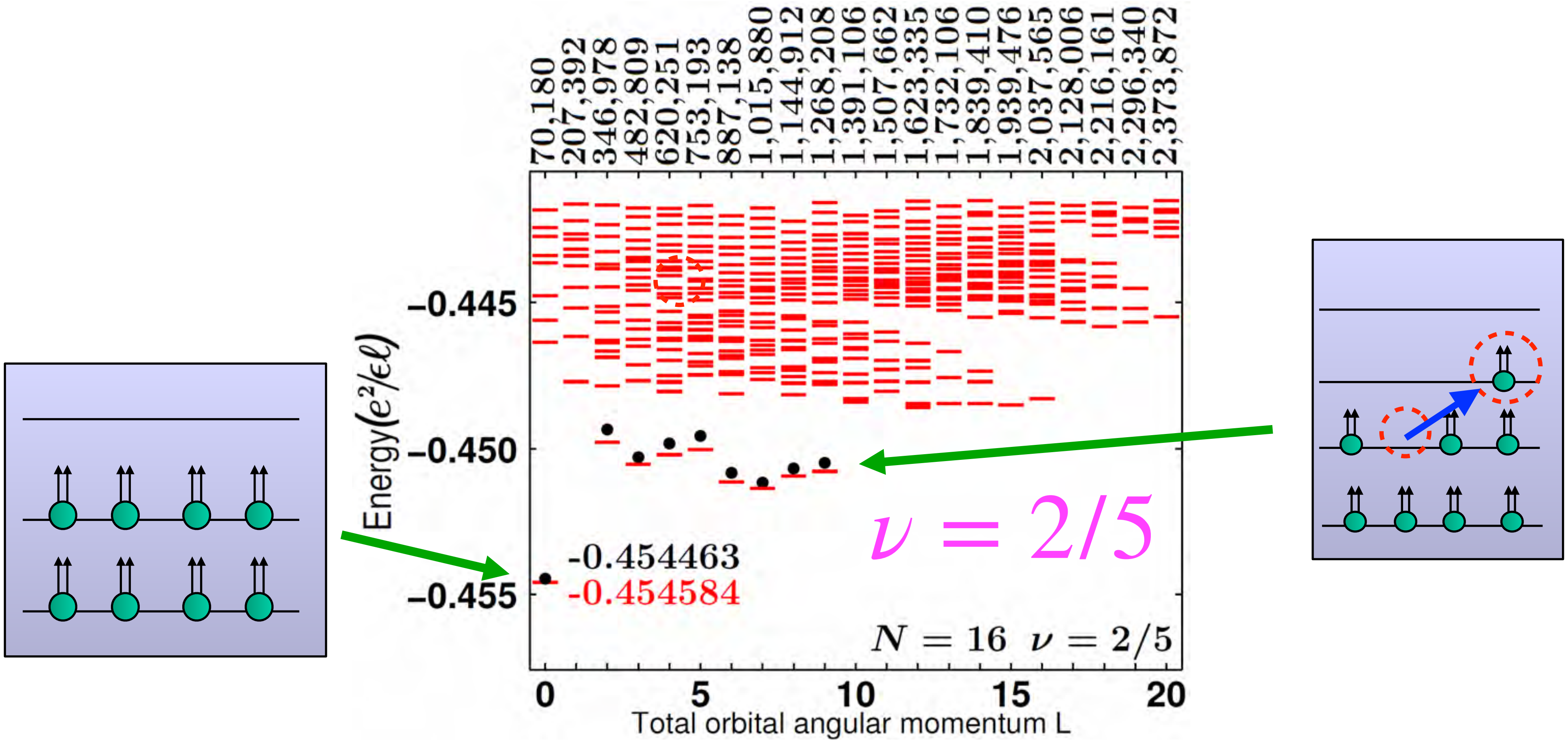
- Successively higher energy spectrum can be obtained in a systematic manner.
- The energies can be obtained to whatever accuracy we wish or need.

Balram, Wójs, Jain 2013



Similar agreement at other fractions

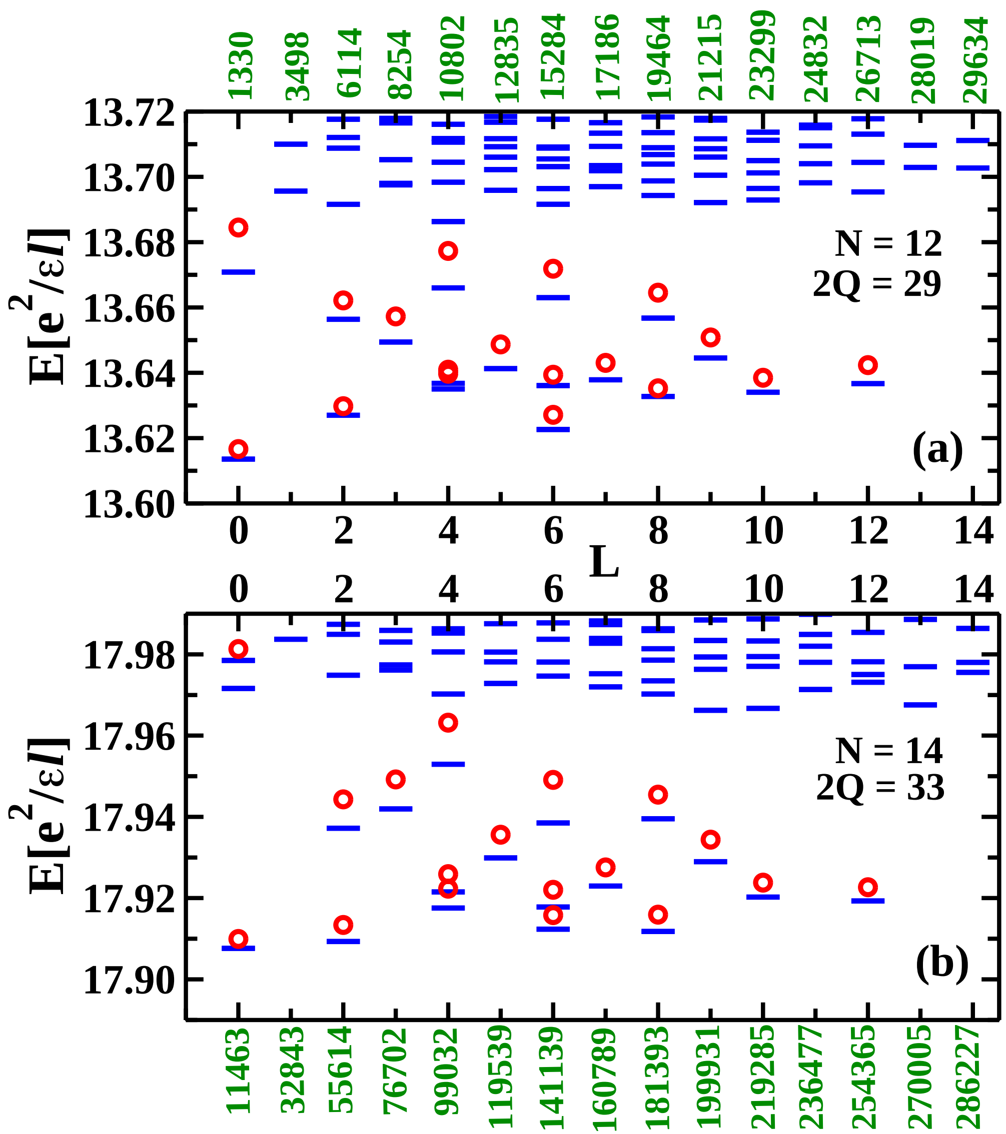
Balram, Wójs, Jain 2013



Similar agreement at other fractions

Balram, Wójs, Jain 2013

Many quasiparticles / quasiholes



$1/3 < \nu < 2/5$

4 quasiparticles of $\nu = 1/3$

6 quasiparticles of $\nu = 1/3$

No free parameters

Mukherjee, Mandal, Wójs, Jain 2012 Balram, Wójs, Jain 2013

Quantitative comparisons with experiments

- The CF theory provides an excellent account of the phenomenology without any calculations, and computer studies demonstrate the CF theory to be extremely accurate.
- Comparison with experiments has been complicated by the fact that the experimental results are modified by finite thickness, Landau level mixing and disorder. which were set to zero in computer studies. (Ironically, we understand the FQHE more accurately than these extraneous effects.) We need to take those into account.
- We have spent a significant amount of effort and resources in pushing theory to its limits to perform detailed quantitative comparisons with experiments.
- It can sometimes also reveal new qualitative physics.

Plan

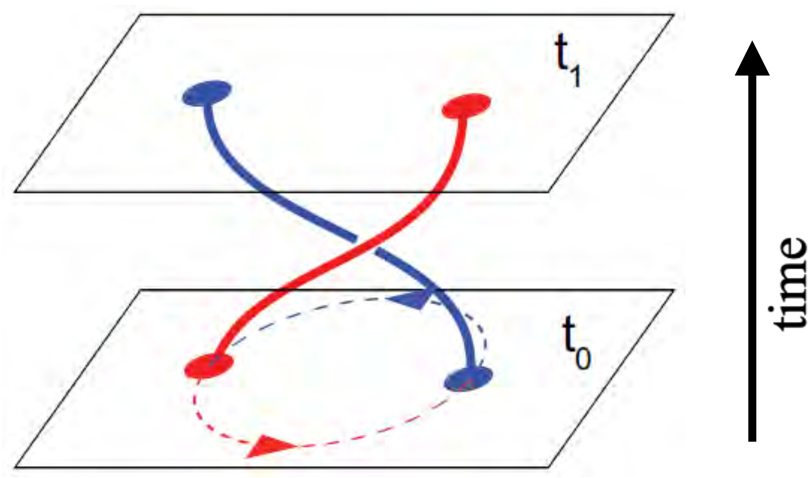
- Abelian and non-Abelian anyons
- Spin / valley polarization transitions
- CF crystal: re-entrant transitions; crystal phase diagram with LL mixing; competing phases at low fillings
- CF pairing: second mechanism of FQHE
- Scaling in FQHE

Anyons

The CF theory gives an account of the FQHE without appealing to fractional charge and fractional statistics.

What about anyons?

Anyons



An exchange of two anyons produces a phase factor of $e^{i\pi\theta^*}$.

They are generalizations of bosons ($\theta^* = 0$) and fermions ($\theta^* = 1$).

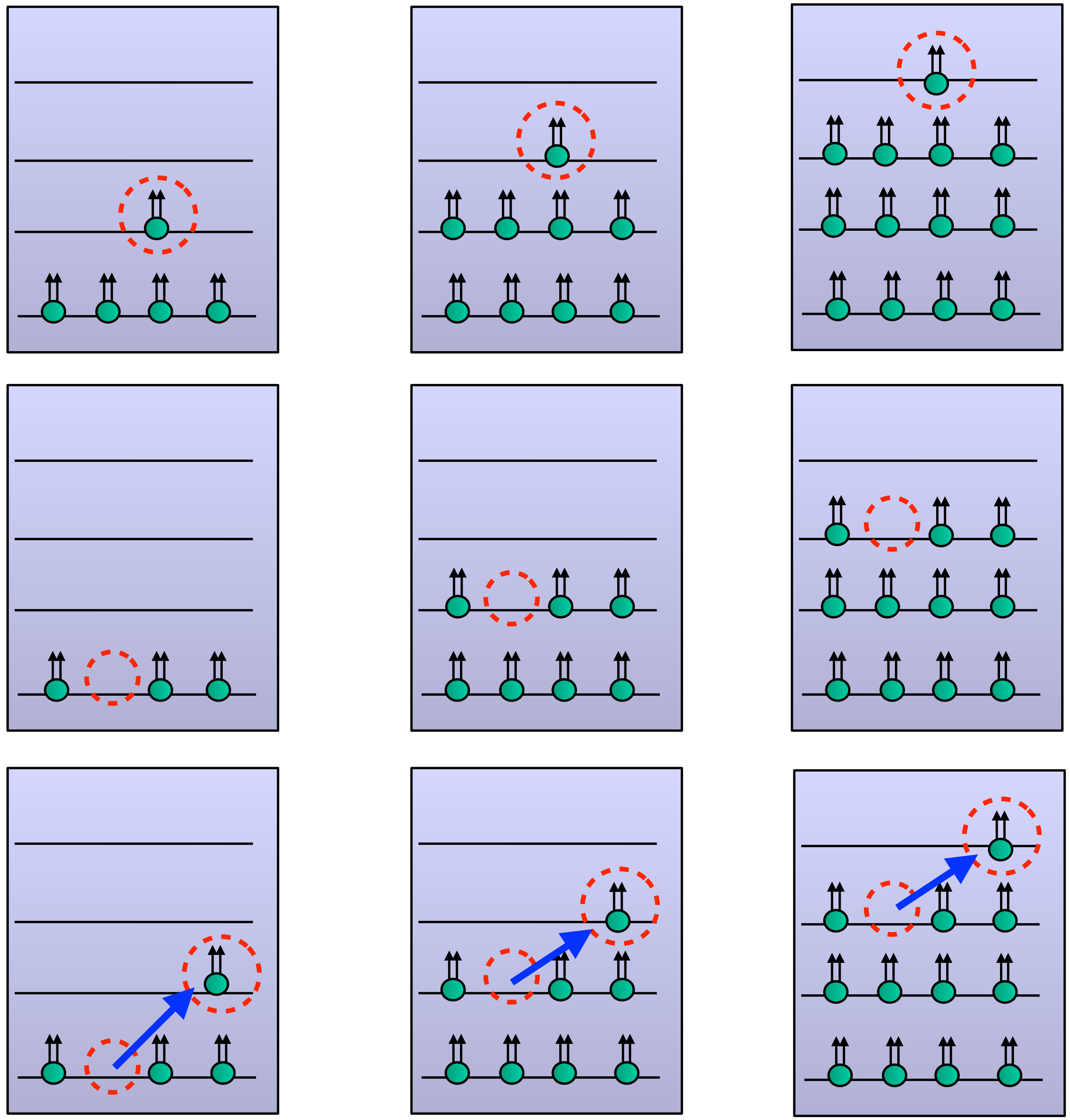
The quasiparticles of the FQHE are fractionally charged anyons (Laughlin 83, Halperin 84).
This follows from general topological arguments and has experimental support.

Quasiparticle = an isolated CF in a Λ level

quasiparticle
= isolated CF

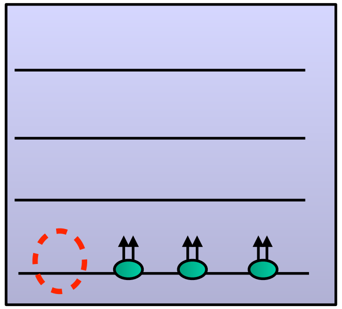
quasihole
= missing CF

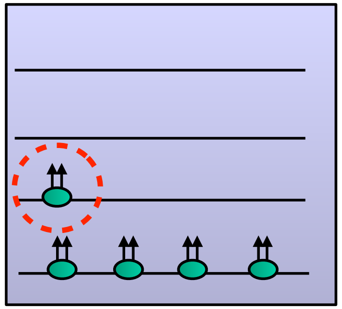
neutral excitation
= CF exciton

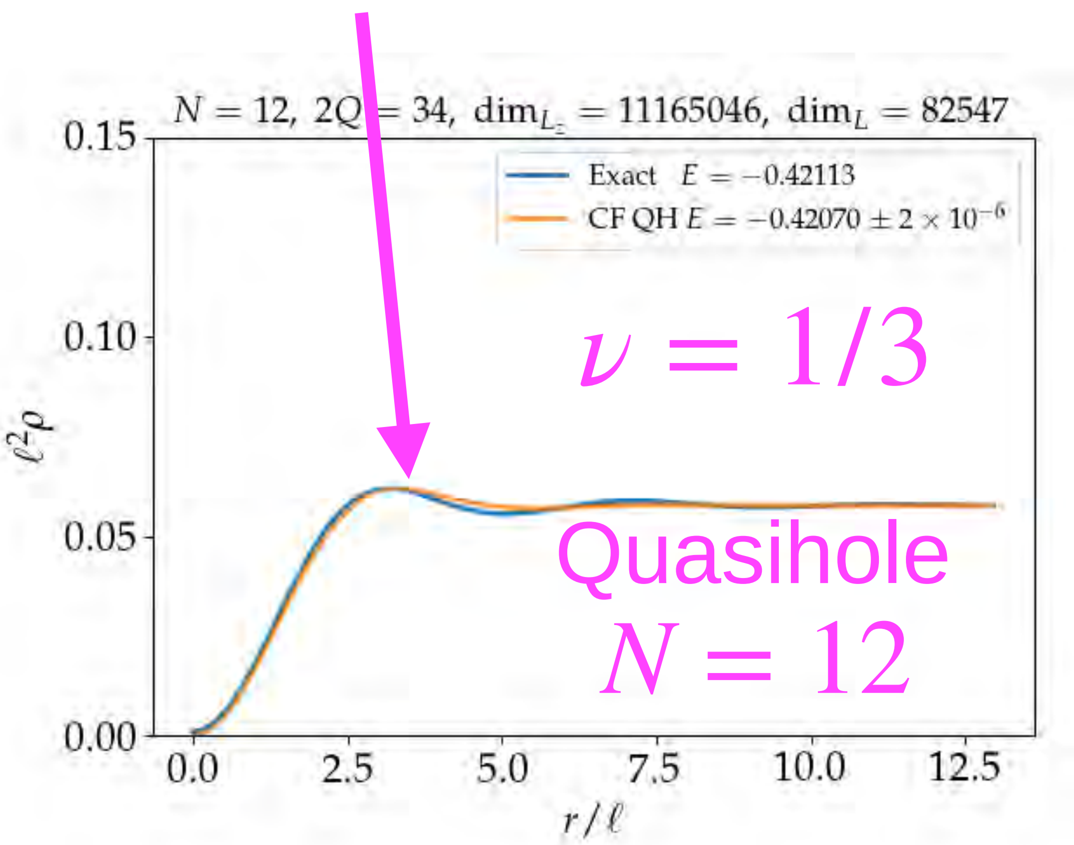


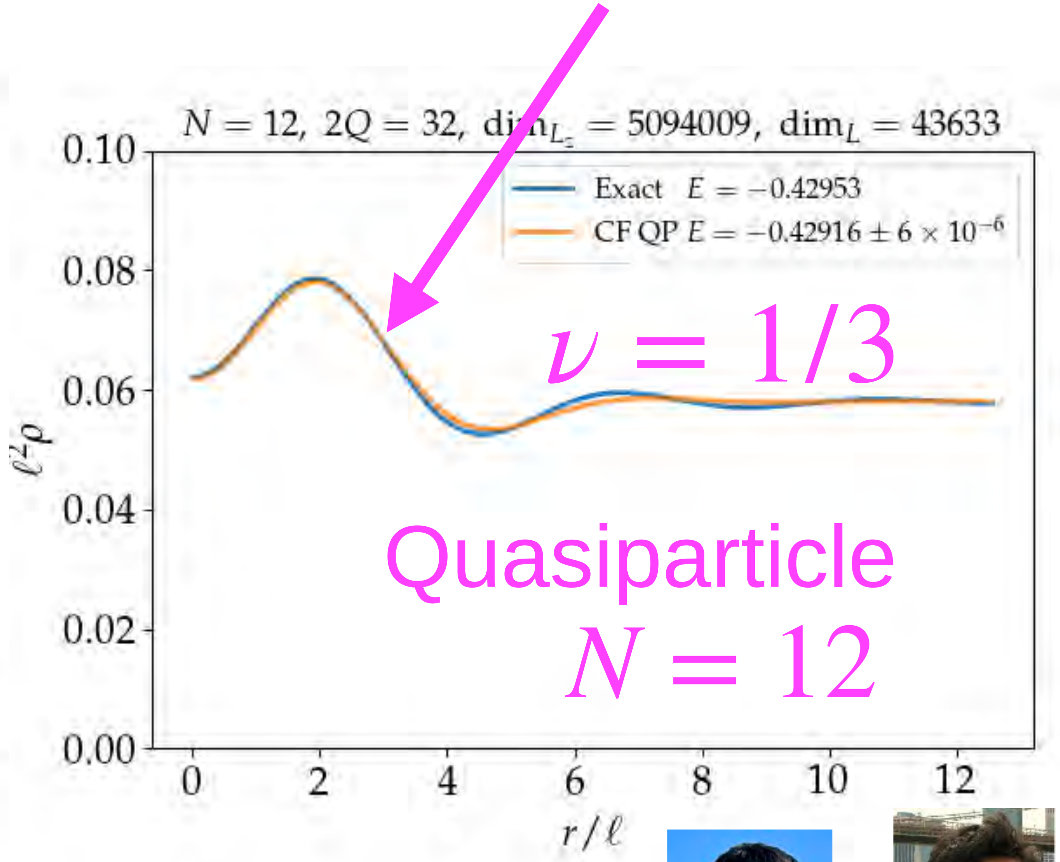
Unified description of all excitations

Quasihole/quasiparticle of 1/3









- There are ~ 6 electrons in a disk of radius 6.

Mytraya Gattu
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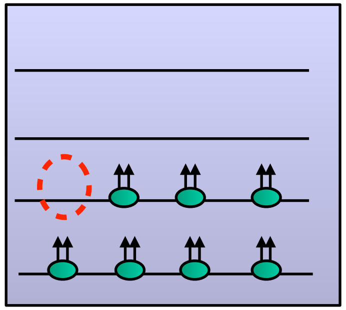
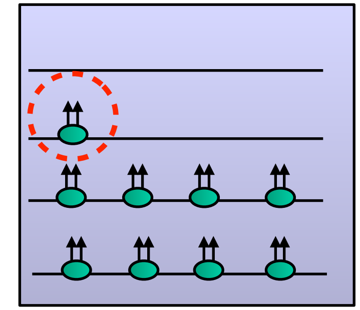
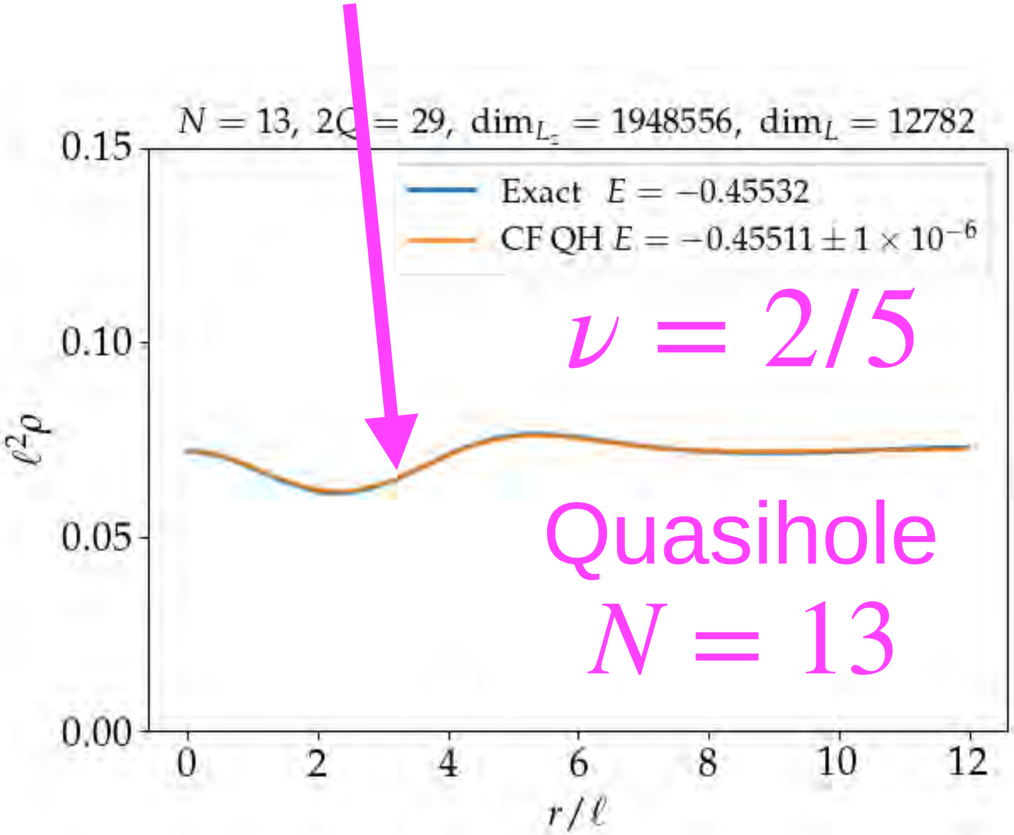
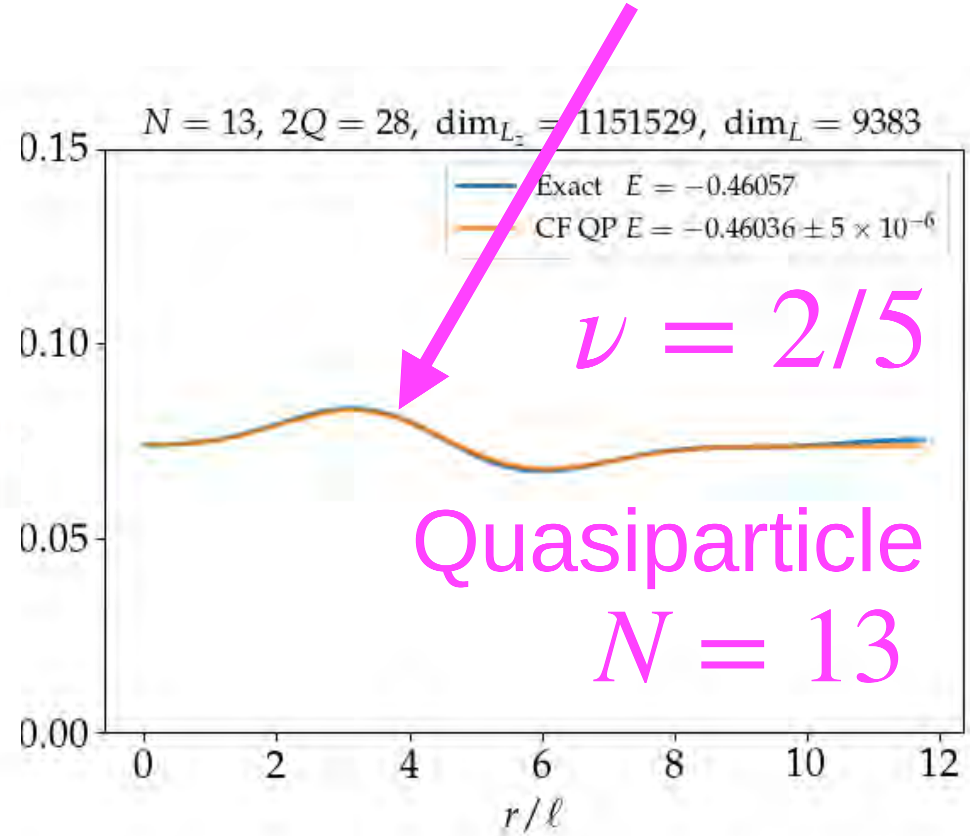
Gattu, Sreejith, JKJ, 2023

Quasihole/quasiparticle of $2/5$





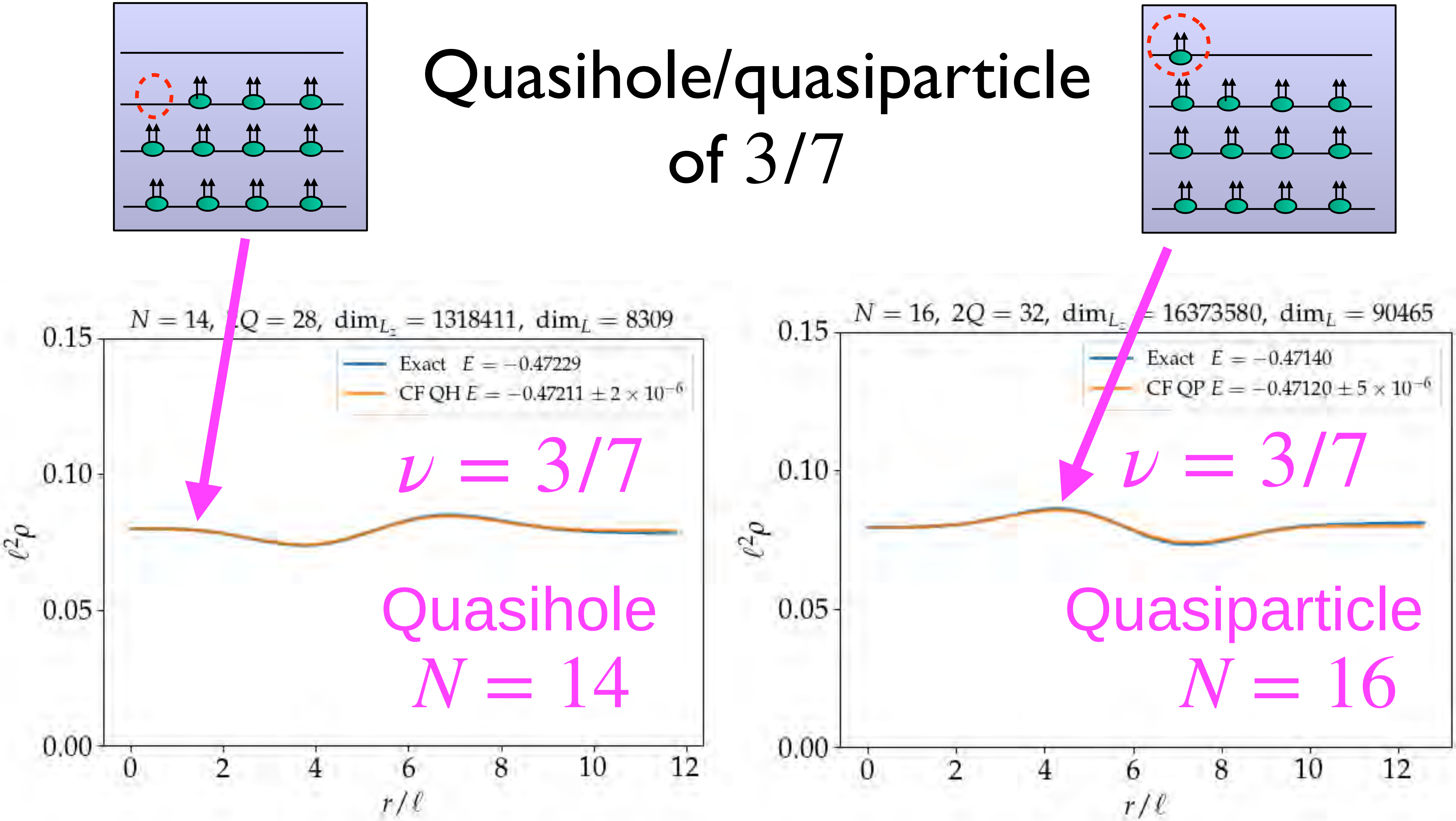




- The radius is $\sim 7 - 8$ magnetic lengths. A single quasiparticle of $2/5$ spreads over approximately $7 - 9$ electrons.

Gattu, Sreejith, JKJ, 2023

Quasihole/quasiparticle of $3/7$



- Even a single quasiparticle / quasihole is a very complex collective state. For $3/7$, it has a radius $\sim 8\ell$ and spreads over a region containing $13-14$ electrons.

Gattu, Sreejith, JKJ, 2023

A paradox?

Quasiparticle = an excited CF

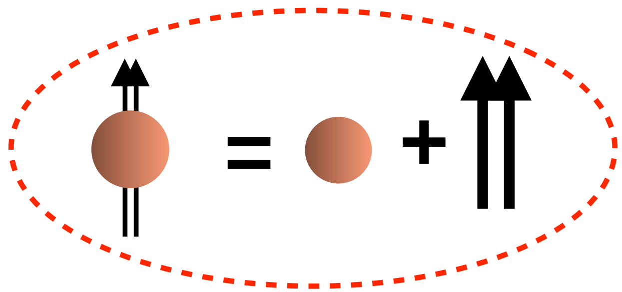
Is it a charge-one fermion or a fractionally charged anyon?

No paradox really. It's a question of what's the reference state — the state with no particles, or the background FQH state — and what's the measurement.

The fractional charge and braid statistics can be derived straightforwardly with the CF theory.

Fractional charge

- When we add an electron to a uniform density FQH system, we add a unit charge overall.
- However, as it gets dressed by vortices to become a CF, the unit charge is screened into a fractional charge, with the remainder leaking out to the edge.



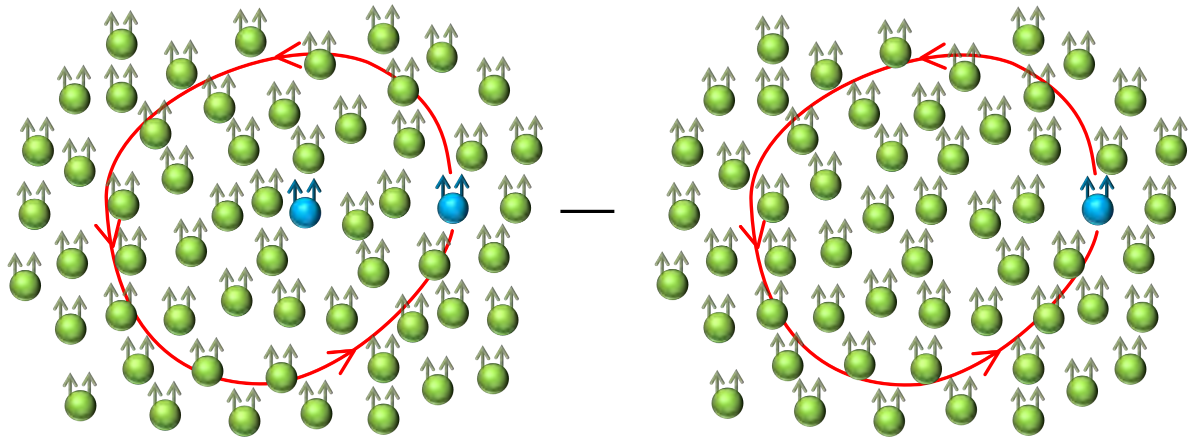
$$q^* = -1 + 2m\nu = -1 + 2m\frac{p}{2mp \pm 1} = \mp\frac{1}{2mp \pm 1}$$

Charge of an electron

Charge of $2m$ vortices

q^* can also be obtained by integrating the density.

Fractional statistics



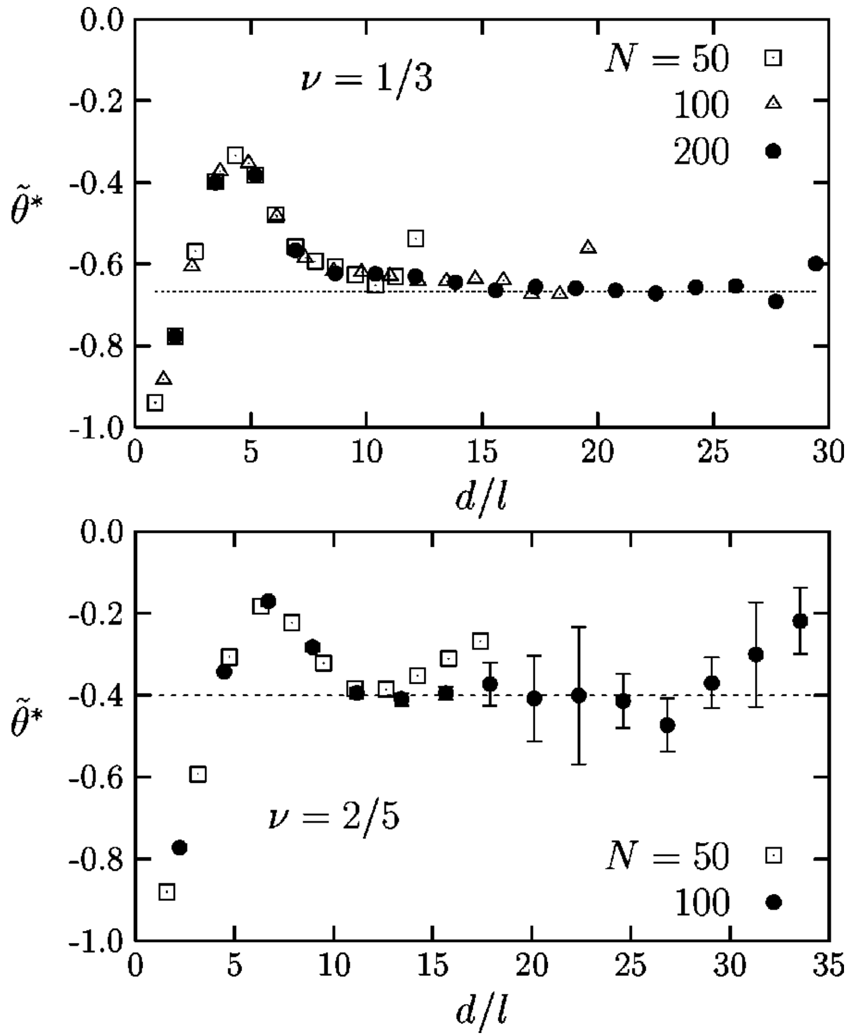
- Berry phase for a closed loop of a CF: $\Phi^* = -2\pi\left(BA/\phi_0 - 2mN_e\right)$, where N_e is the number of enclosed particles. For uniform density $\Phi^* = -2\pi\left(BA/\phi_0 - 2m\rho A\right) \equiv -2\pi B^{\mathrm{CF}}A/\phi_0$ gives $B^{\mathrm{CF}} = B - 2m\rho\phi_0$.

- The change when another quasiparticle is inserted inside the loop:
$$\Delta\Phi^* = 2\pi \times 2m \times \Delta N_e = 2\pi \times 2m \times q^* = 2\pi\frac{2m}{2mp \pm 1}.$$
This is precisely what interference experiments measure.

- It may be also interpreted in terms of fractional statistics $\Delta\Phi^* \equiv 2\pi\theta^*$.

Fractional Statistics in the Fractional Quantum Hall Effect

Gun Sang Jeon, Kenneth L. Graham, and Jainendra K. Jain
Physics Department, 104 Davey Laboratory, The Pennsylvania State University, University Park, Pennsylvania 16802, USA
(Received 14 March 2003; published 15 July 2003)



The fractional statistics is well defined only when CFs are 15ℓ or more away.

CFs are always well defined.

Finite width and LL mixing

Finite width effects

- Finite width is relatively straightforward to incorporate. It effectively modifies the interaction, weakening the short range part of it.
- We obtain the transverse wave function within a local density approximation to determine the effective interaction.

$$V^{\text{eff}}(r) = \frac{e^2}{\epsilon} \int \mathrm{d}z_1 \int \mathrm{d}z_2 \frac{|\xi(z_1)|^2 |\xi(z_2)|^2}{[r^2 + (z_1 - z_2)^2]^{1/2}}$$

Landau level mixing

$$\kappa = \frac{e^2/\epsilon\ell}{\hbar\omega_c}$$

- Typically $\kappa \sim 0.5 - 2.5$ for n-doped GaAs systems, and $\kappa \sim 2 - 20$ for p-doped GaAs, ZnO, or AlAs quantum wells.
- We will employ the non-perturbative method of "fixed phase diffusion Monte Carlo" to treat LL mixing. (Ortiz, Ceperley, Martin, 1993; Melik-Alaverdian, Bonesteel, Ortiz, 1997; Guclu, Umrigar, 2005)

Diffusion Monte Carlo (DMC)

- Consider a wave function that is everywhere real and non-negative. For imaginary time, the Schrodinger equation turns into a diffusion equation, where the wave function corresponds to the density of the diffusing particles. DMC is a stochastic projector method for solving this equation.

$$-\partial_t \Psi(\mathbf{R}, t) = (\hat{H} - E_T)\Psi(\mathbf{R}, t) \qquad \mathbf{R} = (\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$$

- Expand in a complete set of eigenfunctions ϕ_i

$$\begin{aligned} \Psi(\mathbf{R}, t) &= \sum_i N_i \exp[-(E_i - E_T)t]\phi_i(\mathbf{R}) \\ &= N_0 \exp[-(E_0 - E_T)t]\phi_0(\mathbf{R}) \text{ when } t \to \infty \end{aligned}$$

- The evolution operator thus projects the initial trial wave function into the ground state provided the two have non-zero overlap.

Reynolds, Ceperley, Alder, and Lester Jr., J. Chem. Phys. 77, 5593 (1982).

Fixed phase DMC

Ortiz, Ceperley, and Martin, Phys. Rev. Lett. 71, 2777 (1993).

$$H\Psi = E\Psi \qquad H = \sum_j [\vec{p}_j + \overrightarrow{A}(\vec{r}_j)]^2 + V_{\rm int}$$

Chosen phase sector

Ψ is complex.

Substitute: $\Psi(\{\vec{r}_j\}) = \Phi_{\rm T}(\{\vec{r}_j\})\, e^{i\phi_{\rm T}(\{\vec{r}_j\})}, \;\; \Phi_{\rm T}(\{\vec{r}_j\}) = |\Psi_{\rm T}(\{\vec{r}_j\})|;$
take the phase $\phi_{\rm T}(\{\vec{r}_j\})$ to be fixed; and write

$$H'\Phi = E\Phi \qquad H' = \sum_j [\vec{p}_j + \overrightarrow{A}(\vec{r}_j) + \overrightarrow{\nabla}_j \phi(\{\vec{r}_k\})]^2 + V_{\rm int}$$

- DMC on this wave function produces the lowest energy in the chosen phase sector. The result depends on the choice of the phase.
- The energies are obtained as a function of $\kappa = (e^2/\epsilon\ell)/(\hbar\omega_c)$. The method is non-perturbative.

What's a good choice for the phase?

We will use the accurate lowest Landau level wave functions to fix the phase sector in our DMC. There is evidence that the phase of the many-body wave function is not affected much by Landau level mixing (Guclu and Umrigar, 2005).

Spin / Valley polarization phase transitions

CFs with spin / valley degree of freedom

- For small Zeeman energies, spin can play a role. The CF theory makes definite predictions for the possible spin polarizations at any given filling.

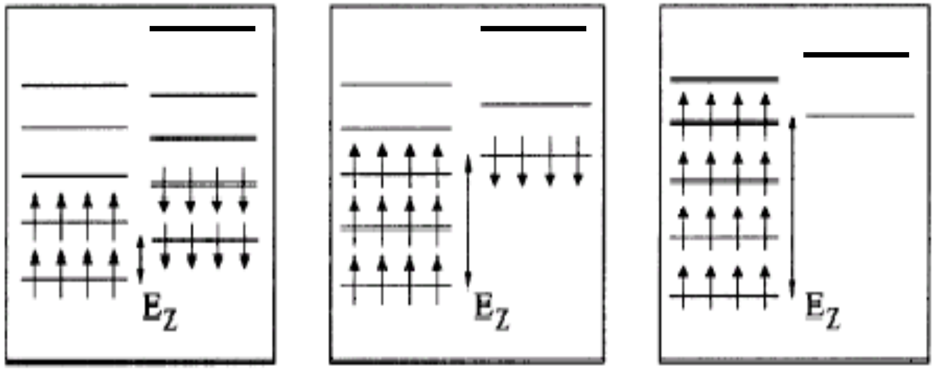
Example

$$\nu^{CF} = 4$$

$$[\nu = 4/9,\ 4/7;$$

$$\nu = 2 - 4/9 = 14/9;$$

$$\nu = 2 - 4/7 = 10/9]$$



Unpolarized **(2,2)**

Partially polarized **(3,1)**

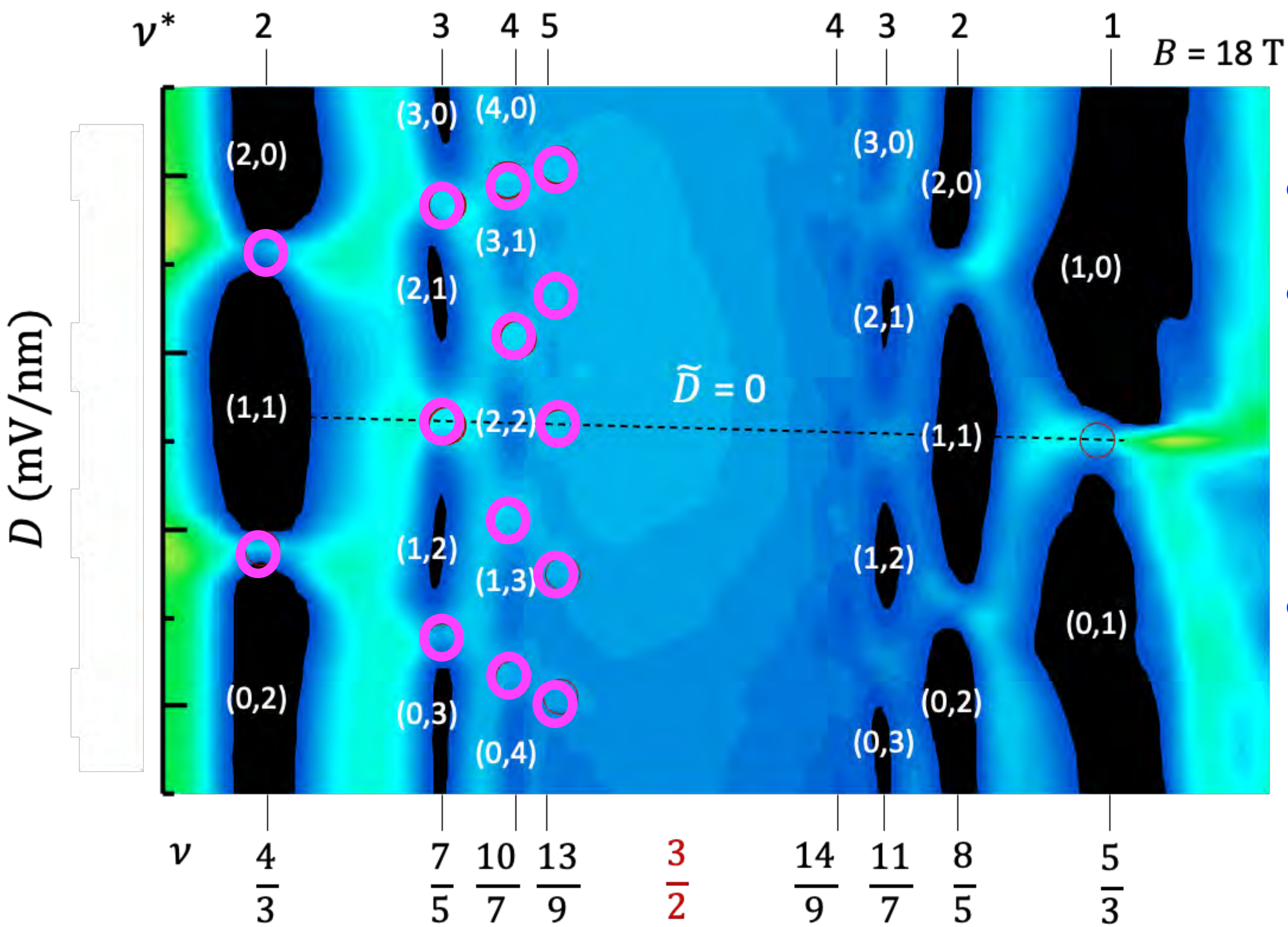
Fully-spin polarized **(4,0)**

- Transitions between them occur as a function of the Zeeman energy — the critical Zeeman energies can be predicted in a model of non-interacting CFs with a single parameter (CF mass).
- The valley degree of freedom in graphene is analogous (although the Zeeman energy now can be negative).

Valley polarization in bilayer graphene*

Valley isospin polarizations of the N=0 Jain states in BLG

- Two valley isospin components: |+ 0> and |- 0> behave like spin



- All predicted states seen.
- Excellent account of the phase diagram with a single fitting parameter: CF mass.
- Mass off by a factor of 4 from its zeroth order value.

Du et al, PRL 75, 3926 (1995), Padamanabhan et al, PRB 80, 035423 (2009), Feldman et al, PRL 11, 076802 (2013)

Huang… JZ, PRX 12, 031019 (2022)

*Slide from Jun Zhu

Phase Diagram of the Spin Polarization of Composite Fermions and a New Effective Mass

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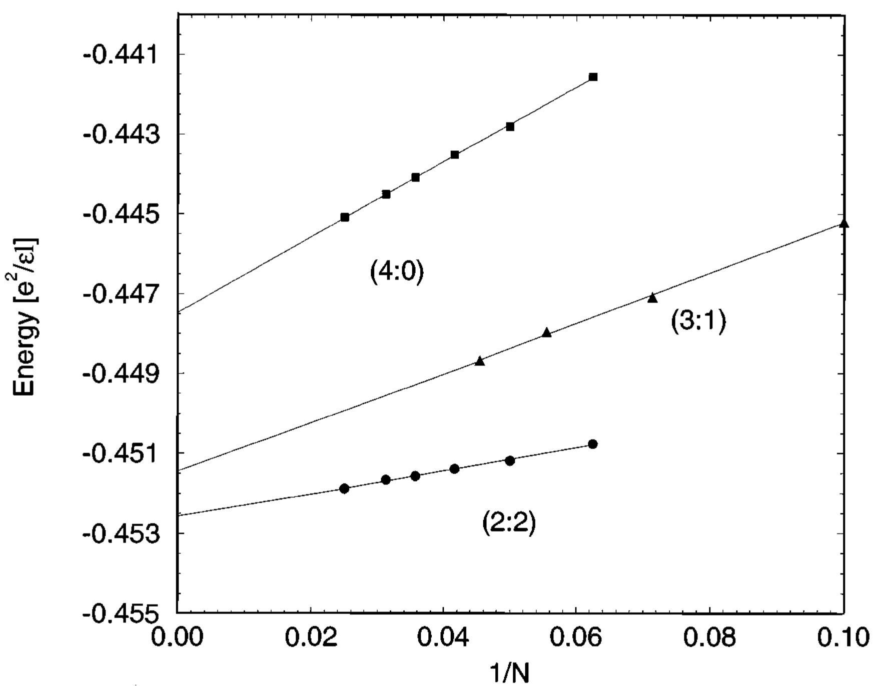
(Received 6 November 1997)

$$\Psi_{\nu=\frac{n}{2mn\pm 1}} = \mathscr{P}_{\mathrm{LLL}}\Phi_{n_\uparrow,n_\downarrow}(\{z_i^{\mathrm{CF}}, z_i\})\prod_{j<k}(z_j - z_k)^{2m}, \quad n = n_\uparrow + n_\downarrow$$



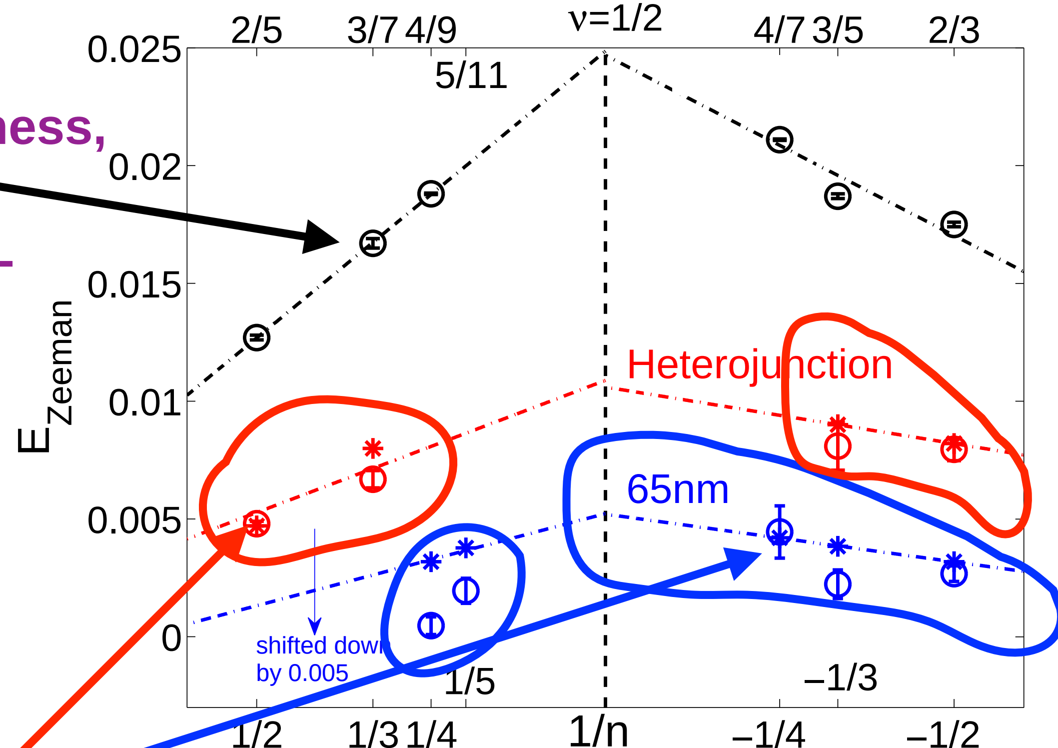
Critical Zeeman energies

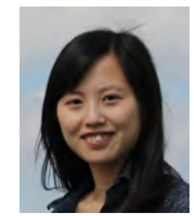
PRL **117,** 116803 (2016) PHYSICAL REVIEW LETTERS week ending 9 SEPTEMBER 2016

Landau-Level Mixing and Particle-Hole Symmetry Breaking for Spin Transitions in the Fractional Quantum Hall Effect

Yuhe Zhang,[1] A. Wójs,[2] and J. K. Jain[1,3]

theory: zero thickness, no LL mixing Park and Jain, PRL (1998)





Yuhe Zhang

L. W. Engel et al. PRB 45, 3418 (1992)
W. Kang et al. PRB 56, R12776 (1997)
Y. Liu et al. PRB 90, 085301 (2014)

Theory including finite width and Landau level mixing in a fixed phase diffusion Monte Carlo study Zhang, Wojs, Jain, PRL (2016)

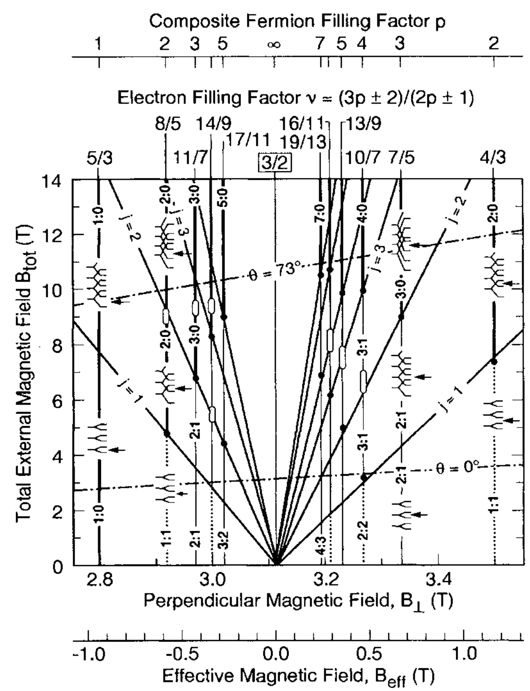
The CF theory obtains the ~ 1 % Coulomb energy difference between the competing states to within a few %.

Breaking of particle-hole symmetry

VOLUME 75, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1995

Fractional Quantum Hall Effect around $\nu = \frac{3}{2}$: Composite Fermions with a Spin

R. R. Du,[1,3] A. S. Yeh,[1] H. L. Stormer,[2] D. C. Tsui,[1] L. N. Pfeiffer,[2] and K. W. West[2]



VOLUME 62, NUMBER 13 PHYSICAL REVIEW LETTERS 27 MARCH 1989

Evidence for a Phase Transition in the Fractional Quantum Hall Effect

J. P. Eisenstein, H. L. Stormer, L. Pfeiffer, and K. W. West

$\nu = 8/5$

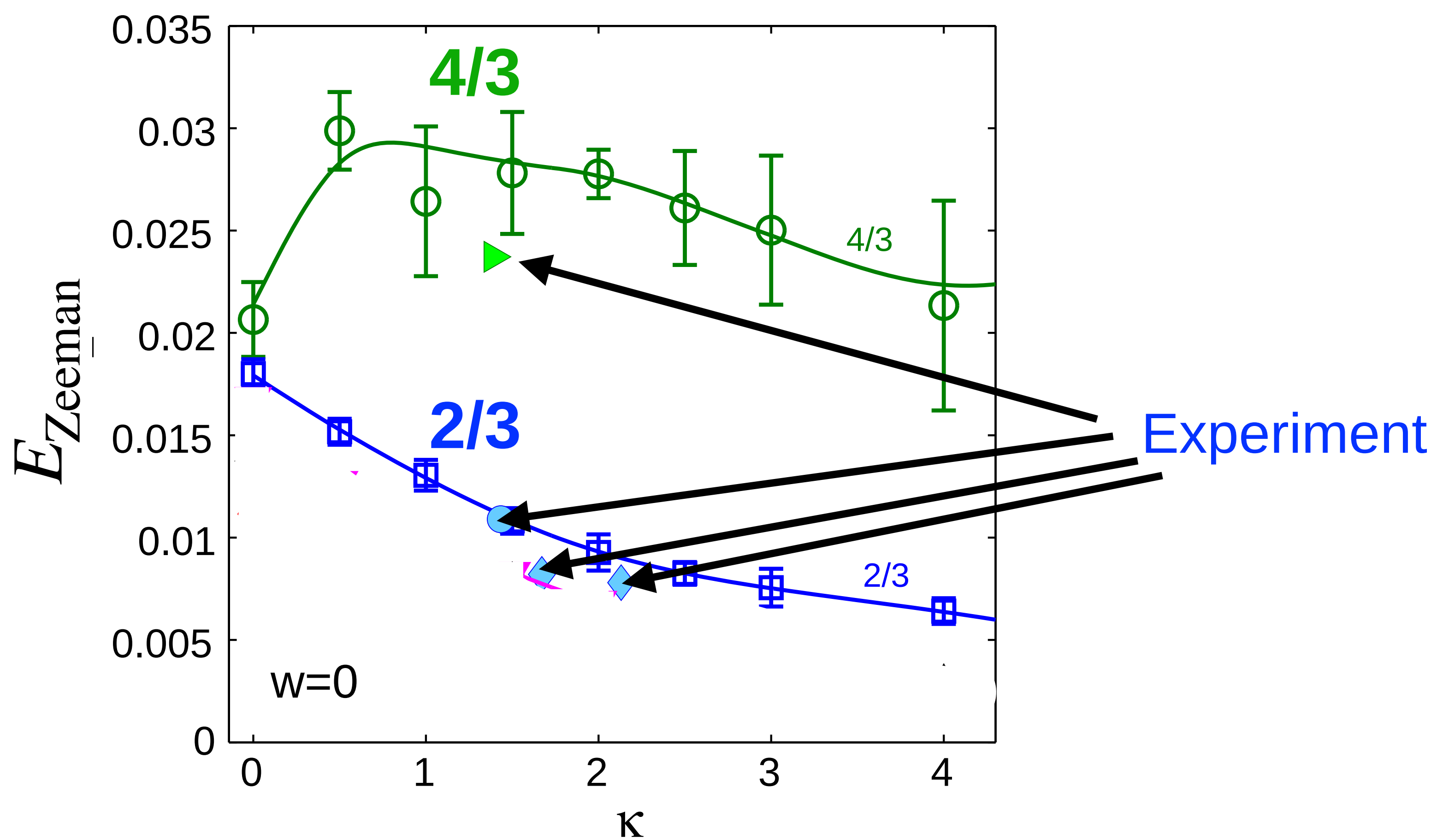
In GaAs quantum wells, spin physics is readily seen at $\nu = 2 - p/(2mp \pm 1)$ but not at $\nu = p/(2mp \pm 1)$.

Breaking of particle-hole symmetry

PRL 117, 116803 (2016) PHYSICAL REVIEW LETTERS week ending 9 SEPTEMBER 2016

Landau-Level Mixing and Particle-Hole Symmetry Breaking for Spin Transitions in the Fractional Quantum Hall Effect

Yuhe Zhang,[1] A. Wójs,[2] and J. K. Jain[1,3]



With Landau level mixing, the critical Zeeman energies are much higher for the $\nu = 2 - p/(2mp \pm 1)$ states than for $\nu = p/(2mp \pm 1)$.

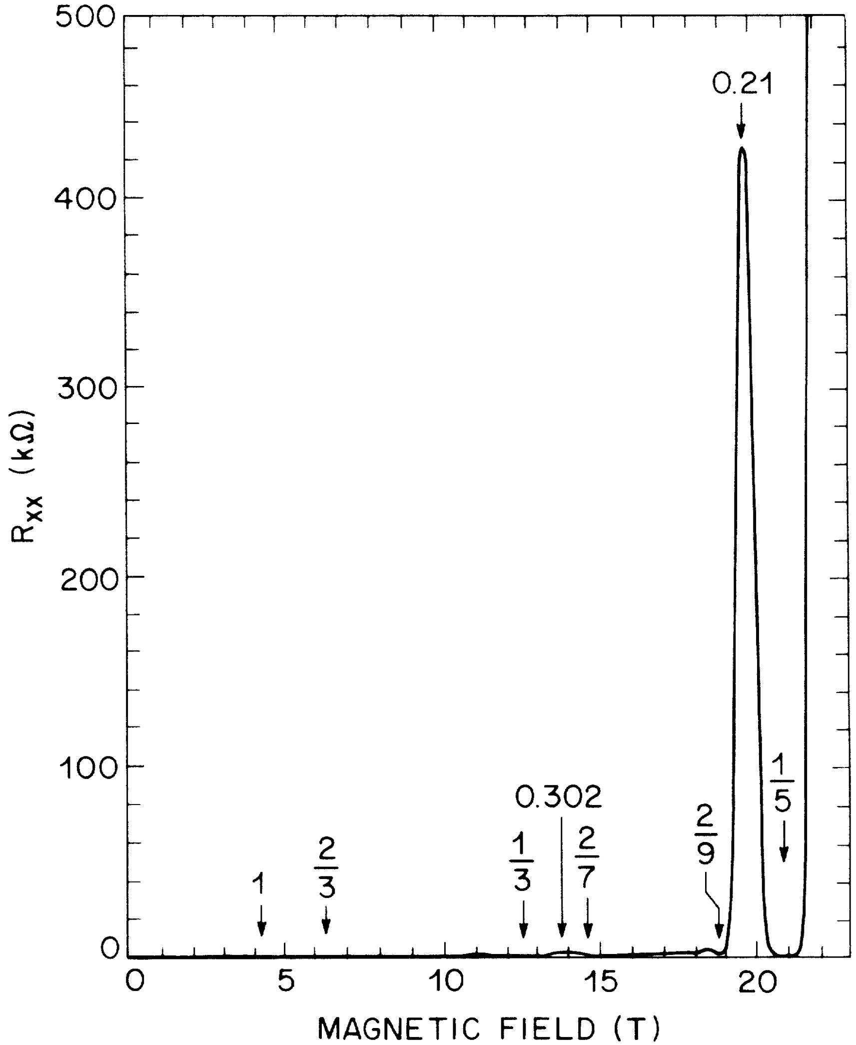
CF crystals

Crystal at low filling factors

VOLUME 65, NUMBER 5 PHYSICAL REVIEW LETTERS 30 JULY 1990

Quantum Liquid versus Electron Solid around $\nu = \frac{1}{5}$ Landau-Level Filling

H. W. Jiang,[1,2] R. L. Willett,[3] H. L. Stormer,[3] D. C. Tsui,[2] L. N. Pfeiffer,[3] and K. W. West[3]
[1] *Francis Bitter National Magnet Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*
[2] *Princeton University, Princeton, New Jersey 08544*
ies, Murray Hill, New Jersey 07974
d 15 February 1990)



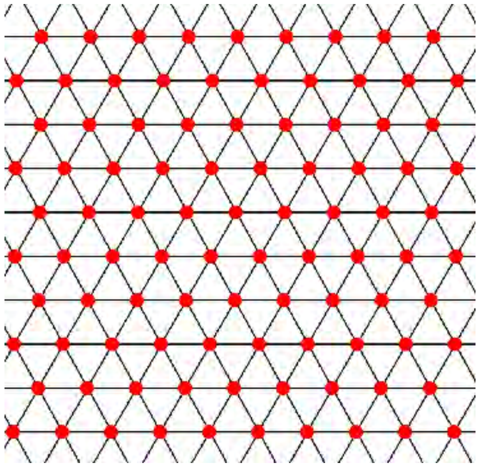
- 1/5 and 2/9 are strong FQH states, but an insulating state appears in-between, and also below 1/5.
- This insulating state has not gone away even when the mobility has gone up significantly.
- It is therefore likely a pinned crystal.

Also Goldman, Santos, Shayegan, Cunningham, PRL 1990

Uncorrelated Hartree-Fock crystal

$$\Psi^{\mathrm{HF}} = \frac{1}{\sqrt{N!}} \sum_P \epsilon_P \prod_{j=1}^{N} \phi_{\boldsymbol{R}_j}(\boldsymbol{r}_{Pj})$$

$$\phi_{\boldsymbol{R}}(\boldsymbol{r}) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{4}(\boldsymbol{r}-\boldsymbol{R})^2 + \frac{\mathrm{i}}{2}(xY - yX)\right)$$



The CF crystal

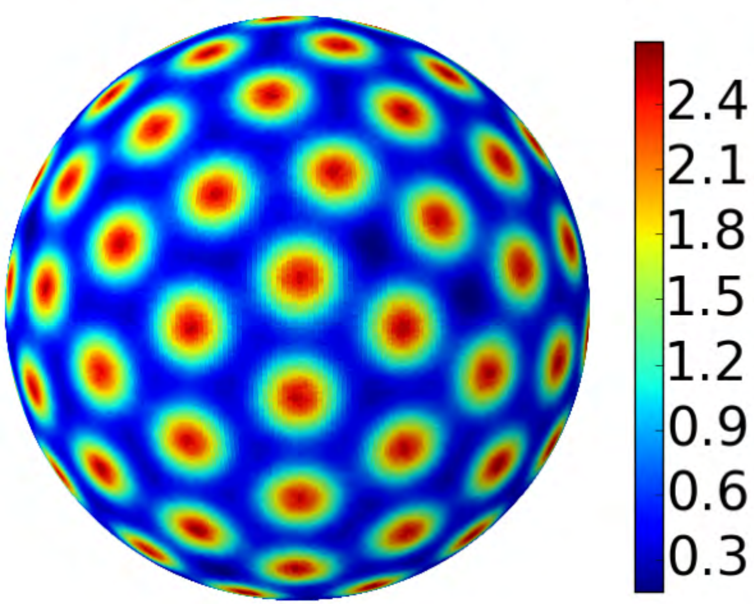
$$\Psi^{\text{CFC}}_{2p} = \prod_{j<k} (z_j - z_k)^{2p} \Psi^{\text{HF}}$$

- It would seem natural for nature to take advantage of *both* the CF and crystalline correlations to find the minimum energy.
- In this wave function, due to the Jastrow factor, the zero point fluctuations at nearby sites are correlated.
- Now $2p$ is a variational parameter.

Yi and Fertig, PRB 58, 4019 (1998)
Narevich, Murthy, Fertig, PRB 64, 245326 (2001)
Chang, Jeon, Jain, PRL 94, 016809 (2005)
Archer, Park, Jain, PRL 111, 146804 (2013)

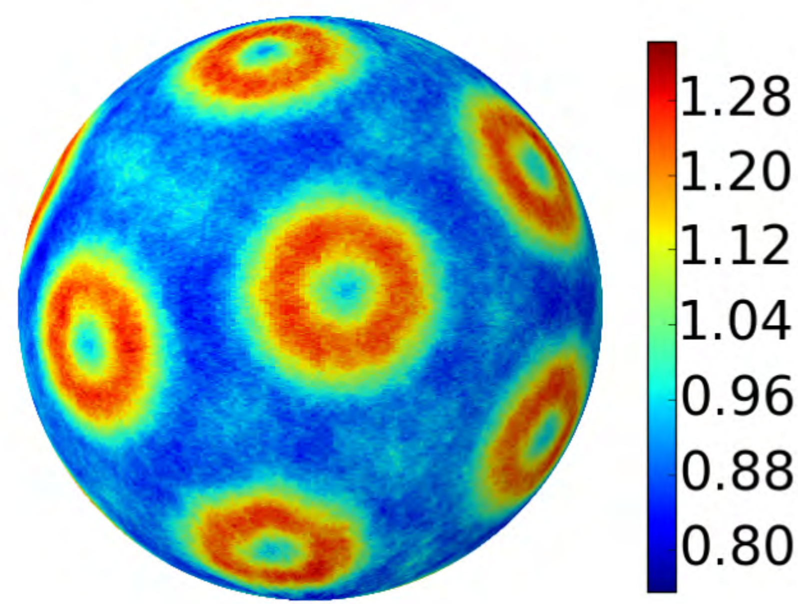
Type I and type II crystals

On a sphere, we approximate the crystal by the Thompson crystal.



v=0.394

Insulator



v=0.351
crystal of CFs in the second Λ level

FQH state

When all CFs form a crystal, its pinning produces an insulator.

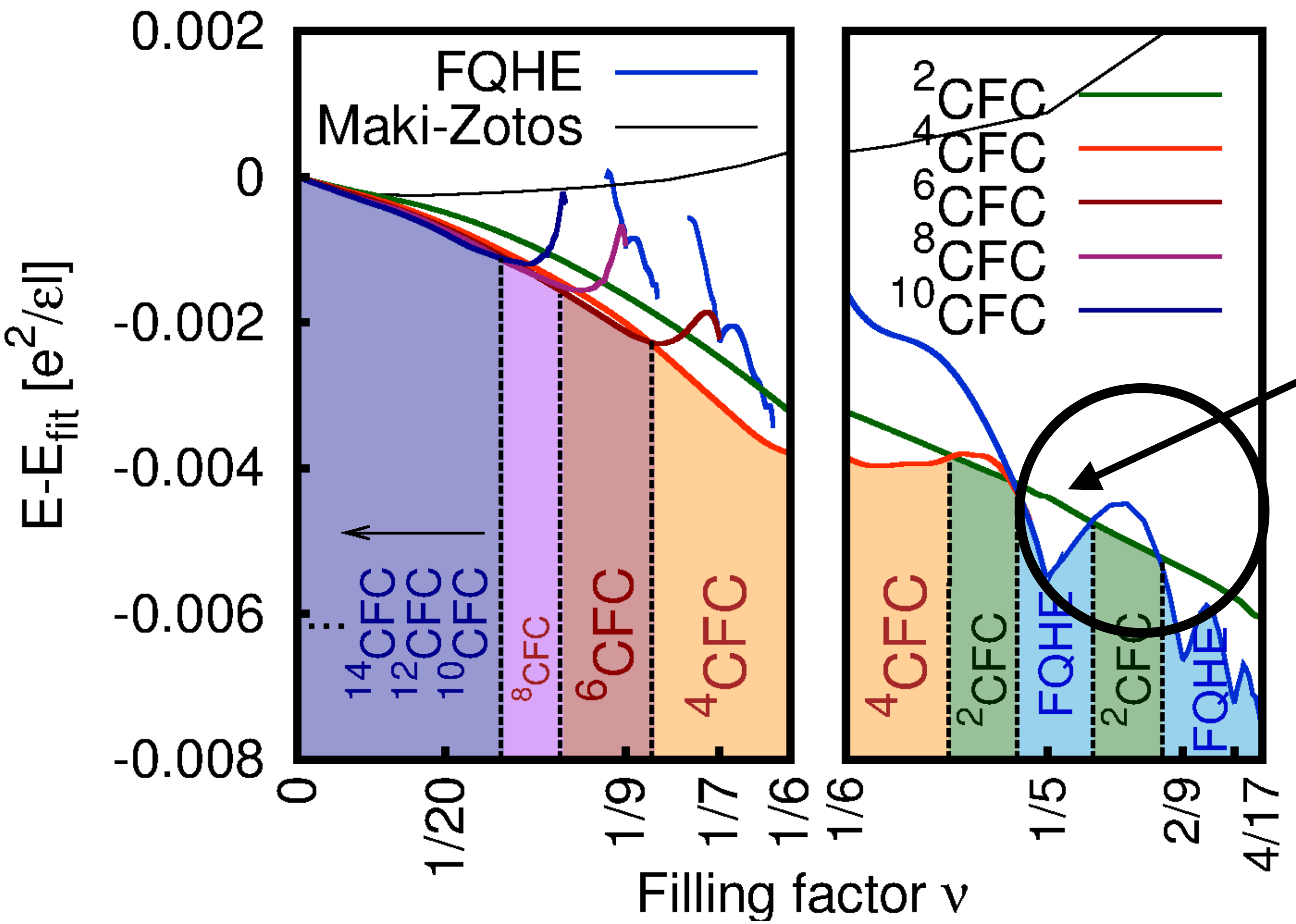
When only the CFs in the partially filled Λ-level form a crystal, its pinning produces a FQHE state.

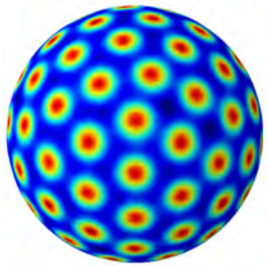
PRL 111, 146804 (2013) PHYSICAL REVIEW LETTERS week ending 4 OCTOBER 2013

Competing Crystal Phases in the Lowest Landau Level

Alexander C. Archer,[1] Kwon Park,[2] and Jainendra K. Jain[1]

A variational study





Re-entrant transition

The crystal between 1/5 and 2/5 is not the ordinary Wigner crystal of electrons, but a crystal of CFs carrying two vortices.

LL mixing and crystal phase diagram

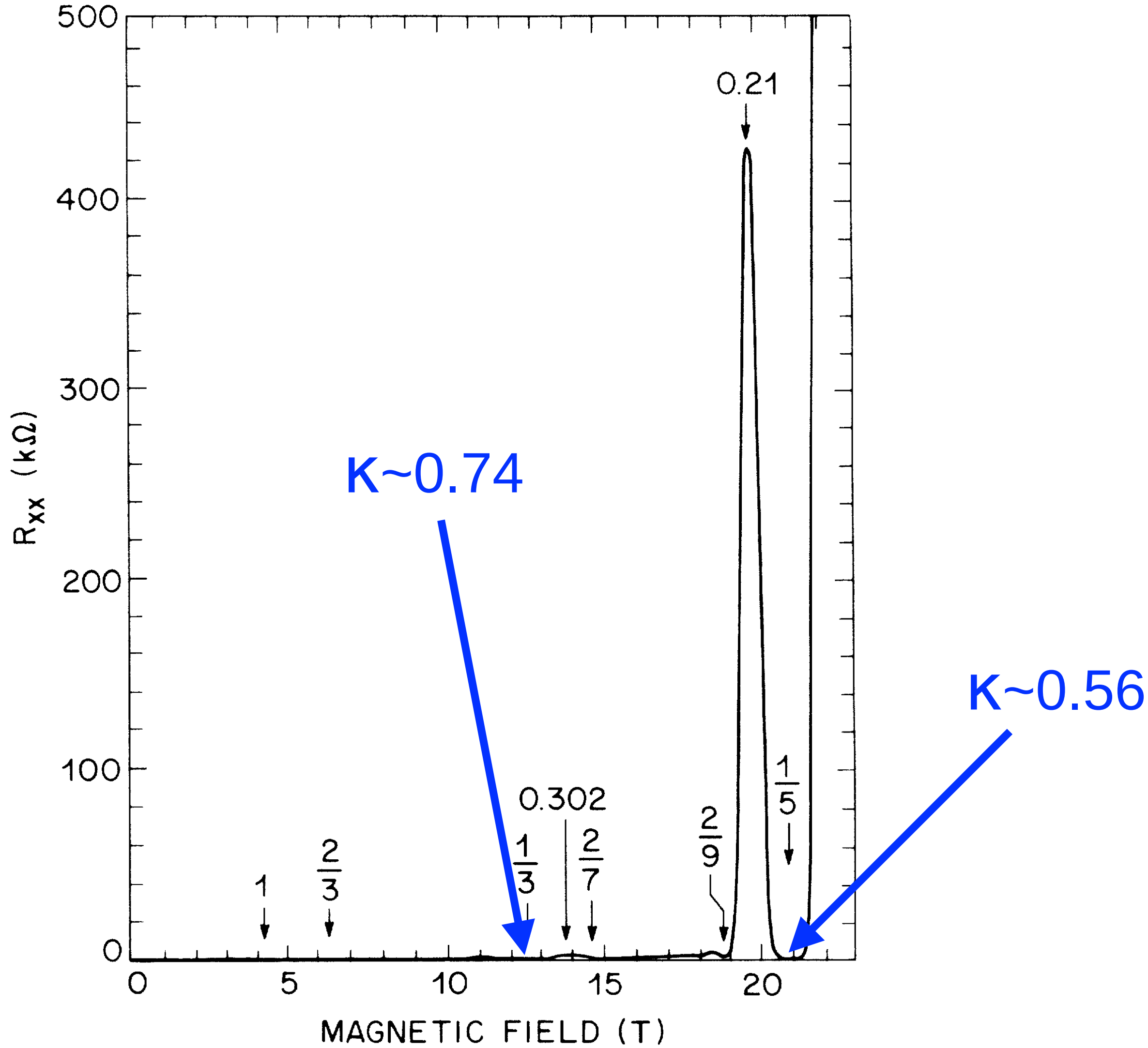
Crystal at low filling factors

VOLUME 65, NUMBER 5 PHYSICAL REVIEW LETTERS 30 JULY 1990

Quantum Liquid versus Electron Solid around $\nu = \frac{1}{5}$ Landau-Level Filling

H. W. Jiang,[1,2] R. L. Willett,[3] H. L. Stormer,[3] D. C. Tsui,[2] L. N. Pfeiffer,[3] and K. W. West[3]
[1] *Francis Bitter National Magnet Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*
[2] *Princeton University, Princeton, New Jersey 08544*
[3] *AT&T Bell Laboratories, Murray Hill, New Jersey 07974*
(Received 15 February 1990)

$$\kappa = \frac{e^2/\epsilon\ell}{\hbar\omega_c}$$



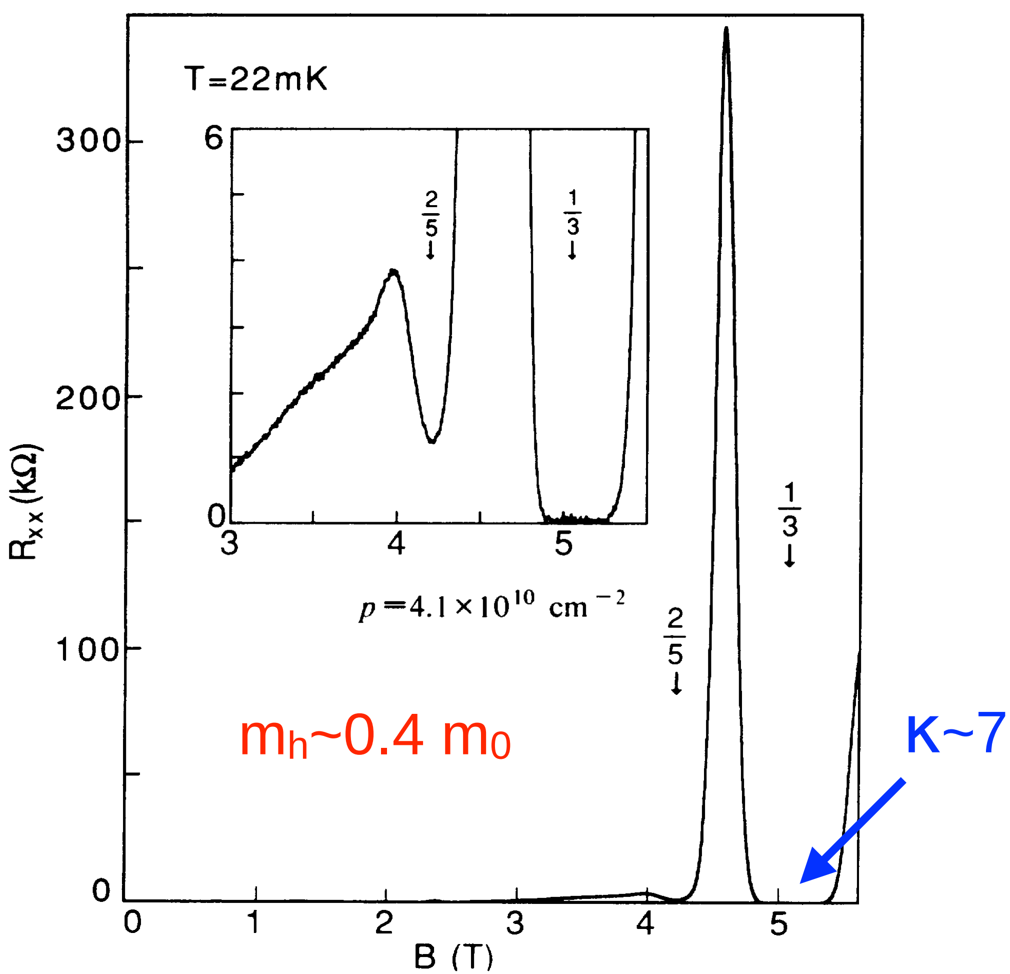
Crystal in hole-type samples

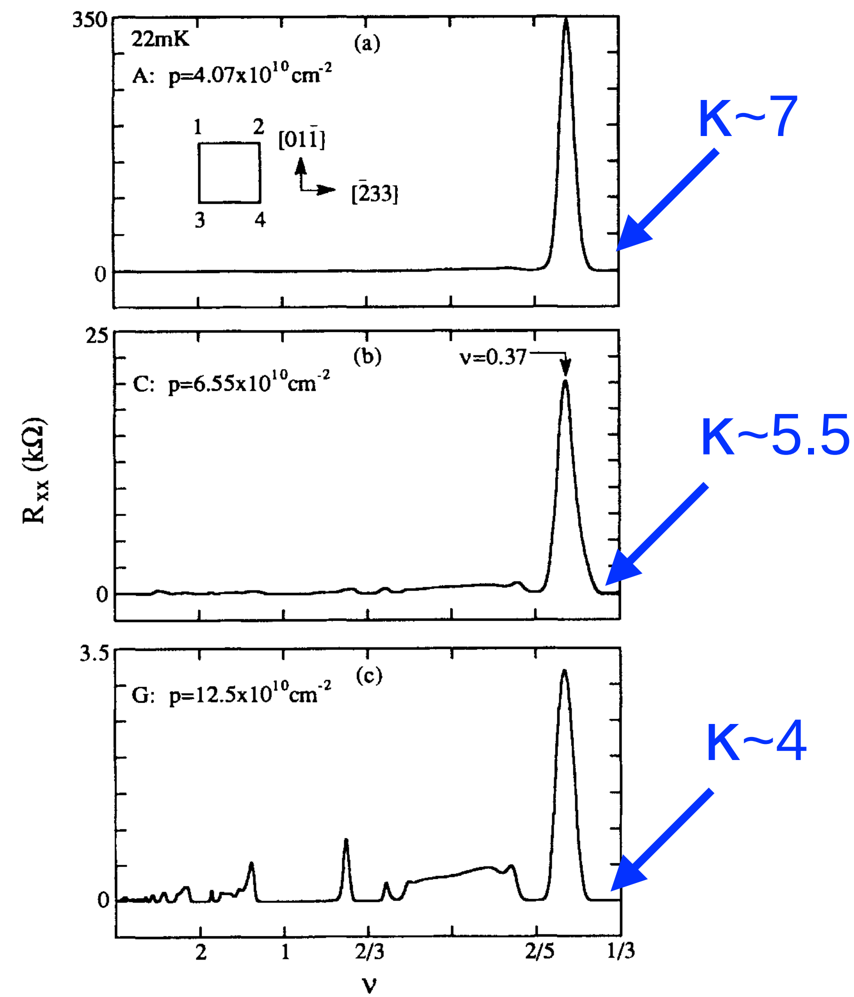
VOLUME 68, NUMBER 8 PHYSICAL REVIEW LETTERS 24 FEBRUARY 1992

Observation of a Reentrant Insulating Phase near the $\frac{1}{3}$ Fractional Quantum Hall Liquid in a Two-Dimensional Hole System

M. B. Santos, Y. W. Suen, M. Shayegan, Y. P. Li, L. W. Engel, and D. C. Tsui
Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544
(Received 4 October 1991)

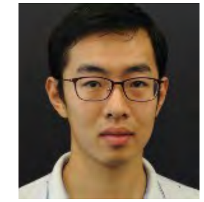
LL mixing can induce a crystal.





Santos et al. PRB 46, 13639 (1992)

Theoretical phase diagram

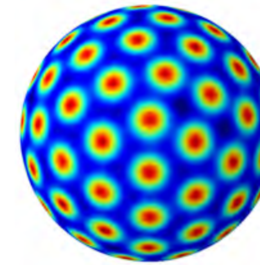



Jianyun Zhao Yuhe Zhang

PHYSICAL REVIEW LETTERS **121,** 116802 (2018)

Crystallization in the Fractional Quantum Hall Regime Induced by Landau-Level Mixing

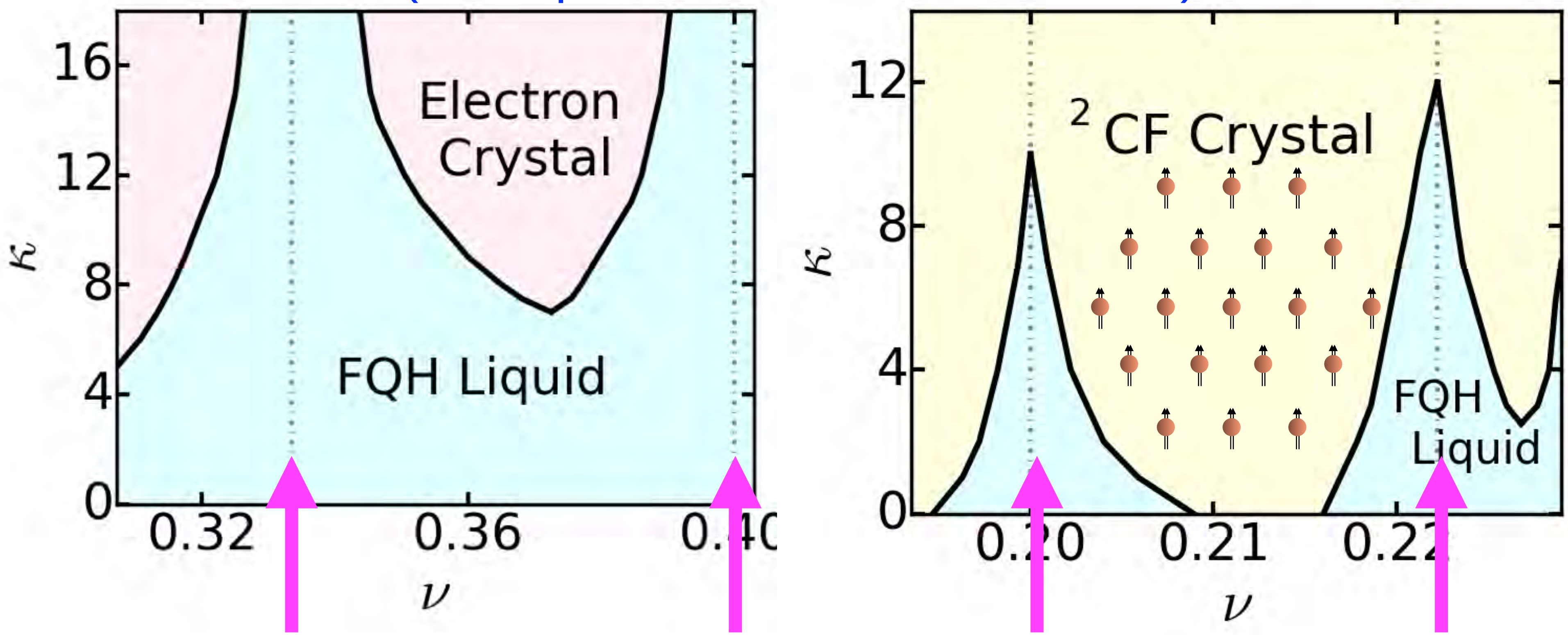
Jianyun Zhao, Yuhe Zhang, and J. K. Jain
Department of Physics, 104 Davey Laboratory, The Pennsylvania State University, University Park, Pennsylvania 16802, USA



LL mixing parameter

$$\kappa = \frac{e^2/\epsilon\ell}{\hbar\omega_c}$$

(Fixed phase diffusion Monte Carlo)



ν=1/3 ν=2/5 ν=1/5 ν=2/9

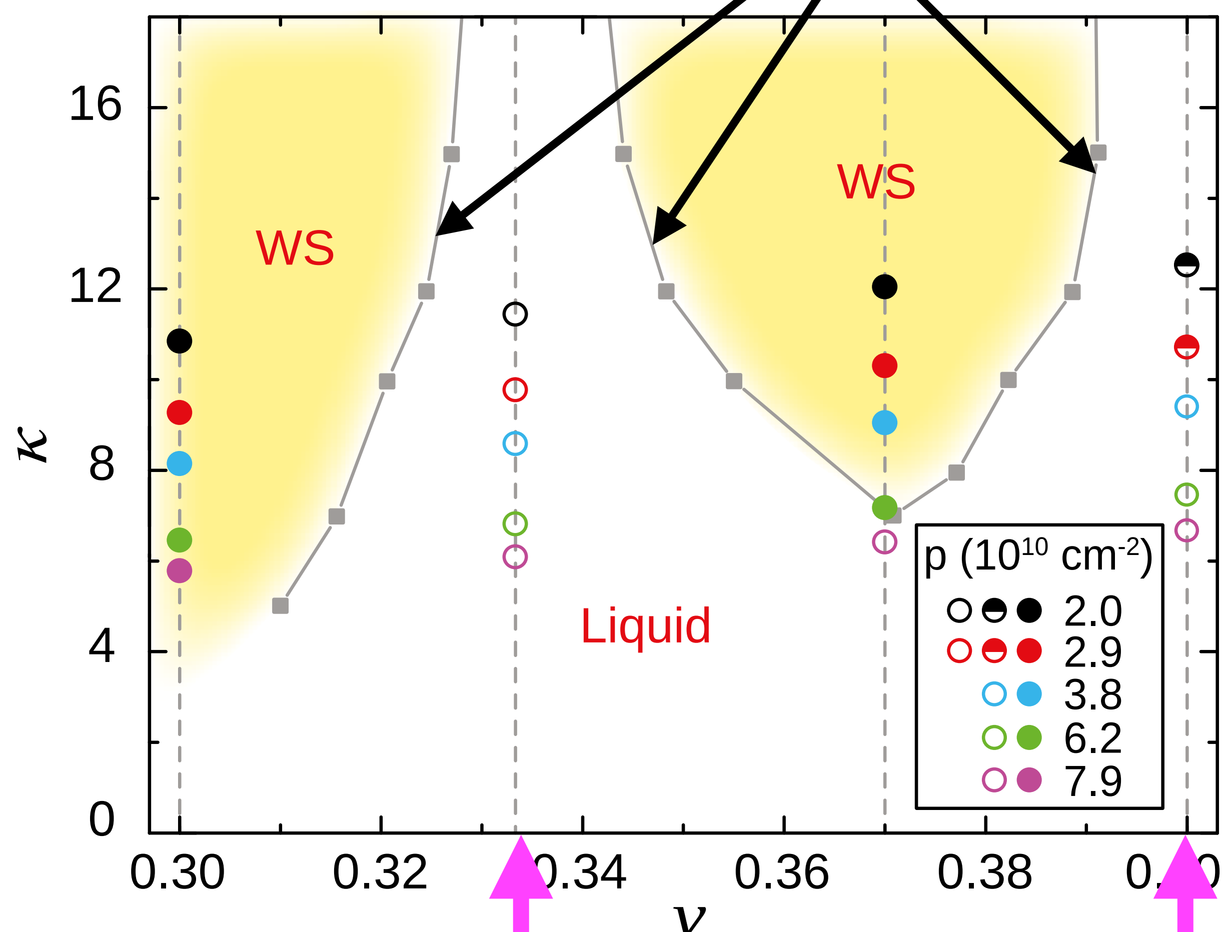
Comparison with experiment

LL mixing parameter

$$\kappa = \frac{e^2/\epsilon\ell}{\hbar\omega_c}$$

Theoretically predicted phase boundary

Zhao, Zhang, Jain (PRL 2018)



ν=1/3 **ν=2/5**

Hole type sample. Hole mass a factor of ~6 times larger than the electron mass.

Ma, Shayegan, et al. (PRL 2020)

PHYSICAL REVIEW RESEARCH **3**, 013181 (2021)

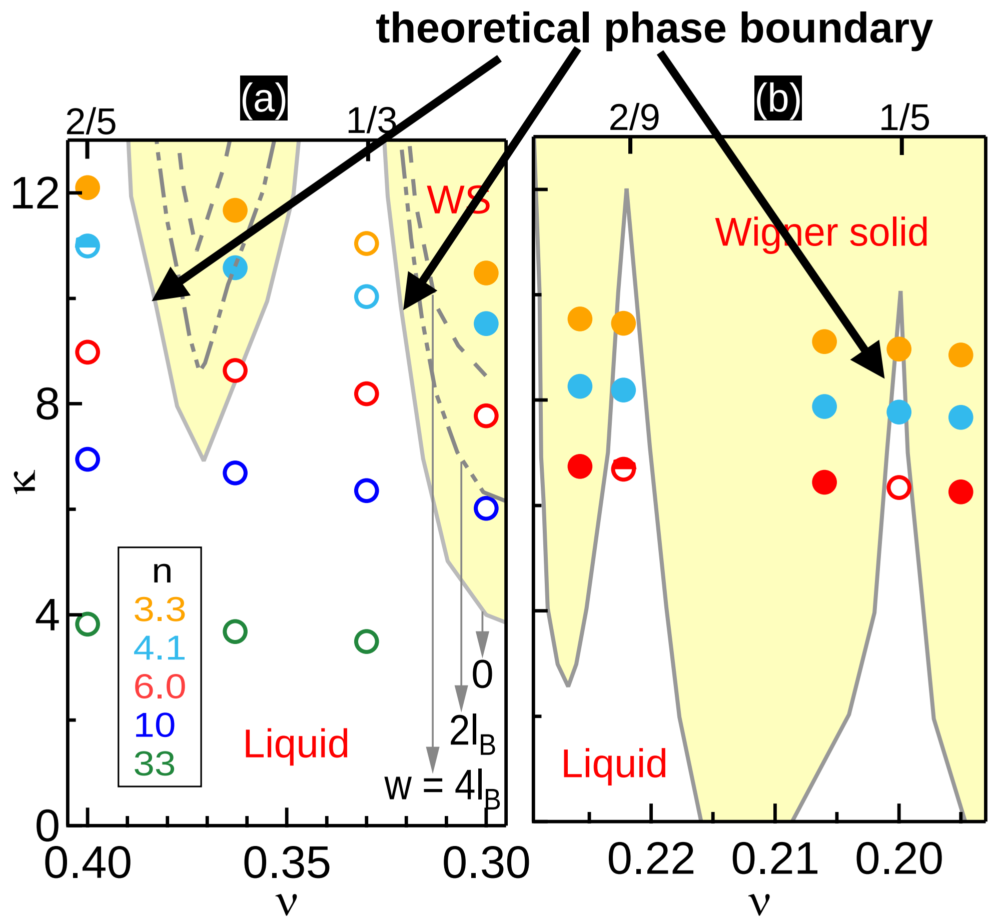
Competition between fractional quantum Hall liquid and Wigner solid at small fillings: Role of layer thickness and Landau level mixing

K. A. Villegas Rosales, S. K. Singh, Meng K. Ma, Md. Shafayat Hossain, Y. J. Chung, L. N. Pfeiffer, K. W. West, K. W. Baldwin, and M. Shayegan
Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA

(Zhao, Zhang, JKJ, PRL 2018)

LL mixing parameter

$$\kappa = \frac{e^2/\epsilon\ell}{\hbar\omega_c}$$



n (10^{10} cm^{-2})	3.3	4.1	6.0	10	33
w (nm)	35	20	50	35	20
w/l_B	2.8	1.8	5.3	4.8	5.0

These experiments are performed in AlAs quantum wells, where κ is a factor of 9 larger than in GaAs for a given density.

Also very likely the reason for crystal in graphene between 1/3 and 2/5…

Why does fixed-phase diffusion Monte Carlo work so well?

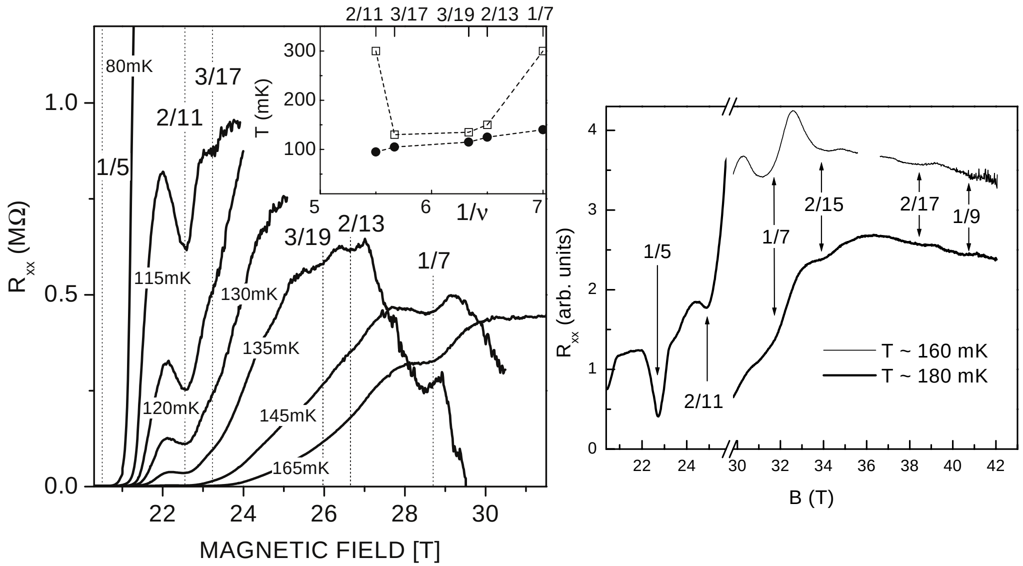
Dressing composite fermions with artificial intelligence

Mytraya Gattu
Department of Physics, 104 Davey Lab, Pennsylvania State University, University Park, Pennsylvania 16802, USA
(Dated: December 2, 2025)

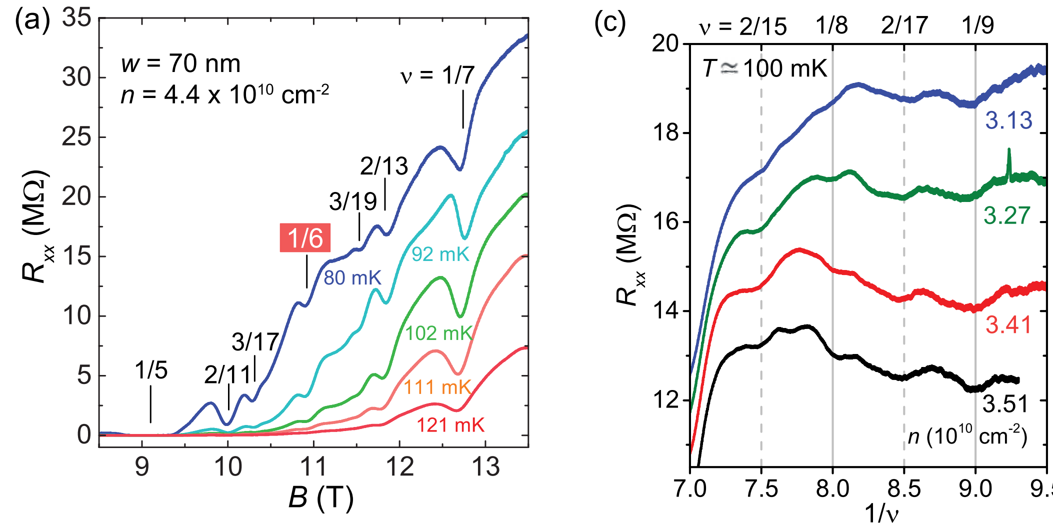
$\gtrsim$ 26 electrons. At fillings $\nu = 1/3$ and $2/5$, as a function of Landau-level mixing strength, CF-Flow produces ground-state energies with low local-energy variance that are nearly indistinguishable from those obtained using the fixed-phase diffusion Monte Carlo (fp-DMC) method, even though the latter constrains the wavefunction phase to that of the lowest Landau level—thereby providing insight into why fp-DMC has been successful in giving an accurate quantitative account of several experiments. Finally, the symmetry-preserving architecture of CF-Flow enables access to excited

Low fillings

^{2}CFs, ^{4}CFs, ^{6}CFs and ^{8}CFs observed



Pan, Stormer, … PRL 2002

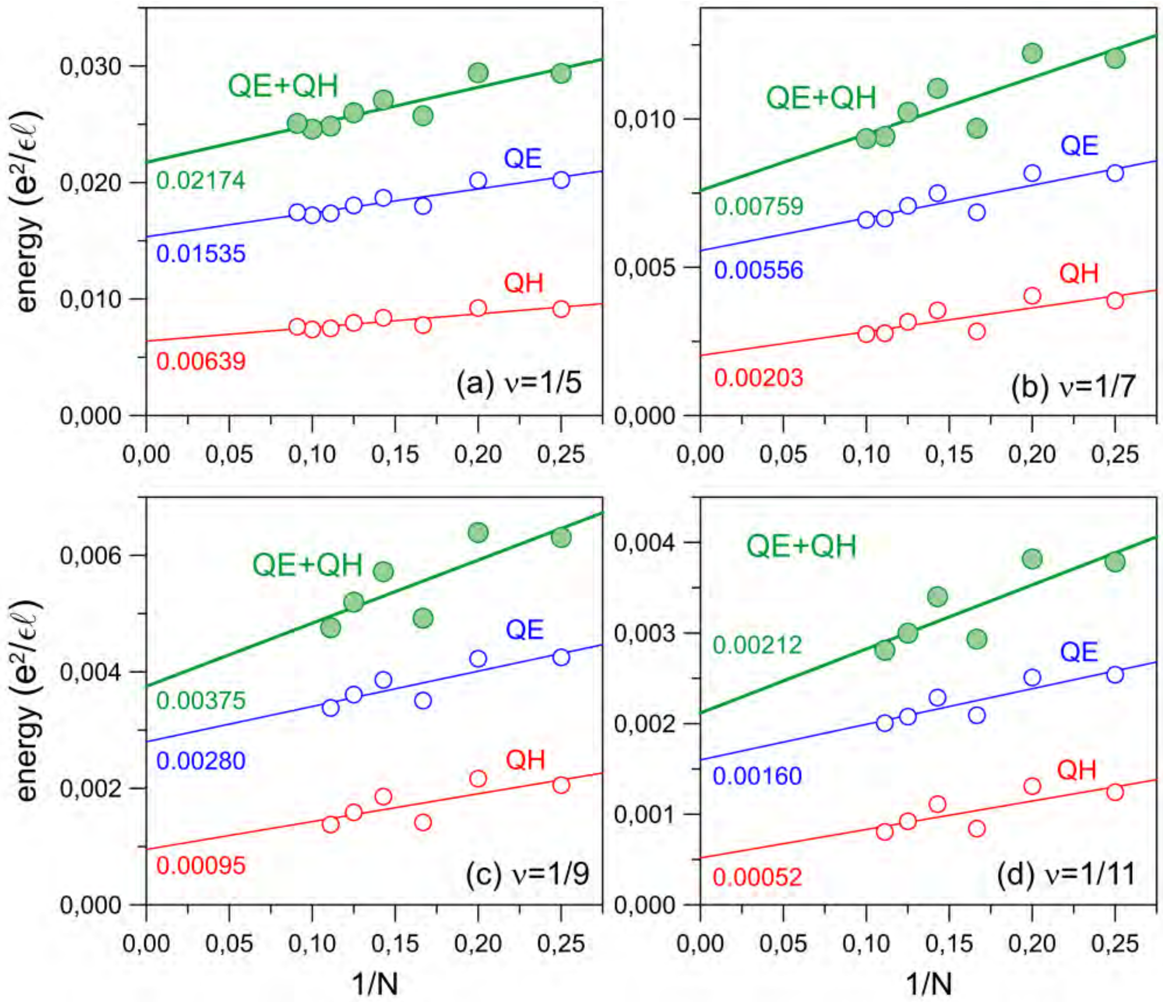


Wang, … Shayegan, PRL 2025

- At low filing factors, the system is insulating in the sense that the resistance increases with decreasing temperature.
- However, on top of the rising resistance, R_{xx} exhibits minima that nicely correlate with FQHE states.
- What is the underlying physics?

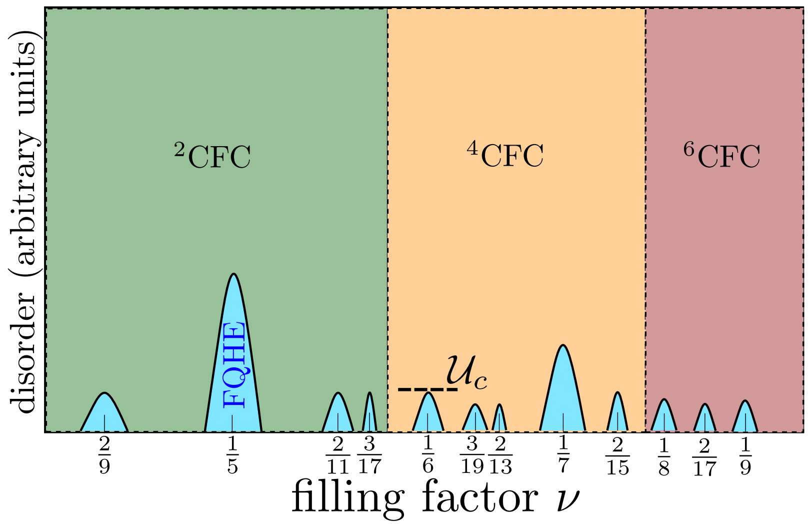
Interplay between fractional quantum Hall liquid and crystal phases at low filling

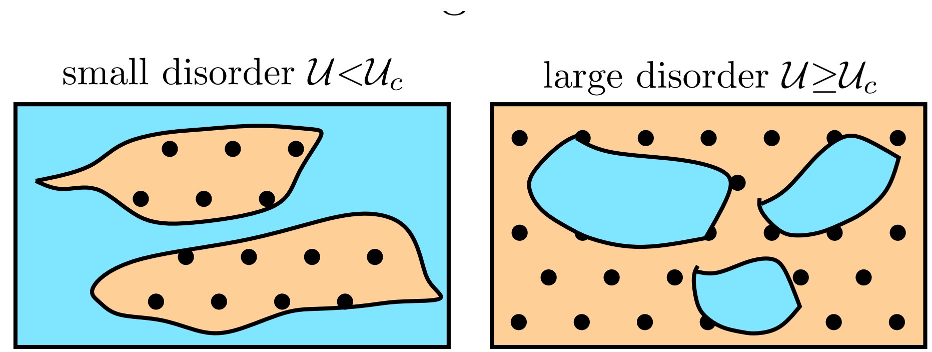
Zheng-Wei Zuo,[1,2] Ajit C. Balram,[3] Songyang Pu,[1] Jianyun Zhao,[1] Thierry Jolicoeur,[4] A. Wójs,[5] and J. K. Jain[1]



Exact diagonalization studies at 1/7 (up to 10 particles), 2/13 (up to 10 particles), 1/9 (up to 9 particles) and 1/11 (up to 8 particles) are consistent with a liquid state, with gaps extrapolating to finite values.

Our view: a cascade of re-entrant transitions





- Exactly at $n/(6n \pm 1)$, $n/(8n \pm 1)$ and $n/(10n \pm 1)$, the actual state is an incompressible FQHE liquid. In between, we have CF crystals.

- Disorder favors crystals.

- At finite disorder, treated as variation in filling factor, we have regions of crystal and FQHE liquid. At low-disorder, the liquid percolates, and perfect FQHE will be seen.

- At higher disorder, the crystal percolates, producing an insulator. Nonetheless, having large puddles of the FQHE liquid can depress the resistance at a finite temperature.

- Eventually, as disorder is eliminated, FQHE will be seen at these fillings.

Stripe and bubble crystal phases of CFs

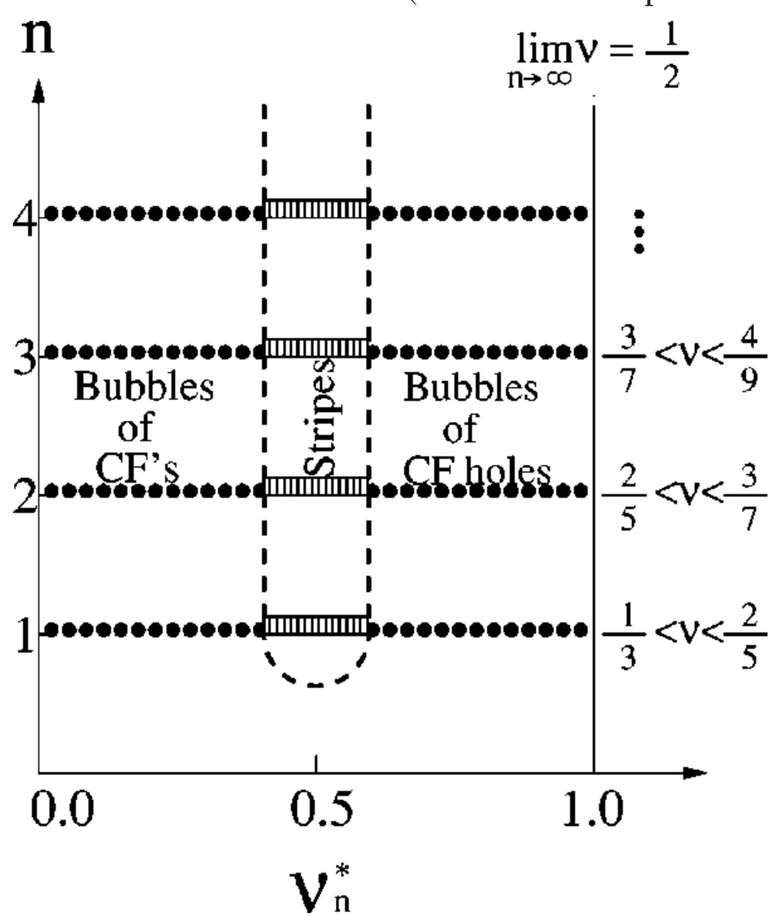
PHYSICAL REVIEW B **66**, 085336 (2002)

Structures for interacting composite fermions: Stripes, bubbles, and fractional quantum Hall effect

Seung-Yeop Lee, Vito W. Scarola, and J.K. Jain
Department of Physics, 104 Davey Laboratory, The Pennsylvania State University, University Park, Pennsy

(Received 15 April 2002; published 30 August 2002)

Vito Scarola
Virginia Tech

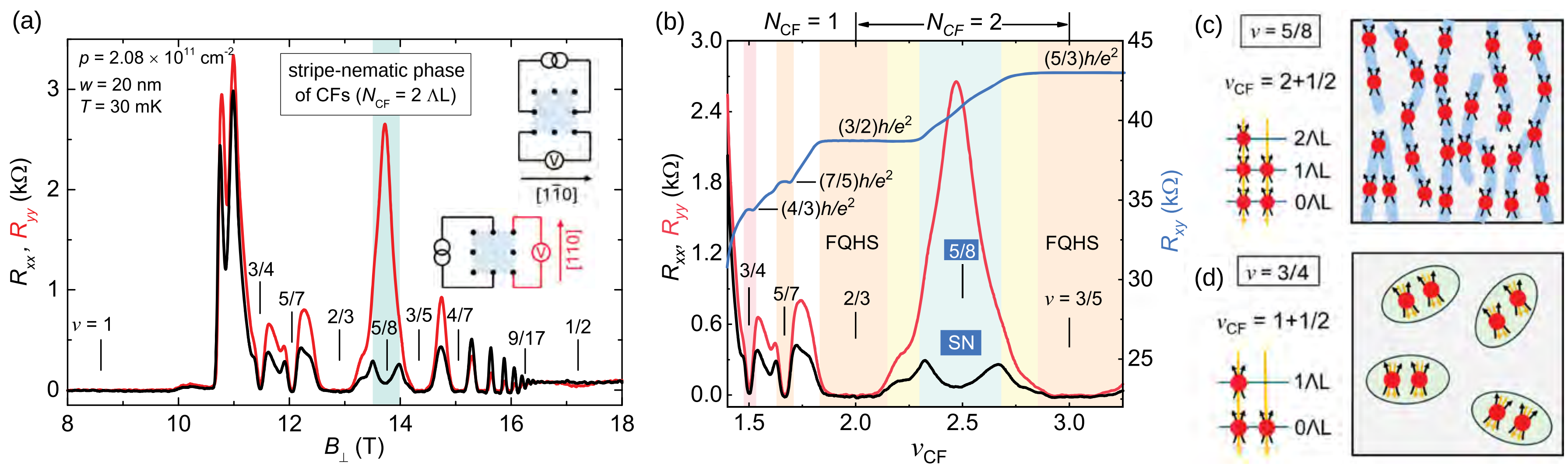


We determined the interaction between CFs in a given Λ-level, and then determined the best variational state.

We predicted the possibility of stripes and bubble crystals.

Stripe-Nematic Phase of Composite Fermions

Chengyu Wang, S. K. Singh, C. T. Tai, A. Gupta, L. N. Pfeiffer, K. W. Baldwin, and M. Shayegan
Department of Electrical and Computer Engineering, Princeton University, Princeton, New Jersey 08544, USA



nature physics

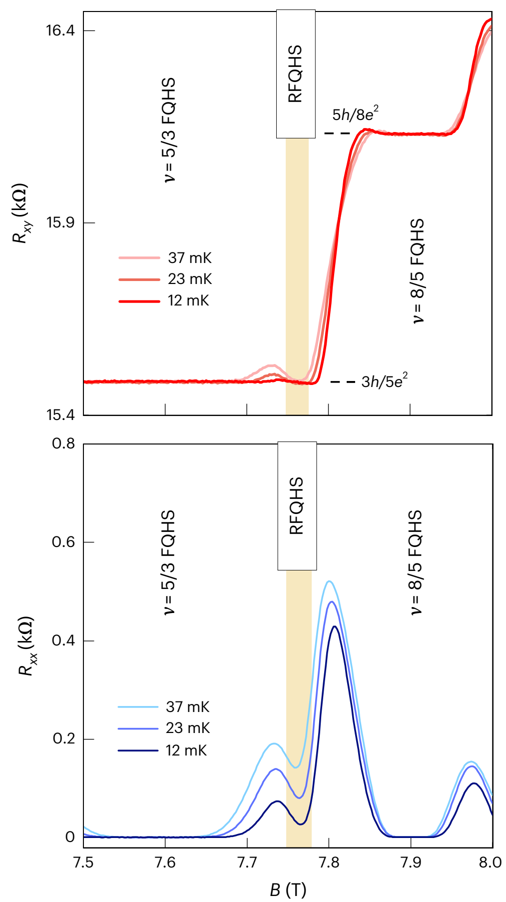
Article https://doi.org/10.1038/s41567-023-01939-2

A highly correlated topological bubble phase of composite fermions

Received: 7 June 2022

Accepted: 23 December 2022

Vidhi Shingla[1,4], Haoyun Huang[1,4], Ashwani Kumar[2], Loren N. Pfeiffer[3], Kenneth W. West[3], Kirk W. Baldwin[3] & Gábor A. Csáthy[1] ✉

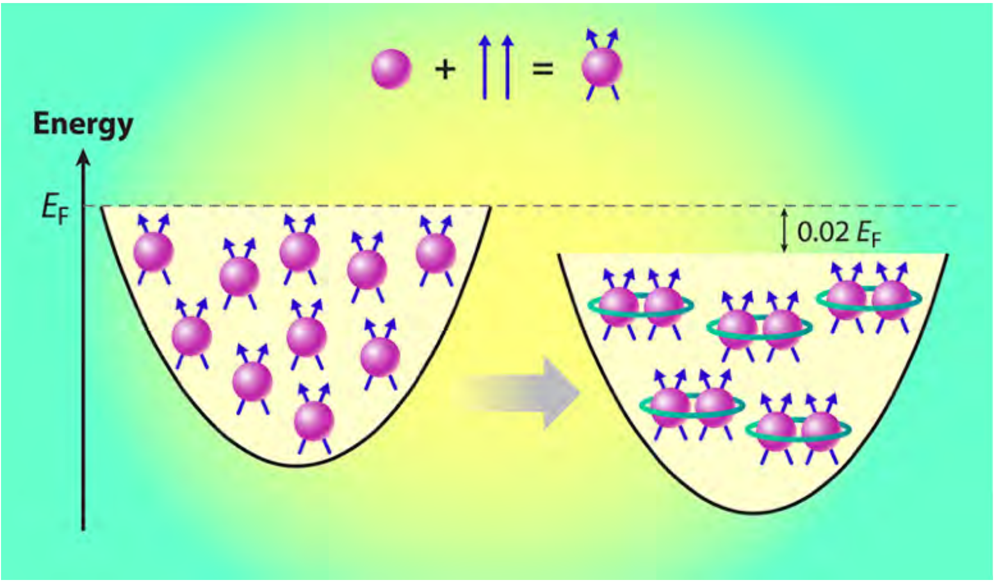


CF “superconductivity”

Even-denominator FQHE: CF pairing

- FQHE has been observed at many even-denominator fractions: 5/2,1/2,1/4,3/4,3/8,3/10,1/6,1/8. These cannot be understood in terms of noninteracting CFs.

- These FQHE states emerge from a CF metal when the weak residual interaction between the CFs causes them to form pairs, opening a gap. This provides a second mechanism for FQHE.



Pairing from purely repulsive interaction?

Empirically: The inter-CF interaction becomes attractive as the strength of the short range repulsion between the electrons is reduced. This may be done in three ways:

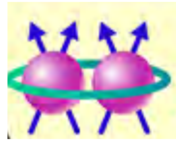
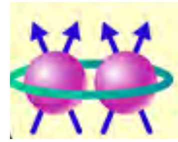
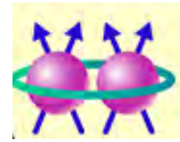
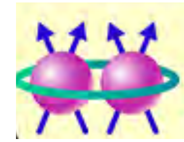
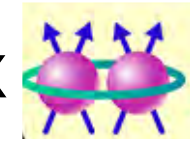
- By going to a higher LL
- By increasing the quantum well width / density
- By enhancing LL mixing

Previous approaches

- Trial wave functions: Pfaffian / anti-Pfaffian (Moore and Read, 1991), parton (Jain 1989, Balram, Barkeshli, Rudner, 2018; Faugno et al. PRL; …)
- These do not have any variational parameters and do not have any explicit relation to the CF metal.

BCS wave function of composite fermions

Sharma, Pu, Jain, PRB 2021

$\Psi^{\mathrm{CF-BCS}}_{1/2} =$ **A** [ x  x  x  x ]

$$\Psi^{\mathrm{el-BCS}}(\{\vec{r}_j\}) = A[g^{(l)}(\vec{r}_1 - \vec{r}_2)g^{(l)}(\vec{r}_3 - \vec{r}_4)\cdots]$$

$$\boxed{\Psi^{\mathrm{CF-BCS}}_{1/2} = P_{\mathrm{LLL}}\ \Psi^{\mathrm{el-BCS}}(\{\vec{r}_j\}) \prod_{j<k}(z_j - z_k)^2}$$

Anirban Sharma
Industry

Songyang Pu
WUSTL

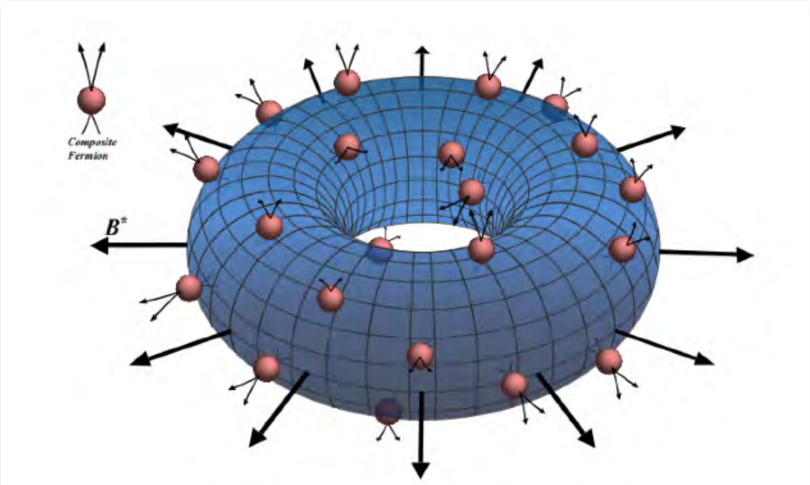
$$g^{(l)}(\vec{r}_i - \vec{r}_j) = \sum_{\vec{k}}^{|\vec{k}|\le k_{\mathrm{cutoff}}} g^{(l)}_{\vec{k}} e^{i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)} \qquad g^{(l)}_{\vec{k}} \equiv \frac{v_{\vec{k}}}{u_{\vec{k}}} = \frac{\epsilon_{\vec{k}} - \sqrt{\epsilon^2_{\vec{k}} + |\Delta^{(l)}_{\vec{k}}|^2}}{\Delta^{(l)*}_{\vec{k}}} = -g^{(l)}_{-\vec{k}}$$

$$\Delta^{(l)}_{\vec{k}} = \Delta\ |\vec{k}|^l e^{-il\theta}$$

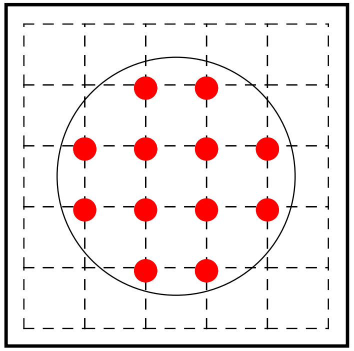
$l = 1$: **p-wave**
$l = 3$: **f-wave**

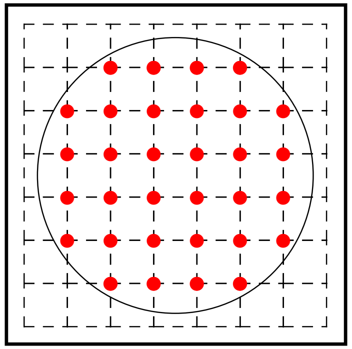
- Two variational parameters: Δ and $k_{\mathrm{cutoff}}(\ge k_{\mathrm{F}})$.
- The CF-BCS wave function reduces to the CF Fermi sea for $\Delta = 0$ or $k_{\mathrm{cutoff}} = k_{\mathrm{F}}$.

Composite fermions on a torus



Pu, Wu, Jain, PRB 96, 195302 (2017)





- Natural geometry for pairing; take a square torus; $N = 12,32$
- Technical hurdle: the projection $\bar{z}_j \rightarrow 2\partial/\partial z_j$ spoils the periodic boundary conditions. Needs to be modified.

Haldane, Rezayi (1985); Rezayi, Haldane (2000); Hermanns, Surosa, Bergholtz, Hansson, Karlhede (2008); Hermanns (2013); Greiter, Schnells, Thomale (2016); Pu, Wu, Jain (2017); Sharma, Pu, Jain (2021)

$$\Psi^{\mathrm{CF-BCS}}_{\frac{1}{2}} = e^{\sum_i \frac{z_i^2 - |z_i|^2}{4\ell^2}} \left\{ \vartheta \begin{bmatrix} \frac{\phi_1}{4\pi} + \frac{N-1}{2} \\ -\frac{\phi_2}{2\pi} + (N-1) \end{bmatrix} \left(\frac{2Z}{L} \middle| 2\tau \right) \right\} \ \mathrm{Pf}(\tilde{M}_{ij})$$

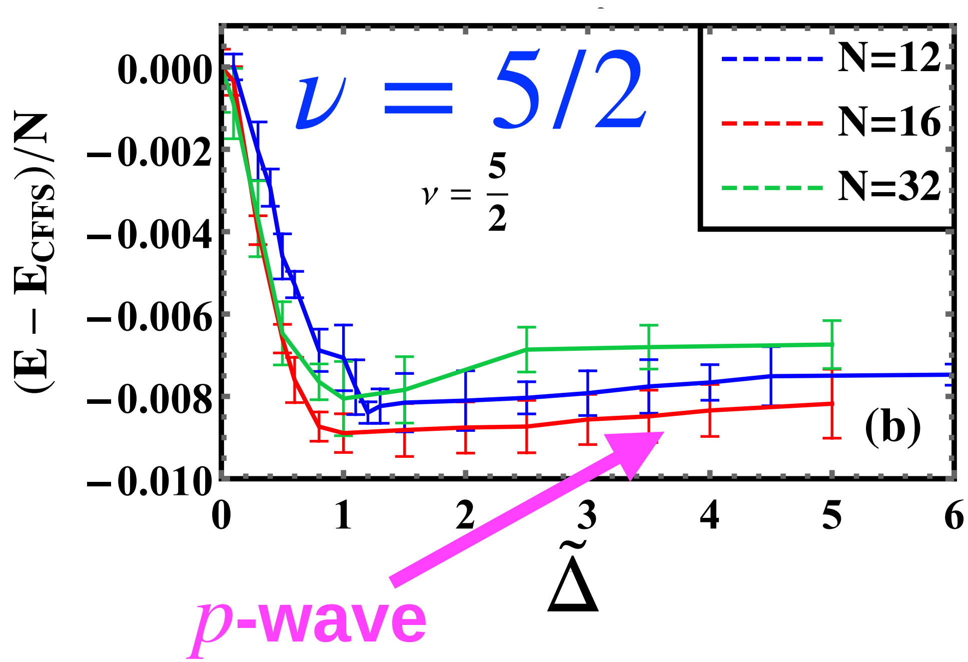
$$\tilde{M}_{ij} = \sum_{k_n} g^{(l)}_{\boldsymbol{k}_n} e^{-\frac{\ell^2}{2} k_n (k_n + 2\bar{k}_n)} e^{\frac{i}{2}(z_i - z_j)(k_n + \bar{k}_n)} \prod_{\substack{m \\ m \neq i,j}} \vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \left(\frac{z_i + i2k_n\ell^2 - z_m}{L} \middle| \tau \right)$$
$$\prod_{\substack{n \\ n \neq i,j}} \vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \left(\frac{z_j - i2k_n\ell^2 - z_n}{L} \middle| \tau \right) \left\{ \vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \left(\frac{z_i + i2k_n\ell^2 - z_j}{L} \middle| \tau \right) \right\}^2$$

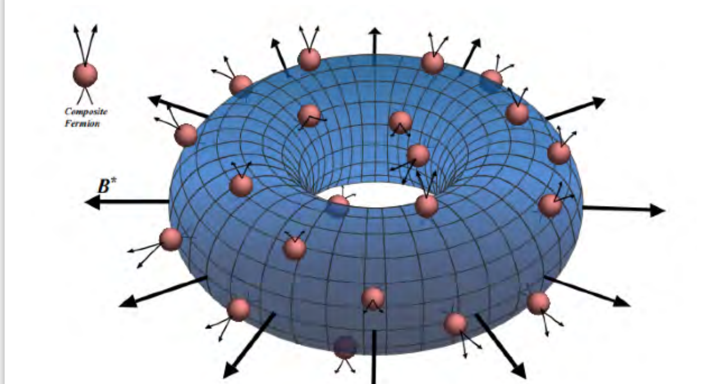
CF pairing at 5/2

PHYSICAL REVIEW B **104**, 205303 (2021)

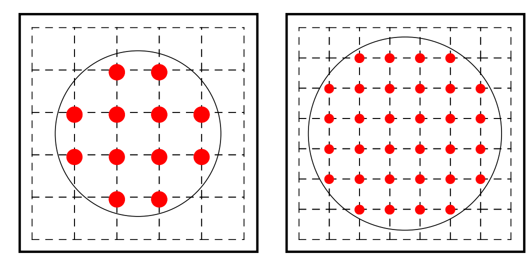
Bardeen-Cooper-Schrieffer pairing of composite fermions

Anirban Sharma, Songyang Pu, and J. K. Jain
Department of Physics, 104 Davey Lab, Pennsylvania State University, University Park, Pennsylvania 16802, USA





Pu, Wu, Jain, PRB 96, 195302 (2017)



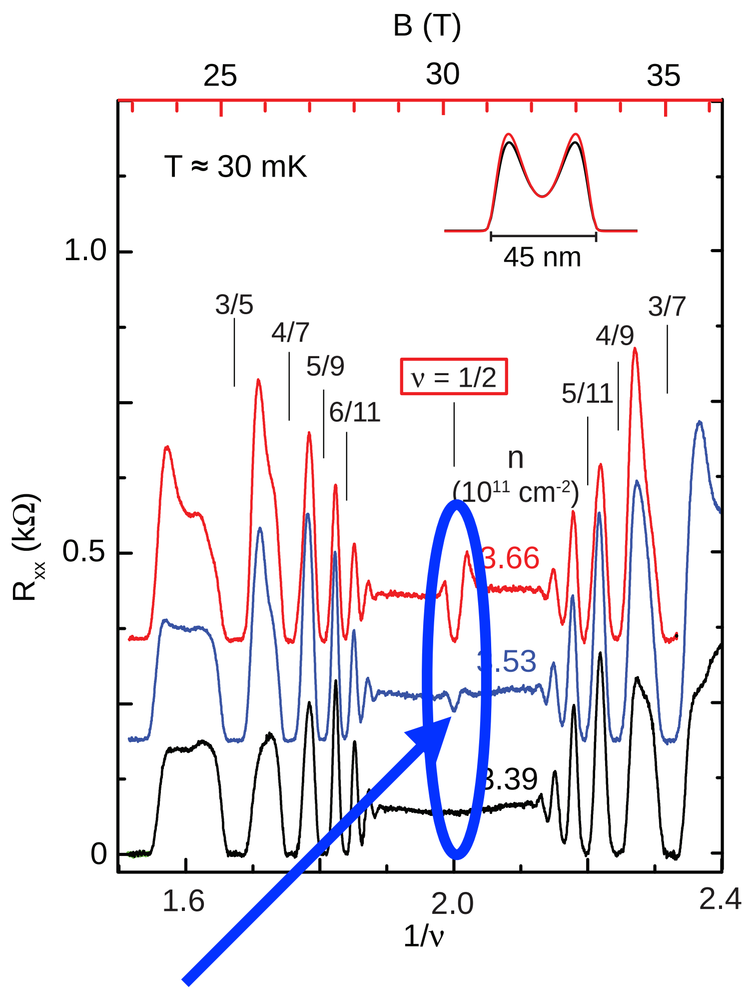
- A p-wave pairing instability occurs at $\nu = 5/2$.
- No instability at $\nu = 1/2$.

FQHE at $\nu = 1/2$ in wide quantum wells

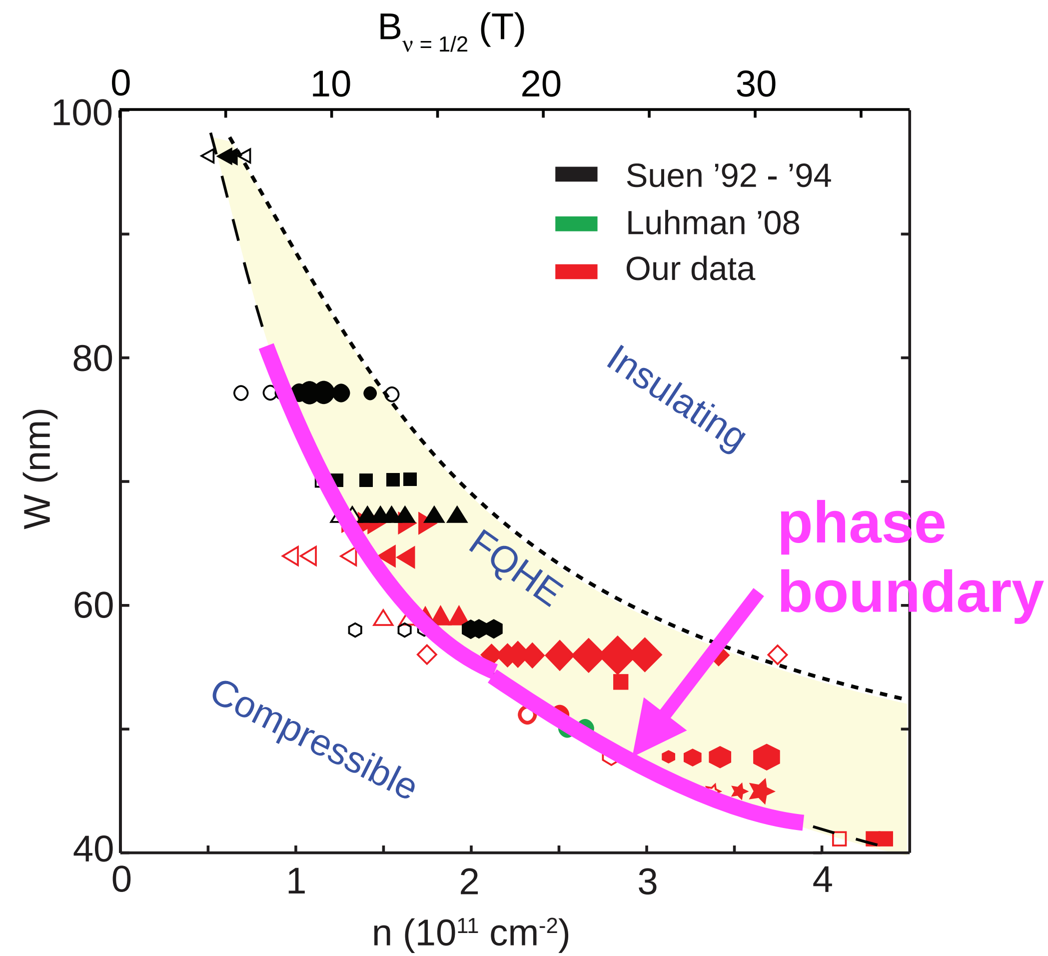
PHYSICAL REVIEW B **88**, 245413 (2013)

Phase diagrams for the stability of the $\nu = \frac{1}{2}$ fractional quantum Hall effect in electron systems confined to symmetric, wide GaAs quantum wells

J. Shabani, Yang Liu, M. Shayegan, L. N. Pfeiffer, K. W. West, and K. W. Baldwin



$\nu = 1/2$ FQHE



Suen, Engel, Santos, Shayegan, Tsui, PRL (1992)

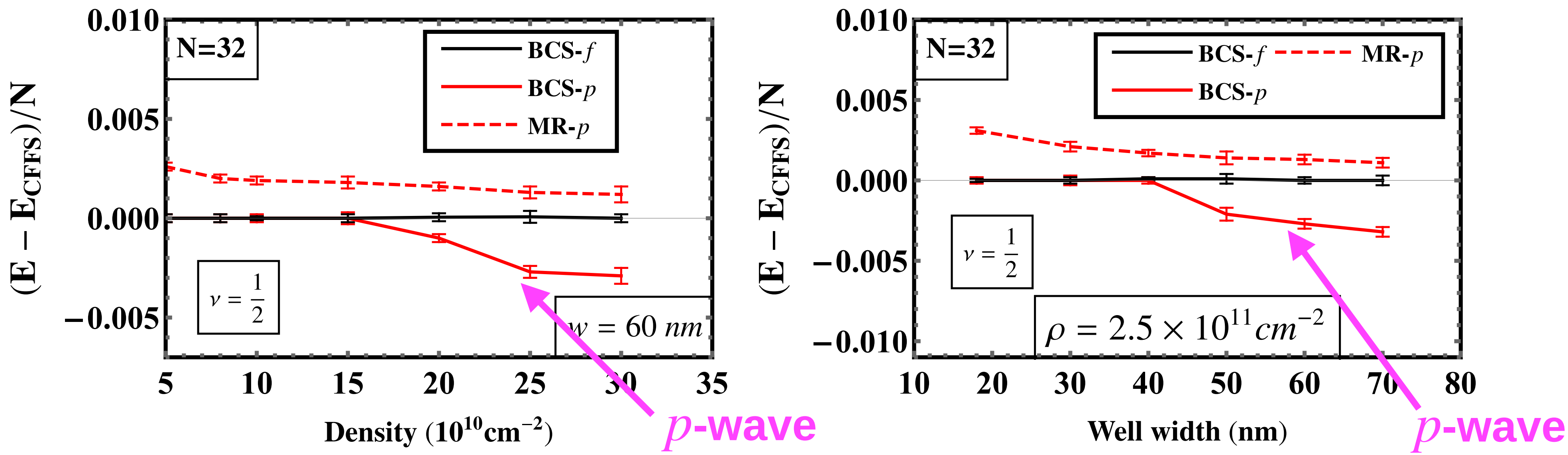
PHYSICAL REVIEW B **109**, 035306 (2024)

Editors' Suggestion | Featured in Physics

Composite-fermion pairing at half-filled and quarter-filled lowest Landau level

Anirban Sharma,[1] Ajit C. Balram,[2,3] and J. K. Jain[1]

Pairing instability at $\nu = 1/2$ in wide quantum wells



- For electrons in the lowest subband, a p-wave pairing instability occurs as either the quantum well width or the density is increased.
- The effect is subtle as indicated by the small energy gain of 0.002-0.003 $e^2/\epsilon\ell$ per particle.

CF pairing at $\nu = 1/2$ in wide quantum wells

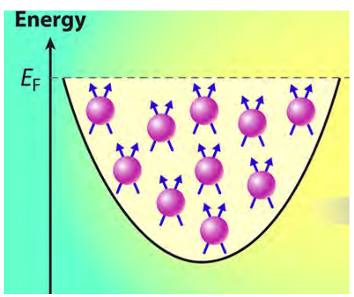
PHYSICAL REVIEW B **109**, 035306 (2024)

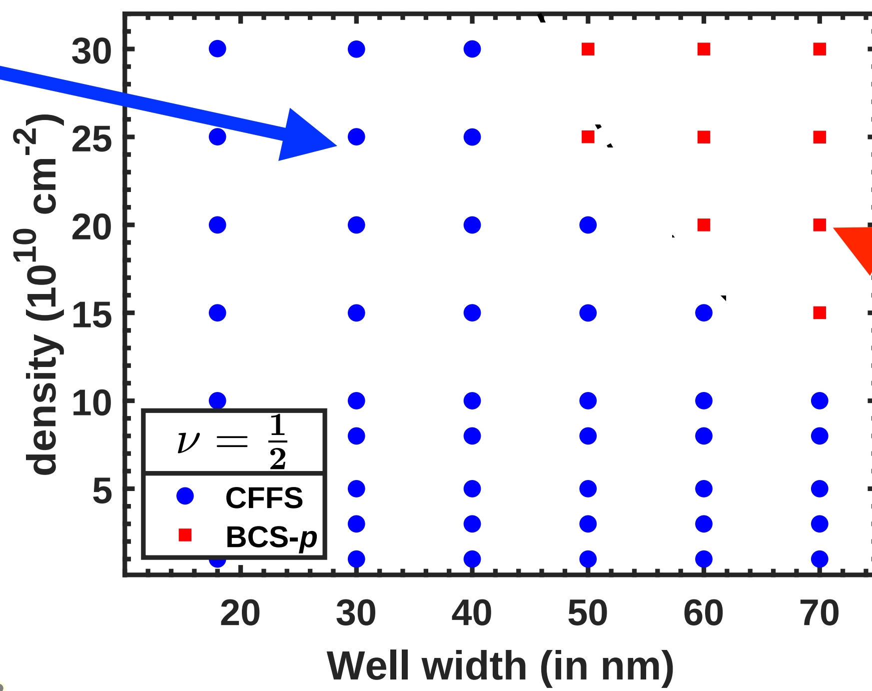
Editors' Suggestion | Featured in Physics

Composite-fermion pairing at half-filled and quarter-filled lowest Landau level

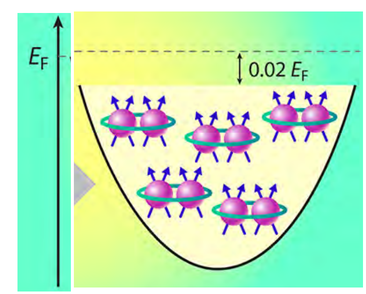
Anirban Sharma,[1] Ajit C. Balram,[2,3] and J. K. Jain[1]

CF metal





CF p-wave pairing



CF pairing at $\nu = 1/2$ in wide quantum wells

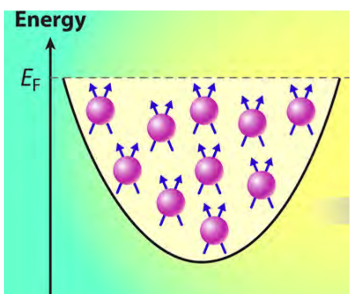
PHYSICAL REVIEW B **109**, 035306 (2024)

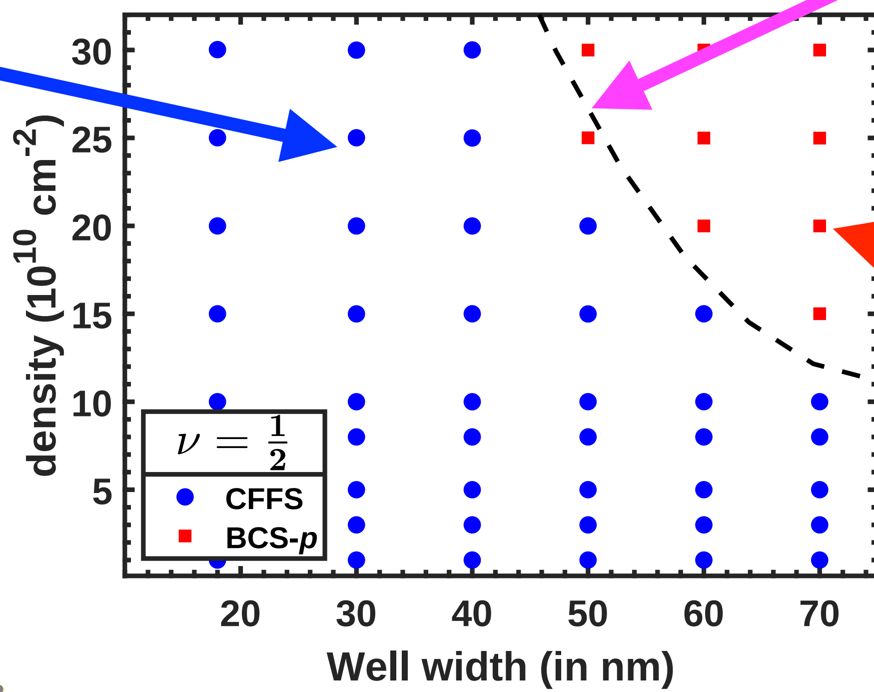
Editors' Suggestion   Featured in Physics

Composite-fermion pairing at half-filled and quarter-filled lowest Landau level

Anirban Sharma,[1] Ajit C. Balram,[2,3] and J. K. Jain[1]

CF metal

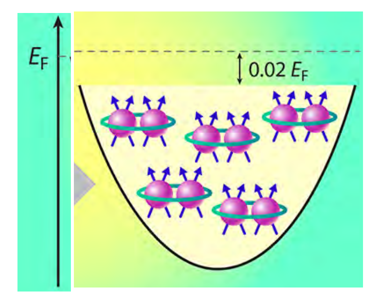




experimental phase boundary

Shabani, Liu Shayegan, et al. Phys. Rev. B (2013)

CF superconductor
p-wave pairing



FQHE at $\nu = 1/4$ in wide quantum wells

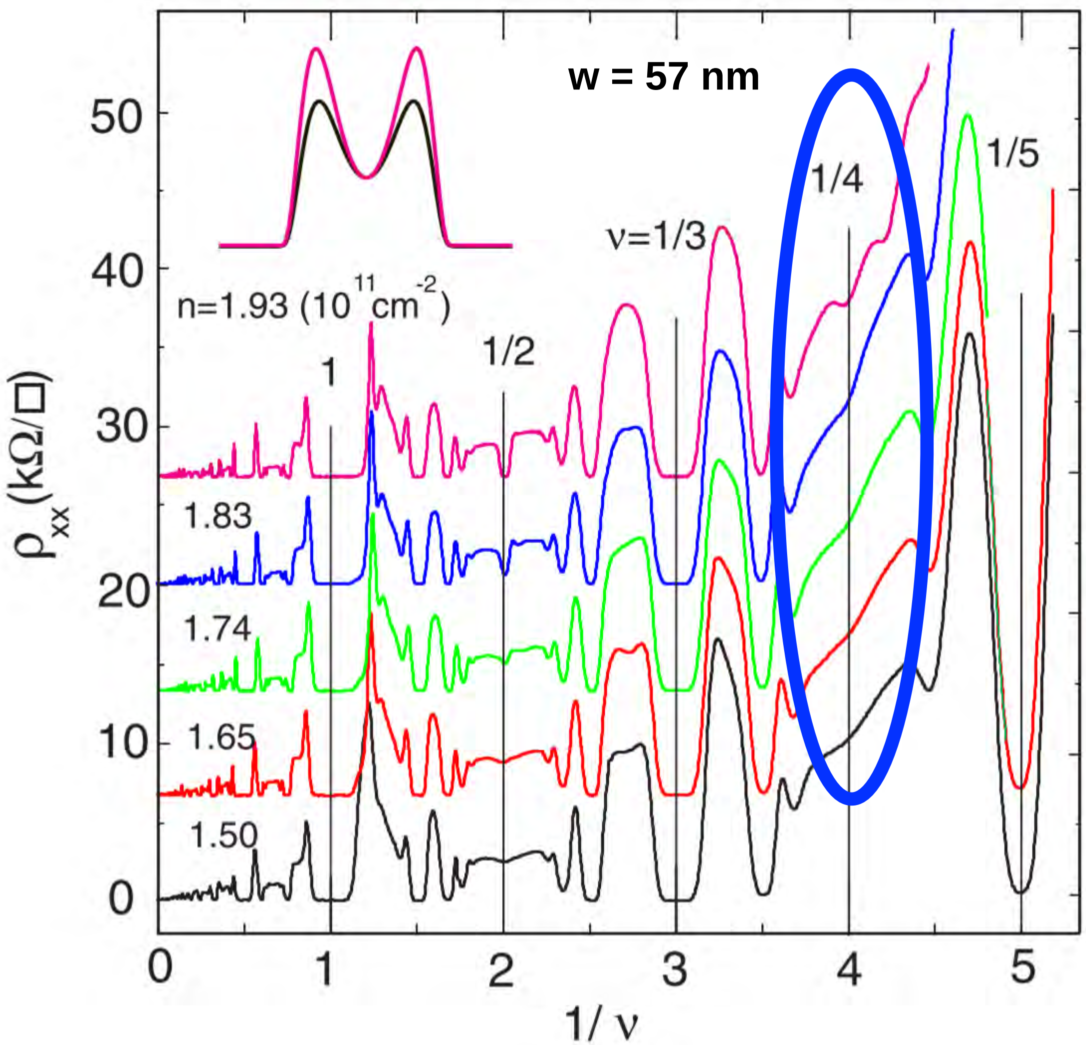
PRL 103, 046805 (2009) PHYSICAL REVIEW LETTERS week ending 24 JULY 2009

Correlated States of Electrons in Wide Quantum Wells at Low Fillings: The Role of Charge Distribution Symmetry

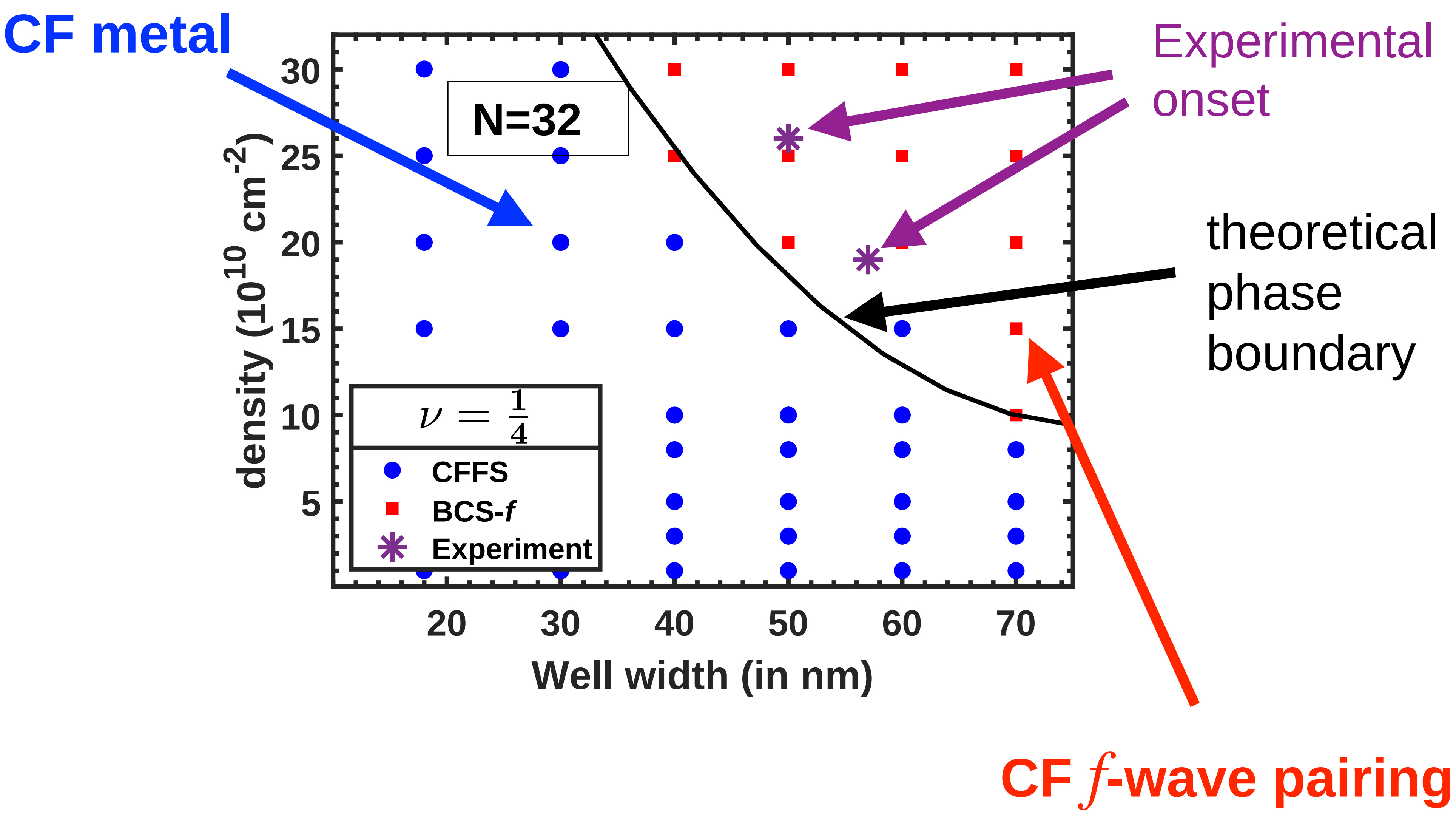
J. Shabani, T. Gokmen, and M. Shayegan

Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA

(Received 24 April 2009; published 22 July 2009)



CF pairing at $\nu = 1/4$ in wide quantum wells

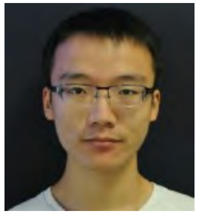


CF pairing at $\nu = 1/4$ induced by LL mixing

PHYSICAL REVIEW LETTERS **130**, 186302 (2023)

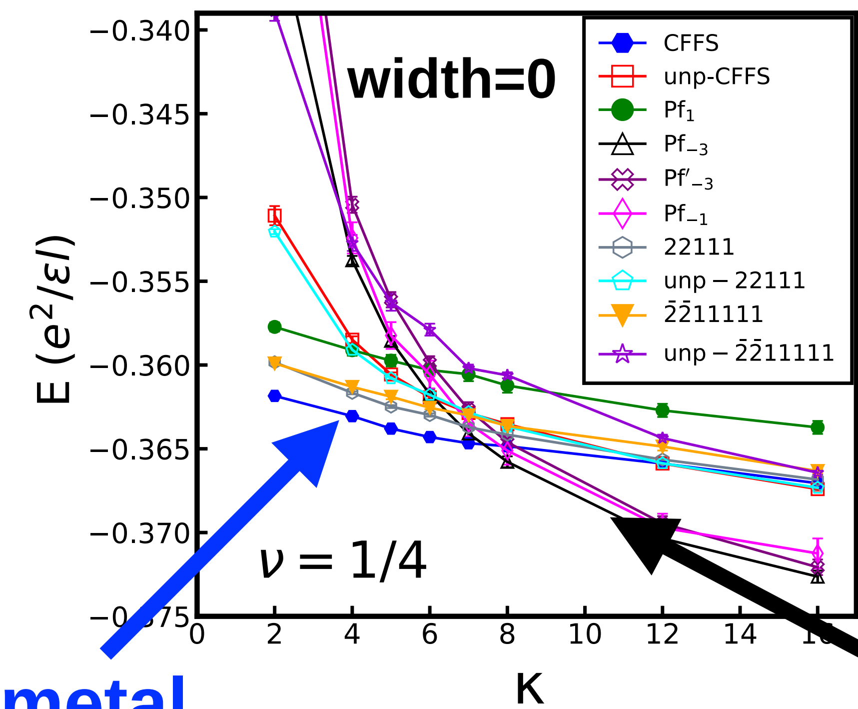
Composite Fermion Pairing Induced by Landau Level Mixing

Tongzhou Zhao,[1] Ajit C. Balram,[2,3] and J. K. Jain[4]



Tongzhou Zhao
IOP-CAS Beijing

(Fixed phase diffusion Monte Carlo)



CF metal

$\kappa = (e^2/\epsilon\ell)/(\hbar\omega_c)$

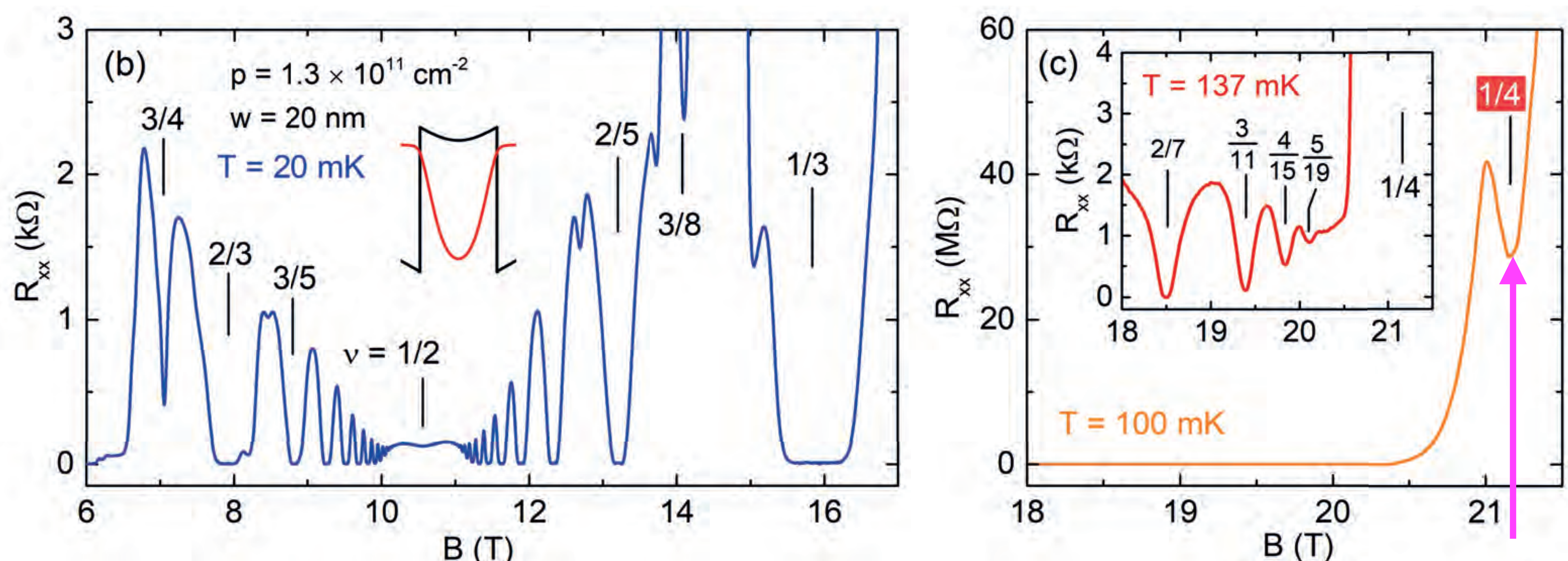
CF pairing
$l = -3$ **wave**

FQHE at $\nu = 1/4$ at large LL mixing

PHYSICAL REVIEW LETTERS **131**, 266502 (2023)

Fractional Quantum Hall State at Filling Factor $\nu = 1/4$ in Ultra-High-Quality GaAs Two-Dimensional Hole Systems

Chengyu Wang,[1] A. Gupta,[1] S. K. Singh,[1] P. T. Madathil,[1] Y. J. Chung,[1] L. N. Pfeiffer,[1]
K. W. Baldwin,[1] R. Winkler,[2] and M. Shayegan[1]



$w = 20$ nm, $\rho = 1.3 \times 10^{11}$ cm^{-2}

$1/4$ **FQHE**

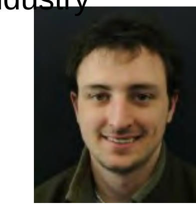
- Evidence for FQHE at $\nu = 1/4$ is seen in high quality hole-type samples with $\kappa = 3 - 6$, riding on an insulating background.

Predicted pairing channel

Anirban Sharma
Industry

- $\nu = 5/2$: p-wave pairing

Moore, Read, Nucl. Phys. (1991);
Read, Green, PRB (2000);
Moller, Simon, PRB (2008);
Sharma, Pu, Jain , PRB (2021)



Bill Faugno
Collage de France

- $\nu = 1/2$ in wide quantum wells: p-wave pairing

Sharma, Balram, Jain, PRB (2024)

- $\nu = 1/4$ in wide quantum wells: f-wave pairing

Faugno, Balram, Barkeshli, Jain, PRL (2019);
Sharma, Balram, Jain, PRB (2024)

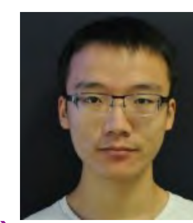
Ajit Balram
IMSc, Chennai

- $\nu = 1/6$ in wide quantum wells: f-wave pairing

Balram, Sharma, Jain, PRB (2025)

- $\nu = 1/4$ with high Landau level mixing: $l = -3$-wave pairing

Zhao, Balram, Jain, PRL (2023)



Tongzhou Zhao
IOP-CAS Beijing

- $\nu = 1/2$ in $N = 3$ graphene Landau level: f-wave pairing

Sharma, Pu, Balram, Jain, PRL (2023)

Songyang Pu
WUSTL

The chiral central charge is given by $c = 1 + l/2$, which can be deduced experimentally from thermal Hall conductance.

Emergence in FQHE

FQHE at
$\nu = 3/8,\ 3/10,\ 3/4$

PNAS RESEARCH ARTICLE | PHYSICS

Next-generation even-denominator fractional quantum Hall states of interacting composite fermions

Chengyu Wang[a,1], Adbhut Gupta[a,1], Pranav T. Madathil[a], Siddharth K. Singh[a], Yoon Jang Chung[a], Loren N. Pfeiffer[a,2], Kirk W. Baldwin[a], and Mansour Shayegan[a,2]

Mukherjee, Mandal, Wojs, Jain, PRL 2012

Emergence in FQHE

FQHE at
$\nu = 3/8, 3/10, 3/4$

PNAS RESEARCH ARTICLE | PHYSICS

Next-generation even-denominator fractional quantum Hall states of interacting composite fermions

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Mukherjee, Mandal, Wojs, Jain 2012