A Fuzzy Sphere Regularization of 3D CFT: Lecture I

Yin-Chen He
(何寅琛)
Perimeter Institute

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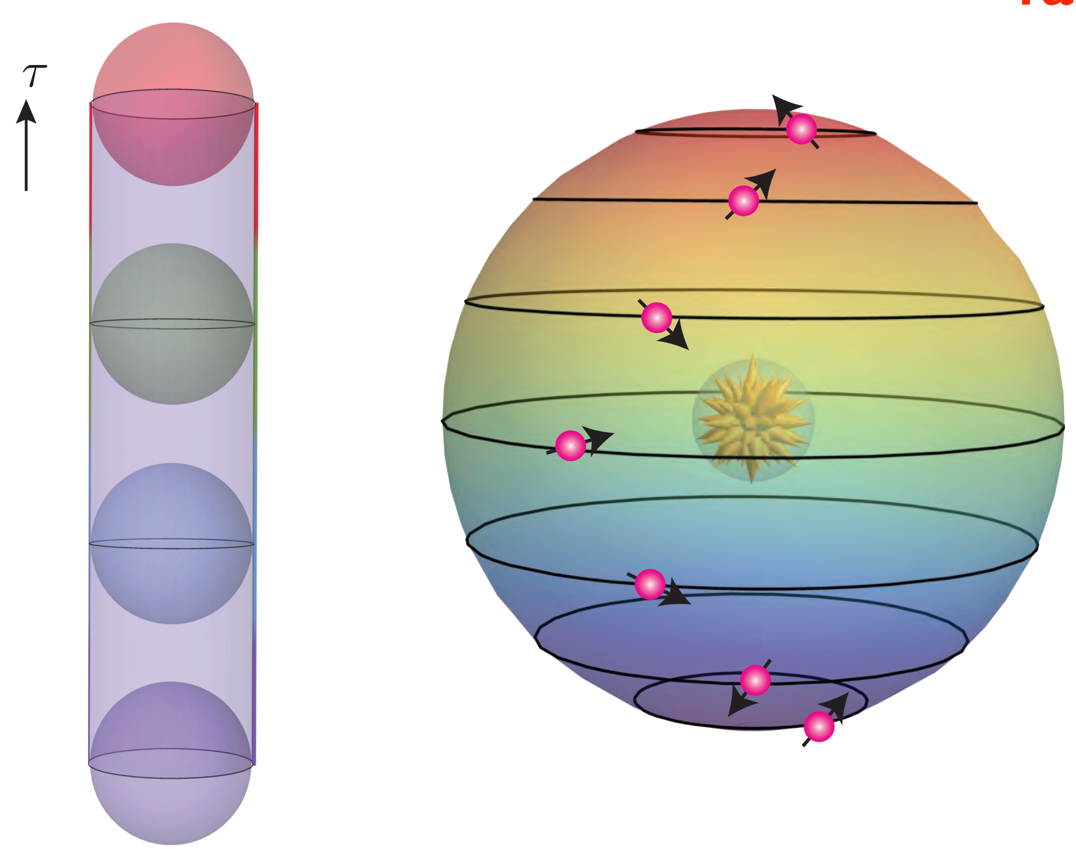
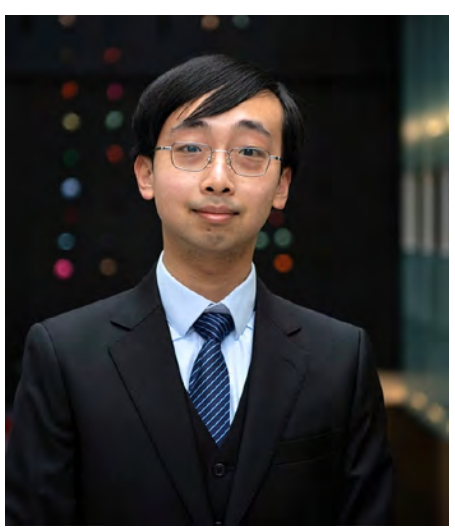


Collaborators

Westlake University: Wei Zhu, Liangdong Hu, Chao Han
Perimeter Institute: Zheng Zhou, Yijian Zou, Davide Gaiotto, Chong Wang
Stony Brook: Zohar Komargodski, Tzu-Chieh Wei, Wenhan Guo
Johannes Hofmann (MPI-PKS), Emilie Huffman (Wake Forrest), Gabriel Cuomo (SISSA)





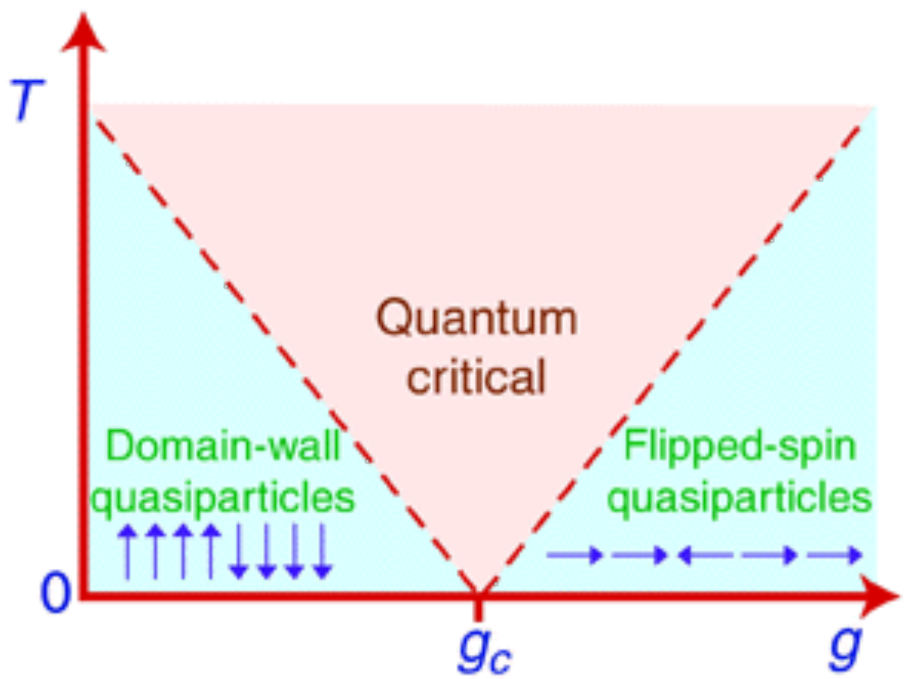




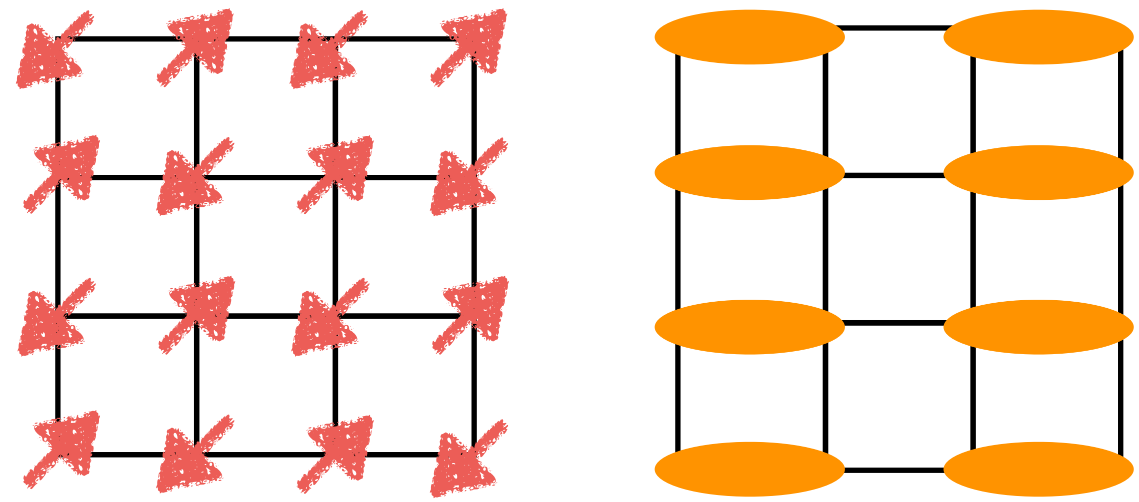

Outline

- Lecture 1: Introduction: motivation and fuzzy sphere regularization
- Lecture 1: 3D Ising CFT
  - State-operator correspondence
  - Operators, correlators, OPEs, F-function
- Lecture 2: Recent progress:
  - Chern-Simons-matter theories: phase transitions between quantum Hall states
  - Fermionic CFT and super-Ising
  - Conformal defect

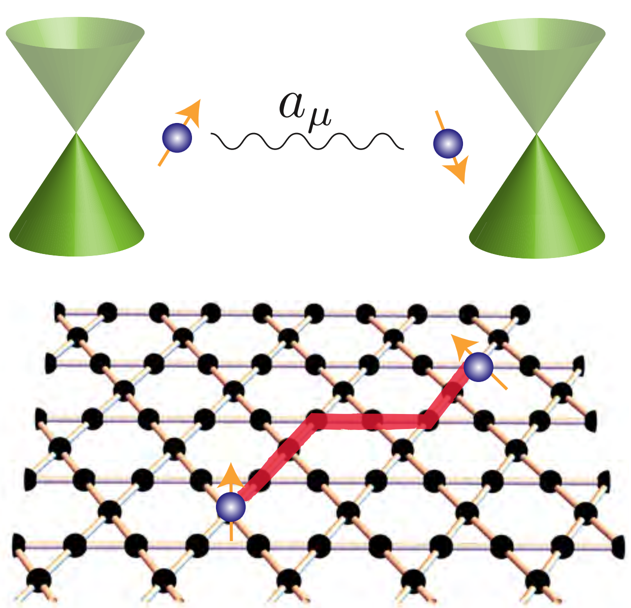
Quantum critical phenomena



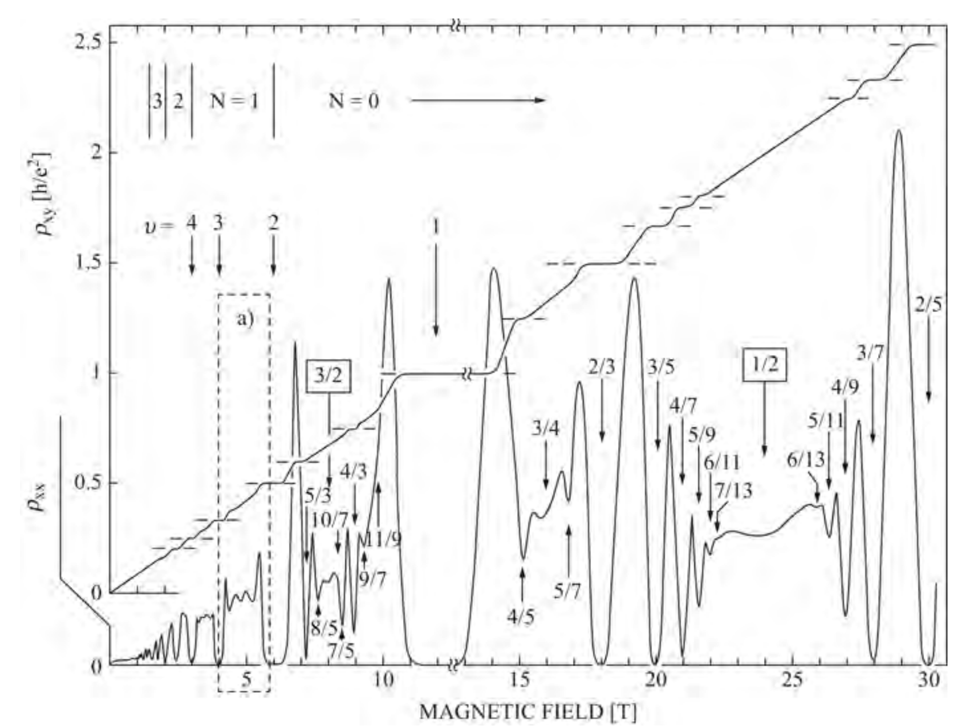
Order-disorder transition



Deconfined phase transition



Gapless spin liquid



Phase transition of topological order

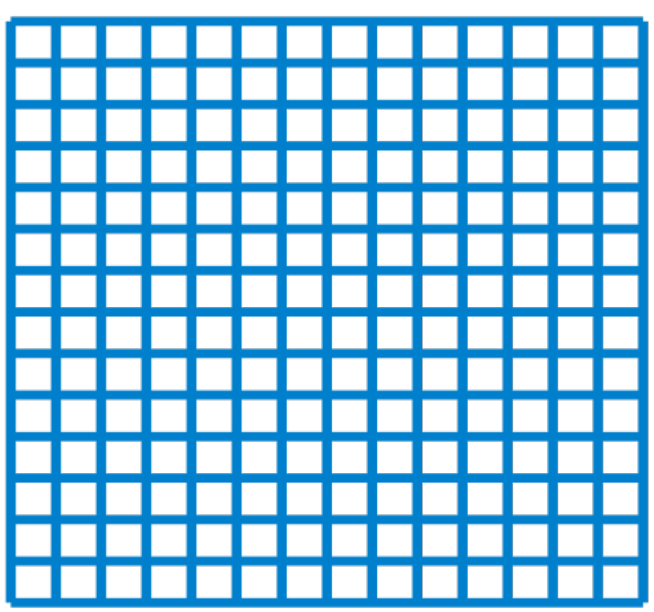
Critical phenomena in modern physics

- The study of critical phenomena gives birth to a number of fundamentally important physics concepts/theories:

  A. Universality.

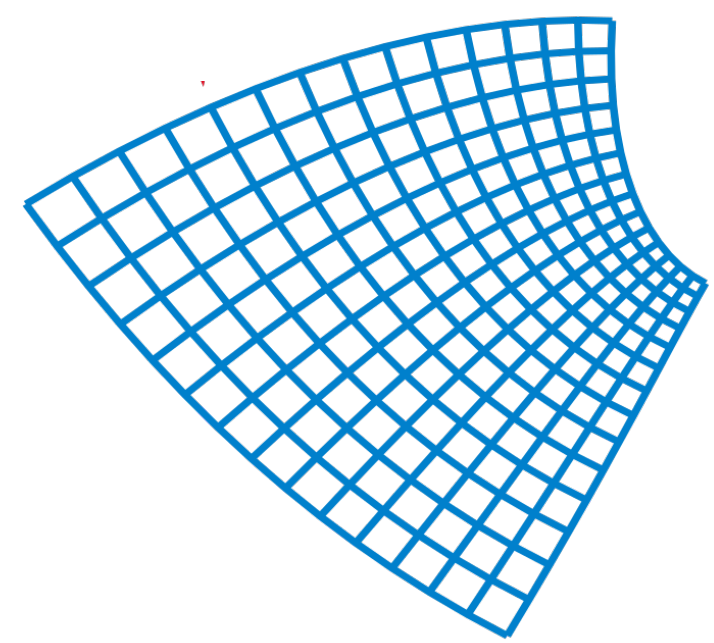
  B. Renormalization group.

  C. Conformal field theory (CFT).



Conformal transformation
(angle preserving transformation)

$$x^\mu \rightarrow \frac{x^\mu - x^2 b^\mu}{1 - 2b \cdot x + b^2 x^2}$$

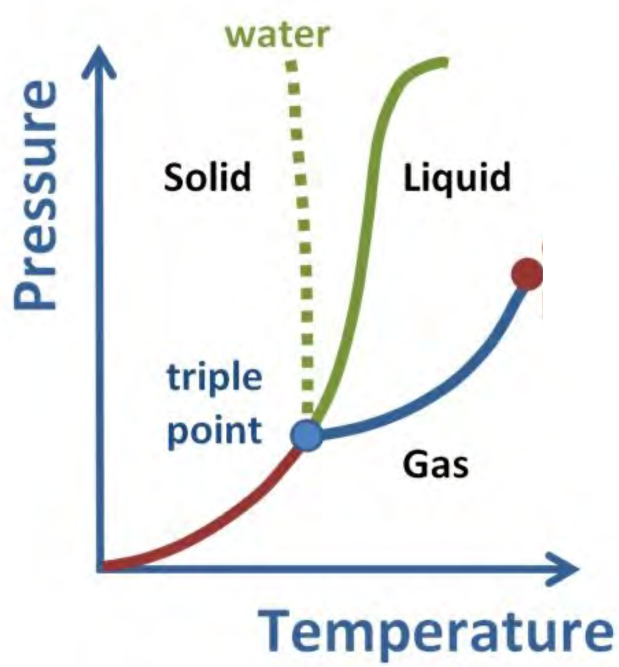


Polyakov 1970: discovered the 2D Ising transition has an emergent conformal symmetry, and conjectured it is also true for the 3D Ising transition.

Conformal field theories (CFTs)

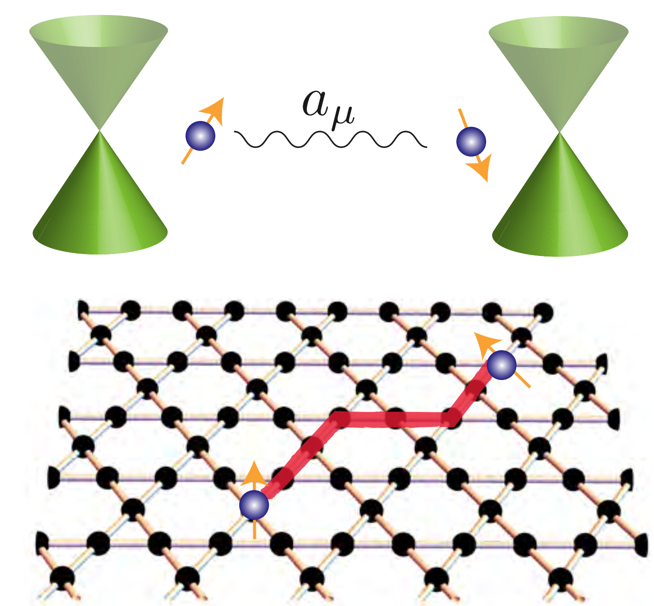
Statistical mechanics

Example:
Ising model,
liquid-gas
transition



Quantum matter

Example:
Quantum criticality,
gapless spin liquid

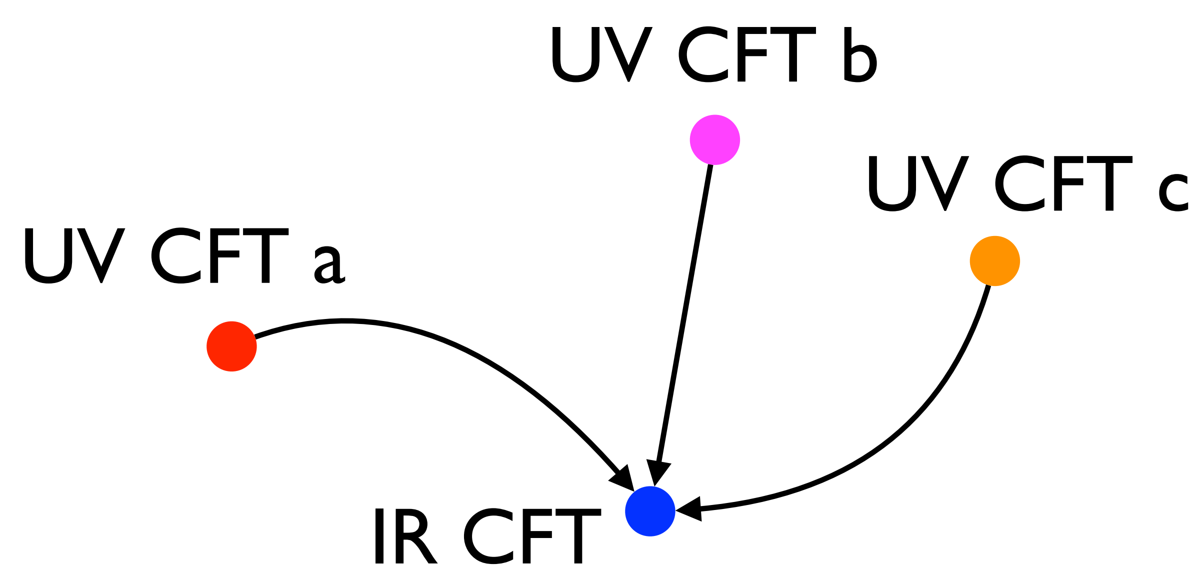


Quantum gravity, String theory

Example:
AdS/CFT

Quantum field theory

RG fixed points of QFTs



2D (1+1D) CFT

Many 2D CFTs are exactly solvable quantum field theories. Belavin, Polyakov & Zamolodchikov (1984)

CFT $\longrightarrow$  $\longrightarrow$ Quantum Hall effect

CFT correlators

- Exact algebraic structures

$$\left\langle \prod_i \psi(z_i) \right\rangle_{\text{Ising}} = \text{Pf}\left(\frac{1}{z_i - z_j}\right)$$

$$\left\langle \prod_i e^{i\sqrt{m}\,\phi(z_i)} \right\rangle_{U(1)} = \prod_{i<j}(z_i - z_j)^m$$

- QH Wavefunctions

$$\text{Pf}\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^m$$

Laughlin 1983; Moore & Read 1991

- Universal CFT data

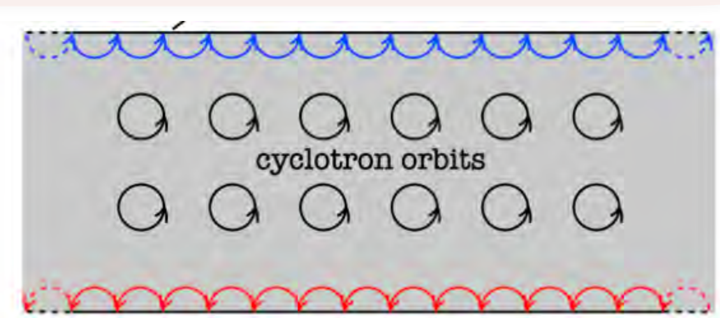
Scaling dimensions, central charge, etc.

- Gapless chiral edges

Wen 1990

Shift: scaling dimension of electron operator.

Read 2009



3D CFTs: a central open challenge

- Central to critical phenomena in statistical mechanics, quantum matter.
- Strongly interacting, no exact solutions (yet) — only a limited set of complementary methods, each probing a corner of the CFT landscape:

1. Perturbative RG Wilson, Fisher 1972
2. Lattice simulations (mostly Monte Carlo).
3. Conformal bootstrap Polyakov 1974; Rattazzi, Rychkov, Tonni, Vichi 2008

What is missing: a versatile nonperturbative framework for 3D CFT.

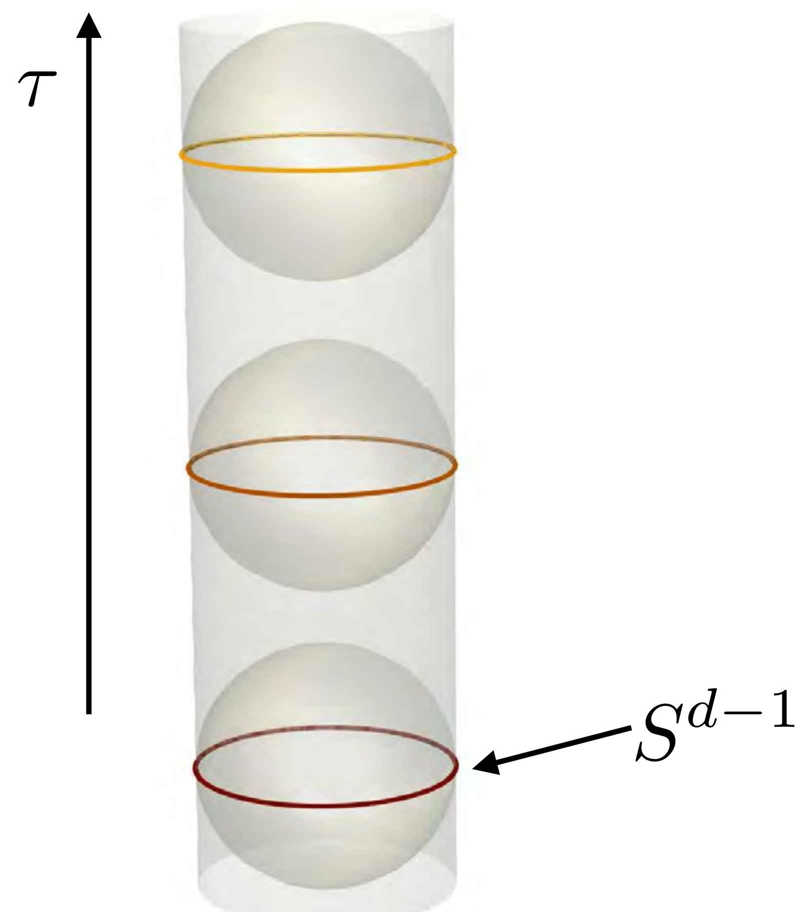
In this lecture

Quantum Hall $\longrightarrow$  $\longrightarrow$ 3D CFT

The gift returns —
in an unexpected way.

State-operator correspondence

Radial quantization
of d-dimensional CFT
$S^{d-1} \times \mathbb{R}$



Quantum Hamiltonian on S^{d-1}

Eigenstates of the quantum Hamiltonian.

One-to-one correspondence

$$\delta E_n = E_n - E_0 = \frac{v}{R}\Delta_n$$

Scaling operators in the CFT.

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle = \frac{\delta_{ij}}{|x_1 - x_2|^{2\Delta}}$$

State-operator correspondence

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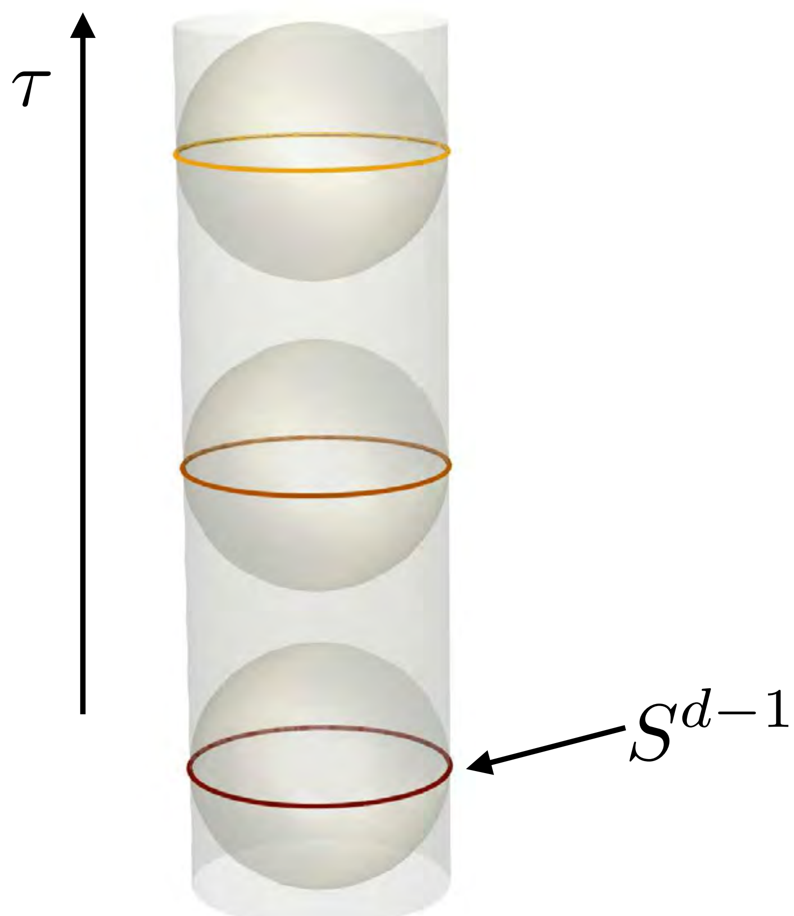

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Primaries and descendants

Conformal multiplet

$$\mathcal{O} \longrightarrow \partial_{\mu_1}\mathcal{O} \longrightarrow \partial_{\mu_1}\partial_{\mu_2}\mathcal{O} \quad \cdots$$

$$\Delta \qquad \Delta + 1 \qquad \Delta + 2 \qquad \cdots$$

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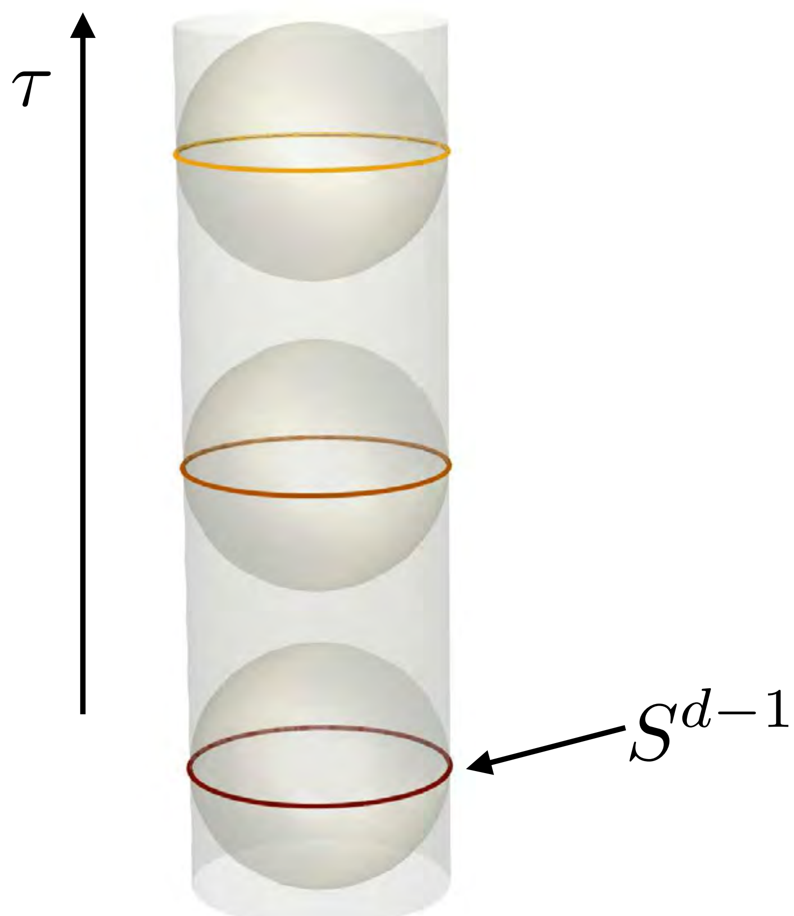

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$$\Delta \qquad \Delta + 1 \qquad \Delta + 2 \quad \cdots$$

There are infinite number of primary operators in any 3D CFT!

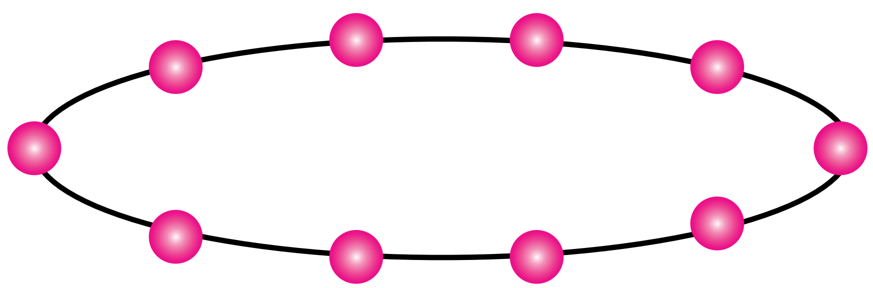
3D Ising	$\Delta_\sigma \approx 0.5184189(10)$	$\Delta_\epsilon \approx 1.412625(10)$	$\Delta_{\epsilon'} \approx 3.82968(23)$
	$\eta = 2\Delta_\sigma - 1$	$\nu = 1/(3 - \Delta_\epsilon)$	$\omega = \Delta_{\epsilon'} - 3$

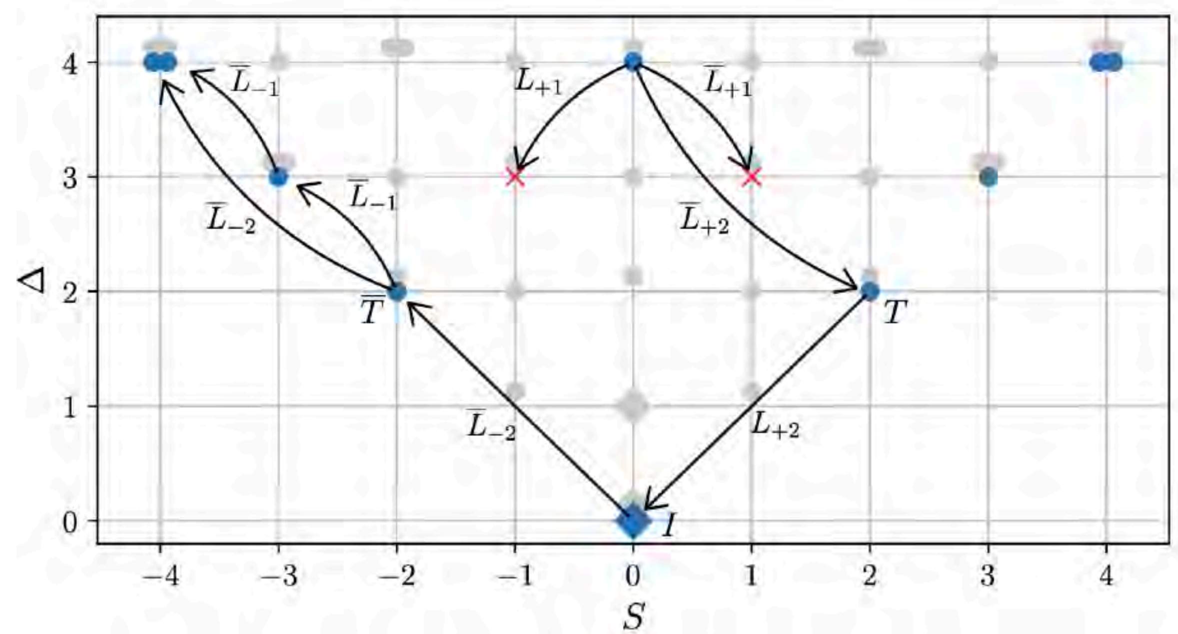
Radial quantization on a lattice

2D CFT: We can just study a quantum Hamiltonian on a circle.

Most conformal data can be extracted.

Cardy

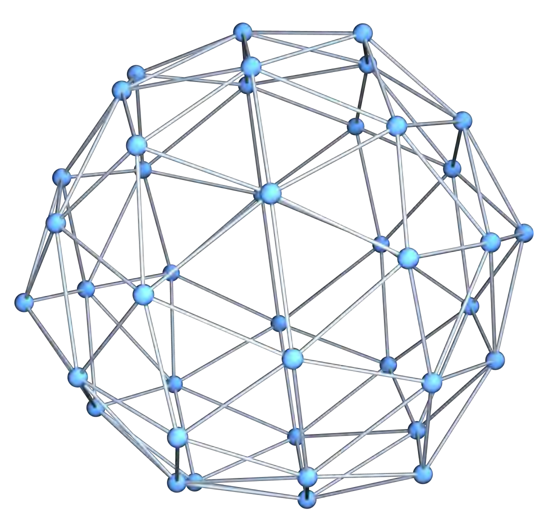


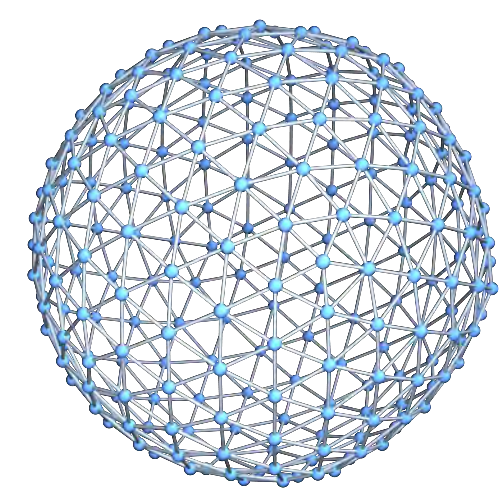


Milsted and Vidal 2017

3D CFT: We need to put a quantum Hamiltonian on a two-sphere.

But a regular lattice won't fit since two-sphere has a curvature…

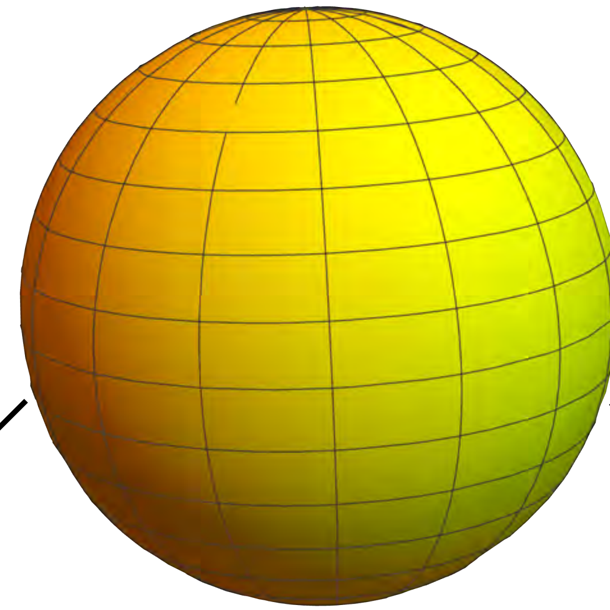




Efforts along this line: Brower…

Our recipe: make it fuzzy

Sphere is a curved space.



Discretize

Fuzzify

Lowest Landau level projection

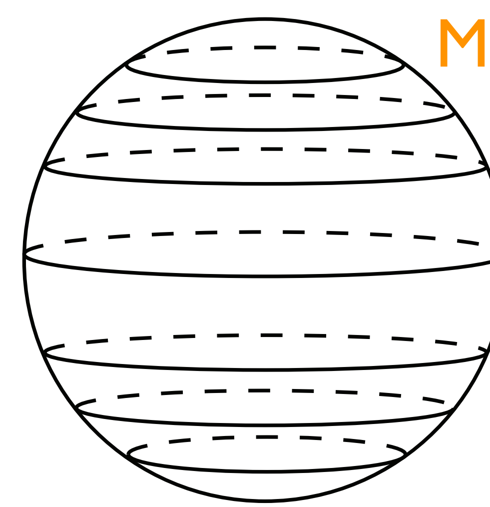
Haldane 1983

Spherical tiling

Spherical rotation is broken badly.

fuzzy (non-commutative) sphere

Madore 1992

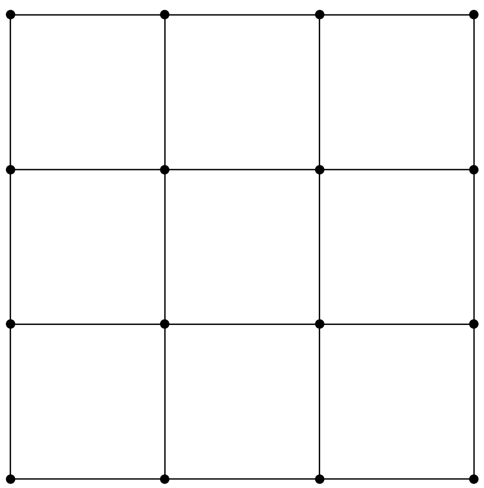


Spherical rotation is kept exactly.

Fuzzy sphere regularization of 3D CFTs

Quantum mechanical model realizations of 2+1D CFTs.

Lattice model



Spin-1/2 on each site $\begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}$ $\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

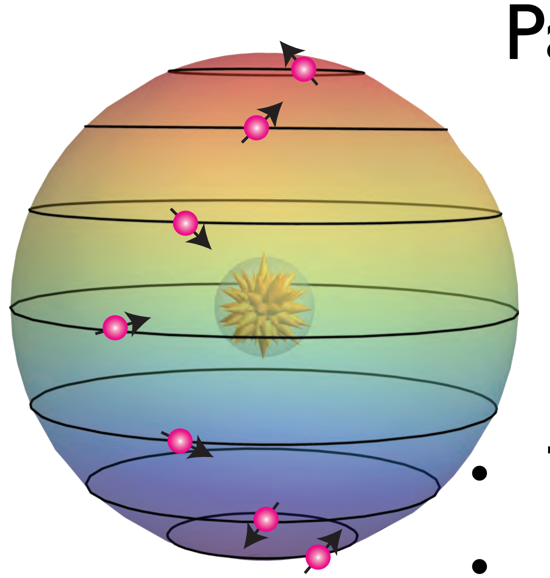
$$H = -\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

Spins point to +z or -z   Spins point to +x

h

2+1D Ising CFT

Fuzzy sphere model



Particles moving on sphere in the presence of a monopole.

$$H = \frac{1}{2m} \sum_{i=1}^{N_e} (\vec{p}_i + \vec{A}(\vec{x}_i))^2 + \sum_{i,j=1}^{N_e} U(\vec{x}_i - \vec{x}_j) + \cdots$$

Interaction term

- The model is local if interactions are local.
- 2+1D CFTs can be realized by tuning the interaction form.

This idea was firstly pursued on the torus. Ippoliti, Mong, Assaad, Zaletel (2018)

Even 4 particles work!!!

Zhu, Han, Huffman, Hofmann, YCH, arXiv:2210.13482 (PRX)

Gaps of ALL the excited states of the system with N=4 "spin".

	CB	4 spins	Errors
σ	0.518	0.530	2.3%
$\partial_{\mu_1}\sigma$	1.518	1.522	0.3%
$\Box\sigma$	2.518	2.427	3.6%
$\partial_{\mu_1}\partial_{\mu_2}\sigma$	2.518	2.428	3.6%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\sigma$	3.518	2.847	20%
$\partial_{\mu_1}\Box\sigma$	3.518	3.291	6.5%
$\sigma_{\mu_1\mu_2}$	4.180	4.241	1.5%
$\sigma_{\mu_1\mu_2\mu_3}$	4.638	4.618	0.4%

	CB	4 spins	Errors
ϵ	1.413	1.382	2.2%
$\partial_{\mu_1}\epsilon$	2.413	2.337	3.1%
$T_{\mu_1\mu_2}$	3	3	NA
$\partial_{\mu_1}\partial_{\mu_2}\epsilon$	3.413	3.126	8.4%
$\Box\epsilon$	3.413	3.577	4.8%
$\partial_{\mu_3}T_{\mu_1\mu_2}$	4	3.663	8.4%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho T_{\mu_1\mu_2}$	4	4.054	1.4%
ϵ'	3.830	4.019	4.9%
$\partial_{\mu_3}\partial_{\mu_4}T_{\mu_1\mu_2}$	5	4.856	2.9%

6 primaries and 11 descendants in the fuzzy sphere model with 4 "spins".

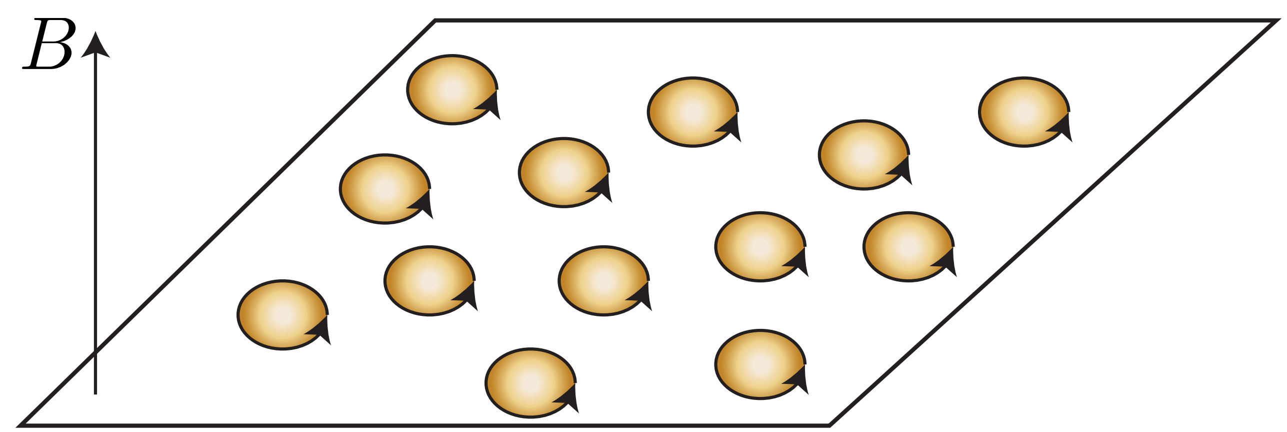
Landau level and non-commutative geometry

Particles moving in a strong magnetic field leads to non-commutative geometry.

Read 1998; Susskind 2001; Polychronakos 2001; Haldane 2011; Dong & Senthil 2020; Du, Mehta & Son 2021…

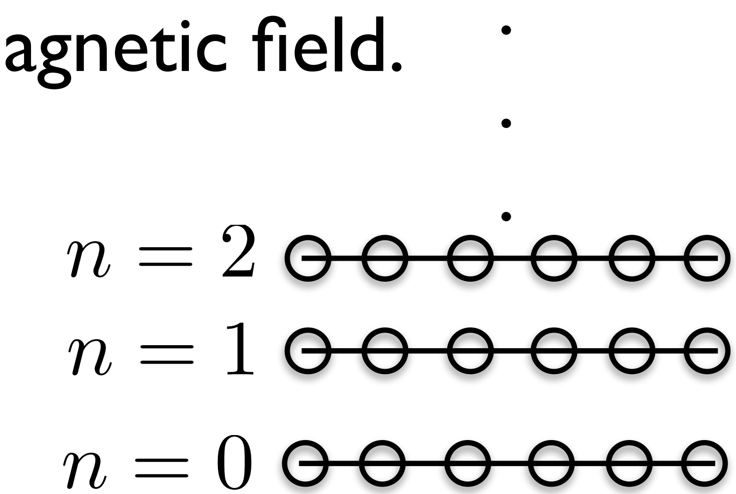
$$\mathcal{L} = \frac{M}{2}\dot{\vec{x}}^2 - \dot{\vec{x}} \cdot \vec{A}$$

$$A_i = -\frac{B}{2}\epsilon_{ij}x^j$$



Landau level: single particle states in the presence of magnetic field.

- Quantized energy: $E_n = \dfrac{B}{M}(n + 1/2)$
- Complete flat.
- Massive degeneracy at each level: $\dfrac{B\mathcal{A}}{2\pi}$



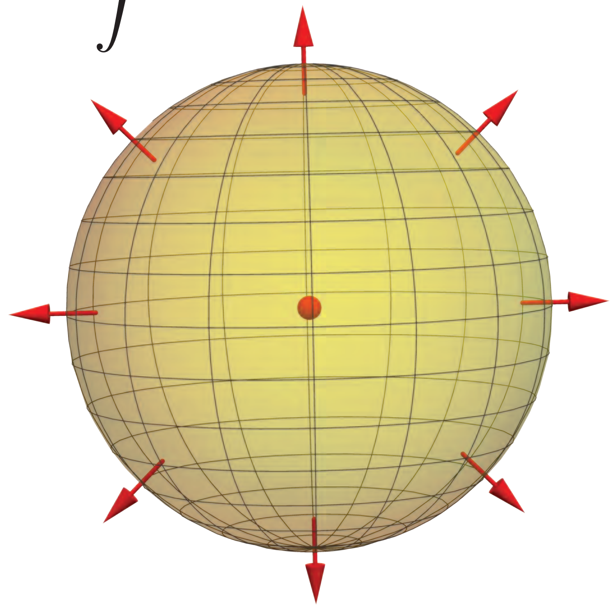
Restrict/Project to the lowest Landau level:

$$\mathcal{L}_0 = -\dot{\vec{x}} \cdot \vec{A} = \frac{B}{2}\epsilon_{ij}\dot{x}^i x^j \Rightarrow [x^i, x^j] = \frac{i}{B}\epsilon^{ij}$$

Spherical Landau levels

$$\int \vec{B} \cdot d\vec{r} = 4\pi \cdot s$$

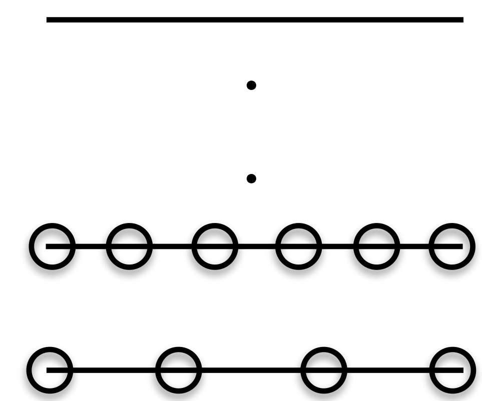
Electrons moving under a magnetic monopole.



Single particle kinetic term

$$\frac{1}{2Mr^2}(\partial_\mu + iA_\mu)^2$$

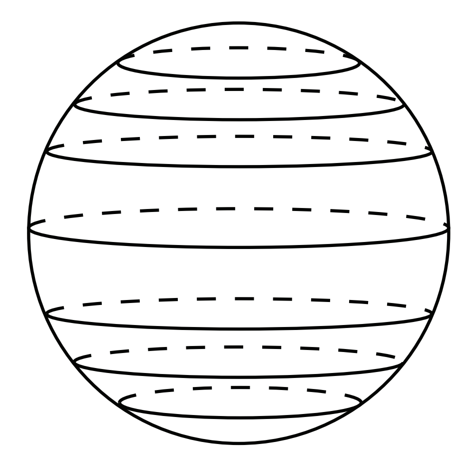
Landau levels (LLs)



- Each LL has a level dependent quantized energy $\dfrac{n(n+1) + (2n+1)s}{2Mr^2}$
- The states (orbitals) in each LL form a spin-(n+s) SO(3) representation.
- The wavefunctions of each LL are monopole Harmonics $Y^{(s)}_{n+s,m}(\theta, \varphi)$.

Lowest LL wavefunction $\quad m = -s, -s+1, \cdots, s$

$$Y^{(s)}_{s,m}(\theta, \varphi) = \mathcal{N}_{s,m} e^{im\varphi} \cos^{s+m}\left(\frac{\theta}{2}\right) \sin^{s-m}\left(\frac{\theta}{2}\right)$$



Lowest Landau level (LLL) projection

$$H = \int d\vec{x} \left(\psi_\sigma^\dagger(\vec{x}) \frac{(\partial_\mu + iA_\mu)^2}{2M} \psi_\sigma(\vec{x}) + \mathcal{H}_{int}(\vec{x}) \right)$$

E.g. $H_{int} = U \int d\vec{x} \, (\psi^\dagger(\vec{x}) \partial_\mu \psi(\vec{x}))^2$

$$\{\psi^\dagger(\vec{x}_a), \psi(\vec{x}_b)\} = \delta(\vec{x}_{ab})$$
$$\{\psi(\vec{x}_a), \psi(\vec{x}_b)\} = 0$$

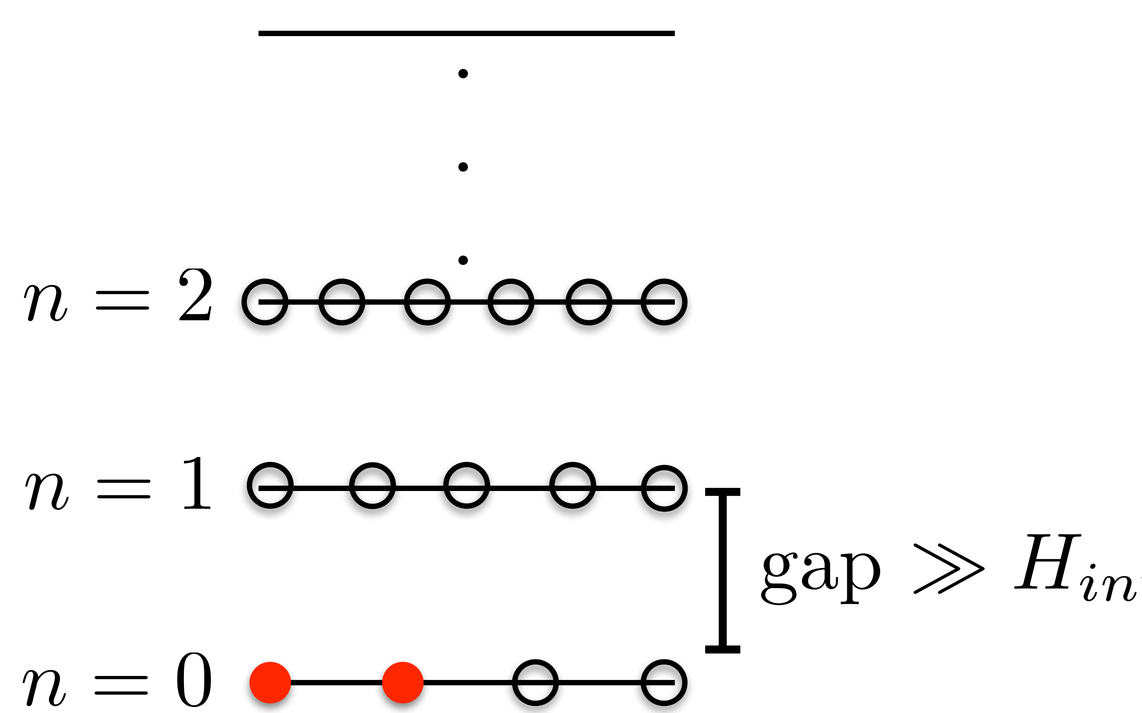
$$\psi^\dagger(\vec{x}) = \sum_{l=s}^{\infty} \sum_{m=-s}^{s} c^\dagger_{l,m} Y^{(s)}_{l,m}(\vec{x})$$

$$\{c^\dagger_{l_a,m_a}, c_{l_b,m_b}\} = \delta_{l_a,l_b} \delta_{m_a,m_b}$$
$$\{c_{l_a,m_a}, c_{l_b,m_b}\} = 0$$

LLL projection $\big\downarrow$ Haldane 1983

$$\psi^\dagger(\vec{x}) = \sum_{m=-s}^{s} c^\dagger_{s,m} Y^{(s)}_{s,m}(\vec{x})$$

Landau levels



LLL projection and fuzzy sphere

On the LLL the sphere coordinates $x_1^2 + x_2^2 + x_3^2 = 1$ become:

$$(X_i)_{m_1,m_2} = \int d\vec{x}\, x_i \bar{Y}^{(s)}_{s,m_1}(\vec{x}) Y^{(s)}_{s,m_2}(\vec{x})$$

$$[X_i, X_j] = \frac{1}{s+1} i\varepsilon_{ijk} X_k \qquad \sum_{i=1}^{3} X_i X_i = \frac{s}{s+1} \mathbf{1}_{2s+1}$$

Fuzzy two-sphere: $[\hat{x}_i, \hat{x}_j] = i\varepsilon_{ijk}\hat{x}_k, \quad \sum_{i=1}^{3} \hat{x}_i \hat{x}_i = \text{const} \cdot \mathbf{1}$

Madore 1992

$$Y^{(s)}_{s,m}(\theta, \varphi) = \mathcal{N}_{s,m} e^{im\varphi} \cos^{s+m}\left(\frac{\theta}{2}\right) \sin^{s-m}\left(\frac{\theta}{2}\right)$$

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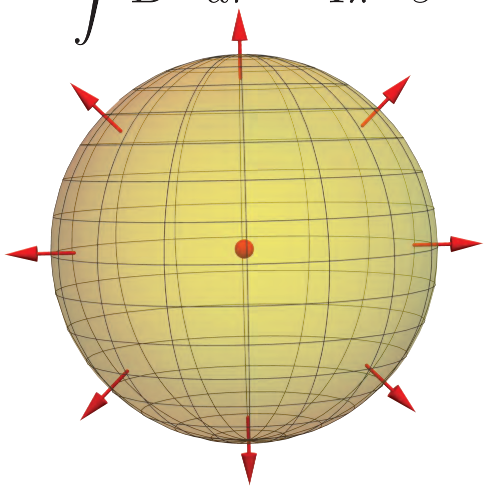
Fuzzy sphere model for the 2+1D Ising CFT

Zhu, Han, Huffman, Hofmann, YCH, arXiv:2210.13482 (PRX)

$\int \vec{B} \cdot d\vec{r} = 4\pi \cdot s$



Non-relativistic fermions with an isospin.

$$n^{\alpha}(\vec{x}) = (\hat{\psi}^{\dagger}_{\uparrow}(\vec{x}),\, \hat{\psi}^{\dagger}_{\downarrow}(\vec{x}))\, \sigma^{\alpha} \begin{pmatrix} \hat{\psi}_{\uparrow}(\vec{x}) \\ \hat{\psi}_{\downarrow}(\vec{x}) \end{pmatrix}$$

$$H = \int d\vec{x} \left(\psi^{\dagger}_{\sigma}(\vec{x}) \frac{(\partial_{\mu} + iA_{\mu})^2}{2M} \psi_{\sigma}(\vec{x}) + \mathcal{H}_{int}(\vec{x}) \right)$$

$$\mathcal{H}_{int}(x) = n^0(\vec{x}) n^0(\vec{x}) - U n^z(\vec{x}) \nabla^2 n^z(\vec{x}) - h\, n^x(\vec{x})$$

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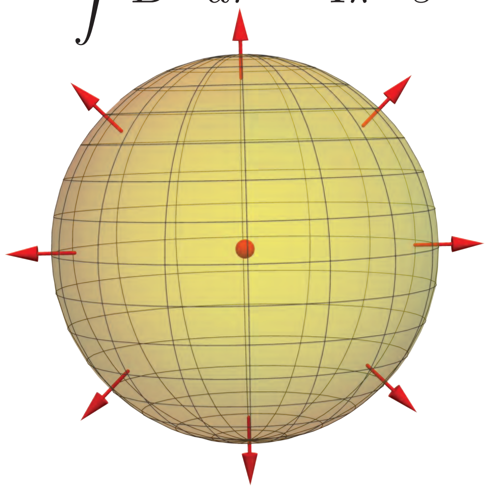

Non-relativistic fermions with an isospin.

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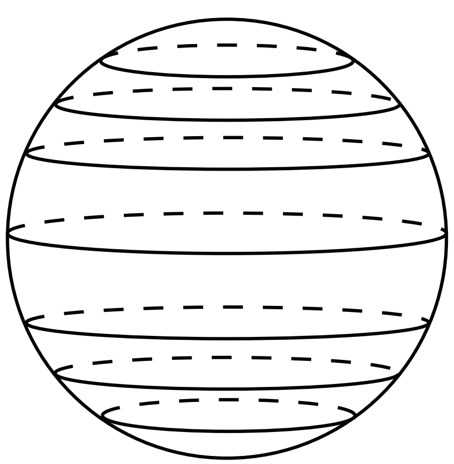
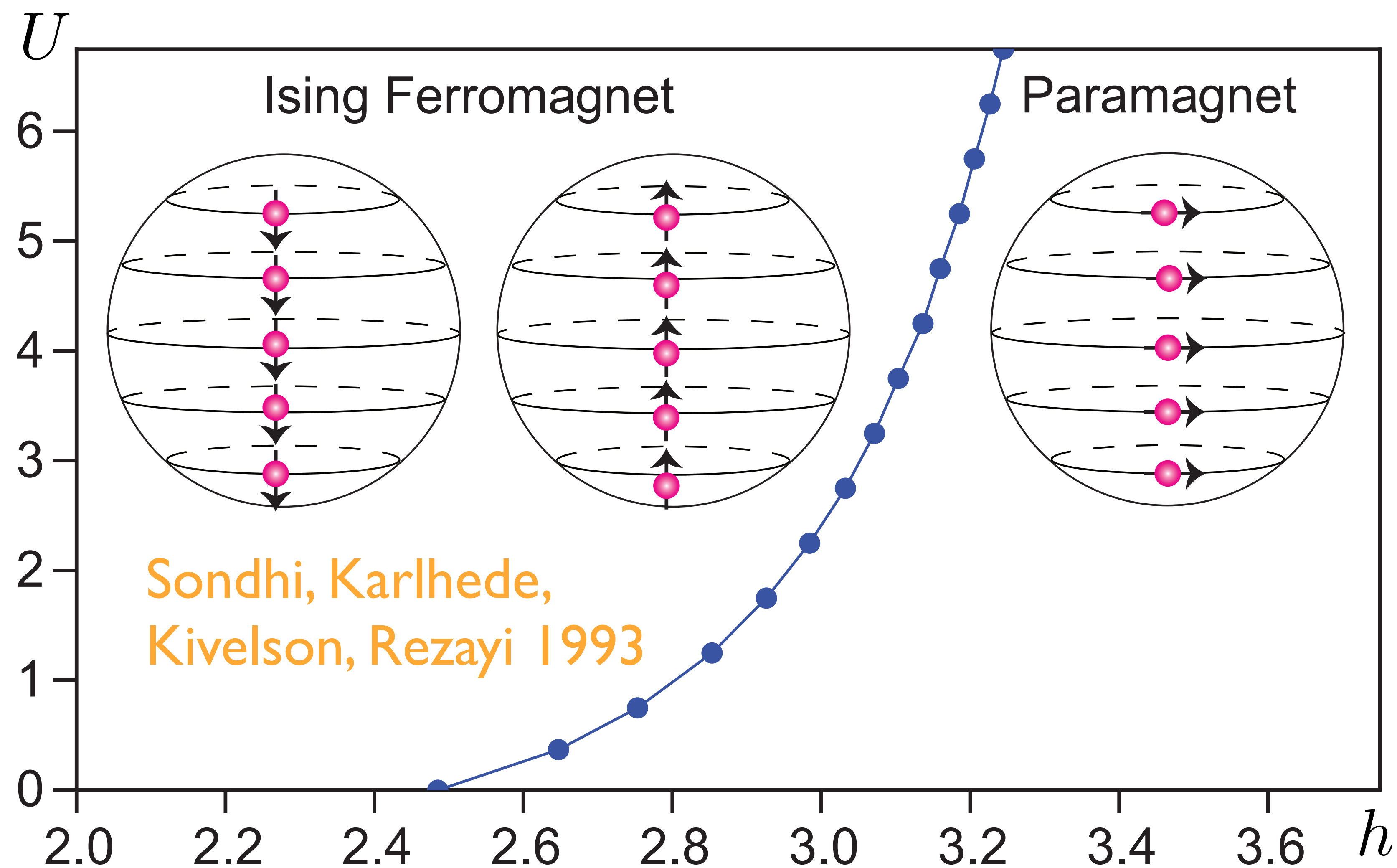
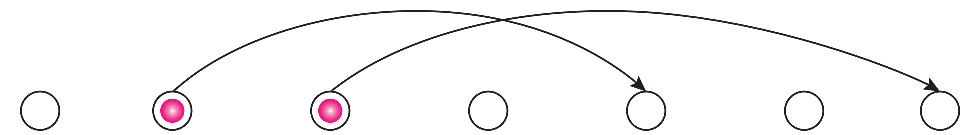
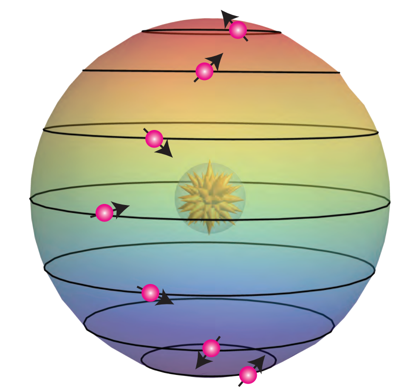




A closer look at the fuzzy sphere model

$2s+1$-site fermionic model





Many-body Hilbert space $\prod_{i=1}^{2s+1} c^\dagger_{m_i,\alpha_i}|0\rangle$

$m_i = -s, -s+1, \cdots, s$
spin-s rep of $SO(3)$

$\alpha_i = \uparrow, \downarrow$

Continuum limit: $s \to \infty$

Hamiltonian for the 2+1D Ising model $\mathbf{c}^\dagger_m = (c^\dagger_{m,\uparrow}, c^\dagger_{m,\downarrow})$

$$H = -\sum_{m_{1,2},m=-s}^{s} V_{m_1,m_2,m_2-m,m_1+m} \left(\mathbf{c}^\dagger_{m_1}\sigma^z\mathbf{c}_{m_1+m}\right)\left(\mathbf{c}^\dagger_{m_2}\sigma^z\mathbf{c}_{m_2-m}\right) - h\sum_{m=-s}^{s}\mathbf{c}^\dagger_m\sigma^x\mathbf{c}_m$$

$$V_{m_1,m_2,m_3,m_3} = \sum_l V_l\,(4s-2l+1)\begin{pmatrix} s & s & 2s-l \\ m_1 & m_2 & -m_1-m_2\end{pmatrix}\begin{pmatrix} s & s & 2s-l \\ m_3 & m_4 & -m_3-m_4\end{pmatrix}$$

Haldane

$$\delta(x_i - x_j) \to V_0 \qquad \nabla^2\delta(x_i-x_j) \to V_1 - V_0$$

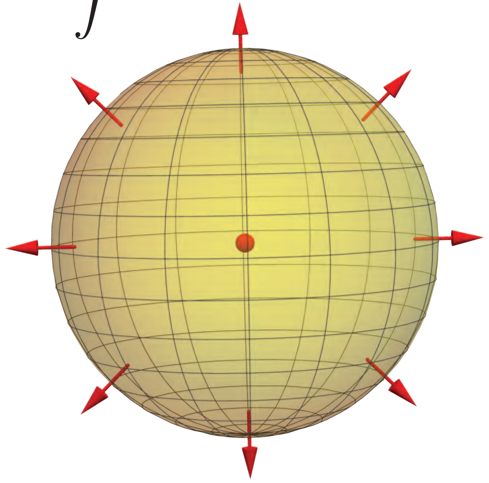
Outline

- Lecture 1: Introduction: motivation and fuzzy sphere regularization
- Lecture 1: 3D Ising CFT
    - State-operator correspondence
    - Operators, correlators, OPEs, F-function
- Lecture 2: Recent progress:
    - Chern-Simons-matter theories: phase transitions between quantum Hall states
    - Conformal defect
    - Fermionic CFT and super-Ising

Fuzzy sphere model for the 2+1D Ising CFT

Zhu, Han, Huffman, Hofmann, YCH, arXiv:2210.13482 (PRX)

$\int \vec{B}\cdot d\vec{r} = 4\pi \cdot s$



Non-relativistic fermions with an isospin.

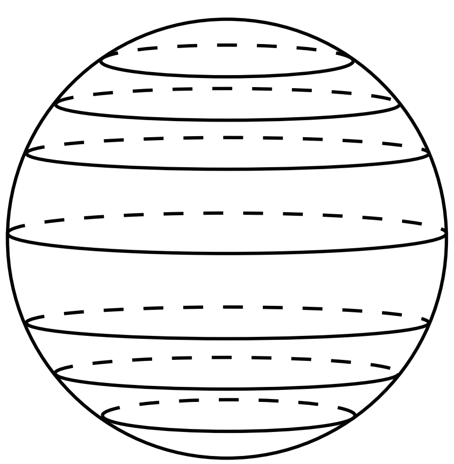
$$n^\alpha(\vec{x}) = (\hat{\psi}^\dagger_\uparrow(\vec{x}),\, \hat{\psi}^\dagger_\downarrow(\vec{x}))\,\sigma^\alpha \begin{pmatrix}\hat{\psi}_\uparrow(\vec{x})\\ \hat{\psi}_\downarrow(\vec{x})\end{pmatrix}$$

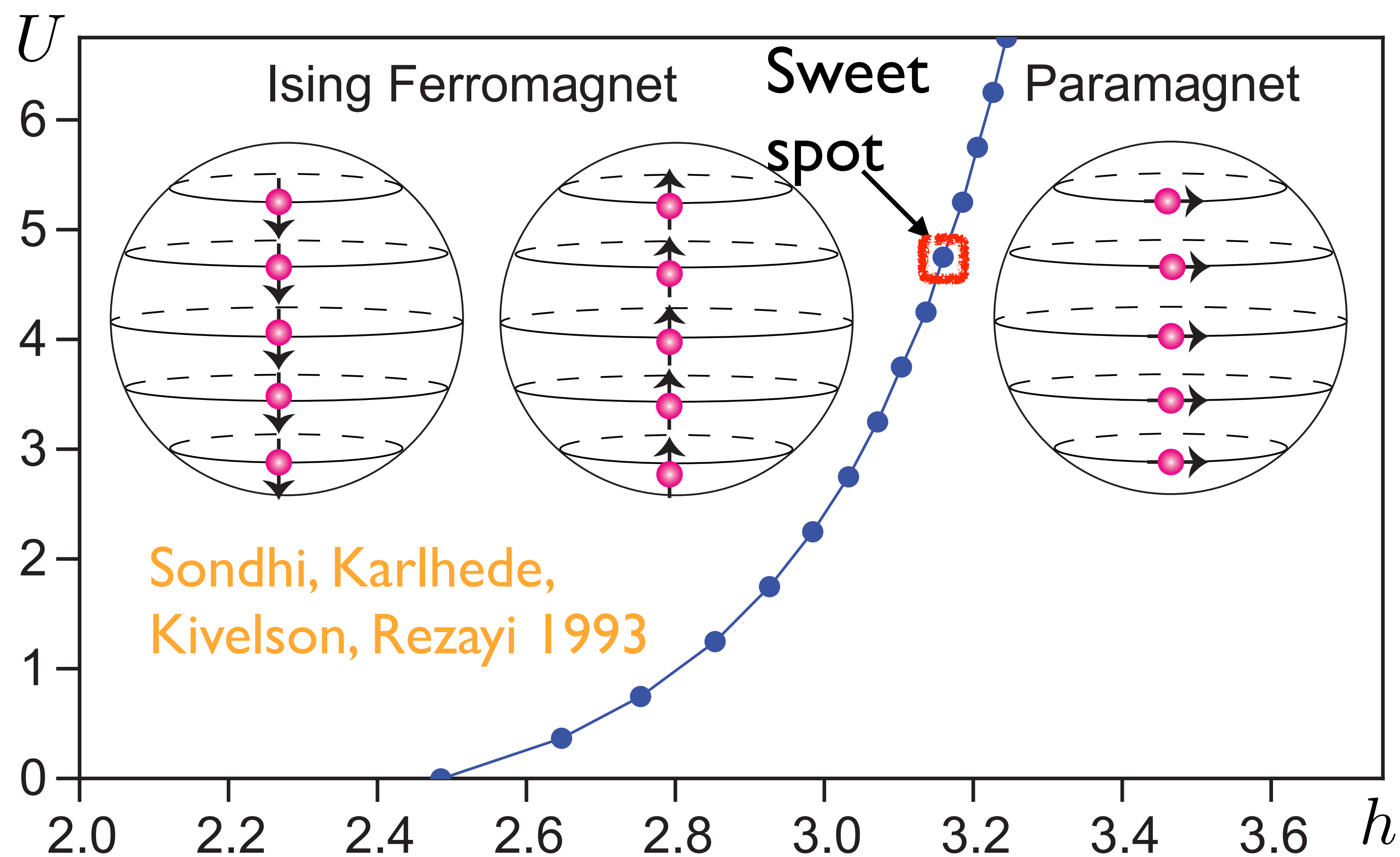
$$H = \int d\vec{x}\left(\psi^\dagger_\sigma(\vec{x})\frac{(\partial_\mu + iA_\mu)^2}{2M}\psi_\sigma(\vec{x}) + \mathcal{H}_{int}(\vec{x})\right)$$

$$\mathcal{H}_{int}(x) = n^0(\vec{x})n^0(\vec{x}) - U n^z(\vec{x})\nabla^2 n^z(\vec{x}) - h\, n^x(\vec{x})$$

LLL projection

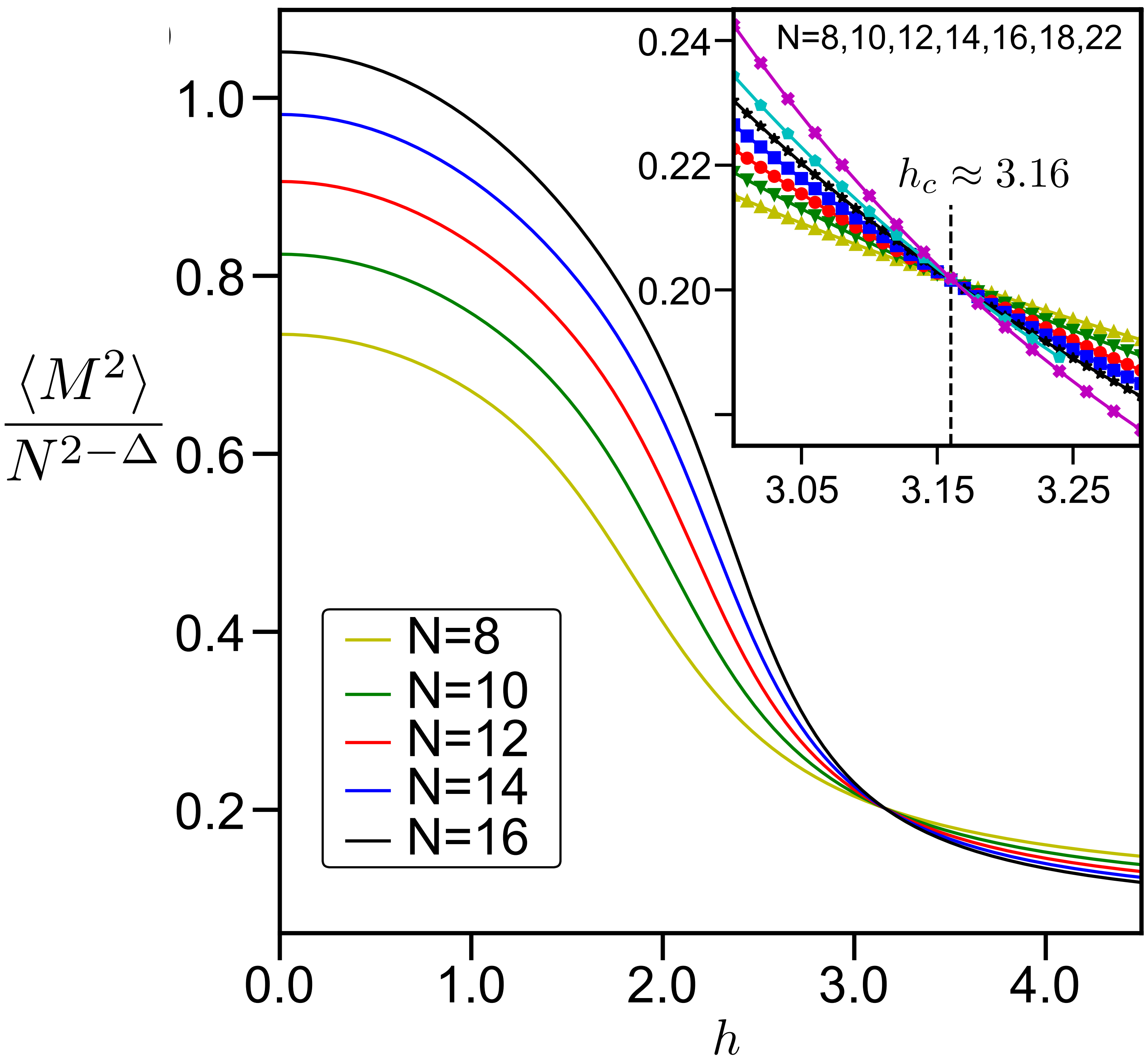
$$\psi^\dagger_\alpha(\vec{x}) = \sum_{m=-s}^{s} c^\dagger_{m,\alpha}\, Y^{(s)}_{s,m}(\vec{x})$$





Finite size scaling: order parameter

$V_1 = 1, V_0 = 4.75$



$$M = \sum_{m=-s}^{s} \mathbf{c}_m^\dagger \frac{\sigma^z}{2} \mathbf{c}_m$$

Ordered phase:

$$\langle M^2 \rangle \sim L_x^4 = N^2$$

Phase transition:

$$\langle M^2 \rangle \sim L_x^{4-2\Delta} = N^{2-\Delta}$$

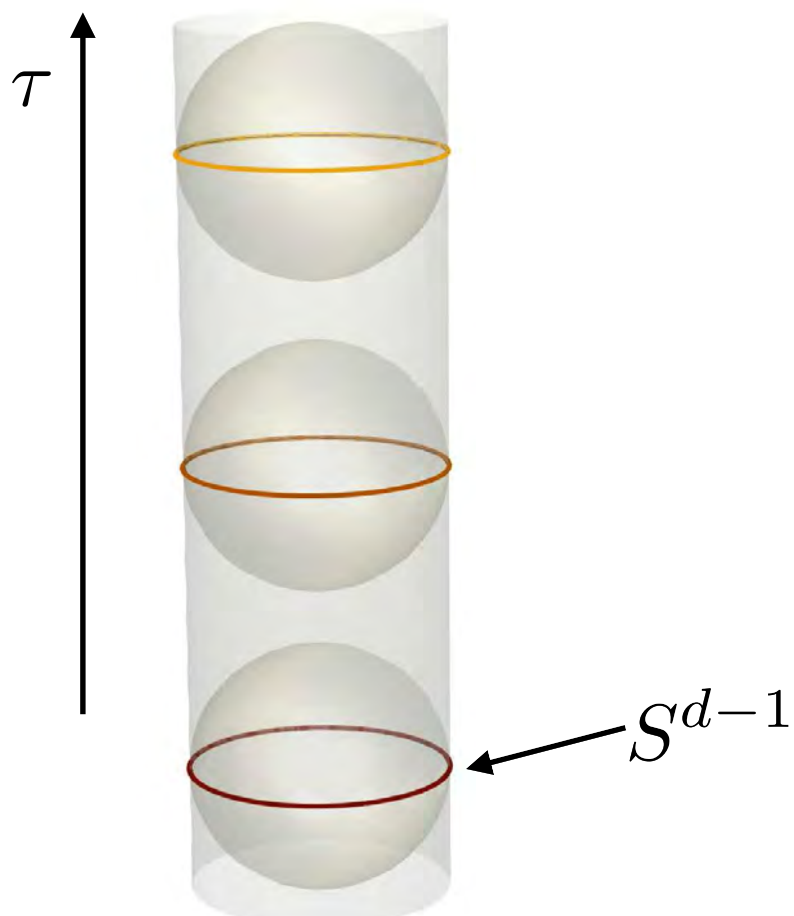
$$\Delta \approx 0.518148$$

Disorder phase:

$$\langle M^2 \rangle \sim O(N^0)$$

State-operator correspondence

Radial quantization
of d-dimensional CFT
$S^{d-1} \times \mathbb{R}$



Quantum Hamiltonian on S^{d-1}

Eigenstates of the quantum Hamiltonian.

One-to-one correspondence

$$\delta E_n = E_n - E_0 = \frac{v}{R}\Delta_n$$

Scaling operators in the CFT.

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle = \frac{\delta_{ij}}{|x_1 - x_2|^{2\Delta}}$$

Primaries and descendants

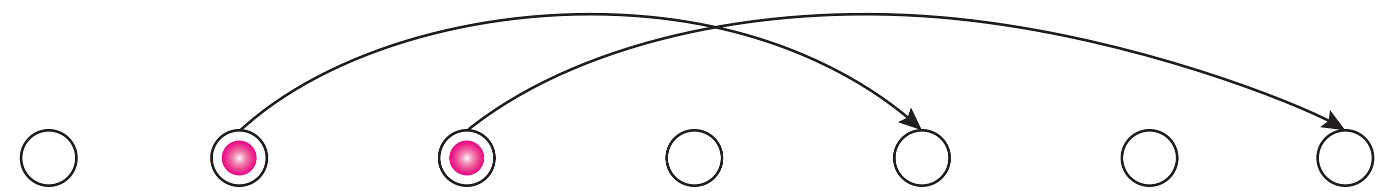
Conformal multiplet	$\mathcal{O} \longrightarrow$	$\partial_{\mu_1}\mathcal{O} \longrightarrow$	$\partial_{\mu_1}\partial_{\mu_2}\mathcal{O}$	$\cdots$
	Δ	$\Delta + 1$	$\Delta + 2$	$\cdots$

There are infinite number of primary operators in any 3D CFT!

3D Ising	$\Delta_\sigma \approx 0.5184189(10)$	$\Delta_\epsilon \approx 1.412625(10)$	$\Delta_{\epsilon'} \approx 3.82968(23)$
	$\eta = 2\Delta_\sigma - 1$	$\nu = 1/(3 - \Delta_\epsilon)$	$\omega = \Delta_{\epsilon'} - 3$

Symmetries

$2s+1$-site "fermionic chain"



Fermions are at half filling: $N = 2s+1$

$$\mathbf{c}_m^\dagger = (c_{m,\uparrow}^\dagger, c_{m,\downarrow}^\dagger)$$

$$m = -s, -s+1, \cdots, s$$

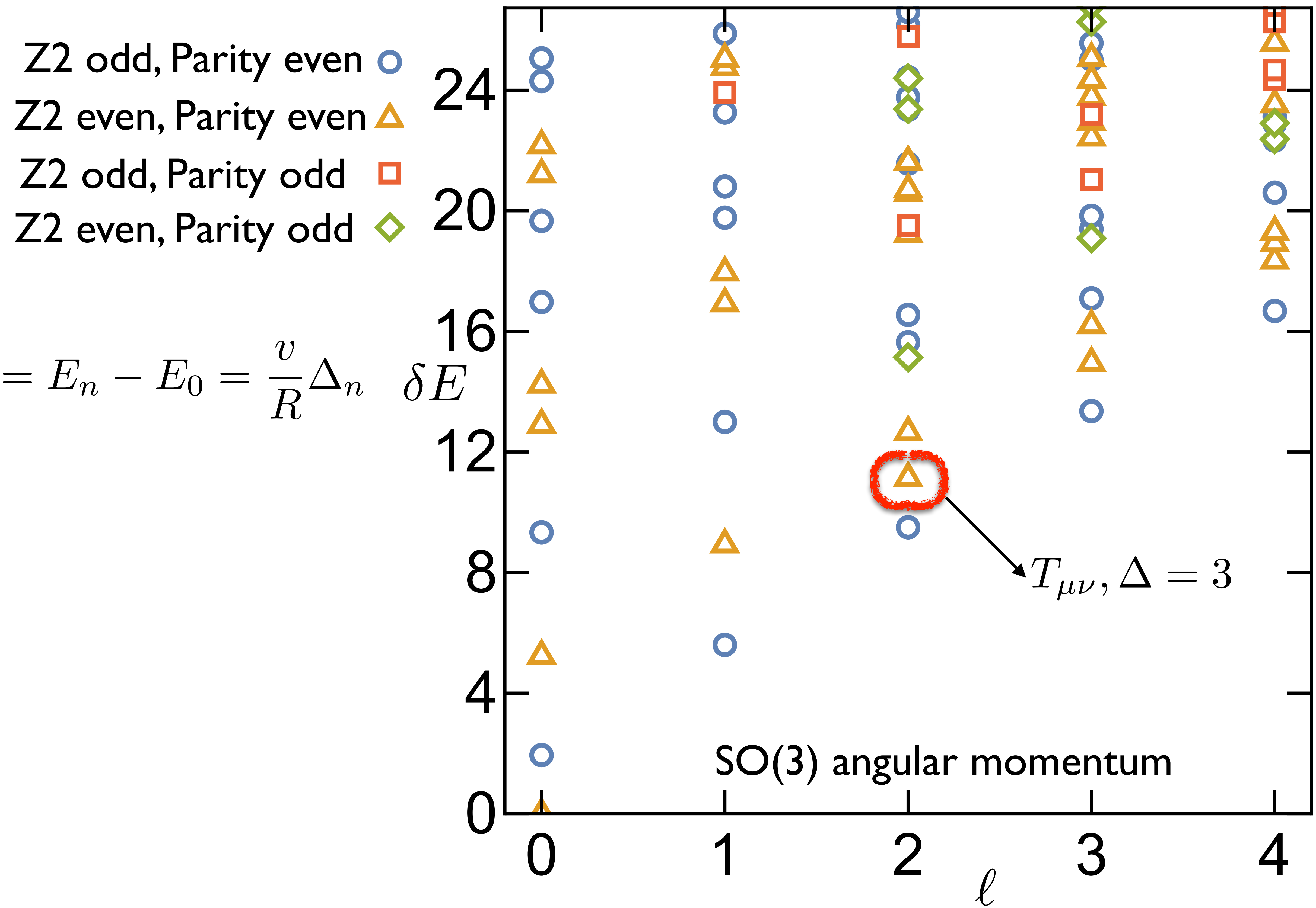
$N = 2s+1$ is the space volume, so $\sqrt{N} \sim R/\delta$.

UV model	IR Ising transition
$\mathbf{c}_m \to \sigma^x \mathbf{c}_m$	Z2 Ising symmetry
SO(3): $\mathbf{c}_{m=-s,\cdots,s}$ spin-s irrep	SO(3) Lorentz rotation
Particle-hole symmetry $\mathbf{c}_m \to i\sigma^y \mathbf{c}_m^\dagger$	Space-time parity symmetry

State-operator correspondence

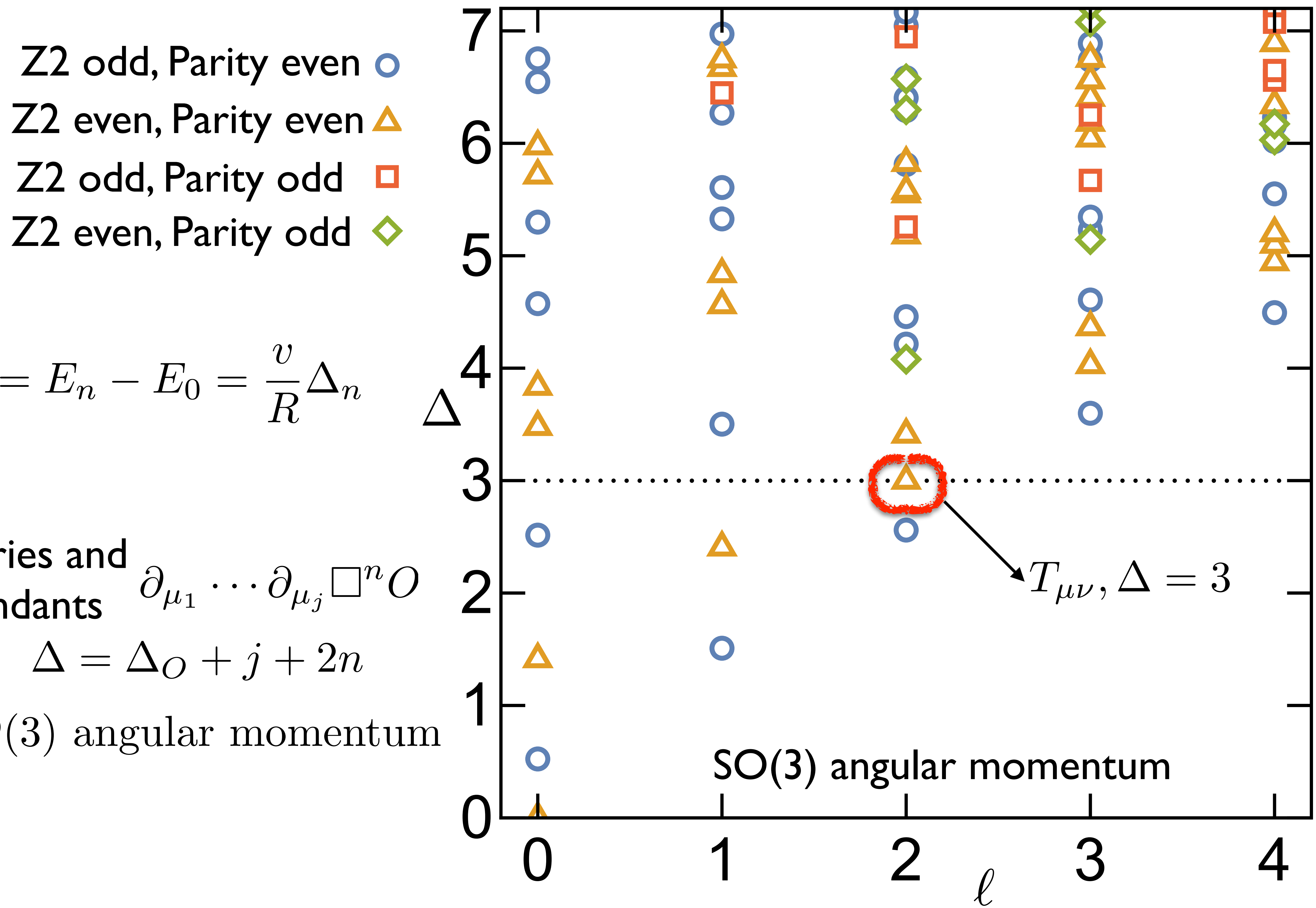
Energy gaps with quantum numbers: N=15 electrons (spins)

$$\delta E_n = E_n - E_0 = \frac{v}{R}\Delta_n$$



State-operator correspondence

Energy gaps with quantum numbers: N=15 electrons (spins)



$$\delta E_n = E_n - E_0 = \frac{v}{R}\Delta_n$$

Primaries and descendants $\partial_{\mu_1}\cdots\partial_{\mu_j}\Box^n O$

$$\Delta = \Delta_O + j + 2n$$

j: $SO(3)$ angular momentum

State-operator correspondence

Energy gaps with quantum numbers: N=15 electrons (spins)

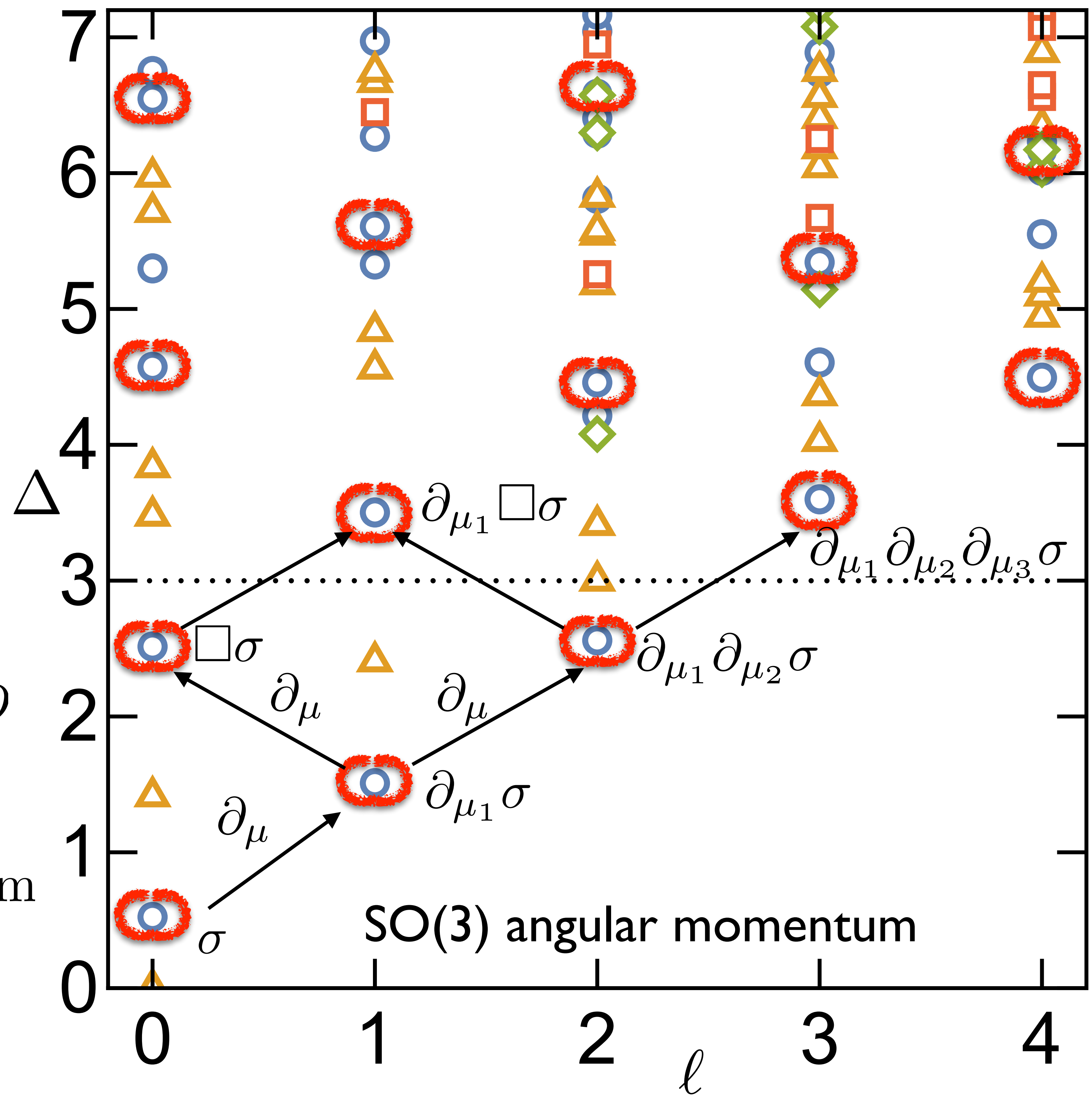
Z2 odd, Parity even ○
Z2 even, Parity even △
Z2 odd, Parity odd □
Z2 even, Parity odd ◇

$$\delta E_n = E_n - E_0 = \frac{v}{R}\Delta_n$$

Primaries and descendants $\partial_{\mu_1}\cdots\partial_{\mu_j}\Box^n O$

$$\Delta = \Delta_O + j + 2n$$

j: $SO(3)$ angular momentum



State-operator correspondence

Energy gaps with quantum numbers: N=15 electrons (spins)

Z2 odd, Parity even ○

Z2 even, Parity even △

Z2 odd, Parity odd □

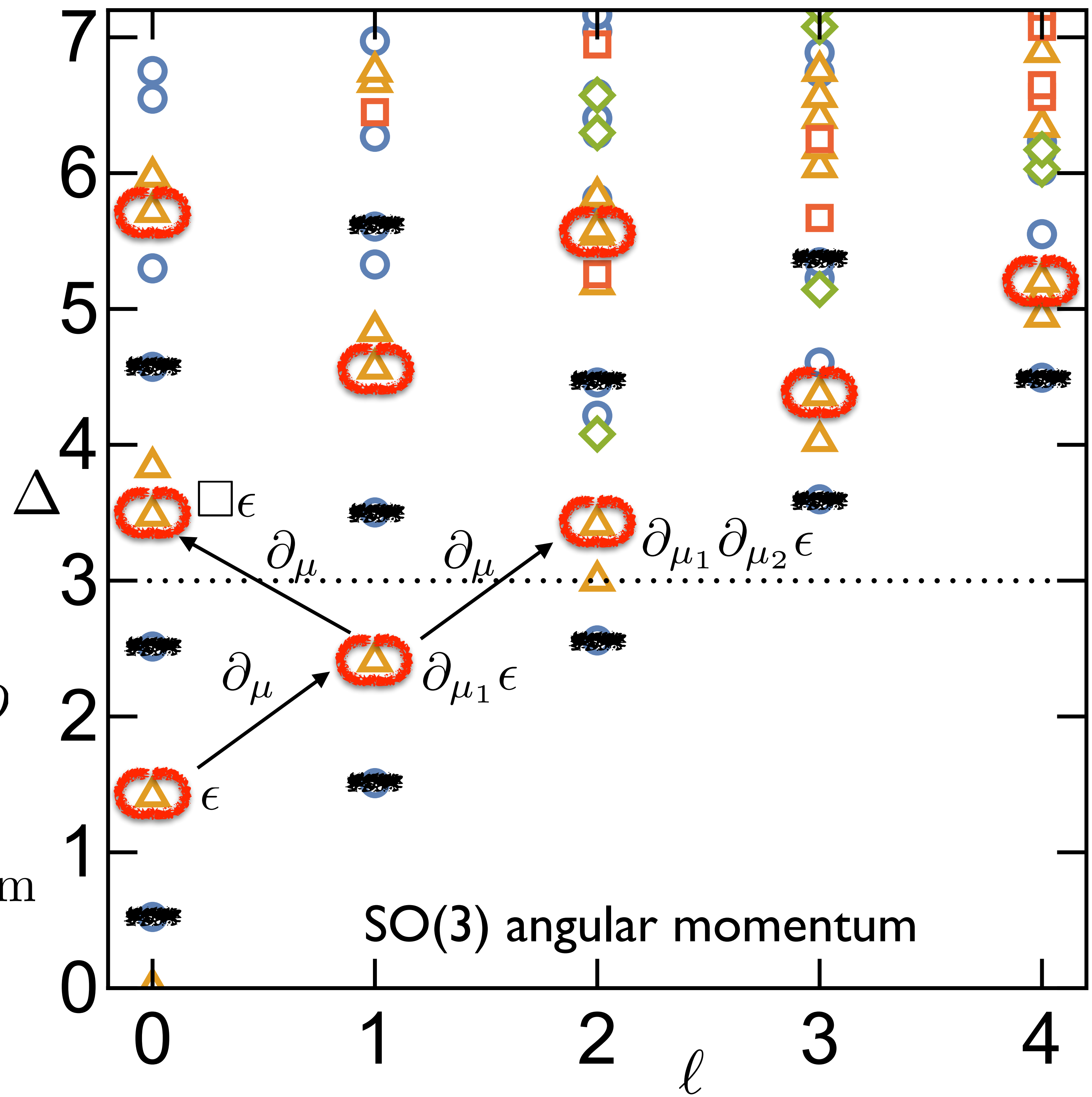
Z2 even, Parity odd ◇

$$\delta E_n = E_n - E_0 = \frac{v}{R}\Delta_n$$

Primaries and descendants $\partial_{\mu_1}\cdots\partial_{\mu_j}\Box^n O$

$$\Delta = \Delta_O + j + 2n$$

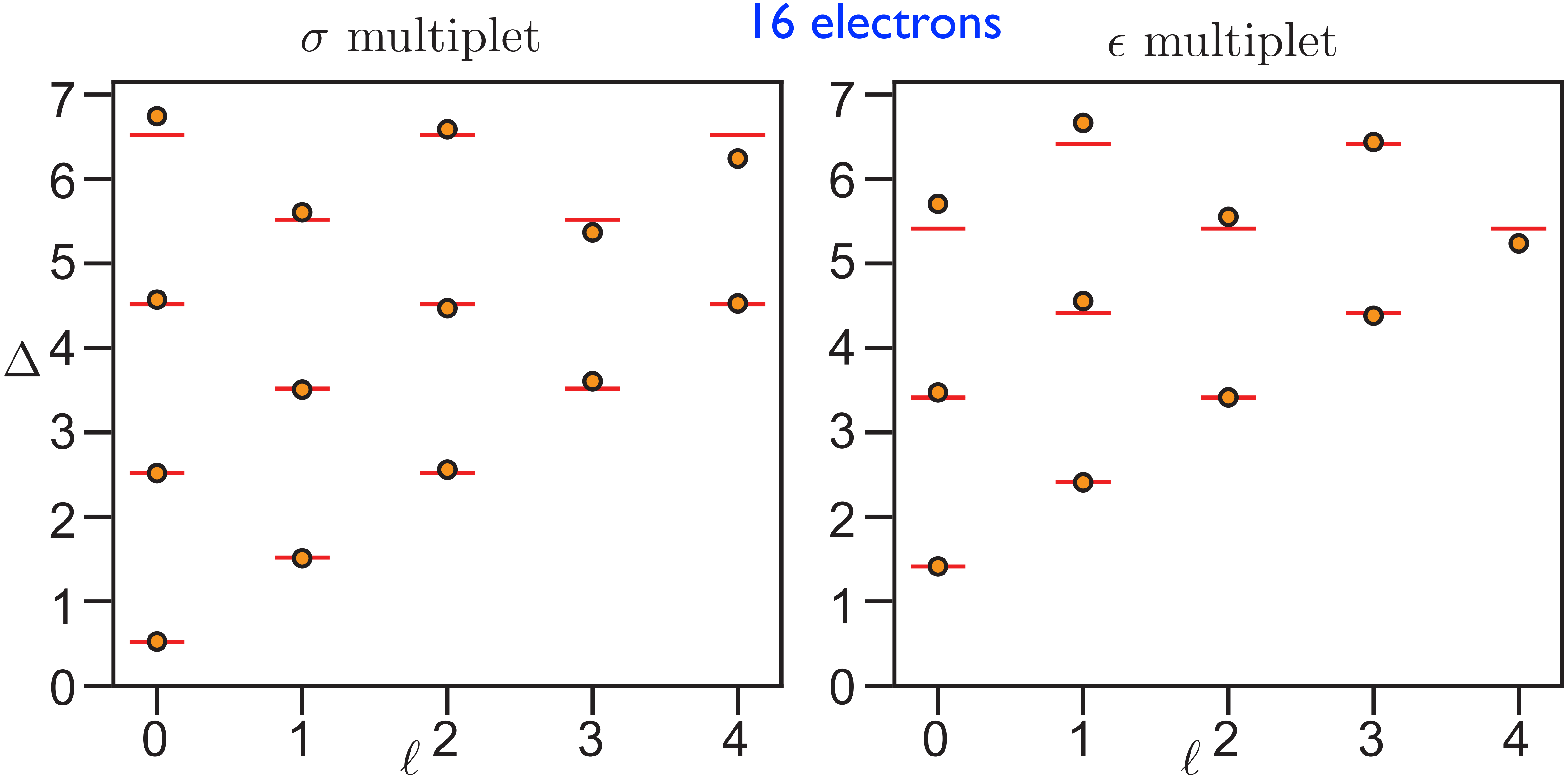
j: $SO(3)$ angular momentum



State-operator correspondence

descendents: $\partial_{\mu_1} \cdots \partial_{\mu_j} \Box^n O, \quad n, j \geq 0 \qquad (\Delta + 2n + j, j)$

16 electrons



State-operator correspondence

Zhu, Han, Huffman, Hofmann, YCH, arXiv:2210.13482 (PRX)

- We identified 15 primary operators, the numerical errors of all primaries are within 1.6%.
- We looked at 70 lowest lying states with L<5, all of them match theoretical expectations with small errors~3%.

	CB	16 spins	Error
σ	0.518	0.524	1.2%
σ'	5.291	5.303	0.2%
$\sigma_{\mu_1\mu_2}$	4.180	4.214	0.8%
$\sigma'_{\mu_1\mu_2}$	6.987	7.048	0.9%
$\sigma_{\mu_1\mu_2\mu_3}$	4.638	4.609	0.6%
$\sigma_{\mu_1\mu_2\mu_3\mu_4}$	6.113	6.069	0.7%
σ^{P-}	NA	11.19	–

Bootstrap data from Simmons-Duffin, 2017

	CB	16 spins	Error
ϵ	1.413	1.414	0.07%
ϵ'	3.830	3.838	0.2%
ϵ''	6.896	6.908	0.2%
$T_{\mu\nu}$	3	3	–
$T'_{\mu\nu}$	5.509	5.583	1.3%
$\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	5.023	5.103	1.6%
$\epsilon'_{\mu_1\mu_2\mu_3\mu_4}$	6.421	6.347	1.2%
ϵ^{P-}	≤ 11.2	10.01	–

TABLE II. Conformal mulitplet of ϵ.

Operator	Quantum Number	CB data	$N=16$	Errors
ϵ	$\ell=0$	1.412625(10)	1.41355766	0.066%
$\partial_\mu \epsilon$	$\ell=1$	2.412625(10)	2.40776449	0.201%
$\partial_{\mu_1}\partial_{\mu_2}\epsilon$	$\ell=2$	3.412625(10)	3.41455749	0.057%
$\Box\epsilon$	$\ell=0$	3.412625(10)	3.47303235	1.770%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\epsilon$	$\ell=3$	4.412625(10)	4.38113022	0.714%
$\Box\partial_\mu\epsilon$	$\ell=1$	4.412625(10)	4.55437869	3.212%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4}\epsilon$	$\ell=4$	5.412625(10)	5.2379631	3.227%
$\partial_{\mu_1}\partial_{\mu_2}\Box\epsilon$	$\ell=2$	5.412625(10)	5.5514904	2.566%
$\Box^2\epsilon$	$\ell=0$	5.412625(10)	5.70570641	5.415%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\Box\epsilon$	$\ell=3$	6.412625(10)	6.43712303	0.382%
$\partial_{\mu_1}\Box^2\epsilon$	$\ell=1$	6.412625(10)	6.66423677	3.924%

TABLE III. Conformal mulitplet of ϵ'.

Operator	Quantum Number	CB data	$N=16$	Errors
ϵ'	$\ell=0$	3.82968(23)	3.83772859	0.210%
$\partial_{\mu_1}\epsilon'$	$\ell=1$	4.82968(23)	4.83973617	0.208%
$\partial_{\mu_1}\partial_{\mu_2}\epsilon'$	$\ell=2$	5.82968(23)	5.82918219	0.009%
$\Box\epsilon'$	$\ell=0$	5.82968(23)	5.9605325	2.245%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\epsilon'$	$\ell=3$	6.82968(23)	6.76617638	0.930%
$\partial_{\mu_1}\Box\epsilon'$	$\ell=1$	6.82968(23)	7.05458433	3.293%

TABLE VI. Conformal mulitplet of $\epsilon_{\mu_1\mu_2\mu_3\mu_4}$.

Operator	Quantum Number	CB data	$N=16$	Errors
$\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	$\ell=4$	5.022665(28)	5.1029942	1.599%
$\varepsilon_{\mu_4\rho\tau}\partial_\rho\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	$\ell=4,\ P=-1$	6.022665(28)	6.17684693	2.560%
$\partial_{\mu_1}\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	$\ell=3$	6.022665(28)	6.19439341	2.851%

TABLE VIII. Conformal mulitplet of σ'.

Operator	Quantum number	CB data	$N=16$	Errors
σ'	$\ell=0$	5.2906(11)	5.30346641	0.243%
$\partial_{\mu_1}\sigma'$	$\ell=1$	6.2906(11)	6.27713785	0.214%

TABLE IV. Conformal mulitplet of $T_{\mu_1\mu_2}$.

Operator	Quantum Number	Exact value	$N=16$	Errors
$T_{\mu_1\mu_2}$	$\ell=2$	3	3	0.000%
$\partial_{\nu_1}T_{\mu_1\mu_2}$	$\ell=3$	4	4.03219819	0.805%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho T_{\mu_1\mu_2}$	$\ell=2,\ P=-1$	4	4.07392075	1.848%
$\partial_{\nu_1}\partial_{\nu_2}T_{\mu_1\mu_2}$	$\ell=4$	5	4.96734107	0.653%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho\partial_{\nu_1}T_{\mu_1\mu_2}$	$\ell=3,\ P=-1$	5	5.14602926	2.921%
$\Box T_{\mu_1\mu_2}$	$\ell=2$	5	5.17292963	3.459%
$\partial_{\nu_1}\Box T_{\mu_1\mu_2}$	$\ell=3$	6	6.04586808	0.764%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho\partial_{\nu_1}\partial_{\nu_2}T_{\mu_1\mu_2}$	$\ell=4,\ P=-1$	6	6.06221026	1.037%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho\Box T_{\mu_1\mu_2}$	$\ell=2,\ P=-1$	6	6.29074558	4.846%

TABLE V. Conformal mulitplet of $T'_{\mu_1\mu_2}$.

Operator	Quantum Number	CB data	$N=16$	Errors
$T'_{\mu_1\mu_2}$	$\ell=2$	5.50915(44)	5.5827144	1.335%
$\partial_{\nu_1}T'_{\mu_1\mu_2}$	$\ell=3$	6.50915(44)	6.57137975	0.956%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho T'_{\mu_1\mu_2}$	$\ell=2,\ P=-1,$	6.50915(44)	6.57557892	1.020%
$\partial_{\mu_1}T'_{\mu_1\mu_2}$	$\ell=1$	6.50915(44)	6.74639599	3.645%

TABLE VII. Conformal mulitplet of σ.

Operator	Quantum number	CB data	$N=16$	Errors
σ	$\ell=0$	0.5181489(10)	0.52428857	1.185%
$\partial_\mu\sigma$	$\ell=1$	1.5181489(10)	1.50941793	0.575%
$\Box\sigma$	$\ell=0$	2.5181489(10)	2.51722181	0.037%
$\partial_{\mu_1}\partial_{\mu_2}\sigma$	$\ell=2$	2.5181489(10)	2.55937503	1.637%
$\Box\partial_\mu\sigma$	$\ell=1$	3.5181489(10)	3.50635346	0.335%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\sigma$	$\ell=3$	3.5181489(10)	3.6059226	2.495%
$\Box\partial_{\mu_1}\partial_{\mu_2}\sigma$	$\ell=2$	4.5181489(10)	4.47002281	1.065%
$\Box^2\sigma$	$\ell=0$	4.5181489(10)	4.57231367	1.199%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4}\sigma$	$\ell=4$	4.5181489(10)	4.52727499	0.202%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\Box\sigma$	$\ell=3$	5.5181489(10)	5.36761913	2.728%
$\partial_\mu\Box^2\sigma$	$\ell=1$	5.5181489(10)	5.60563429	1.585%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4}\Box\sigma$	$\ell=4$	6.5181489(10)	6.24268467	4.226%
$\partial_{\mu_1}\partial_{\mu_2}\Box^2\sigma$	$\ell=2$	6.5181489(10)	6.58905267	1.088%
$\Box^3\sigma$	$\ell=0$	6.5181489(10)	6.74334514	3.455%

TA

Operator	Quar
ϵ	
$\partial_\mu\epsilon$	
$\partial_{\mu_1}\partial_{\mu_2}\epsilon$	
$\Box\epsilon$	
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\epsilon$	
$\Box\partial_\mu\epsilon$	
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4}\epsilon$	
$\partial_{\mu_1}\partial_{\mu_2}\Box\epsilon$	
$\Box^2\epsilon$	
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\Box\epsilon$	
$\partial_{\mu_1}\Box^2\epsilon$	

TABL

Operator	Quantu
ϵ'	
$\partial_{\mu_1}\epsilon'$	
$\partial_{\mu_1}\partial_{\mu_2}\epsilon'$	
$\Box\epsilon'$	
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\epsilon'$	
$\partial_{\mu_1}\Box\epsilon'$	

TABLE

Operator	Qu
$\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	
$\varepsilon_{\mu_4\rho\tau}\partial_\rho\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	ℓ
$\partial_{\mu_1}\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	

TABLE

Operator	Quantu
σ'	ℓ
$\partial_{\mu_1}\sigma'$	ℓ

TABLE IX. Conformal multiplet of $\sigma_{\mu_1\mu_2}$.

Operator	Quantum number	CB data	$N = 16$	Errors
$\sigma_{\mu_1\mu_2}$	$\ell = 2$	4.180305(18)	4.21382989	0.802%
$\partial_{\nu_1}\sigma_{\mu_1\mu_2}$	$\ell = 3$	5.180305(18)	5.23649044	1.085%
$\partial_{\mu_1}\sigma_{\mu_1\mu_2}$	$\ell = 1$	5.180305(18)	5.31575894	2.615%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho\sigma_{\mu_1\mu_2}$	$\ell = 2,\ P = -1$	5.180305(18)	5.25415317	1.426%
$\partial_{\nu_1}\partial_{\nu_2}\sigma_{\mu_1\mu_2}$	$\ell = 4$	6.180305(18)	6.18724938	0.112%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho\partial_{\nu_1}\sigma_{\mu_1\mu_2}$	$\ell = 3,\ P = -1$	6.180305(18)	6.26160085	1.315%
$\partial_{\nu_1}\partial_{\mu_1}\sigma_{\mu_1\mu_2}$	$\ell = 2$	6.180305(18)	6.29114975	1.794%
$\Box\sigma_{\mu_1\mu_2}$	$\ell = 2$	6.180305(18)	6.39595149	3.489%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho\partial_{\mu_1}\sigma_{\mu_1\mu_2}$	$\ell = 1,\ P = -1$	6.180305(18)	6.42999132	4.040%
$\partial_{\mu_1}\partial_{\mu_2}\sigma_{\mu_1\mu_2}$	$\ell = 0$	6.180305(18)	6.52321841	5.548%

TABLE X. Conformal multiplet of $\sigma_{\mu_1\mu_2\mu_3}$.

Operator	Quantum number	CB data	$N = 16$	Errors
$\sigma_{\mu_1\mu_2\mu_3}$	$\ell = 3$	4.63804(88)	4.60892045	0.628%
$\partial_{\nu_1}\sigma_{\mu_1\mu_2\mu_3}$	$\ell = 4$	5.63804(88)	5.56345584	1.323%
$\varepsilon_{\mu_3\rho\tau}\partial_\rho\sigma_{\mu_1\mu_2\mu_3}$	$\ell = 3,\ P = -1$	5.63804(88)	5.6704459	0.575%
$\partial_{\mu_1}\sigma_{\mu_1\mu_2\mu_3}$	$\ell = 2$	5.63804(88)	5.79746571	2.828%
$\Box\sigma_{\mu_1\mu_2\mu_3}$	$\ell = 3$	6.63804(88)	6.74065848	1.546%
$\partial_{\nu_1}\partial_{\mu_1}\sigma_{\mu_1\mu_2\mu_3}$	$\ell = 3$	6.63804(88)	6.88182226	3.672%
$\varepsilon_{\mu_3\rho\tau}\partial_\rho\partial_{\nu_1}\sigma_{\mu_1\mu_2\mu_3}$	$\ell = 4,\ P = -1$	6.63804(88)	6.57417625	0.962%
$\varepsilon_{\mu_3\rho\tau}\partial_\rho\partial_{\mu_1}\sigma_{\mu_1\mu_2\mu_3}$	$\ell = 2,\ P = -1$	6.63804(88)	6.93133276	4.418%
$\partial_{\mu_1}\partial_{\mu_2}\sigma_{\mu_1\mu_2\mu_3}$	$\ell = 1$	6.63804(88)	6.9490099	4.685%

μ_2.

$N = 16$	Errors
3	0.000%
4.03219819	0.805%
4.07392075	1.848%
4.96734107	0.653%
5.14602926	2.921%
5.17292963	3.459%
6.04586808	0.764%
6.06221026	1.037%
6.29074558	4.846%

$\mu_1\mu_2$.

$N = 16$	Errors
.5827144	1.335%
.57137975	0.956%
.57557892	1.020%
.74639599	3.645%

$N = 16$	Errors
0.52428857	1.185%
1.50941793	0.575%
2.51722181	0.037%
2.55937503	1.637%
3.50635346	0.335%
3.6059226	2.495%
4.47002281	1.065%
4.57231367	1.199%
4.52727499	0.202%
5.36761913	2.728%
5.60563429	1.585%
6.24268467	4.226%
6.58905267	1.088%
6.74334514	3.455%

Even 4 electrons work!!!

Gaps of ALL the excited states of the system with N=4 electrons.

6 primaries are found!!

	CB	4 spins	Errors
σ	0.518	0.530	2.3%
$\partial_{\mu_1}\sigma$	1.518	1.522	0.3%
$\Box\sigma$	2.518	2.427	3.6%
$\partial_{\mu_1}\partial_{\mu_2}\sigma$	2.518	2.428	3.6%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\sigma$	3.518	2.847	20%
$\partial_{\mu_1}\Box\sigma$	3.518	3.291	6.5%
$\sigma_{\mu_1\mu_2}$	4.180	4.241	1.5%
$\sigma_{\mu_1\mu_2\mu_3}$	4.638	4.618	0.4%

	CB	4 spins	Errors
ϵ	1.413	1.382	2.2%
$\partial_{\mu_1}\epsilon$	2.413	2.337	3.1%
$T_{\mu_1\mu_2}$	3	3	NA
$\partial_{\mu_1}\partial_{\mu_2}\epsilon$	3.413	3.126	8.4%
$\Box\epsilon$	3.413	3.577	4.8%
$\partial_{\mu_3}T_{\mu_1\mu_2}$	4	3.663	8.4%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho T_{\mu_1\mu_2}$	4	4.054	1.4%
ϵ'	3.830	4.019	4.9%
$\partial_{\mu_3}\partial_{\mu_4}T_{\mu_1\mu_2}$	5	4.856	2.9%

Outline

- Lecture 1: Introduction: motivation and fuzzy sphere regularization
- Lecture 1: 3D Ising CFT
  - State-operator correspondence
  - Operators, correlators, OPEs, F-function
- Lecture 2: Recent progress:
  - Chern-Simons-matter theories: phase transitions between quantum Hall states
  - Conformal defect
  - Fermionic CFT and super-Ising

Let us continue with fuzzy journey

The core of the story is the State-Operator correspondence.

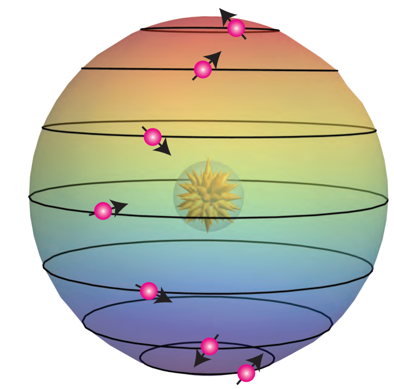
- We have explored the energy gaps of the states.
- A lot of information ready for exploration:

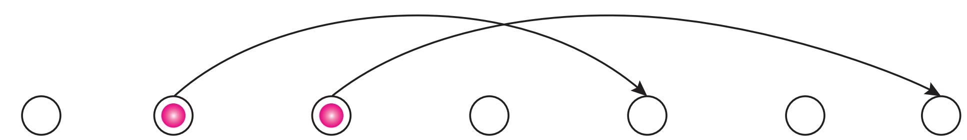
  A. Wave-functions of the states.

  B. Operators.

From orbital space to real space

All the computations are done in the orbital space



$2s+1$-site "fermionic chain" 

Many-body Hilbert space $\prod_{i=1}^{2s+1} c^{\dagger}_{m_i,\alpha_i}|0\rangle$ $\quad m_i = -s, -s+1, \cdots, s$ spin-s rep of $SO(3)$ $\quad \alpha_i = \uparrow, \downarrow$

Real space is continuous (NOT discrete like lattice models).

$$\psi^{\dagger}_a(\theta,\varphi) = \sum_{m=-s}^{s} c^{\dagger}_{m,a} Y^{(s)}_{s,m}(\theta,\varphi)$$

$$Y^{(s)}_{s,m}(\theta,\varphi) = \mathcal{N}_{s,m} e^{im\varphi} \cos^{s+m}\left(\frac{\theta}{2}\right) \sin^{s-m}\left(\frac{\theta}{2}\right)$$

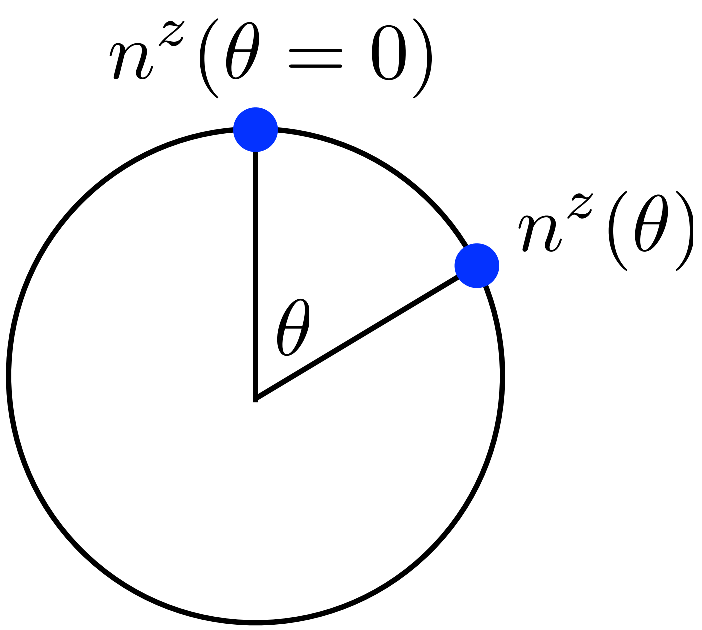
Any observables can be computed in real space!

Numerical data of 2-point correlator

We get a function defined in the continuum:

$$0.6941 + 0.3724\cos\theta + 0.2840\cos^2\theta + 0.2091\cos^3\theta + \cdots$$

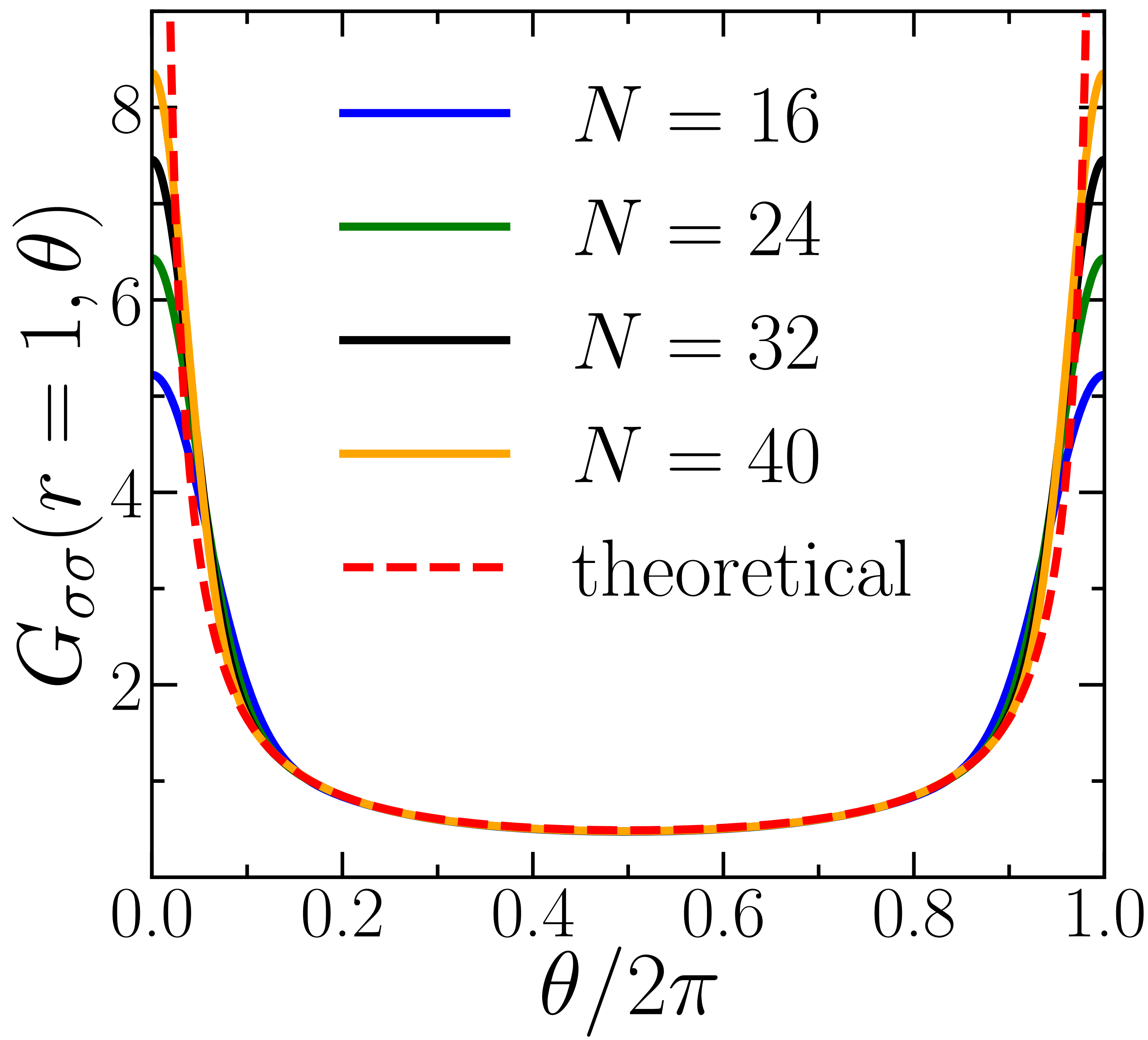
Han, Hu, Zhu, YCH, arXiv: 2306.04681



$$\langle n^z n^z \rangle = \frac{\langle 0|n^z(\theta=0)n^z(\theta)|0\rangle}{\langle \sigma|n^z(\theta=0)|0\rangle^2}$$

CFT prediction: $\dfrac{1}{(2\sin(\theta/2))^{2\Delta}}$

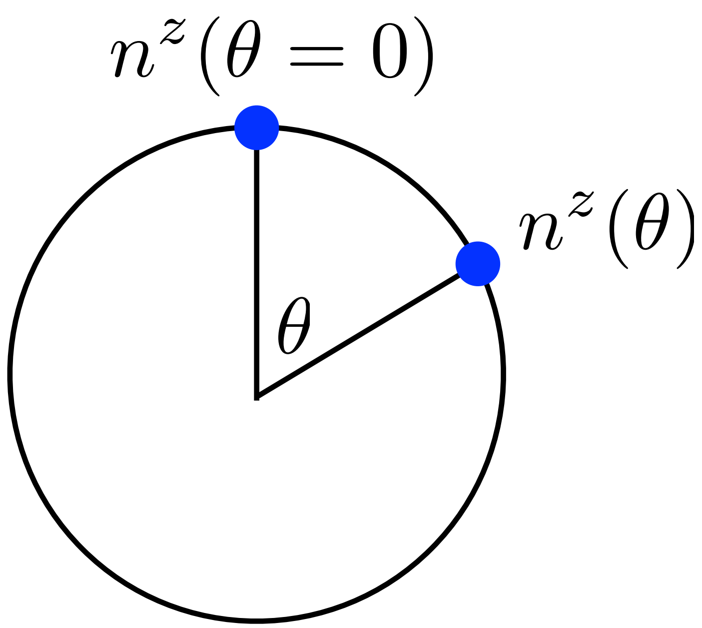
$\Delta \approx 0.518149$



Operators and their correlators are sharp, continuous and conformal.

Four-point correlator

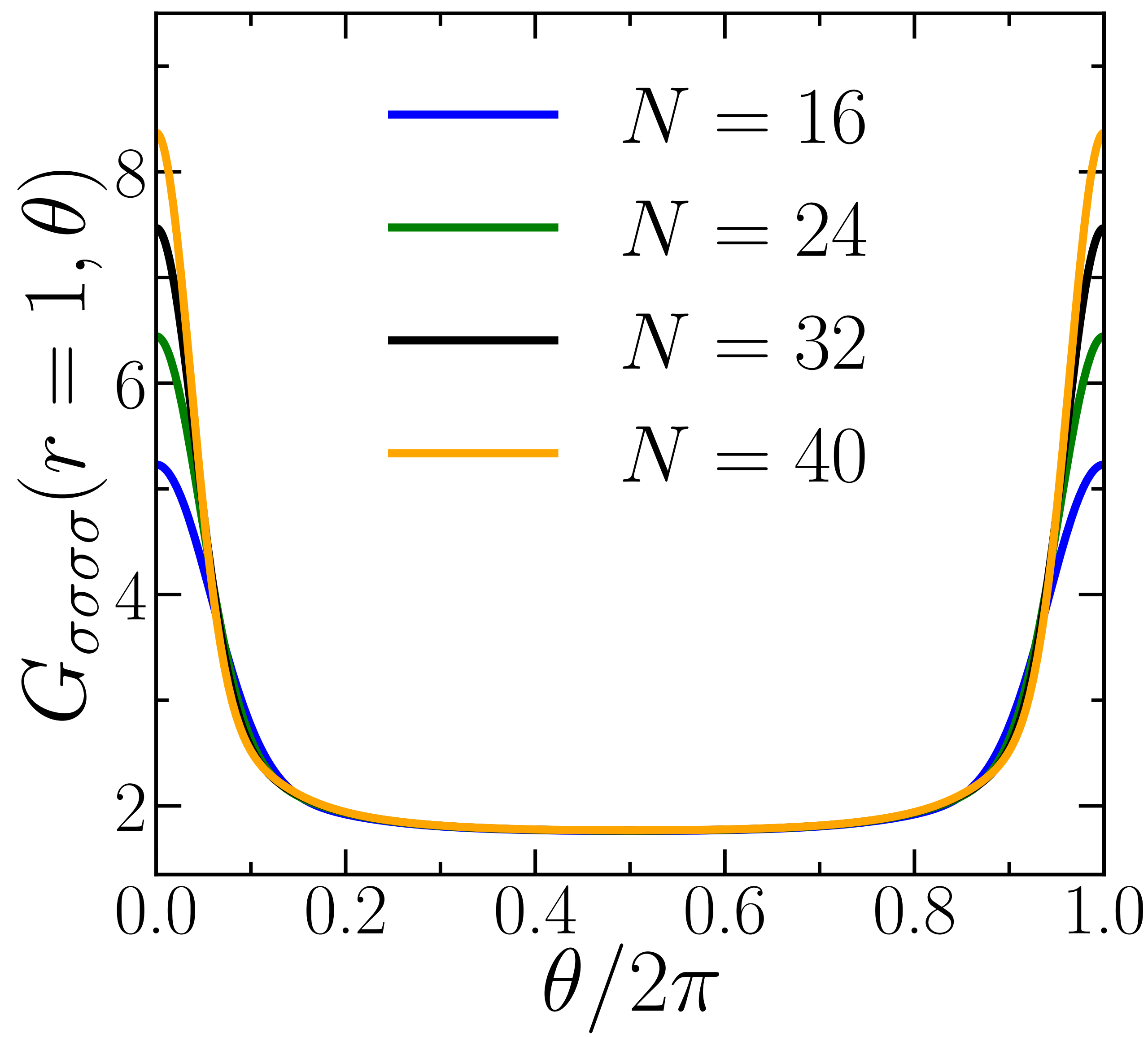
Han, Hu, Zhu, YCH, arXiv: 2306.04681



$$G(z = e^{i\theta}, \bar{z} = e^{-i\theta})$$

$$\frac{\langle \sigma | n^z(\theta = 0) n^z(\theta) | \sigma \rangle}{\langle \sigma | n^z(\theta = 0) | 0 \rangle^2}$$

$$1.846 + 0.171 \cos\theta + 0.152 \cos^2\theta$$
$$+0.109 \cos^3\theta + 0.109 \cos^4\theta + \cdots$$



0.06% difference

	Bootstrap	$N = 40$	$N = 32$	$N = 24$	$N = 16$
$\theta = \pi$	1.76855	1.76742	1.76671	1.76549	1.76244
$\theta = \pi/3$	2.049	2.03921	2.03495	2.02470	2.01212

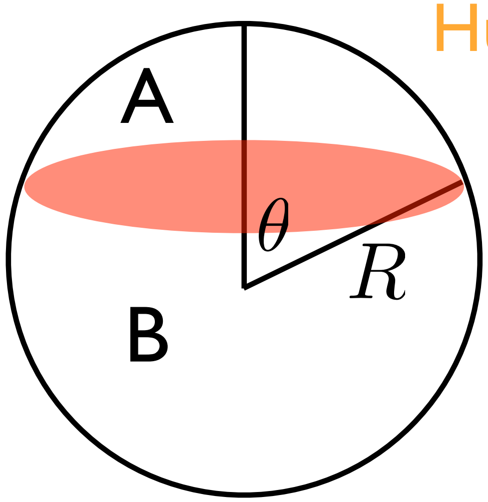
Numerical data: OPE coefficients

We have computed 13 OPE coefficients including a few new ones that have not been computed by any approach.

Hu, YCH, Zhu, arXiv:2303.08844 (PRL)

Operators	Spin	Z_2	$f_{\alpha\beta\gamma}$ (Fuzzy Sphere)	$f_{\alpha\beta\gamma}$ (Bootstrap)
σ	0	$-$	$f_{\sigma\sigma\epsilon} \approx 1.0539(18)$	$f_{\sigma\sigma\epsilon} \approx 1.0519$
ϵ	0	$+$	$f_{\epsilon\epsilon\epsilon} \approx 1.5441(23)$	$f_{\epsilon\epsilon\epsilon} \approx 1.5324$
ϵ'	0	$+$	$f_{\sigma\sigma\epsilon'} \approx 0.0529(16)$	$f_{\sigma\sigma\epsilon'} \approx 0.0530$
			$f_{\epsilon\epsilon\epsilon'} \approx 1.566(68)$	$f_{\epsilon\epsilon\epsilon'} \approx 1.5360$
σ'	0	$-$	$f_{\sigma'\sigma\epsilon} \approx 0.0515(42)$	$f_{\sigma'\sigma\epsilon} \approx 0.0572$
			$f_{\sigma'\sigma\epsilon'} \approx 1.294(51)$	NA
			$f_{\sigma'\epsilon\sigma'} \approx 2.98(13)$	NA
$T_{\mu\nu}$	2	$+$	$f_{\sigma\sigma T} \approx 0.3248(35)$	$f_{\sigma\sigma T} \approx 0.3261$
			$f_{\sigma'\sigma T} \approx -0.00007(96)$	$f_{\sigma'\sigma T} = 0$
			$f_{\epsilon\epsilon T} \approx 0.8951(35)$	$f_{\epsilon\epsilon T} \approx 0.8892$
			$f_{T\epsilon T} \approx 0.8658(69)$	NA
$\sigma_{\mu\nu}$	2	$-$	$f_{\sigma\epsilon\sigma_{\mu\nu}} \approx 0.400(33)$	$f_{\sigma\epsilon\sigma_{\mu\nu}} \approx 0.3892$
			$f_{\sigma\epsilon'\sigma_{\mu\nu}} \approx 0.18256(69)$	NA

F-function of 3D Ising CFT



Hu, Zhu, YCH, arXiv:2401.17362

$$S_A(\theta) = -Tr(\rho_A \ln \rho_A)$$
$$= \alpha \frac{R}{\delta} \sin\theta - F$$

$$F = (\tan\theta\partial_\theta - 1)S_A|_{\theta=\pi/2}$$
$$F(R) = F_{Ising} + aR^{-\omega} + O(R^{-1})$$
$$R/\delta \sim \sqrt{N} = \sqrt{2s+1}$$

Our estimates on fuzzy sphere:

$$F_{Ising} \approx 0.0614(7)$$

$$F_{scalar} = \frac{\log 2}{8} - \frac{3\zeta(3)}{16\pi^2} \approx 0.0638$$

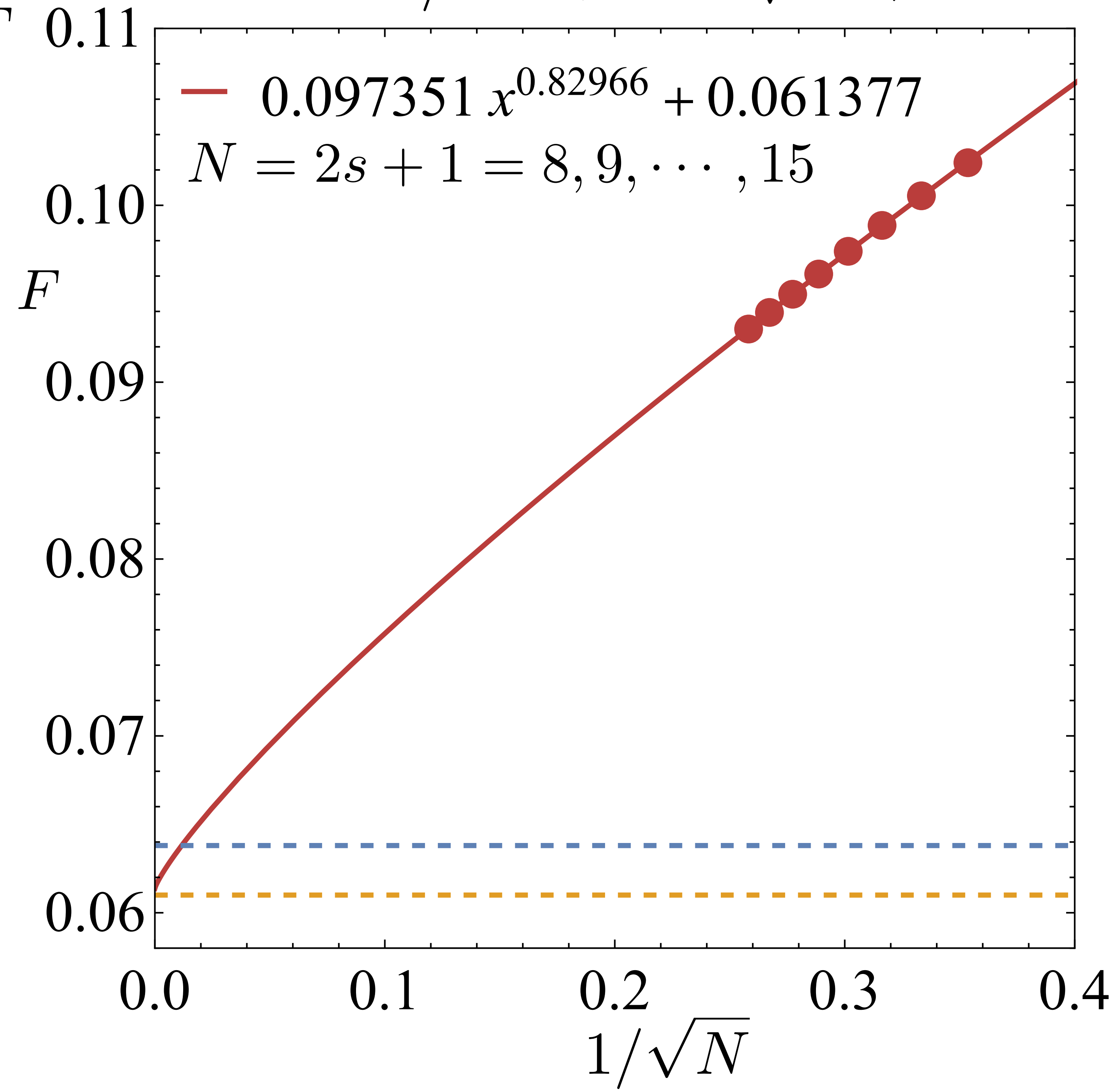
ϵ expansion

Giobmbi, Klebanov 2015;

Fei, Giombi, Klebanov, Tarnopolsky 2015

$$F_{Ising} \approx 0.957 F_{scalar} \approx 0.0610$$

$$F_{Ising} \approx 0.979 F_{scalar} \approx 0.0622$$



Almost everything can be studied

A partial list of results in this new "fuzzy" world.

- Operators, OPE coefficients, correlators Hu, YCH, Zhu; Han, Hu, Zhu, YCH
- RG monotonic F-function Hu, Zhu, YCH
- Conformal generators Fardelli, Fitzpatrick, Katz
- Conformal defect and boundaries Hu, YCH, Zhu; Zhou, Gaiotto, YCH, Zou; Zhou, Zou; Dedushenko
- Free boson and free fermion and Super-Ising CFT YCH; Taylor, Voinea, Papic, Fan; Zhou, Gaiotto, YCH; Tang, Voinea, Papic, Zhu
- Gauged Ising, Majorana Zhou, Gaiotto, YCH; Voinea, Zhu, Regnault, Papic
- O(N) WF Han, Hu, and Zhu; Lauchli, Rychkov, Herviou; Guo, Zhou, Wei, YCH
- Deconfined phase transition and non-Abelian gauge theories Zhou, Hu, Zhu, YCH; Zhou, YCH
- 3D Yang-Lee non-unitary CFT Fan, Dong, Vishwanath; Cruz, Klebanov, Tarnopolsky, Xin; Miro, Delouche
- Chern-Simons matter theories Zhou, Wang, YCH

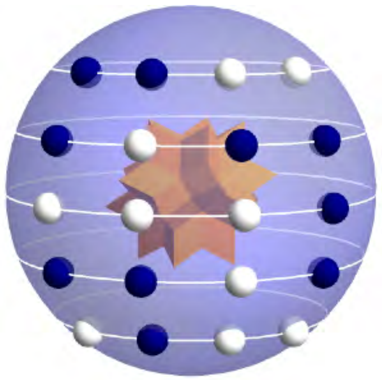
Welcome to the era of fuzzy sphere

Simulating 3D (2+1D) CFT? Easier than ever!

Lattice model simulation	Fuzzy sphere
1000~100,000 spins	4~20 spins
Millions of CPU hours	30 mins on a laptop
Very limited information	Almost everything
No access to conformal symmetry	Fingerprint of conformal symmetry

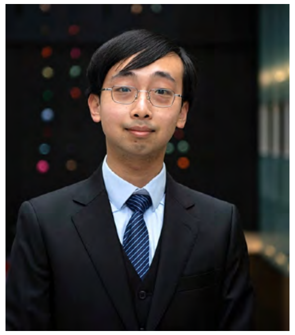
Zhu, Han, Huffman, Hofmann, YCH, arXiv:2210.13482 (PRX); Hu, YCH, Zhu, arXiv:2303.08844 (PRL); Han, Hu, Zhu, YCH, arXiv:2306.04681 (PRB); Zhou, Hu, Zhu, YCH, arXiv:2306.16435 (PRX); Hu, YCH, Zhu, arXiv:2308.01903 (Nat. Comm.); Zhou, Gaiotto, YCH, Zou, arxiv:2401.00039 (Sci. Post); Hu, Zhu, YCH, arXiv:2401.17362; Cuomo, YCH, Komargodski, arXiv:2406.10186 (JHEP); Zhou, YCH arXiv:2410.00087 (PRL); YCH arXiv:2506.14904; Zhou, Wang, YCH arXiv:2507.19580; Zhou, Gaiotto, YCH, arXiv: 2509.08038

Everything made easy by FuzzifiED



Numerical package (FuzzifiED): www.fuzzified.world

10mins~1hr coding, 1min-1hr simulation on a laptop!



Zheng Zhou

FuzzifiED.jl is a Julia package designed to do exact diagonalisation (ED) calculation on the fuzzy sphere, and also facilitates the DMRG calculations by ITensors. It can also be used for generic fermionic and bosonic models. Using this package, you can reproduce almost all the results in fuzzy sphere works.

To install the package, please enter the commands in Julia :

```
using Pkg ; Pkg.add("FuzzifiED")
```

Find below some useful links :

- Documentation : https://docs.fuzzified.world/ (*Also contains an introduction for the fuzzy sphere and a collection of examples*)
- Documentation (PDF) : https://docs.fuzzified.world/assets/FuzzifiED_Documentation.pdf
- Registry at JuliaHub : https://juliahub.com/ui/Packages/General/FuzzifiED
- Source code : https://github.com/FuzzifiED/FuzzifiED.jl

If this package is helpful in your research, we would appreciate it if you mention in the acknowledgement. If you have any questions, please contact Zheng Zhou (周正) at physics@zhengzhou.page.

Can wavefunction be the hero once again?

Two wavefunctions reshaped condensed matter physics:

Superconductors Bardeen, Cooper, Schrieffer	$\prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} \right) \lvert 0 \rangle$
Fractional quantum Hall Laughlin	$\prod_{i<j} (z_i - z_j)^m \exp\left(-\frac{1}{4\ell_B^2} \sum_{i=1}^{N} \lvert z_i \rvert^2 \right)$

A wavefunction ansatz for 3D Ising on fuzzy sphere: YCH arXiv:2506.14904

$$\exp\left(\sum_{\ell=0}^{2s} \frac{1}{4} \log\left(\frac{2(1-\Delta)_{\ell+1}}{(\Delta)_\ell} K_\ell \right) \sum_{m=-\ell}^{\ell} (-1)^m \frac{n^-_{\ell,m} n^-_{\ell,-m} - n^+_{\ell,m} n^+_{\ell,-m}}{2s+1} \right) \lvert \varphi_0 \rangle$$

1. Power law correlation function.
2. High wavefunction overlap with the Ising CFT.

	$N=12$	$N=16$	$N=20$	$N=24$	$N=28$	$N=32$
Ising CFT	0.9967	0.9951	0.9935	0.9918	0.9902	0.9885

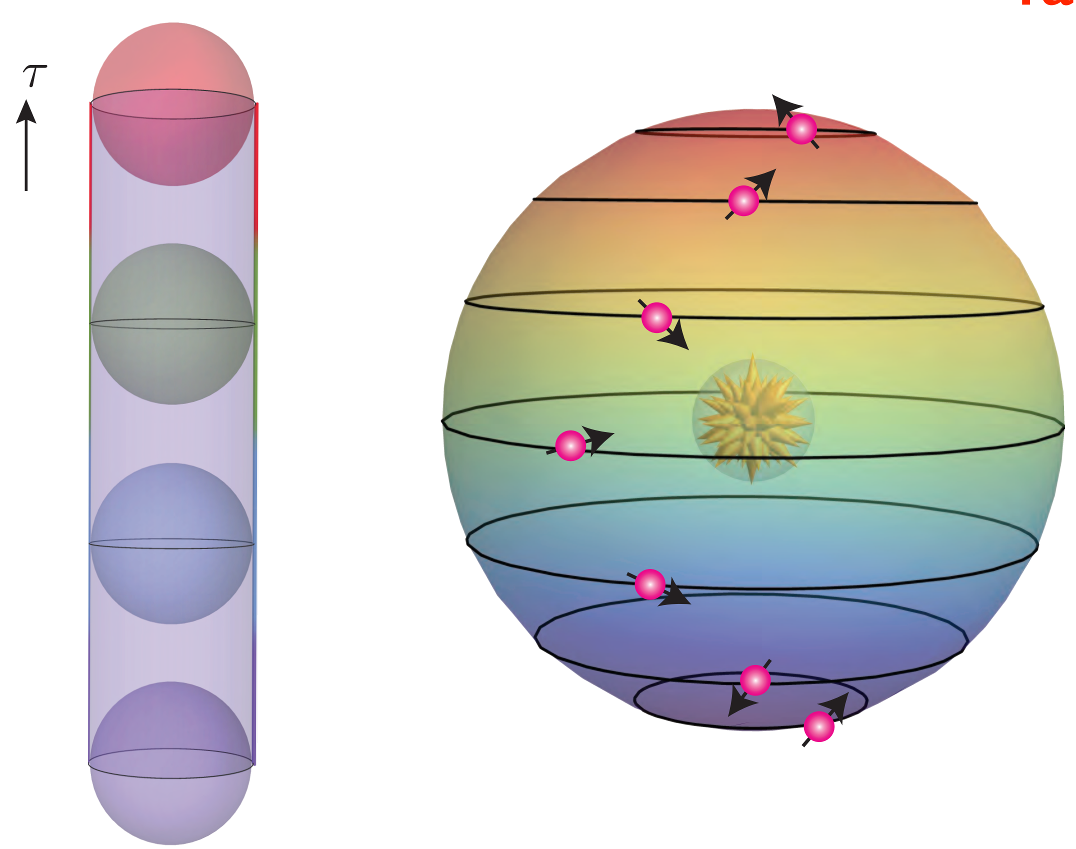
Thank you!

A Fuzzy Sphere Regularization of 3D CFT: Lecture II

Yin-Chen He
(何寅琛)
Perimeter Institute

PERIMETER INSTITUTE

Tallahassee, 2026 January

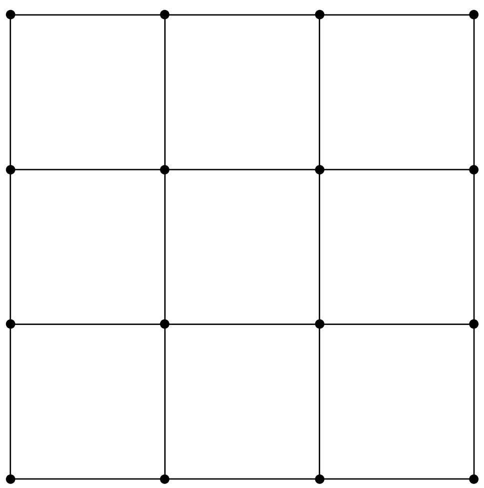


Fuzzy sphere regularization of 3D CFTs

Zhu, Han, Huffman, Hofmann, YCH, arXiv:2210.13482 (PRX)

Quantum mechanical model realizations of 2+1D CFTs.

Lattice model



Spin-1/2 on each site $\begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}$ $\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

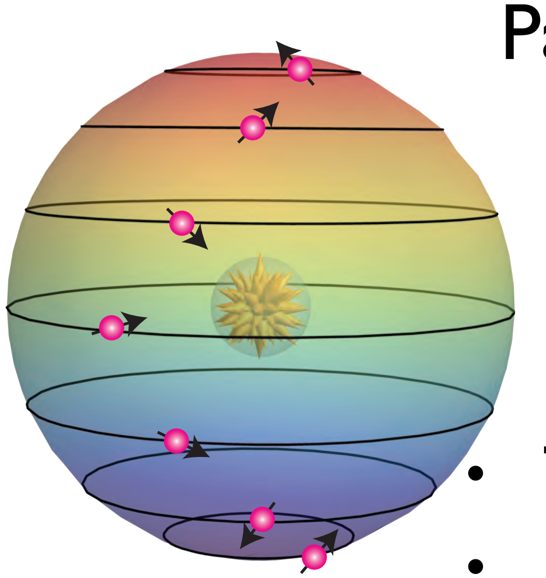
$$H = -\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

Spins point to +z or -z Spins point to +x

h

2+1D Ising CFT

Fuzzy sphere model



Particles moving on sphere in the presence of a monopole.

$$H = \frac{1}{2m} \sum_{i=1}^{N_e} (\vec{p}_i + \vec{A}(\vec{x}_i))^2 + \sum_{i,j=1}^{N_e} U(\vec{x}_i - \vec{x}_j) + \cdots$$

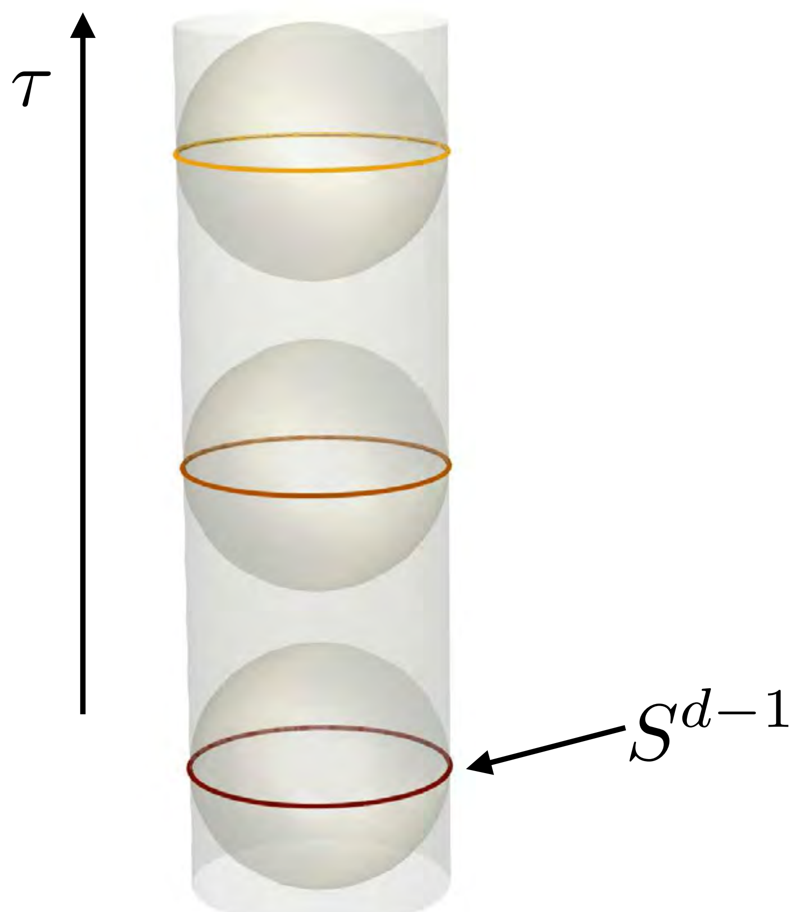
Interaction term

- The model is local if interactions are local.
- 2+1D CFTs can be realized by tuning the interaction form.

This idea was firstly pursued on the torus. Ippoliti, Mong, Assaad, Zaletel (2018)

State-operator correspondence

Radial quantization
of d-dimensional CFT
$S^{d-1} \times \mathbb{R}$



Quantum Hamiltonian on S^{d-1}

Eigenstates of the quantum Hamiltonian.

One-to-one correspondence

$$\delta E_n = E_n - E_0 = \frac{v}{R}\Delta_n$$

Scaling operators in the CFT.

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle = \frac{\delta_{ij}}{|x_1 - x_2|^{2\Delta}}$$

Primaries and descendants

Conformal multiplet	$\mathcal{O} \longrightarrow$	$\partial_{\mu_1}\mathcal{O} \longrightarrow$	$\partial_{\mu_1}\partial_{\mu_2}\mathcal{O}$	$\cdots$
	Δ	$\Delta + 1$	$\Delta + 2$	$\cdots$

There are infinite number of primary operators in any 3D CFT!

3D Ising	$\Delta_\sigma \approx 0.5184189(10)$	$\Delta_\epsilon \approx 1.412625(10)$	$\Delta_{\epsilon'} \approx 3.82968(23)$
	$\eta = 2\Delta_\sigma - 1$	$\nu = 1/(3 - \Delta_\epsilon)$	$\omega = \Delta_{\epsilon'} - 3$

State-operator correspondence

Zhu, Han, Huffman, Hofmann, YCH, arXiv:2210.13482 (PRX)

- We identified 15 primary operators, the numerical errors of all primaries are within 1.6%.
- We looked at 70 lowest lying states with L<5, all of them match theoretical expectations with small errors~3%.

	CB	16 spins	Error
σ	0.518	0.524	1.2%
σ'	5.291	5.303	0.2%
$\sigma_{\mu_1\mu_2}$	4.180	4.214	0.8%
$\sigma'_{\mu_1\mu_2}$	6.987	7.048	0.9%
$\sigma_{\mu_1\mu_2\mu_3}$	4.638	4.609	0.6%
$\sigma_{\mu_1\mu_2\mu_3\mu_4}$	6.113	6.069	0.7%
σ^{P-}	NA	11.19	$-$

Bootstrap data from Simmons-Duffin, 2017

	CB	16 spins	Error
ϵ	1.413	1.414	0.07%
ϵ'	3.830	3.838	0.2%
ϵ''	6.896	6.908	0.2%
$T_{\mu\nu}$	3	3	$-$
$T'_{\mu\nu}$	5.509	5.583	1.3%
$\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	5.023	5.103	1.6%
$\epsilon'_{\mu_1\mu_2\mu_3\mu_4}$	6.421	6.347	1.2%
ϵ^{P-}	≤ 11.2	10.01	$-$

Welcome to the era of fuzzy sphere

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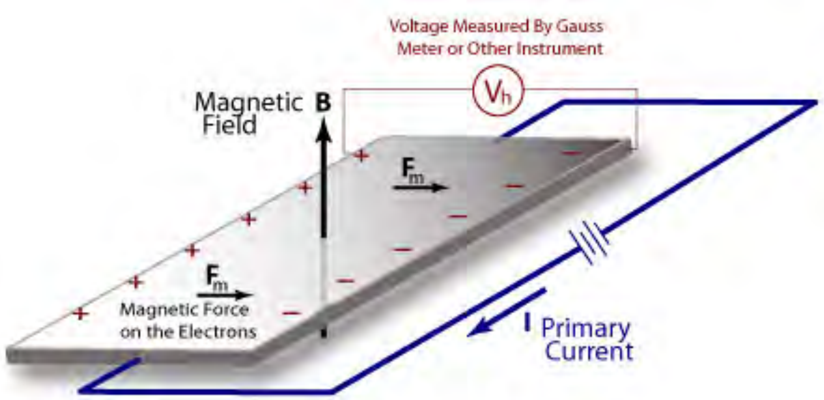
Outline

- Application of fuzzy sphere scheme to study open problems:
  - Chern-Simons-matter theories: phase transitions between quantum Hall states
  - Fermionic CFT and super-Ising
  - Conformal defect
- Conclusion and outlook

Fractional quantum Hall state



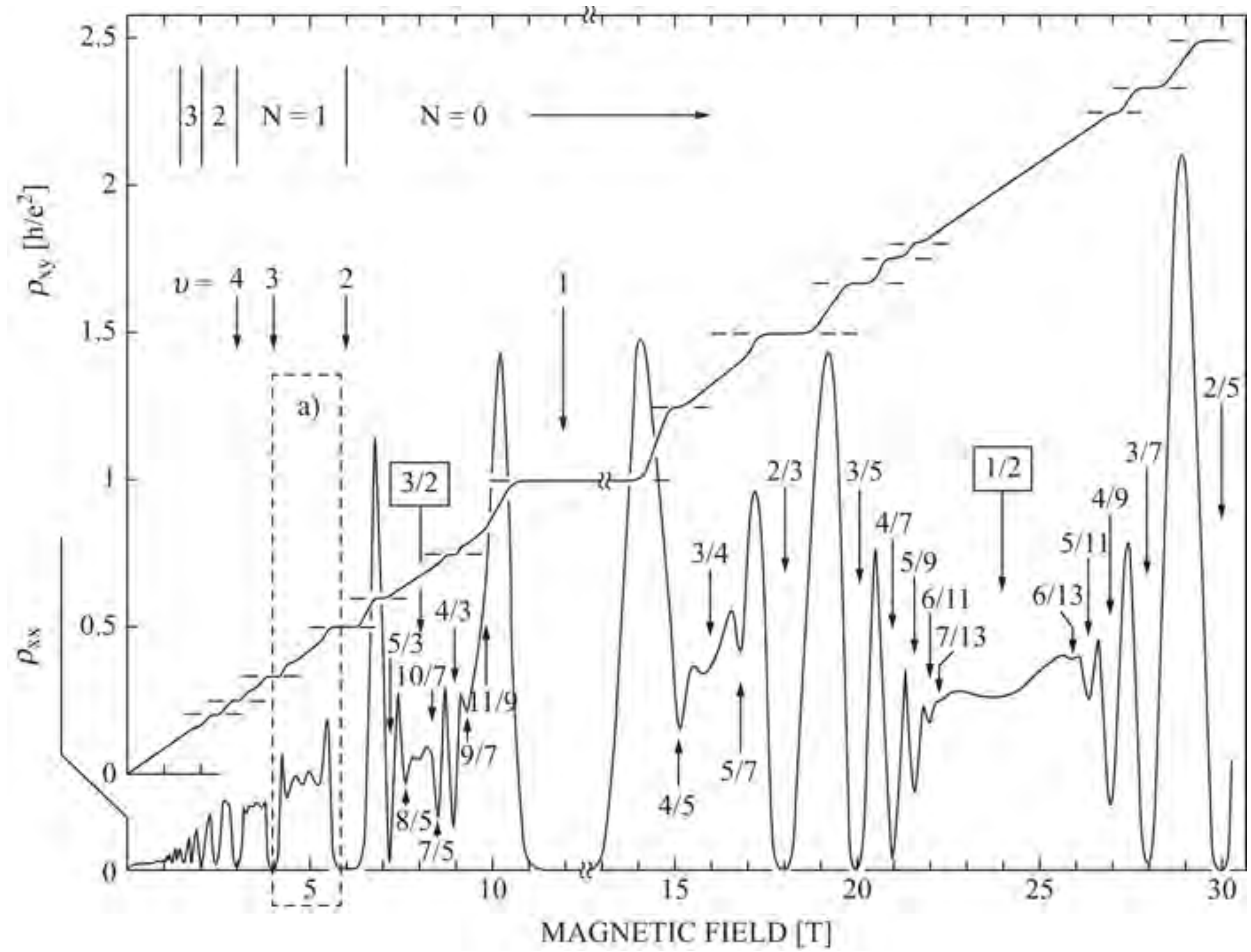
Classical

$$R_{xy} = 1/\sigma_{xy} \sim B$$

Quantum $\sigma_{xy} = \nu \dfrac{e^2}{h}$

Stormer, Tsui, Laughlin

TQFT $\dfrac{k}{4\pi} ada$



Jain sequence: 1/3, 2/5,…, 3/5, 2/3

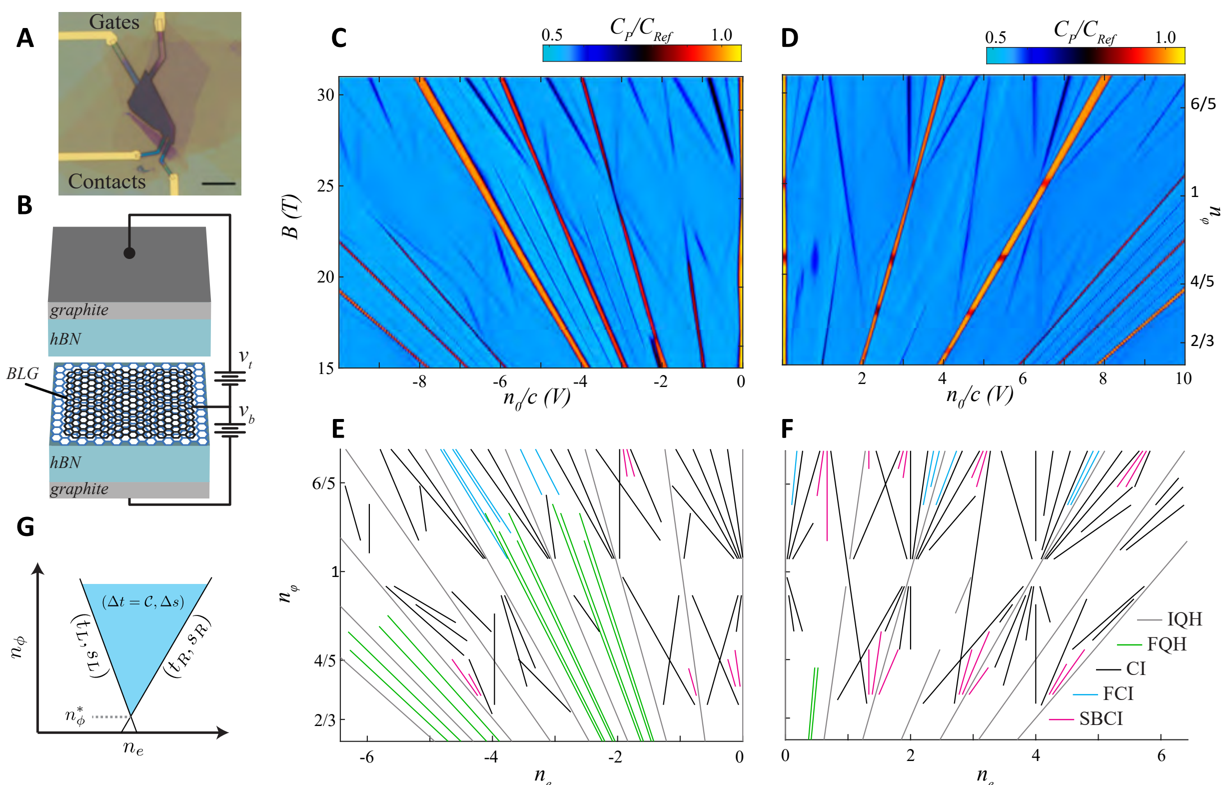
Phase transition?

Fractional quantum Hall on the lattice

Fractional Chern Insulators (FCI)

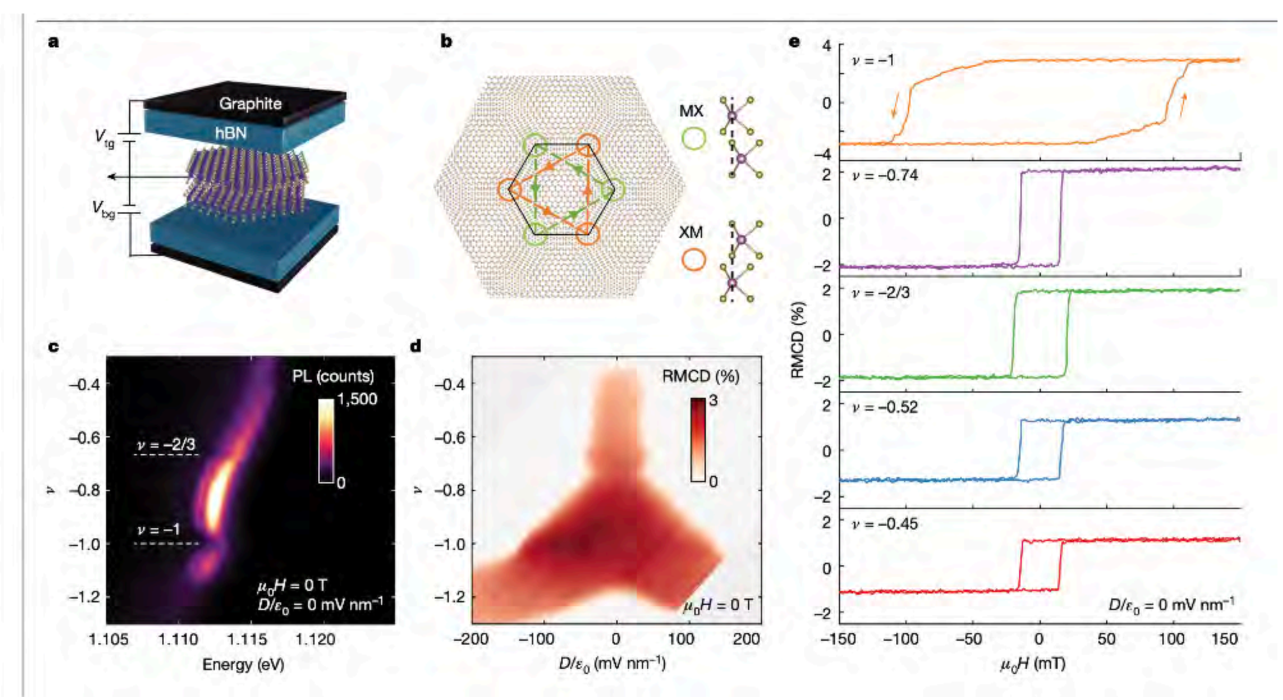
A. Young group, Science 2018

Lattice + Magnetic field



X. Xu group, Nature 2023

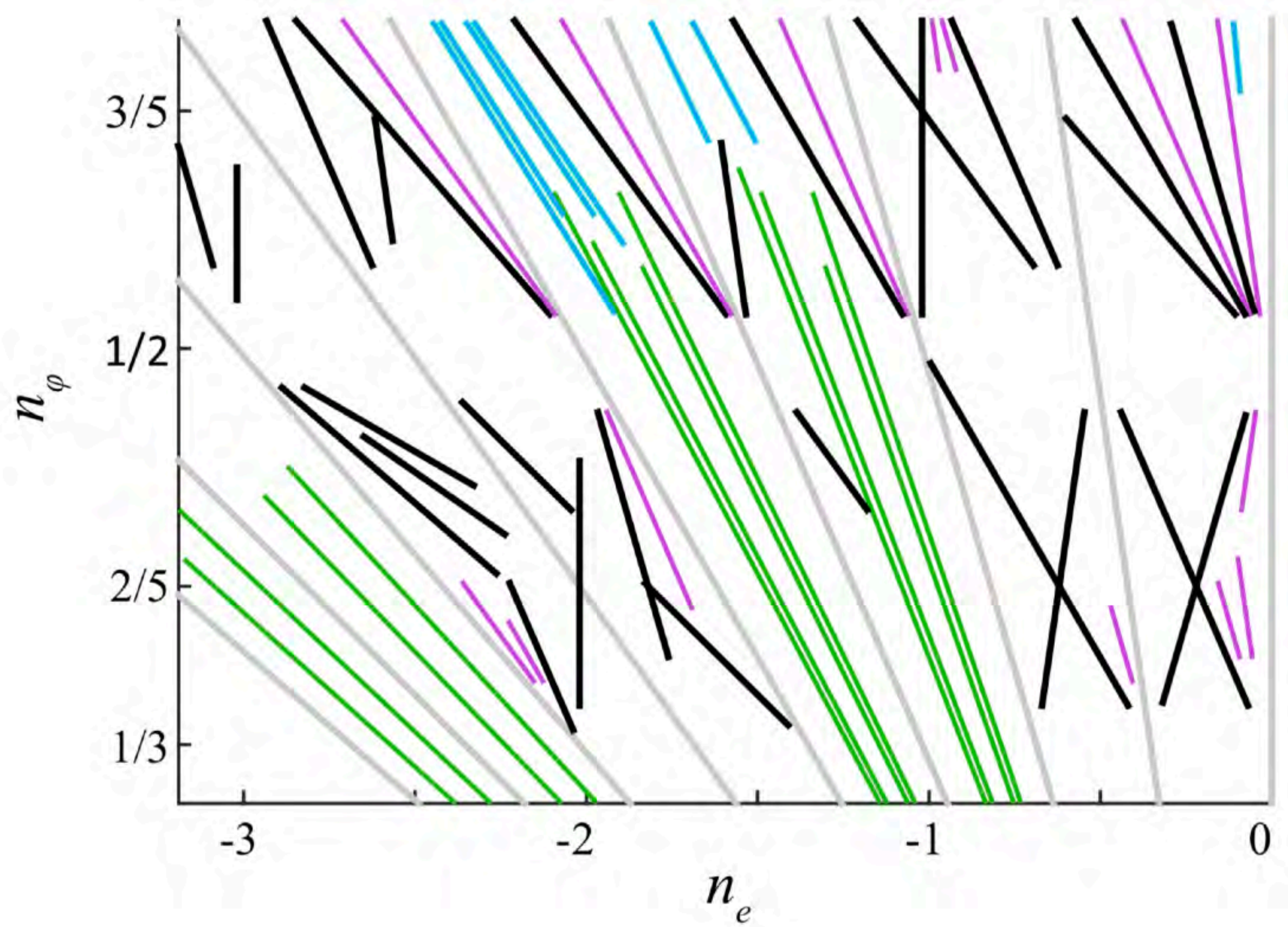
No magnetic field



New opportunities for studying quantum phase transitions (CFTs) in experiments.

Phase transition between FCI/FQH

$n = C\phi + s$



- Disorder is not necessary
- Lattice symmetry

New family of quantum critical points

Lee, Wang, Zaletel, Vishwanath, YCH 2018

$$\mathcal{L} = \sum_{I=1}^{N_f} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/ - m)\psi_I + \frac{K}{4\pi} ada + \cdots$$

Transition between Jain QH states.

Chern-Simons-matter theories

Confinement transition of chiral spin liquid

Kalmeyer-Laughlin chiral spin liquid
a.k.a. 1/2 bosonic Laughlin state

?

Trivial phase: gapped, no SSB,
no topological order

Candidate theories: gapless boson/fermion coupled to CS field

Wen, Wu 1993; Chen, Fisher, Wu 1993

One boson with $U(1)_2$ $$|(\partial_\mu - a_\mu)\phi|^2 - \frac{2}{4\pi} ada + \frac{1}{2} m^2 |\phi|^2 + u|\phi|^4$$

One fermion with $U(1)_{-3/2}$ $$\bar{\psi}\gamma_\mu (i\partial_\mu + a_\mu)\psi + \frac{3}{8\pi} ada + m\bar{\psi}\psi$$

"Simplest" interacting CS-matter theory

Is this a CFT, or is it even a continuous transition?

Duality conjectures

Confinement transition of Chiral spin liquid (1/2 bosonic Laughlin state)

Conjecture: a SO(3) CFT with four dual Lagrangians

Benini, Hsin, Seiberg 2017

One boson with $U(1)_2$

One fermion with $U(1)_{-3/2}$

A CFT with SO(3) symmetry?

One boson with $SU(2)_1$

One fermion with $SU(2)_{-1/2}$

Duality conjectures

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One fermion with $U(1)_{-3/2}$

A CFT with SO(3) symmetry?

One boson with $SU(2)_1$

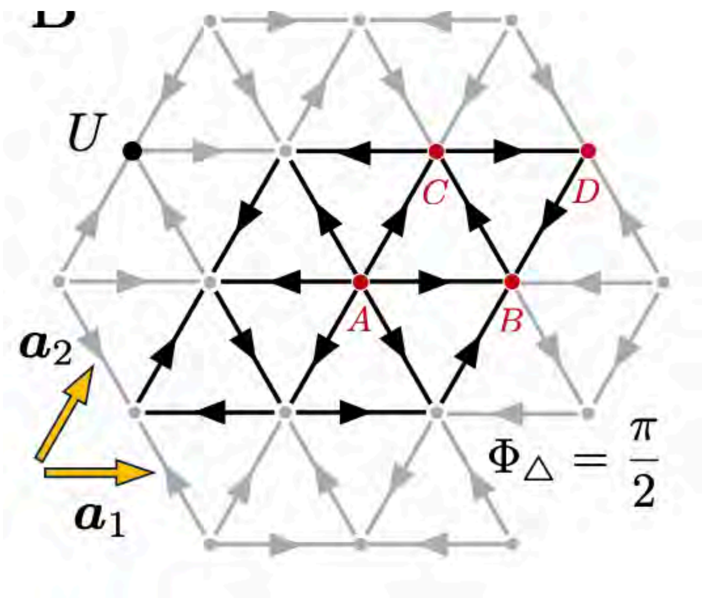
One fermion with $SU(2)_{-1/2}$

Doping leads to superconductors!!

Anyon Superconductivity from Topological Criticality in a Hofstadter-Hubbard Model

Stefan Divic,[1] Valentin Crépel,[2] Tomohiro Soejima (副島智大),[3] Xue-Yang Song,[4] Andrew Millis,[2,5] Michael P. Zaletel,[1,6] and Ashvin Vishwanath[3]

Superconductors from quantum criticality?



Hofstadter-Hubbard model

$$H = -\sum_{ij}(e^{i\theta_{ij}} c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i (c^\dagger_{i\uparrow} c_{i\uparrow} + c^\dagger_{i\downarrow} c_{i\downarrow})^2$$

Kuhlenkamp, Kadwo, Imamoglu, Knap, 2022;
Divic, Crepel, Soejima, Song, Millis, Zaletel, Vishwanath 2025

C=2 Chern insulator

$\kappa_{xy} = 2$

Chiral spin liquid

Semions -1

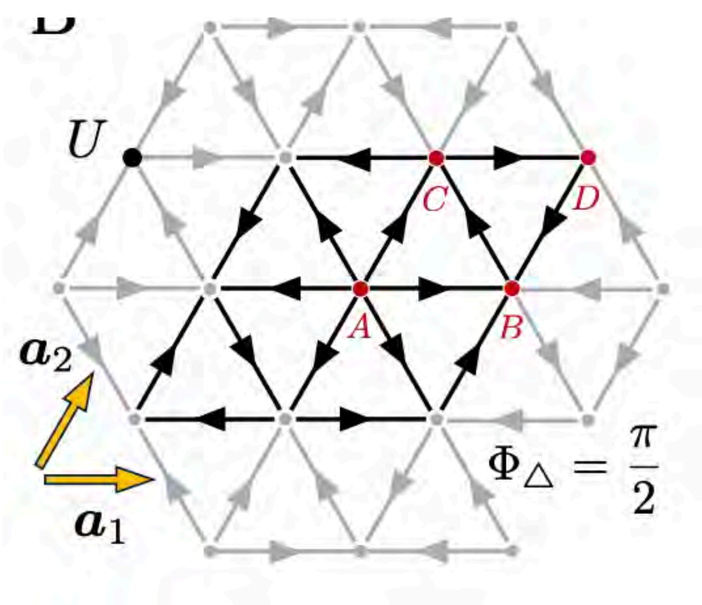
U

$\kappa_{xy} = 1$

At phase transition: Single electron gap: $\Delta E_{c_\sigma} > 0$
Cooper pair gap: $\Delta E_{c_\uparrow c_\downarrow} = 0$

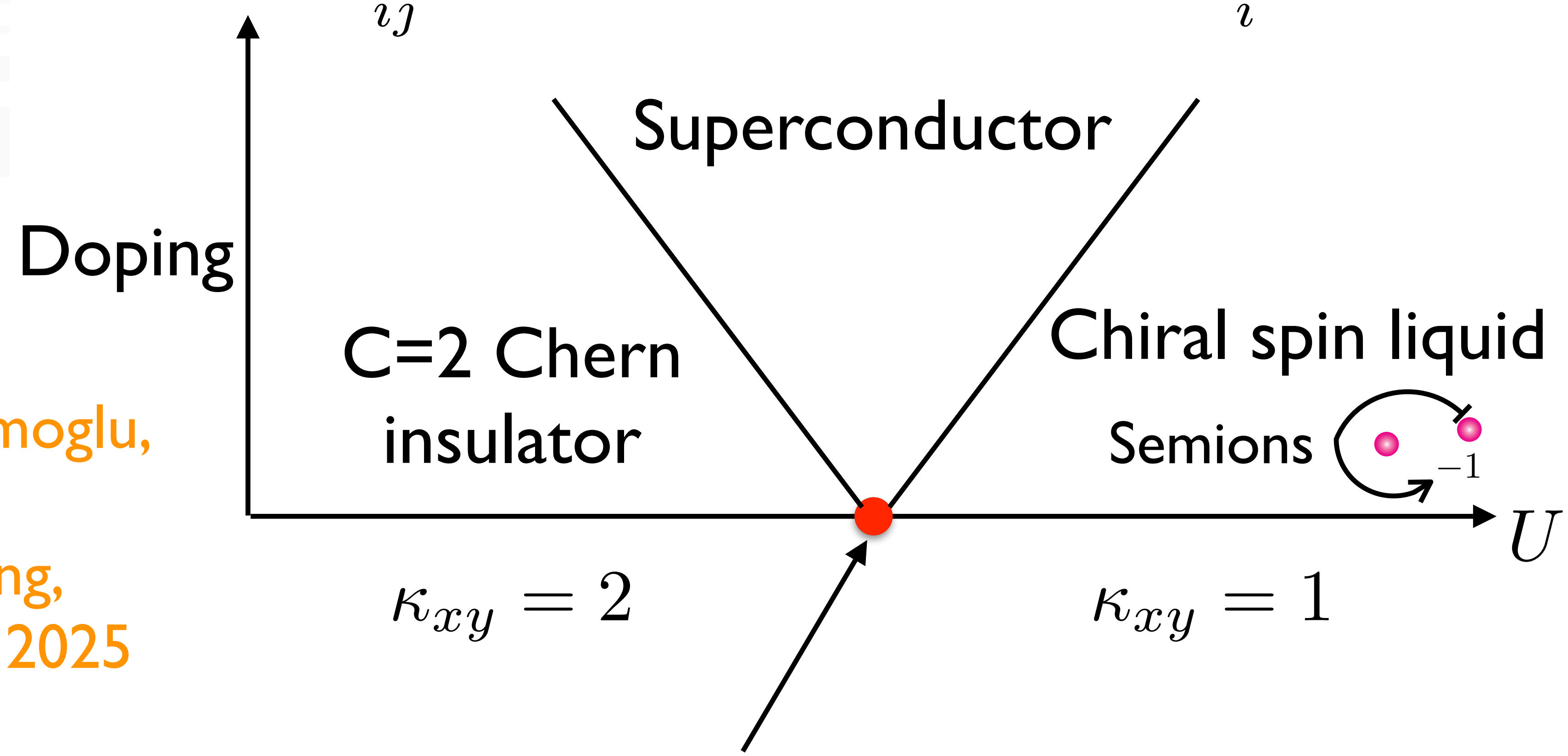
Key question: Is it really a continuous phase transition?

Superconductors from quantum criticality?



Hofstadter-Hubbard model

$$H = -\sum_{ij} (e^{i\theta_{ij}} c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i (c^\dagger_{i\uparrow} c_{i\uparrow} + c^\dagger_{i\downarrow} c_{i\downarrow})^2$$



Kuhlenkamp, Kadwo, Imamoglu, Knap, 2022;
Divic, Crepel, Soejima, Song, Millis, Zaletel, Vishwanath 2025

At phase transition:
Single electron gap: $\Delta E_{c_\sigma} > 0$
Cooper pair gap: $\Delta E_{c_\uparrow c_\downarrow} = 0$

Key question: Is it really a continuous phase transition?

Make it fuzzified

Zhou, Wang, YCH arXiv:2507.19580

Boson-fermion mixture system

Relevant to Feshbach resonance in cold atom systems.

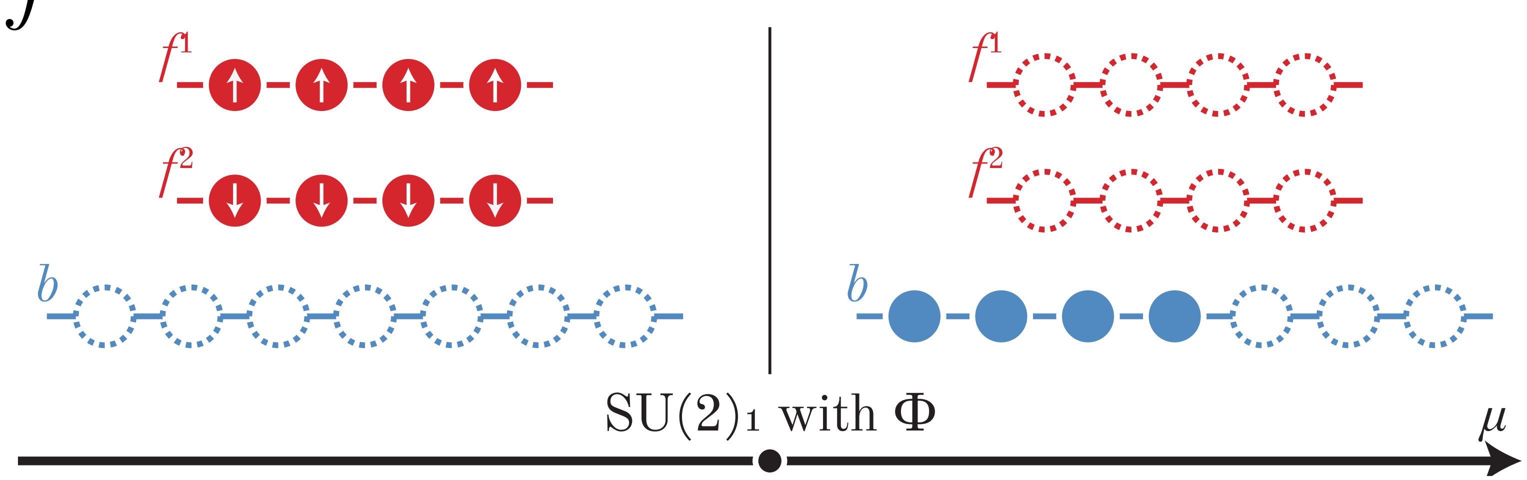
Liou, Hu, Yang 2018

Two flavours of fermions | One flavour of boson

$b^\dagger f^1 f^2 + \text{h.c.}$

Electric charge Q=1 | Electric charge Q=2

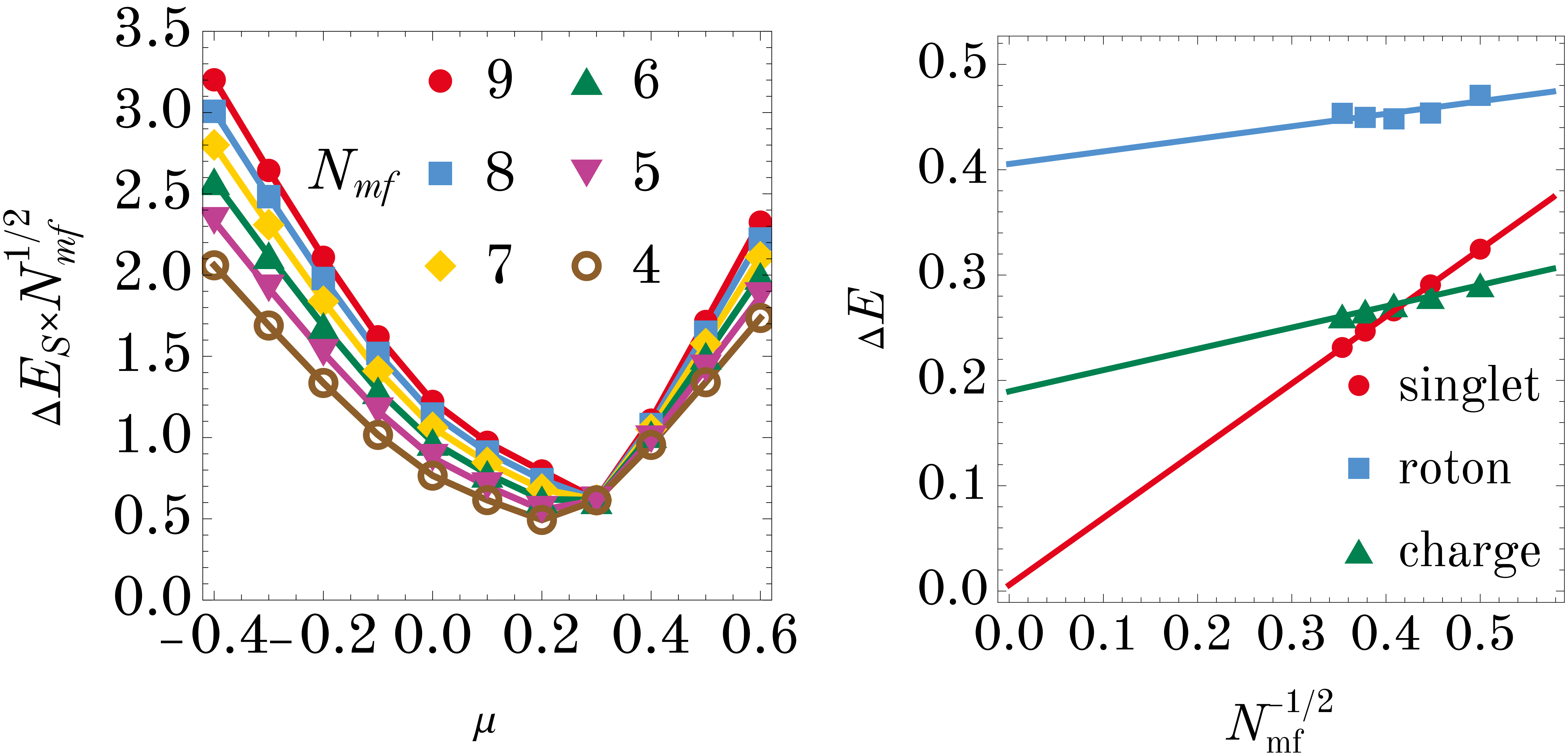
$$H = \int d^2\vec{r}\left[(n_b + 2n_f)^2 - \frac{1}{2}(b^\dagger f^1 f^2 + h.c.) + \mu\, n_f\right]$$



$\nu = 2$ Fermionic Integer QH | $\nu = 1/2$ Bosonic Laughlin state

$$H = \int d^2\vec{r}\,\left[(n_b + 2n_f)^2 - \tfrac{1}{2}(b^\dagger f^1 f^2 + h.c.) + \mu\, n_f\right],$$

Gap closing for CFT states $\Delta_{\text{singlet}} \sim \dfrac{1}{R} \sim N_{mf}^{-1/2}$



$$H = \int d^2\vec{r}\left[(n_b + 2n_f)^2 - \tfrac{1}{2}(b^\dagger f^1 f^2 + h.c.) + \mu\, n_f\right],$$

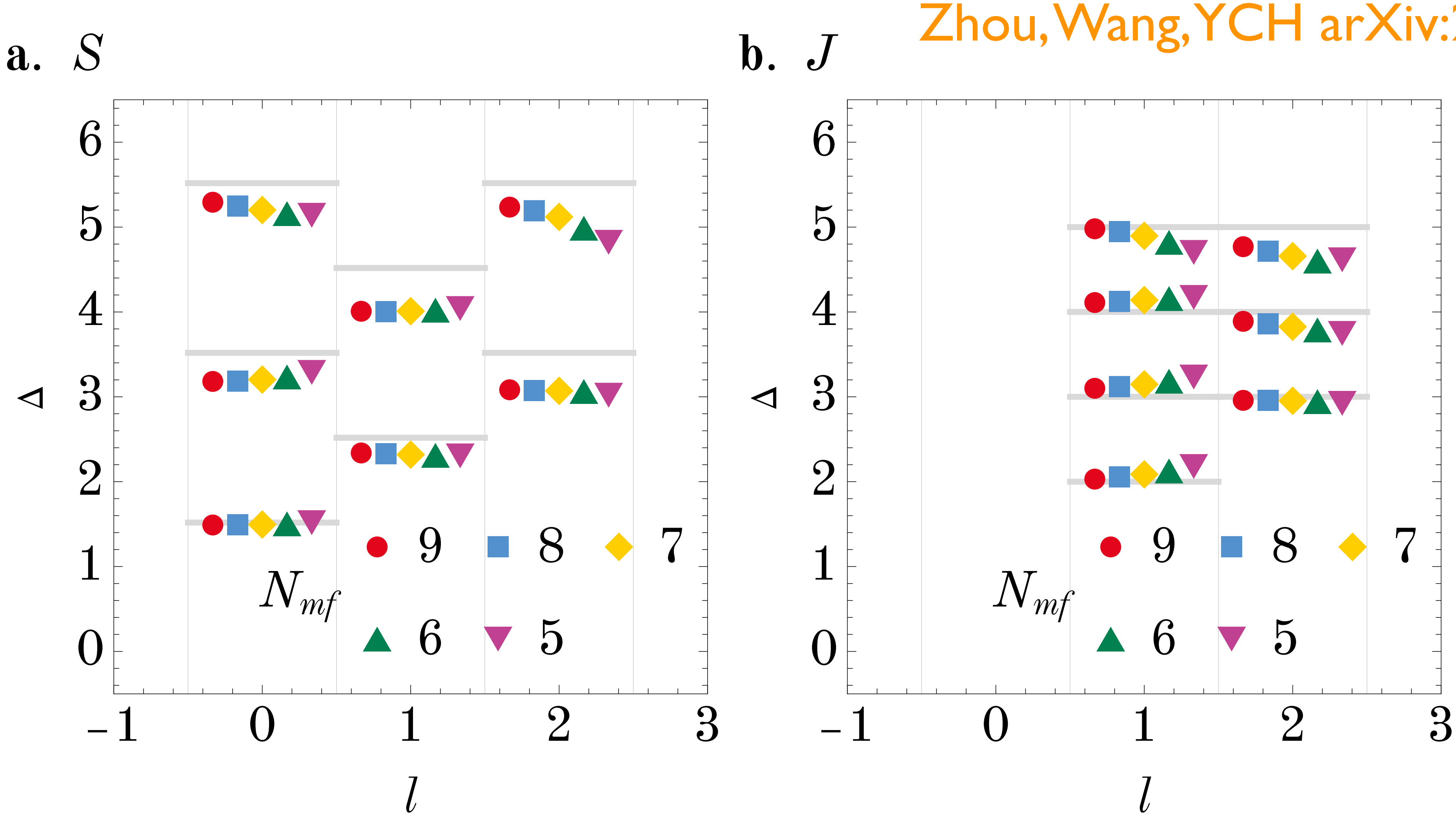
$$\mu_c = 0.312$$

Two relevant operators:

S: tuning operator for the transition $\Delta_S = 1.52(18)$

J_μ: $SO(3)$ conserved current, $\Delta = 2$

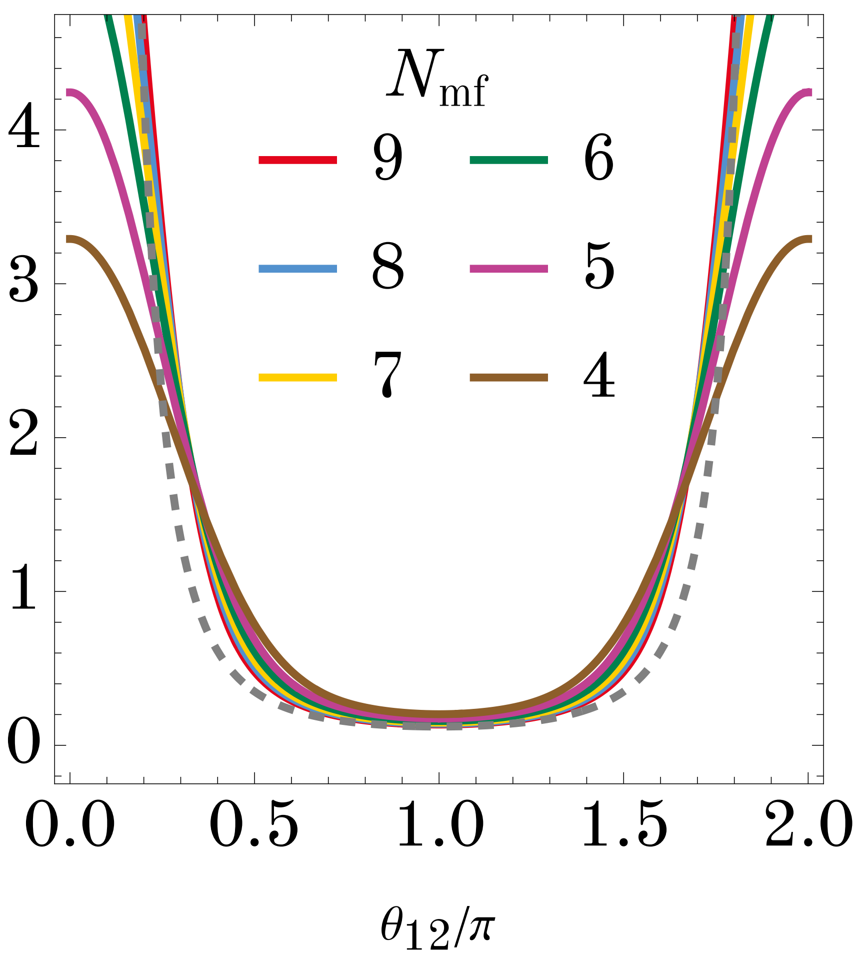
Zhou, Wang, YCH arXiv:2507.19580

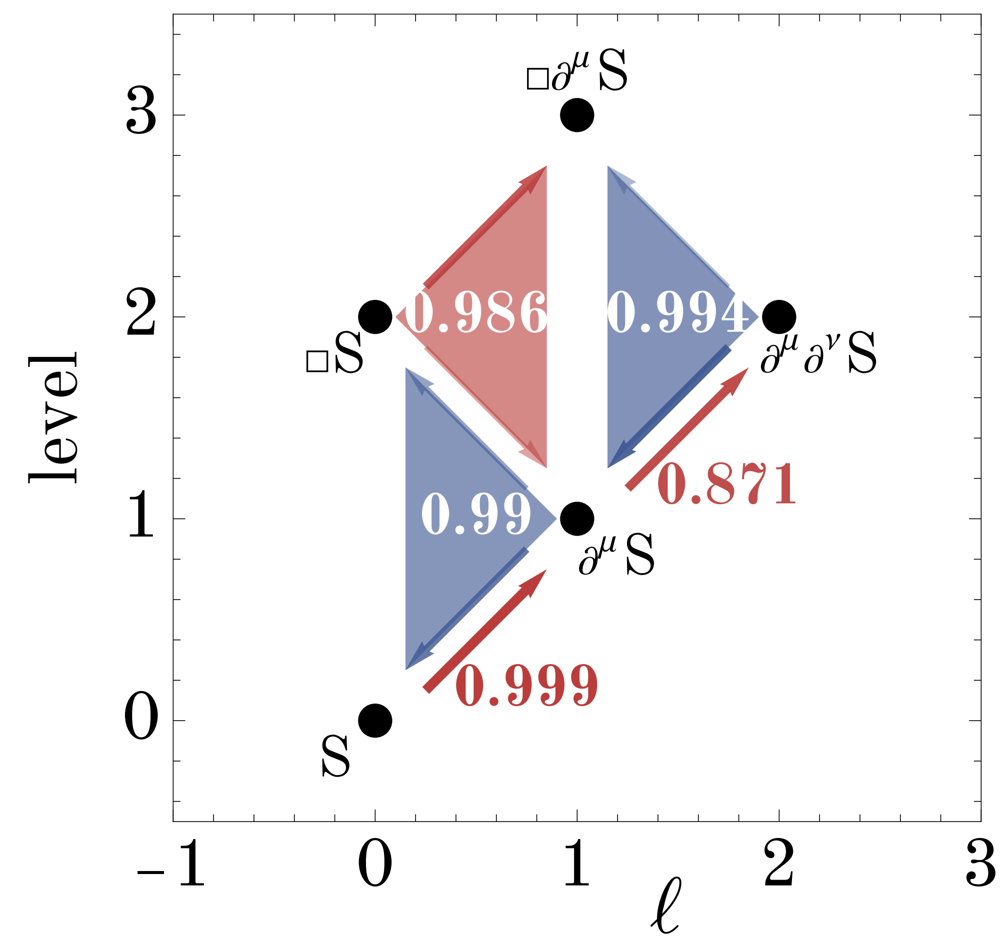


This transition is continuous and conformal.

More evidence

- Conformal correlators of local operators
- Robustness of the scaling dimensions at different parameter points.
- Conformal generators. $O \underset{K_\mu}{\overset{P_\mu}{\rightleftarrows}} \partial_\mu O \qquad P^\mu + K^\mu = \int \mathbf{d}^2\mathbf{x}\, x^\mu \mathcal{H}(\mathbf{x})$





Outline

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  - Fermionic CFT and super-Ising
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Fermionic CFT

- CFTs with local fermionic operators.

$$\mathcal{L} = \frac{1}{2}\bar{\chi}(i\gamma^\mu \partial_\mu)\chi$$

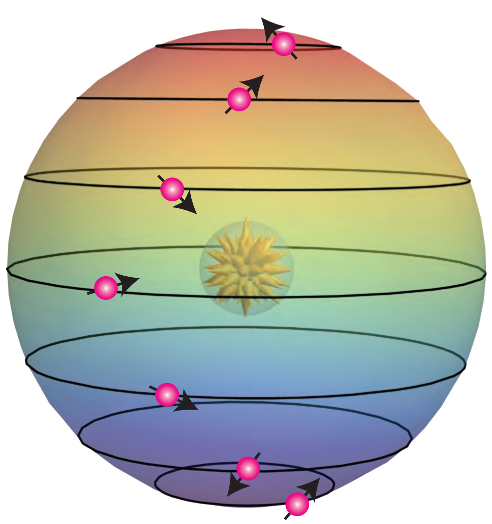
Free Majorana fermion

$$\mathcal{L} = \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}\bar{\chi}i\gamma^\mu\partial_\mu\chi + \frac{g}{2}\sigma\bar{\chi}\chi + \frac{\lambda}{4!}\sigma^4$$

Gross-Neveu-Yukawa CFT, super-Ising

- More relevant for materials and has cleaner experimental diagnostics.

- Challenging for fuzzy sphere approach.



Magnetic spatial symmetries forbids microscopic particles (fermions) to be be a part of CFT.

CFT operators are charge neutral.

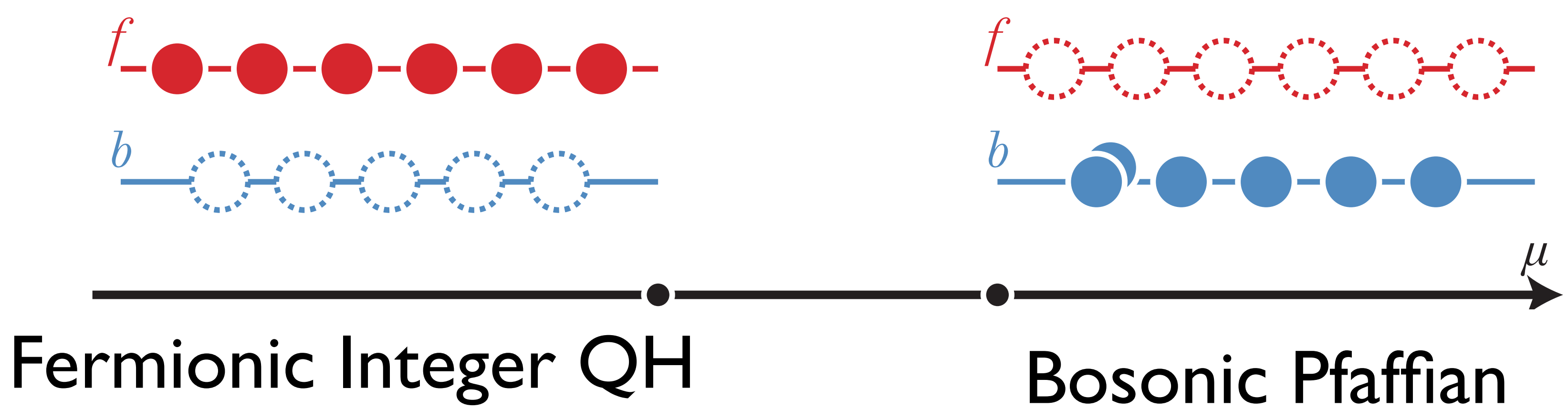
$$n^\alpha(\vec{x}) = (\hat{\psi}^\dagger_\uparrow(\vec{x}),\, \hat{\psi}^\dagger_\downarrow(\vec{x}))\, \sigma^\alpha \begin{pmatrix} \hat{\psi}_\uparrow(\vec{x}) \\ \hat{\psi}_\downarrow(\vec{x}) \end{pmatrix}$$

Fermionic CFT on fuzzy sphere

Boson-fermion mixture

$$H = \int d^2\vec{x}\ \left[(n_b+n_f)^2 + t\left(b^\dagger b^\dagger f(D_\theta + iD_\phi)f + h.c.\right) + \mu n_f\right]$$

Zhou, Gaiotto, YCH, arXiv: 2509.08038

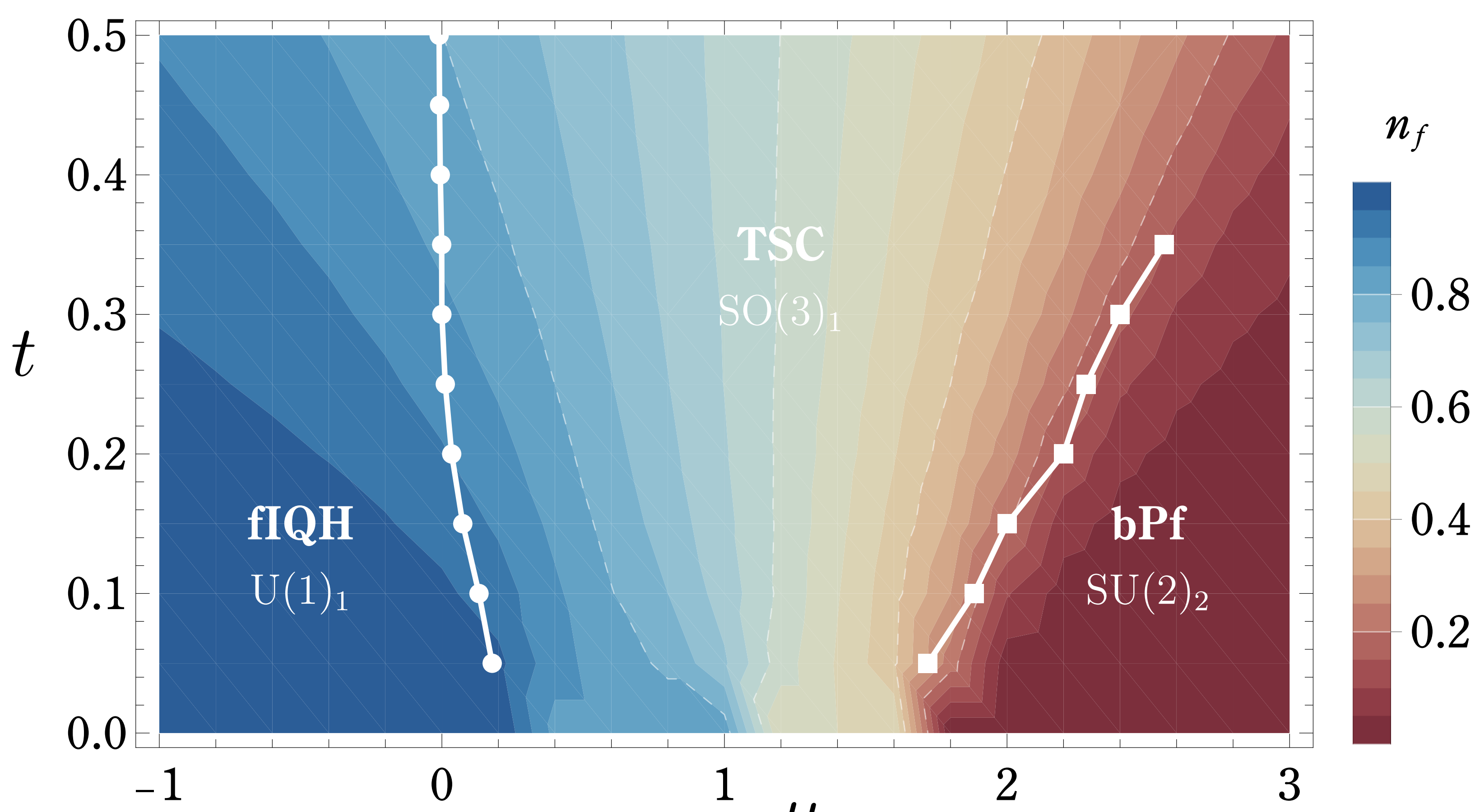


Local fermion $f^\dagger b, b^\dagger f$

Angular momentum of boson and fermion has 1/2 mismatch*.

↓

Spin-statistics theorem

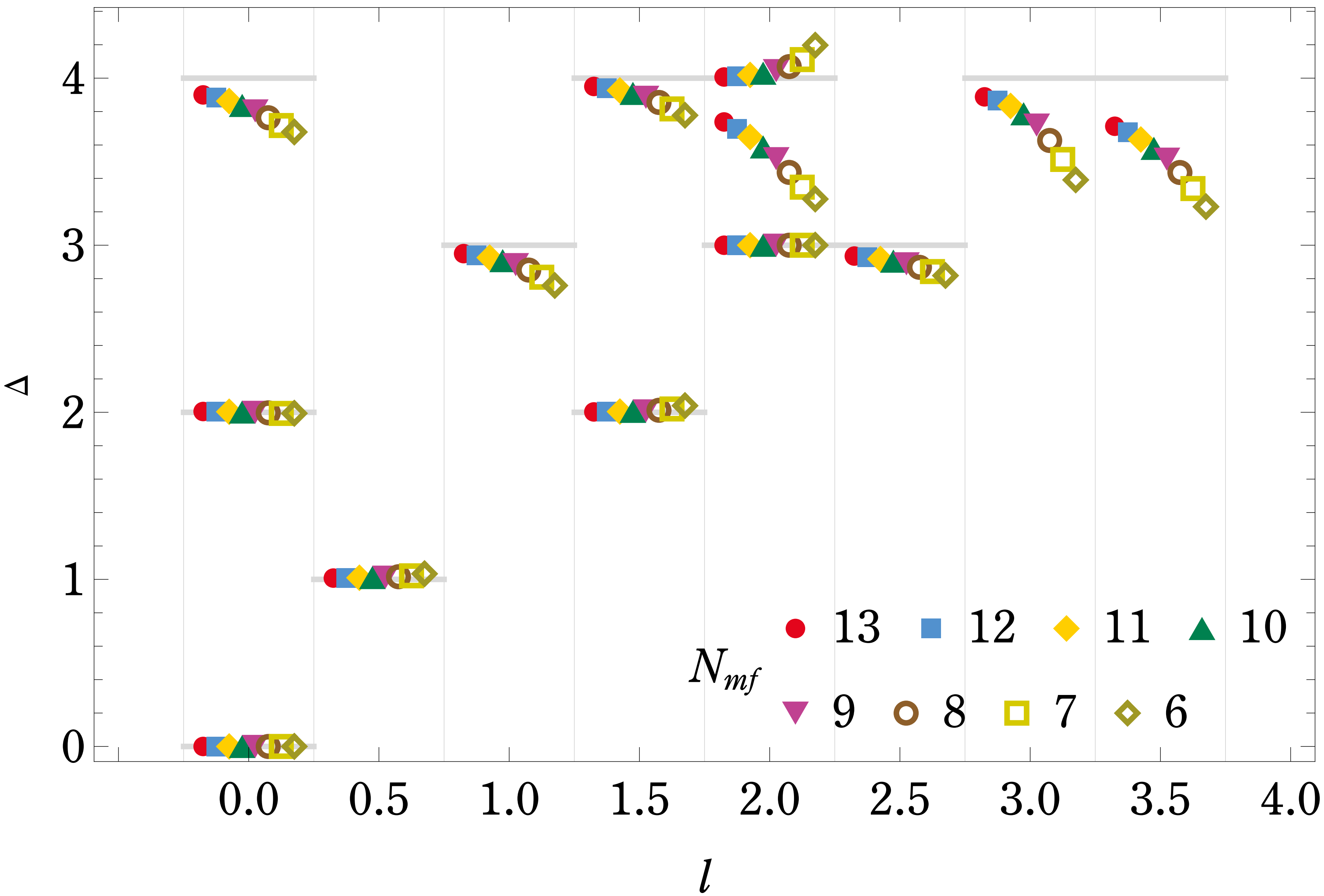


*Wen-Zee shift

Operator spectrum

$$H = \int d^2\vec{x}\ \left[(n_b+n_f)^2 + t\left(b^\dagger b^\dagger f(D_\theta + iD_\phi)f + h.c.\right) + \mu n_f\right]$$

For $t \in (0.2, 0.5)$ the free Majorana CFT appears at $\mu = 0$.



Two-component Fermionic spinor

$$\chi_{1/2} \sim b^\dagger f$$

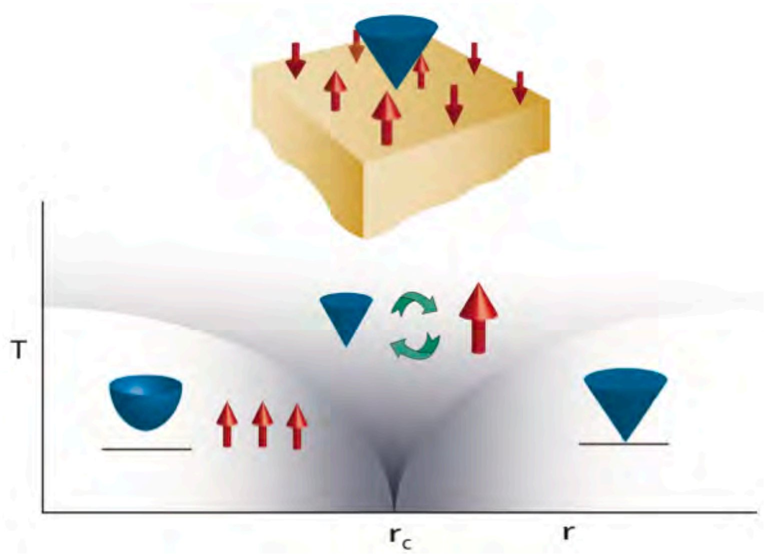
$$\chi_{-1/2} \sim f^\dagger b$$

$$H = \int d^2\vec{x}\ \left(\chi^T \gamma^\mu \partial_\mu \chi\right)$$

Super-Ising

Emergent supersymmetry

Grover, Sheng, Vishwanath, 2014

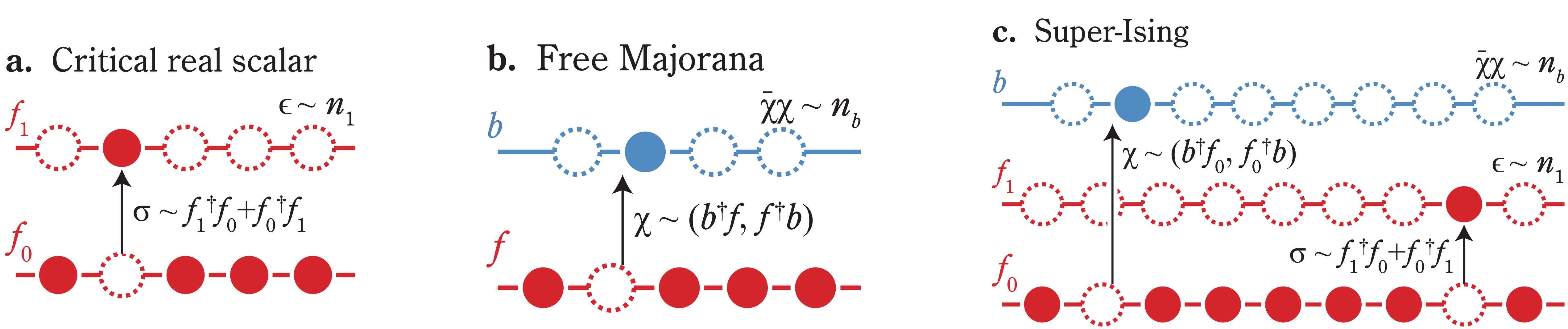


Majorana fermion coupled to critical scalar (Ising)

$$\mathcal{L} = \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}\bar{\chi} i\gamma^\mu\partial_\mu\chi + \frac{g}{2}\sigma\bar{\chi}\chi + \frac{\lambda}{4!}\sigma^4$$

Potential realization on the surface of 3+1D topological superconductor.

Fuzzy sphere realization



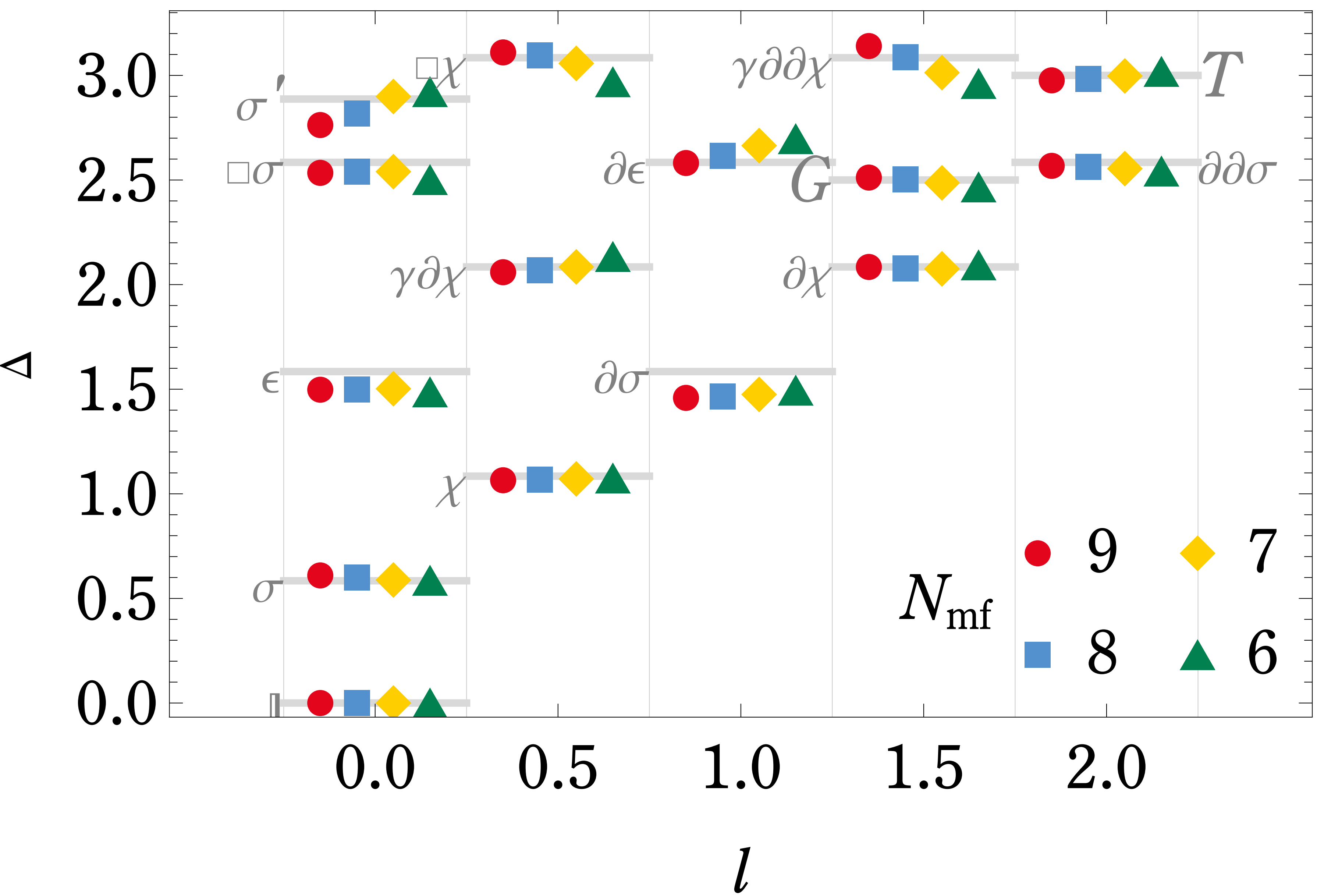
$$H = H_1 + H_2$$

$$H_1 = \int d^2\vec{r}\, \left[n_e^2(\vec{r}) + t\,(\eta(\vec{r})D_+\eta(\vec{r}) + h.c.) + U n_x(\vec{r})\nabla^2 n_x(\vec{r}) + g n_x(\vec{r}) n_b(\vec{r})\right]$$

Fermi kinetic term   Scalar interaction   Yukawa

$$H_2 = \int d^2\vec{r}(-h n_x(\vec{r}) - \mu_1 n_1(\vec{r}) - \mu_b n_b(\vec{r}))$$

Tuning of 3 relevant perturbation.



$t = 1.5, U = 0.25, g = 1$

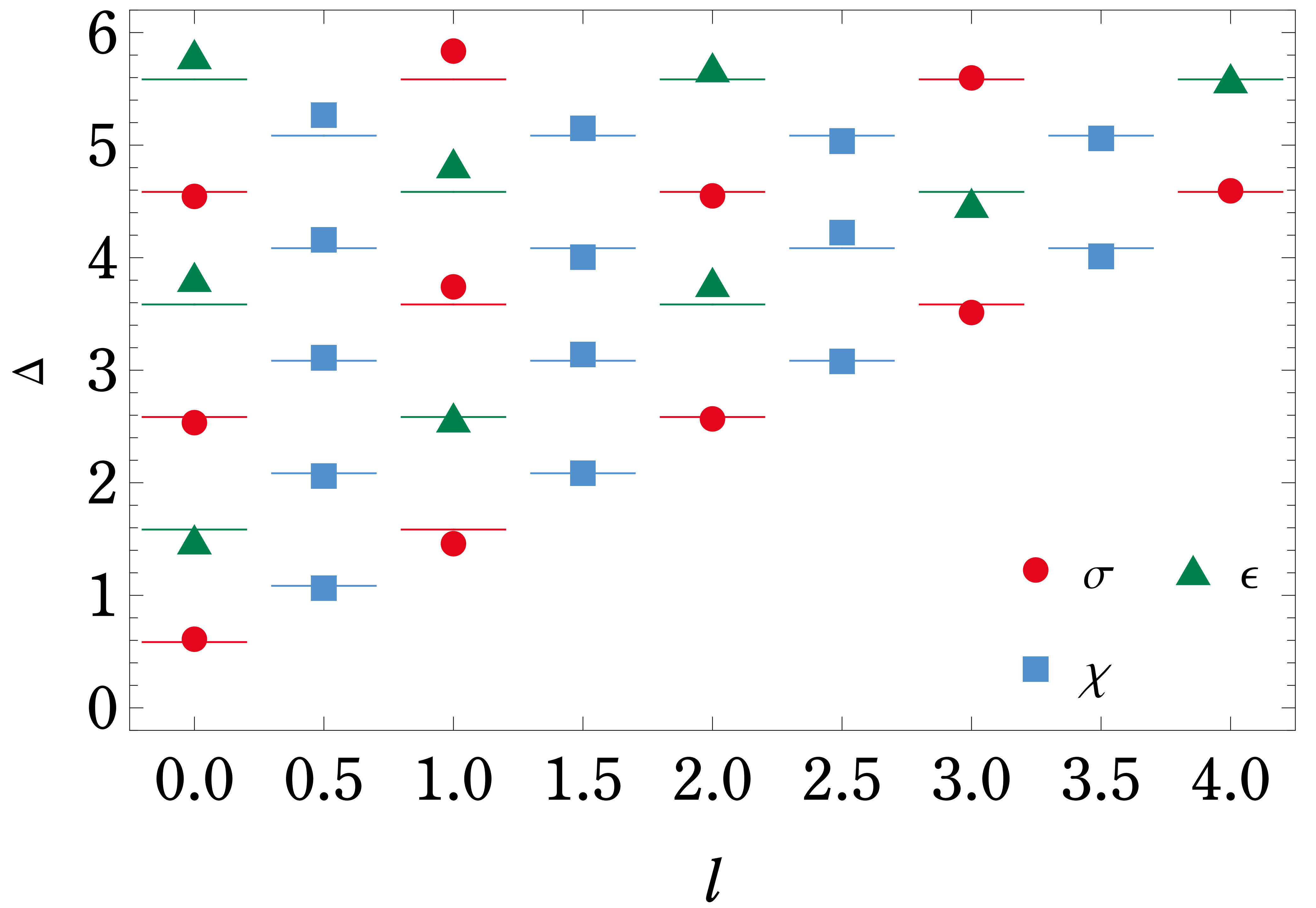
Zhou, Gaiotto, YCH 2025

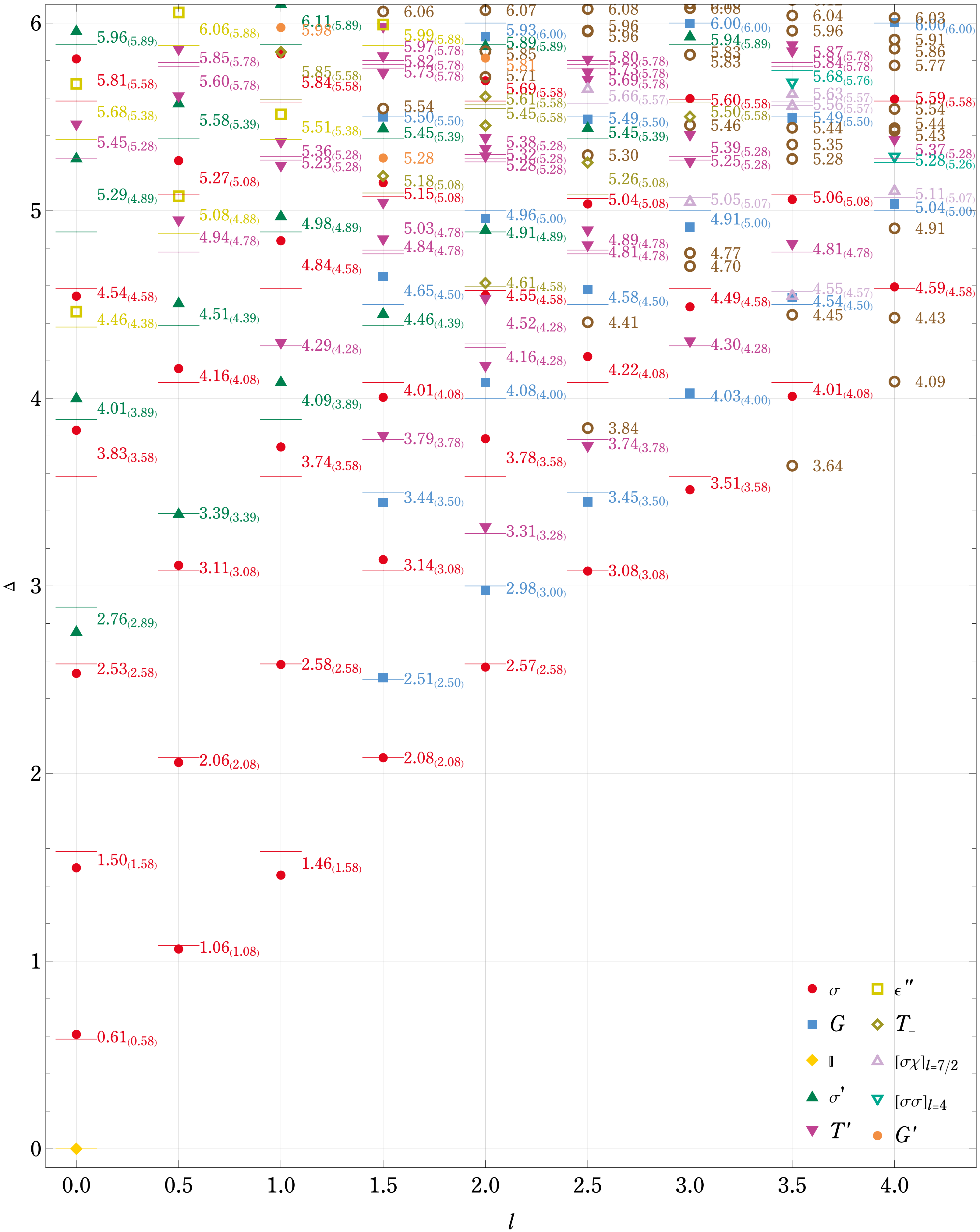
N_{mf}	h	μ_1	μ_b
9	0.0680	0.0809	0.0776
8	0.0690	0.0792	0.0757
7	0.0698	0.0774	0.0722
6	0.0662	0.0793	0.0615

Bootstrap data

Atanasov, Hillman, Poland, Rong, Su

a. Super-multiplet of σ





$\Delta \leq 6$ states are all CFT states

Outline

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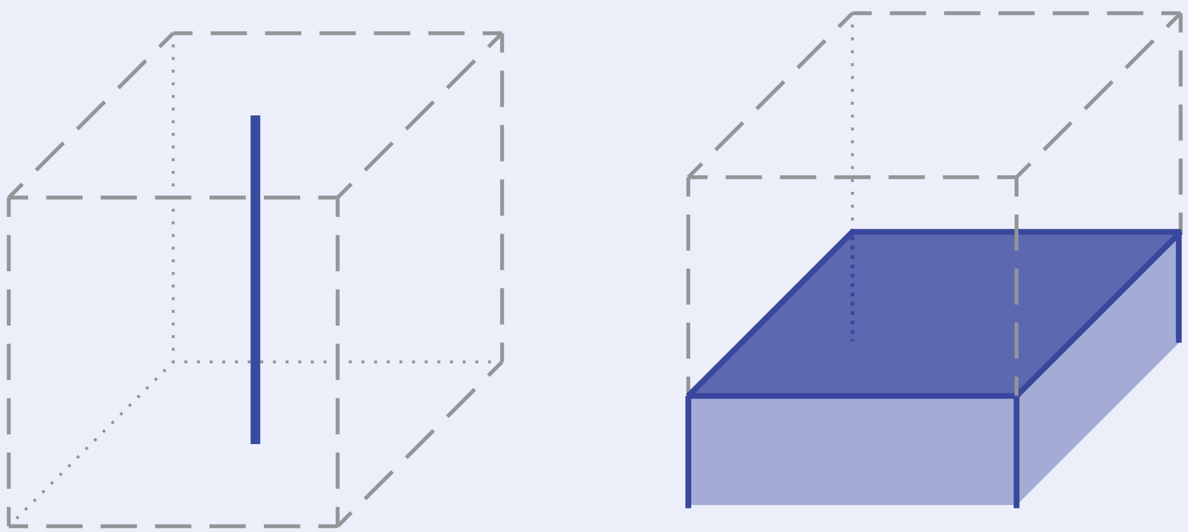
Conformal defect and boundary

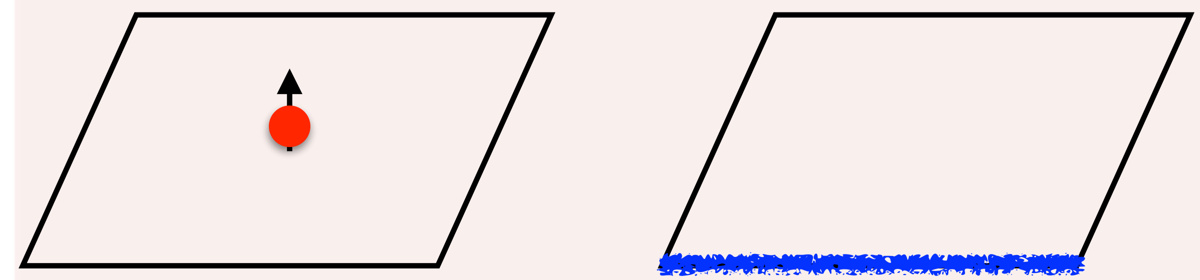
Classical system



Line defect

Surface criticality

Quantum system



Kondo impurity

Surface criticality

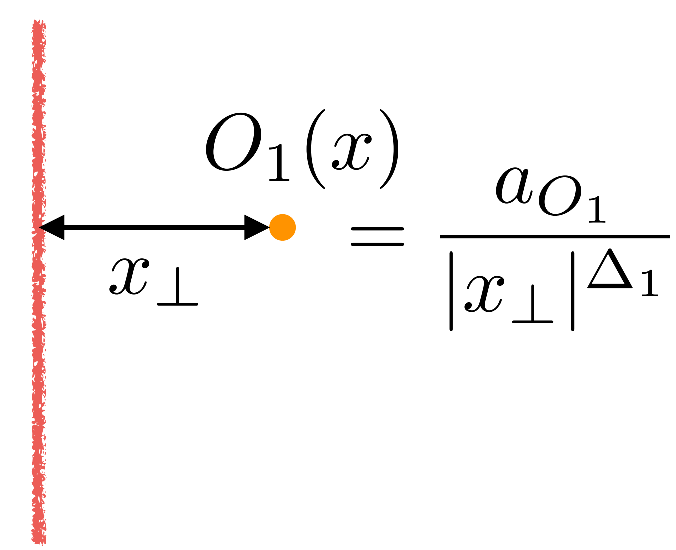
Bulk conformal symmetry

Defect conformal symmetry

$$SO(d+1,1) \to SO(p+1,1) \times SO(d-p)$$

- New operators living on the defect.

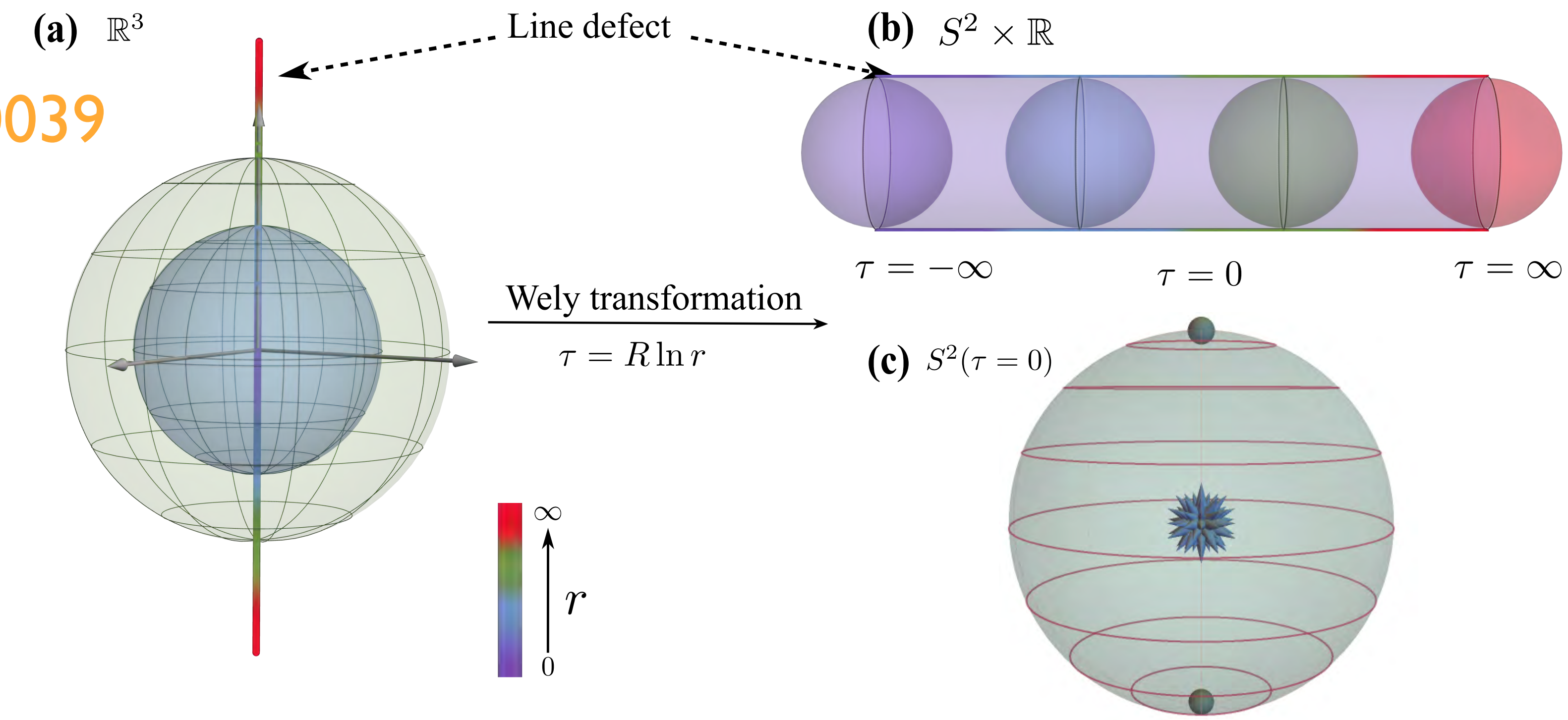
- Non-trivial interplay between bulk and defect.



Solving conformal defect using fuzzy sphere

Hu, YCH, Zhu, arXiv:2308.01903
Zhou, Gaiotto, YCH, Zou, arXiv:2401.00039
YCH, Komargodski, arXiv:2406.10186

State-operator correspondence still works for conformal defects.



Magnetic line defect of 3D Ising: $H = H_{\text{bulk}} + H_{\text{defect}}$

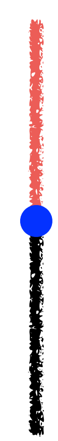
$$H_{\text{defect}} = -h_N n^z(N) - h_S n^z(S)$$

Defect operators

$h_N = h_S > 0$

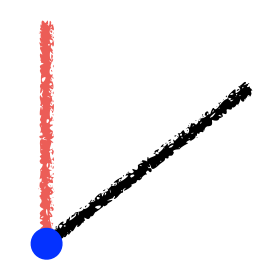
Defect creation

$h_N > 0, h_S = 0$



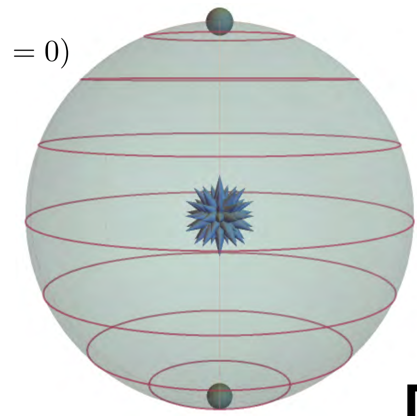
Defect changing

$h_N > 0, h_S < 0$



Cusp anomalous dimension

State-operator correspondence



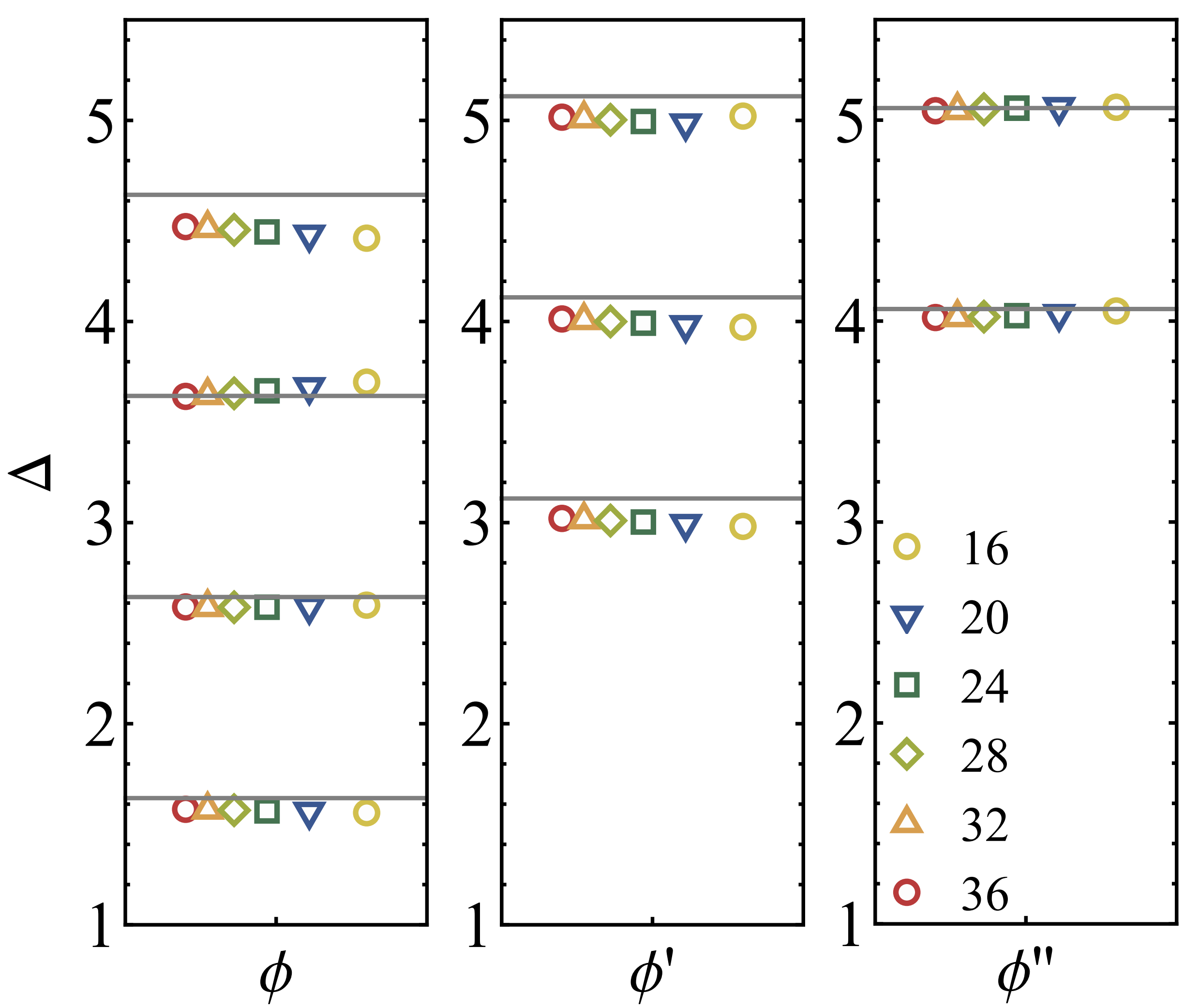
$$H = H_{\mathrm{bulk}} + H_{\mathrm{defect}}$$

$$H_{\mathrm{defect}} = -h_N n^z(N) - h_S n^z(S)$$

Defect conformal symmetry: $SO(2,1)$

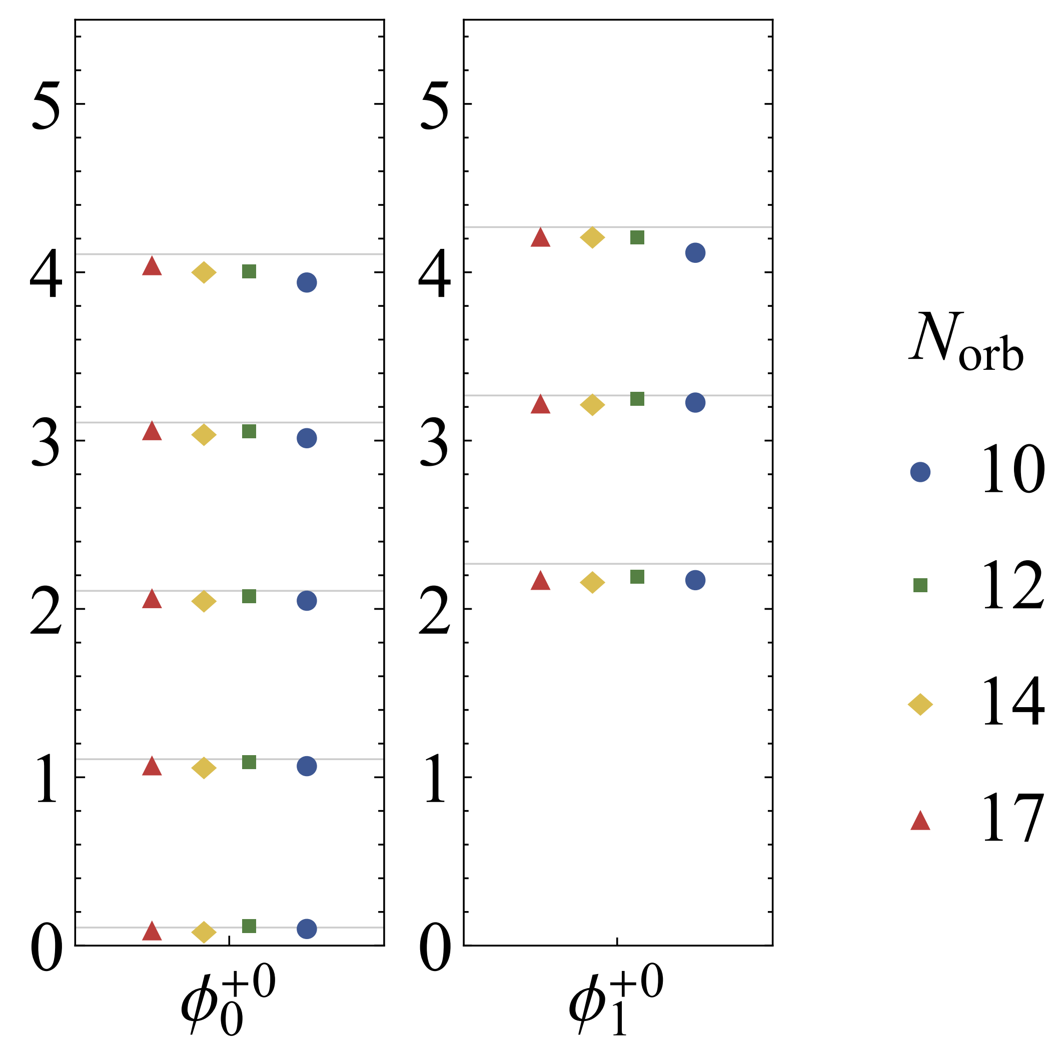
Hu, YCH, Zhu, arXiv:2308.01903

Defect operators $h_N = h_S > 0$



Zhou, Gaiotto, YCH, Zou, arXiv:2401.00039

Defect creation $h_N > 0, h_S = 0$



g-function (defect central charge)

g-function: $g = \frac{Z_{\mathrm{dCFT}}}{Z_{\mathrm{CFT}}}$ Monotonic under RG flow of defect

- 2D bulk: Affleck & Ludwig 1991; Friedan & Konechny 2004
- 3D bulk: Cuomo, Komargodski, Raviv-Moshe 2022
- General dim bulk: Casini, Landea, Torroba 2023

Our results: $g = 0.602(2)$

Zhou, Gaiotto, YCH, Zou, arXiv:2401.00039

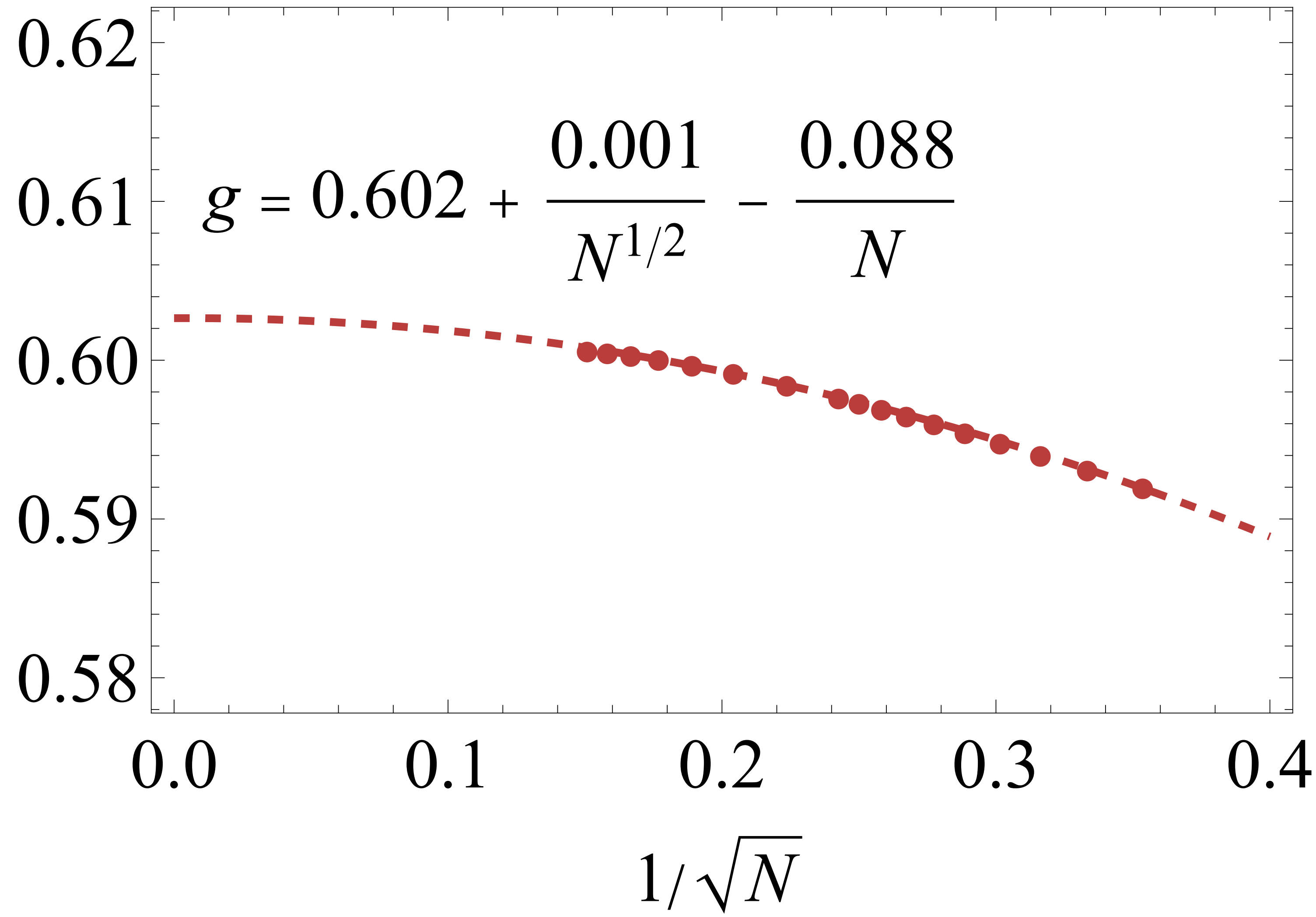
Bootstrap confirms our prediction

$$g = 0.605(32)$$

Lanzetta, Liu, Metlitski 2025

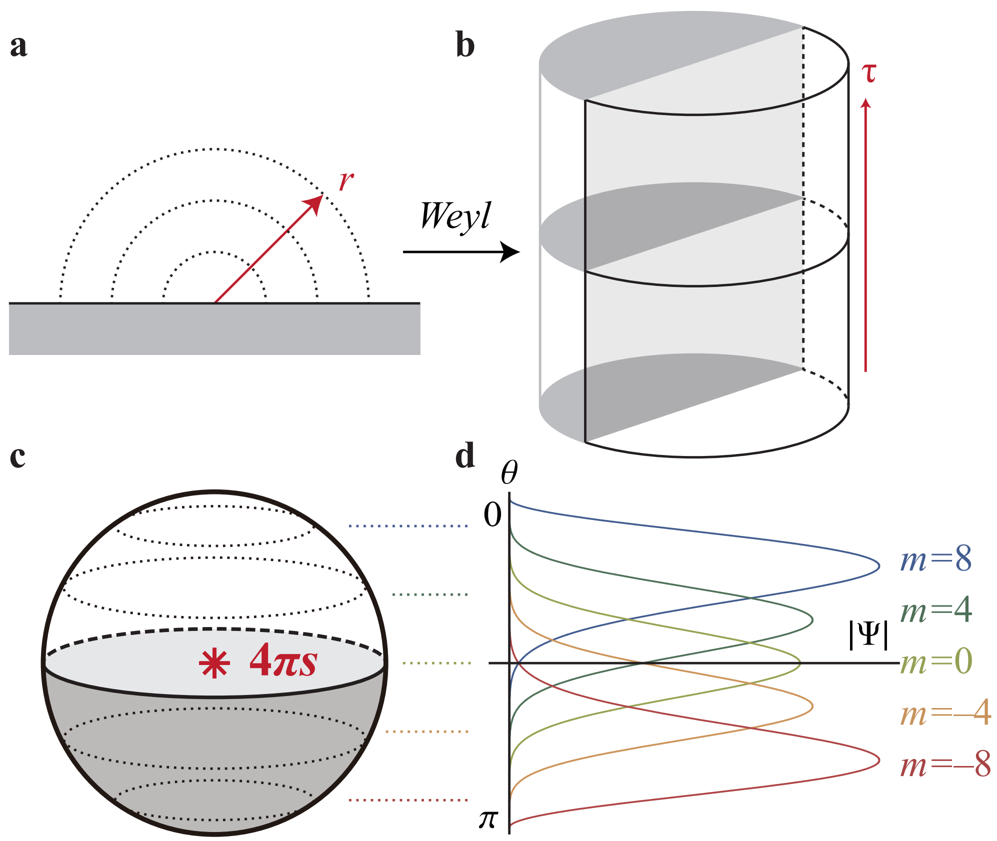
ϵ expansion: $g = 0.57 + O(\epsilon^2)$

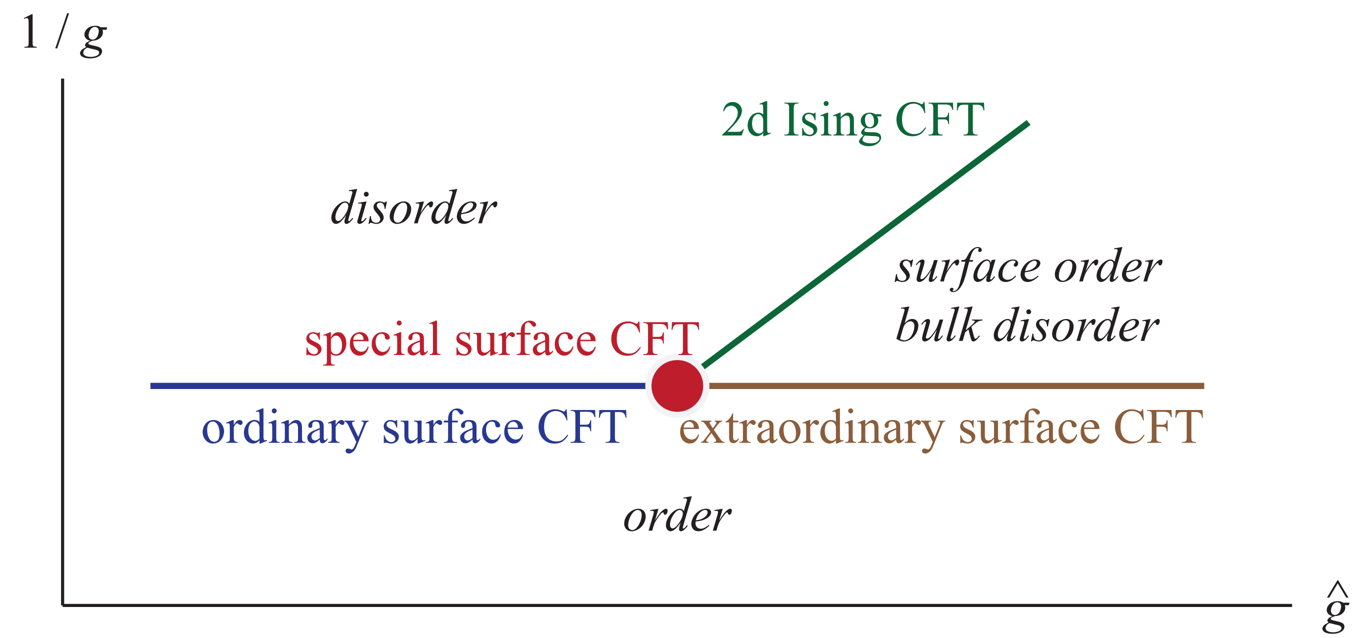
Cuomo, Komargodski & Mezei 2022

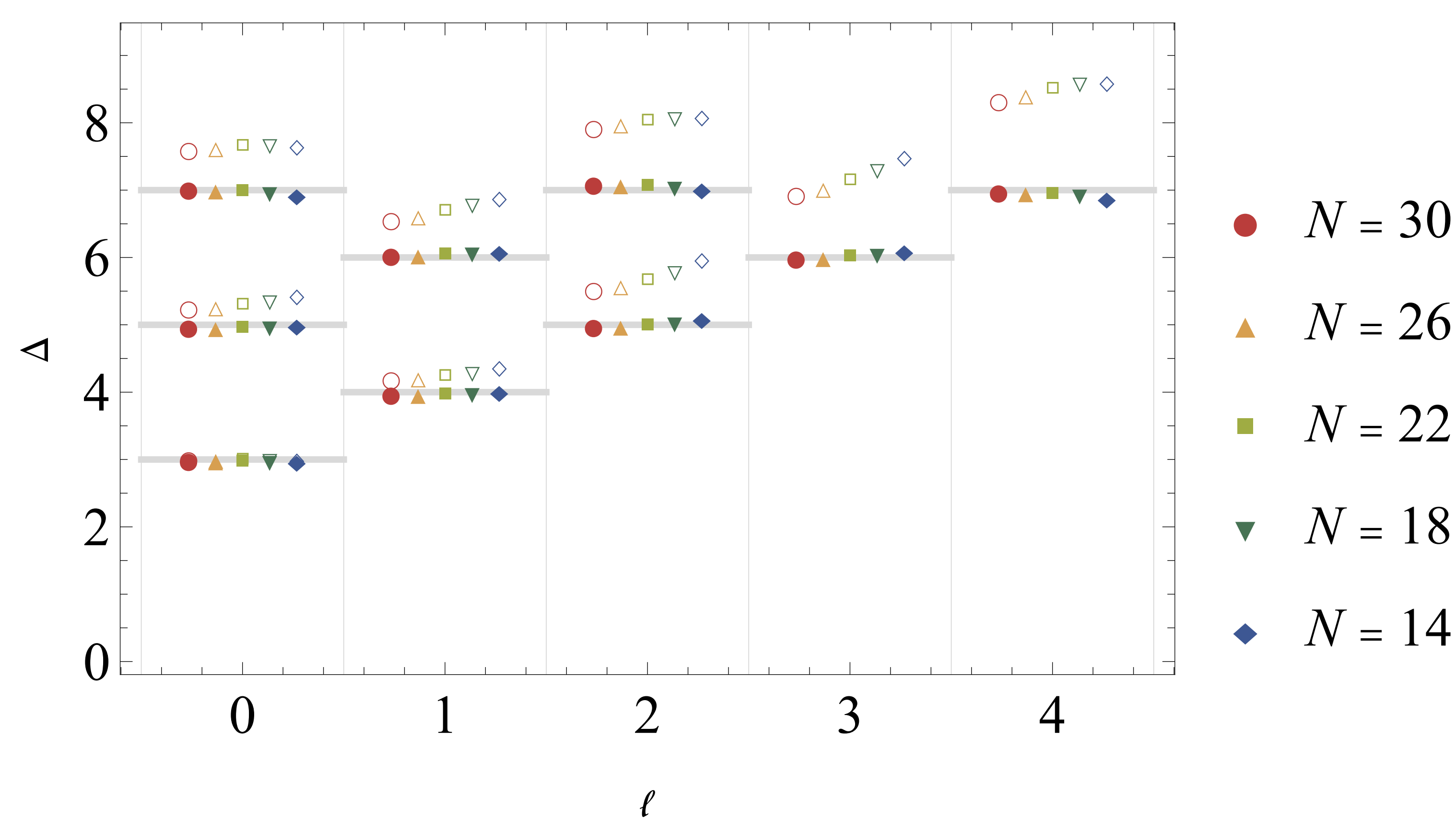


Conformal boundary

Zhou, Zou (2025)







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A lot to explore in this fuzzy world

Direction I

A numerical tool to solve open problems of CFTs/QFTs:

- Critical gauge theories: QED3, QCD3, Chern-Simons matter theories, etc.
- 2+1D CFT at finite temperature, Cardy formula
- Conformal defect
- Non-equilibrium dynamics, quantum chaos
- Complex fixed point, complex CFT
- Landscape of CFTs, new CFTs
- Higher dimensional generalizations
- …

Direction II

Unreasonable effectiveness of mathematics (fuzzy geometry):

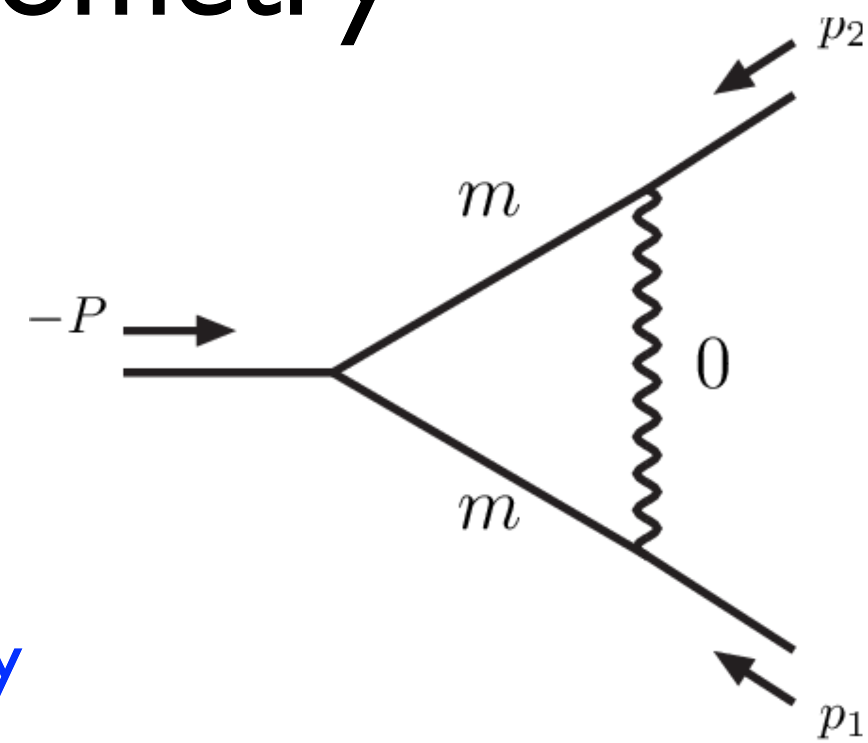
- Regulating QFTs using non-commutative geometry?!
- Exact solution or hidden structure of 3D CFTs?!

Non-commutative geometry

$$[x_i, p_j] = i\pi\delta_{ij} \quad \Longrightarrow \quad [x_i, x_j] = i\theta_{ij}$$

non-commutativity in phase space

non-commutativity in real space



Heisenberg's original idea in 1930s: to cure the infamous UV divergence in quantum field theory

Heisenberg

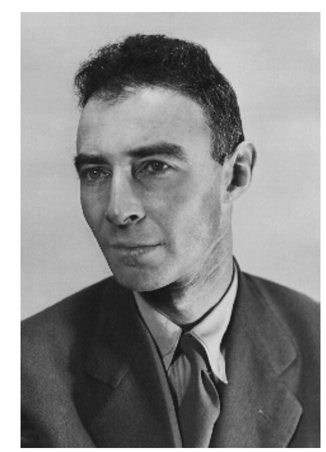
A letter to → 1930

Peierls

→

Pauli

→



Oppenheimer

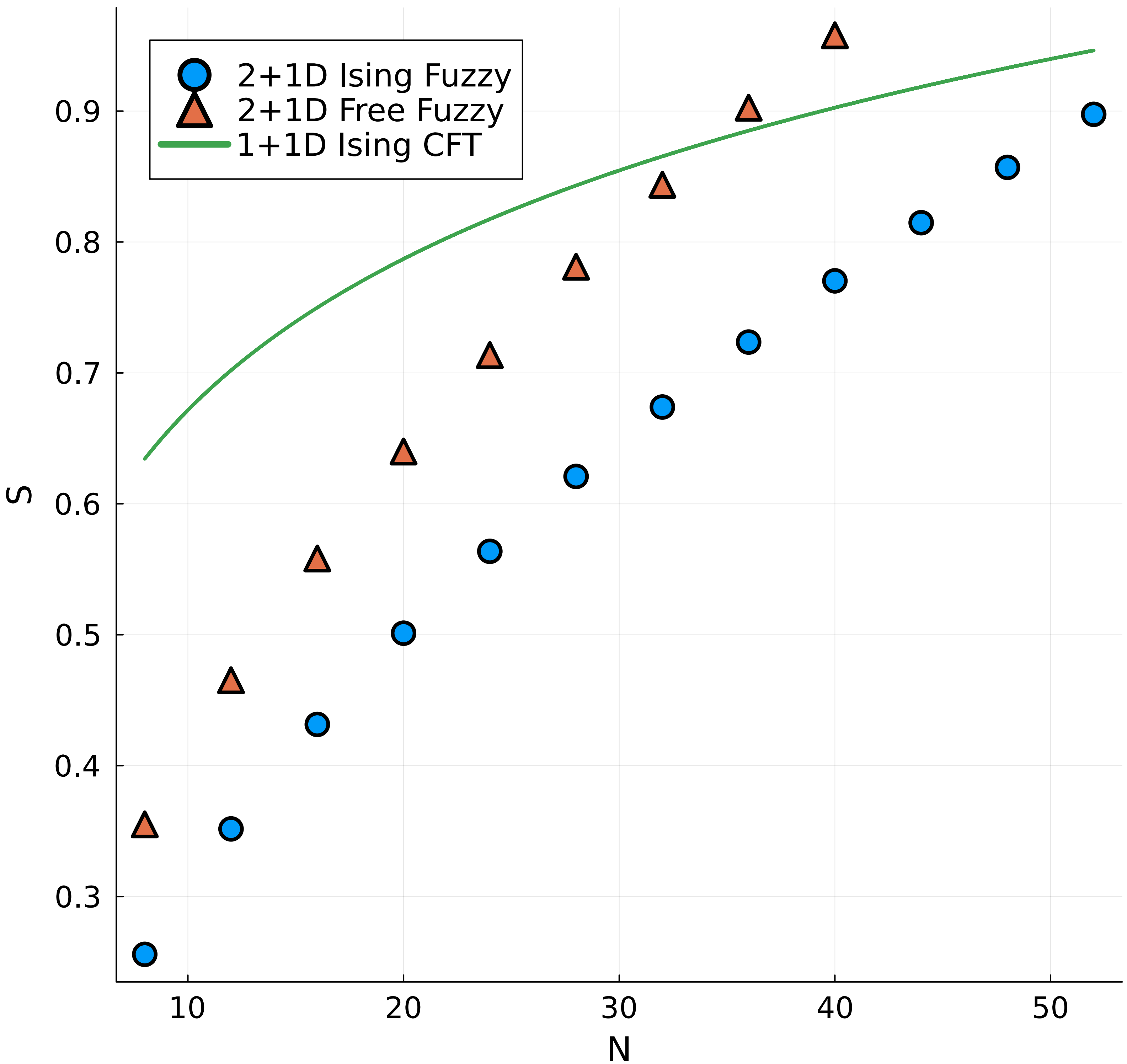
A PhD project → 1947

Snyder

Mathematical foundation was developed by Connes during 1970s-1980s.

Ground state is little entangled

Entanglement entropy with the system cut into two halves.



1+1D Transverse Ising model

$$S = \frac{1}{6} \ln \left[\frac{N}{\pi} \right] + 0.478558014(5)$$

2+1D CFT on fuzzy sphere

$$S = \alpha \sqrt{N} + \beta$$

Can wavefunction be the hero once again?

Two wavefunctions reshaped condensed matter physics:

Superconductors Bardeen, Cooper, Schrieffer	$\prod_{\mathbf{k}}\left(u_{\mathbf{k}} + v_{\mathbf{k}}\, c^{\dagger}_{\mathbf{k}\uparrow}\, c^{\dagger}_{-\mathbf{k}\downarrow}\right)\lvert 0\rangle$
Fractional quantum Hall Laughlin	$\prod_{i<j}(z_i - z_j)^m \exp\left(-\frac{1}{4\ell_B^2}\sum_{i=1}^{N}\lvert z_i\rvert^2\right)$

A wavefunction ansatz for 3D Ising on fuzzy sphere: YCH arXiv:2506.14904

$$\exp\left(\sum_{\ell=0}^{2s}\frac{1}{4}\log\left(\frac{2(1-\Delta)_{\ell+1}}{(\Delta)_\ell}K_\ell\right)\sum_{m=-\ell}^{\ell}(-1)^m\frac{n^-_{\ell,m}n^-_{\ell,-m}-n^+_{\ell,m}n^+_{\ell,-m}}{2s+1}\right)\lvert\varphi_0\rangle$$

1. Power law correlation function.
2. High wavefunction overlap with the Ising CFT.

	$N=12$	$N=16$	$N=20$	$N=24$	$N=28$	$N=32$
Ising CFT	0.9967	0.9951	0.9935	0.9918	0.9902	0.9885

Summary

Thank you!

- We proposed a new scheme called fuzzy sphere regularization to study 3D CFTs by making use of the quantum Hall physics and non-commutative geometry.
- A major surprise is that it miraculously works for a very small system size, i.e. N=4~16 spins.
- A wealth of information (e.g. operator spectrum, OPE coefficients, F-function) as well as different CFTs (e.g. Wilson-Fisher, SO(5) deconfined phase transitions, defect CFTs, QED3-Chern-Simons theory, Majorana fermion, super-Ising) can be computed efficiently in this scheme.

arXiv:2210.13482 (PRX); arXiv:2303.08844 (PRL); arXiv:2306.04681 (PRB); arXiv:2306.16435 (PRX); arXiv:2308.01903 (Nat. Comm.); arXiv:2401.00039 (Sci. Post.); arXiv:2401.17362 (PRB); arXiv:2406.10186 (JHEP); arXiv:2410.00087 (PRL); arXiv:2506.14904; arXiv:2507.19580; Zhou, Gaiotto, YCH, arXiv: 2509.08038

Let's explore the fuzzy world!

Collaborators

Westlake University: Wei Zhu, Liangdong Hu, Chao Han
Perimeter Institute: Zheng Zhou, Yijian Zou, Davide Gaiotto, Chong Wang
Stony Brook: Zohar Komargodski, Tzu-Chieh Wei, Wenhan Guo
Johannes Hofmann (MPI-PKS), Emilie Huffman (Wake Forrest), Gabriel Cuomo (SISSA)