Elements of theory of Heavy Fermion superconductors

(NHMFL Winter school, Jan.2013)

Cooper effect and BCS→ SC order parameter→ Parity→ Spin-orbit interaction →Lattice group representations→ Energy spectrum

Classes: crystalline classes→ magnetic classes→ the formal approach and simple examples → superconducting classes→ Landau functional → two-and three - dimensional representations

Magnetic moments → energy spectrum → difference between the symmetry and topologically stable zeroes

Summary

(Literature)

The Cooper paper, 1956

Consider two interacting particles $\vec{p}_1 + \vec{p}_2 = 0 \quad \Psi(\vec{r}_1 - \vec{r}_2) \to \Psi(\vec{p})$

$$[p^2/m - E]\Psi(\vec{p}) = -\int V(\vec{p},\vec{p}\,')\Psi(\vec{p}\,')[d^3\vec{p}\,'/(2\pi)^3]$$

Let $V(\vec{p},\vec{p}\,') = V$ →with the notation $\Psi(\vec{p}) \equiv \Phi/[p^2/m - E] \Longrightarrow$

$$\Phi = -V\Phi\int \frac{p'^2 dp' d\Omega}{(2\pi)^3}\left(\frac{1}{p'^2/m - E}\right)$$

i.e., the integral converges at large *p*

In 3D to form a bound state one needs a finite *V* !

! Cooper: not so for two electrons near the Fermi surface

$$\Phi = -V\Phi\int \frac{p'^2 dp' d\Omega}{(2\pi)^3}\left(\frac{1}{p'^2/m - E}\right) \Rightarrow |V|\Phi\int \nu(E_F)d\xi\left(\frac{1}{2\xi + \varepsilon}\right) \propto |V/2|\Phi\nu(E_F)\ln(\bar{\omega}/\varepsilon)$$

$$E = 2E_F - \varepsilon; \xi = v_F(p - p_F); \varepsilon > 0$$

! integrated over $\xi \subseteq \{0, \bar{\omega}\}$ One finds: $\varepsilon = \bar{\omega}\exp\{-[2/g\nu(E_F)]\}$ $(g = |V|)$

The solution always exists! The Fermi surface is unstable with respect to pairing at the arbitrary weak attractive interaction !

(BCS, 1958)

the e-e interaction:

$$\hat{H}_{\text{int}} = \frac{1}{2}\sum_{k,k',q} V_{\alpha\beta;\lambda\mu}(\vec{k},\vec{k}')\hat{a}^{+}_{-k+q/2,\alpha}\hat{a}^{+}_{k+q/2,\beta}\hat{a}_{k'+q/2,\lambda}\hat{a}_{-k'+q/2,\mu}$$

$$G_{\alpha\beta}(\vec{k};\tau_1-\tau_2) = -\{\hat{T}_{\tau}(\hat{a}_{k,\alpha}(\vec{k},\tau_1)\hat{a}^{+}_{k,\beta}(\vec{k},\tau_2))\}$$

Now

$$\sum_k < \hat{a}_{k,\alpha}\hat{a}_{-k,\beta} > \neq 0 : \; N/2$$

$$\sum_k < \hat{a}^{+}_{k,\alpha}\hat{a}^{+}_{-k,\beta} > \neq 0 : \; N/2$$

(Gor'kov, 1958)

The anomalous functions:

$$F_{\alpha,\beta}(\vec{k};\tau_1-\tau_2) = \{\hat{T}_{\tau}(\hat{a}_{k,\alpha}(\tau_1)\hat{a}_{-k,\beta}(\tau_2))\}$$

$$F^{+}_{\alpha\beta}(\vec{k};\tau_1-\tau_2) = \{\hat{T}_{\tau}(\hat{a}^{+}_{-k,\alpha}(\tau_1)\hat{a}^{+}_{k,\beta}(\tau_2))\}$$

In the equations for the new Green functions:

$$\begin{cases} (i\omega_n - \xi(\vec{k}))G_{\alpha\beta}(\vec{k},\omega_n) + \Delta_{\alpha\gamma}(\vec{k})F^{+}_{\gamma\beta}(\vec{k},\omega_n) = \delta_{\alpha\beta} \\ (i\omega_n + \xi(\vec{k}))F^{+}_{\alpha\beta}(\vec{k},\omega_n) + \Delta^{+}_{\alpha\gamma}(\vec{k})G_{\gamma\beta}(\vec{k},\omega_n) = 0 \end{cases}$$

the "gaps" $\hat{\Delta}(\vec{k}), \hat{\Delta}^{+}(\vec{k})$ are the superconducting order parameters :

$$\Delta_{\alpha\beta}(\vec{k}) = -\sum_{k'} V_{\beta\alpha,\mu\lambda}(\vec{k},\vec{k}') < \hat{a}_{k',\alpha}(\tau)\hat{a}_{-k',\alpha}(\tau) > \equiv -\sum_{k'} V_{\beta\alpha,\mu\lambda}(\vec{k},\vec{k}')F_{\lambda\mu}(\vec{k}',0+)$$

$$\Delta^{+}_{\alpha,\beta}(\vec{k}) = -\sum_{k'} V_{\lambda\mu,\beta\alpha}(\vec{k},\vec{k}') < \hat{a}^{+}_{-k',\lambda}(\tau)\hat{a}^{+}_{k',\mu}(\tau) > \equiv -\sum_{k'} V_{\lambda\mu,\beta\alpha}(\vec{k},\vec{k}')F^{+}_{\lambda,\mu}(\vec{k}',0+)$$

Definition of the transition temperature *Tc* from the linearized gap equation:

$$\Delta_{\alpha\beta}(\vec{k}) = -T_c \sum_{n;k'} V_{\beta\alpha,\gamma\delta}(\vec{k},\vec{k}')\Delta_{\gamma\delta}(\vec{k}')\{\omega_n^2 + \xi^2(\vec{k}')\}^{-1}$$

$$T\sum_{\omega_n,k'} \hat{V}\hat{\Delta}(\vec{k}') < \ldots > \Rightarrow \int \frac{d^3\vec{k}'}{(2\pi)^3}\hat{V}\hat{\Delta}(\vec{k}')\{\frac{th(\xi_{k'}/2T)}{\xi_{k'}}\} \Rightarrow (\ln(\bar{W}/T_c)\int \hat{V}(\vec{k},\vec{k}')\hat{\Delta}(\vec{k}')d\Omega_{F,k'}$$

Energy spectrum: $i\omega_n \Rightarrow E$

$$(i\omega_n - \xi(\vec{k}))\hat{G}(\vec{k},\omega_n) + \hat{\Delta}(\vec{k})\hat{F}^{+}(\vec{k},\omega_n) = \delta_{\alpha\beta}$$

$$(i\omega_n + \xi(\vec{k}))\hat{F}^{+}(\vec{k},\omega_n) + \hat{\Delta}^{+}(\vec{k})\hat{G}(\vec{k},\omega_n) = 0$$

$$\mathrm{Det}\left\|\begin{matrix} [E-\xi(\vec{k})]\hat{I} & \hat{\Delta}(\vec{k}) \\ \hat{\Delta}^{+}(\vec{k}) & [E+\xi(\vec{k})]\hat{I} \end{matrix}\right\| = 0$$

($\hat{I}$ is the unit spin matrix)

$$Det\,\|\,[E^2-\xi^2(\vec{k})]\hat{I} - \hat{\Delta}(\vec{k})\times\hat{\Delta}^{+}(\vec{k})\,\| = 0$$

$$< \hat{a}_{k,\alpha}\hat{a}_{-k,\beta} > = - < \hat{a}_{-k,\beta}\hat{a}_{k,\alpha} >$$

$$\Delta_{\alpha\beta}(\vec{k}) = -\sum_{k'} V_{\beta\alpha,\mu\lambda}(\vec{k},\vec{k}') < \hat{a}_{k',\alpha}(\tau)\hat{a}_{-k',\alpha}(\tau) > = -\Delta_{\beta\alpha}(-\vec{k})$$

Strong spin-orbit coupling: $S \Rightarrow P$

P even:
a“singlet”, S=0

$$\Delta_{\alpha\beta}(\vec{k}) = i(\hat{\sigma}_2)_{\alpha\beta} f(\vec{k}) \quad \Rightarrow f(-\vec{k}) = f(\vec{k})$$

P odd:
a “triplet”, S=1

$$\Delta_{\alpha\beta}(\vec{k}) = i\{(\vec{\sigma} \cdot \vec{d}(\vec{k}))\hat{\sigma}_2\} \Rightarrow \vec{d}(-\vec{k}) = -\vec{d}(\vec{k})$$

the interaction V expanded over representations of the point group:

$$V_{\alpha,\beta;\mu\lambda}(\vec{k},\vec{k}') \Rightarrow \sum_j A_j \hat{\varphi}_j(\vec{k}) \otimes \hat{\varphi}_j(\vec{k}')$$

$$\hat{\Delta}(\vec{k}) = (\ln(\bar{W}/T_c) \int \hat{V}(\vec{k},\vec{k}')\hat{\Delta}(\vec{k}')d\Omega_{F,k'} \quad \Longrightarrow \quad \hat{\Delta}(\vec{k}) \propto \hat{\varphi}^{g,u}(\vec{k})$$

(Here in $(\ldots)^{g.u}$ *g* stands for an *even* and *u- for an* odd representations)

$\hat{\Delta}(\vec{k}) \propto \hat{\varphi}^{g,u}(\vec{k})$ → Arises only as the solution for the gap at *T*=*Tc*

What is the gap structure?

?Strong coupling (say, higher order corrections in V)

? Non-linear corrections below *Tc* from other representations

? The multi-dimensional representation : what is the structure of the order parameter just below *Tc* ? in the ground state ?

<Common Crystalline classes *and* the Space Group>

At SC pairing ***Q*** =0: ⟶ Search for the superconducting classes !

The total Symmetry Group in the normal phase:

$$G \times R \times U(1)$$

G –the point group of all rotations and reflections

U(1) -multiplication by a phase factor

R- the time reversal ***t→-t*** . Applying to a wave function: corresponds to the complex conjugation

To warm up: how one builds the non-trivial magnetic classes?

Then the Group of Symmetry in the normal phase is:

$$G \times R$$

General (formal) approach: single out a subgroup ***H*** of the group ***G***

Take all elements $G_i \notin \hat{H}$ and form all products

$$G_1\hat{H}, G_2\hat{H}, \ldots G_i\hat{H}$$

These termed the left classes. Similarly, form the right classes :

$$\hat{H}G_1, \hat{H}G_2, \ldots \hat{H}G_i$$

If two manifolds coincide, $\hat{H}$ is the *invariant* sub-group or the *normal divisor* of $\hat{G}$. Let g be the number of elements in $\hat{G}$ and h in $\hat{H}$

.

Then: $g = h(i+1)$ $i+1$ is called the *index* of the sub-group

Multiplication of the classes → multiply as the elements constituting the classes:

$$G_i\hat{H} \times G_k\hat{H} \Rightarrow (G_iG_k)\hat{H}$$

The new group of i+1 elements is called the factor-group: $\hat{F}$

Two transformation *(1, R)* constitute *the two elements forming* the group: $\hat{R}$

The method for building all non-trivial magnetic classes is now clear: first find a sub-group of index 2 and distribute the remaining elements over its classes .

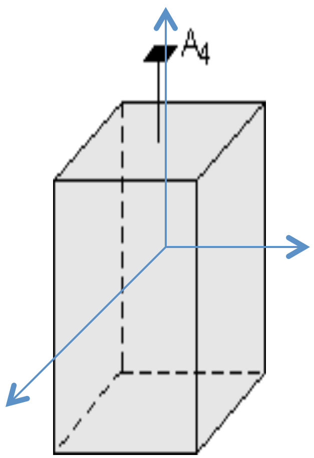
Next step, form the direct product :

$$\hat{F} \times \hat{R}$$

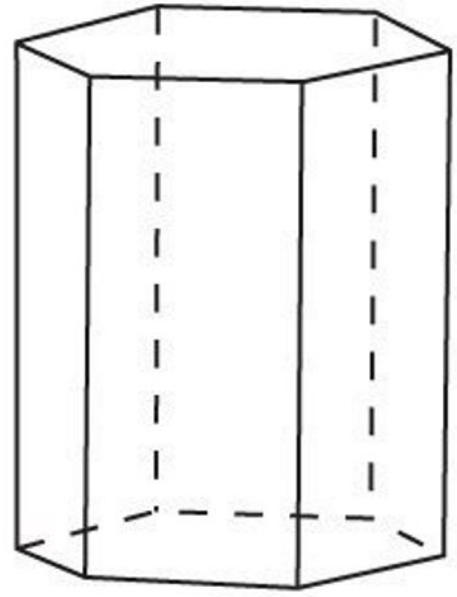
In practice, the method is that all elements from each class, i.e., the elements of the factor group,except the identical class formed by the sub-group $\hat{H}$ Itself, appear combined with the time reversal transformation *R: t→-t.*

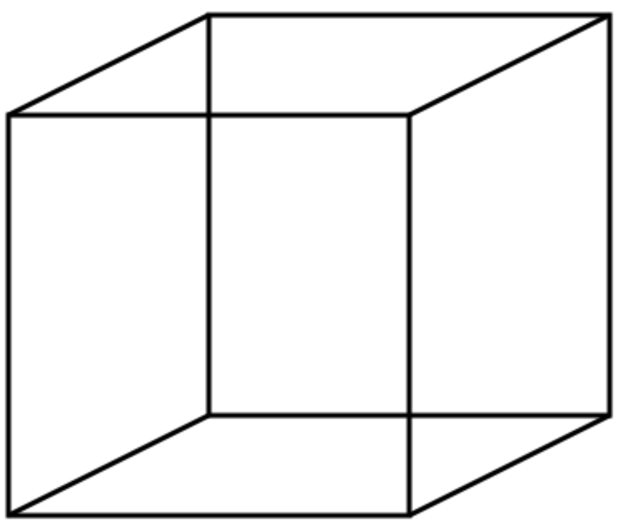
A couple of simple examples below **!**

$D_{4h}=D_4 \times C_i$



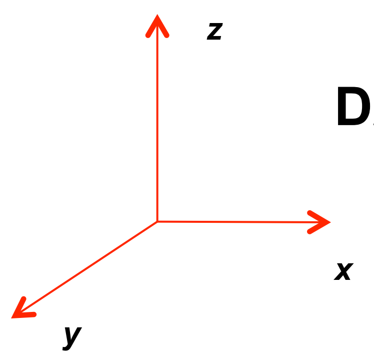
$D_{6h}=D_6 \times C_i$





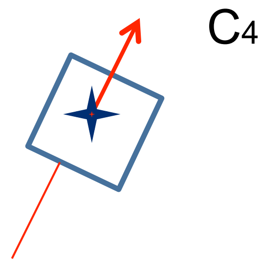
$O_h=O \times C_i$

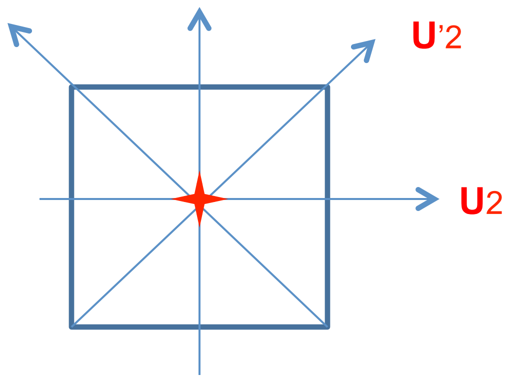
C_i (E, P) $\quad P: \ \vec{r} \Rightarrow -\vec{r}$



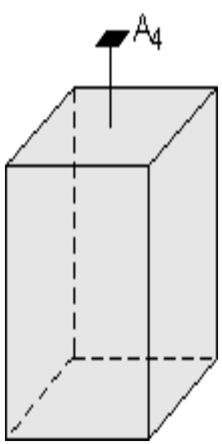
D_2

D_4

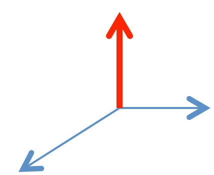




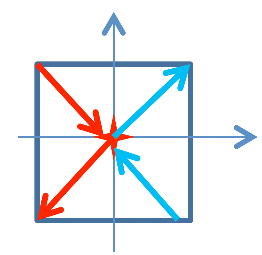
	E	C_2	$2C_4$	$2U_2$	$2U'_2$
A_1	1	1	1	1	1
A_2, z	1	1	1	-1	-1
B_1	1	1	-1	1	-1
B_2	1	1	-1	-1	1
E; x, y	2	-2	0	0	0



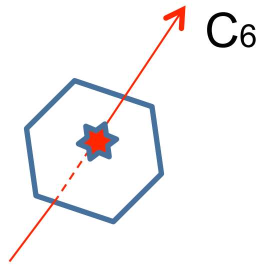
A_2, z $\quad D_4(C_4) \quad (E \quad C_2 \quad 2C_4 \quad 2RU_2 \quad 2RU'_2\ )$

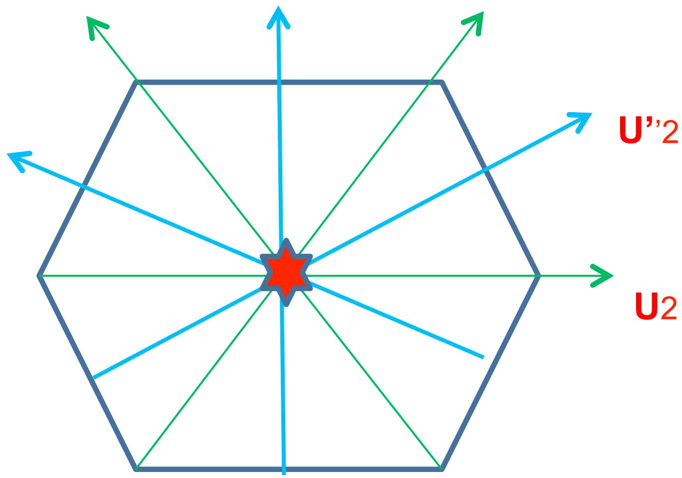


B_2 $\quad D_4(D_2) \quad (E \quad C_2 \quad 2RC_4 \quad 2RU_2 \quad 2U'_2\ )$



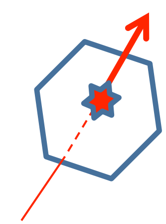
D_6



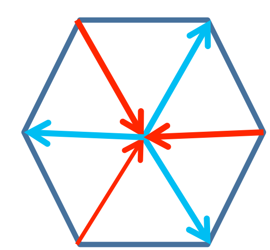


	E	C_2	$2C_3$	$2C_6$	$3U_2$	$3U'_2$
A_1	1	1	1	1	1	1
$A_{2,z}$	1	1	1	1	-1	-1
B_1	1	-1	1	-1	1	-1
B_2	1	-1	1	-1	-1	1
E_2	2	2	-1	-1	0	0
$E_{1;x,y}$	2	-2	-1	1	0	0

A_2: $D_6(C_6)$ $(E \quad C_2 \quad 2C_3 \quad 2C_6 \quad 3RU_2 \quad 3RU'_2)$



B_1 : $D_6(D_3)$ $(E \quad RC_2 \quad 2C_3 \quad 2RC_6 \quad 3U_2 \quad 3RU'_2)$



Return to superconductivity and to the solutions for the gap at T_c

$$V_{\alpha,\beta;\mu\lambda}(\vec{k},\vec{k}') \Rightarrow \sum_j A_j \hat{\varphi}_j(\vec{k}) \otimes \hat{\varphi}_j(\vec{k}')$$

$$\hat{\Delta}(\vec{k}) = (\ln(\bar{W}/T_c)\int \hat{V}(\vec{k},\vec{k}')\hat{\Delta}(\vec{k}')d\Omega_{FS,k'} \Longrightarrow \hat{\Delta}(\vec{k}) \propto \hat{\varphi}^{g,u}(\vec{k})$$

Symmetry Group in the normal phase:

$$G \times R \times U(1)$$

For the crystal groups with the center of inversion one may write:

$$G = G' \times C_i$$

where G' is the group of the rotations only and study cases of the even and the odd parity separately

As one example, consider again D_4 . In the normal state:

$$D_4 \times R \times U(1)$$

From the product $R\times U(1)$ one may construct the following groups:

(In applying to the pair function -> R means taking the complex conjugate)

a) The only two groups with index 2 : R and $U(1)\Rightarrow(1,e^{-i\pi})$

b) the product of $(1,e^{i\pi/2},e^{i\pi},e^{-i\pi/2})\times R$

	E	C_2	$2C_4$	$2U_2$	$2U'_2$
A_1	1	1	1	1	1
A_2, *z*	1	1	1	-1	-1
B_1	1	1	-1	1	-1
B_2	1	1	-1	-1	1
E; *x, y*	2	-2	0	0	0

a) Do as before:

A_2, *z* $D_4(C_4)$ (E C_2 $2C_4$ $2RU_2$ $2RU'_2$)

$D_4(C_4)$ (E C_2 $2C_4$ $2e^{i\pi}U_2$ $2e^{i\pi}U'_2$)

B_2 $D_4(D_2)$ (E C_2 $2RC_4$ $2RU_2$ $2U'_2$)

$D_4(D_2)$ (E C_2 $2e^{i\pi}C_4$ $2e^{i\pi}U_2$ $2U'_2$)

! for one "gap" the magnetic superconducting phases in a) do not appear at T_c

b) !? Non- Abelian group $(1,e^{i\pi/2},e^{i\pi},e^{-i\pi/2})\times R$ is isomorphic D_4 (index 8 !)

(See below)

The wave functions for the representations of the group D_4

$A_1(S=0): Symm.function$ $A_1(S=1): a\vec{z}k_z + b(\vec{x}k_x + \vec{y}k_y)$

$A_2(S=0): k_x k_y(k_x^2 - k_y^2)$ $A_2(S=1): (\vec{x}k_y + \vec{y}k_x)(k_x^2 - k_y^2)$

$$D_4(C_4):(E, C_2, 2C_4, 2e^{i\pi}U_2, 2e^{i\pi}U_2')$$

$B_1(S=0): (k_x^2 - k_y^2)$ $B_1(S=1): \vec{x}k_x - \vec{y}k_y$ “d-wave”!

$B_2(S=0): k_x k_y$ $B_2(S=1): \vec{x}k_y + \vec{y}k_x$

$$D_4(D_2)(E, C_2, 2e^{i\pi}C_4, 2e^{i\pi}U_2, 2U_2')$$

$E(S=0): k_z k_x; k_z k_y$ $E(S=1): \vec{z}k_x; \vec{z}k_y$

b) !? Non- Abelian group $(1, e^{i\pi/2}, e^{i\pi}, e^{-i\pi/2}) \times R$ is isomorphic D_4 (index 8 !)

(For the classes that can be constructed on basis of the two-dimensional representation E ⟹ see below)

Symmetry Class and positions of zeroes

$$\hat{A}\psi(\vec{p}) = \psi(\hat{A}\vec{p}) \Longleftrightarrow \hat{A}\vec{d}(\vec{p}) = \hat{A}\vec{d}(\hat{A}\vec{p})$$

D4(C4) (E , C2 , 2C4, 2 $e^{i\pi}$U2, 2 $e^{i\pi}$U'2)

S=0:

$$\vec{p} = (x, 0, p_z)$$

$$\Delta(\vec{p}) = e^{i\pi}U_{2(x)}\Delta(\vec{p}) = -U_{2(x)}\Delta(\vec{p}) = -\Delta(x, 0, -p_z) = -\Delta(x, 0, p_z) \equiv 0$$

$$\vec{p} = (x, x, p_z)$$

$$\Delta(\vec{p}) = e^{i\pi}U_{2(x=y)}\Delta(\vec{p}) = -U_{2(x=y)}\Delta(\vec{p}) = -\Delta(x, x, -p_z) = -\Delta(x, x, p_z) \equiv 0$$

Gap is zero on intersections of FS with the vertical symmetry planes

S=1:

$$\vec{p}_0 = (0, 0, p_z); \vec{d}_0 = (0, 0, d_z)$$

$$\vec{d}(\vec{p}) = e^{i\pi}U_{2(x)}\vec{d}(\vec{p}) = -[U_{2(x)}\vec{d}(U_{2(x)}\vec{p})] \Rightarrow d_z(0, 0, -p_z) = -\vec{d}(\vec{p}_0)$$

Gap is zero on FS at intersection with the 4-fold axis C4

Now let $\varphi_x(\vec{k});\varphi_y(\vec{k})$ be two functions realizing the representation E. Then the superconducting order parameter,i.e., the “gap” can be presented as:

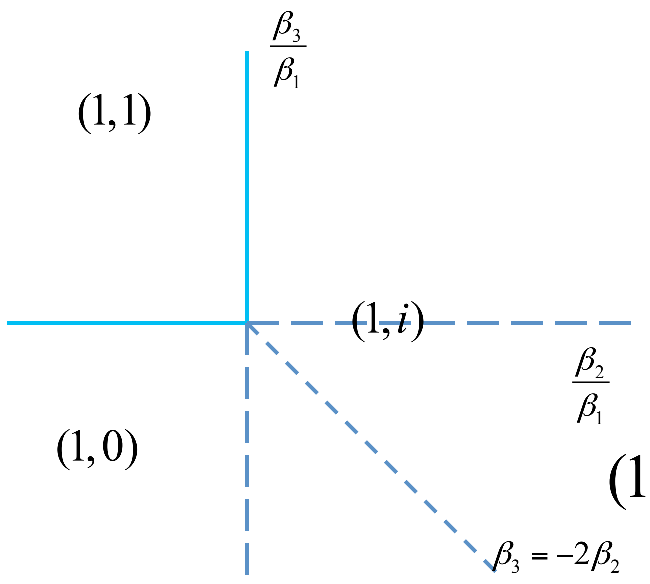
$$\Delta(\vec{k}) = \sum_{i=1,2} \eta_i \varphi_i(\vec{k})$$

Find Free energy minimum**?** Consider the second order transitions from the normal state

Near T_c → the Landau functional $\Phi(T)$ has the following general form:

$$\Phi(T) = \alpha(T-T_c)(\vec{\eta}\cdot\vec{\eta}^*) + \beta_1(\vec{\eta}\cdot\vec{\eta}^*)^2 + \beta_2|\vec{\eta}^2|^2 + \beta_3(|\eta_x|^4 + |\eta_y|^4)$$

Depending on the coefficients, its minimization leads to the following solutions:



$E(S=0): k_z k_x; k_z k_y$

$E(S=1): \vec{z} k_x; \vec{z} k_y$

For instance (S=0):

$$(1,0): k_z k_x (or \rightarrow k_z k_y) \qquad (1,1): k_z(k_x \pm k_y)$$

(1,i): $k_z(k_x+ik_y) \Rightarrow k_z \exp(i\varphi)$ $\vec{z}(k_x+ik_y) \Rightarrow \vec{z}\exp(i\varphi)$

Superconducting class
$D_4(E)$: $$(E, e^{i\pi}C_2, e^{i\pi//2}C_4, e^{-i\pi//2}C_4^3, e^{i\pi}RU_{2x}, RU_{2y}, 2e^{\pm i\pi//2}RU_2')$$

Magnetic class! (the moment is along the z-axis)

? !Omit the U(1)-elements
and compare with $D_4(C_4)$: $(E \quad C_2 \quad 2C_4 \quad 2RU_2 \quad 2RU'_2)$

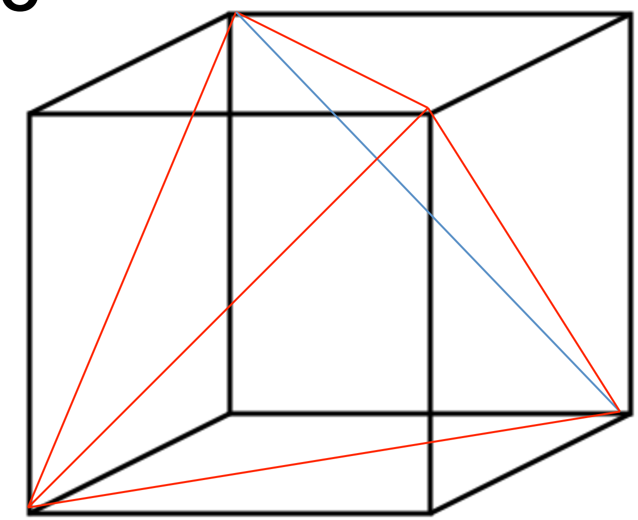
$D_4(E)$ is the most symmetric class that can be constructed from this two-dimensional representation without lowering symmetry of the lattice (of the crystalline class)

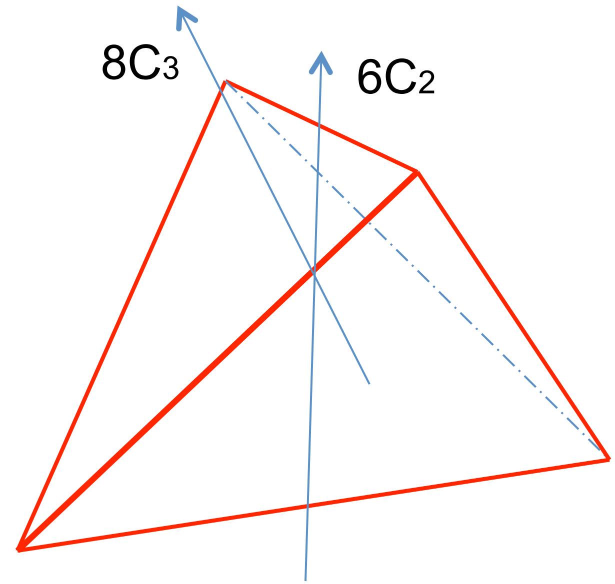
In fact, compare → $(1,0): k_zk_x(or \to k_zk_y)$ $(1,1): k_z(k_x \pm k_y)$

Two symmetric classes preserving the crystalline symmetry for the cubic lattices :

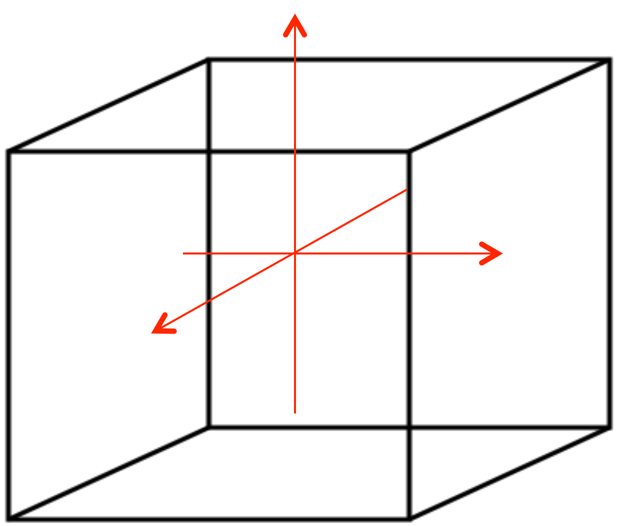
The cubic symmetry

O

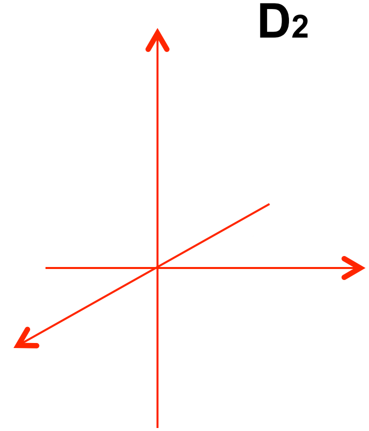




T



D2



Two symmetric new classes that are possible **O(T), O(D_2)**

A_2 $(E, 8C_3, 3C_2, 6e^{-i\pi}C_2, 6e^{-i\pi}C_4)$ **O(T)**

Another high symmetric class formed from E: **O(D_2)**

(Somewhat lengthy !)

$$O(D_2) \Rightarrow (E, 3C_2, 2U_2^{(perp)x}RC_4^xR, 2C_4^y\varepsilon R, 2C_4^z\varepsilon^2R,$$
$$4C_3\varepsilon^2, 4C_3^2\varepsilon, 2U_2^{(perp)x}R, 2U_2^{(perp)y}\varepsilon R, 2U_2^{(perp)z}\varepsilon^2R)$$

	E	$8C_3$	$3C_2$	$6C_2$	$6C_4$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
F_2	3	0	-1	1	-1
F_1 *x, y; z*	3	0	-1	-1	1

Symmetry phases for representations E, F_1, F_2 just below T_c → from the Landau functional

! Qualitative new results from F_1 and F_2

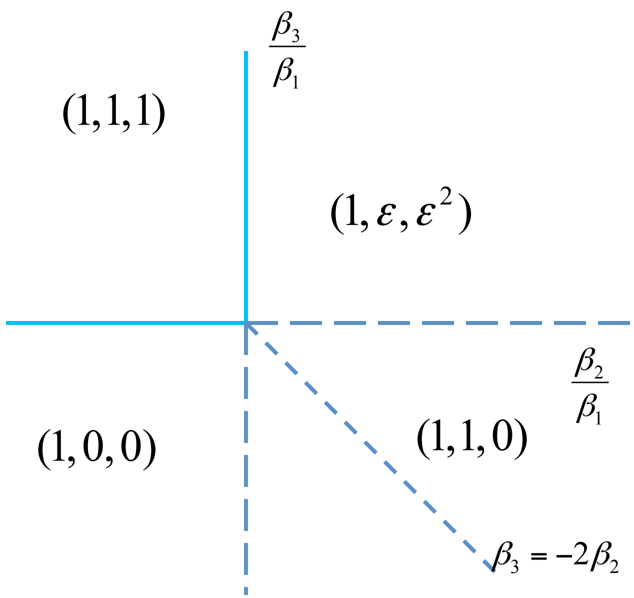
→ For the three dimensional representations there are three parameters in

$$\Delta(\vec{k}) = \sum_{i=1,2,3} \eta_i \varphi_i(\vec{k})$$

The Landau functional at T_c is analogous to that one for the 2D representation of D_4:

$$\Phi(T) = \alpha(T - T_c)(\vec{\eta}\cdot\vec{\eta}^*) + \beta_1(\vec{\eta}\cdot\vec{\eta}^*)^2 + \beta_2|\vec{\eta}^2|^2 + \beta_3(|\eta_x|^4 + |\eta_y|^4 + |\eta_z|^4)$$

The analysis leads to the phase diagram:



In $(1,\varepsilon,\varepsilon^2)$ $\varepsilon = e^{i\pi/3}$

Three components (η_x,η_y,η_z) play role of the vector $\vec{\eta}$ in the 3D space of F_1 or F_2.

In the above case the components are complex:

$\vec{\eta} = \vec{\eta}' + i\vec{\eta}''$ and one may form the third vector

$$\vec{m} = [\vec{\eta}' \times \vec{\eta}'']$$

that has the meaning of a magnetic moment

Excitations to be found from

$$Det \,\|\, [E^2 - \xi^2(\vec{k})]\hat{I} - \hat{\Delta}(\vec{k}) \times \hat{\Delta}^+(\vec{k}) \,\| = 0$$

$$Det\,\|\,[E^2-\xi^2(\vec{k})]\hat{I}-\hat{\Delta}(\vec{k})\times\hat{\Delta}^+(\vec{k})\,\|=0$$

For P-even ("singlet") $\quad E^2=\xi^2(\vec{k})+|\Delta(\vec{k})|^2$

For P-odd ("triplet"):

$\vec{\eta}(\vec{k})=\vec{\eta}'(\vec{k})+i\vec{\eta}''(\vec{k})$ and $\vec{m}(\vec{k})=[\vec{\eta}'(\vec{k})\times\vec{\eta}''(\vec{k})]$

with $\hat{\Delta}(\vec{k})=i\{(\vec{\sigma}\vec{d}(\vec{k}))\hat{\sigma}_2\}$ using: $(\vec{\sigma}\cdot\vec{d})(\vec{\sigma}\cdot\vec{d}^*)=(\vec{d}\cdot\vec{d}^*)\hat{I}+i(\vec{\sigma}\cdot[\vec{d}\times\vec{d}^*])$

→ $\det\|\,[E^2-\xi^2(\vec{k})-|\Delta(\vec{k})|^2]\hat{I}-\vec{m}(\vec{k})\,\|=0$

one finds that the excitations are split into the two and two branches

$$E^{\pm}_{1,2}(\vec{k})=\pm\sqrt{\xi^2_{\vec{k}}+|\vec{d}(\vec{k})|^2\pm|\vec{m}(\vec{k})|}$$

(compare with two directions of spin for the "s-wave" superconductors)

The basis functions:

F_1 ("singlet"): $[k_x k_z (k_x^2 - k_z^2)];[k_z k_x (k_z^2 - k_x^2)];[k_y k_x (k_x^2 - k_y^2)]$

("triplet"): $[\vec{y} k_z - \vec{z} k_y];[\vec{z} k_x - \vec{x} k_z];[\vec{x} k_y - \vec{y} k_x]$

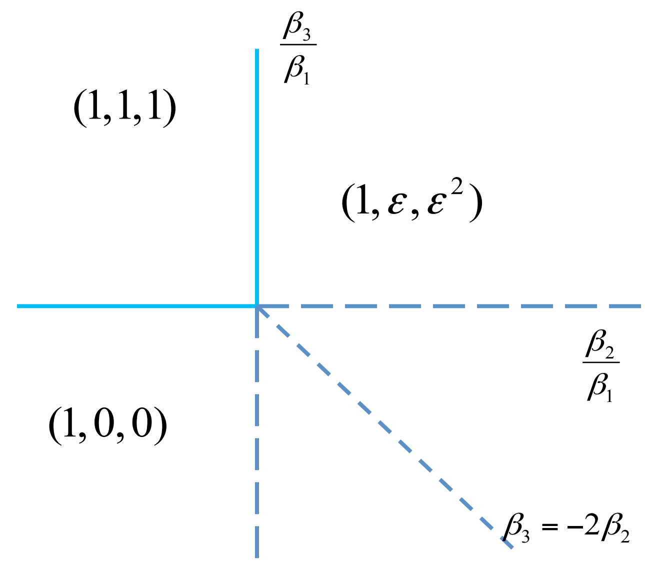
F_2 ("singlet"): $k_y k_z ; k_z k_x ; k_x k_y$

("triplet"): $\vec{y} k_z + \vec{z} k_y ; \vec{z} k_x + \vec{x} k_z ; \vec{x} k_y + \vec{y} k_x$

Symmetry zeros *versus* the point zeroes:

"singlet" gaps may have zeroes on the symmetry elements of the group (unstable at perturbations)

"triplet" gaps may have zeroes at the symmetry points on the Fermi surface. In the magnetic classes zeroes correspond to the non-zero magnetic moments: hence, are topologically stable



What is achieved by the above methods?

a) Knowing the symmetry class allows to identify the positions of the gap zeroes without model assumptions concerning the basis functions

b) Lines of zeroes possible for "singlet" phases; "triplet" phases may possess zeroes only at the points on the Fermi surface. *T-square or T-cube* dependence of the specific heat at low *T*, correspondingly.

c) Topologically stable magnetic moments in some "triplet" phases

d) For two-and three-dimensional representations the phase transitions at T_c can be split by external perturbations (? UBe_{13} and UPt_3)

e) Ordinary impurities decrease T_c and may result in "gapless" SC.Lines of zeroes are absolutely unstable at the arbitrary small impurity concentration→ nonzero DOS

f)The upper critical field can be anisotropic for some symmetry directions directly at T_c in the cubic and tetragonal lattices → $H_{c2}(\varphi)$

g) Non-trivial (phase-sensitive) boundary conditions with significant implications to the Josephson effect $F_S \propto w(\Delta_L \Delta_R^* + c.c)$

Literature(textbooks)

L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics*
a) Elements of Group Theory: *Quantum mechanics, Non-relativistic Theory,* Pergamon, 1977.
b) Crystalline Classes: *Statistical Physics*, *p.1*,Pergamon,1980.
c) Magnetic Classes: *Electrodynamics of Continuous Media*, Pergamon,1983.

A. A. Abrikosov, L. P. Gor’kov and I. E. Dzyaloshinskii ,*Methods of Quantum Field Theory in Statistical Physics,* Prentice-Hall, 1963.

V. P. Mineev and K. V. Samokhin,*Introduction to Unconventional Superconductivity,* Gordon and Breach (1999)

Original papers

UBe13: P. W. Anderson, Phys. Rev. B**30**,1549, 4000 (1984); G. E. Volovik and L. P. Gor’kov, JETP Lett. **39,** 674 (1984);

Superconducting Classes:G. E. Volovik and L. Gor’kov, JETP **61**, 843 (1986)

Anisotropy of Hc2 at Tc:L. P. Gor’kov, JETP Lett. **40**, 1155 (1984)

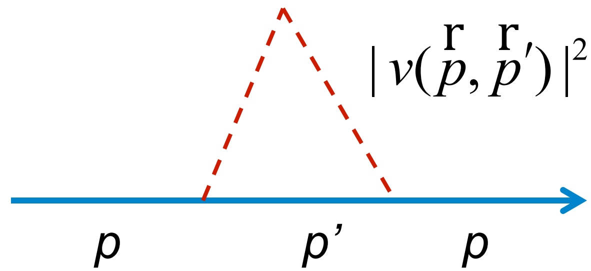
Instability of lines of zeroes in the presence of deffects: L. P. Gor’kov and P. A. Kalugin, JETP Lett. **41**, 253 (1985)

On magnetism of SCs: G. E. Volovik and V. P. Mineev, JETP **56**, 579 (1982); *ibid*. **54**, 524 (1981); **59**, 972 (1984)

Boundary conditions and the Josephson effect : V. B. Geshkenbein and A. I. Larkin JETP Lett.**43**, 395 (1986)

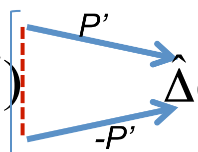
Appendix : Impurities

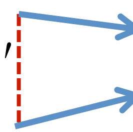
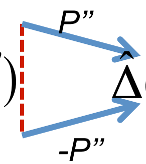
The Green function's self-energy:



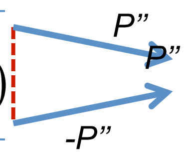
$$\frac{1}{\tau} = n_i \int \frac{d\vec{p}'}{(2\pi)^3} |v(\vec{p},\vec{p}')|^2 \, \mathrm{Im}\, G(E;\vec{p}')$$

Why MUST *Tc* decrease?

$\hat{\Delta}(\vec{p}) \propto \int V(\vec{p},\vec{p}')$ [

$\hat{\Delta}(\vec{p}')$] $d\vec{p}' + \int V(\vec{p},\vec{p}')d\vec{p}'$  [$\int V(\vec{p}',\vec{p}'')$ 

$\hat{\Delta}(\vec{p}'')d\vec{p}''$] $+\ldots$

$\Rightarrow \int V(\vec{p}',\vec{p}'')$ [

$\hat{\Delta}(\vec{p}'')$] $d\vec{p}'' \equiv 0$ If gap belongs to any non-identical representation!

Density of states (DOS): $\nu_S / \nu_N = 4\tau^2\Delta_0^2 \exp(-2\tau\Delta_0)$

Appendix : Symmetry Class and positions of zeroes

$$\hat{A}\psi(\vec{p}) = \psi(\hat{A}\vec{p}) \Longleftrightarrow \hat{A}\vec{d}(\vec{p}) = \hat{A}\vec{d}(\hat{A}\vec{p})$$

Example D4(C4) (E , C2 , 2C4, 2 $e^{i\pi}$ U2, 2 $e^{i\pi}$ U'2)

S=0:

$$\vec{p} = (x, 0, p_z)$$

$$\Delta(\vec{p}) = e^{i\pi} U_{2(x)} \Delta(\vec{p}) = -U_{2(x)} \Delta(\vec{p}) = -\Delta(x, 0, -p_z) = -\Delta(x, 0, p_z) \equiv 0$$

$$\vec{p} = (x, x, p_z)$$

$$\Delta(\vec{p}) = e^{i\pi} U_{2(x=y)} \Delta(\vec{p}) = -U_{2(x=y)} \Delta(\vec{p}) = -\Delta(x, x, -p_z) = -\Delta(x, x, p_z) \equiv 0$$

Gap is zero on intersections of FS with the vertical symmetry planes

S=1:

$$\vec{p}_0 = (0, 0, p_z); \vec{d}_0 = (0, 0, d_z)$$

$$\vec{d}(\vec{p}) = e^{i\pi} U_{2(x)} \vec{d}(\vec{p}) = -[U_{2(x)} \vec{d}(U_{2(x)} \vec{p})] \Rightarrow d_z(0, 0, -p_z) = -\vec{d}(\vec{p}_0)$$

Gap is zero on FS at intersection with the 4-fold axis C4

Appendix: Multi band SCs

Three X points in a cubic lattice.

$$\lambda_{\alpha\beta}=\lambda\delta_{\alpha\beta}+\mu(1-\delta_{\alpha\beta}).$$

$$\Delta_\alpha^*\frac{2\pi^2}{mp_0}=-\sum_\beta \lambda_{\alpha\beta}\Delta_\beta^*\ln\left(\frac{2\gamma\omega_D}{\pi T_c}\right).$$

$$l=(\Delta_1+\Delta_2+\Delta_3)/\sqrt{3}$$

$$\eta_1=(\Delta_1+\epsilon\Delta_2+\epsilon^2\Delta_3)/\sqrt{3},$$

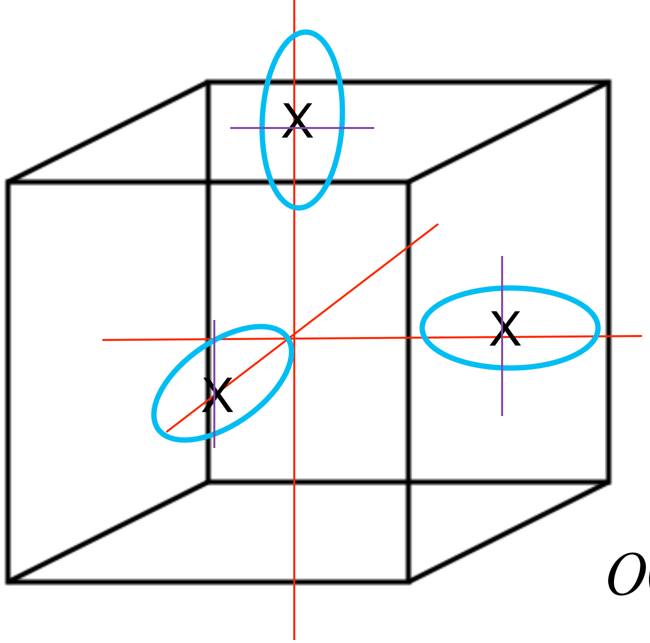
$$\eta_2=(\Delta_1+\epsilon^2\Delta_2+\epsilon\Delta_3)/\sqrt{3},$$

$$T_{c,A}=\frac{2\gamma\omega_D}{\pi}\exp\left(\frac{2\pi^2}{mp_0(\lambda+2\mu)}\right) \quad (1D)$$

$$T_{c,E}=\frac{2\gamma\omega_D}{\pi}\exp\left(\frac{2\pi^2}{mp_0(\lambda-\mu)}\right)$$

$$\frac{2\pi^2}{mp_0}\delta F=\frac{T-T_{c,E}}{T_{c,E}}(|\eta_1|^2+|\eta_2|^2)+\ln(T_{c,E}/T_{c,A})|l|^2$$

$$+\frac{7\zeta(3)}{48\pi^2T_{c,E}^2}(|\eta_1|^4+|\eta_2|^4+4|\eta_1|^2|\eta_2|^2+F_{l\eta}^{(4)}),$$

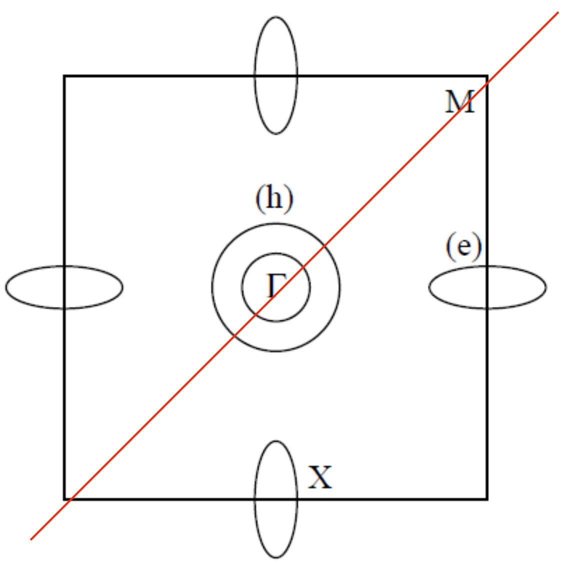


$$T_{c,A} < T_{c,E}$$

E_g Representation of the cubic group

$$\eta_2 = 0; \eta_1 \neq 0 \to O(D_2)$$

$$O(D_2) \Rightarrow (E, 3C_2, 23U_2^{(perp)x} RC_4^x R, 2C_4^y \varepsilon R, 2C_4^z \varepsilon^2 R,$$
$$4C_3\varepsilon^2, 4C_3^2\varepsilon, 2U_2^{(perp)x} R, 2U_2^{(perp)y} \varepsilon R, 2U_2^{(perp)z} \varepsilon^2 R)$$



!?Iron pnictides: “1111”

$$"d-wave" \to \quad x^2 - y^2$$

$$\Delta_{1h} = \Delta_{2h} = 0; \; \Delta_{3e} = -\Delta_{4e} = \Delta$$