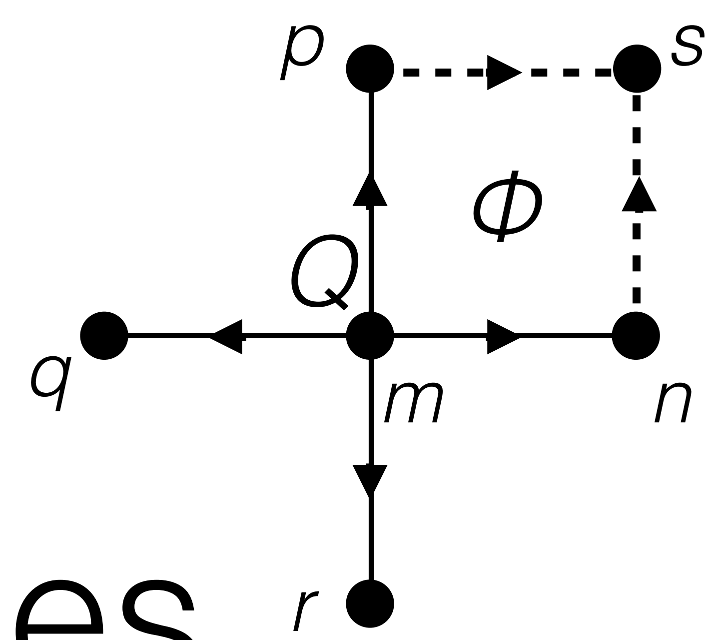
Spin liquids I and II: an introduction to lattice gauge theories



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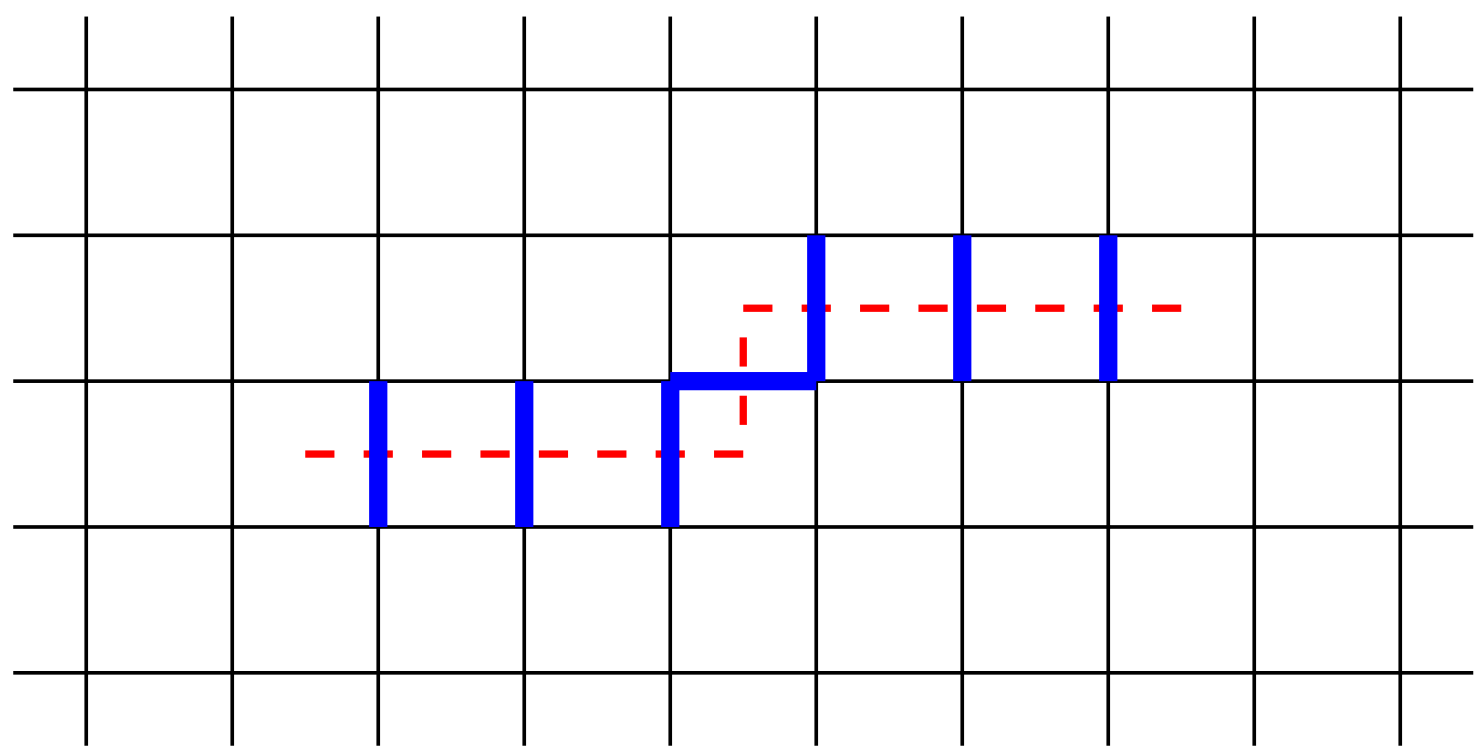
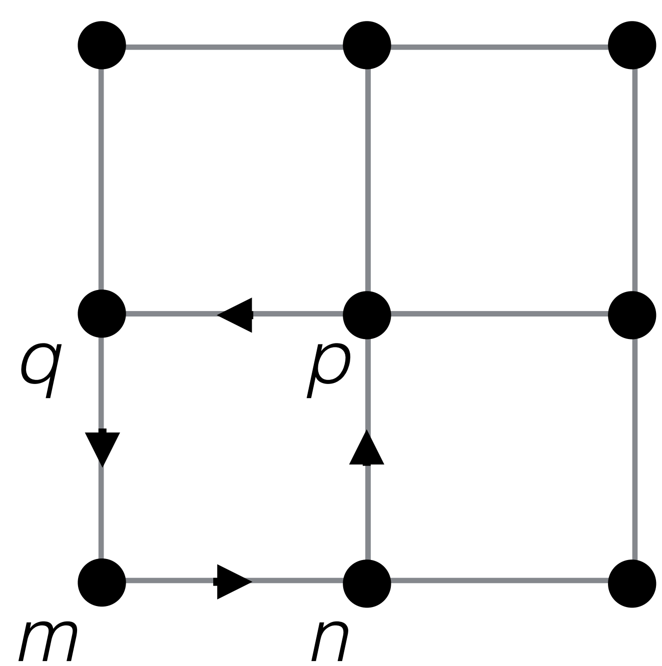
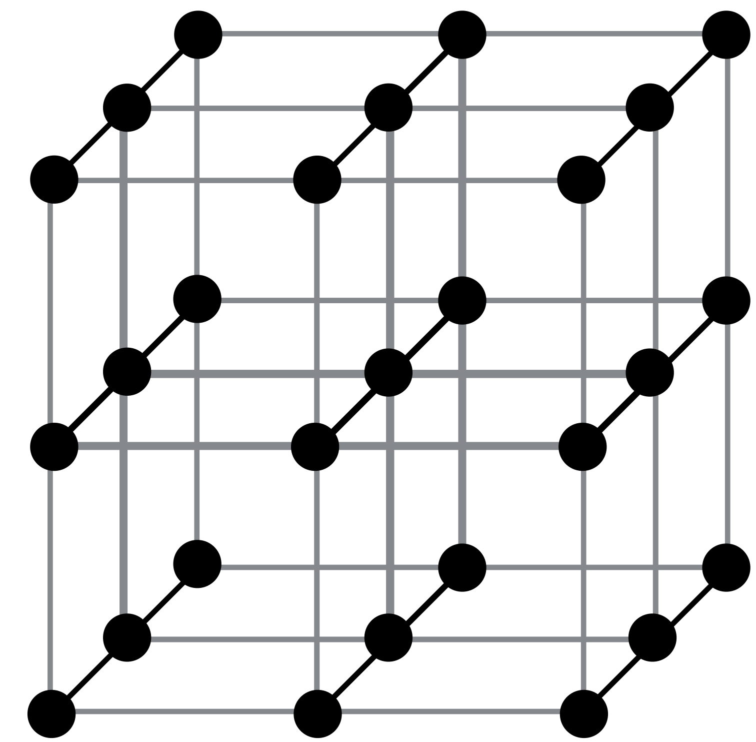
U.S. DEPARTMENT OF ENERGY



U(1) lattice gauge theory
(summary)

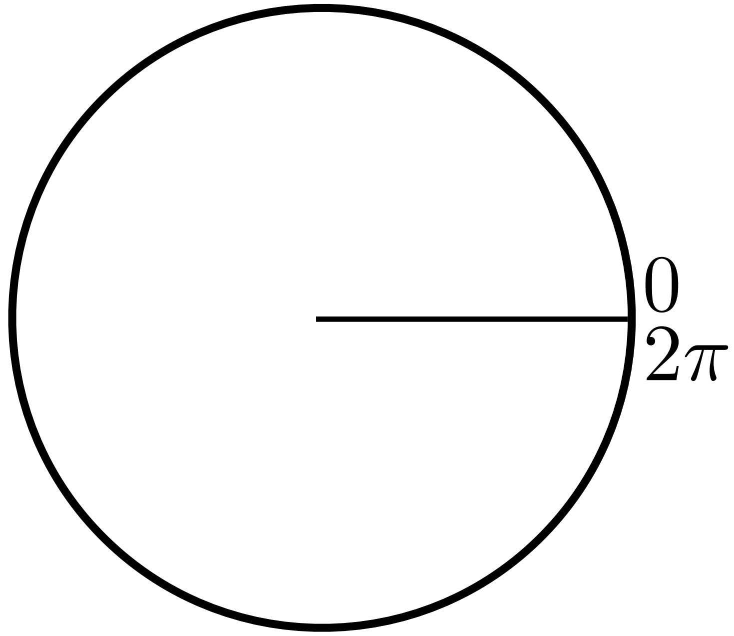




A U(1) gauge theory can be defined on any lattice, in any number of dimensions.

Gauge (unphysical) variables $A_{mn} = - A_{mn}$ live on links *mn*. Physical variables are electric fluxes on links E_{mn} and magnetic fluxes through plaquettes Φ_{mnpq}.

$$E_{mn} = -\dot{A}_{mn} \qquad \Phi_{mnpq} = A_{mn} + A_{np} + A_{pq} + A_{qn}$$



$$0 \leq A \leq 2\pi$$

$$U(A + 2\pi) = U(A)$$

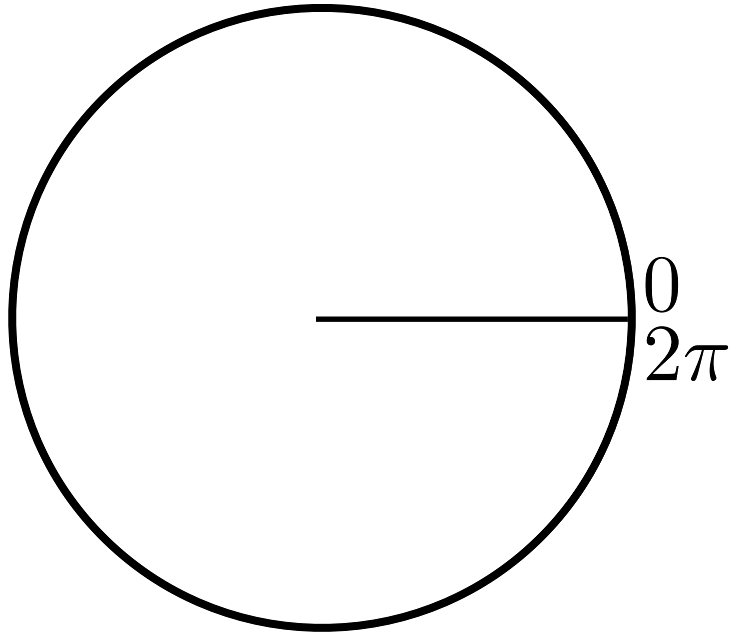
$$U(A) = f(e^{iA})$$

$$H(A, E) = \sum_{\text{links}} \frac{E_{mn}^2}{2I} - \sum_{\text{plaquettes}} \lambda \cos \Phi_{mnpq}$$

$$[E_{mn}, A_{mn}] = i\hbar$$

$$E_{mn} = 0, \pm 1, \pm 2, \ldots \qquad \psi(A + 2\pi) = \psi(A)$$

$e^{\pm i A_{mn}}$ lowering and raising operators for E_{mn}



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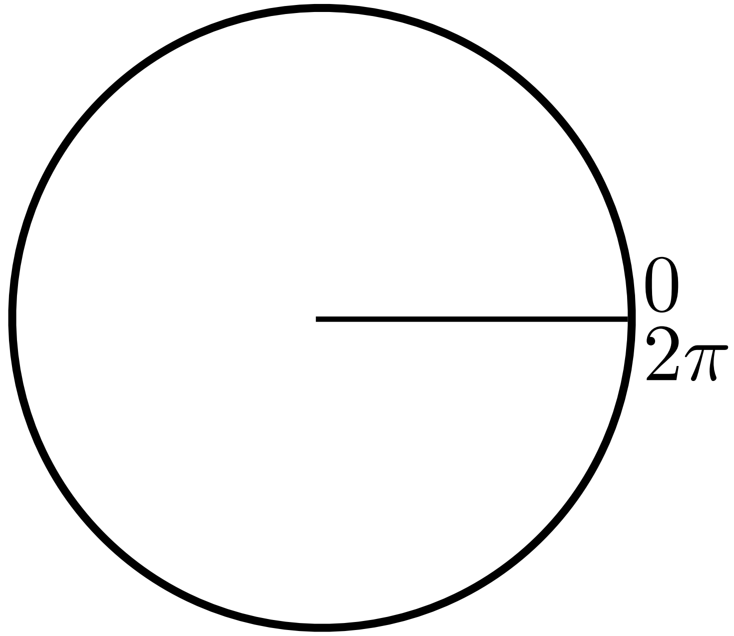

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$$[E_{mn}, A_{mn}] = i$$

$$E_{mn} = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots \qquad \psi(A + 2\pi) = -\psi(A)$$

$e^{\pm iA_{mn}}$ lowering and raising operators for E_{mn}

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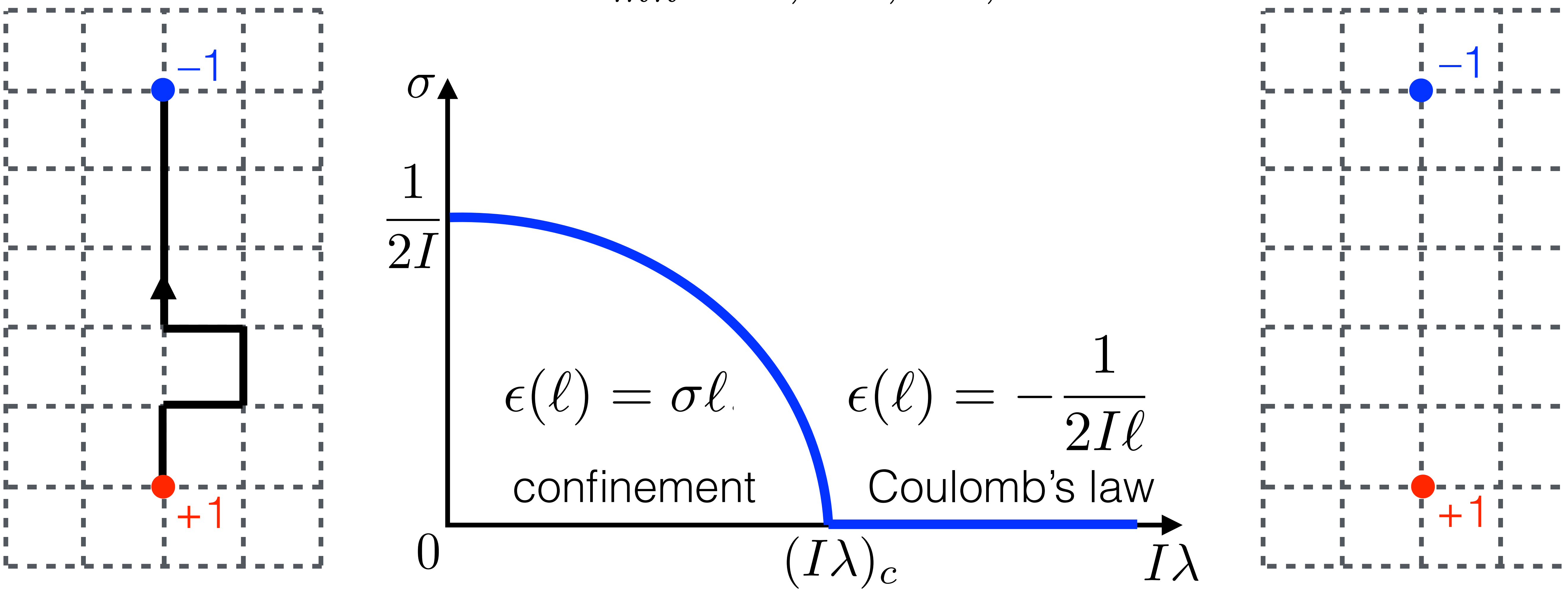

Quantum phase transition in *d*=3 spatial dimensions.
The order parameter is string tension for a pair of test charges.

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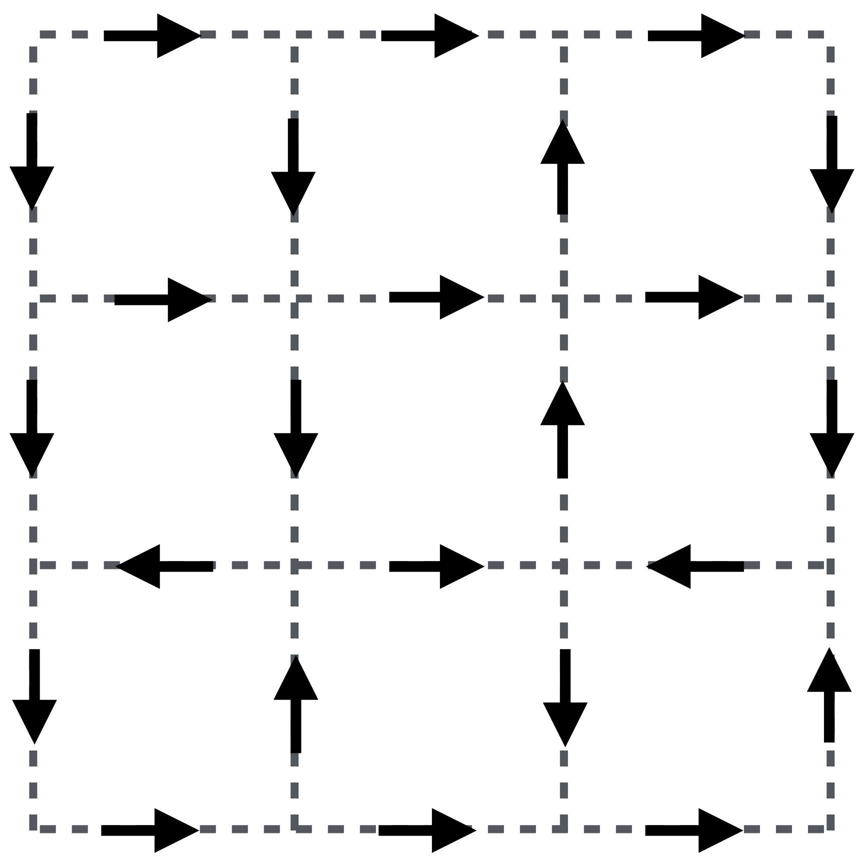
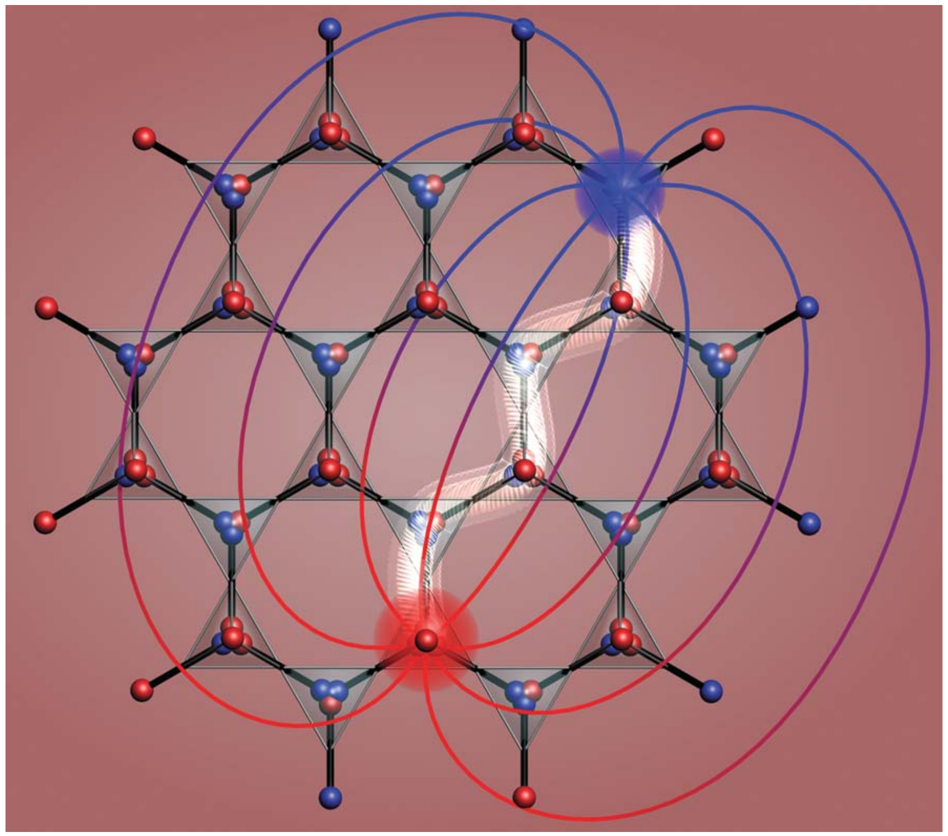
$$E_{mn} = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots$$

U(1) gauge theory of quantum spin ice





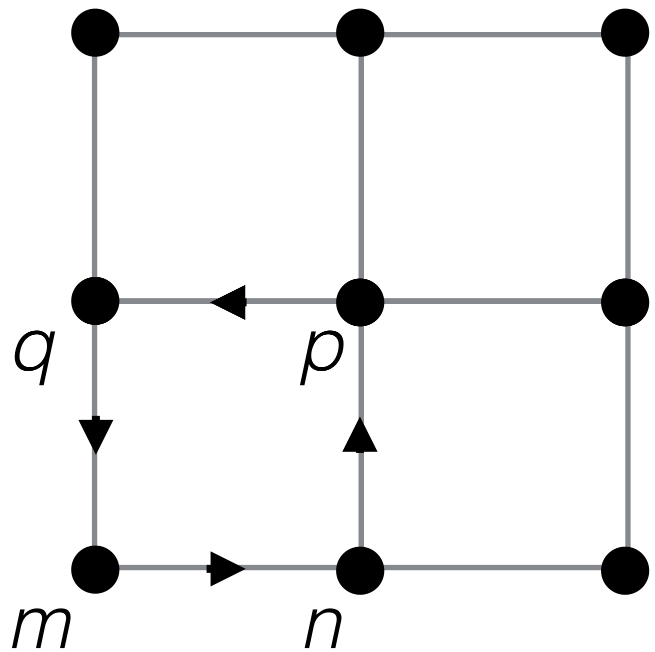
M. Hermele's talk tomorrow.

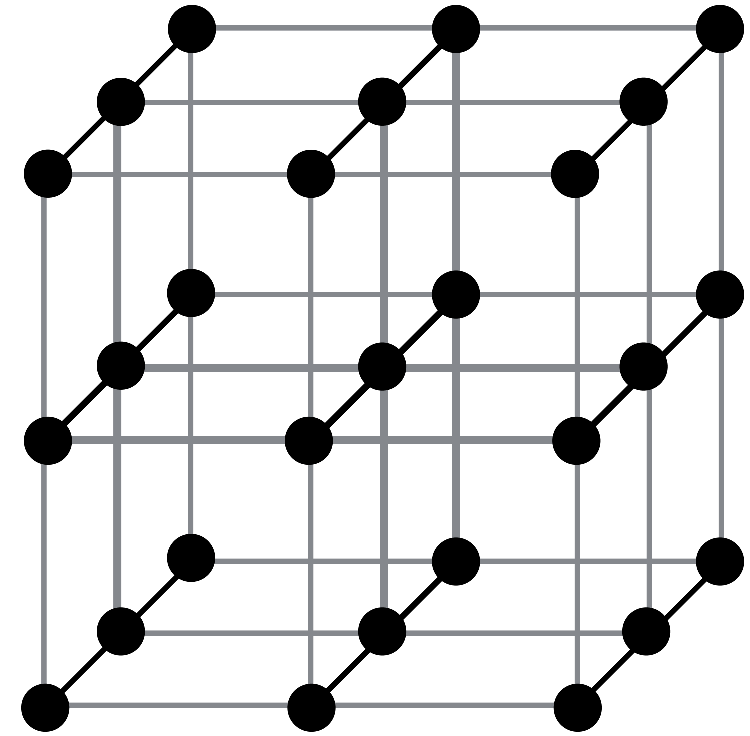
Z_2 lattice gauge theory

The simplest gauge theory ever: uses binary arithmetics!

Relevant to some quantum spin models: Heisenberg model on the square and kagome lattices.

G. Misguich, D. Serban, and V. Pasquier, Phys. Rev. Lett. **89**, 137202 (2002).
H.C. Jiang, H. Yao, and L. Balents, Phys. Rev. B **86**, 024424 (2012).
Y. Wan and O. Tchernyshyov, Phys. Rev. B **87**, 104408 (2013).
H.J. Ju and L. Balents, Phys. Rev. B **87**, 195109 (2013).





A Z₂ gauge theory can be defined on any lattice, in any number of dimensions. We will specialize to d=2 here.

We will jump directly to the quantized version of the theory.

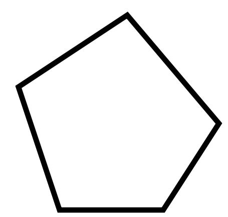
The main idea is to switch from integer arithmetics ($\mathbb{Z}$) to binary one ($\mathbb{Z}_2$) for the electric flux through lattice links.

Compact U(1) gauge theory

$$E = 0, \pm 1, \pm 2, \dots$$

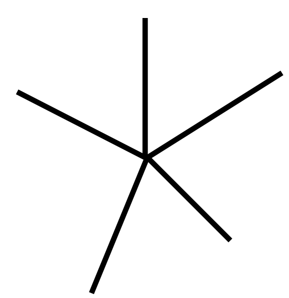
$$0 \leq A \leq 2\pi$$

$$\Phi = \sum_{\text{plaquette}} A$$



$$e^{\pm iA} E e^{\mp iA} = E \pm 1$$

$$Q = \sum_{\text{star}} E$$



Addition

Z_2 gauge theory

$$(-1)^E \cong \sigma^x = \pm 1$$

$$e^{\pm iA} \cong \sigma^z = \pm 1$$

$$e^{i\Phi} \cong \phi = \prod_{\text{plaquette}} \sigma^z = \pm 1$$

$$\sigma^z \sigma^x \sigma^z = -\sigma^x$$

$$(-1)^Q \cong \rho = \prod_{\text{star}} \sigma^x = \pm 1$$

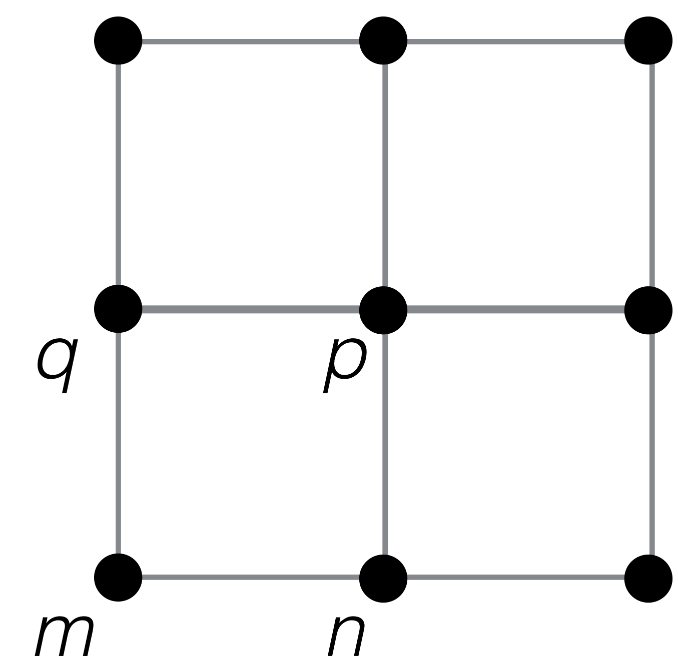
Multiplication

Here $\cong$ means “corresponds to.” $\boldsymbol{\sigma}$ are Pauli operators.

Quantum Hamiltonian

Compact U(1) gauge theory:

$$H = \sum_{\text{links}} \frac{E_{mn}^2}{2I} - \sum_{\text{plaquettes}} \lambda \cos \Phi_{mnpq}$$

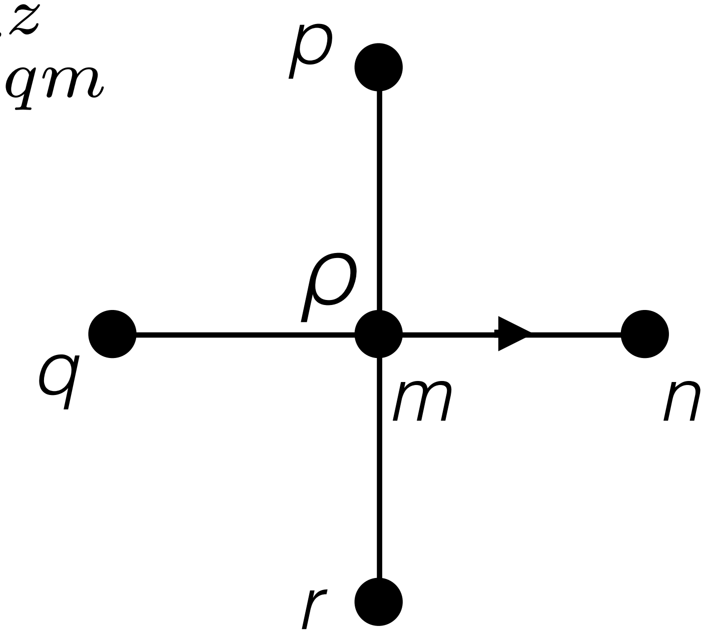


Z_2 gauge theory:

$$H = -\Gamma \sum_{\text{links}} \sigma_{mn}^x - \lambda \sum_{\text{plaquettes}} \sigma_{mn}^z \ldots \sigma_{qm}^z$$

Conserved charges

$$H = -\Gamma \sum_{\text{links}} \sigma^x_{mn} - \lambda \sum_{\text{plaquettes}} \sigma^z_{mn} \dots \sigma^z_{qm}$$



$$\rho_m = \underline{\sigma^x_{mn}}\,\underline{\sigma^x_{mp}}\,\sigma^x_{mq}\sigma^x_{mr}$$

$$\phi_{mnsp} = \underline{\sigma^z_{mn}}\,\sigma^z_{ns}\sigma^z_{sp}\,\underline{\sigma^z_{pm}}$$

$$[\rho_m, \phi_{mnsp}] = 0$$

$$[\rho_m, H] = 0$$

Z_2 electric charges ρ are constants of motion.
States again separate into different charge sectors.

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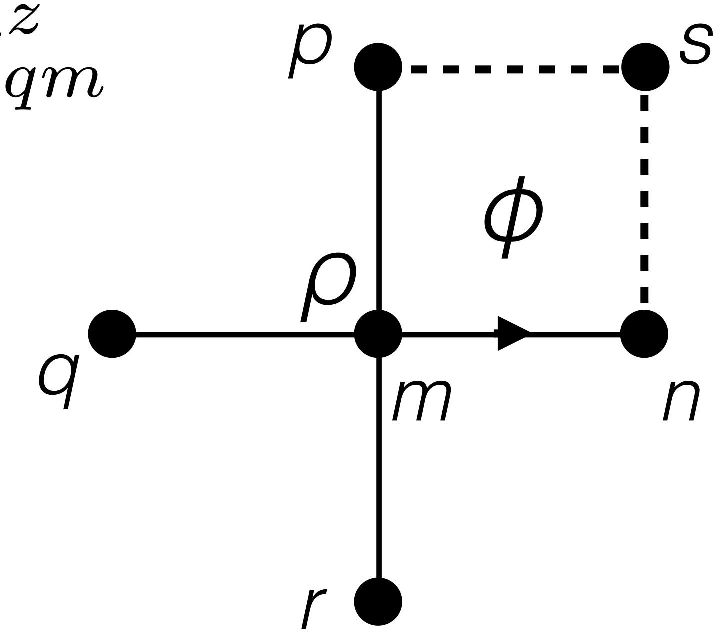

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Z_2 electric charges ρ are constants of motion.
States again separate into different charge sectors.

Electric term dominates: $\Gamma \gg \lambda$

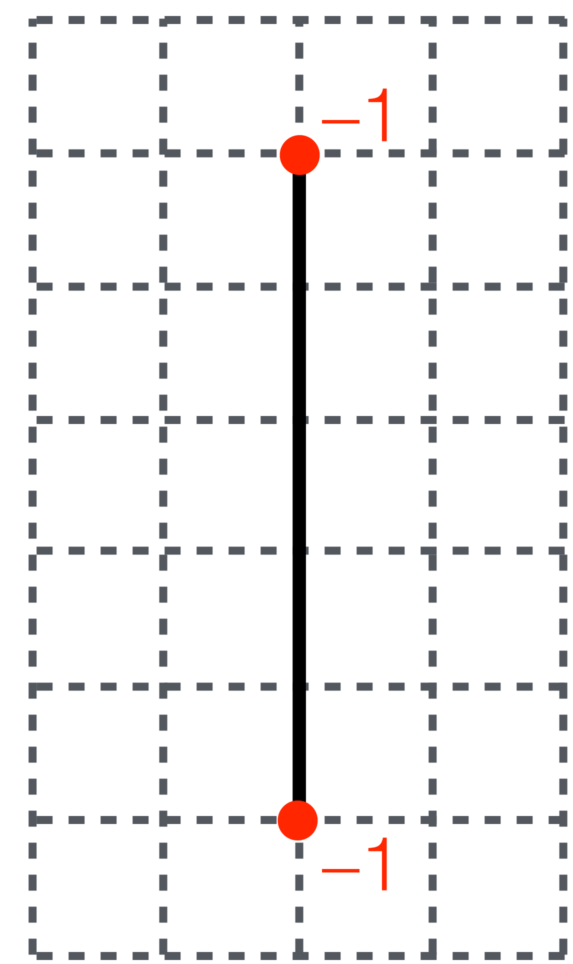
$$H = H_0 + H_1, \quad H_0 = -\Gamma \sum_{\text{links}} \sigma^x_{mn}, \quad H_1 = -\lambda \sum_{\text{plaquettes}} \sigma^z_{mn} \ldots \sigma^z_{qm}$$

Neglect the weak magnetic term.

No-charge sector: $\sigma^x = +1$ everywhere.

Sector with two probe charges $\rho = -1$: ground state with an electric flux line $\sigma^x = -1$ connecting the charges.

Energy grows linearly with the distance. Electric charges are confined.



$$\epsilon(\ell) = 2\Gamma\ell$$

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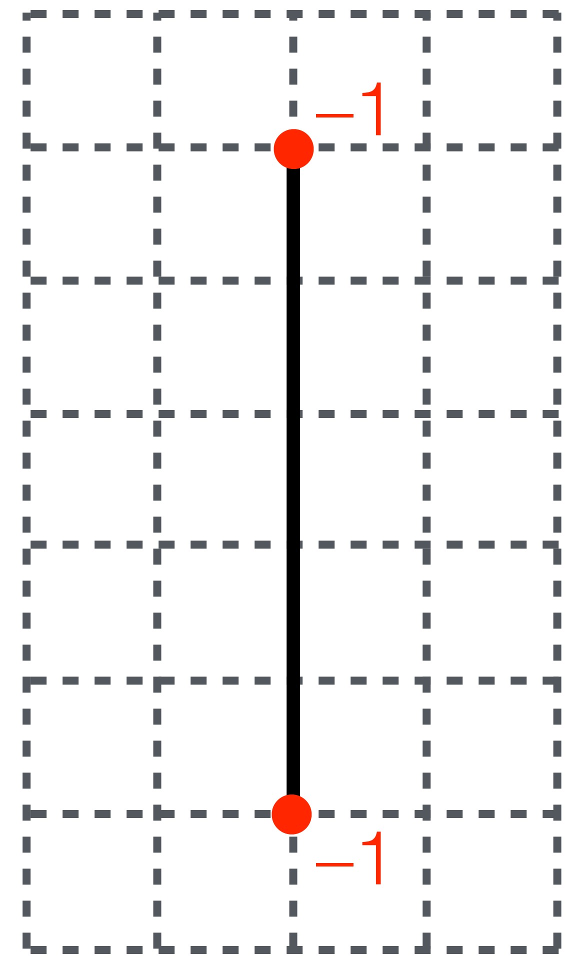
Treat the magnetic term as a perturbation. It induces quantum fluctuations of the electric string connecting the charges.

String tension is reduced. Confinement remains.

$$\sigma = 2\Gamma - \lambda^2/4\Gamma + \ldots$$



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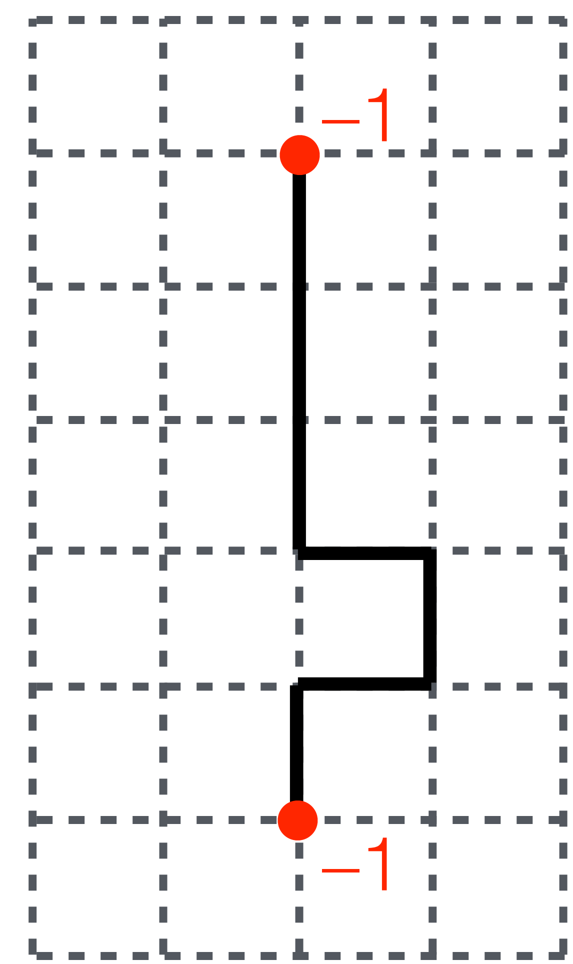

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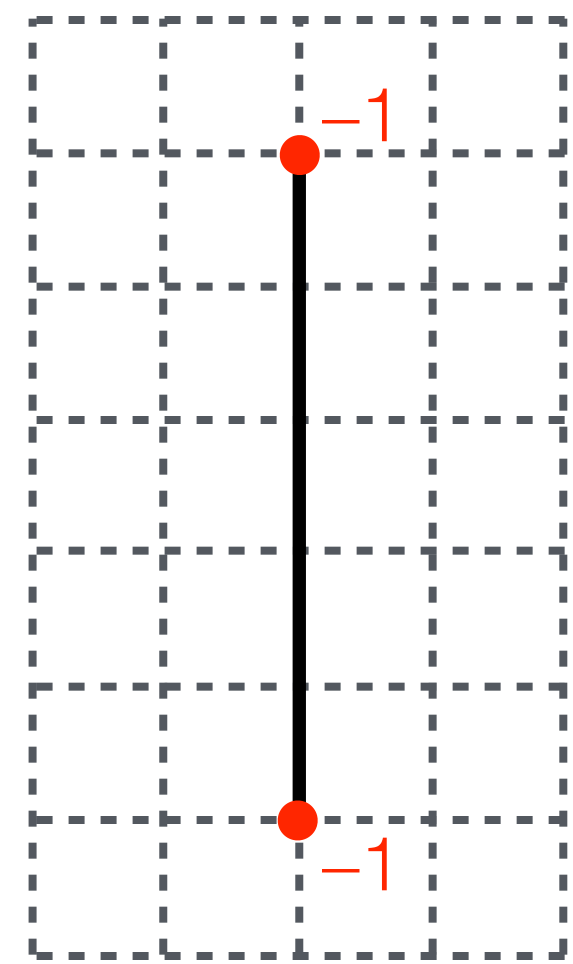

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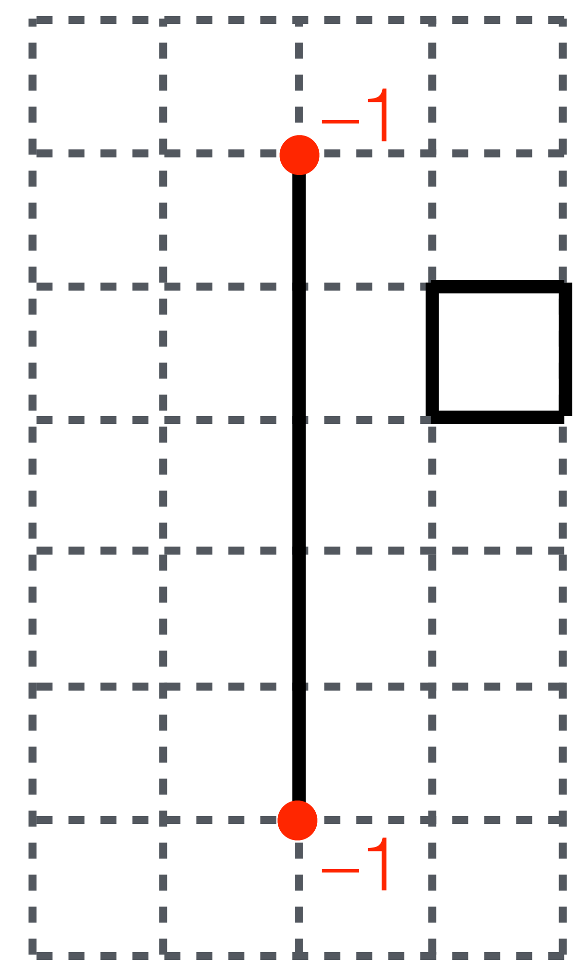

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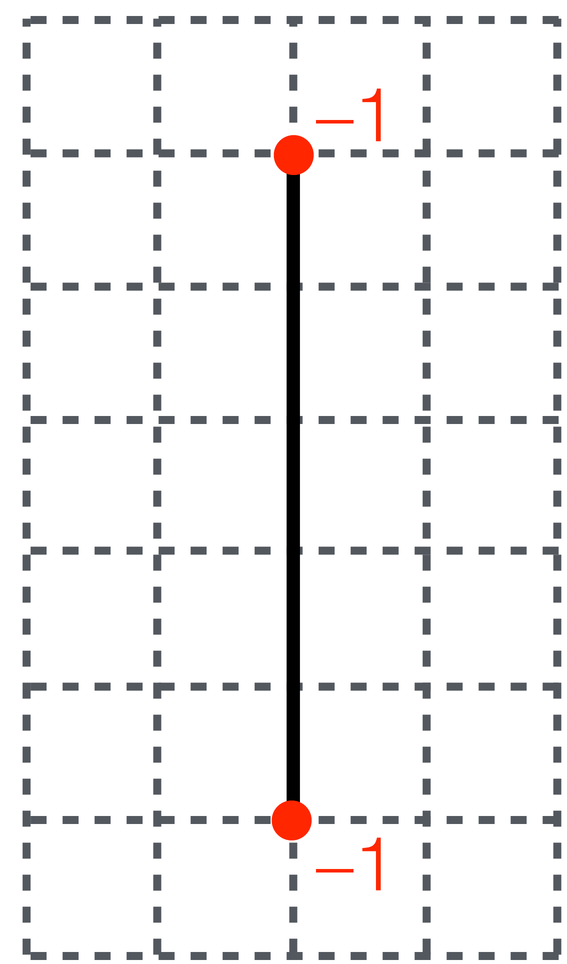

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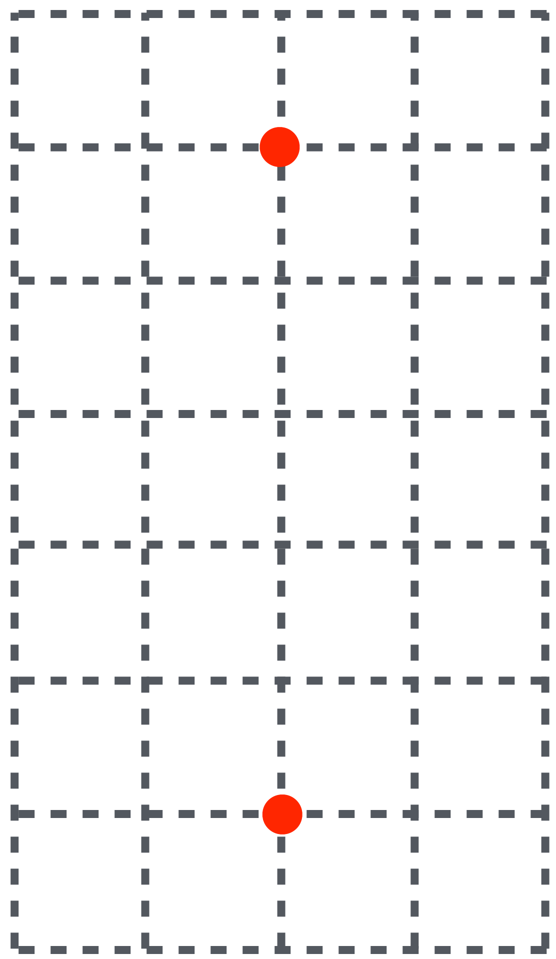
Neglect the weak electric term. The magnetic term is minimized if all $\phi = +1$.

This condition is independent of the charge sector (ϕ and ρ commute).

Energy of two charges does not depend on the distance between them.
Electric charges are not confined.



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Treat the electric term as a perturbation.

It creates virtual excitations: pairs of Z_2 vortices ($\phi = -1$).

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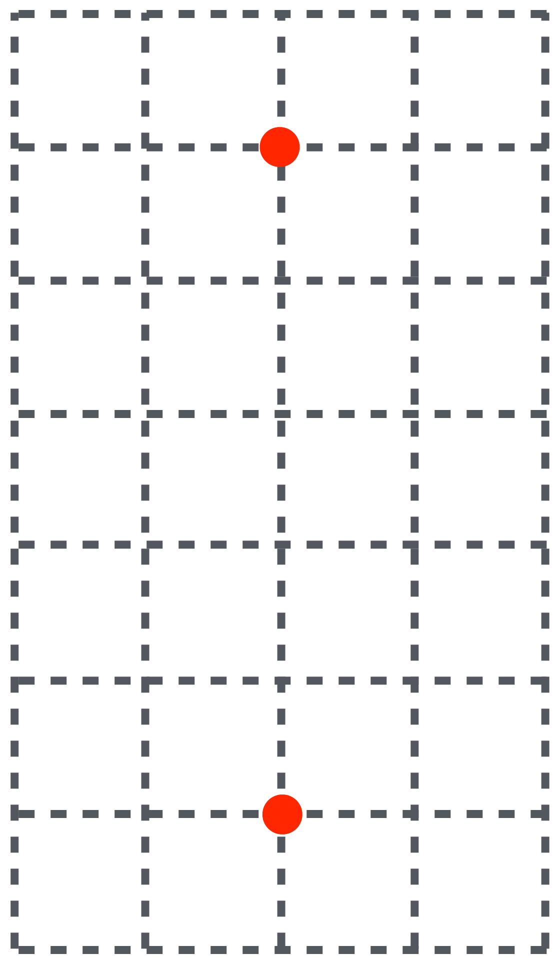

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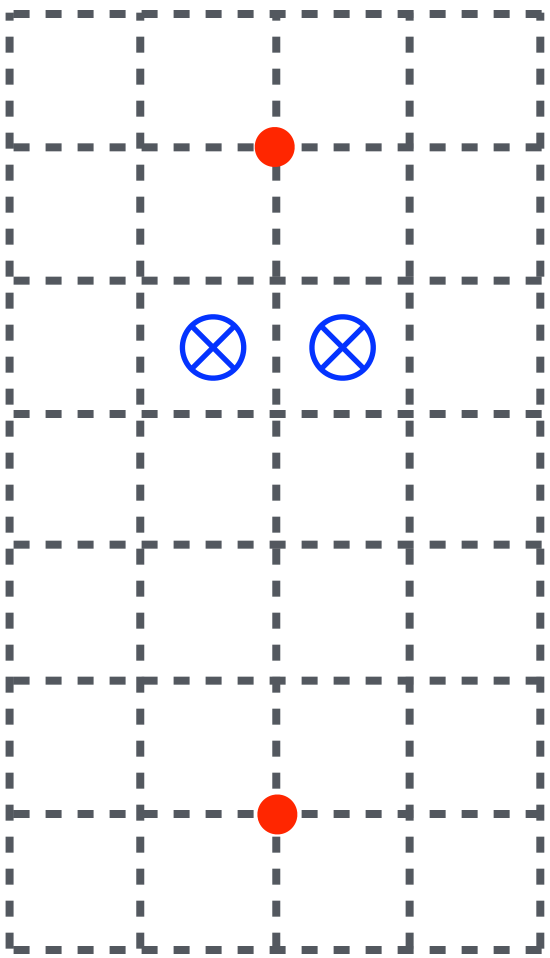

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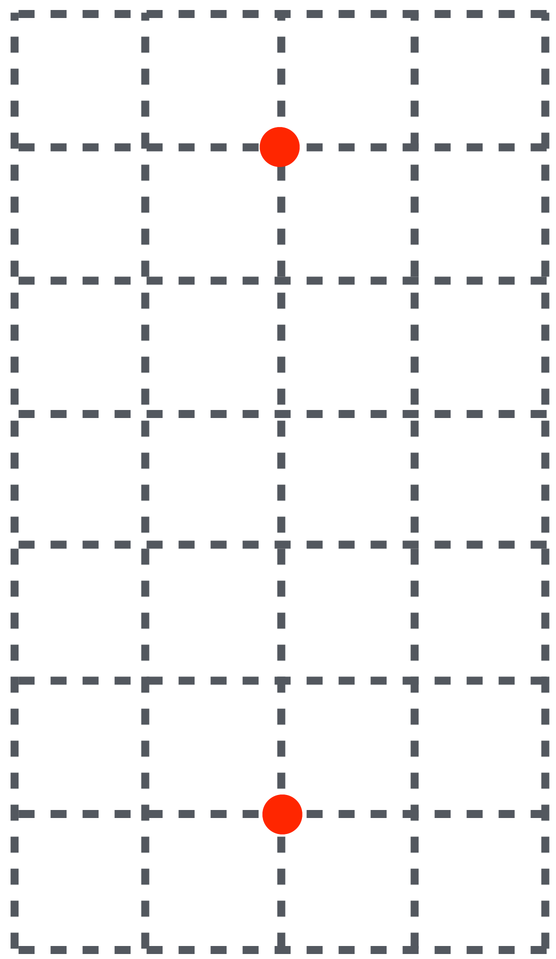

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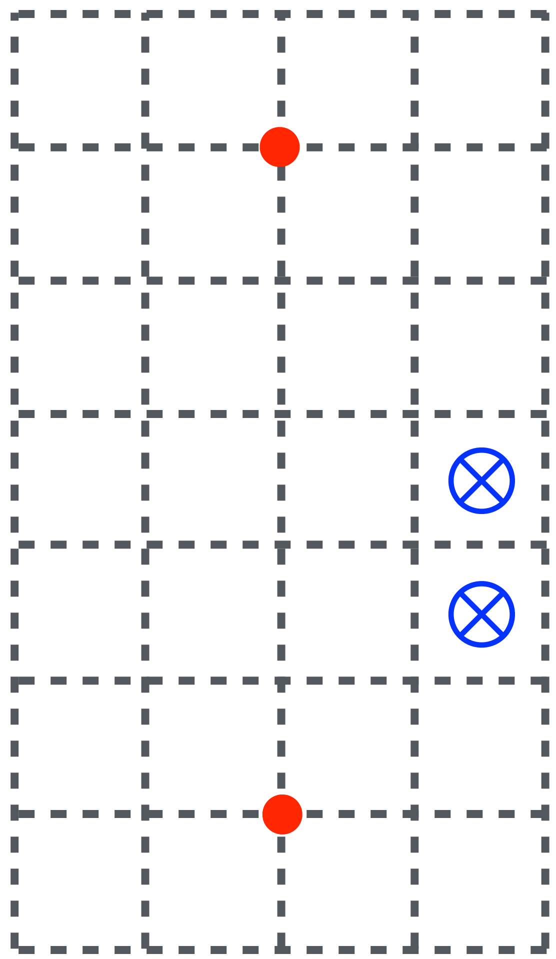

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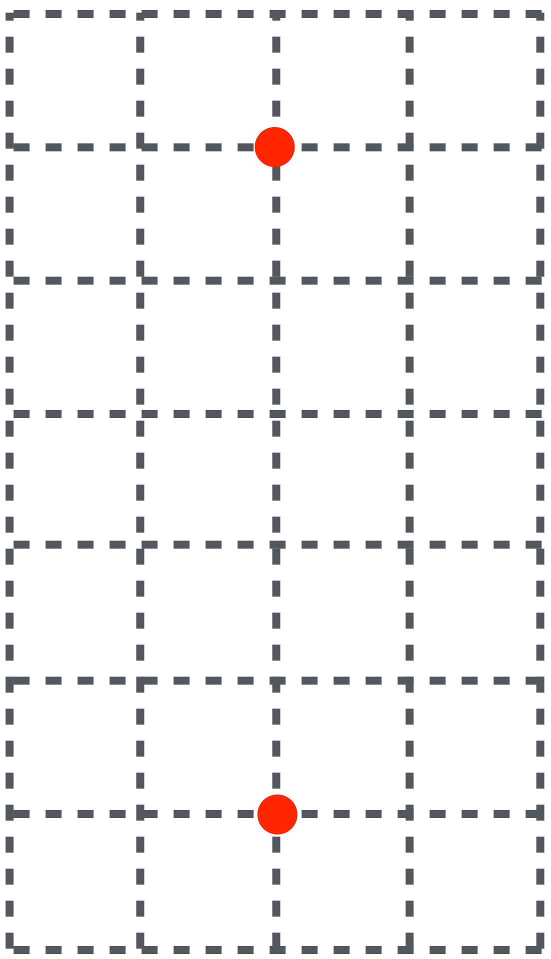

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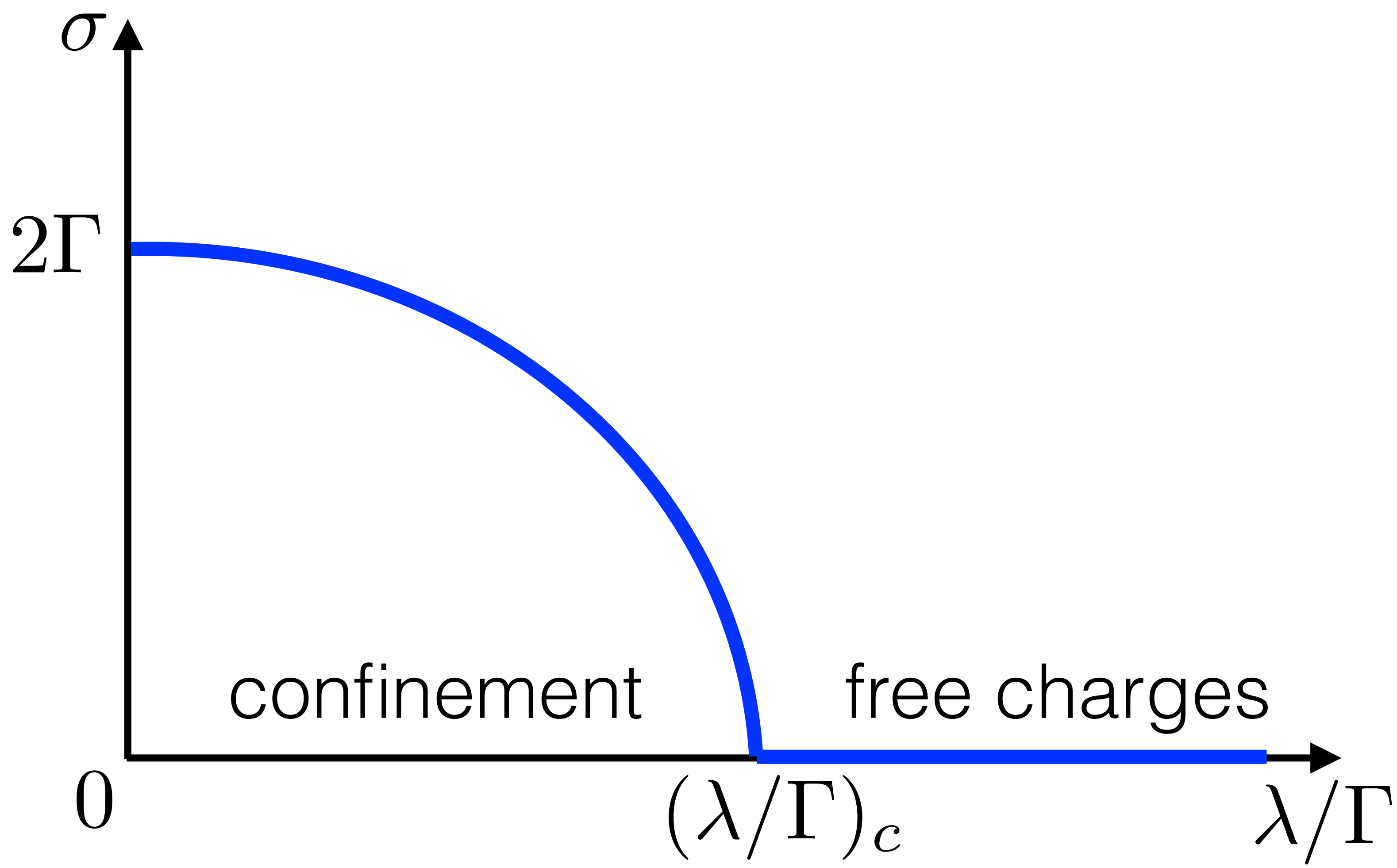
String tension in d=2



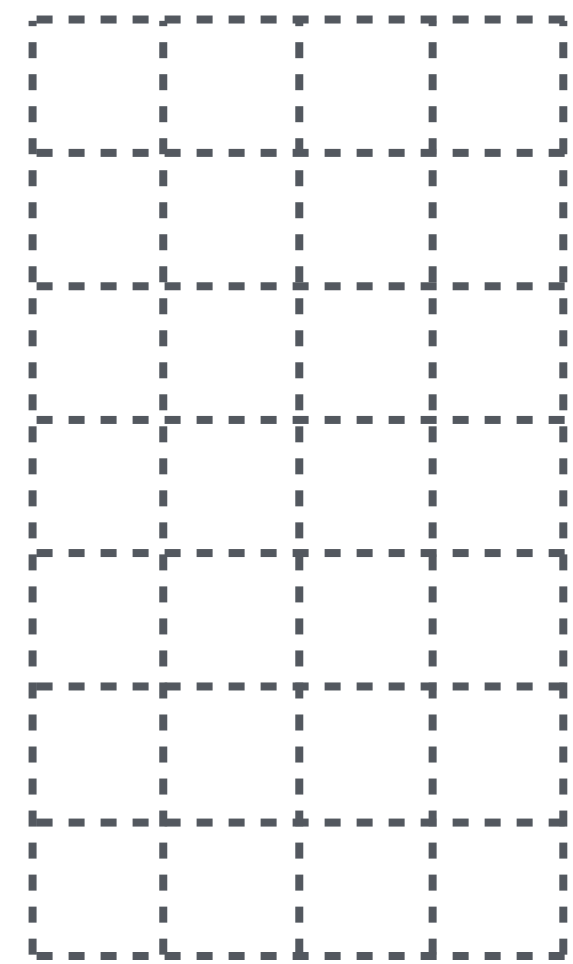
Two distinct phases of matter: confined and deconfined. String tension can be used as an order parameter whose presence or absence determines which phase we are in.

Topological degeneracy

The confined phase of a lattice gauge theory, where electric field dominates, has a simple ground state that is a direct product of individual link states:

$$|\Psi\rangle = \prod_{\text{links}} |E = 0\rangle$$

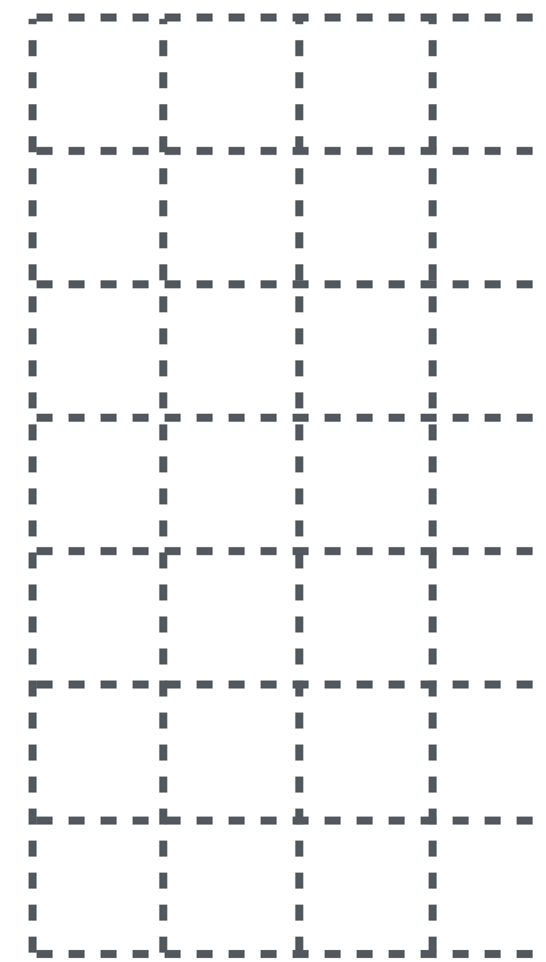
The state is explicitly specified and is unique, not degenerate.



Topological degeneracy

In the deconfined phase, the ground state is described in terms of fluxes through plaquettes of the lattice:

$\Phi = 0$ on every plaquette

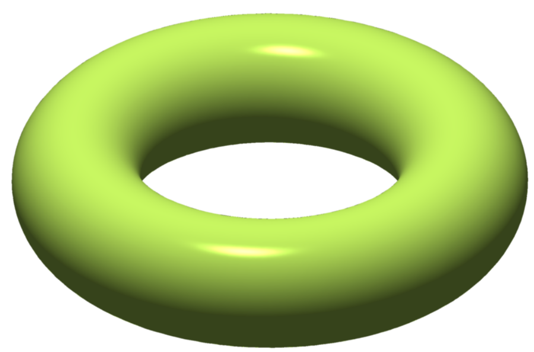


This is an implicit description: we do not know the states of individual links.

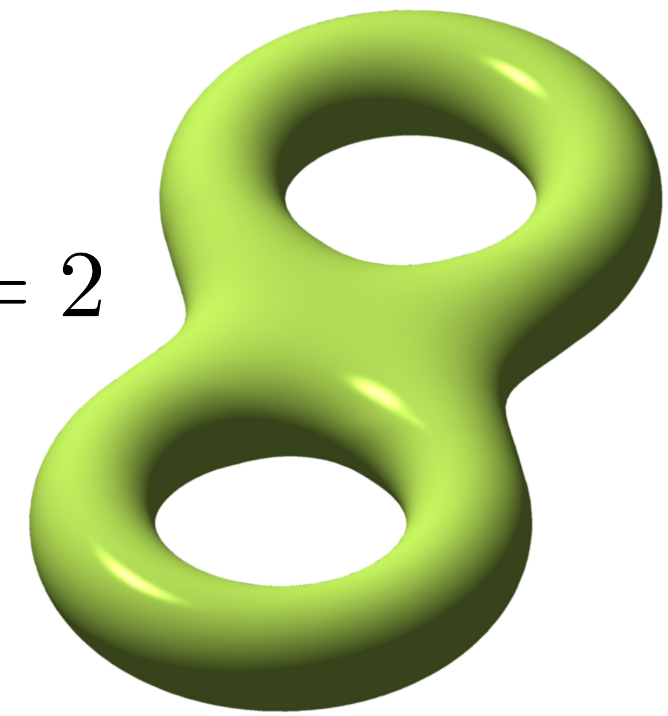
We shall see that all states of the system are degenerate in this phase and that the degeneracy depends on the topology of the sample.

Topological degeneracy

$g = 1$



$g = 2$



In the deconfined phase of the Z_2 gauge theory, all states have the degeneracy 4^g, where g is the genus of the two-dimensional surface (the number of handles).

The genus of a surface is related to its Euler characteristic, which can be calculated for a discrete (lattice) surface:

$$2 - 2g = \chi \equiv V - E + F$$

A Z_2 gauge theory has E qubits (one per edge).

A charge sector is specified by $V-1$ independent charges (one per vertex minus the condition of net neutrality).

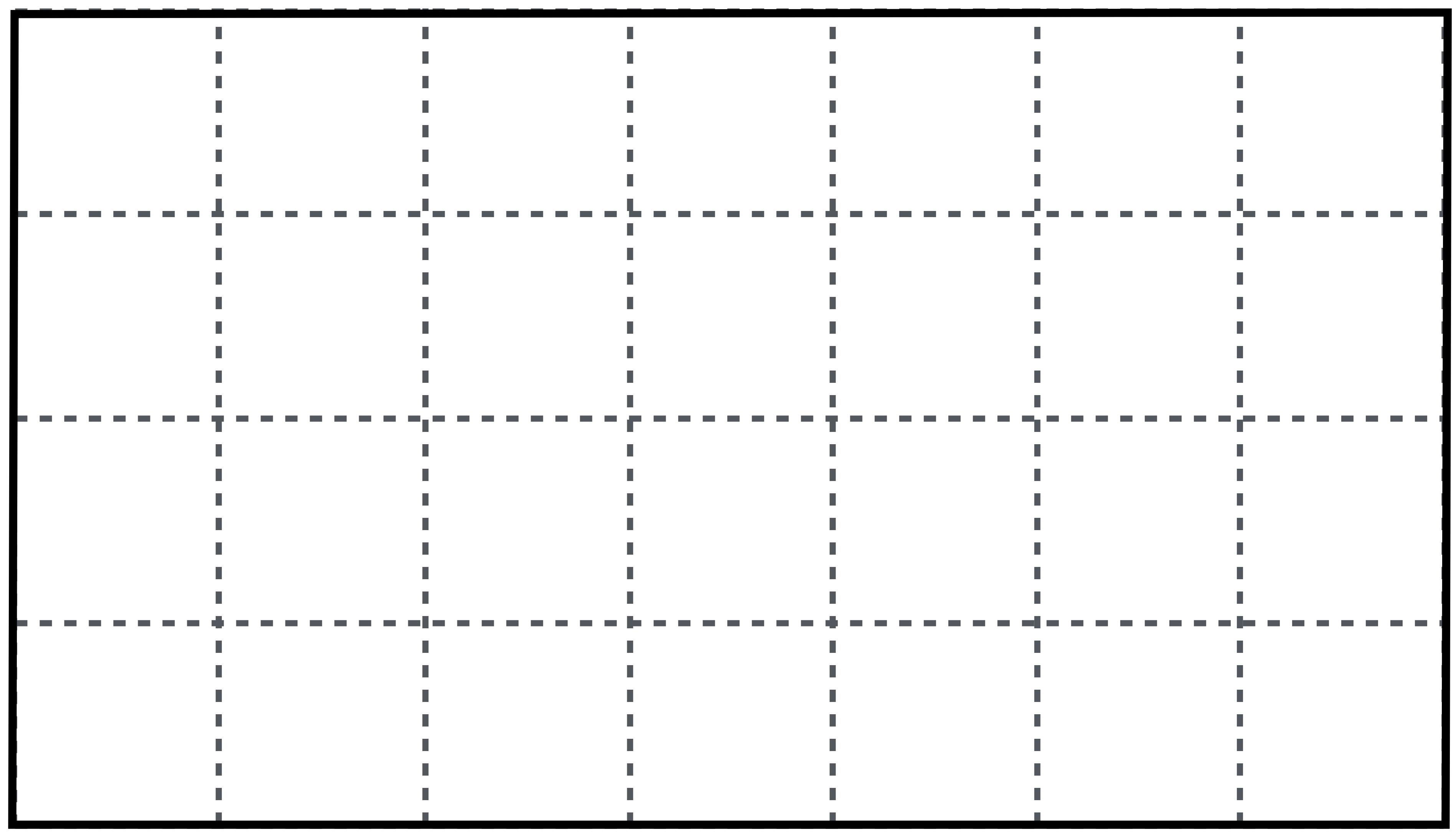
A flux state is specified by $F-1$ independent fluxes (one per face minus the condition of net zero flux).

The number of qubits remaining is

$$E - (V-1) - (F-1) = 2 - \chi = 2g$$

Hence the degeneracy 2^{2g}.

Each elementary plaquette has zero flux. What is the flux through the big loop?



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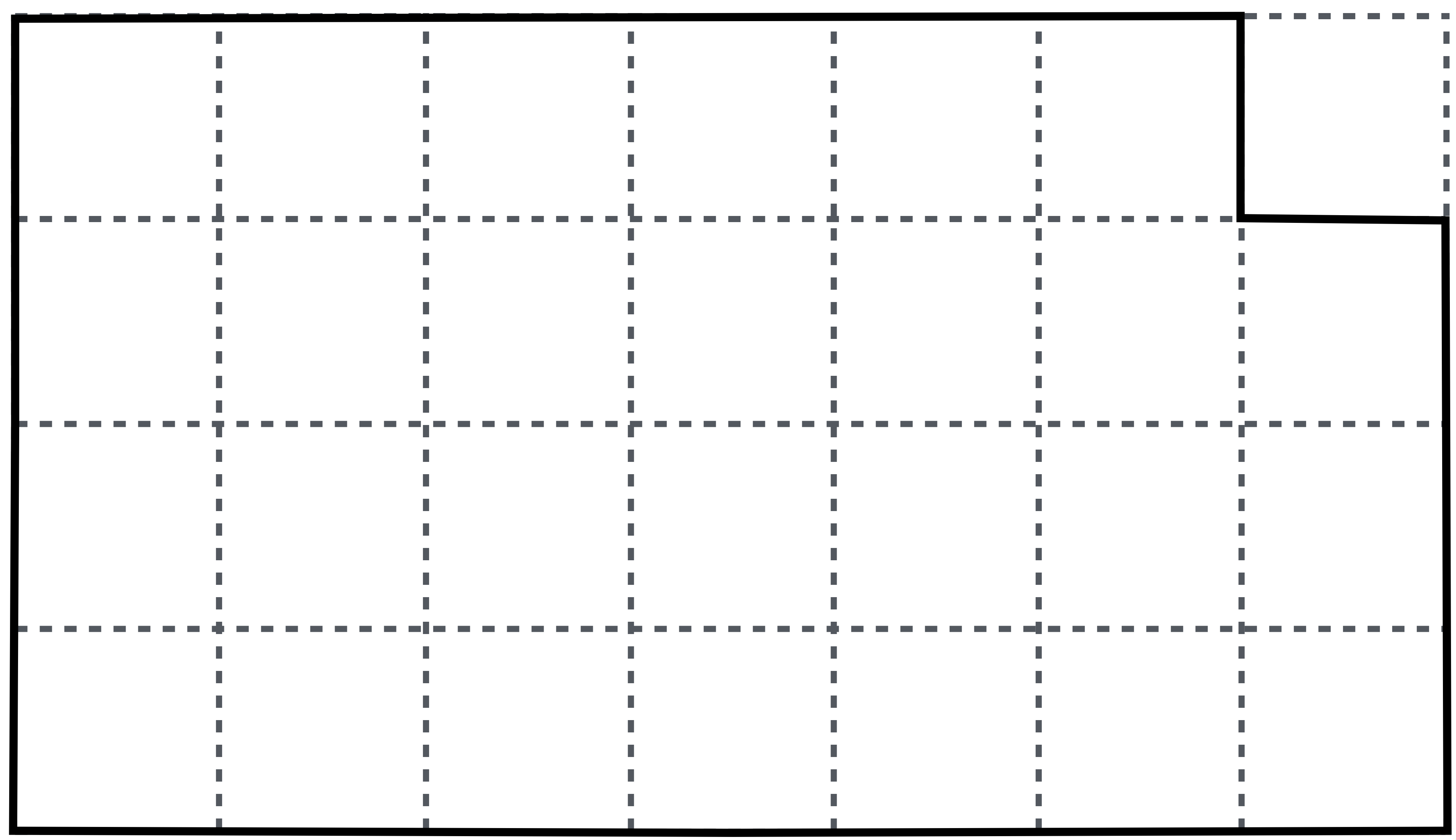

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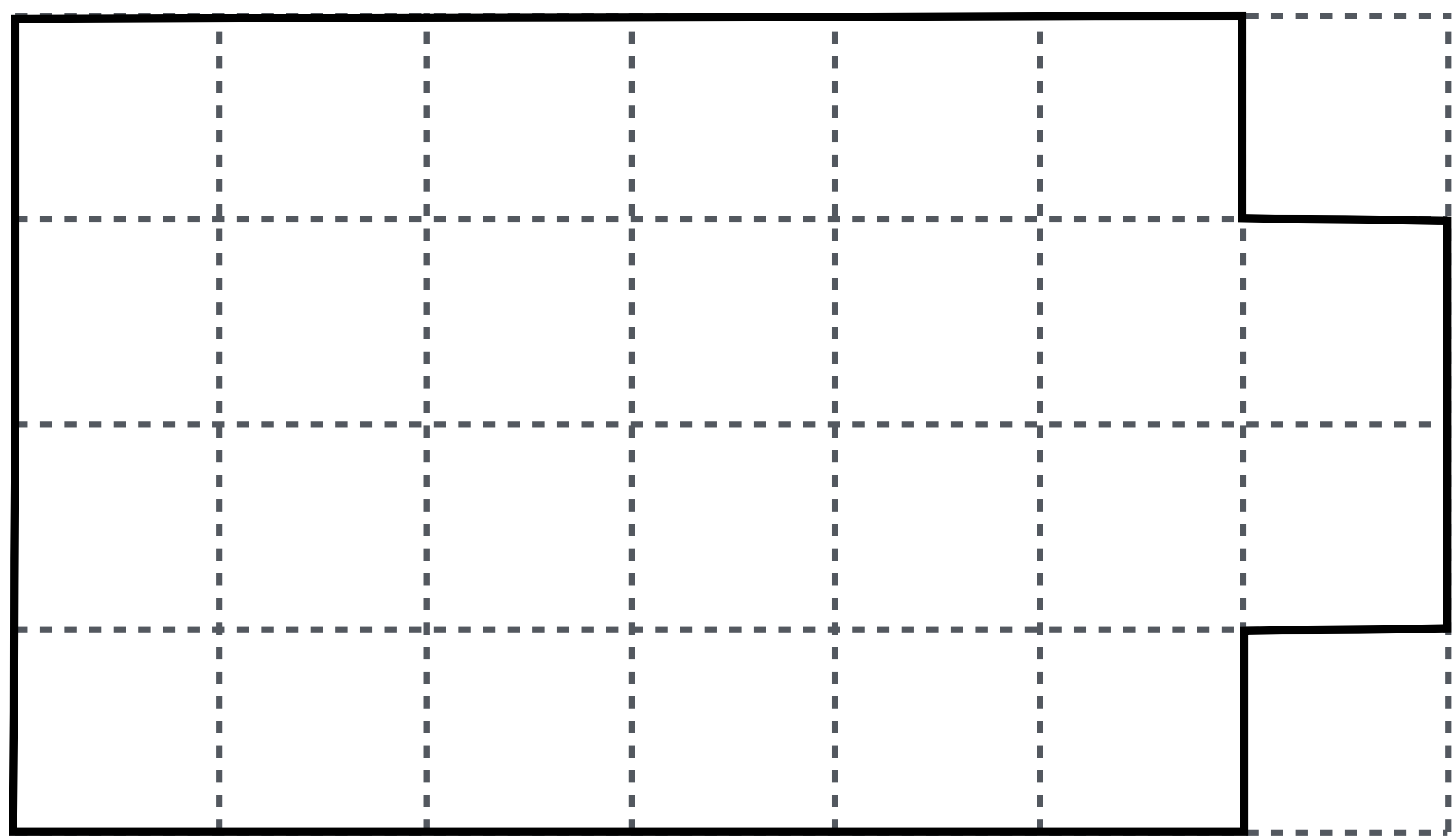

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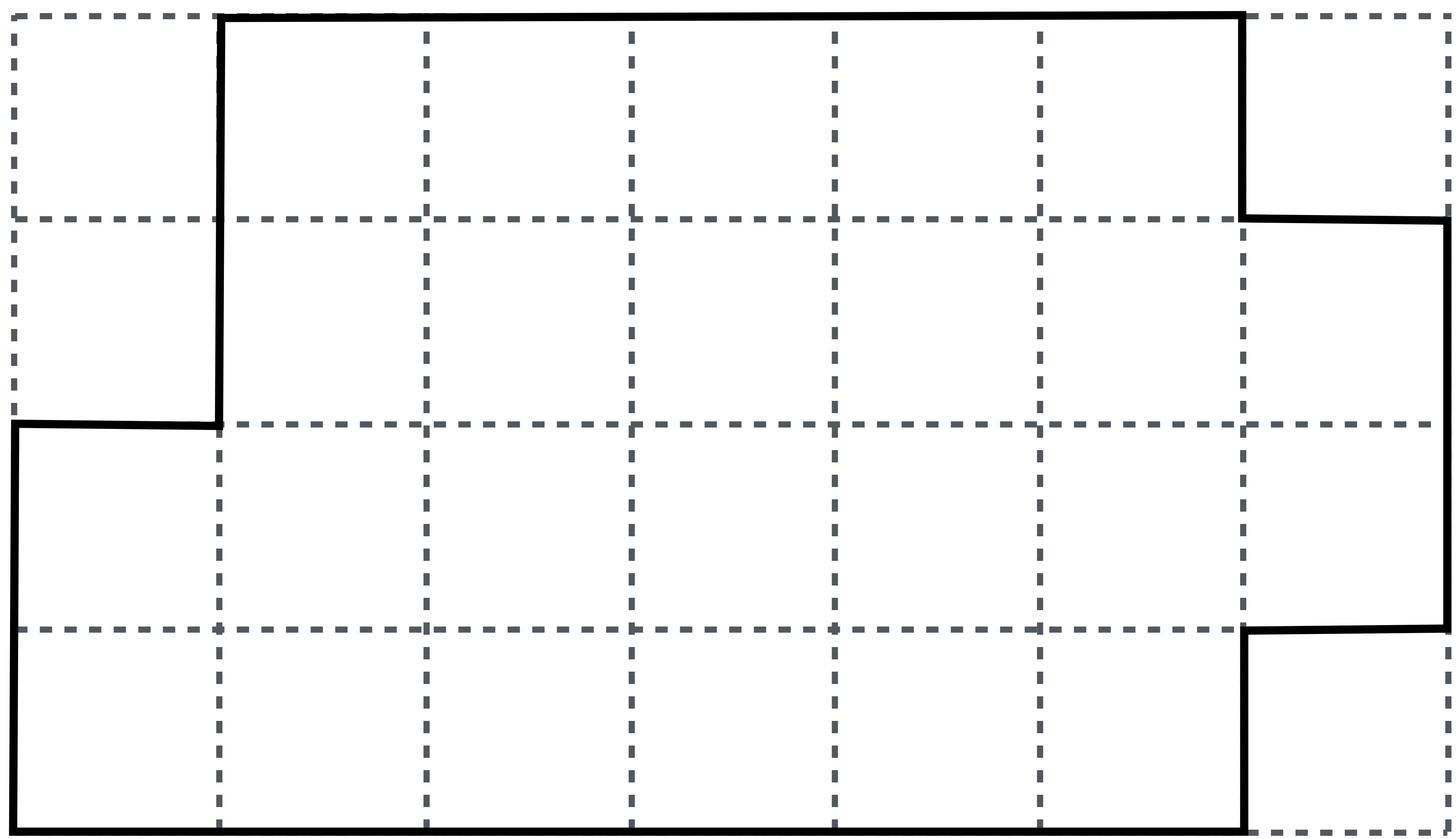

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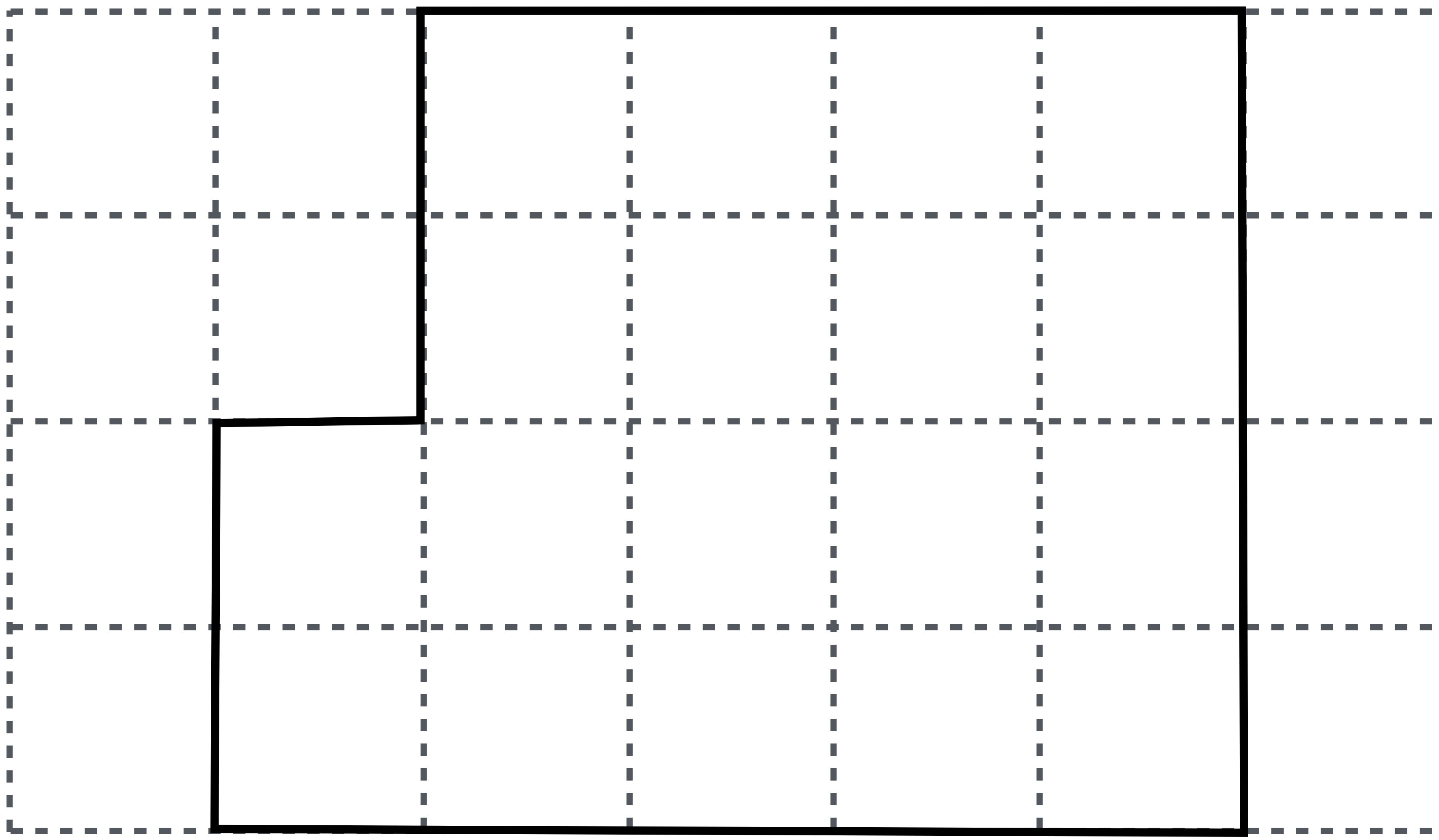

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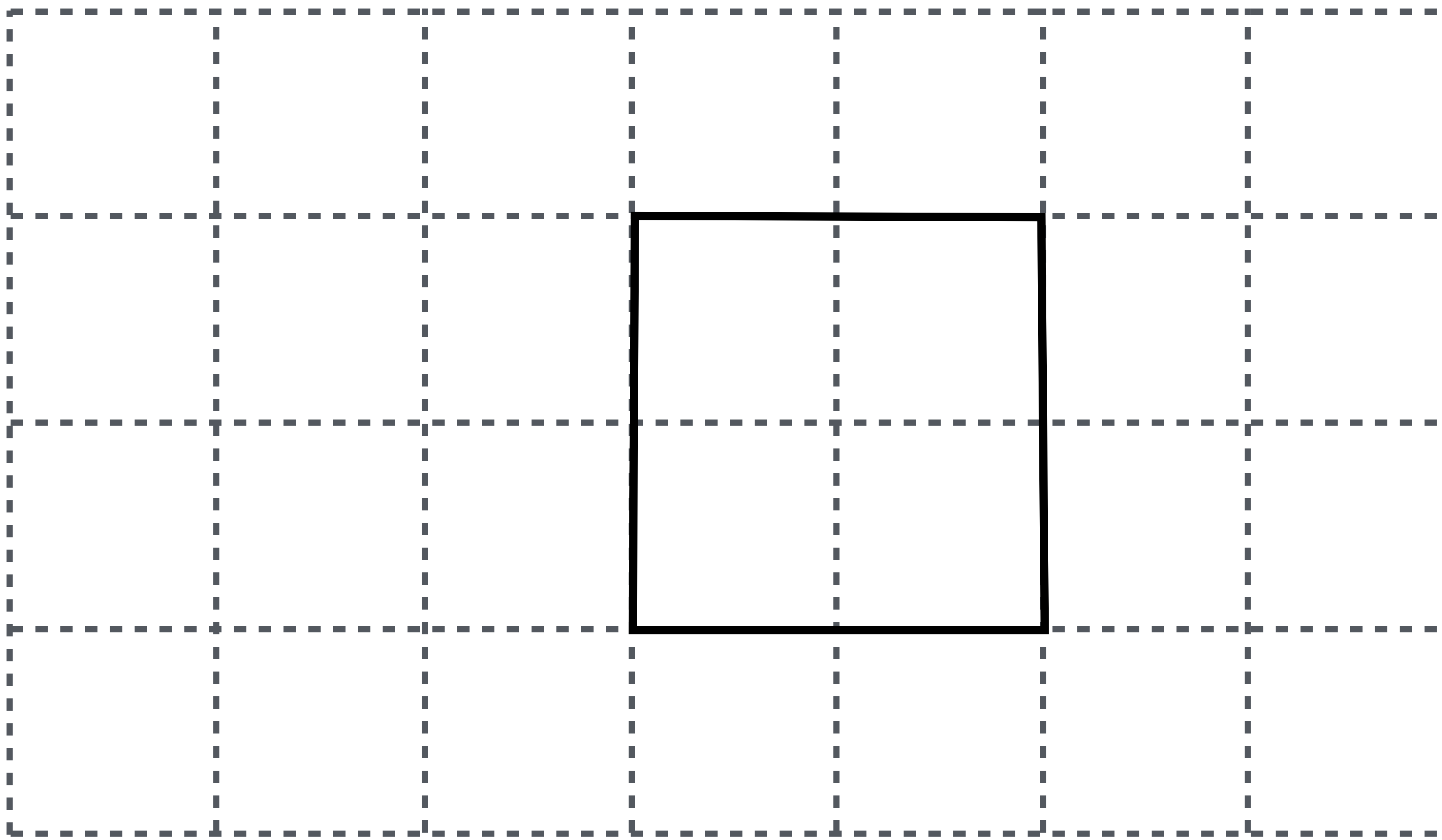

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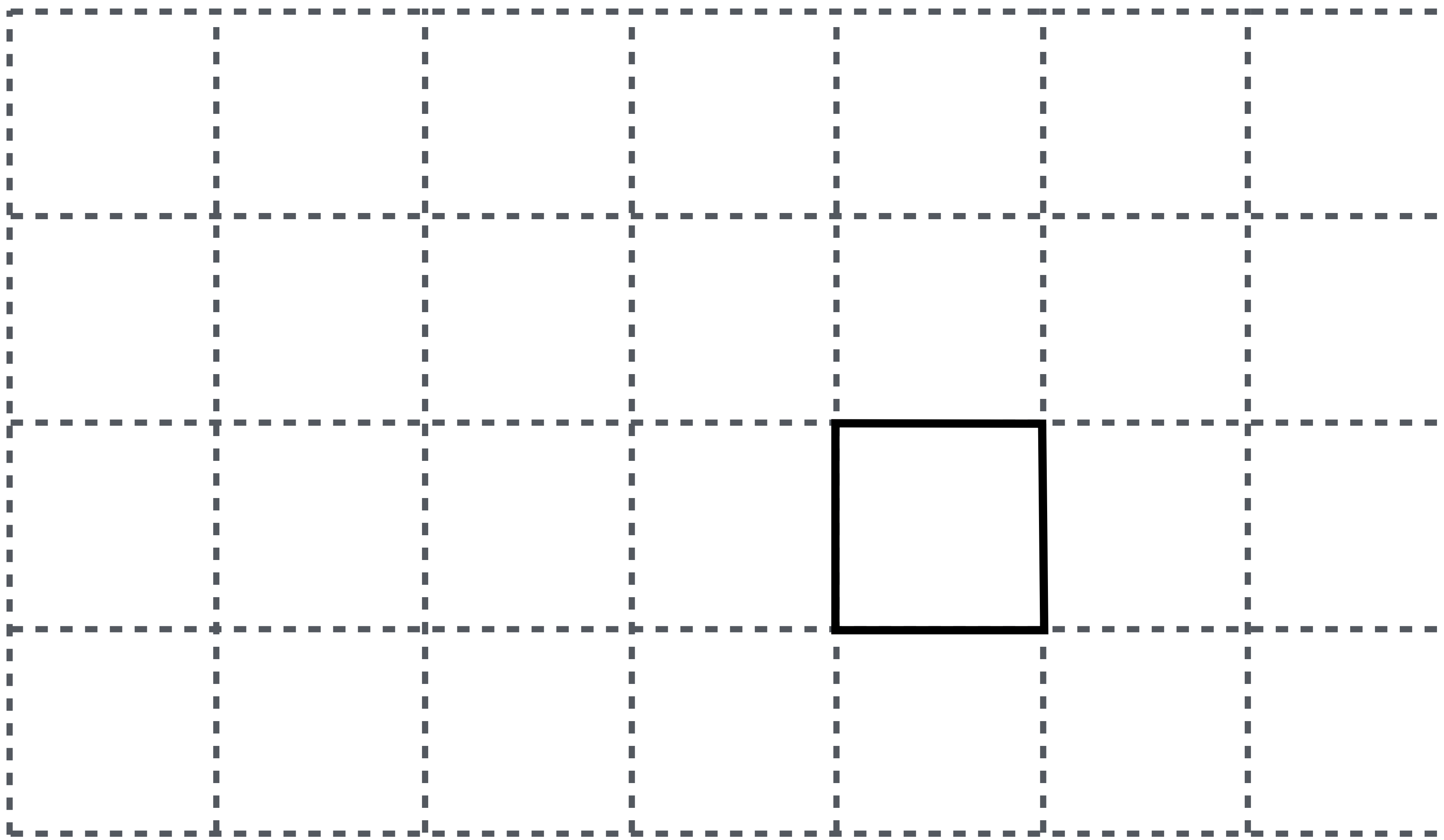

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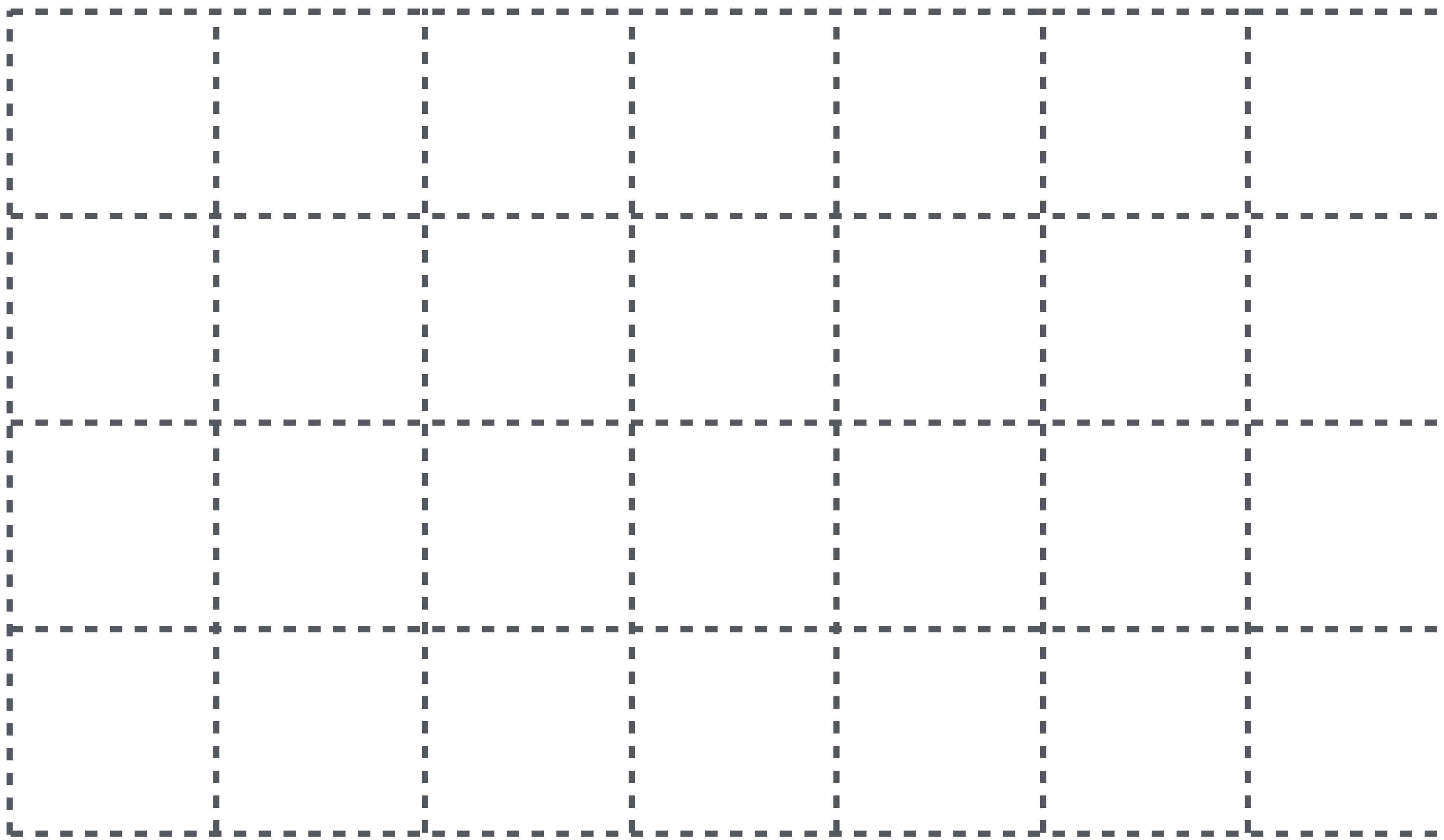
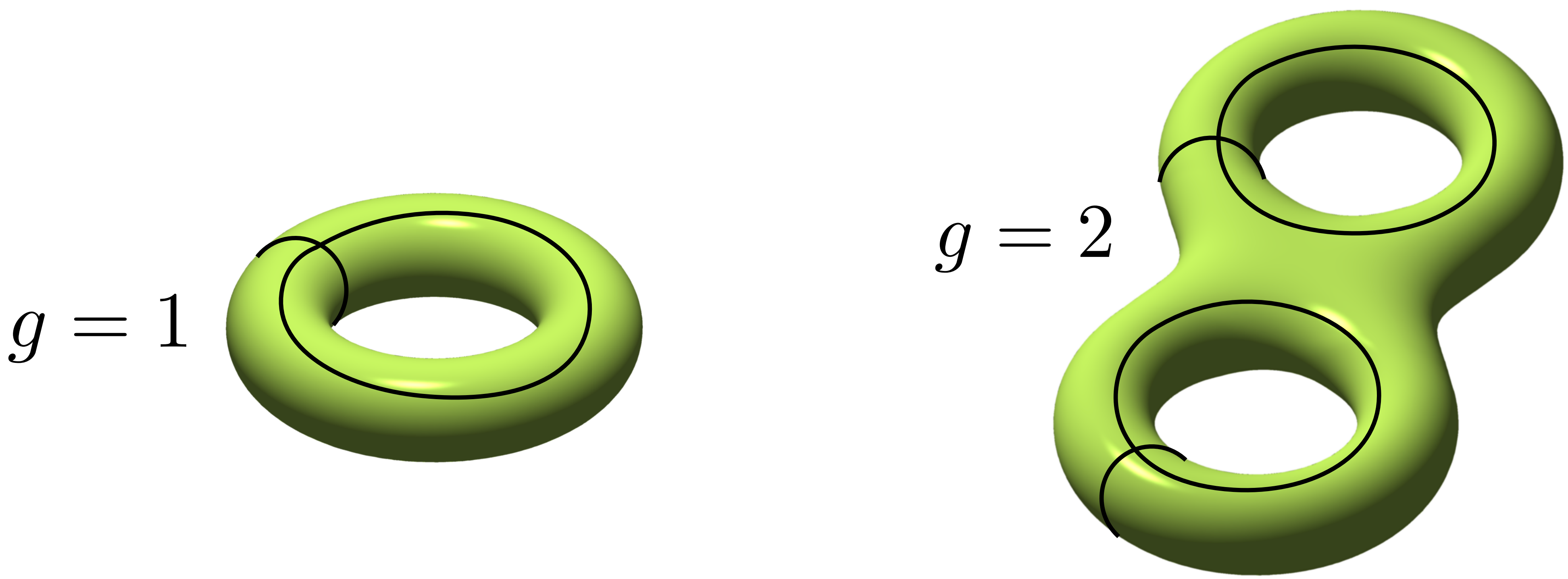


$\Phi = 0$ if the loop is contractible to a point.

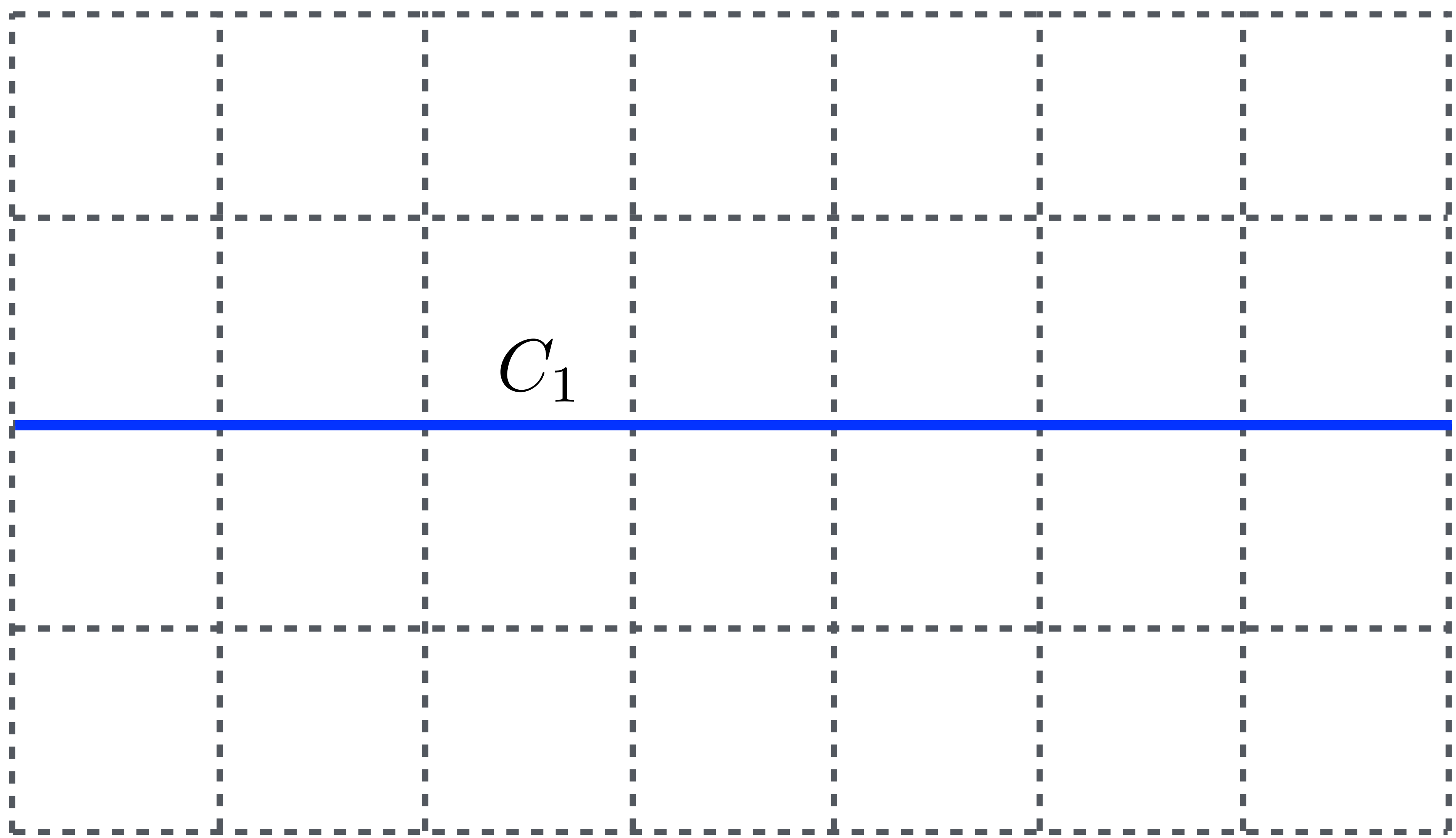
Topological degeneracy



$2g$ is the number of topologically distinct non-contractible loops of a surface. Their flux is not specified when we set the fluxes of contractible loops to zero.

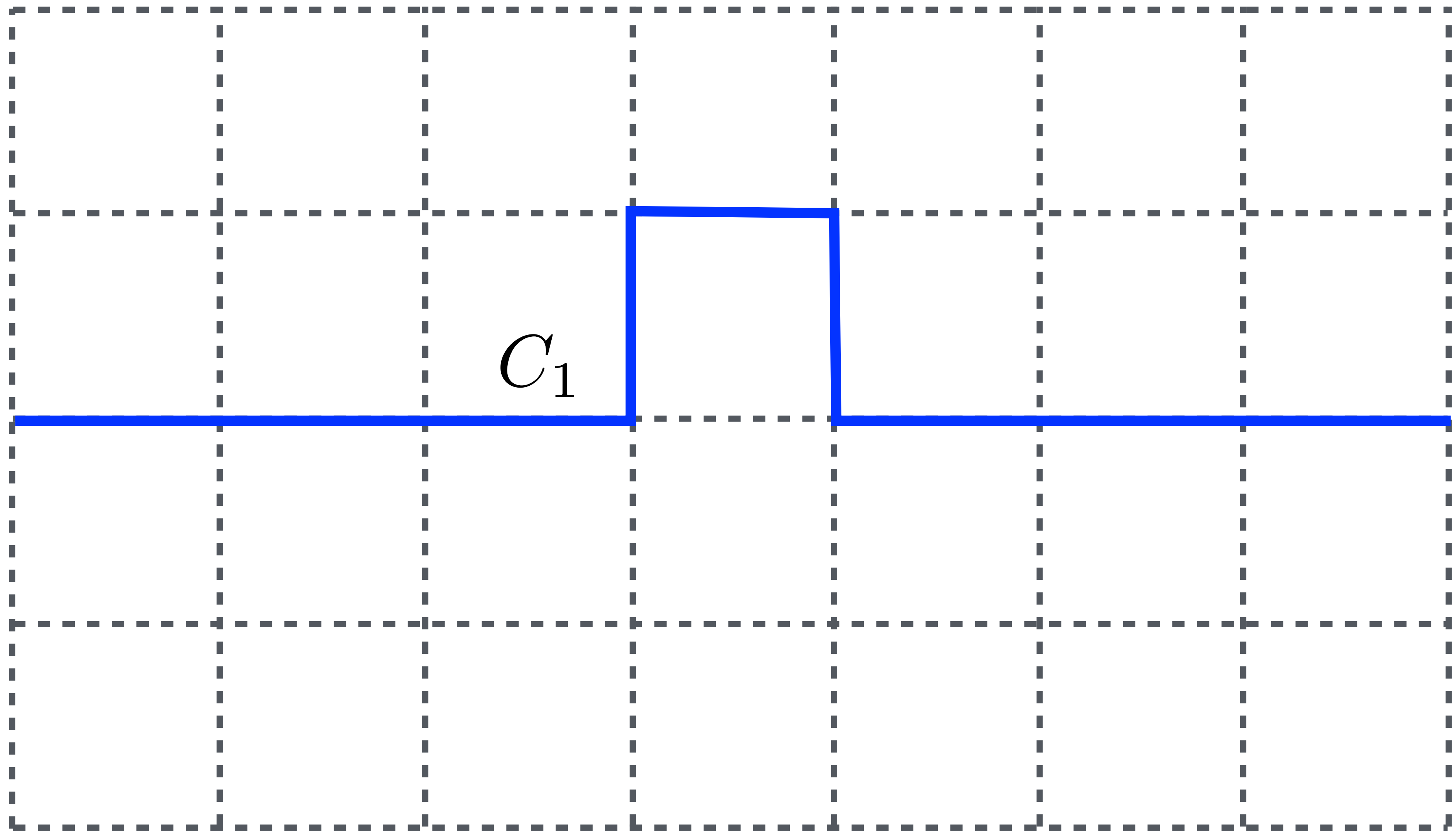
Thus there remain $2g$ degrees of freedom, global fluxes.

Periodic boundary conditions = torus (genus 1)



$$Z_1 = \prod_{C_1} \sigma^z_{mn} = \pm 1$$

Periodic boundary conditions = torus (genus 1)



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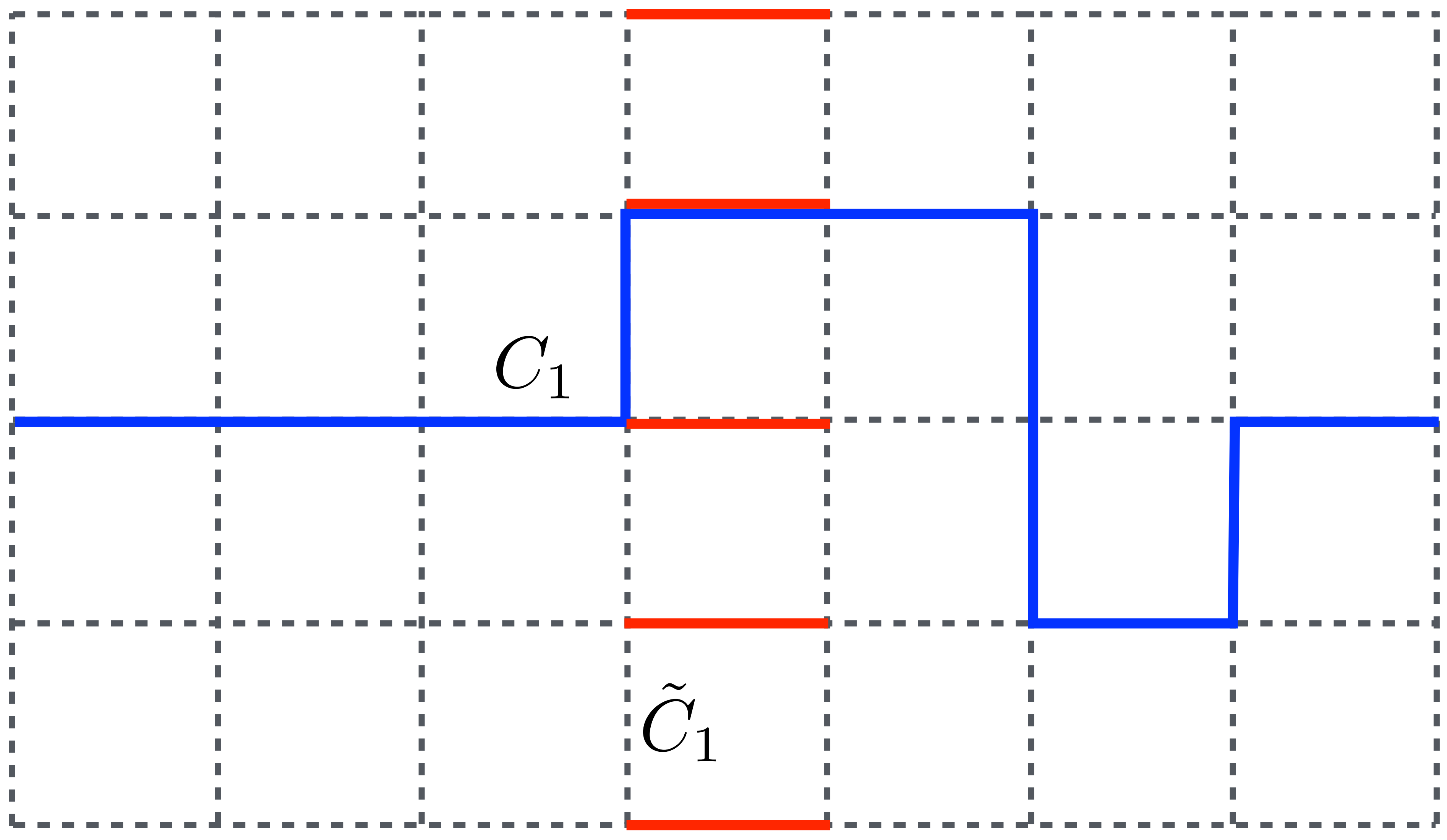
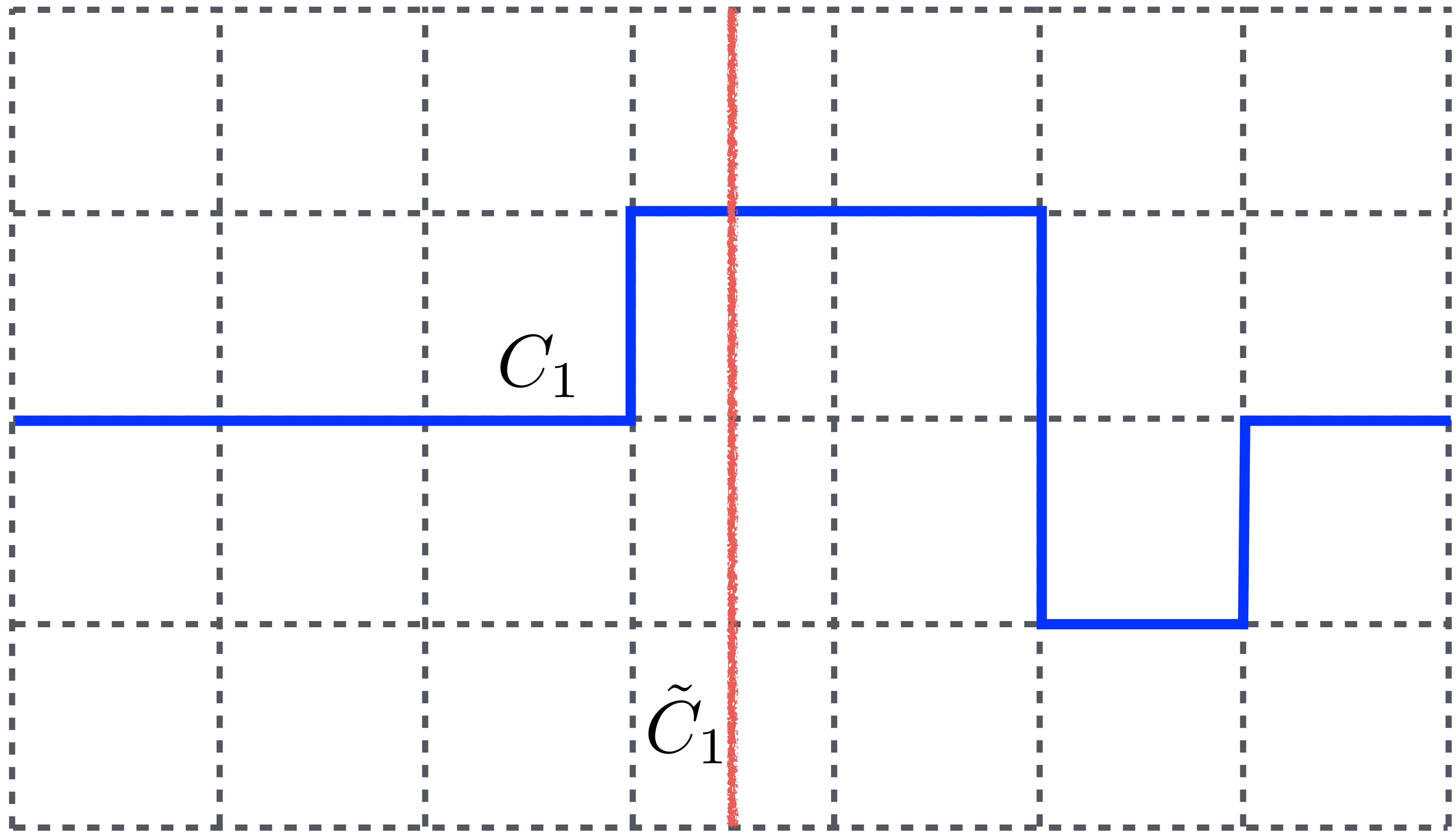


$$Z_1 = \prod_{C_1} \sigma^z_{mn} = \pm 1 \qquad X_1 = \prod_{\tilde{C}_1} \sigma^x_{mn} \qquad X_1^{-1} Z_1 X_1 = -Z_1$$

Periodic boundary conditions = torus (genus 1)



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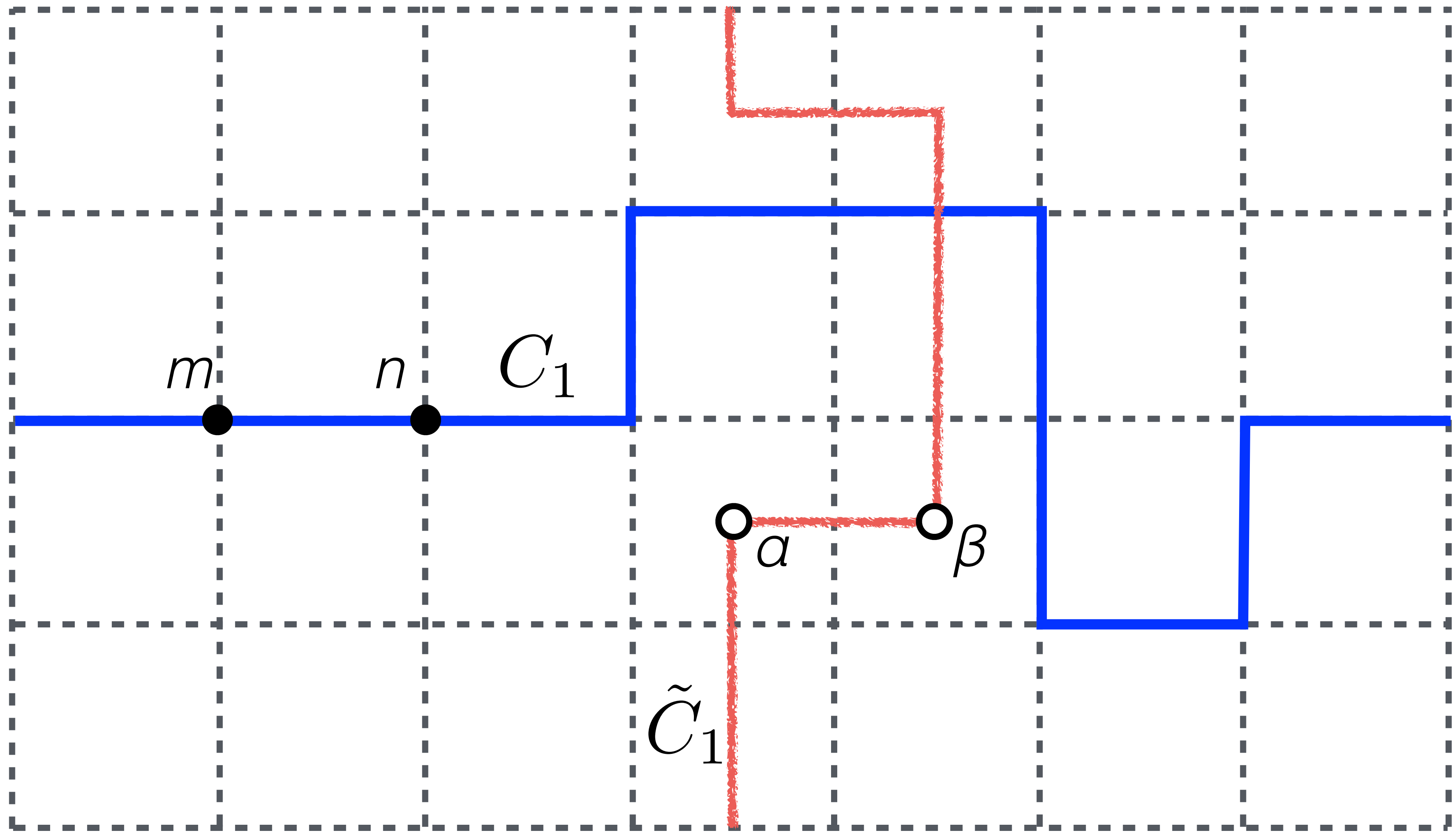
Periodic boundary conditions = torus (genus 1)

● *m, n* – sites of original lattice; ○ *α, β* – sites of dual lattice



$$Z_1 = \prod_{C_1} \sigma^z_{mn} = \pm 1 \qquad X_1 = \prod_{\tilde{C}_1} \sigma^x_{\alpha\beta} \qquad X_1^{-1} Z_1 X_1 = -Z_1$$

Periodic boundary conditions = torus (genus 1)

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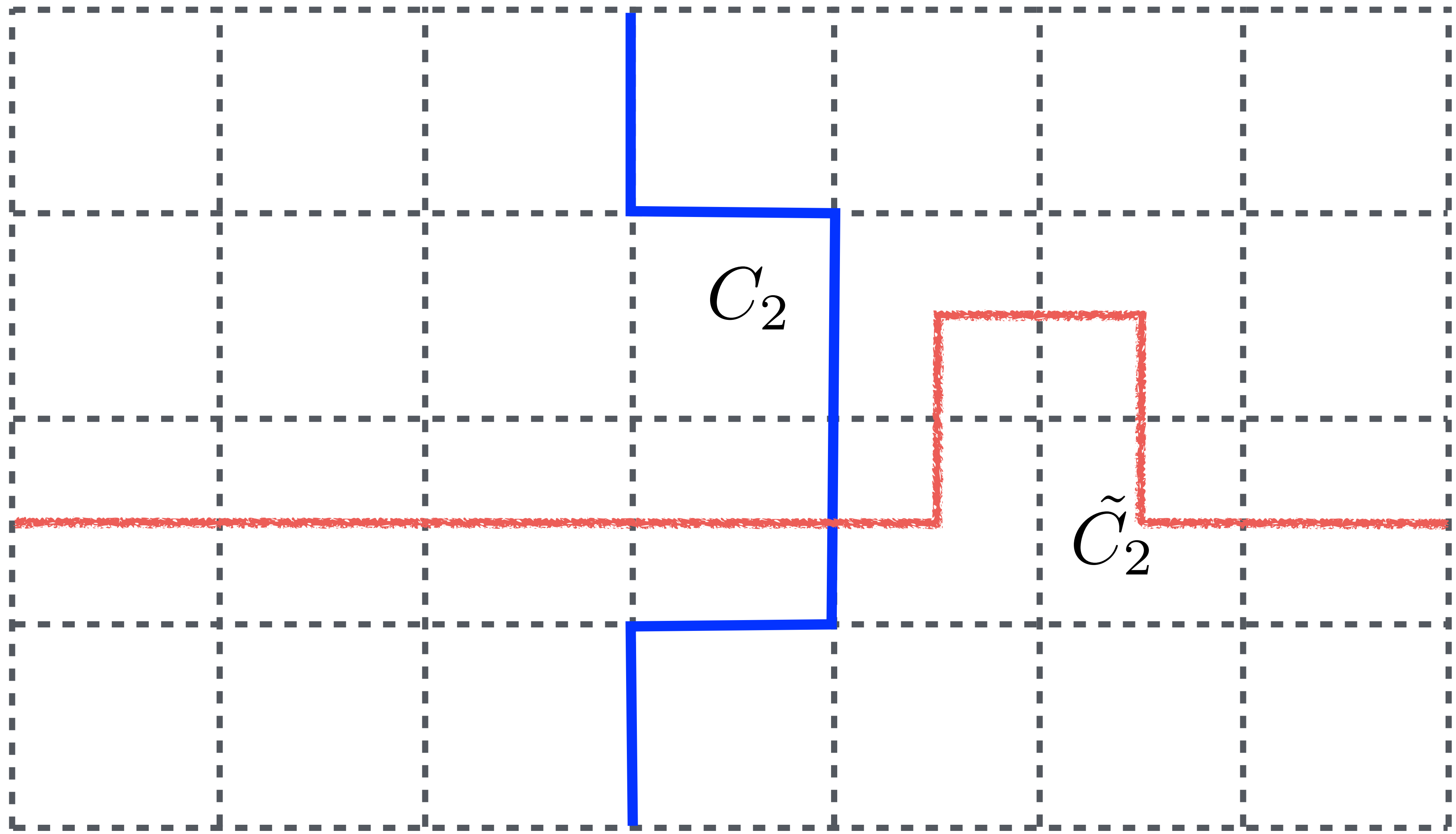
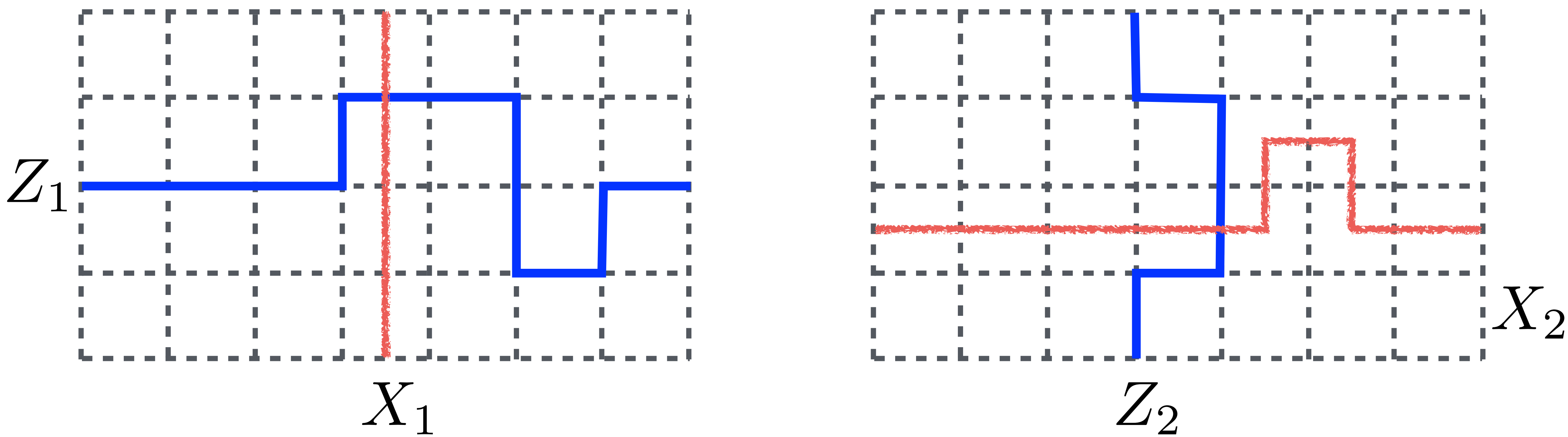


$$Z_2 = \prod_{C_2} \sigma^z_{mn} = \pm 1 \qquad X_2 = \prod_{\tilde{C}_2} \sigma^x_{\alpha\beta} \qquad X_2^{-1} Z_2 X_2 = -Z_2$$

Global qubits



X creates a pair of vortices, moves one of them around the system and annihilates them, returning the system to a ground state. This process occurs spontaneously with an amplitude of the order of

$$\lambda(\Gamma/\lambda)^L = \lambda e^{-L/\xi}, \quad 1/\xi = \ln(\lambda/\Gamma)$$

Degeneracy is observed only in large systems, $L \gg \xi$.

Dual variables

Original Pauli operators:

$\sigma^x = \pm 1$ measures the Z_2 electric field on a link.

σ^z alters the value of the electric field.

Labeled by link (mn).

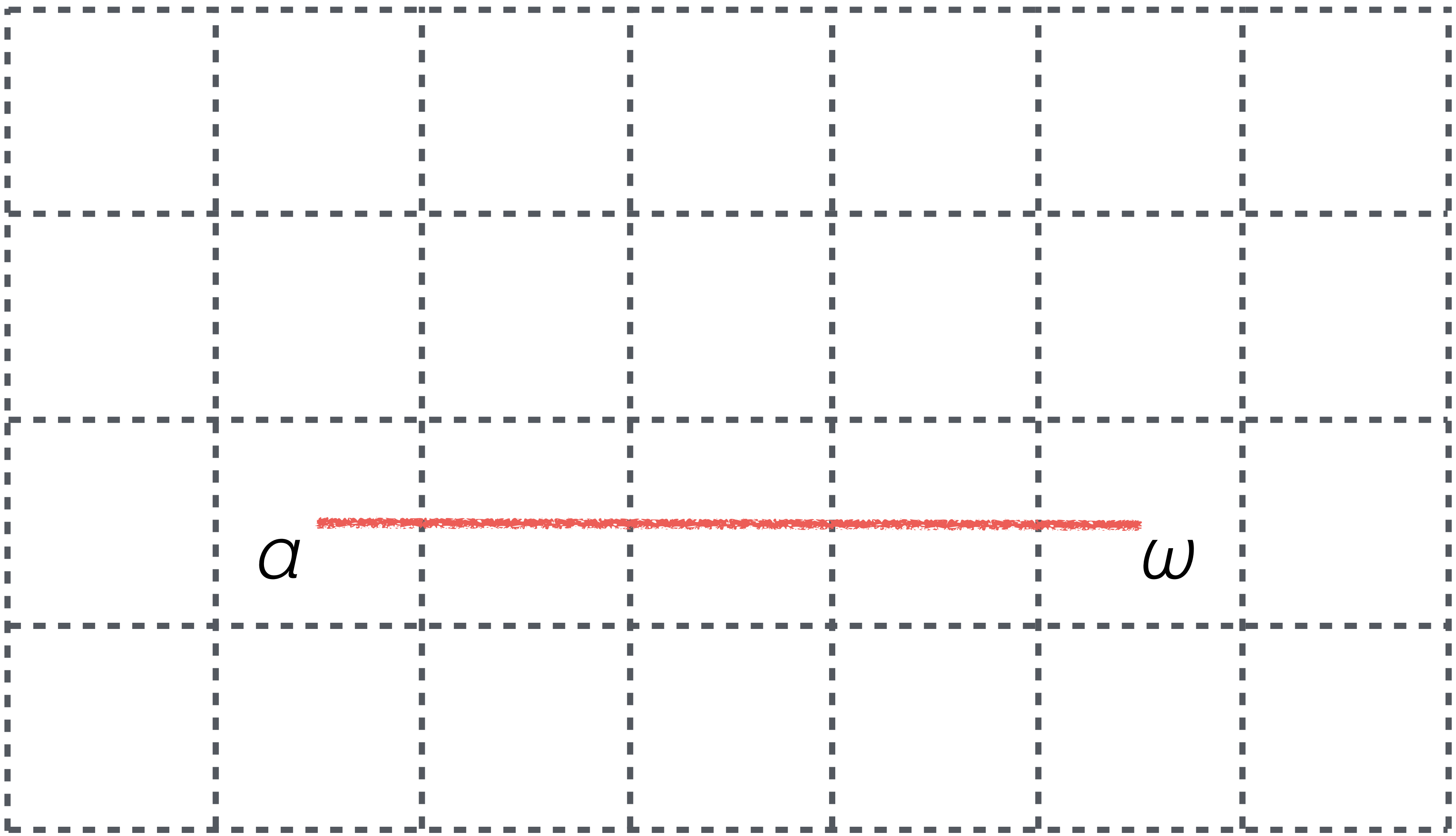
Dual Pauli operators:

$\tau^x = \pm 1$ measures the Z_2 magnetic flux on a plaquette.

τ^z alters the value of the magnetic flux.

Labeled by plaquette (dual site β).

Open electric string $X_{\alpha\omega}$



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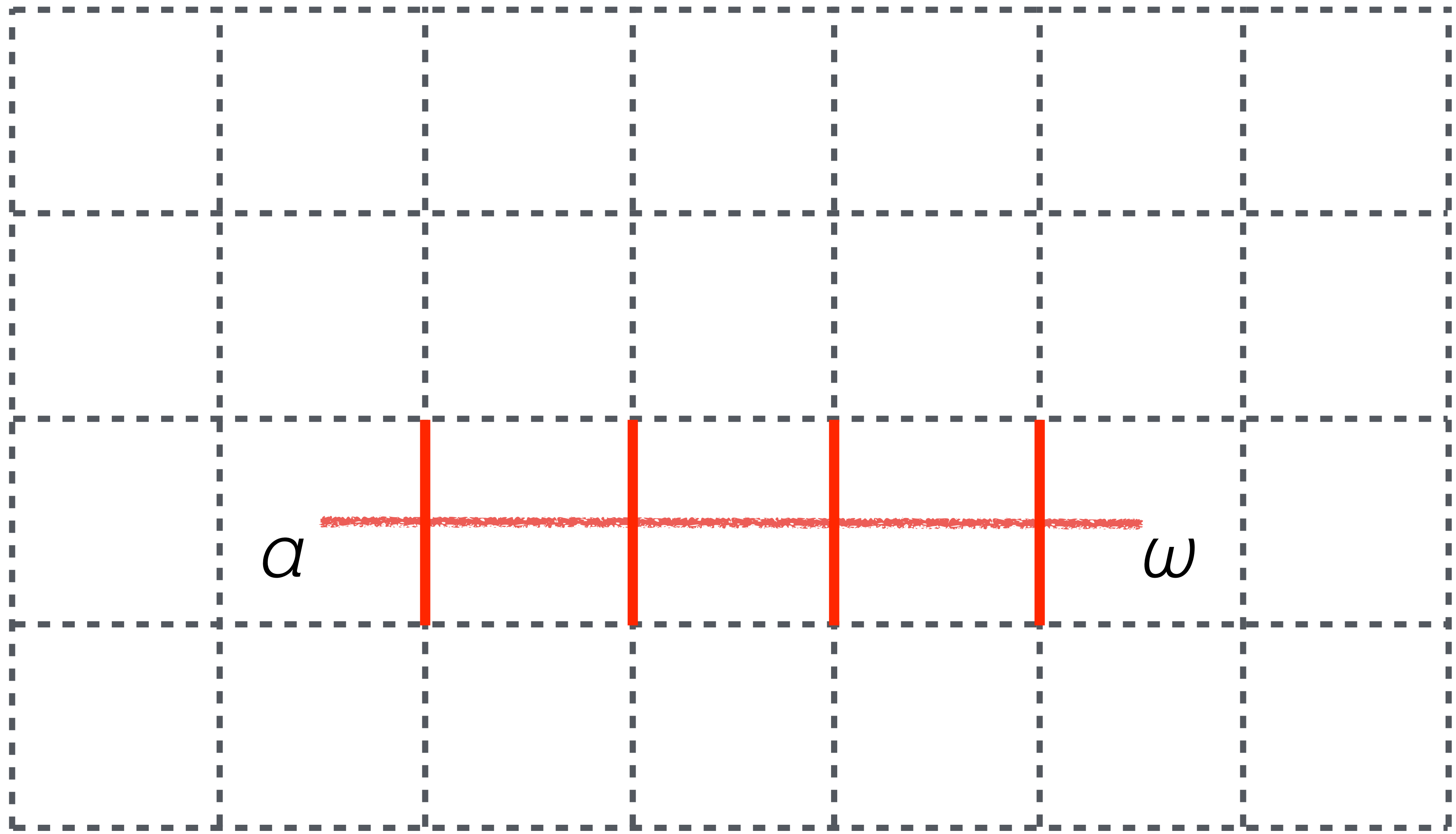

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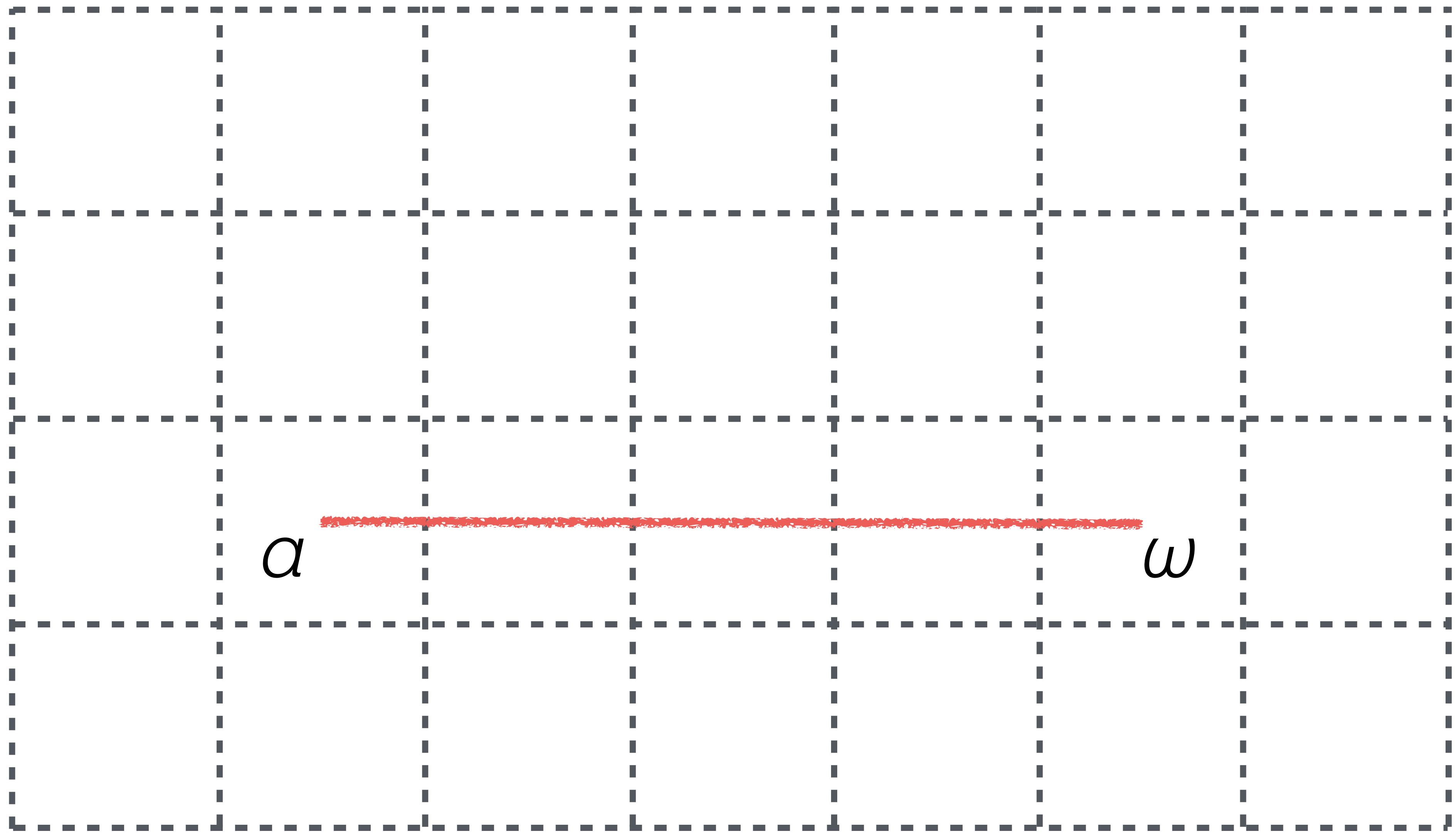
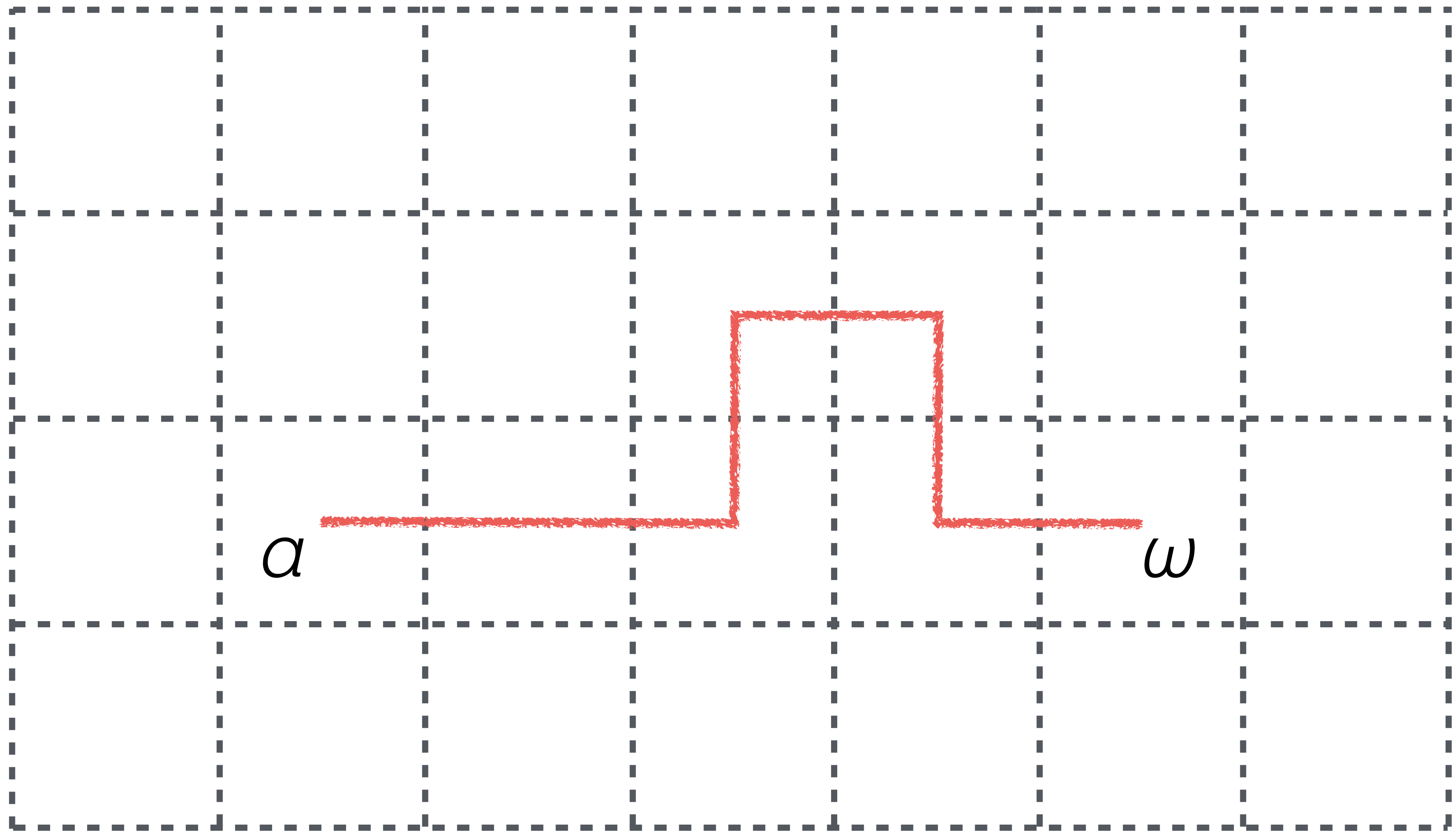
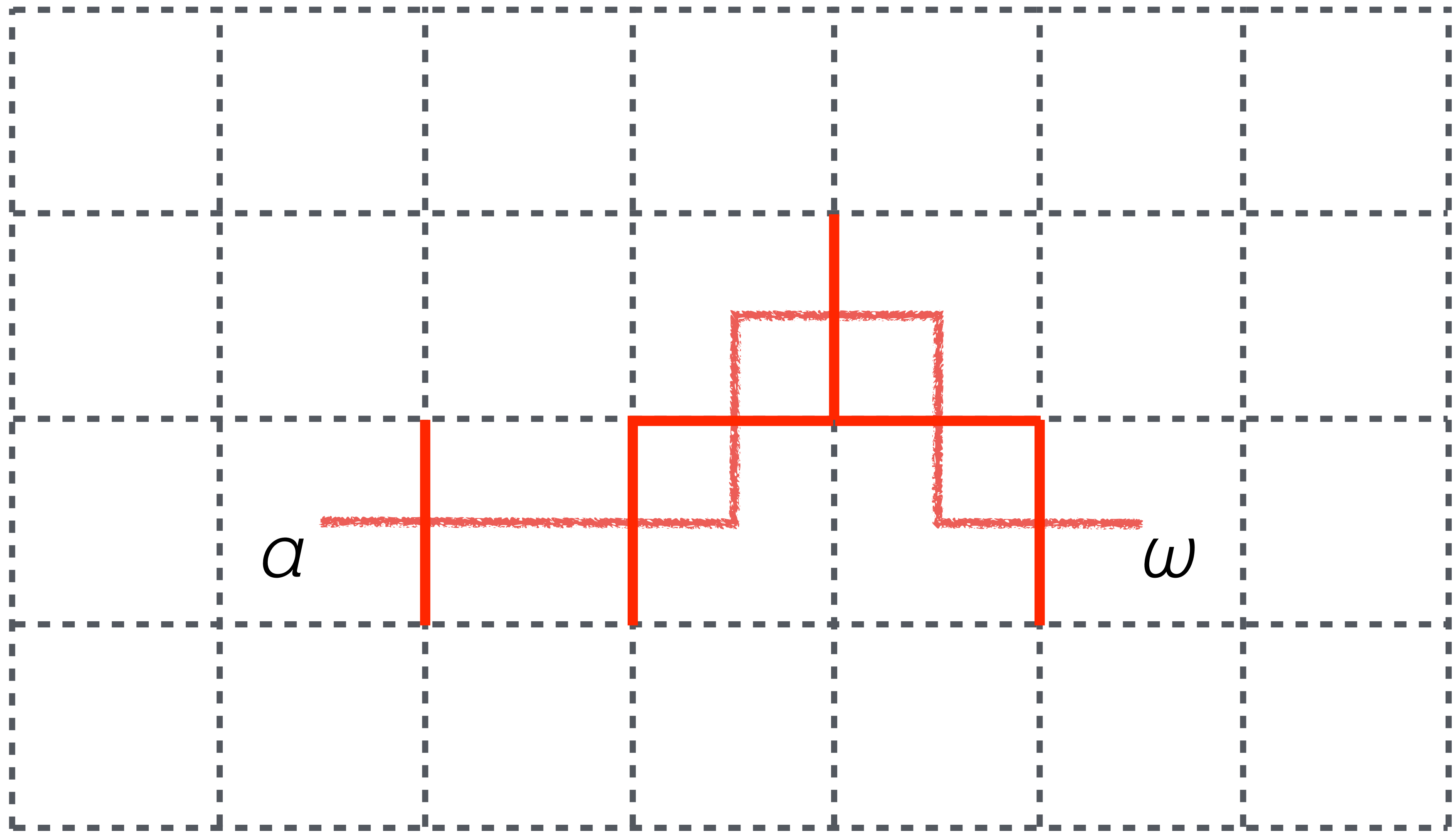


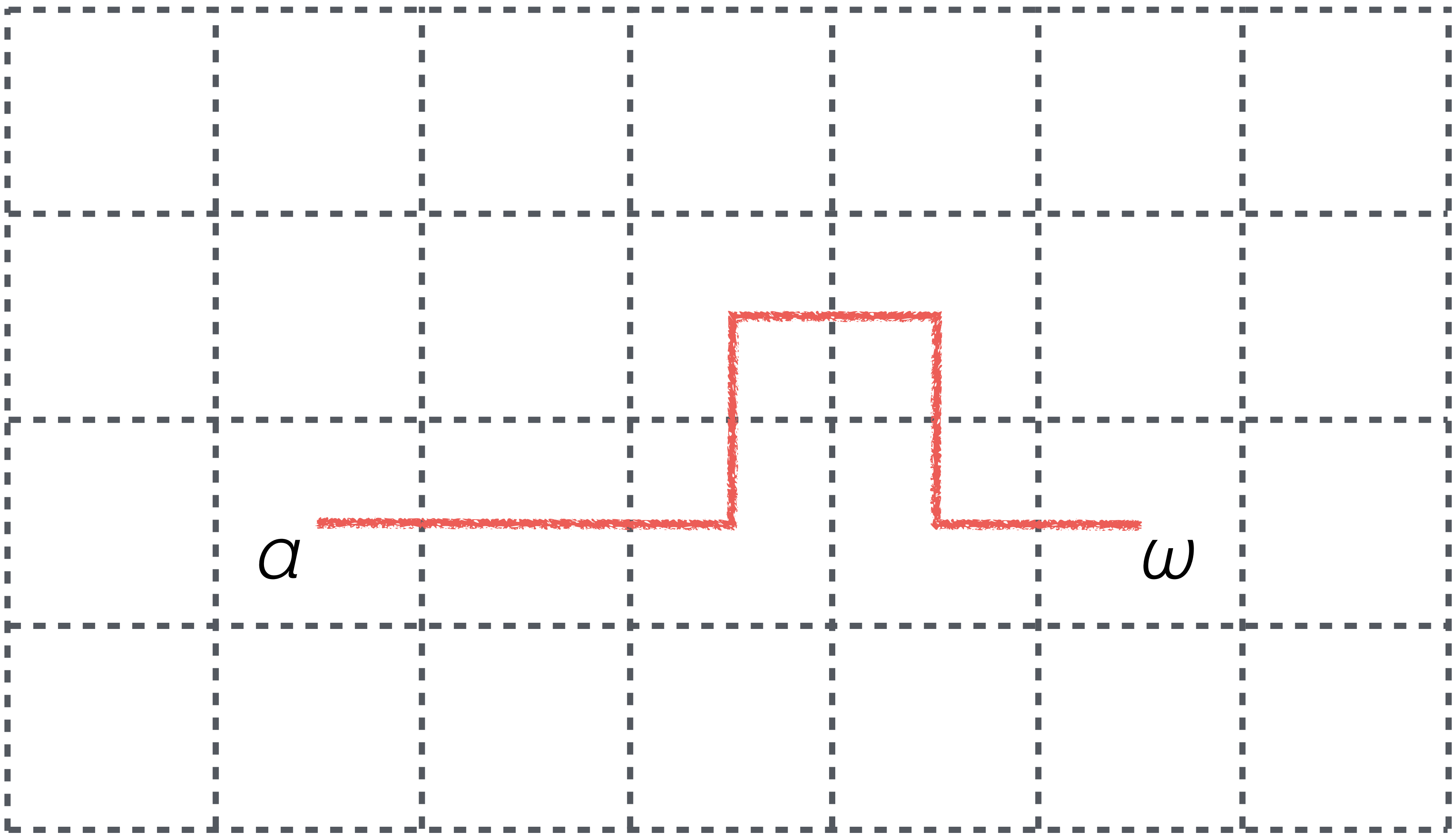
Open electric string $X_{\alpha\omega}$ deformed



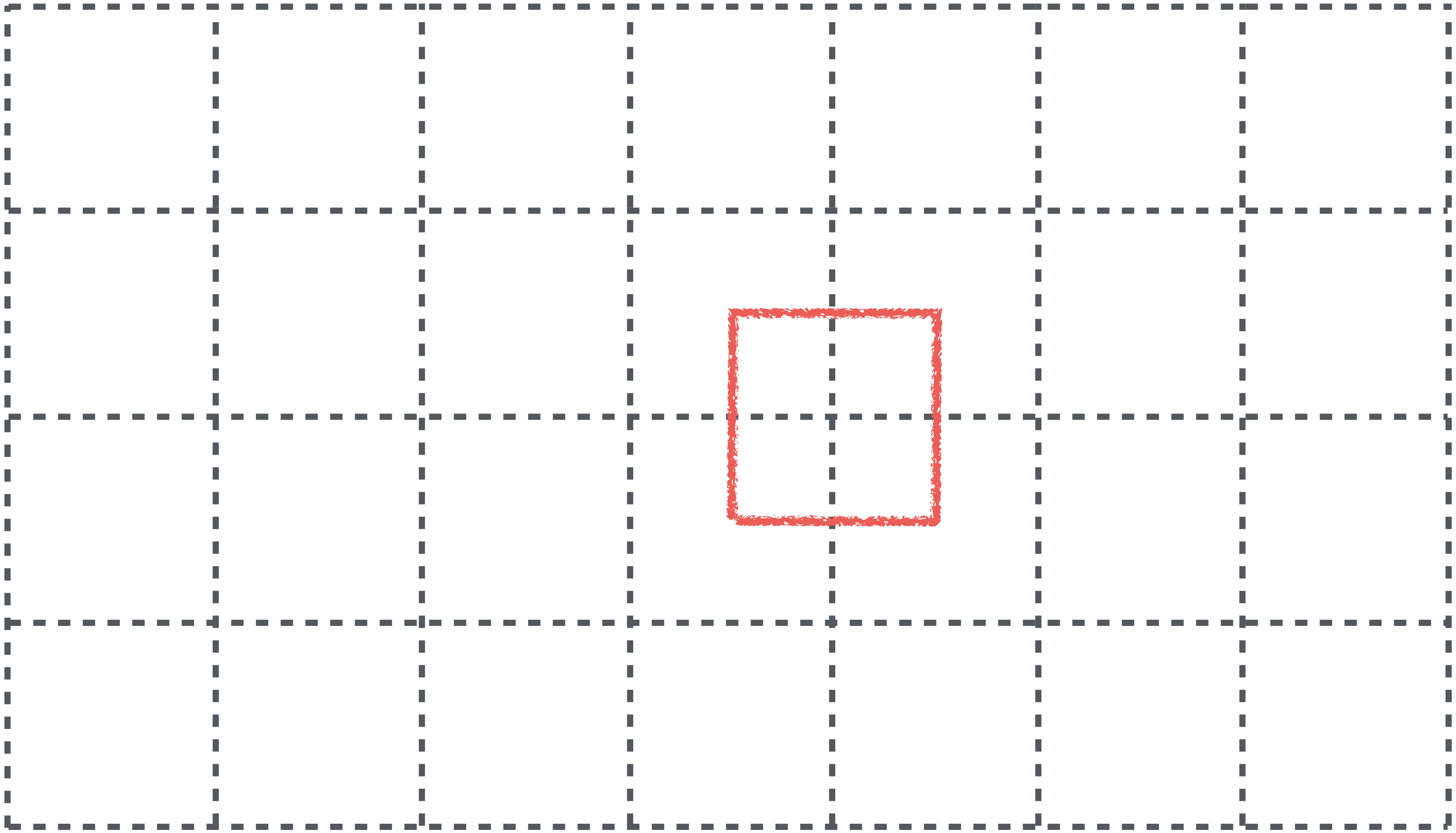
Open electric string $X_{\alpha\omega}$ deformed



Open electric string $X_{\alpha\omega}$ deformed



Difference



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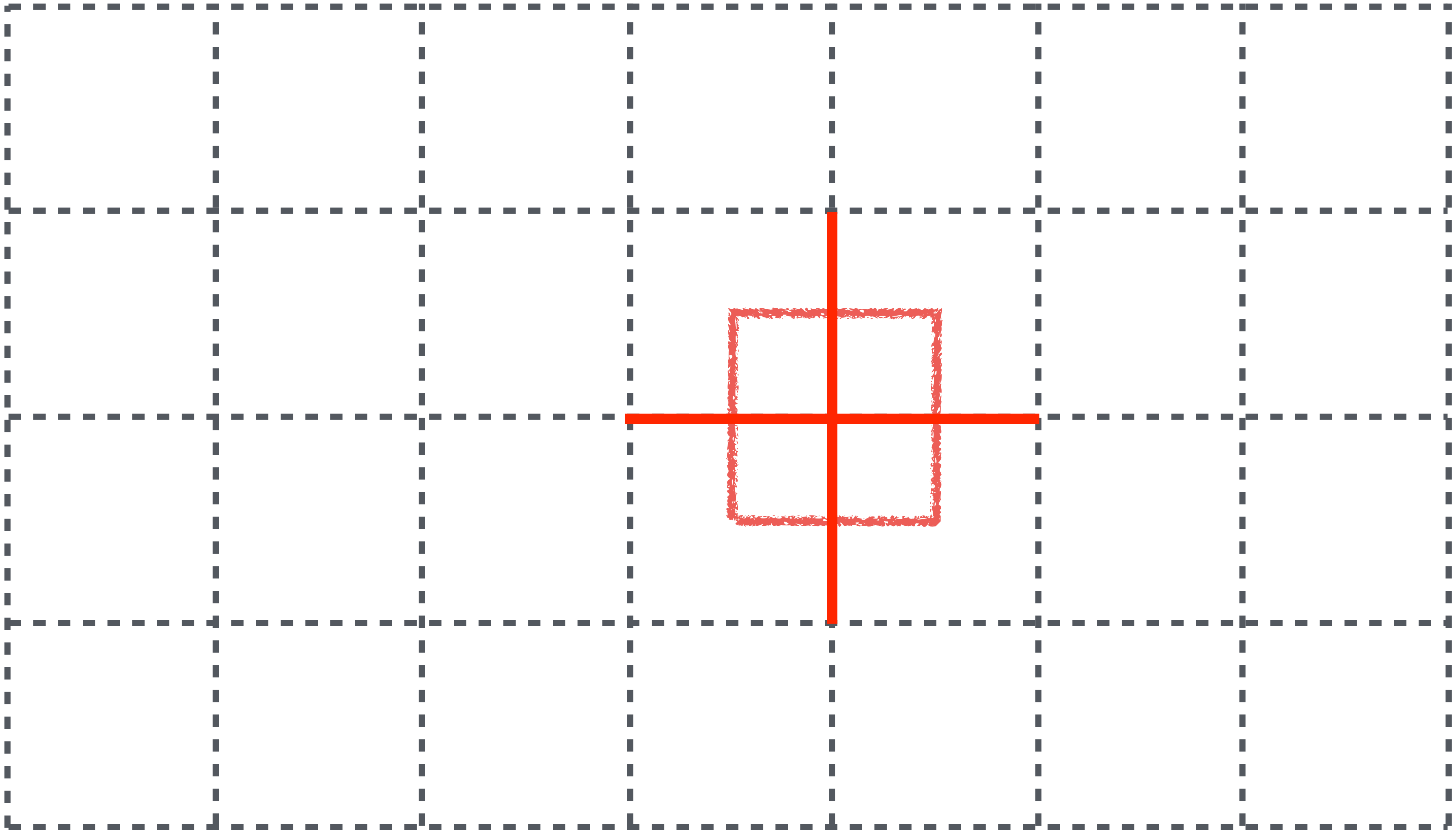

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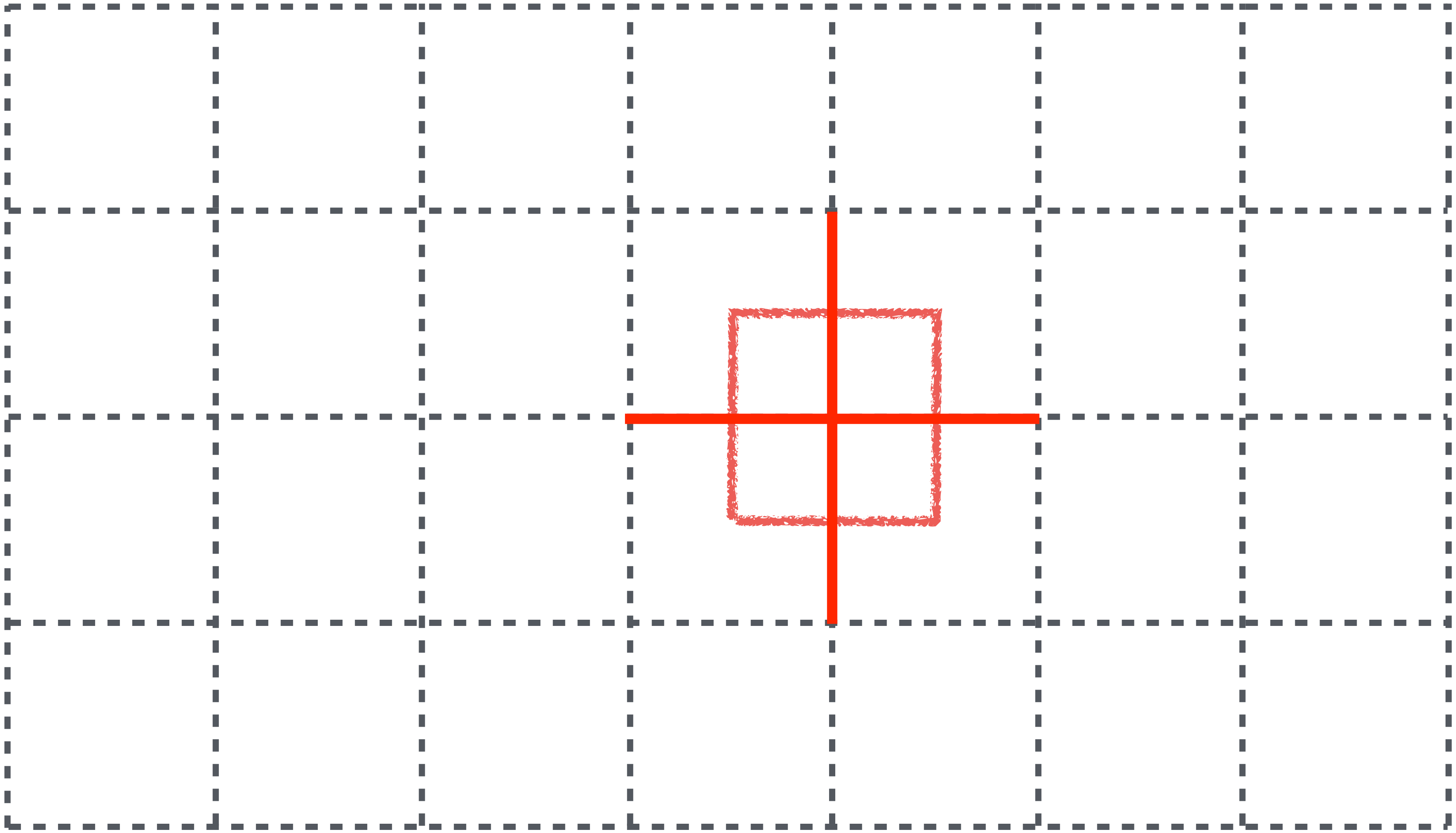
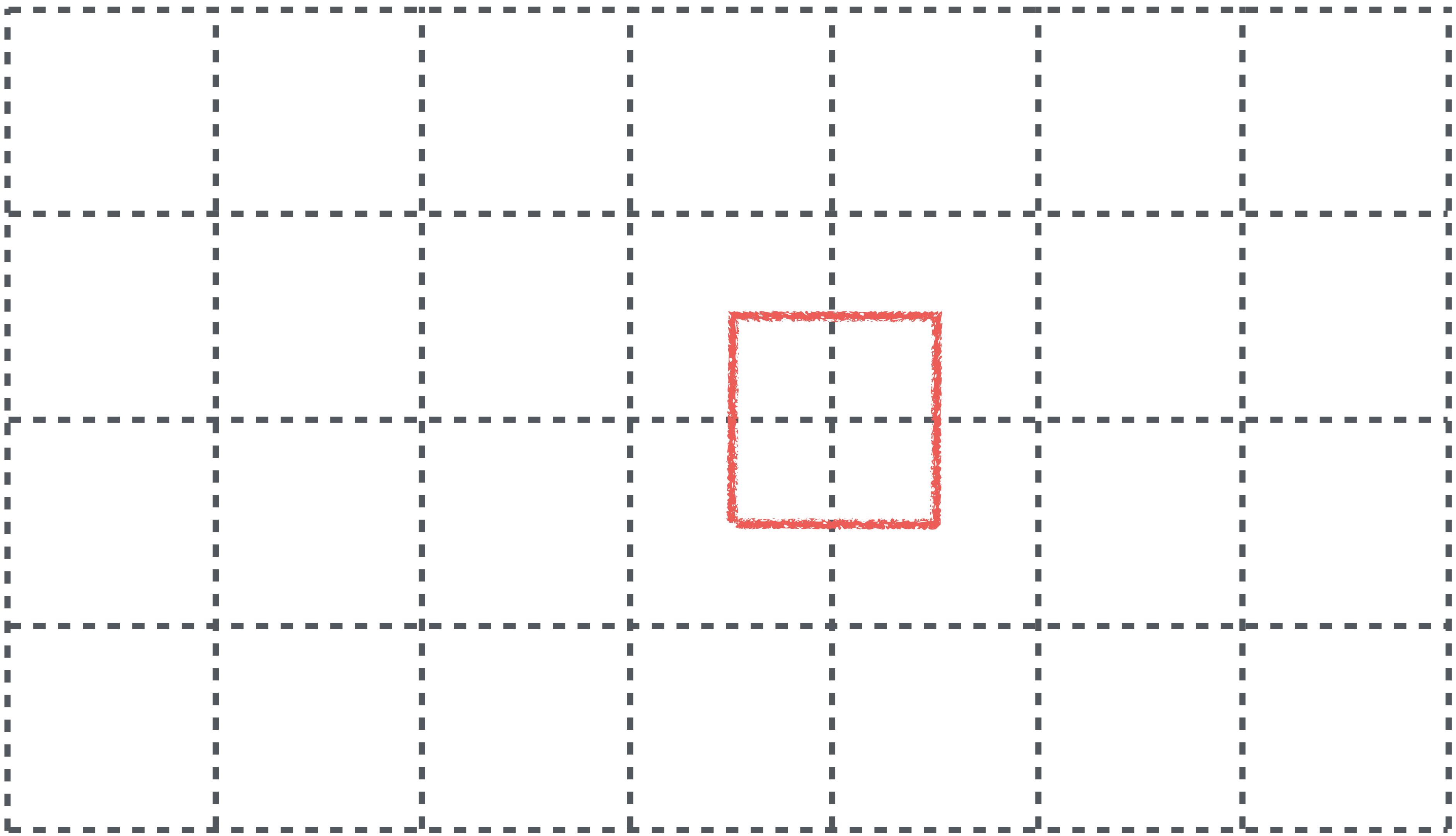


$$\prod_{\text{star}} \sigma_x = 1$$

Difference



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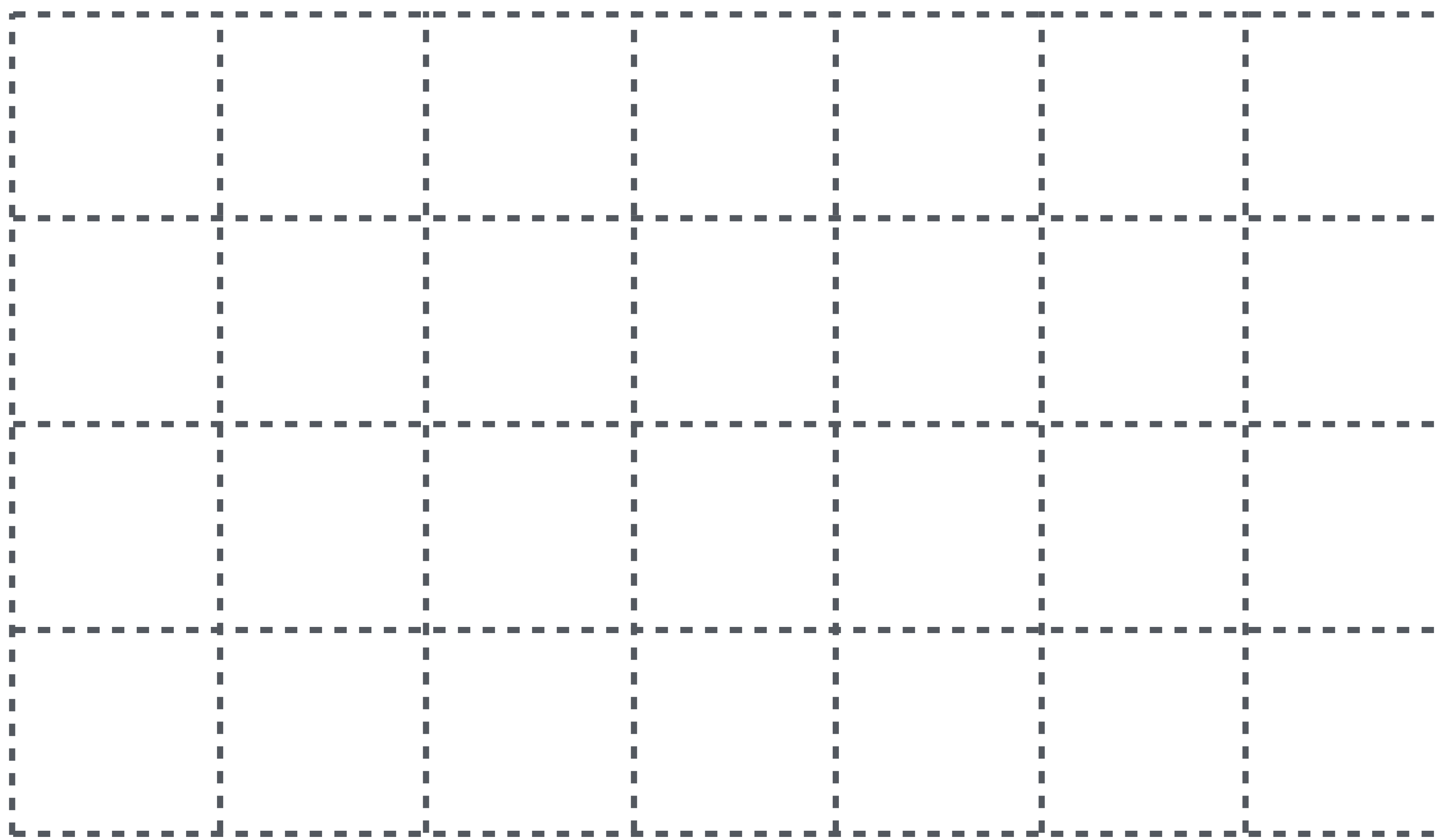

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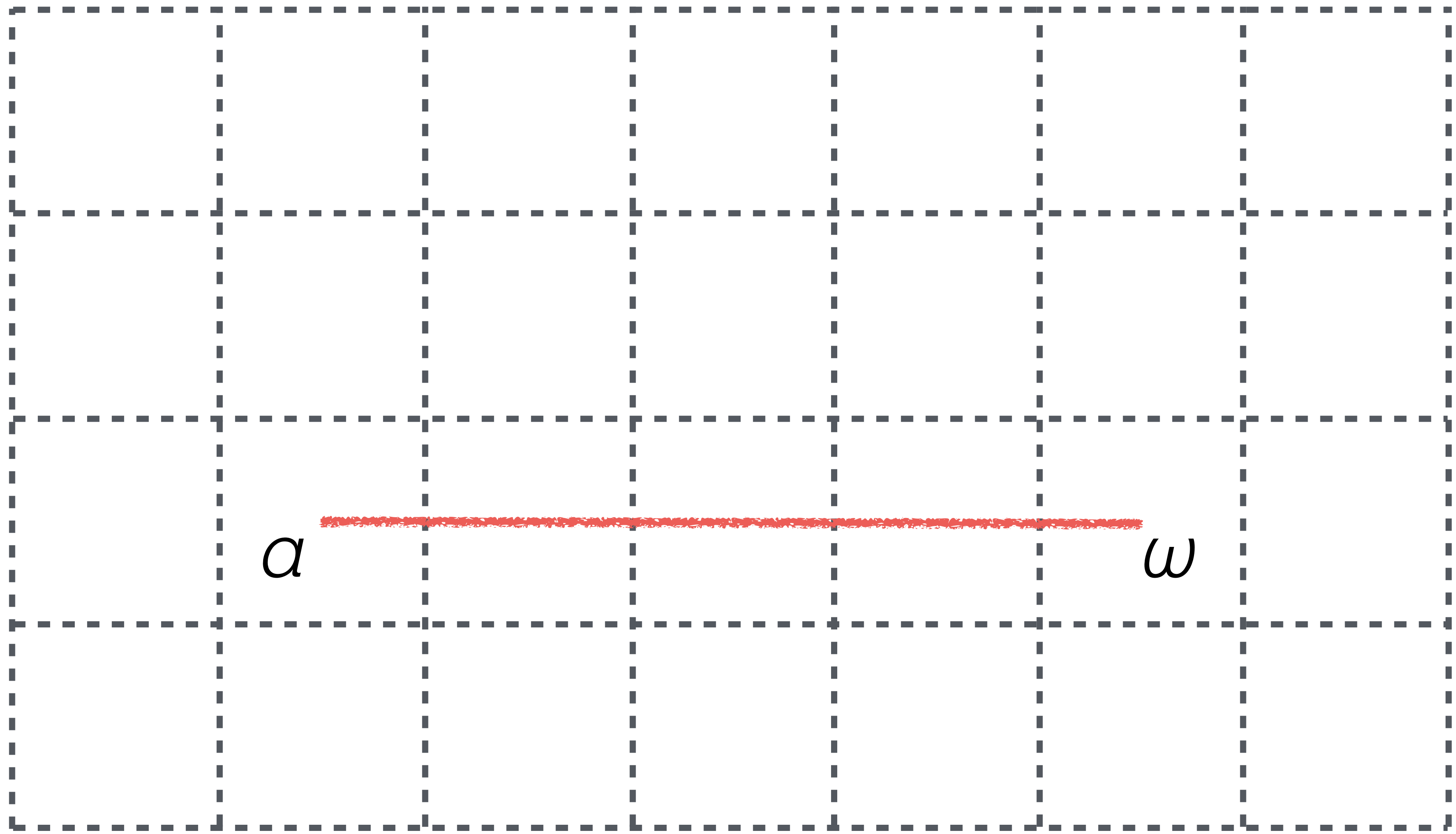
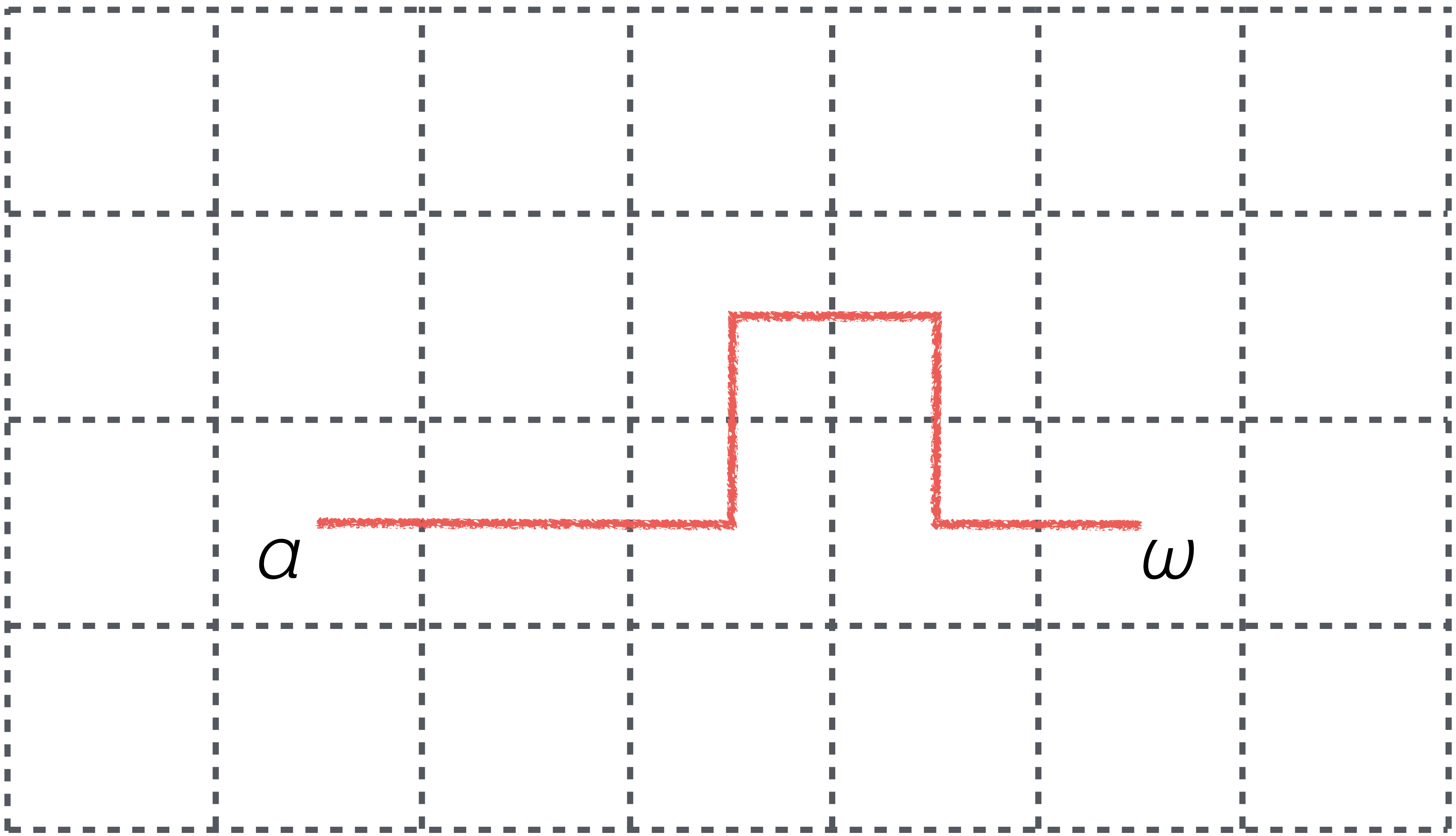


The ends are fixed, the path is arbitrary



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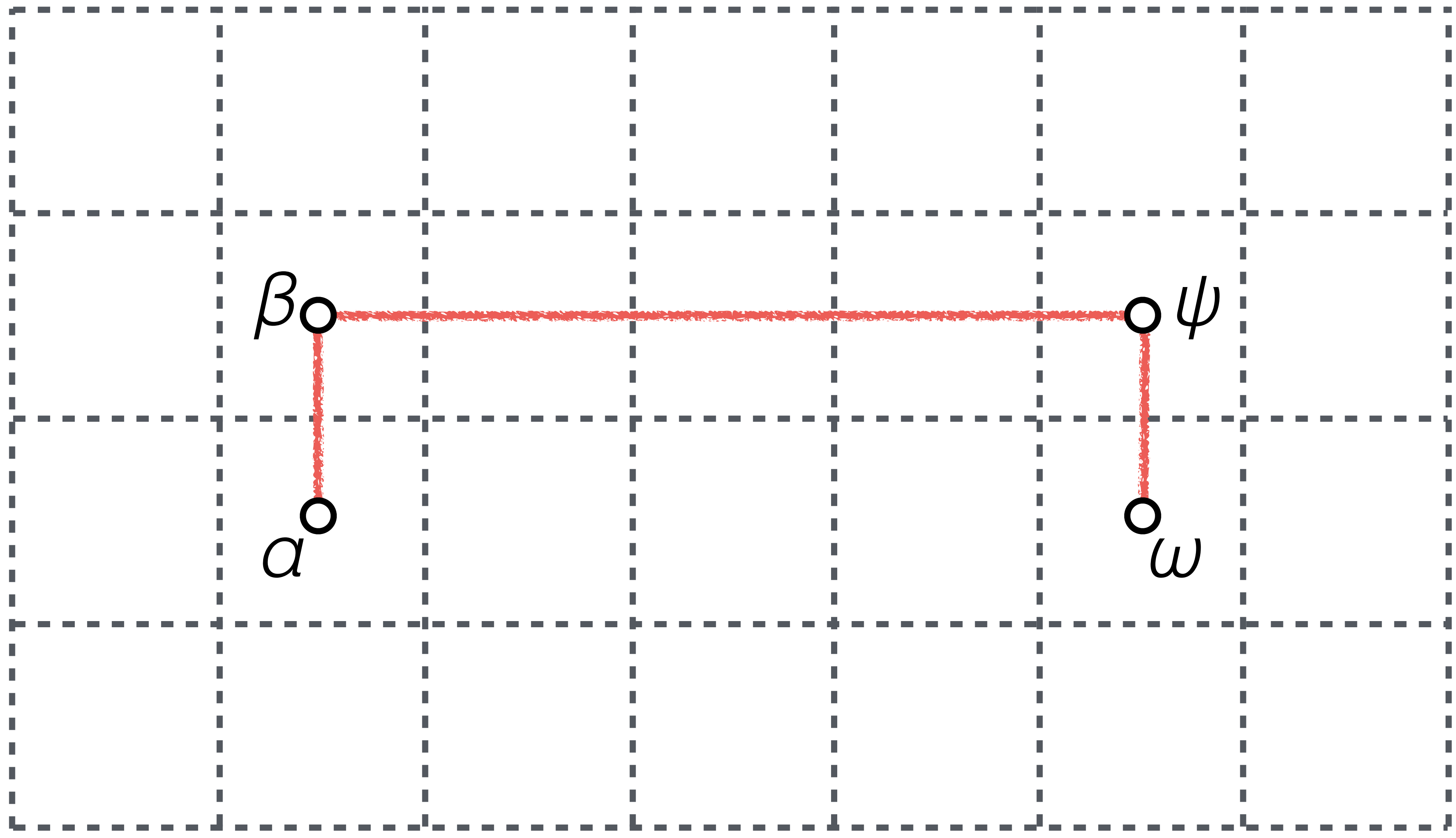

$$X_{\alpha\omega} = \sigma^x_{\alpha\beta}\sigma^x_{\beta\gamma}\ldots\sigma^x_{\psi\omega} = \tau^z_\alpha\tau^z_\omega$$

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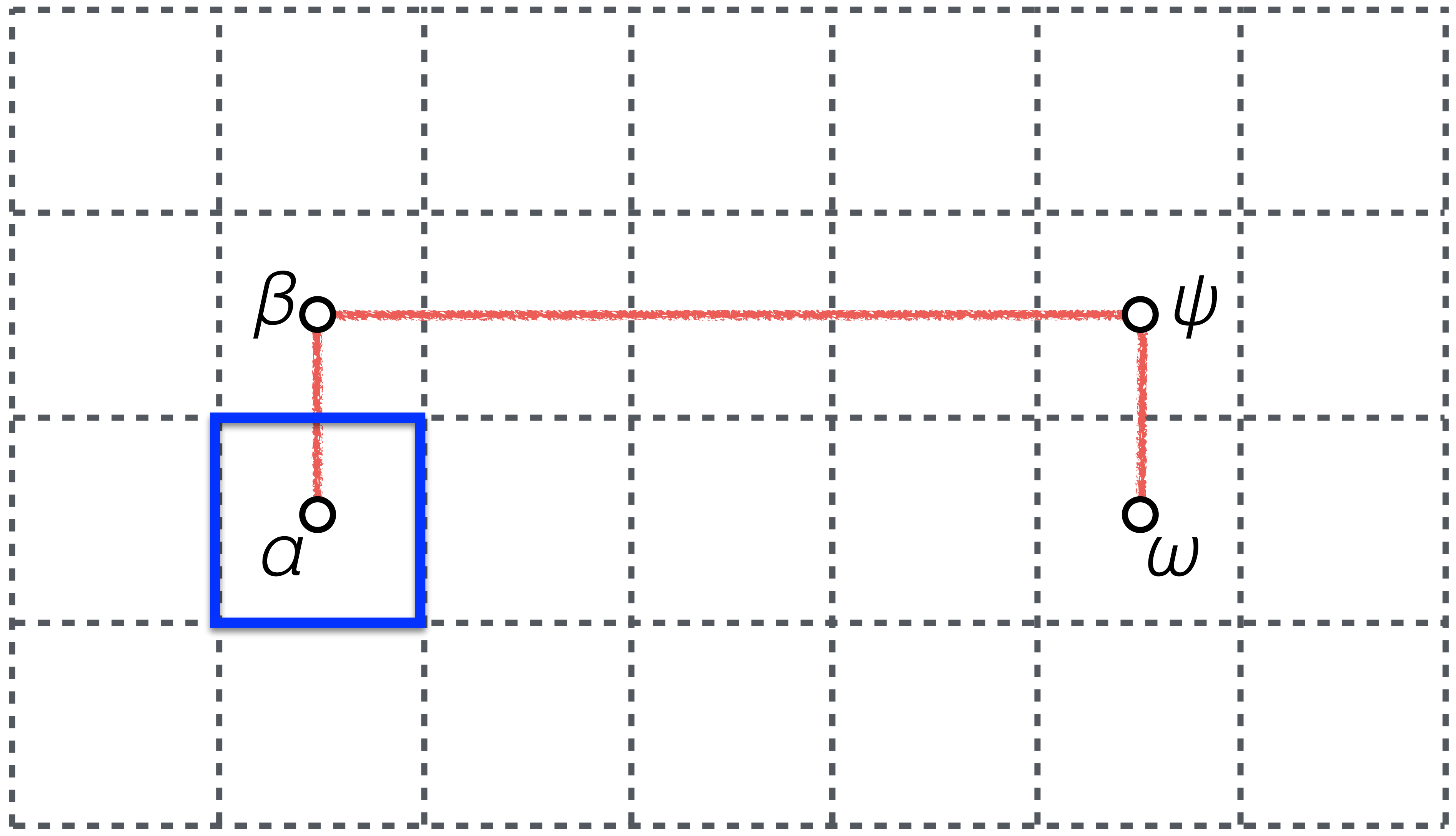

$$X_{\alpha\omega} = \sigma^x_{\alpha\beta}\sigma^x_{\beta\gamma}\ldots\sigma^x_{\psi\omega} = \tau^z_\alpha\tau^z_\omega$$

$$\phi_\alpha = \prod_{\beta(\alpha)} \sigma^z_{\alpha\beta} = \tau^x_\alpha$$

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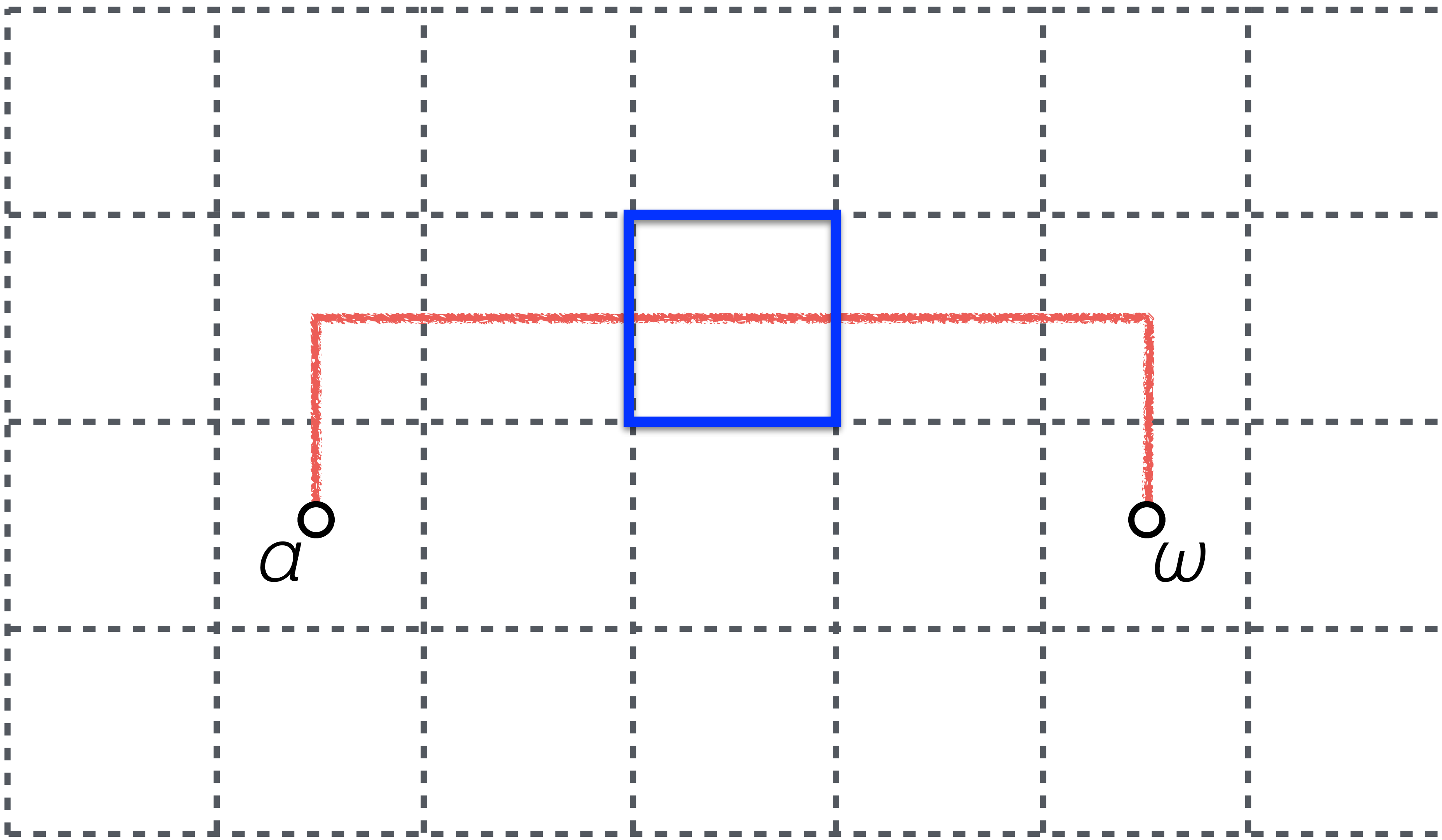

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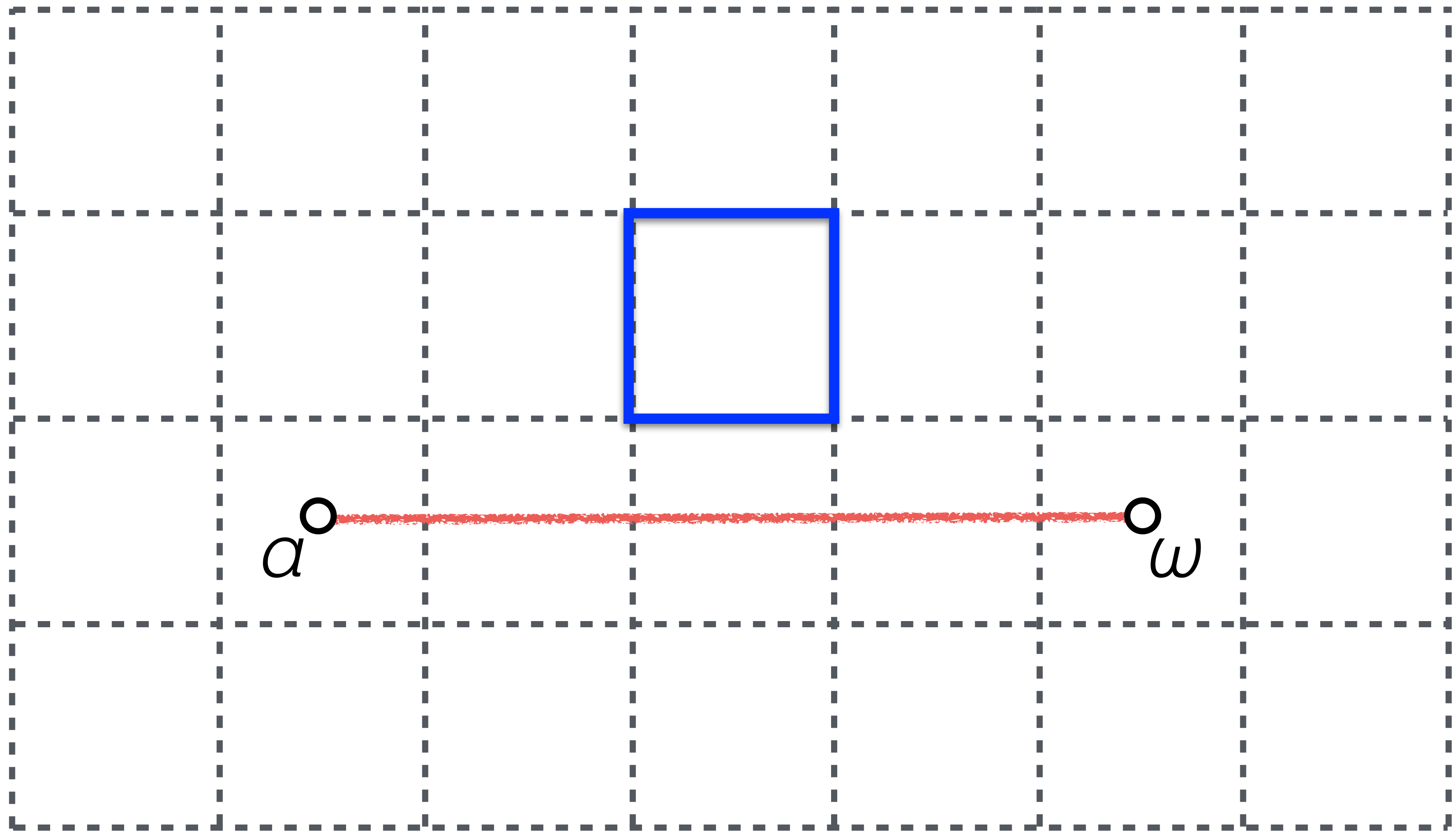
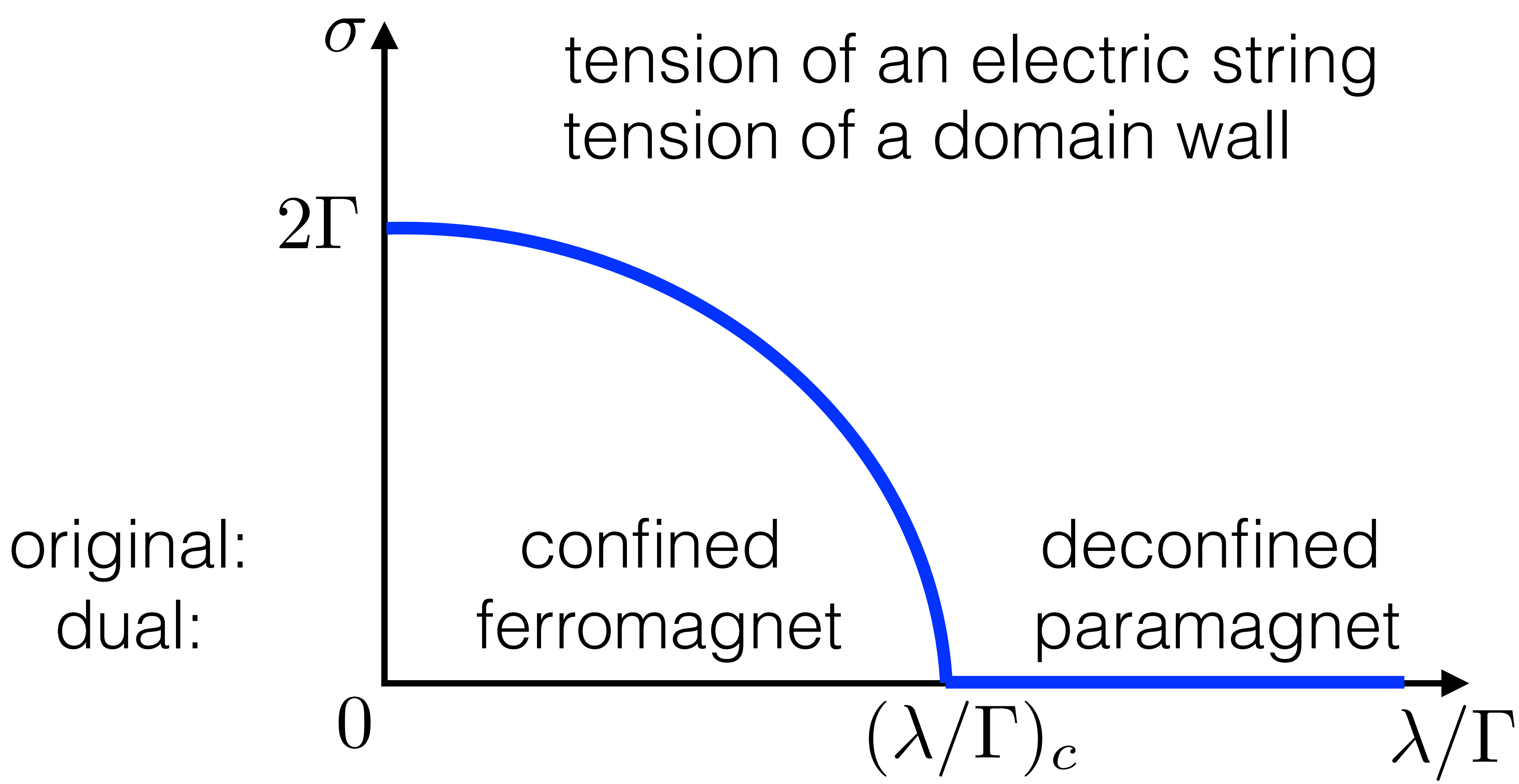


$$X_{\alpha\omega} = \sigma^x_{\alpha\beta}\sigma^x_{\beta\gamma}\ldots\sigma^x_{\psi\omega} = \tau^z_\alpha \tau^z_\omega$$

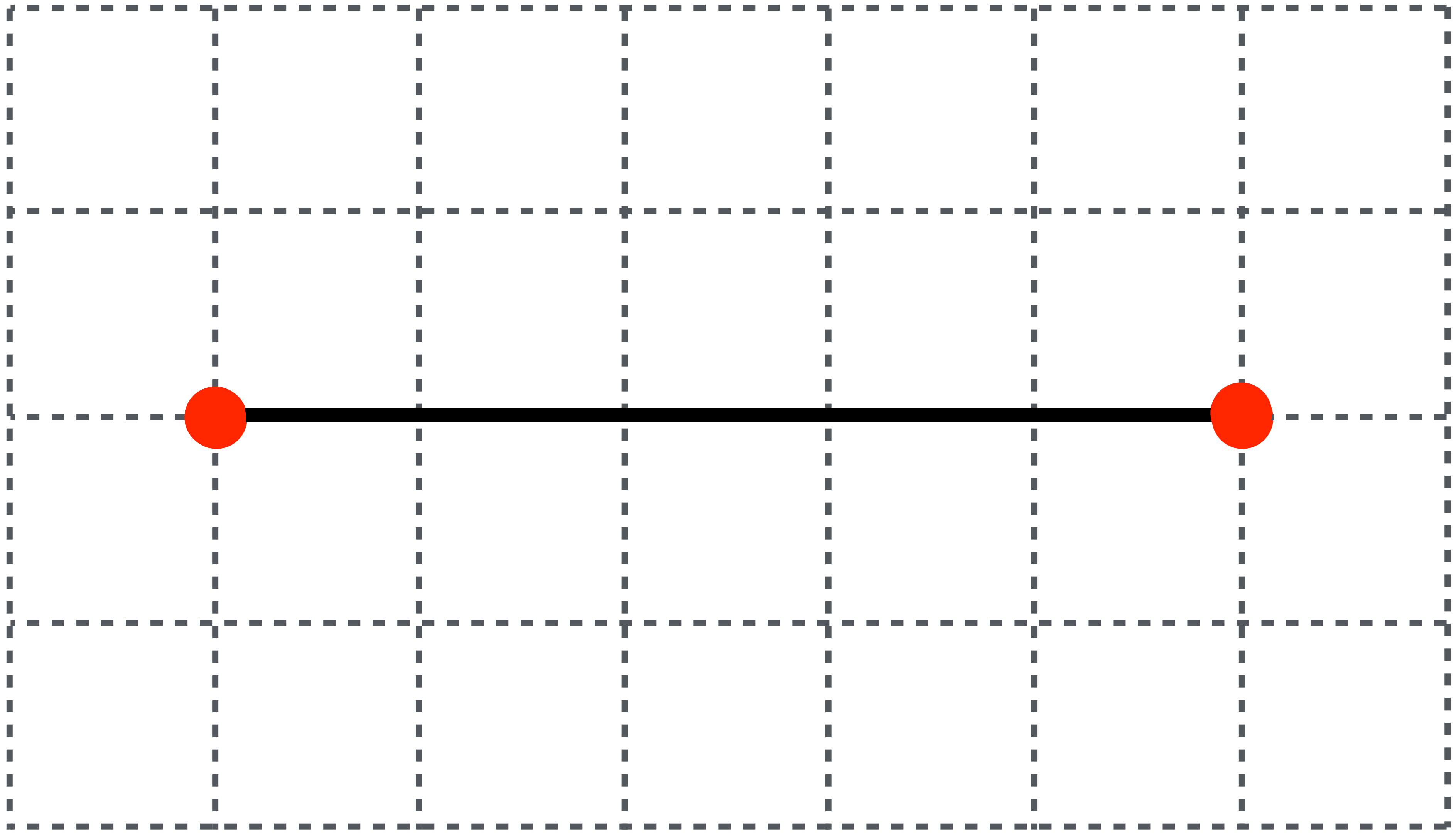
Dual Hamiltonian

$$H = -\Gamma \sum_{\langle\alpha\beta\rangle} \sigma^x_{\alpha\beta} - \lambda \sum_\alpha \prod_{\beta(\alpha)} \sigma^z_{\alpha\beta}$$ Ising gauge theory

$$\tilde{H} = -\Gamma \sum_{\langle\alpha\beta\rangle} \tau^z_\alpha \tau^z_\beta - \lambda \sum_\alpha \tau^x_\alpha$$ Ising ferromagnet



An electric string in the original theory



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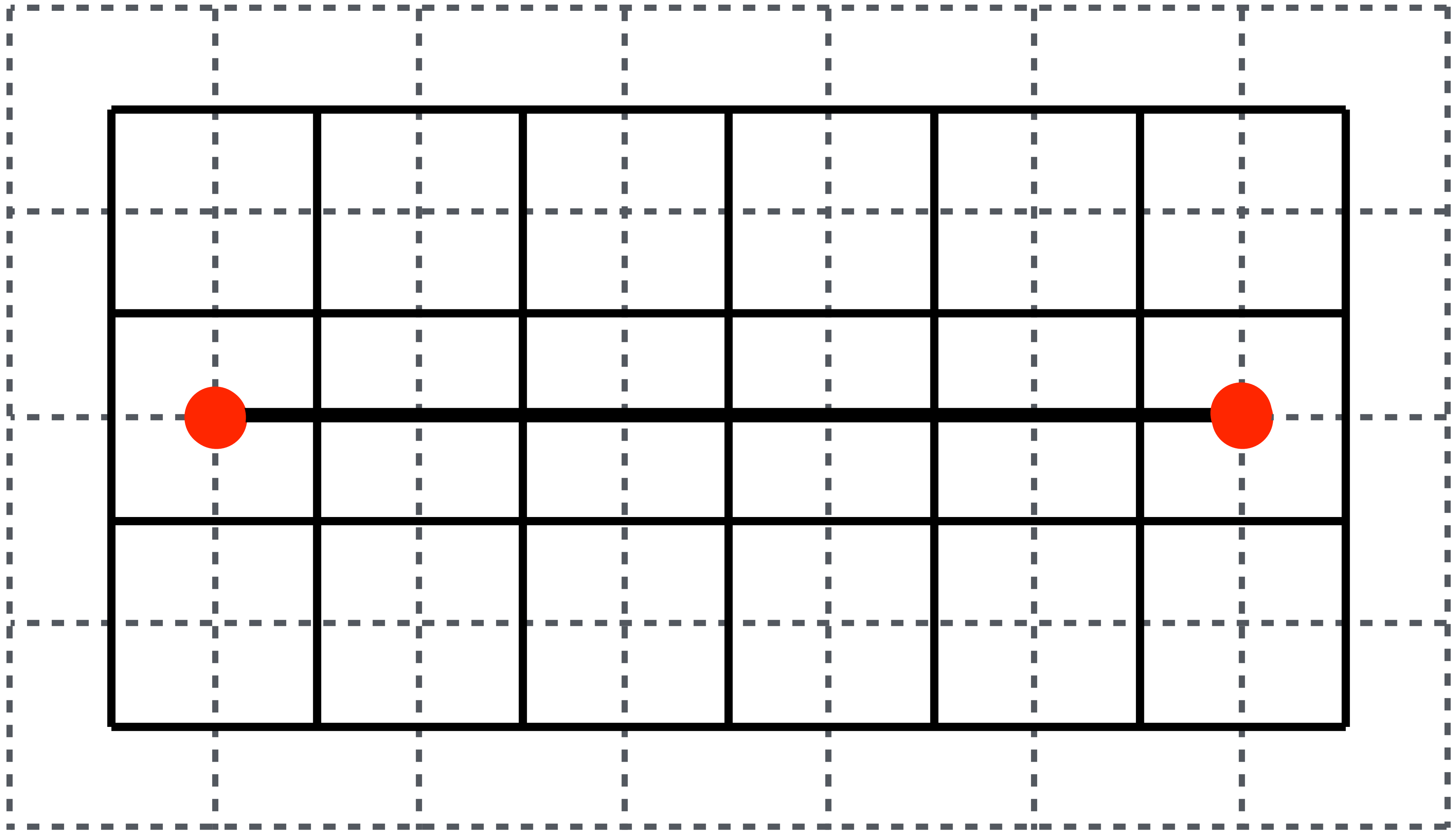

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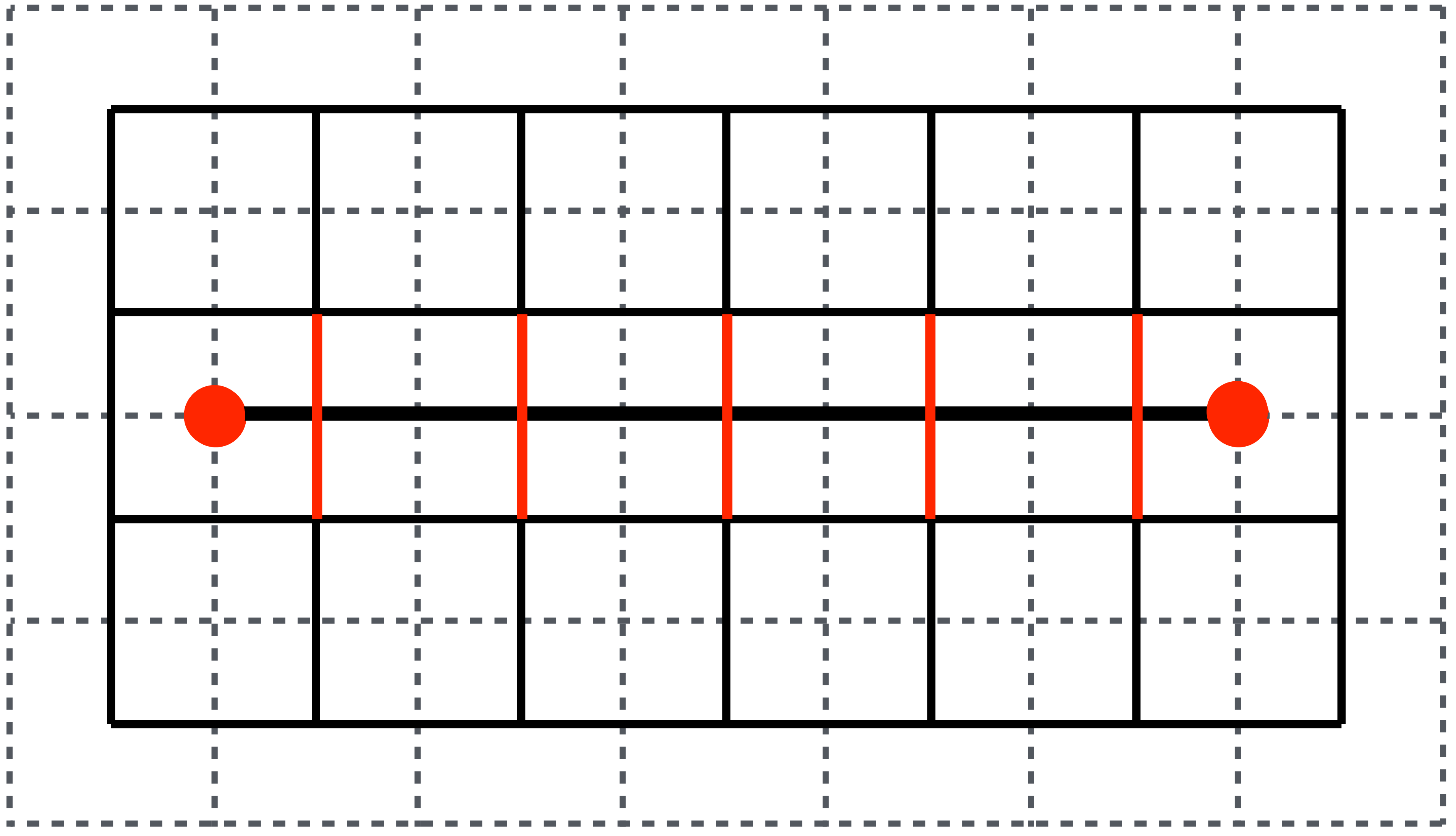

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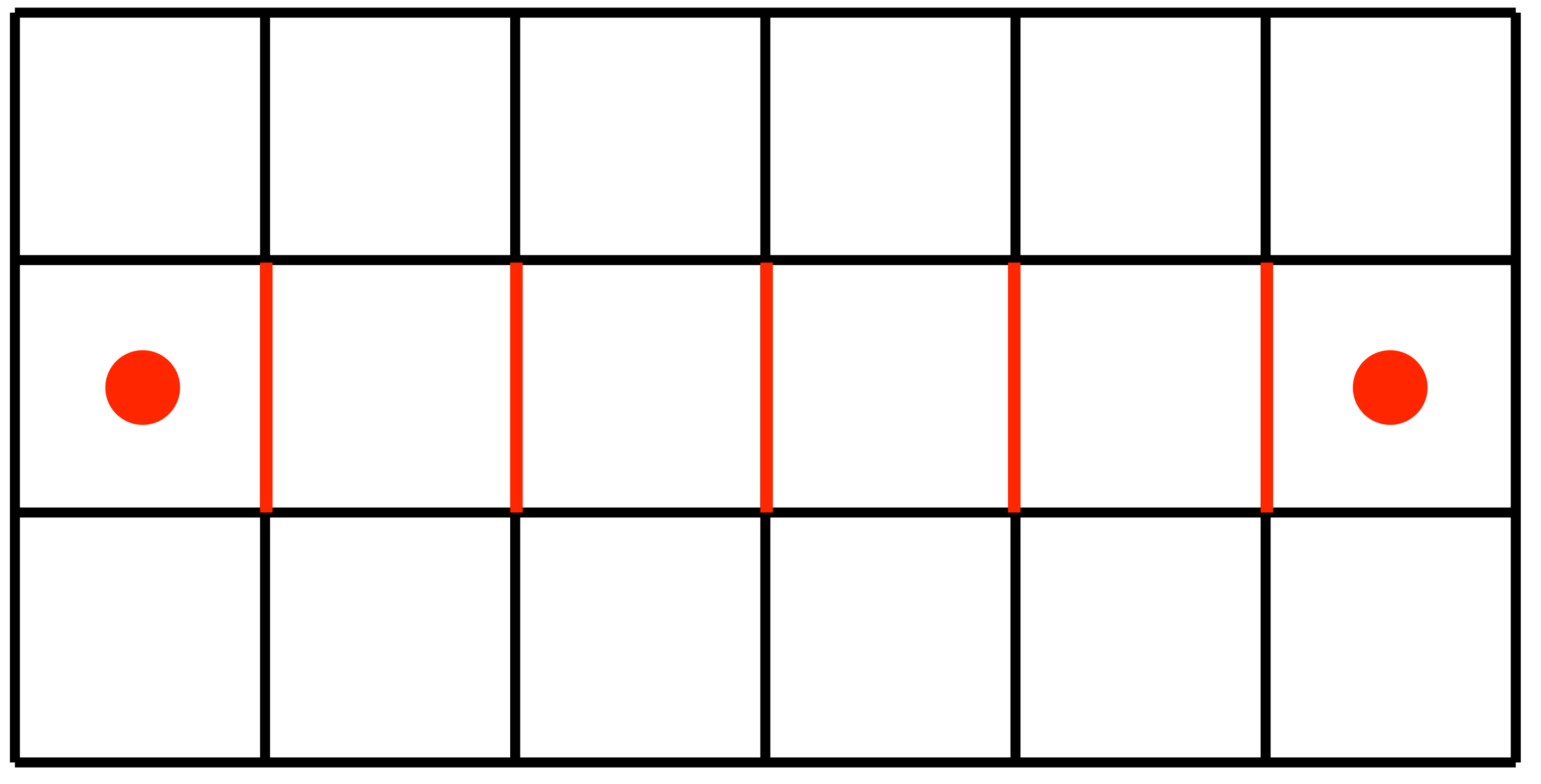

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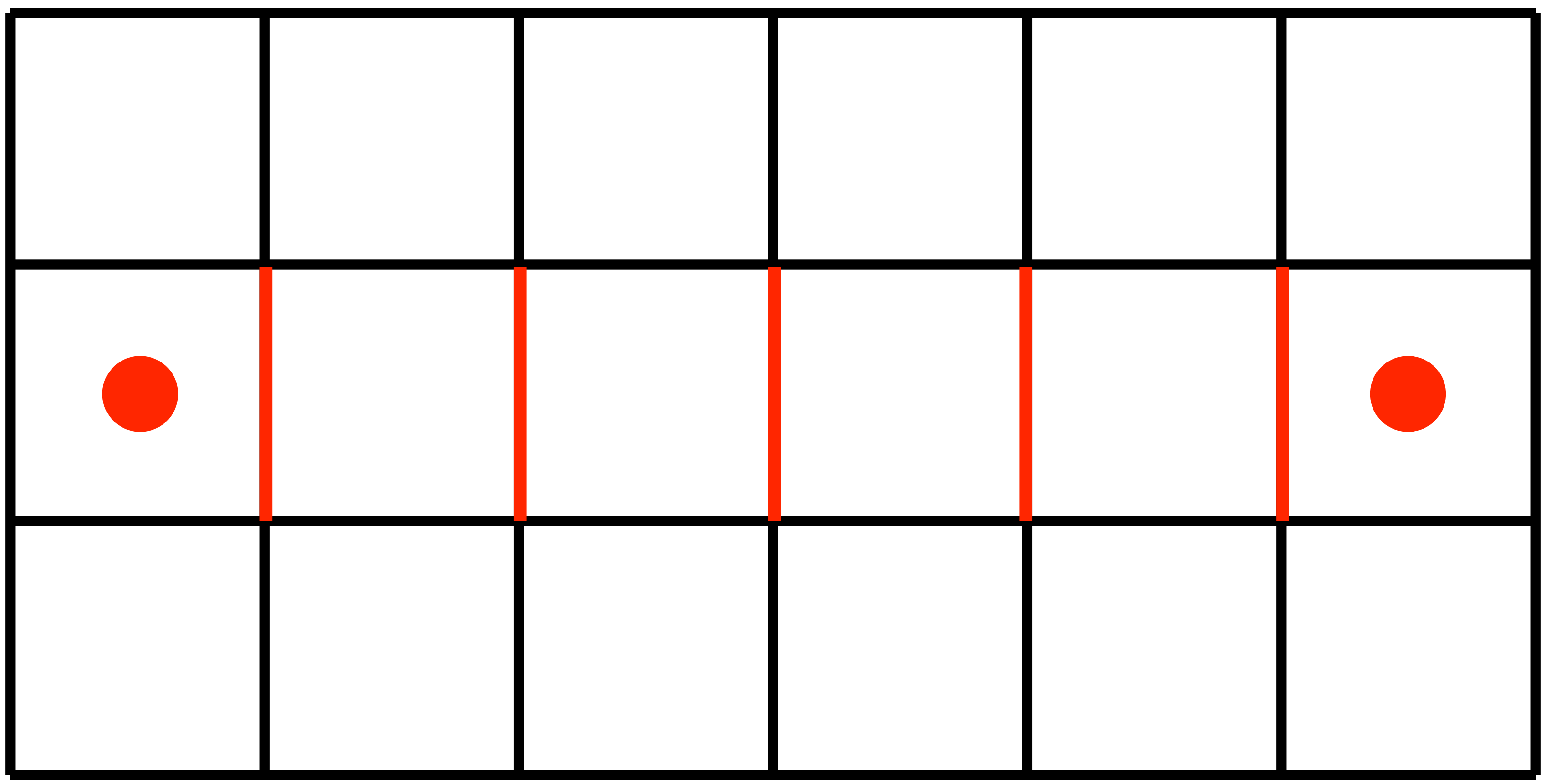
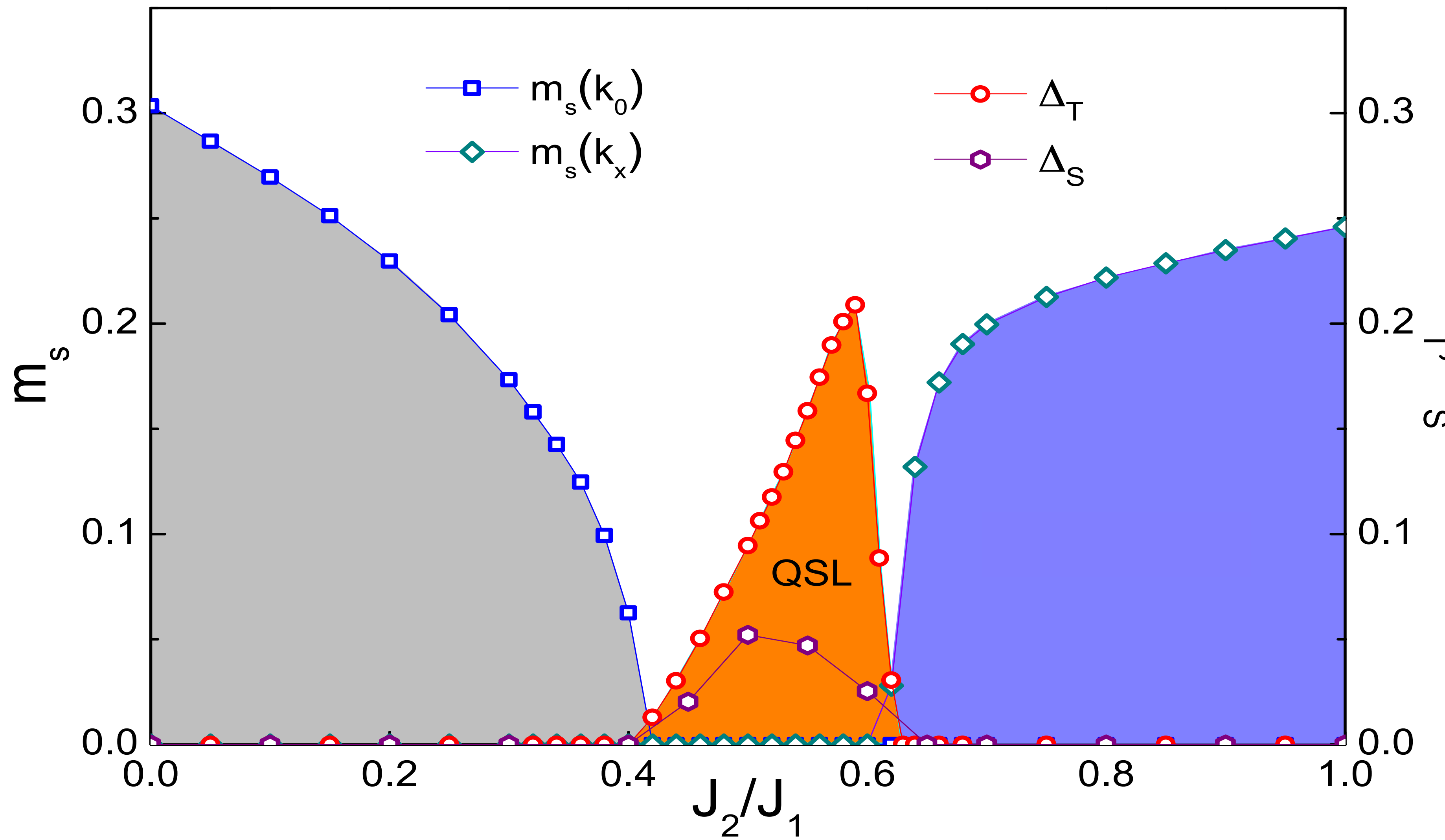


A domain wall in the dual theory

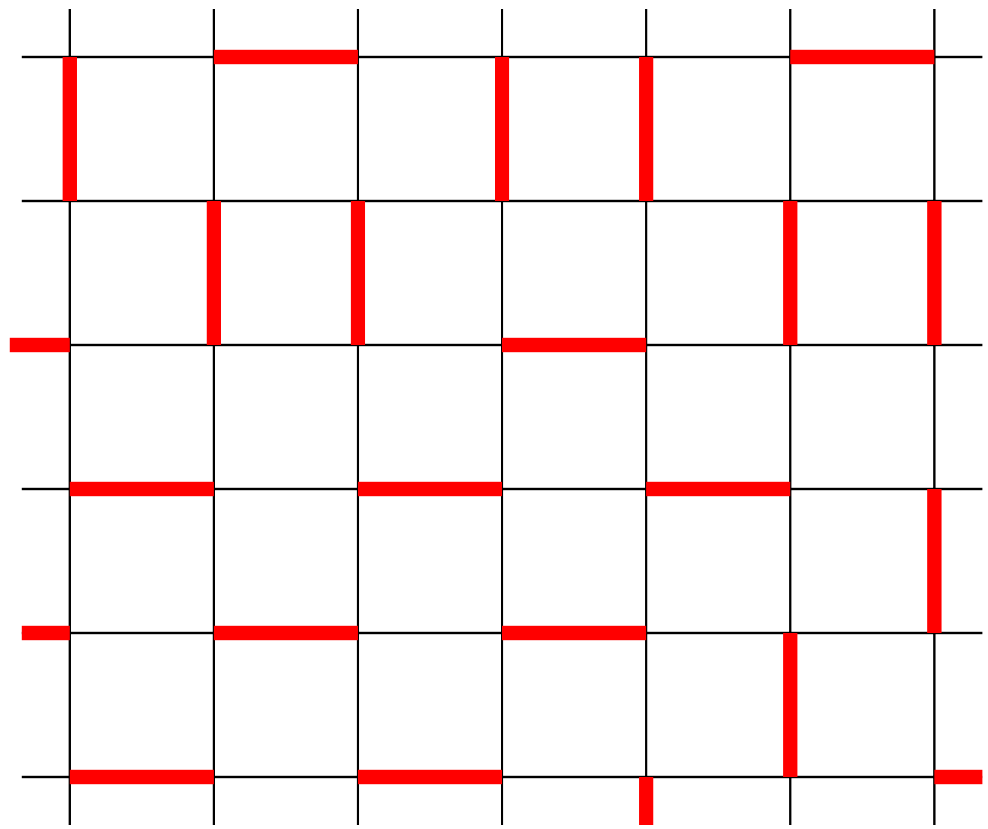
Heisenberg model: square lattice

Phenomenology of Heisenberg models in terms of a Z_2 gauge theory.



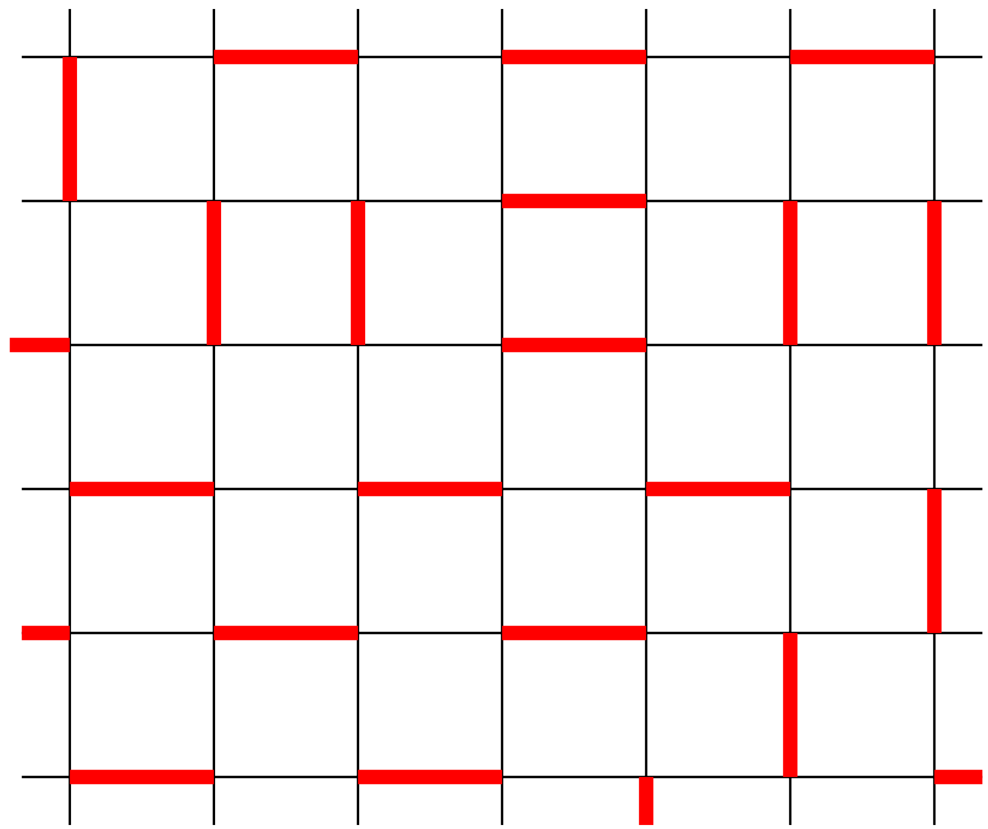
R. Moessner, S.L. Sondhi, and E. Fradkin, Phys. Rev. B **65**, 024504 (2001).
H.C. Jiang, H. Yao, and L. Balents, Phys. Rev. B **86**, 024424 (2012).

Heisenberg model: square lattice



Dimers represent pairs of spins in a singlet state.

Heisenberg model: square lattice



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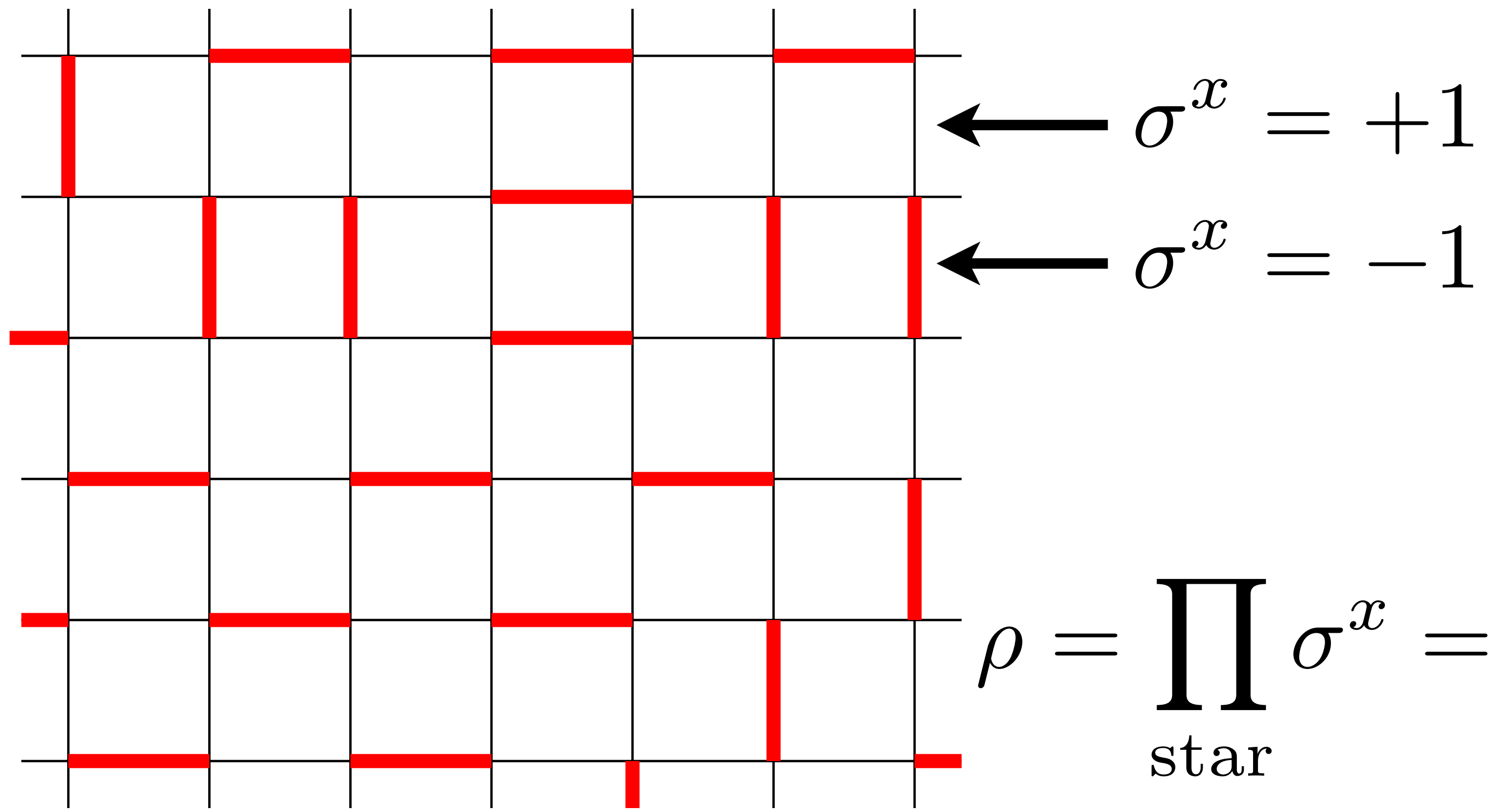
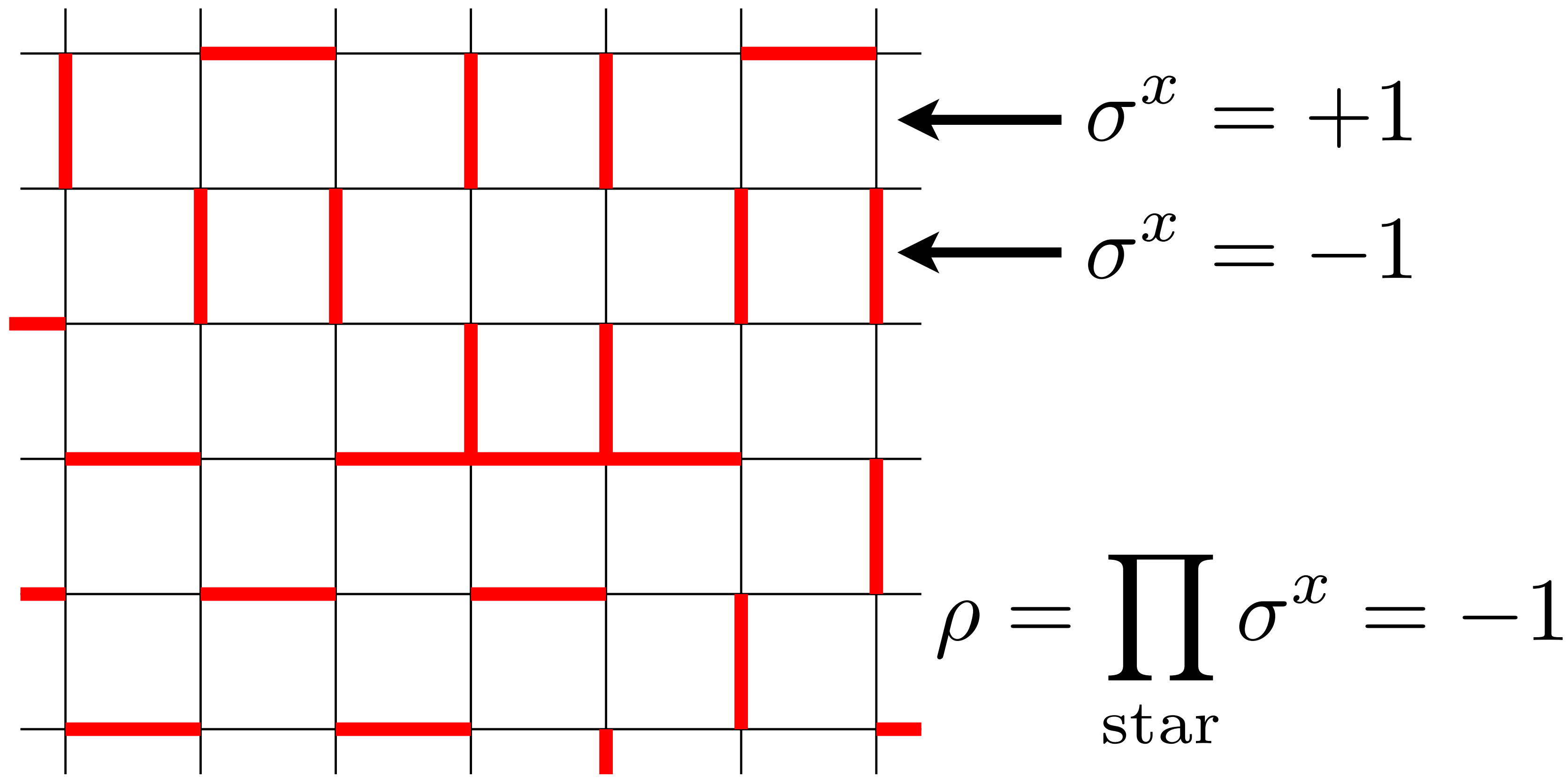
Heisenberg model: square lattice



$$\rho = \prod_{\text{star}} \sigma^x = -1$$

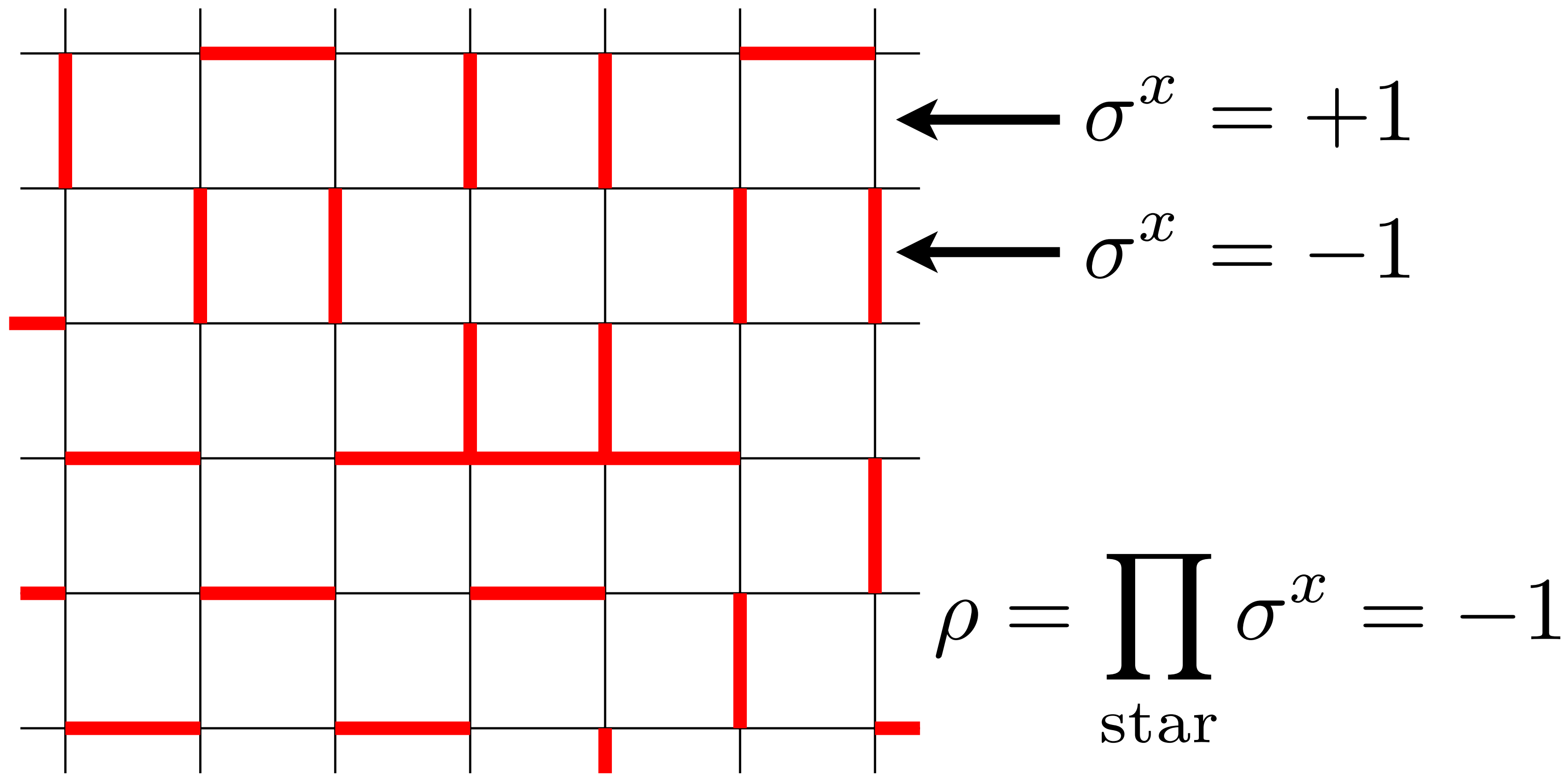
Z_2 gauge theory with static charges

Heisenberg model: square lattice



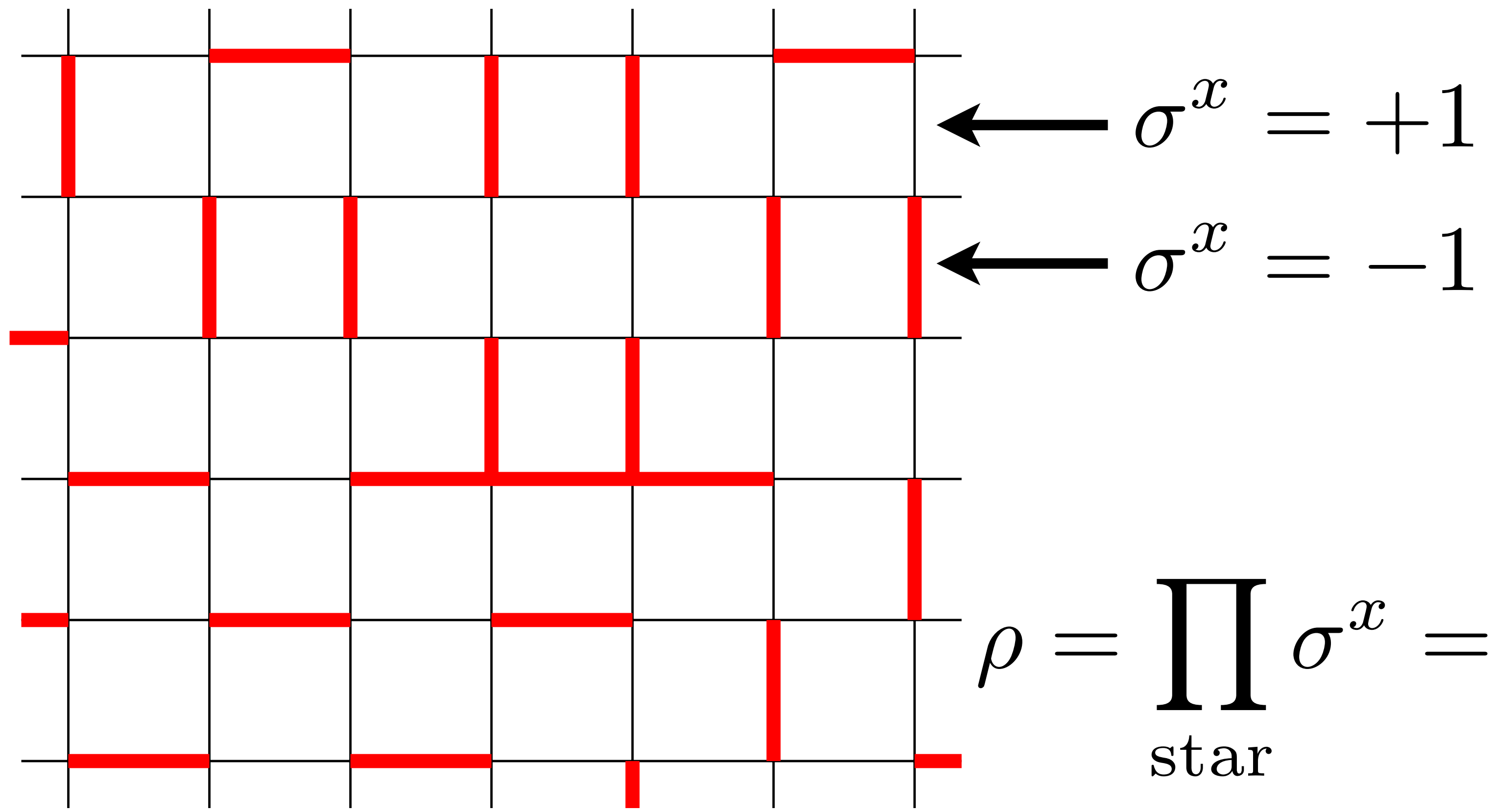
3 dimers at a site are allowed by these constraints.

Heisenberg model: square lattice



Suppress 3-dimer configurations with a $-\Gamma\sigma^x$ term.

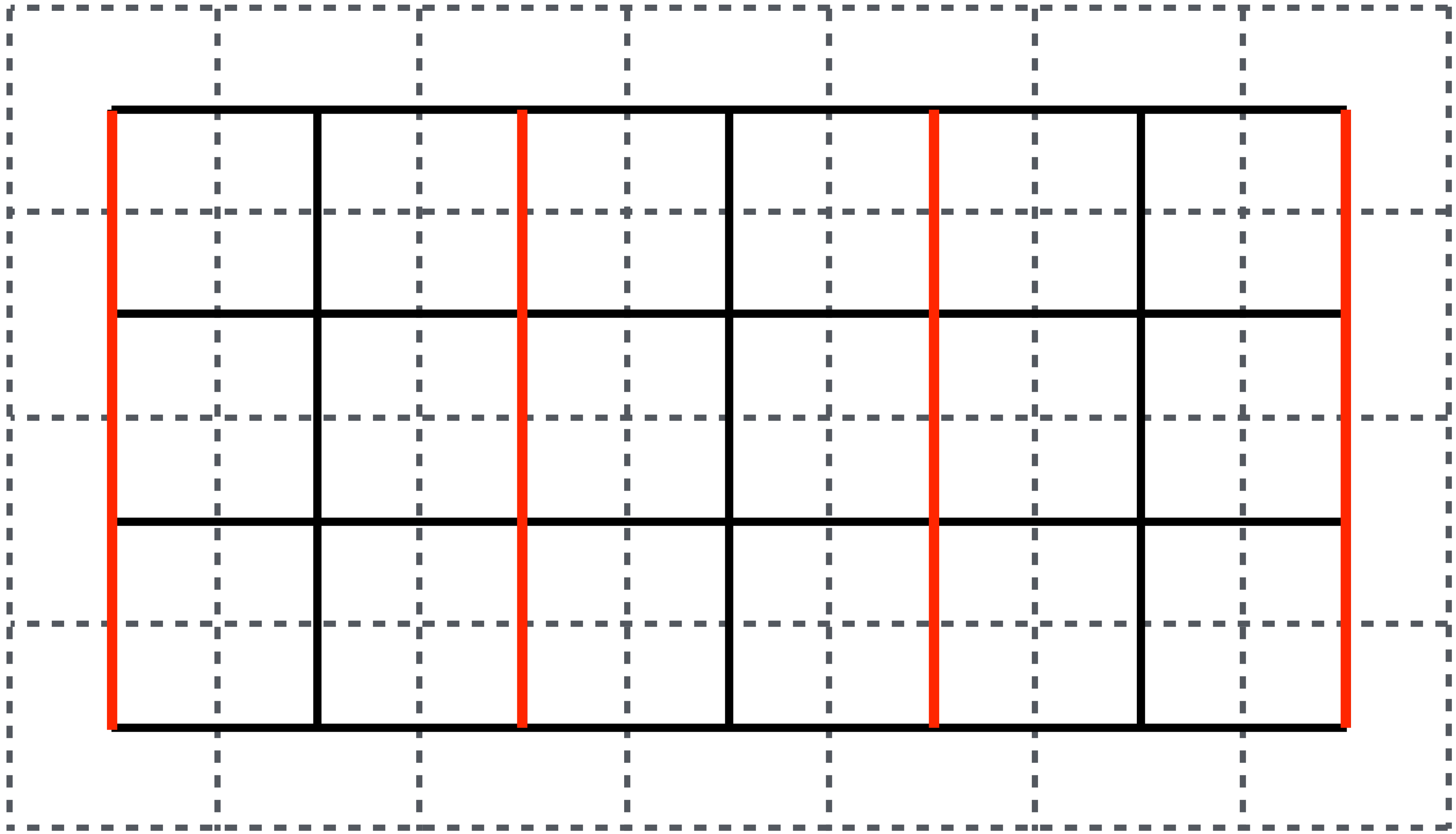
Heisenberg model: square lattice



$$\rho = \prod_{\text{star}} \sigma^x = -1$$

$$H = -\Gamma \sum_{\text{links}} \sigma^x_{mn} - \lambda \sum_{\text{plaquettes}} \sigma^z_{mn} \ldots \sigma^z_{qm}$$

Quantities $\lambda_{\alpha\beta}\langle\tau^z_\alpha\tau^z_\beta\rangle = \langle\sigma^x_{\alpha\beta}\rangle$
are non-uniform on cylinders with odd circumference.



Quantities $\lambda_{\alpha\beta}\langle\tau_{\alpha}^{z}\tau_{\beta}^{z}\rangle = \langle\sigma_{\alpha\beta}^{x}\rangle$
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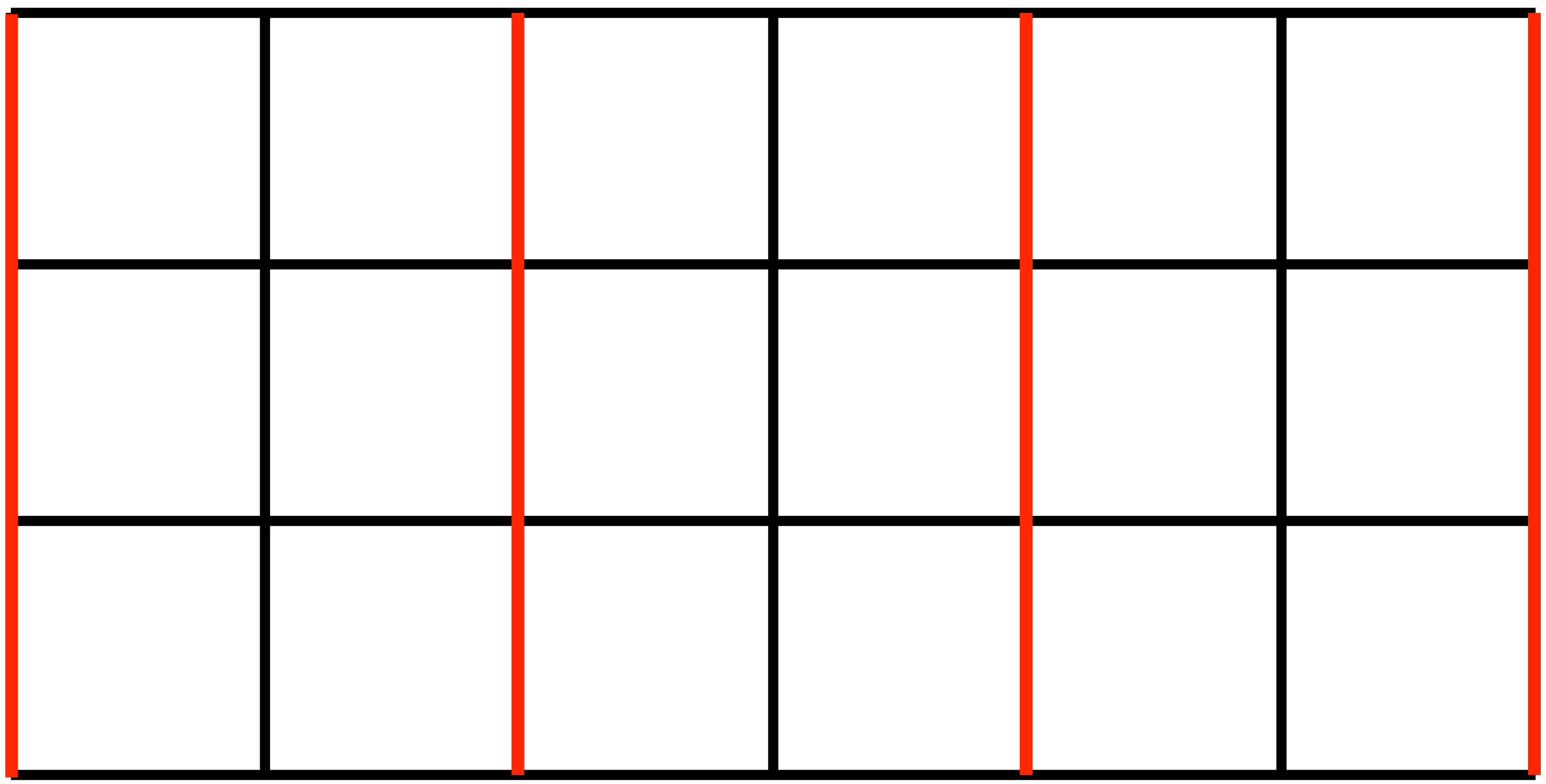
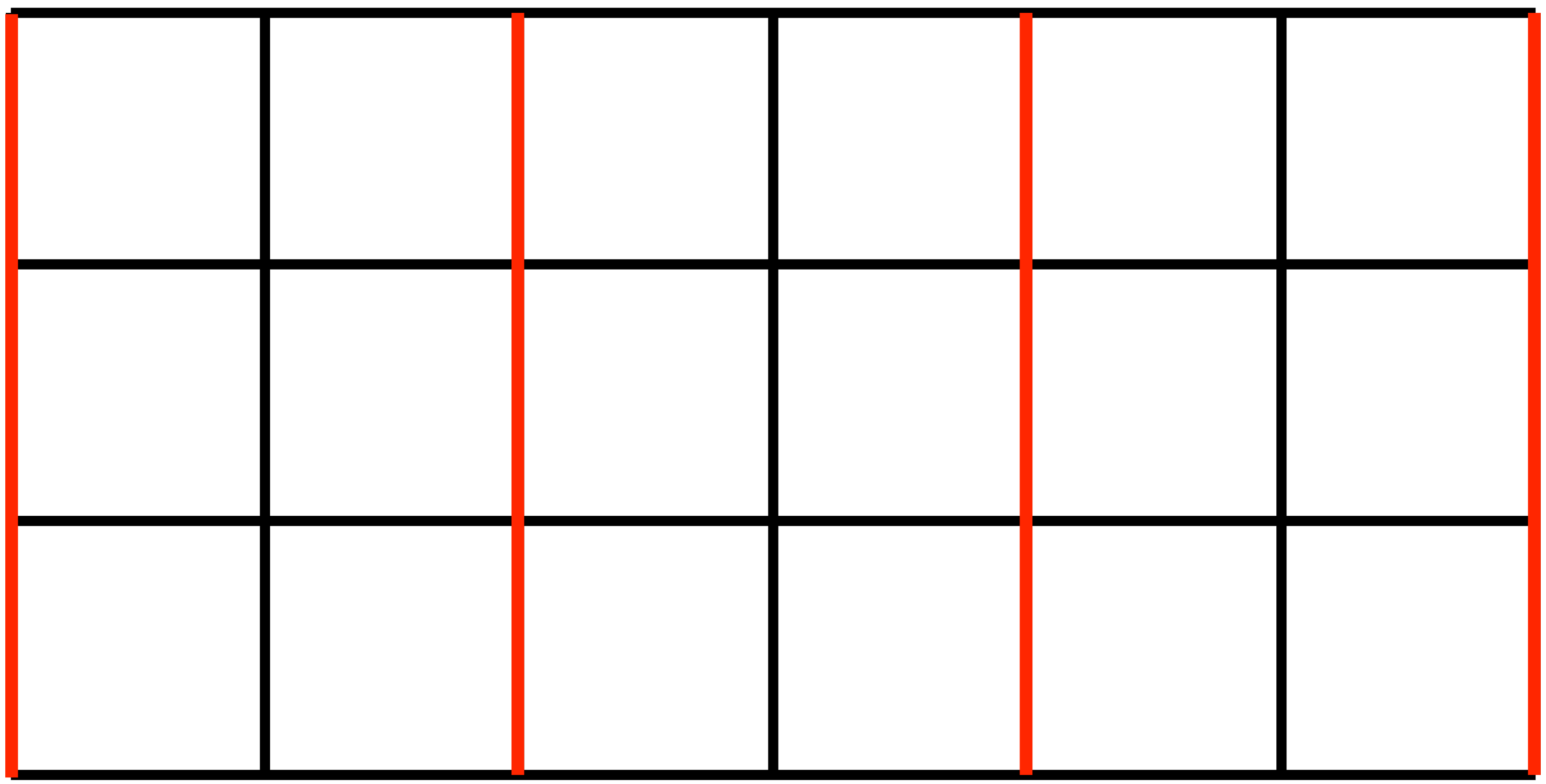


Quantities $\lambda_{\alpha\beta}\langle\tau^z_\alpha\tau^z_\beta\rangle = \langle\sigma^x_{\alpha\beta}\rangle$
are non-uniform on cylinders with odd circumference.

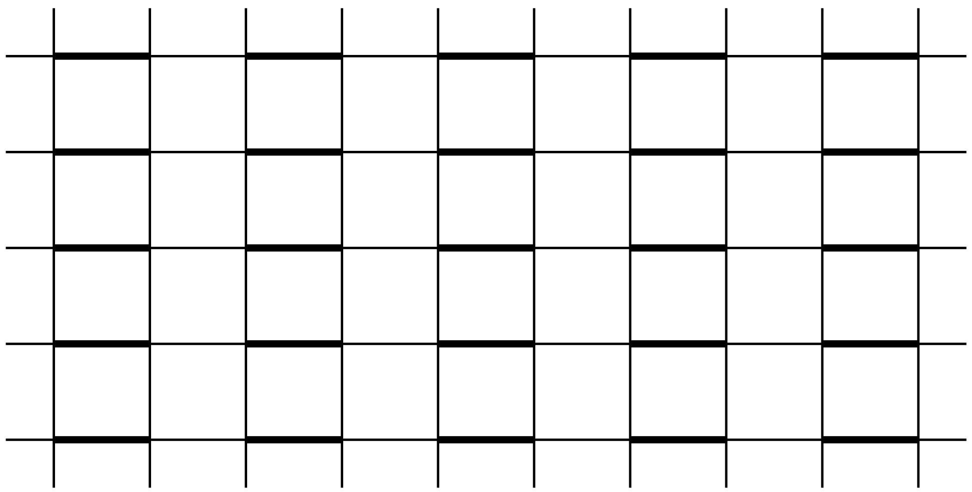
$$\tilde{H} = -\Gamma \sum_{\langle\alpha\beta\rangle} \lambda_{\alpha\beta} \tau^z_\alpha \tau^z_\beta - \lambda \sum_\alpha \tau^x_\alpha, \quad \lambda_{\alpha\beta} = \pm 1$$

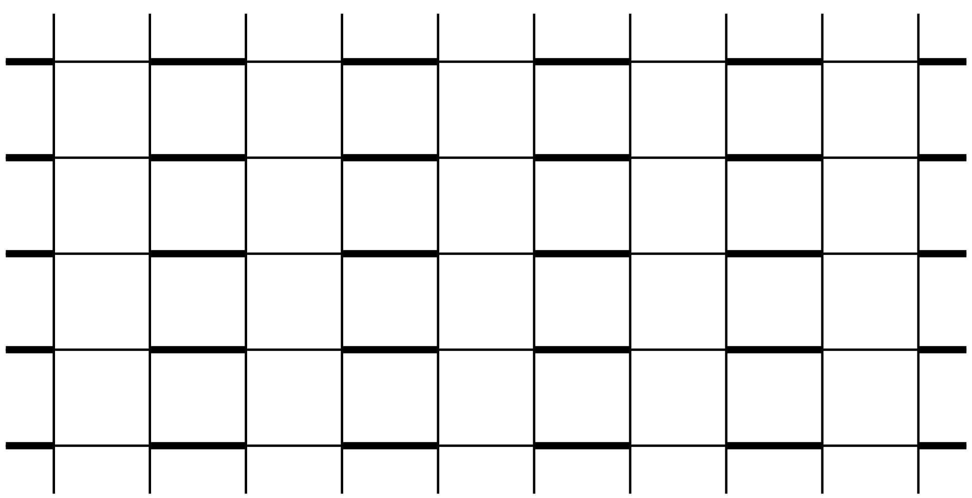


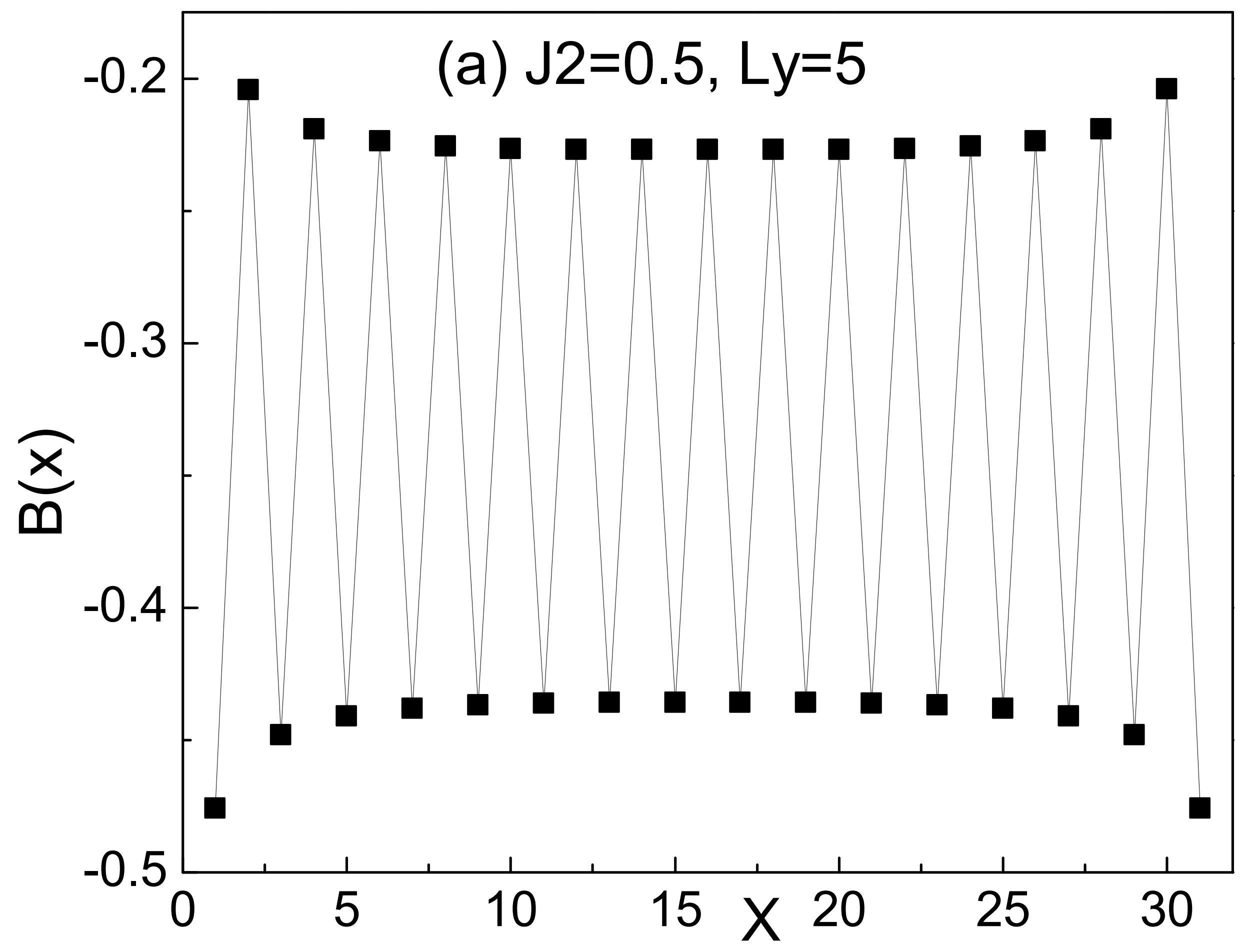
Quantities $\lambda_{\alpha\beta} \langle \tau^z_\alpha \tau^z_\beta \rangle = \langle \sigma^x_{\alpha\beta} \rangle$ are non-uniform on cylinders with odd circumference.

Odd circumference

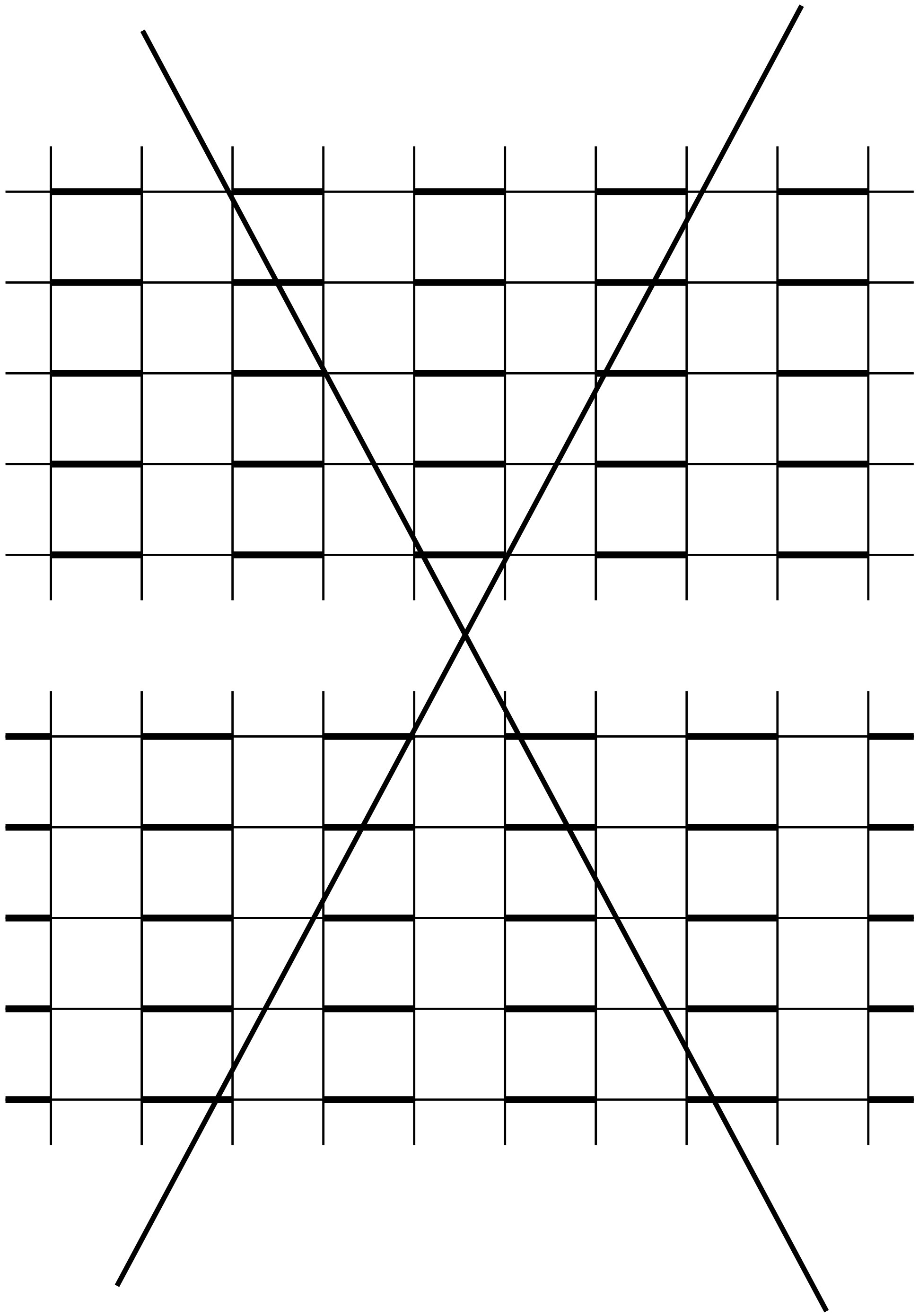
$$B(x) = \langle \mathbf{S}(x,y) \cdot \mathbf{S}(x+a,y) \rangle$$



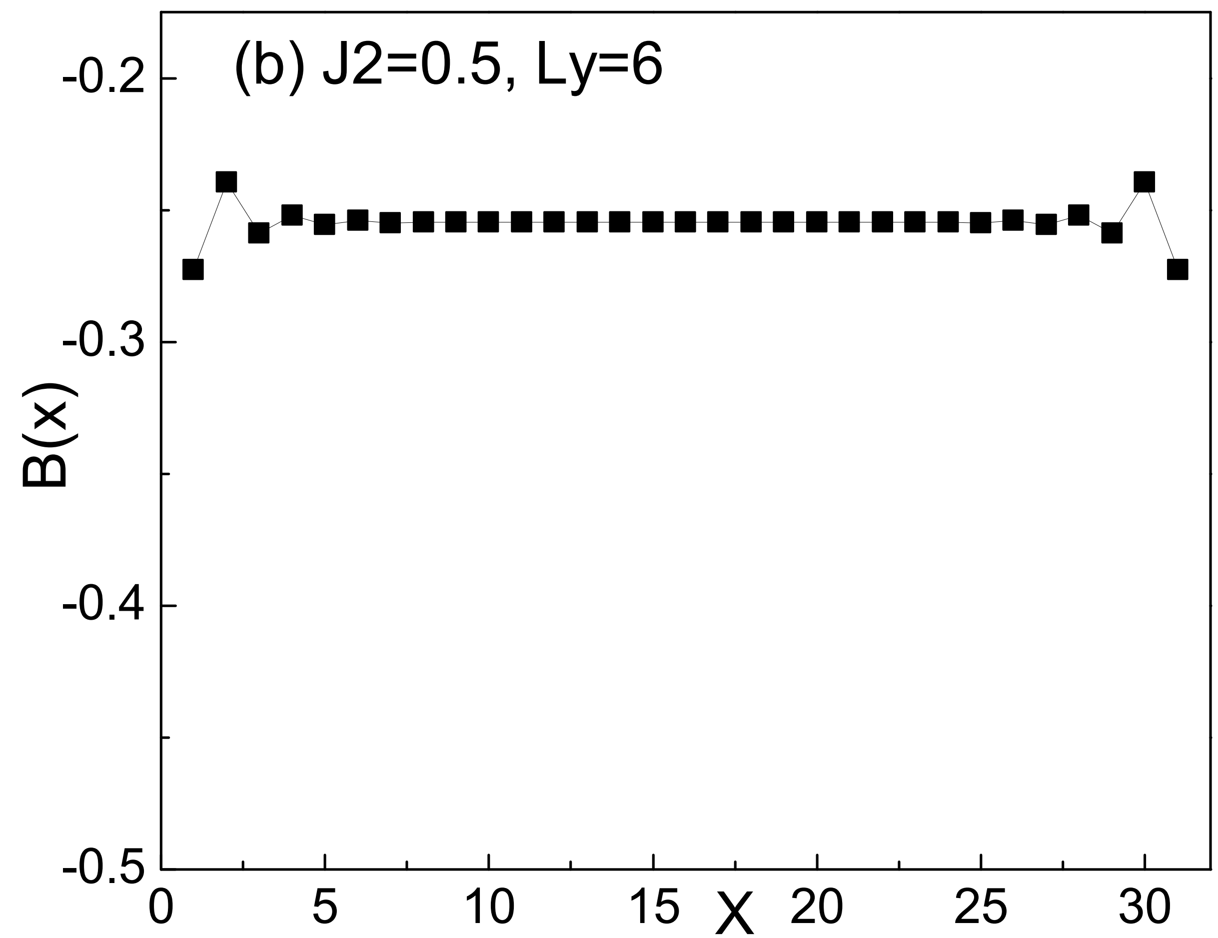




Even circumference

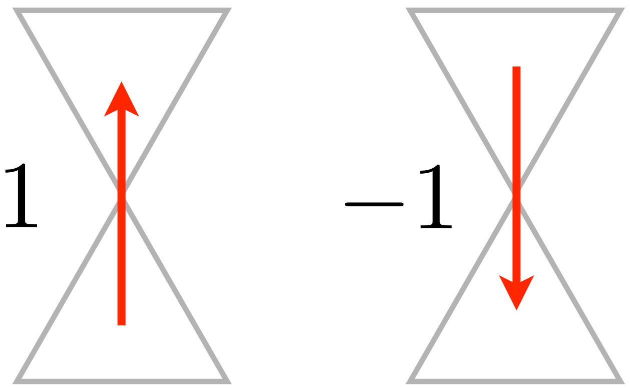


$$B(x) = \langle \mathbf{S}(x,y) \cdot \mathbf{S}(x+a,y) \rangle$$



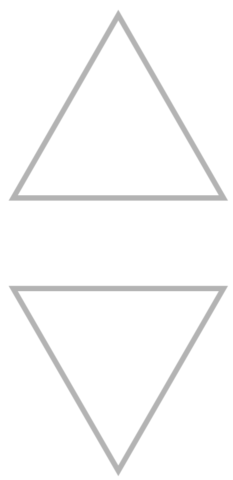
Heisenberg model: kagome lattice

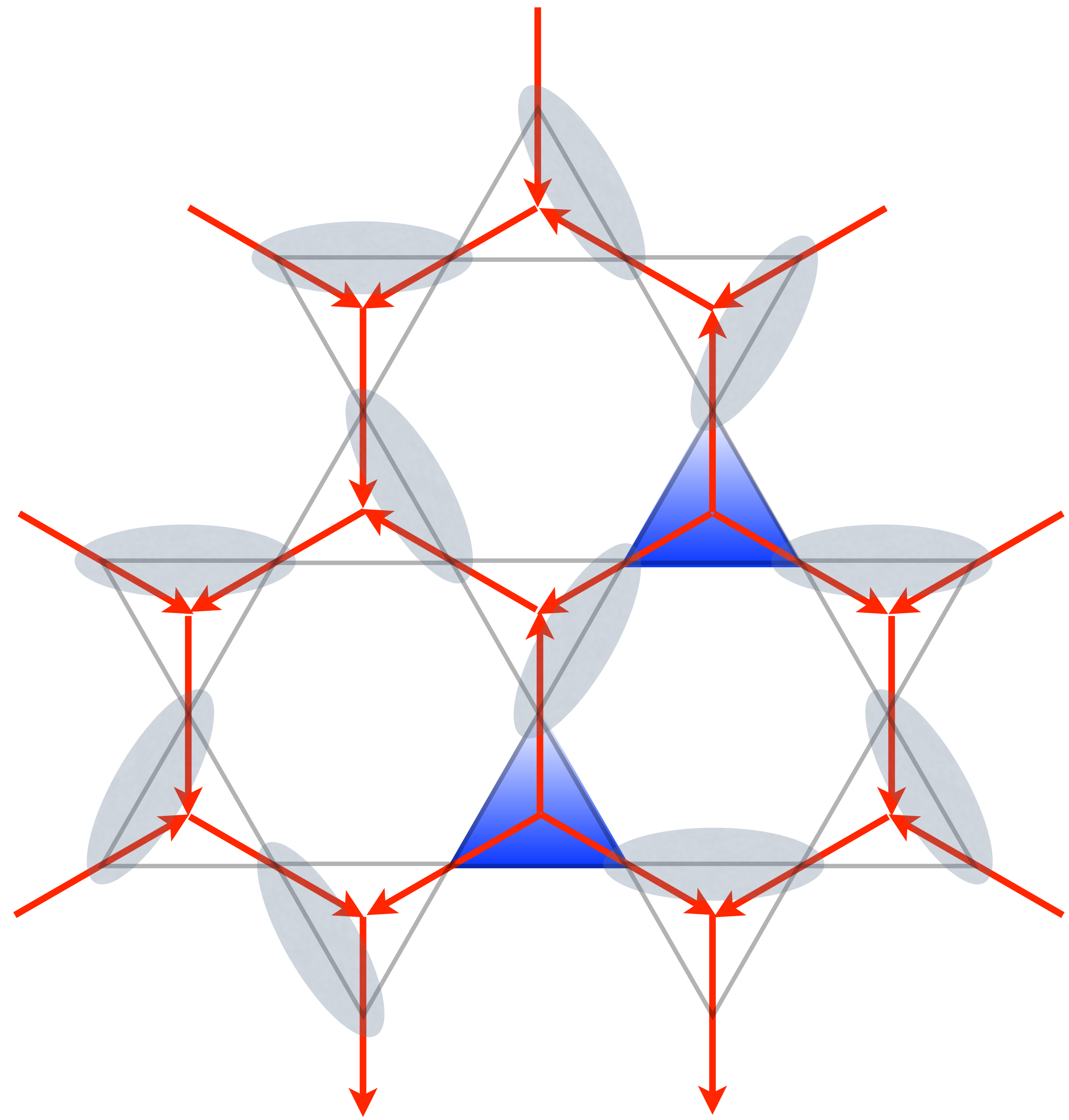
$\sigma^x = +1$ $\quad -1$



$$\rho = \prod_{\text{star}} \sigma^x = +1$$

-1

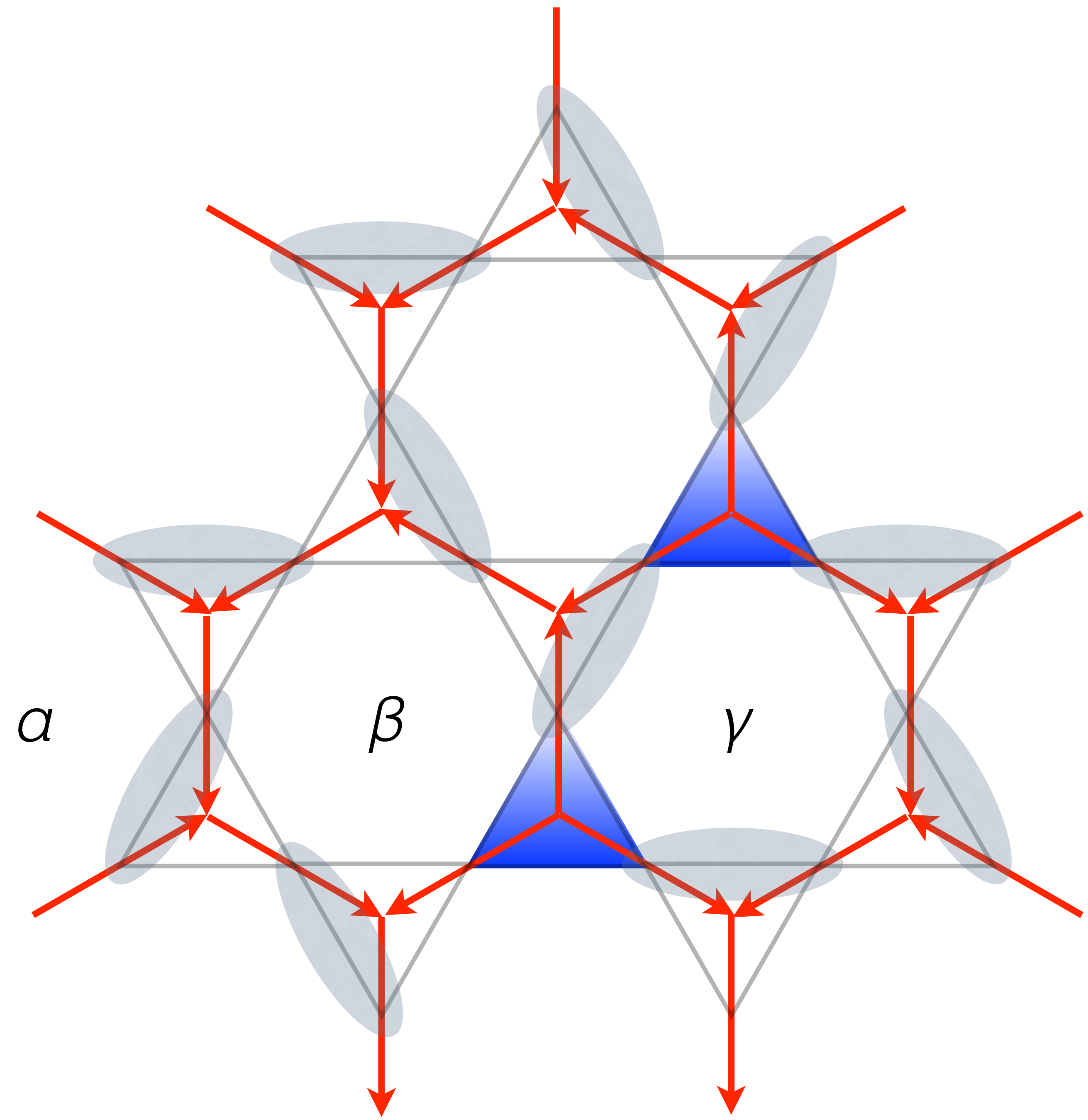




Heisenberg model: kagome lattice

$$H = -\Gamma \sum_{\text{plaquettes}} \phi_\alpha + \lambda \sum_{\text{links}} \sigma^x_{\alpha\beta}$$

$$\phi_\alpha = \prod_{\beta(\alpha)} \sigma^z_{\alpha\beta}$$

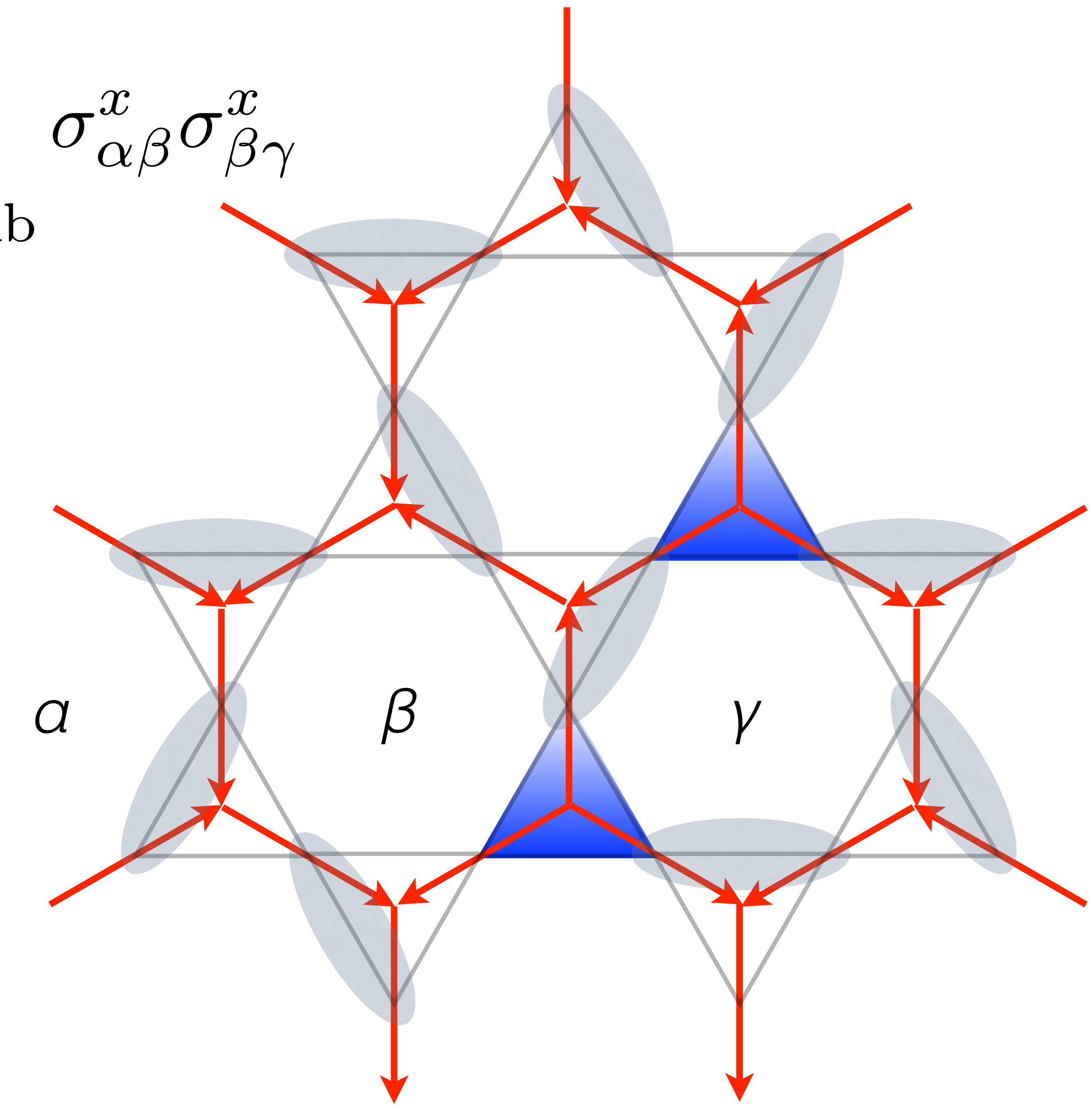


Y. Wan and O. Tchernyshyov (2013).
H.J. Ju and L. Balents (2013).

Heisenberg model: kagome lattice

$$H = -\Gamma \sum_{\text{plaquettes}} \phi_\alpha + \lambda \sum_{\text{3rd neighb}} \sigma^x_{\alpha\beta}\sigma^x_{\beta\gamma}$$

$$\phi_\alpha = \prod_{\beta(\alpha)} \sigma^z_{\alpha\beta}$$



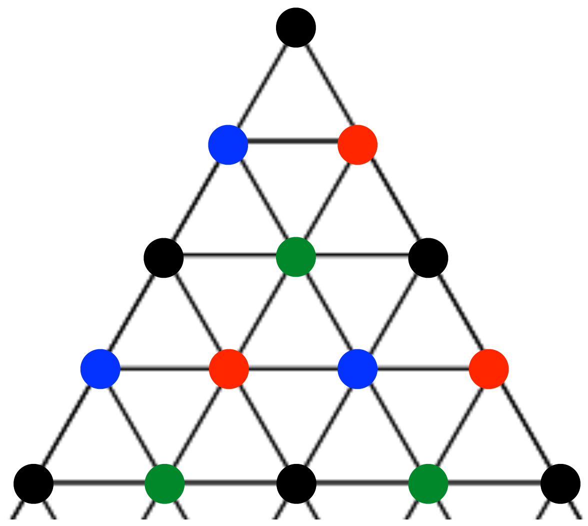
Y. Wan and O. Tchernyshyov (2013).
H.J. Ju and L. Balents (2013).

Heisenberg model: kagome lattice

$$H = -\Gamma \sum_{\text{plaquettes}} \phi_\alpha + \lambda \sum_{\text{3rd neighb}} \sigma^x_{\alpha\beta}\sigma^x_{\beta\gamma}$$

$$\tilde{H} = -\Gamma \sum_{\alpha} \tau^x_\alpha + \lambda \sum_{\text{3rd neighb}} \lambda_{\alpha\beta}\lambda_{\beta\gamma}\tau^z_\alpha\tau^z_\gamma$$

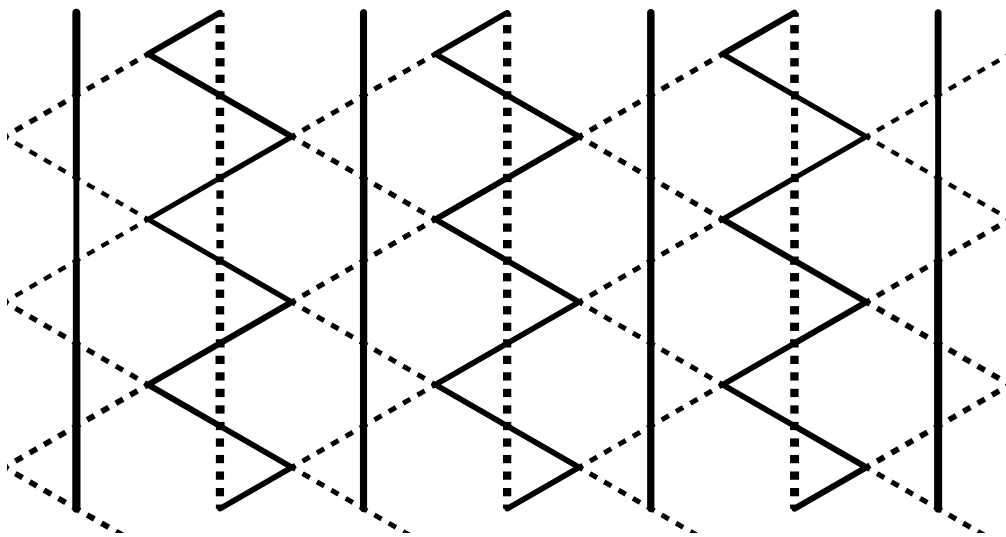
Quantum Ising model with
4 independent sublattices.



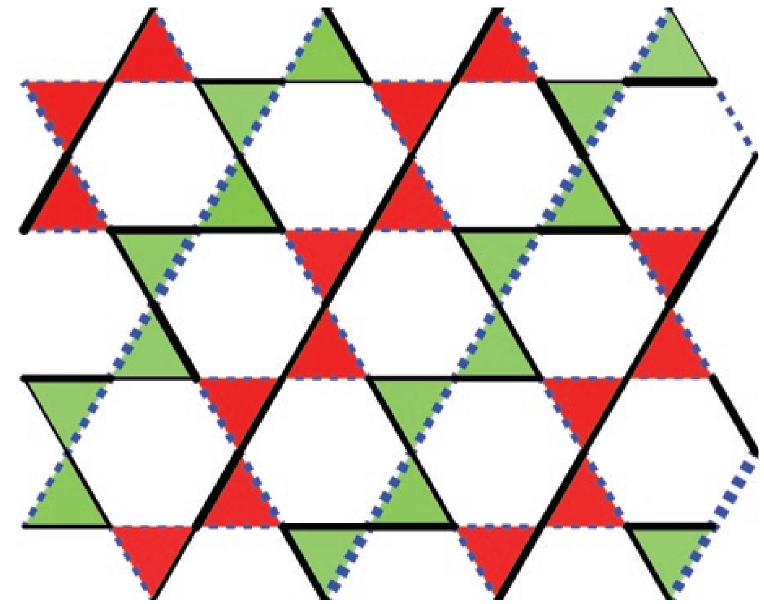
Odd circumference

"YC6" | "YC9-2"

DMRG:

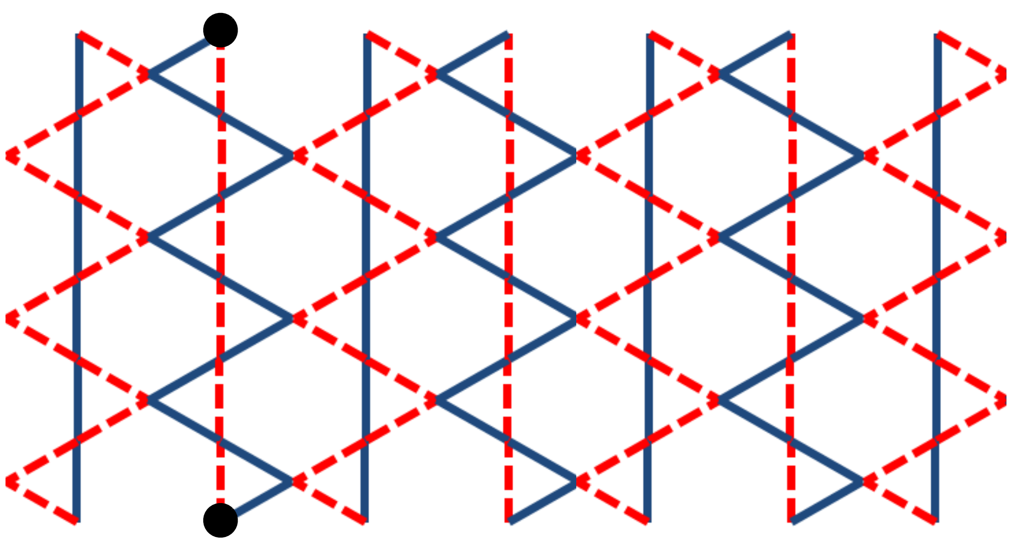


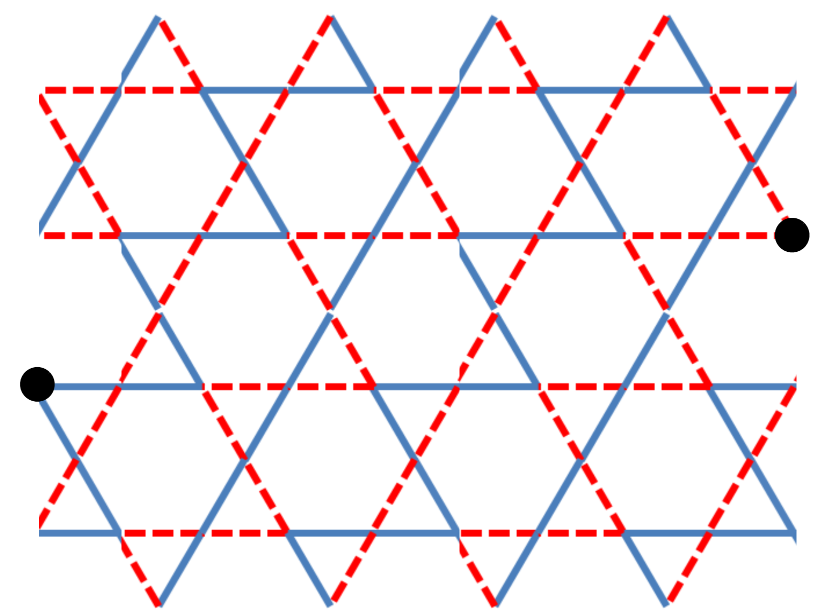
S. R. White, KITP 2010



S. Yan *et al.*, Science (2011)

Z_2 gauge theory:

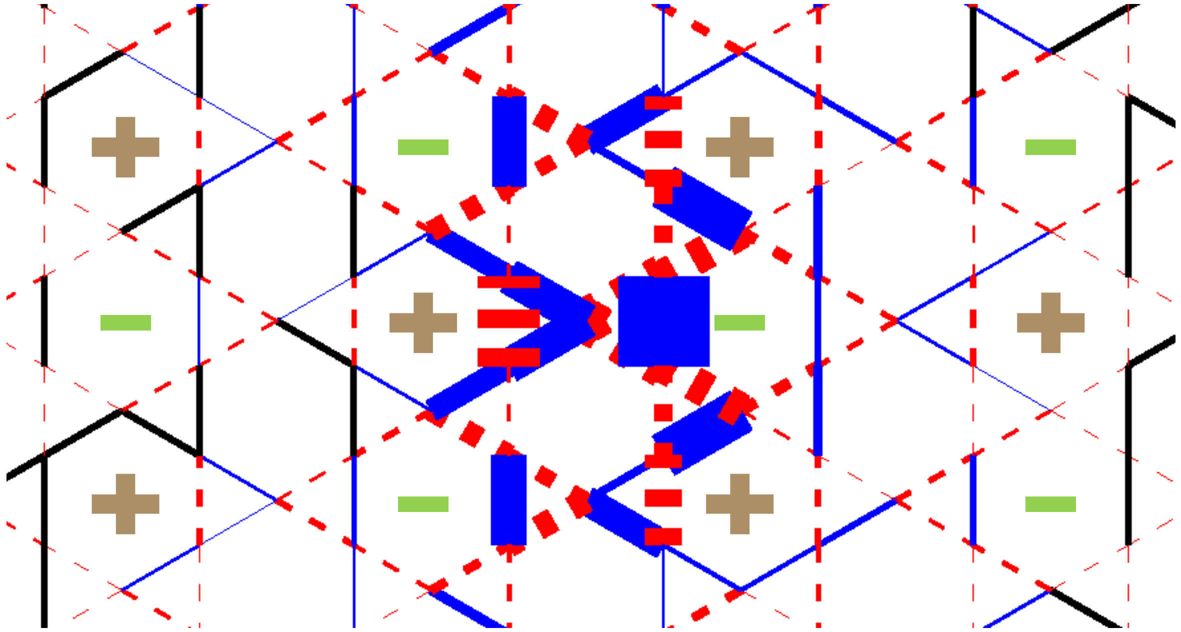




Y. Wan and OT, Phys. Rev. B (2013)

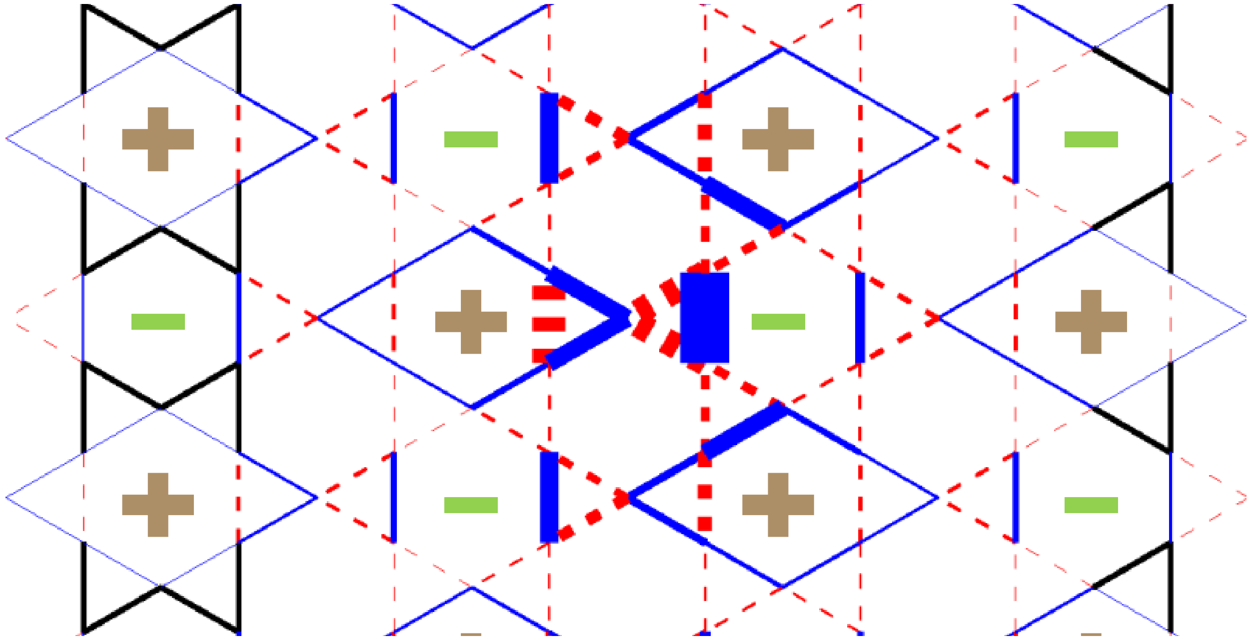
Valence-bond correlations

DMRG:



S. R. White, private comm.

Z_2 gauge theory:



Y. Wan and OT, Phys. Rev. B (2013)