Friendly introduction to AdS/CMT

1. Equilibrium physics

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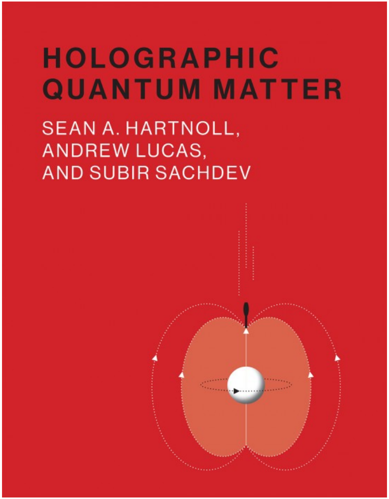

arXiv:1612.07324

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- when $\lambda \gg 1$ and $N \gg 1$, gauge theory is strongly coupled, and gravity is approximately classical: $G_\mathrm{N} \sim N^{-2}$
- no proof, but much evidence, for this duality

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  - effective theory teaches us that the lack of a microscopic model isn't a bad thing

- the "most common" approach to CMT:

$$H \sim \epsilon c^\dagger c + g c^\dagger c^\dagger c c + \cdots$$

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- ...but dynamics is *singular*: e.g. conductivity

$$\sigma(\omega) \sim \frac{1}{g^2 - \mathrm{i}\omega} + \cdots$$

  this is often *unphysical* when $g = 1$ (strong coupling)

[Damle, Sachdev; cond-mat/9705206]

What to look out for

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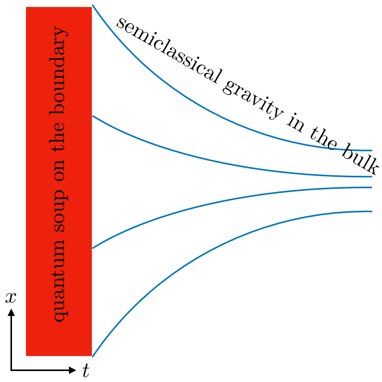
- next lecture: dynamics (more new stuff)

- **conjecture** of holographic duality: [Witten; `hep-th/9802150`]

$$\left\langle \exp\left[\mathrm{i} \int \mathrm{d}^{d+1} x \phi_a(x^\mu) \mathcal{O}_a(x^\mu) \right] \right\rangle = Z_{\mathrm{QG}}[\phi_a(x^\mu)]$$



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- gravity background = (state of the) field theory (temperature, density, disorder, some couplings, etc.)

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- bulk gravity = stress tensor $T^{\mu\nu}$; holographic theory has gravity because boundary theory has energy!

- global symmetry of the field theory, e.g.

$$\langle \mathcal{O}_a(x_1)\mathcal{O}_b(x_2)\cdots\rangle = \langle \mathcal{O}_a(x_1+s)\mathcal{O}_b(x_2+s)\cdots\rangle$$

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- this is a saddle point of Einstein gravity:

$$Z_{\mathrm{QG}} \approx \exp\left[\frac{\mathrm{i}}{16\pi G_{\mathrm{N}}}\int \mathrm{d}^{d+2}x\sqrt{-g}(R-2\Lambda)\right]$$

  with cosmological constant $\Lambda = -d(d+1)/2L_{\mathrm{AdS}}^2$

- given a bulk scalar field ϕ, with $\Delta > (d+1)/2$,

$$\nabla_M \nabla^M \phi = m^2 \phi + g\phi^2 + \cdots$$

and $m^2 = \Delta(d+1-\Delta)$, solution near $r = 0$:

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Source and response

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- example solution is (if $g = 0$)

$$\phi \sim \left(\frac{r}{r^2 + x^2} \right)^\Delta, \quad \phi_0 = \delta(x) \quad \langle \mathcal{O} \rangle = \frac{1}{x^{2\Delta}}$$

which is two-point function of CFT

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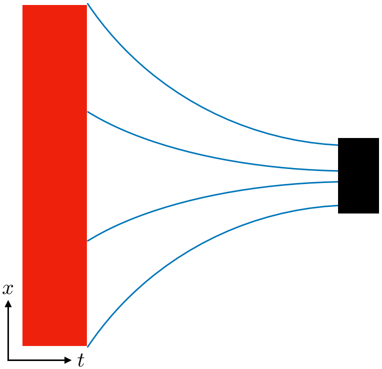
- the holographic geometry:

$$\mathrm{d}s^2 \sim r^{2\theta/d} \left[\frac{\mathrm{d}r^2 + \mathrm{d}\mathbf{x}^2}{r^2} - \frac{\mathrm{d}t^2}{r^{2z}} \right]$$

- black hole = finite T state!

$$\mathrm{d}s^2 \sim r^{2\theta/d}\left[\frac{f(r)^{-1}\mathrm{d}r^2 + \mathrm{d}\mathbf{x}^2}{r^2} - f(r)\frac{\mathrm{d}t^2}{r^{2z}}\right]$$

$$f(r) = 1 - \left(\frac{r}{r_\mathrm{h}}\right)^{d+z-\theta}$$



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- avoid conical singularity! τ is periodic with $\tau \sim \tau + r_\mathrm{h}^z$
- this is the same as finite temperature! $\tau \sim \tau + 1/T$; hence

$$T \sim r_\mathrm{h}^{-z}$$

- horizon area (per unit volume of *boundary theory*):

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- recall the previous scaling of the entropy density

$$s \sim T^{(d-\theta)/z}$$

Bekenstein-Hawking entropy

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$$\text{area} \sim [\text{length scale}]^d \sim g_{xx}^{d/2} \sim r_{\text{h}}^{-d+\theta} \sim T^{(d-\theta)/z}$$

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- entropy of holographic black hole = entropy of QFT!

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$$A_t(r=0) = \mu$$

- field theory with charge disorder:

$$S = S_0 + \int \mathrm{d}^d\mathbf{x}\mathrm{d}t\, \mu(\mathbf{x}) J^t(\mathbf{x}, t)$$

corresponds to inhomogeneous/disordered black hole:

$$A_t(r=0) = \mu(\mathbf{x})$$

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- AdS-Reissner-Nordstrom (AdS-RN) solution:

$$A = \mu\left[1-\left(\frac{r}{r_+}\right)^{d-1}\right]\mathrm{d}t, \quad \mathrm{d}s^2 = \frac{L^2}{r^2}\left[\frac{\mathrm{d}r^2}{f(r)} - f(r)\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2\right],$$

$$f(r) = 1-\left(\frac{r}{r_+}\right)^{d+1} + \frac{r_+^2\mu^2}{\gamma^2}\left(\left(\frac{r}{r_+}\right)^{2d} - \left(\frac{r}{r_+}\right)^{d+1}\right)$$

$$\gamma^2 = \frac{d}{d-1}\frac{e^2L^2}{8\pi G_{\mathrm{N}}}, \quad T = \frac{1}{4\pi r_+}\left(d+1-(d-1)\frac{r_+^2\mu^2}{\gamma^2}\right)$$

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- entropy:

$$s \sim r_+^{-d} \sim \max(T,\mu)^d$$

[Chamblin, Emparan, Johnson, Myers; `hep-th/9902170`]

- what's happening when $T \ll \mu$? consider the metric at $T = 0$...

$$\mathrm{d}s^2 \propto \underbrace{\frac{\mathrm{d}r^2}{(r_+ - r)^2} - (r_+ - r)^2 \mathrm{d}t^2}_{\mathrm{AdS}_2} + \underbrace{\mathrm{d}\mathbf{x}^2}_{\mathbb{R}^d}, \quad (r \approx r_+)$$

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- local criticality at every point in space! $z = \infty$
- note: similar $S \sim T^0$ in SYK model; understood from the many-body energy spacings

$$\Delta E \sim \begin{cases} \mathrm{e}^{-\alpha N} & \text{SYK, AdS-RN?} \\ N^{-\lambda} & \text{quasiparticles} \end{cases}, \text{ when } E - E_{\mathrm{gs}} \sim N^{-1}$$

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at finite momentum k at low T,

$$m_{\text{eff}}^2 \sim m^2 + k^2\frac{\gamma^2}{\mu^2 L^2}$$

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- in general, IR geometry controls measurable properties if $\mu > T$ and $k, \omega < \mu$

- recall that $m^2 < 0$ is OK for scalar ϕ:

$$m^2 = \frac{\Delta(d+1-\Delta)}{L^2} \implies m^2 \geq -\frac{(d+1)^2}{4L^2}.$$

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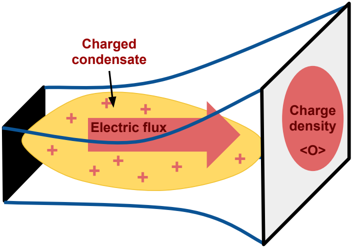
- condensate of ϕ will form (superfluid!):

[Hartnoll, Herzog, Horowitz; 0803.3295]



- in $d = 2$, algebraic long range order for $T > 0$:

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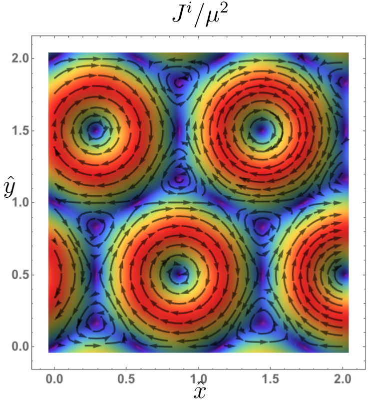
- this calculation requires *quantum gravity* in the bulk – very hard! [Anninos, Hartnoll, Iqbal; 1005.1973]

- complicated bulk action + numerics = spontaneous formation of "lattices", charge/magnetization density waves, etc...



[Donos, Gauntlett; 1512.06861]

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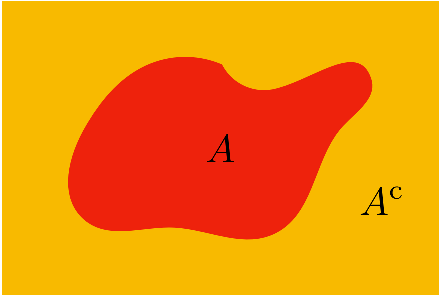

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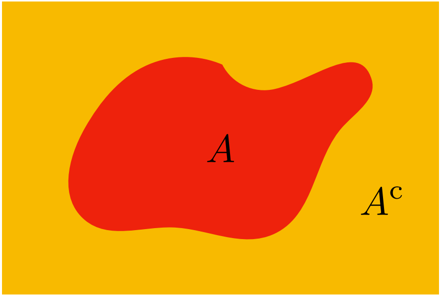
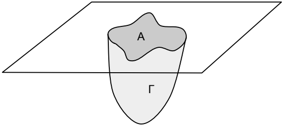


- entanglement entropy

$$S_A = -\operatorname{tr}\left(\rho_A \log \rho_A\right)$$

- for a static geometry, entanglement entropy of region A is (regularized) *minimal area of membrane*:

$$S_A = \min \frac{\mathrm{Area}(\Gamma_A)}{4G_\mathrm{N}} + \mathrm{O}\left(\frac{1}{N}\right)$$



[Ryu, Takayanagi; hep-th/0603001]

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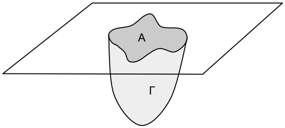

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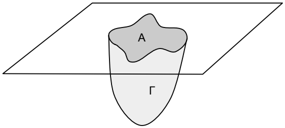

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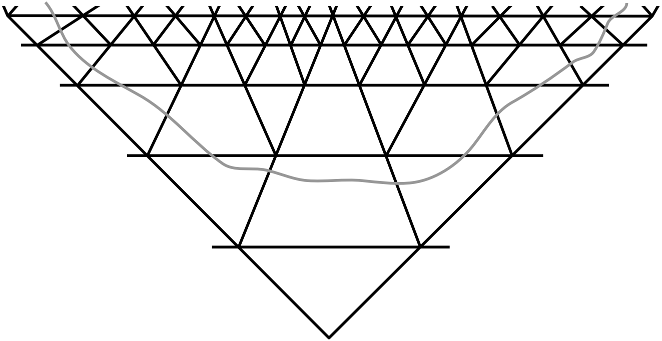
- can generalize to time-dependent problems

- technical point: AdS geometry appears qualitatively identical to MERA network: [Swingle; 0905.1317]



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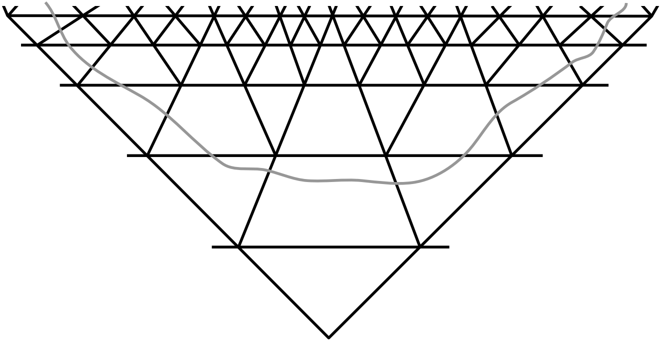

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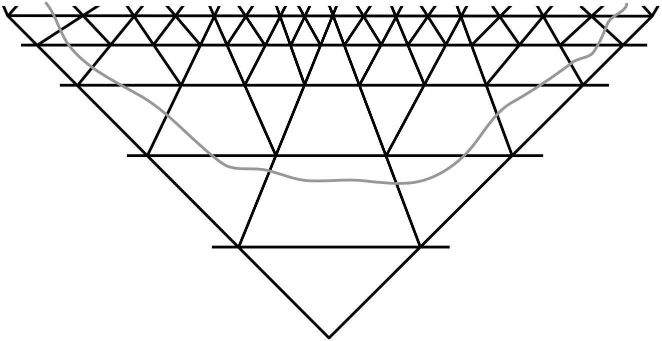

- entanglement entropy = number of broken bonds
- conjecture: building geometry from entanglement structure?