

## AP Calculus 2.1

### Rates of Change: Federal Public School Spending

#### Lesson Objectives

- SWBAT analyze public education budget data to understand the difference between average and instant rate of change
- 

#### Next Generation Science Standard

- N/A
- 

#### STEM Rationale for Lesson:

AP Calculus Standard: CHA-2.A: Determine average rates of change using different quotients

Students must understand the difference between the Average Rate of Change and the Instant Rate of Change in order to conceptualize a derivative measurement. This lesson assesses students' prior knowledge of describing data sets and slope, and then builds the understanding that change in data is described differently depending on specific intervals. In the next lesson, students will derive the formal limit definition of the derivative by making the interval approach a width of zero.

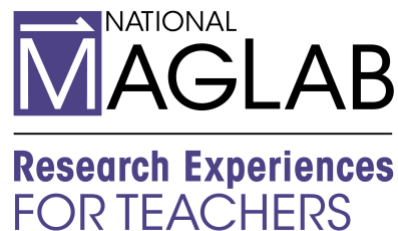
Other key understandings:

- Students will understand that the average rate of change estimates the slope of a curve over a period of time. As the endpoints of the intervals get closer together (forcing smaller intervals of time) we can get a more accurate estimate of the change happening at a single moment.
- The average rate of change tells us an overall trend of change, while the instant rate of change tells us the change in a shorter time period (or even a single moment).
- Individual government decisions can drastically change the overall trend over time. Media can tell a story different ways depending on which endpoints they choose.

#### Culturally responsive connection:

# National MagLab

## STEM Lesson Plan:



This lesson allows students to analyze public education budget data for trends over time. All of my students currently attend a public school and therefore all are affected by this data, and many have been affected by it over time. The lesson pushes students to think about whether or not this data affects different races and regions differently, and asks them to think about ways in which their elected officials impact the amount of money they personally receive and how that affects their overall community. It also gives students the opportunity to learn more about their parents' experiences with public education in the extension homework activity, which may or may not have taken place in the US.

---

### Materials Needed:

Provided by Teacher (Available in Appendix A):

1. [Discussion Prep Worksheet](#)
  2. [Slides](#)
- 

### Activate Prior Knowledge:

1. Students will be shown the first graph of the education budget and asked to participate in [Project Zero's See, Think, Wonder](#) thinking routine. Instructors should emphasize the terms "increasing, decreasing, maximum, minimum, slope, rate of change, and interval" as students introduce them into discussion.
- 

### Lesson Introduction:

1. Students individually answer the question "How would you describe the TRENDS in the data?". Students will answer independently on a post-it.
  2. Turn and Talk: Share your responses with your partner.
  3. Teacher will gather post-its onto poster paper and read aloud trends. Ask students to share any additional thoughts that they have based on their own responses or the responses shared out.
- 

### Lesson Activity:

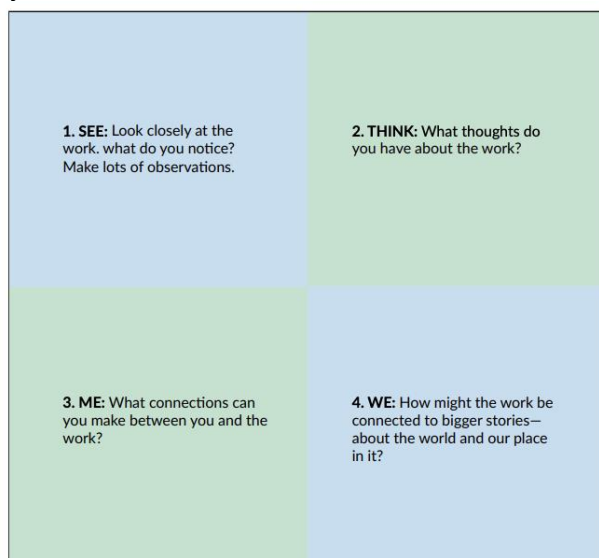
1. Vocabulary: Stop and summarize. What terms are coming up for us in this discussion? Have students share out terms and draw pictures next to them (EL support / stamping academic language expectations). Terms to highlight: increasing / decreasing / maximum / minimum / interval / rate of change / slope / steepness
2. Task: In your groups, work to complete the front page of the task. Prepare to share your thoughts in our group discussion.
3. Group discussion will vary based on student interest, offerings and experiences. As this is an introductory lesson, the purpose of the discussion is to have students connect the data to their own experience; instructor will have to be flexible based on what information

# National MagLab

## STEM Lesson Plan:

students volunteer. Instructor is encouraged to share as well. Some guiding questions could be:

- Do we think that this data affects every student the same in the US? Why or why not?
  - What factors might affect changes in government spending?
  - Who has the power in these decisions?
  - How might this data be used to tell different stories?
  - What story does this data tell you?
  - What personal connection do you have with this data?
  - When was the most drastic change? How do you know?
4. Discussion Summary: Students will answer the following questions as a discussion summary:



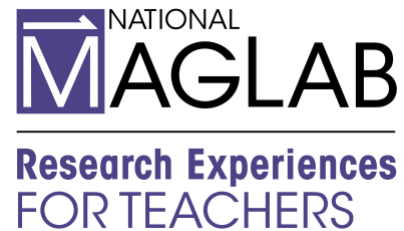
5. Connection to Content: Define the Average Rate of Change as the Derivative
- Practice calculating the average rate of change on different intervals.
  - Practice writing the equation of the tangent line at a given year using the data.
  - *Teacher can choose random years, or ask students to use a year that has special significance to them.*
  - AP Calculus Example Question: Writing the Equation of the Tangent Line

---

## Lesson Assessment

1. Skill Assessment: Khan Academy Practice:
- Average Rate of Change
  - Comparing and Estimating Derivatives

# National MagLab STEM Lesson Plan:

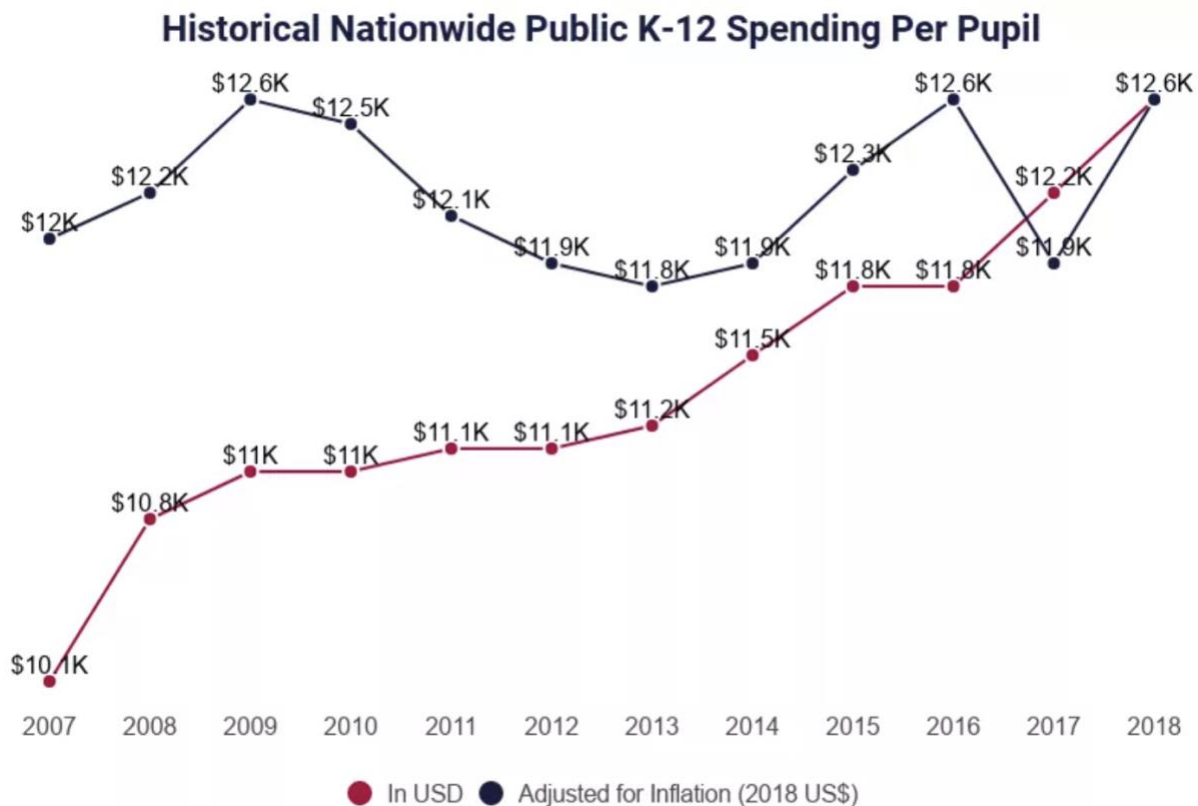


- Writing the Equation of the Tangent Line.
2. Conceptual Understanding (written response):
- How might your understanding of data trends affect the way that you view data in the future?

# National MagLab STEM Lesson Plan:



## Appendix A: Discussion Prep Worksheet:



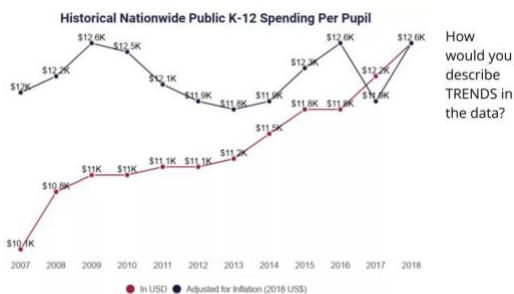
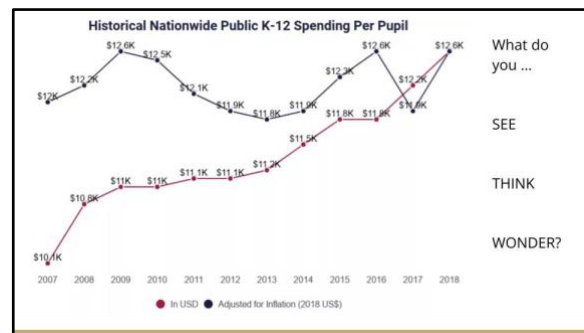
1. What do you SEE? THINK? WONDER?
2. What TRENDS do you see in the data?
3. What is the OVERALL NET CHANGE PER PUPIL in the data from 2007 to 2018?
4. What is the OVERALL CHANGE PER PUPIL PER YEAR in the data from 2007 to 2018?
5. What is the difference between the two previous questions?
6. When was the most significant change in the spending per pupil per year? How do you know?

# National MagLab STEM Lesson Plan:

## Slides:

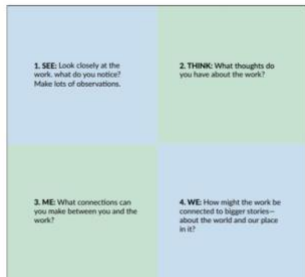
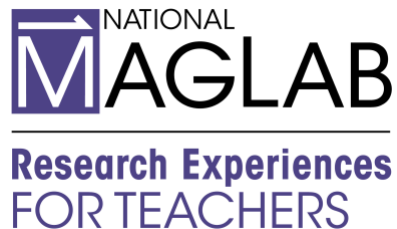
<p><b>1. SEE:</b> Look closely at the work. what do you notice? Make lots of observations.</p>	<p><b>2. THINK:</b> What thoughts do you have about the work?</p>
<p><b>3. ME:</b> What connections can you make between you and the work?</p>	<p><b>4. WE:</b> How might the work be connected to bigger stories—about the world and our place in it?</p>

2.1 -  
Understanding the  
Derivative



What vocabulary is coming up for us in this discussion?

# National MagLab STEM Lesson Plan:



## Big Idea #1: Change

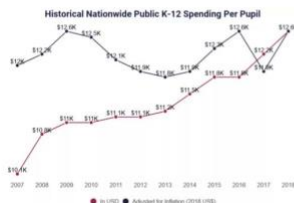
How can a state determine the rate of change in high school graduates at a particular level of public investment in education (in graduation per dollar) based on a model for the number of graduates as a function of the state's education budget?

What vocabulary is coming up for us in this discussion?

Average Rate of Change:

Instant Rate of Change:

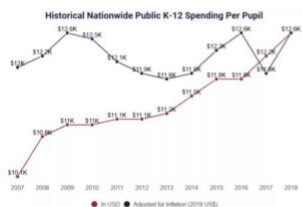
### Practice: Average Rate of Change:



What was the average change in public education spending from the years 2007 to 2018?

What was the average change in public education spending from the years 2009 to 2013?

### Practice: Average Rate of Change:

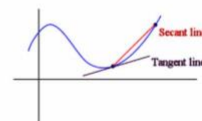


What was the average change in public education spending from the years 2017 to 2018?

What do we notice about the differences in the rates of change we just calculated? How might they tell different stories?

## THE AVERAGE RATE OF CHANGE

The AVERAGE RATE OF CHANGE is an ESTIMATE of the slope at a given point, otherwise known as the SLOPE of the SECANT LINE



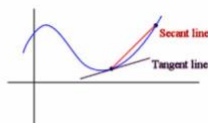
# National MagLab STEM Lesson Plan:



## THE DERIVATIVE

The DERIVATIVE is the INSTANT RATE OF CHANGE of a function, which is the SLOPE of the TANGENT LINE

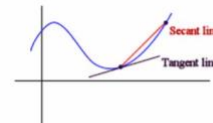
$$y'(x) \quad dy / dx$$



## THE DERIVATIVE

The DERIVATIVE is the INSTANT RATE OF CHANGE of a function, which is the SLOPE of the TANGENT LINE

How might we get the most accurate rate of change for a given year?



### Writing the Equation of the Tangent Line

$$y - y_1 = m(x - x_1)$$

### Review: Writing the Equation of a Line in Point-Slope

Write the equation of the tangent line that represents the change in the budget in the year 2010.

Based on this equation, what might you expect the budget to be in 2018?

### Review: Writing the Equation of a Line in Point-Slope

Write the equation of the tangent line that represents the change in the budget in the year 2010.

Based on this equation, what might you expect the budget to be in 2018?

### Review: Writing the Equation of a Line in Point-Slope

Write the equation of the tangent line that represents the change in the budget in the year 2015.

Based on this equation, what might you expect the budget to be in 2018?

## Practice: Khan Academy

[Skill #1: AROC](#)

[Skill #2: Comparing & Estimating derivatives](#)

[Skill #3: Writing the Equation of Tangent Lines](#)

**Lesson by:** Katherine Risbrough