A non-Fermi liquid: Quantum criticality of metals near the Pomeranchuk instability

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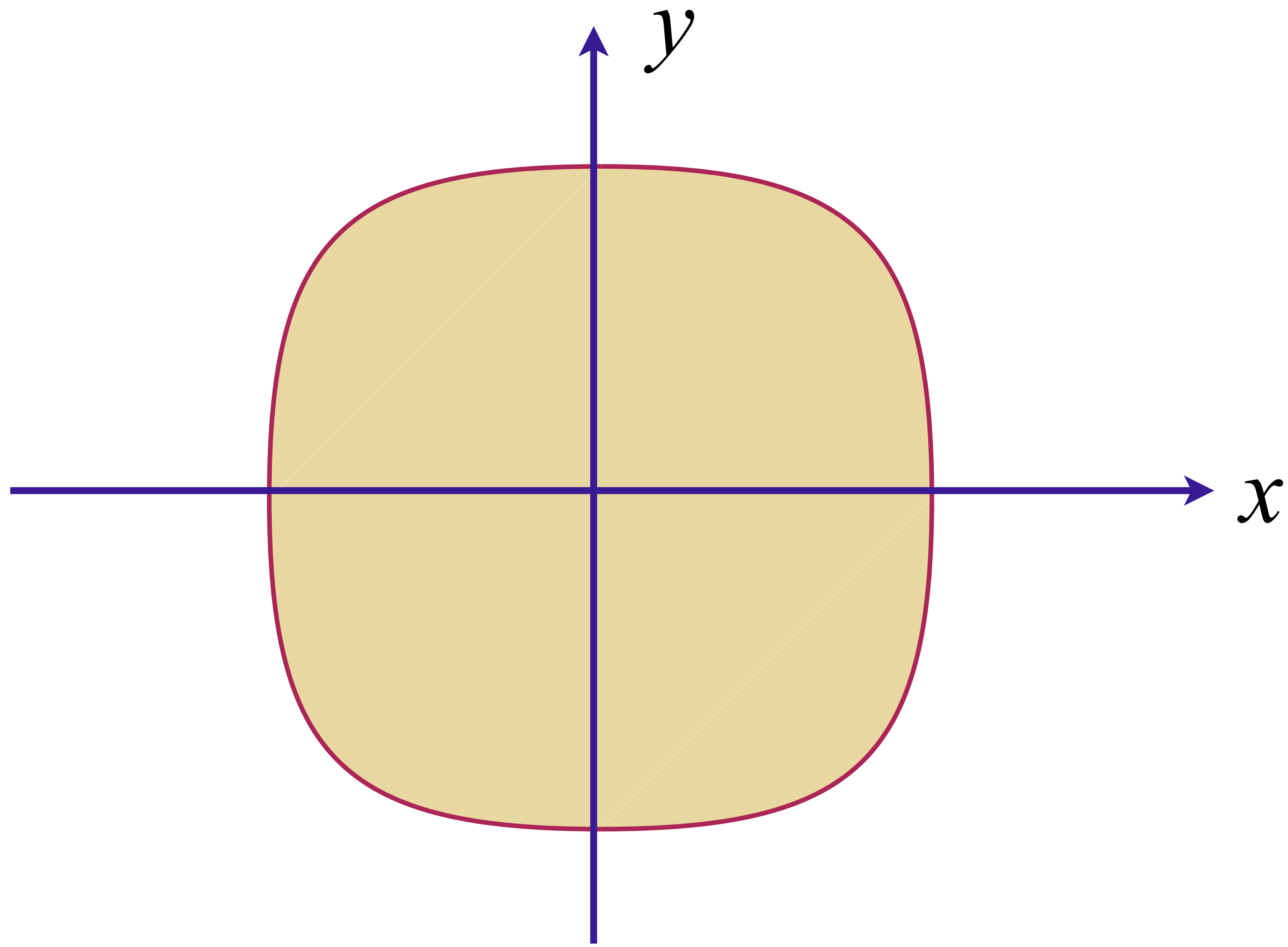
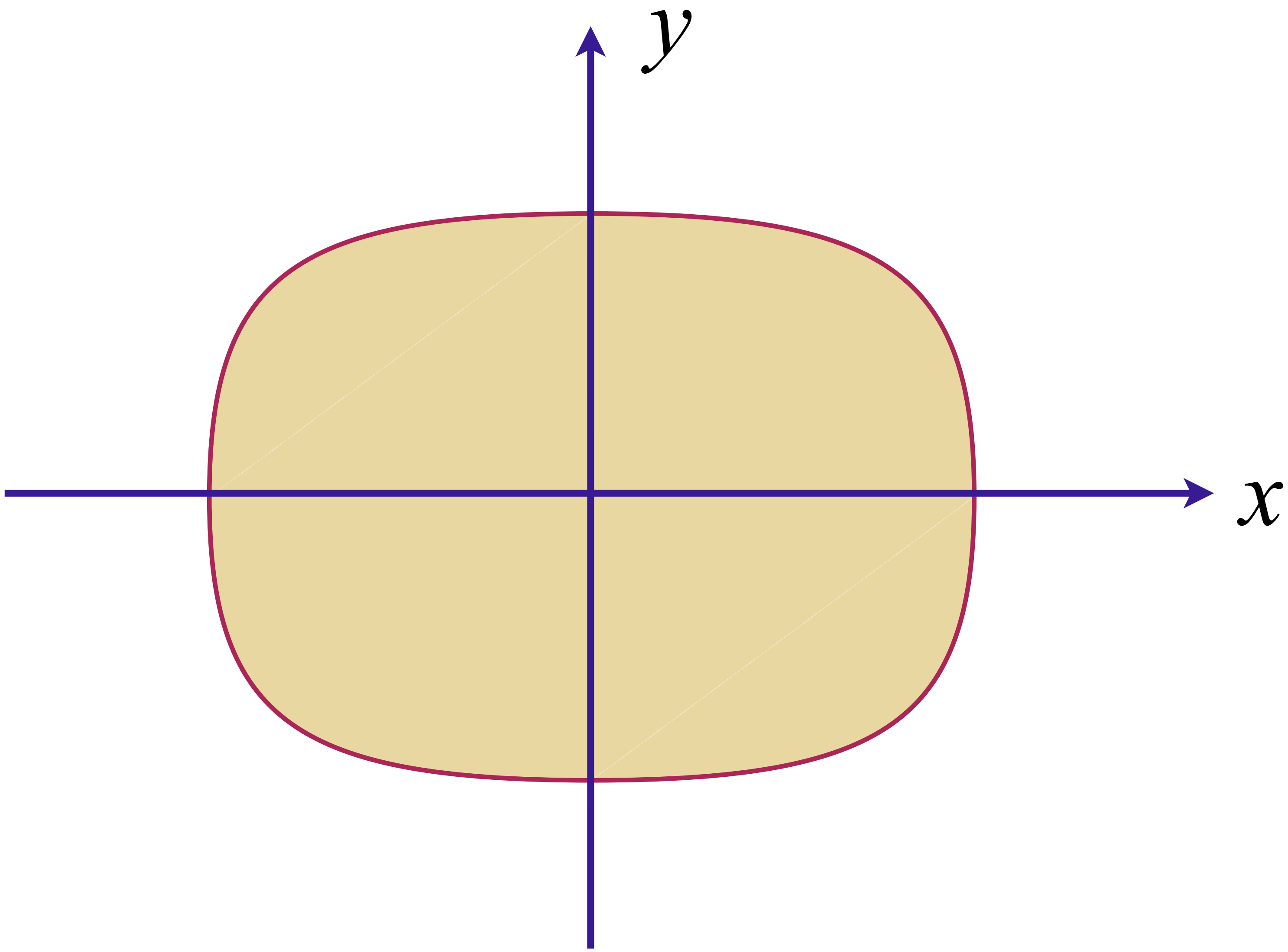
Quantum criticality of Ising-nematic ordering



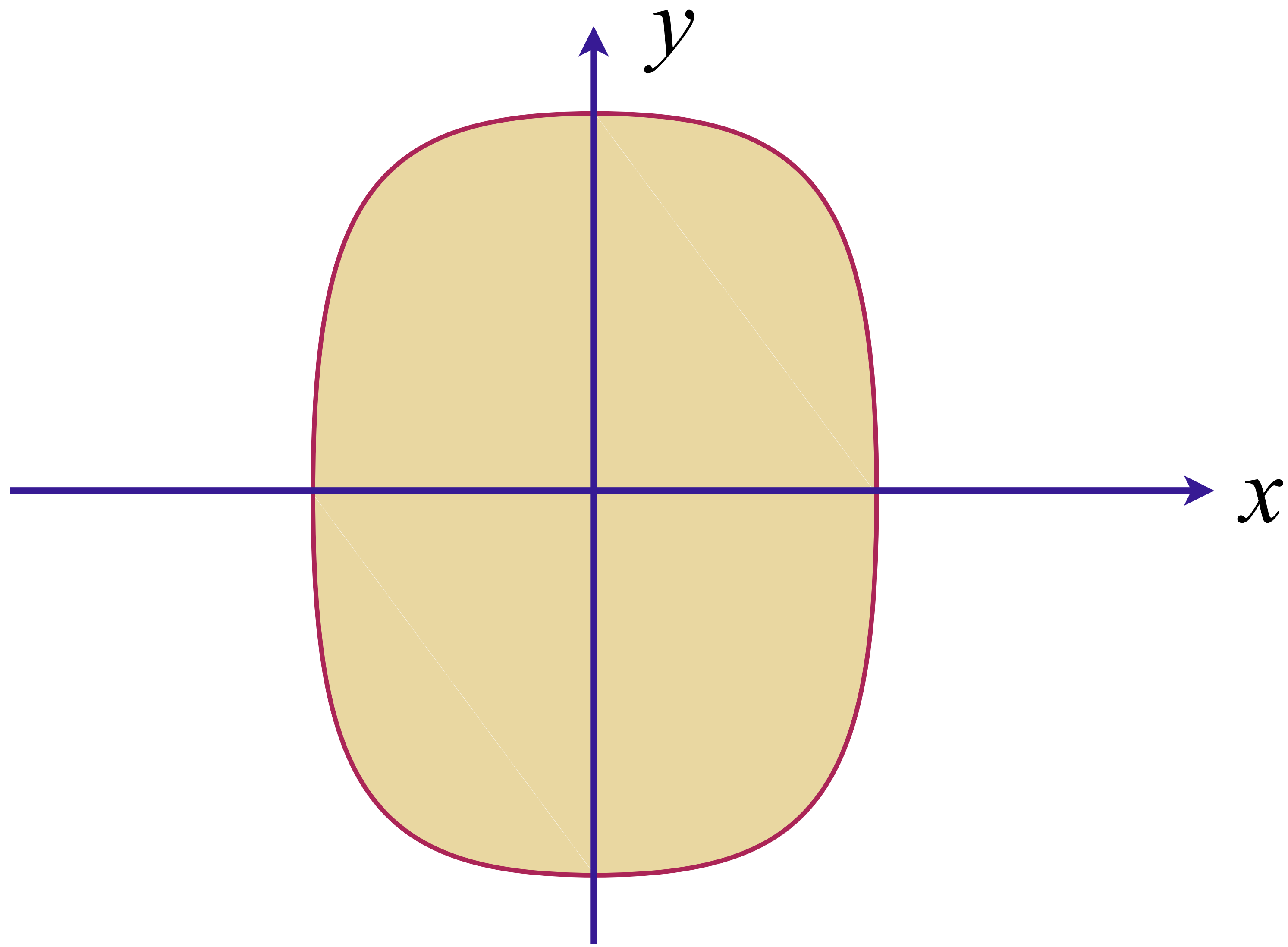
Fermi surface with full square lattice symmetry

Quantum criticality of Ising-nematic ordering



Spontaneous elongation along x direction:

Quantum criticality of Ising-nematic ordering



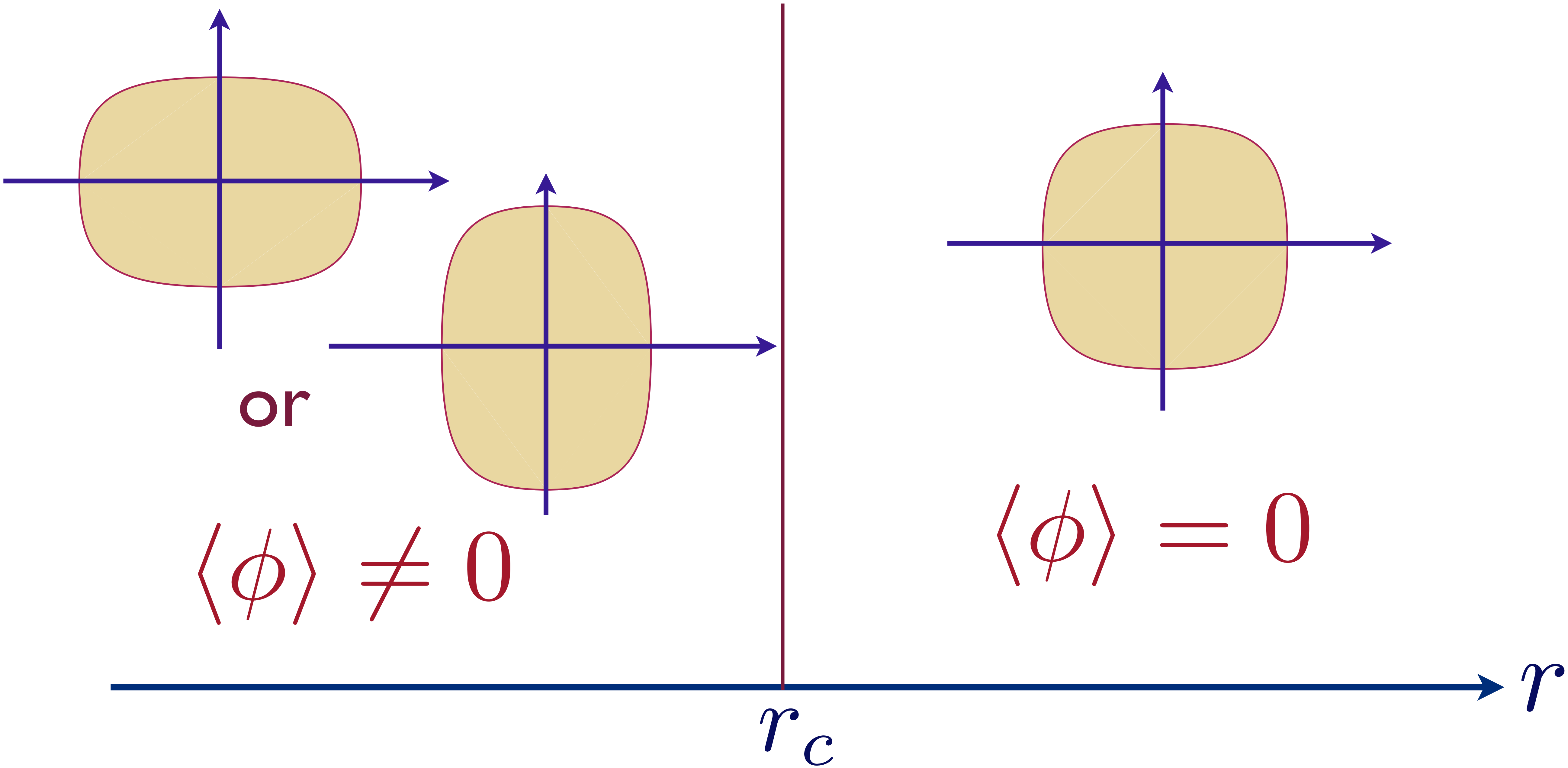
Spontaneous elongation along y direction:

Ising-nematic order parameter

$$\phi \sim \int d^2 k \, (\cos k_x - \cos k_y) \, c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$

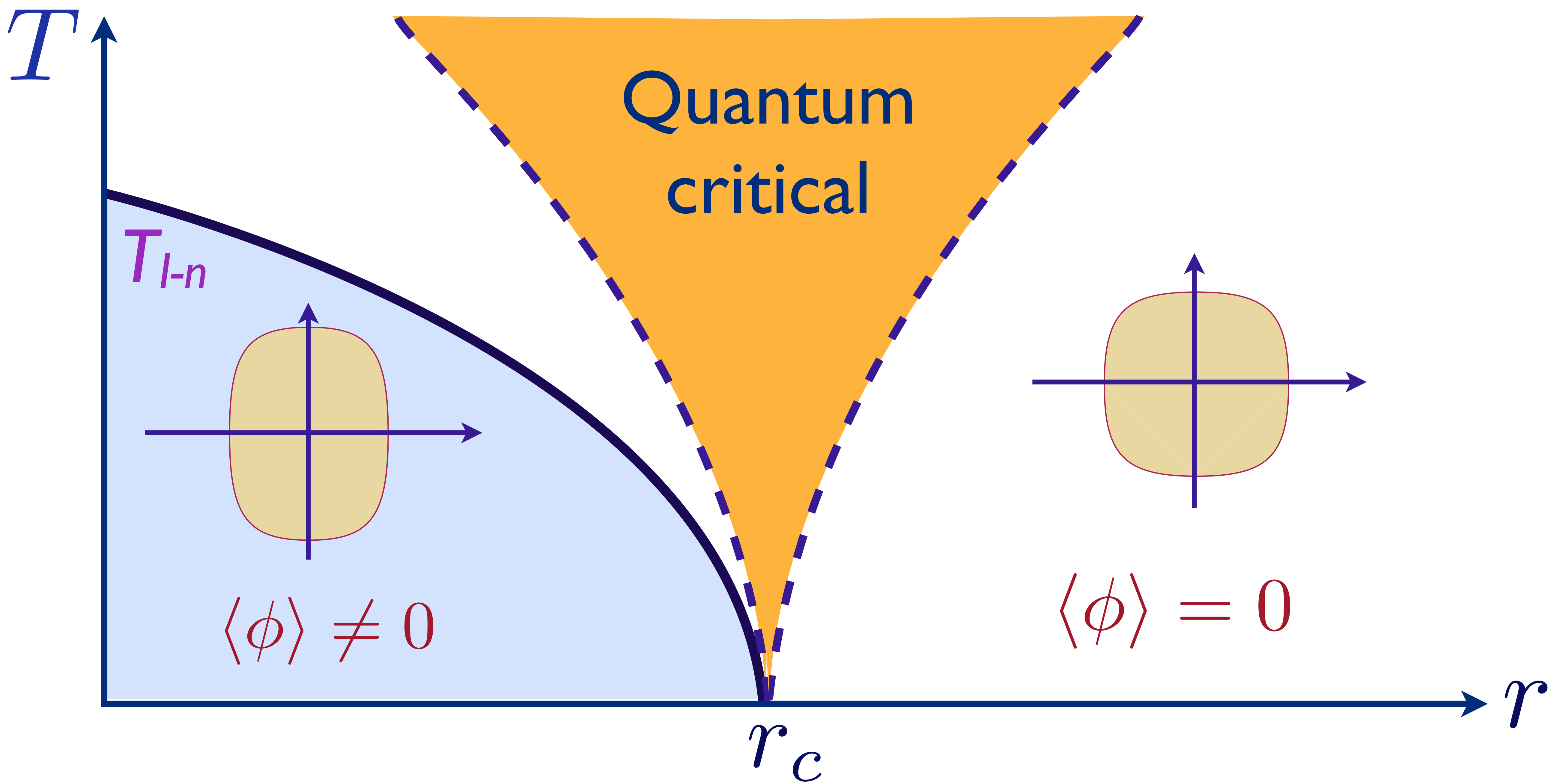
Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

Quantum criticality of Ising-nematic ordering



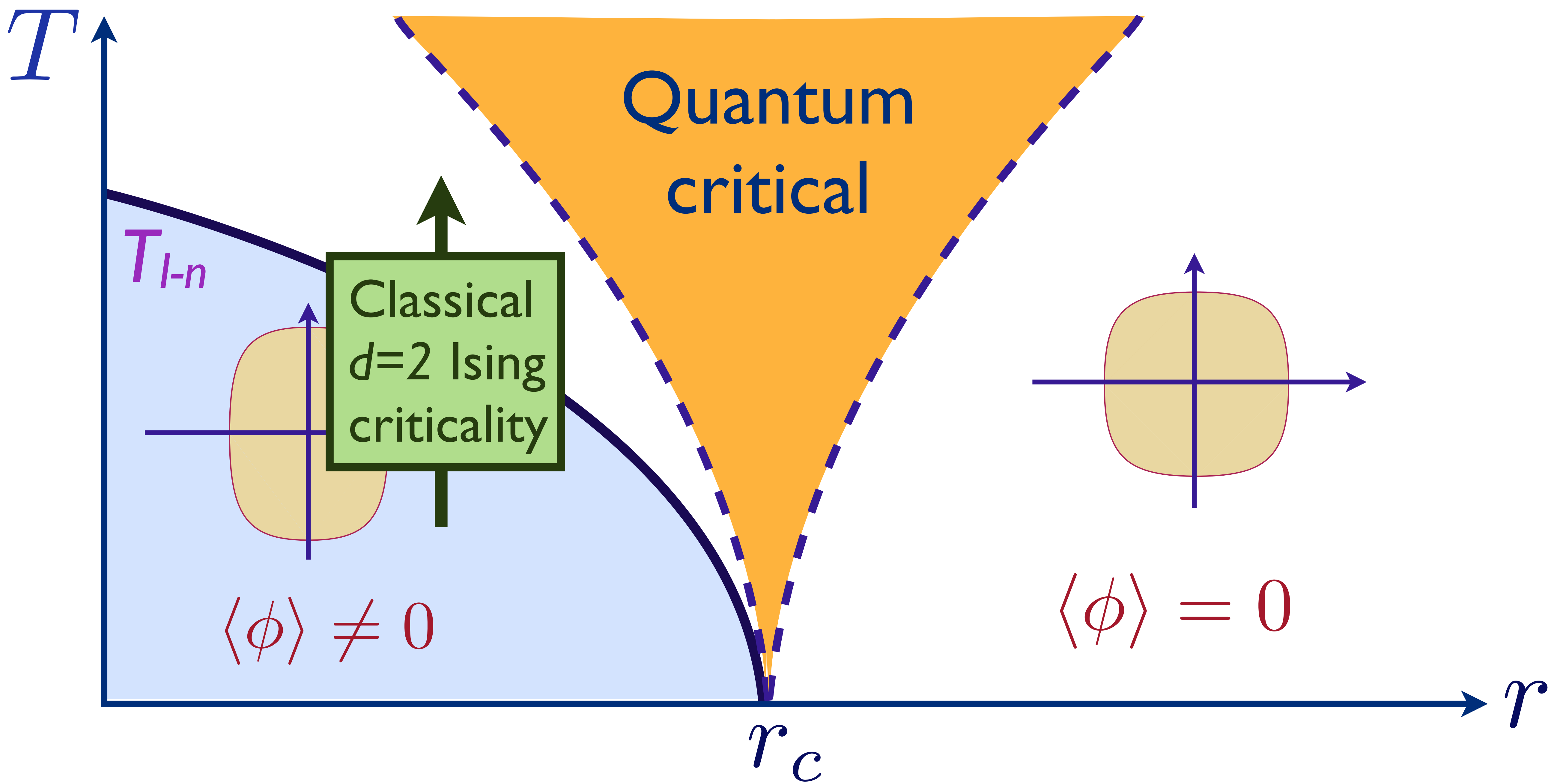
Pomeranchuk instability as a function of coupling r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



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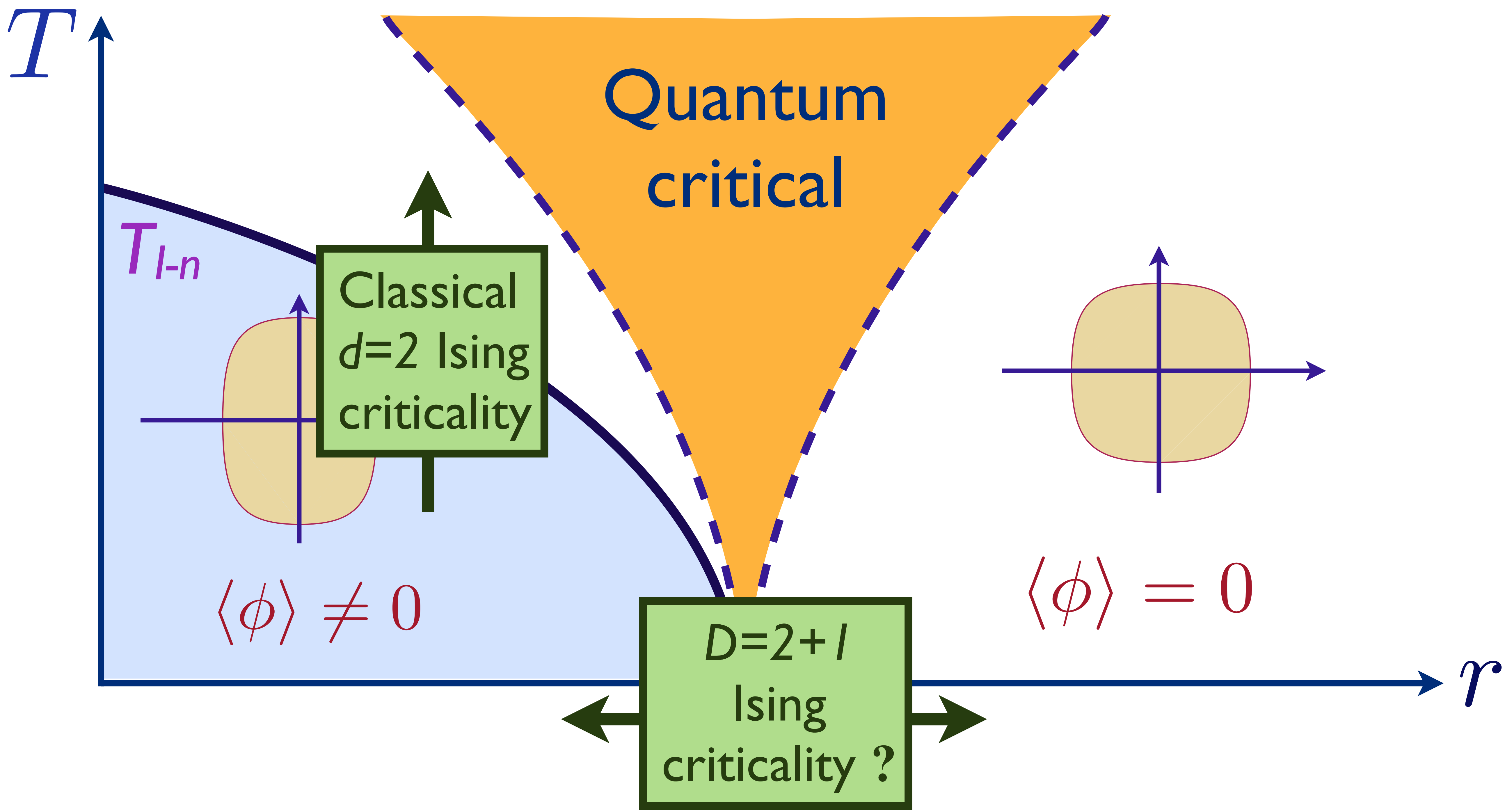

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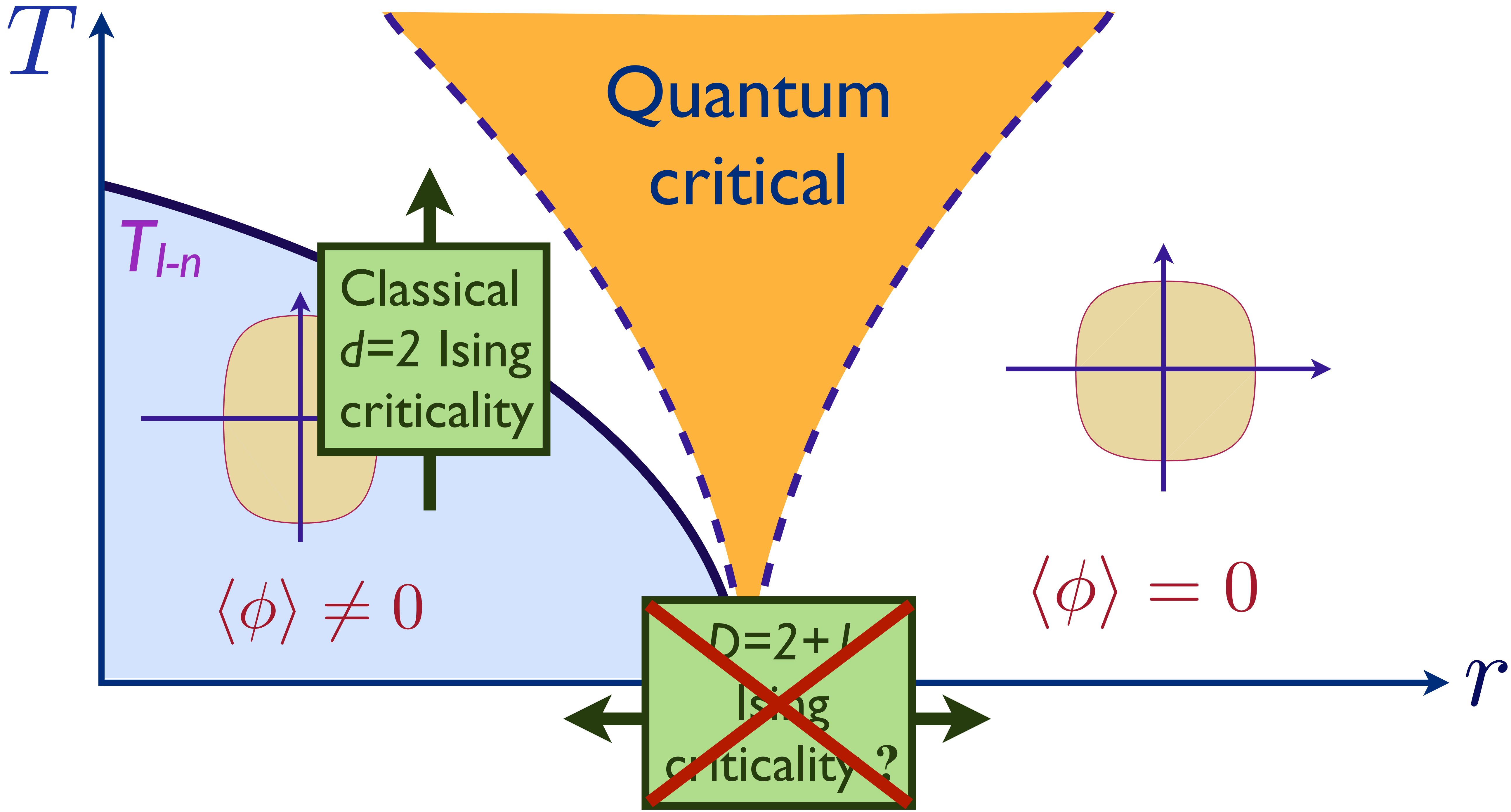

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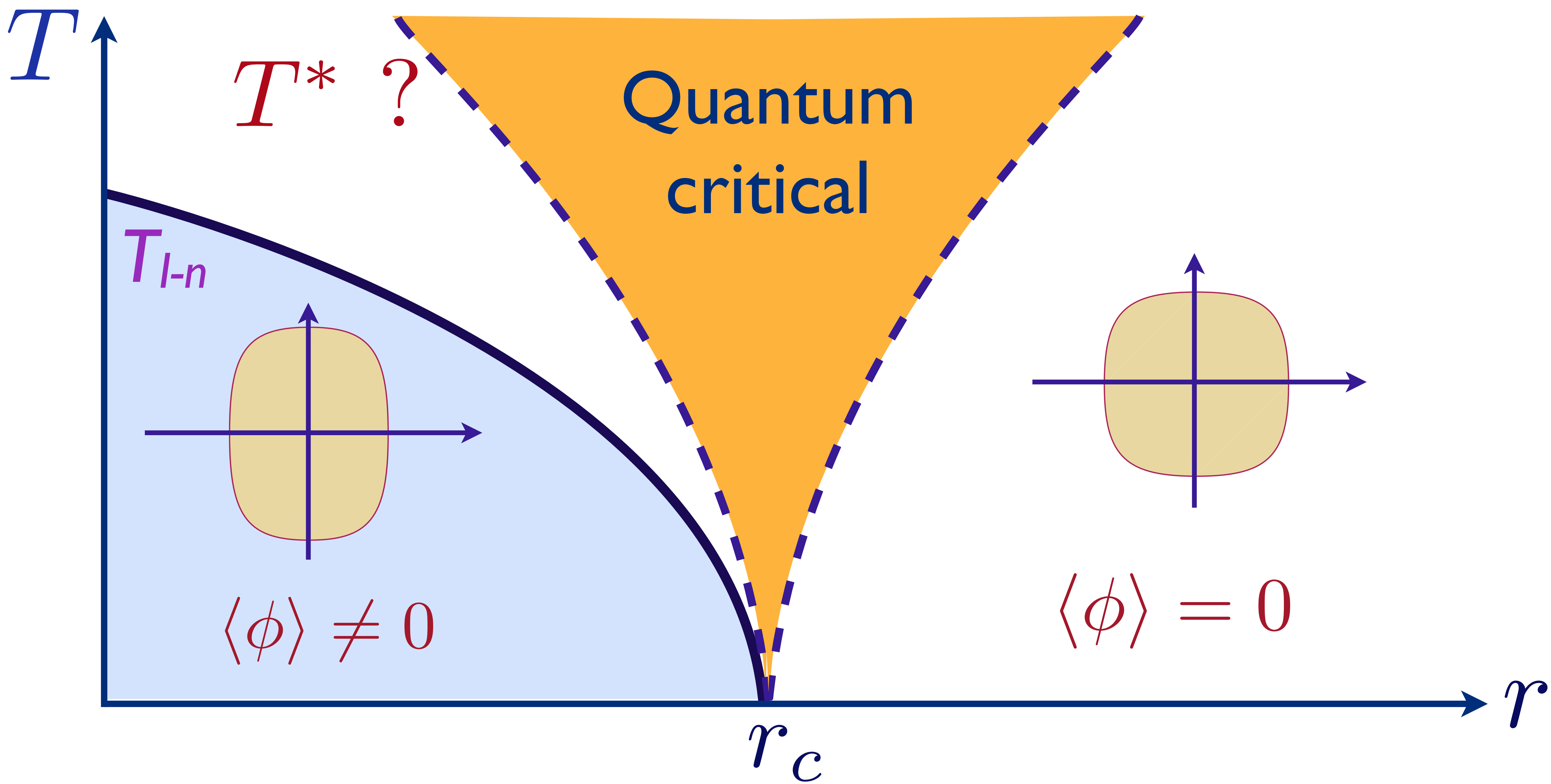

Phase diagram as a function of T and r

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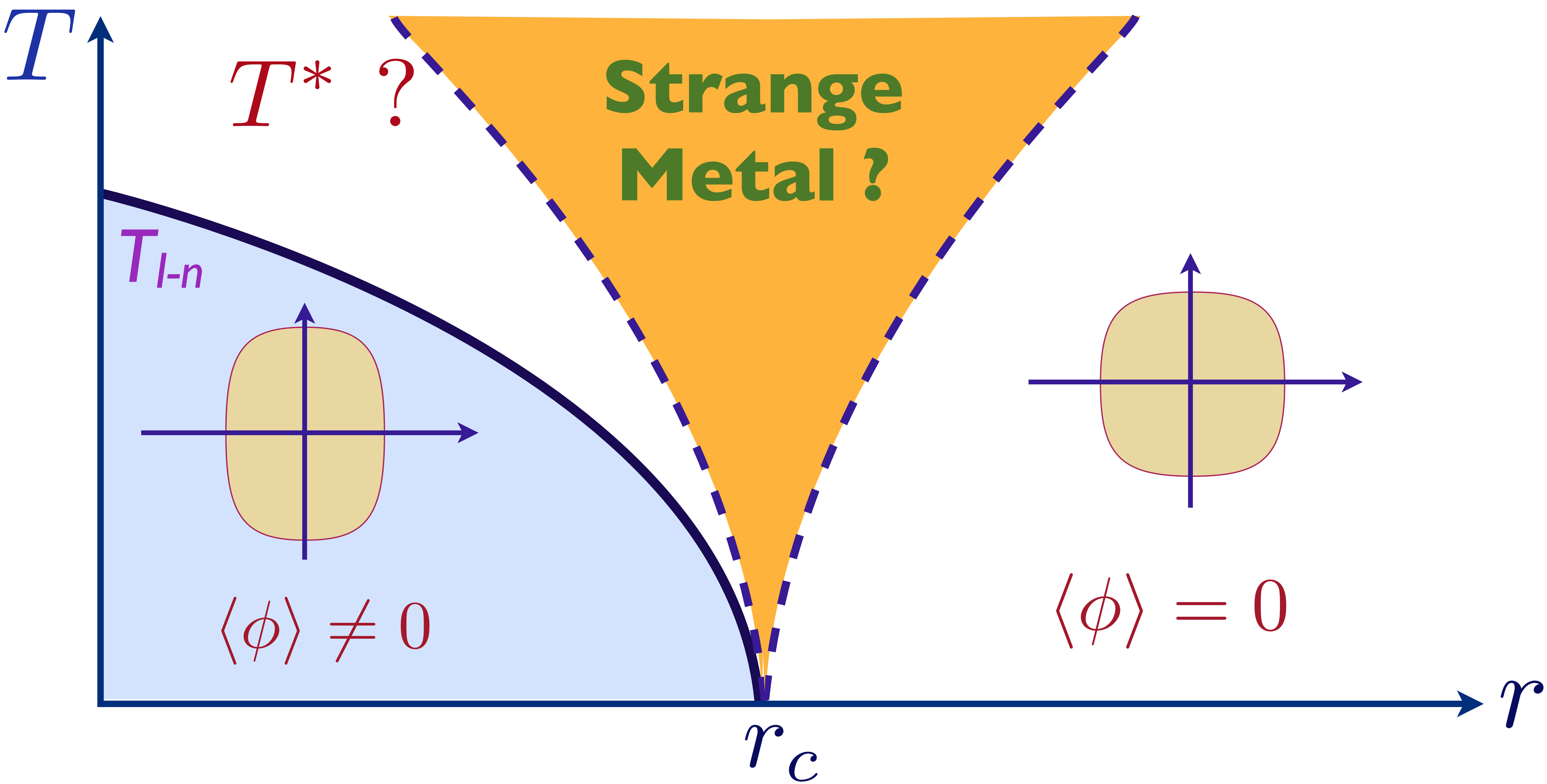

Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

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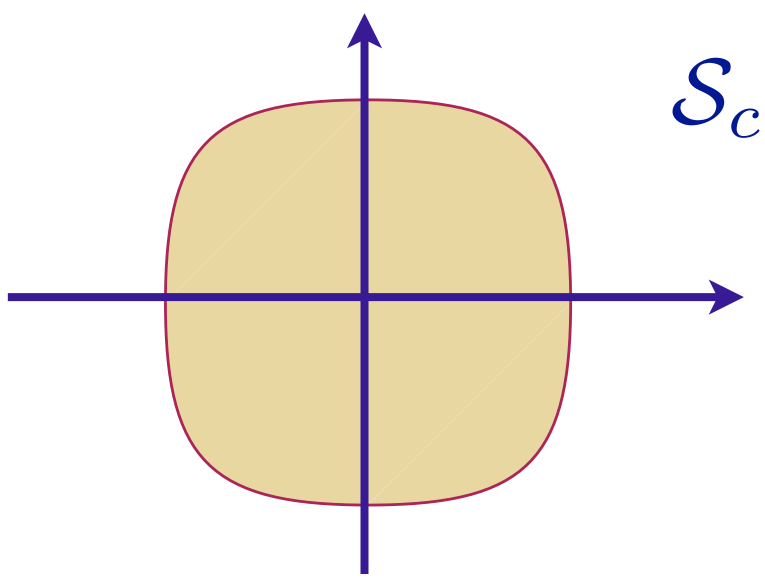
Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

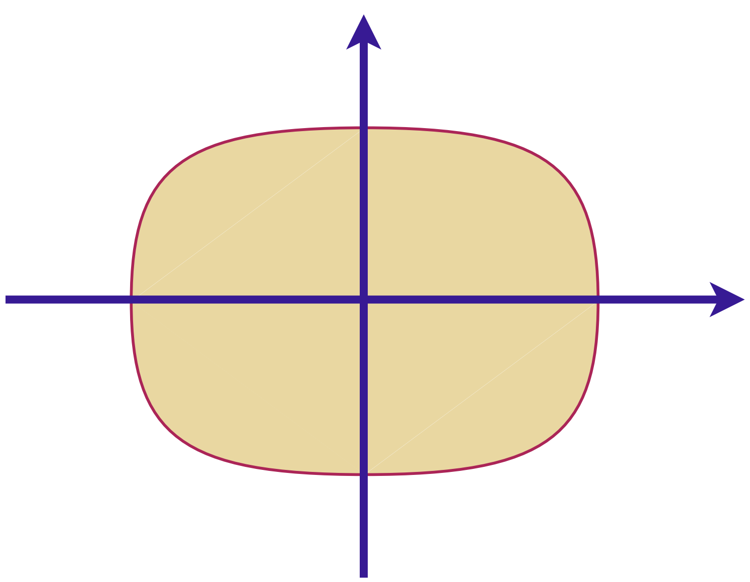
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Ising-nematic ordering

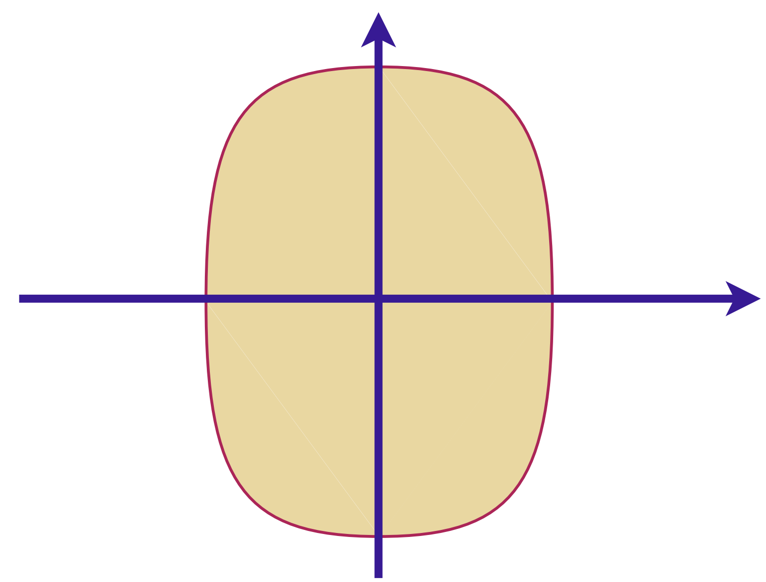
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^\dagger_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



$\langle \phi \rangle > 0$



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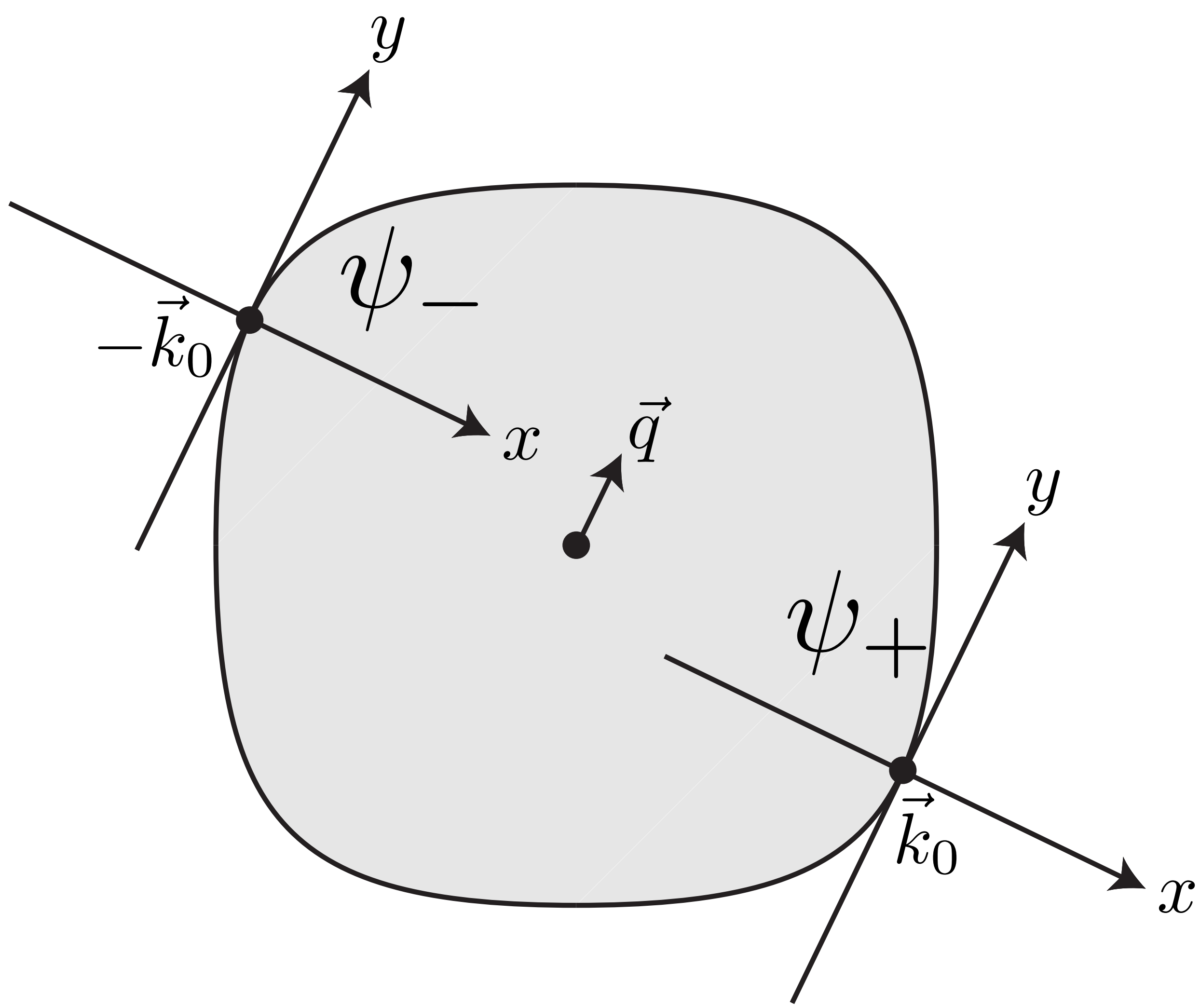
Quantum criticality of Ising-nematic ordering

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

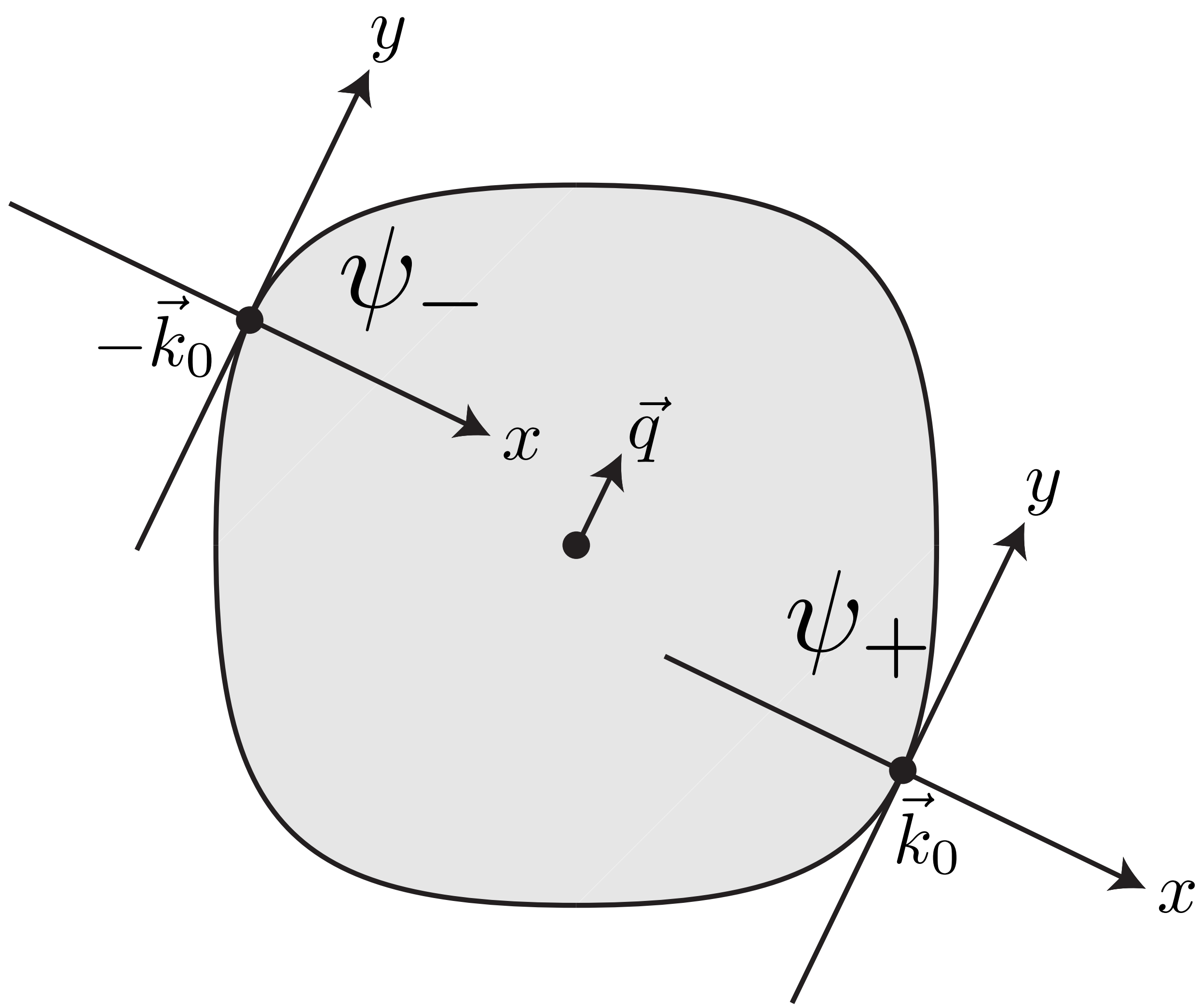
$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

Quantum criticality of Ising-nematic ordering



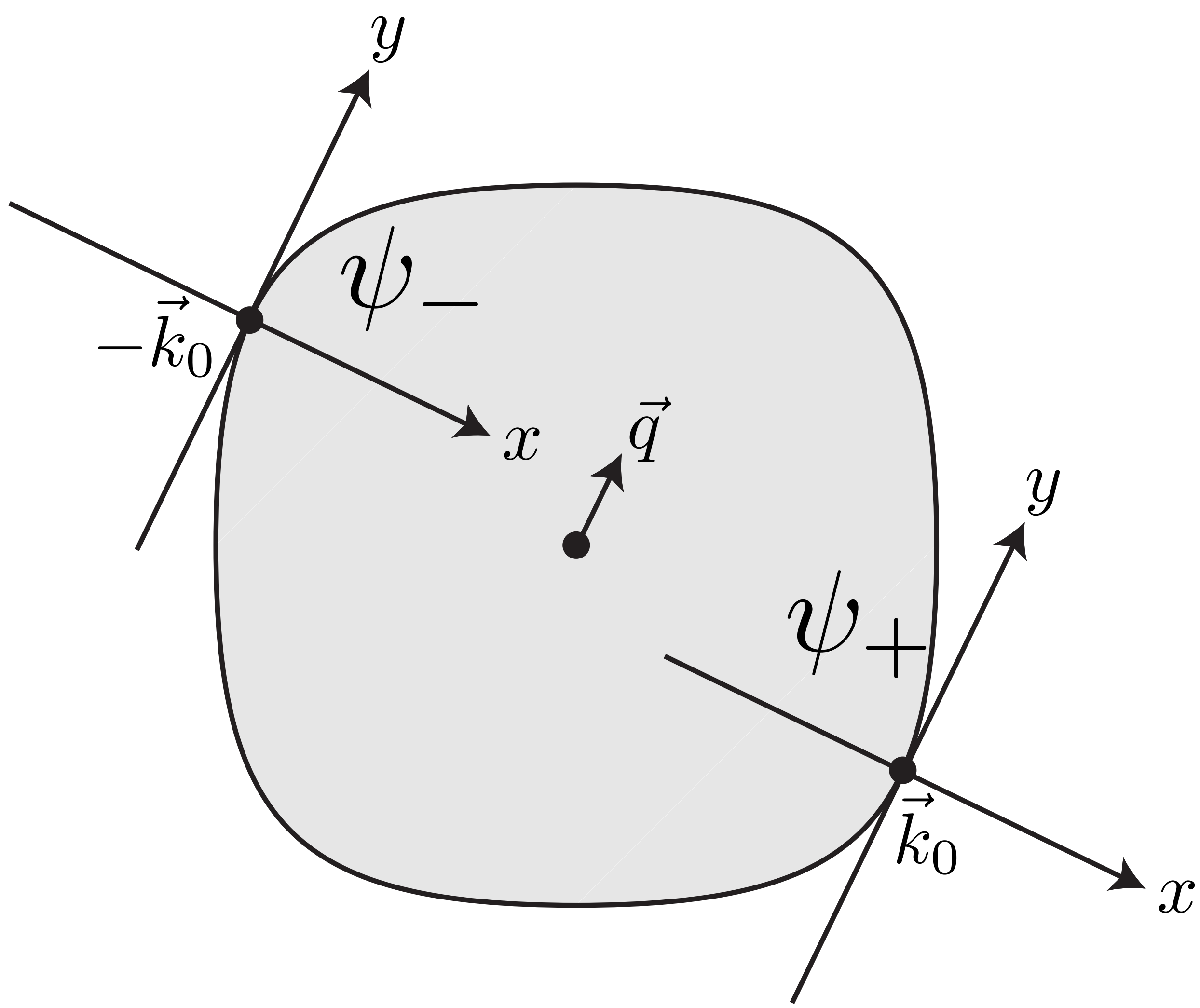
- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

Quantum criticality of Ising-nematic ordering



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Quantum criticality of Ising-nematic ordering



$$\mathcal{L}[\psi_\pm, \phi] =$$

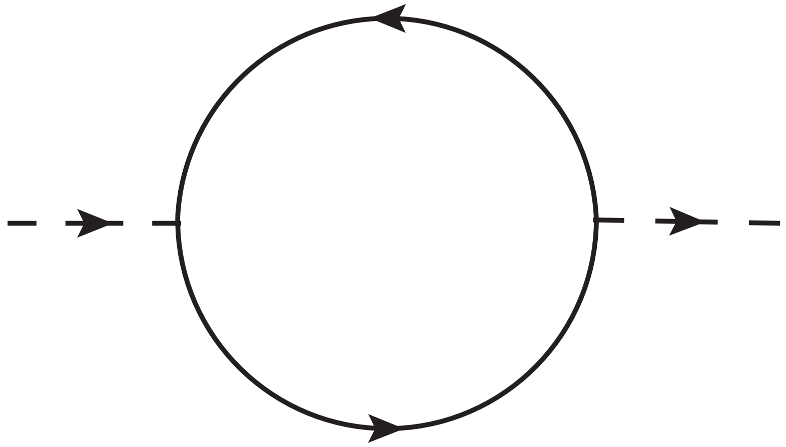
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$



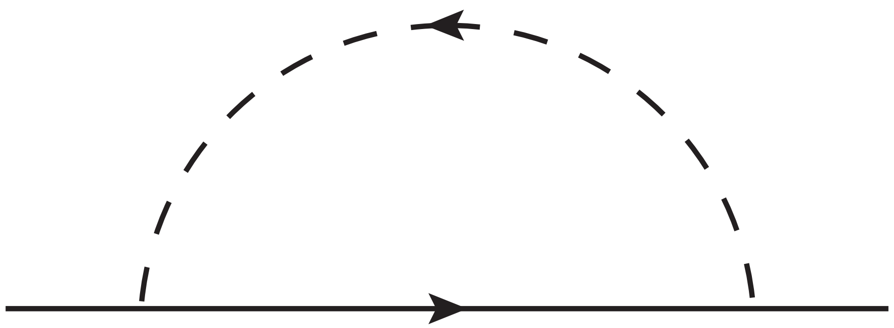
One loop ϕ self-energy with N_f fermion flavors:

$$\Sigma_\phi(\vec{q}, \omega) = N_f \int \frac{d^2k}{4\pi^2} \frac{d\Omega}{2\pi} \frac{1}{\left[-i(\Omega+\omega) + k_x + q_x + (k_y+q_y)^2\right]\left[-i\Omega - k_x + k_y^2\right]} = \frac{N_f}{4\pi} \frac{|\omega|}{|q_y|}$$

Landau-damping

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$



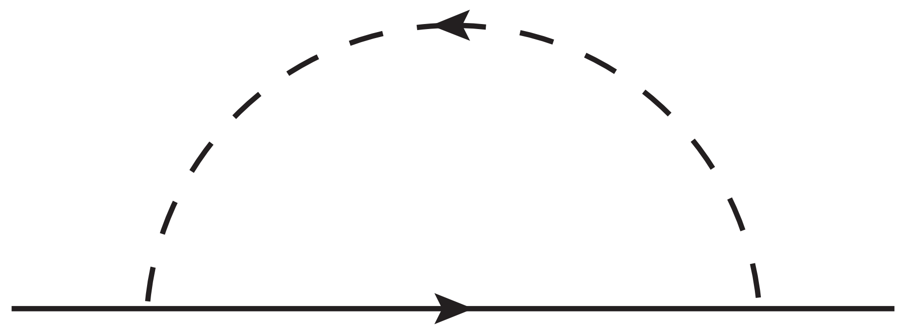
Electron self-energy at order $1/N_f$:

$$\Sigma(\vec{k},\Omega) = -\frac{1}{N_f}\int\frac{d^2q}{4\pi^2}\frac{d\omega}{2\pi}\frac{1}{\left[-i(\omega+\Omega)+k_x+q_x+(k_y+q_y)^2\right]\left[\frac{q_y^2}{g^2}+\frac{|\omega|}{|q_y|}\right]}$$

$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\mathrm{sgn}(\Omega)|\Omega|^{2/3}$$

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$



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$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\text{sgn}(\Omega)|\Omega|^{2/3}$$

$\sim |\Omega|^{d/3}$ in dimension d.

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$

Schematic form of ϕ and fermion Green's functions in d dimensions

$$D(\vec{q},\omega) = \frac{1/N_f}{q_\perp^2 + \dfrac{|\omega|}{|q_\perp|}} \quad , \quad G_f(\vec{q},\omega) = \frac{1}{q_x + q_\perp^2 - i\text{sgn}(\omega)|\omega|^{d/3}/N_f}$$

In the boson case, $q_\perp^2 \sim \omega^{1/z_b}$ with $z_b = 3/2$.
In the fermion case, $q_x \sim q_\perp^2 \sim \omega^{1/z_f}$ with $z_f = 3/d$.

Note $z_f < z_b$ for $d > 2$ $\Rightarrow$ Fermions have *higher* energy than bosons, and perturbation theory in g is OK.
Strongly-coupled theory in $d = 2$.

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$

Schematic form of ϕ and fermion Green's functions in $d = 2$

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_y^2 + \dfrac{|\omega|}{|q_y|}} \quad , \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_y^2 - i\,\mathrm{sgn}(\omega)|\omega|^{2/3}/N_f}$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Quantum criticality of Ising-nematic ordering

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- \\ &\quad - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2\end{aligned}$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering

$$\mathcal{L}_{\rm scaling} = \psi_+^\dagger \left(-i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(+i\partial_x - \partial_y^2\right) \psi_- - g\,\phi \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \left(\partial_y \phi\right)^2$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering

$$\mathcal{L}_{\text{scaling}} = \psi_+^\dagger \left(-i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(+i\partial_x - \partial_y^2\right) \psi_- \\ - g\,\phi \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \left(\partial_y \phi\right)^2$$

Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} \phi &\to \phi\, s^{(2z+1)/4} \\ \psi &\to \psi\, s^{(2z+1)/4} \\ g &\to g\, s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

Quantum critical metals near the onset of antiferromagnetism: superconductivity and other instabilities

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Max Metlitski

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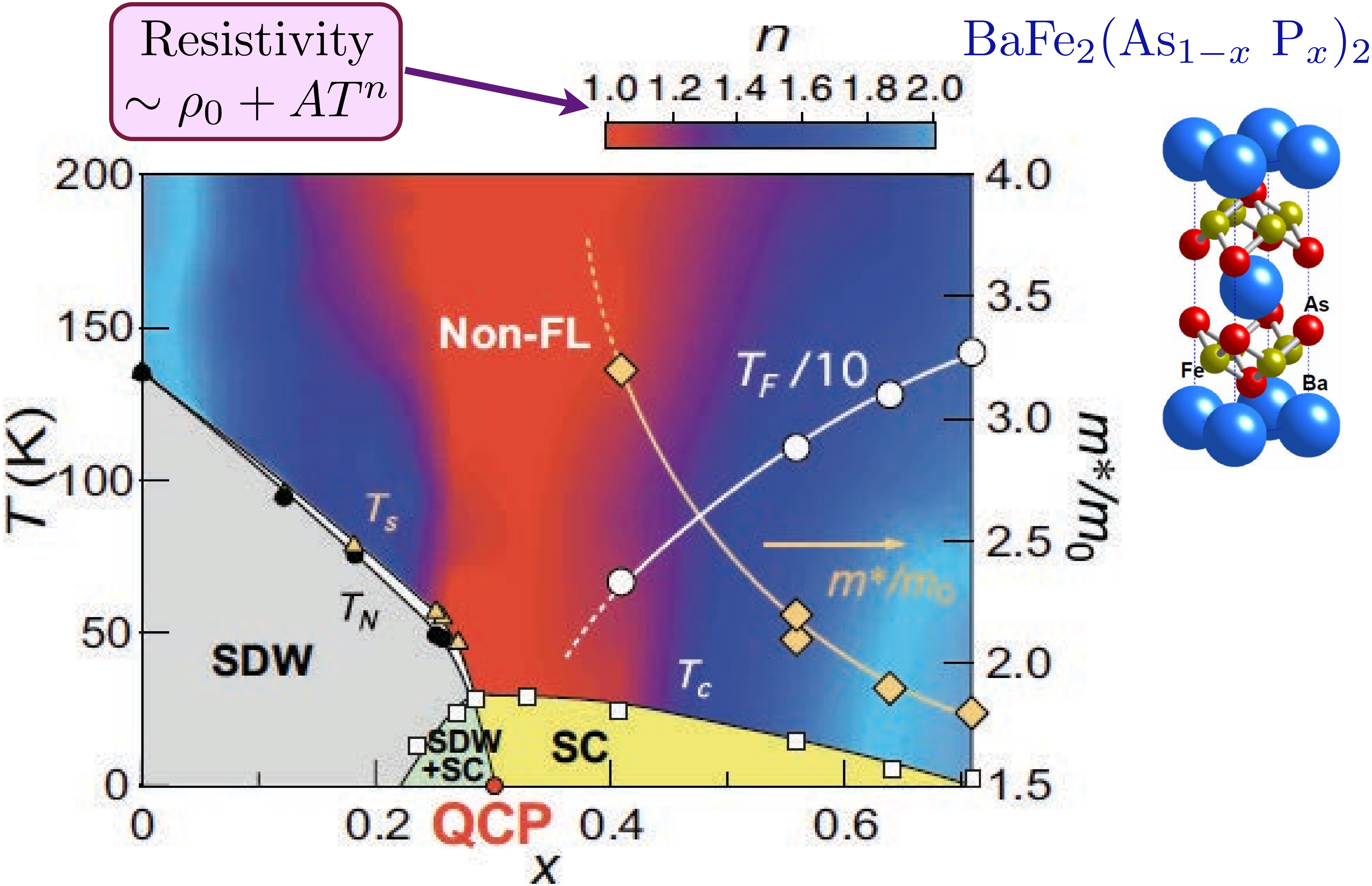

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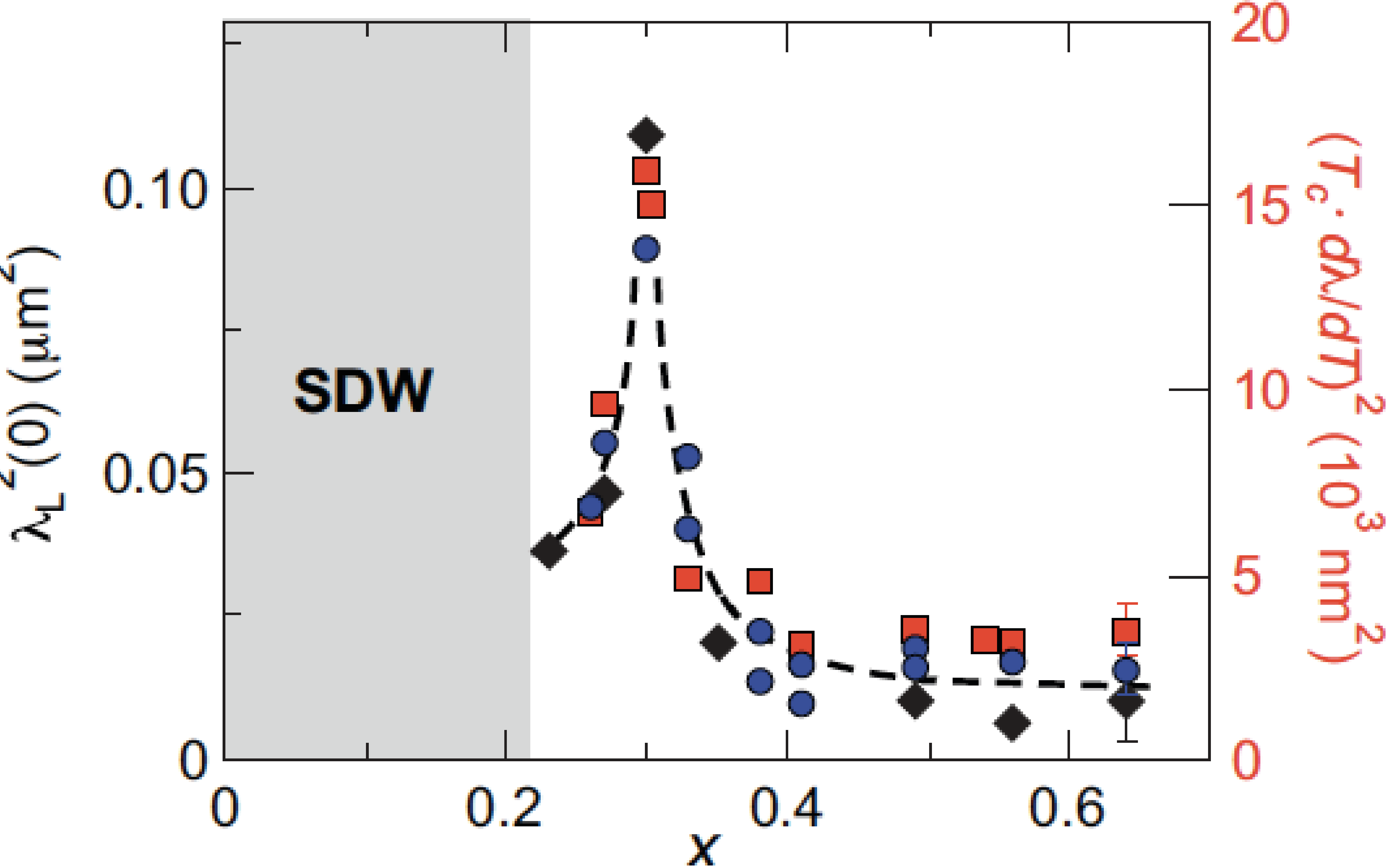
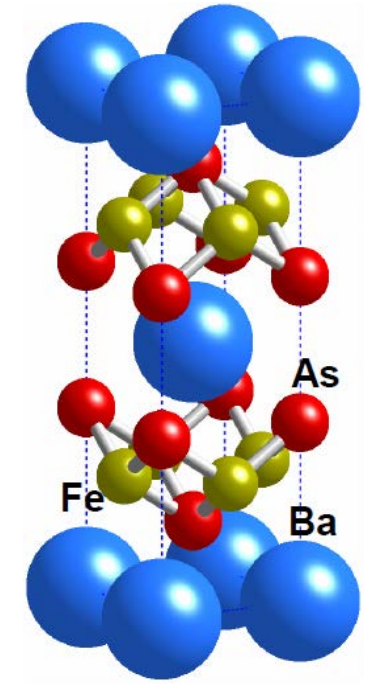
$BaFe_2(As_{1-x}\,P_x)_2$





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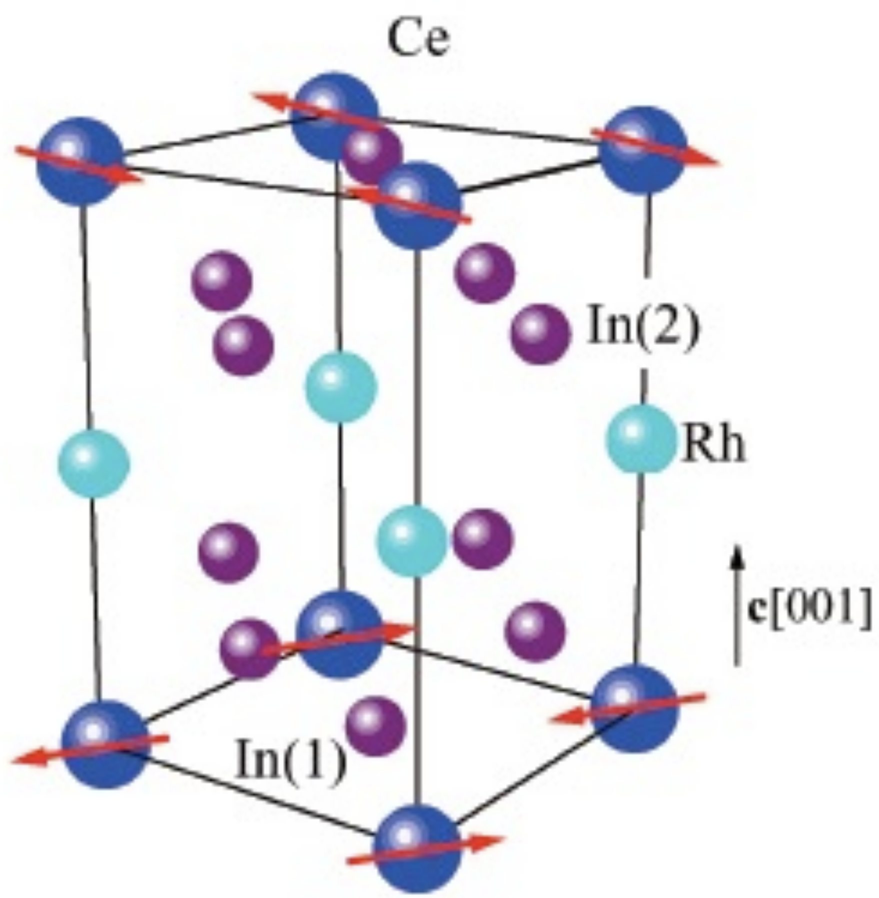
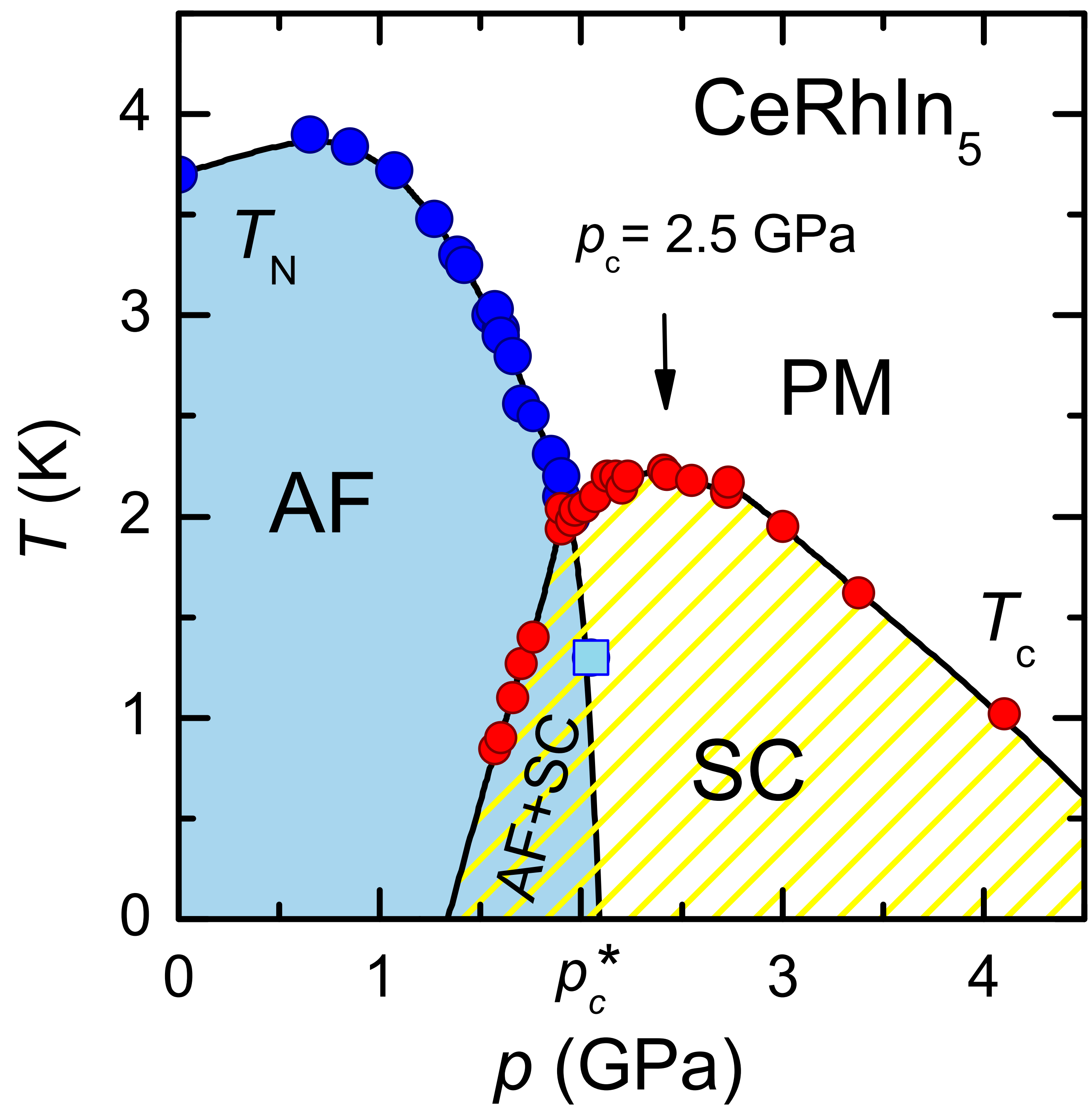
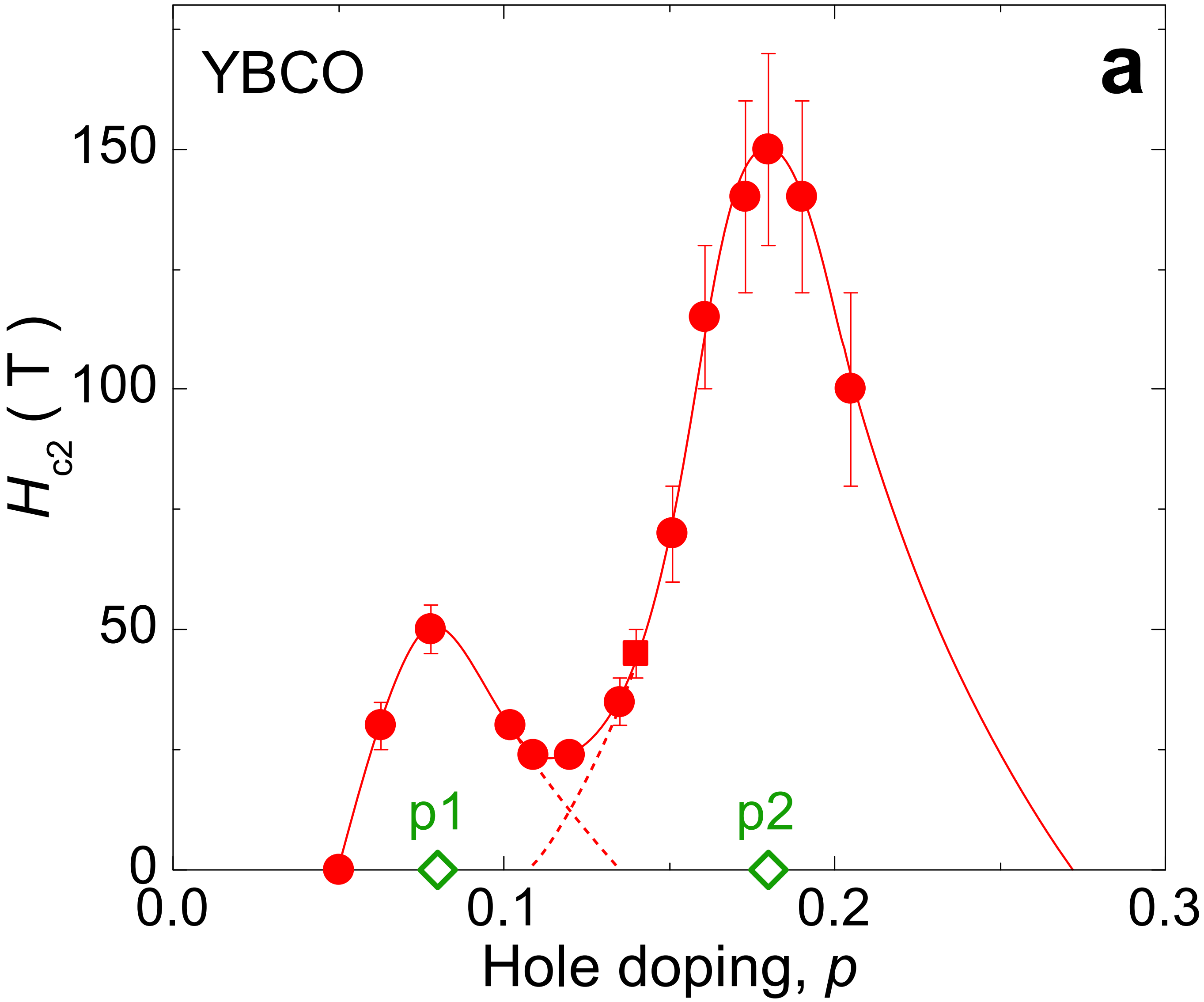
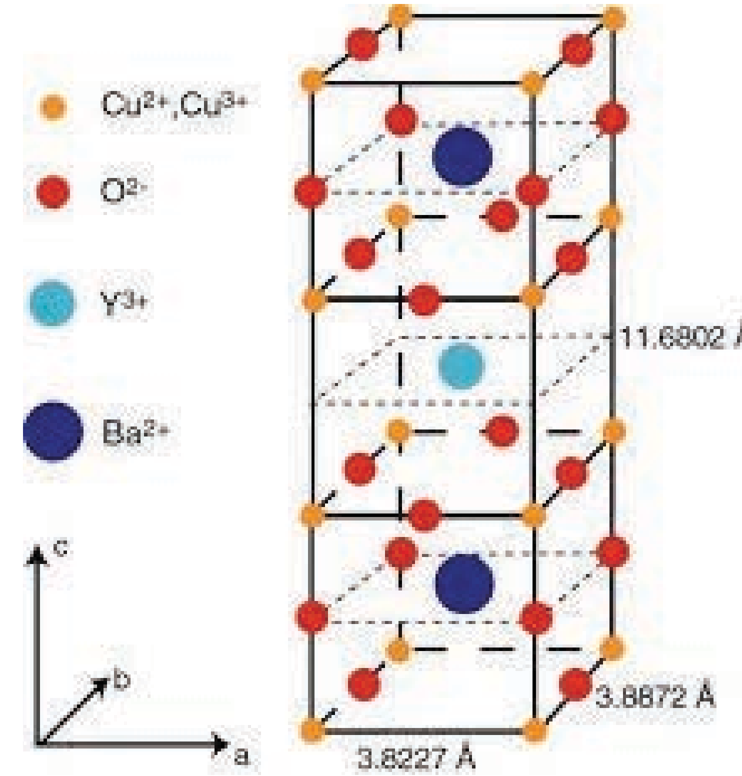
Lower T_c superconductivity in the heavy fermion compounds

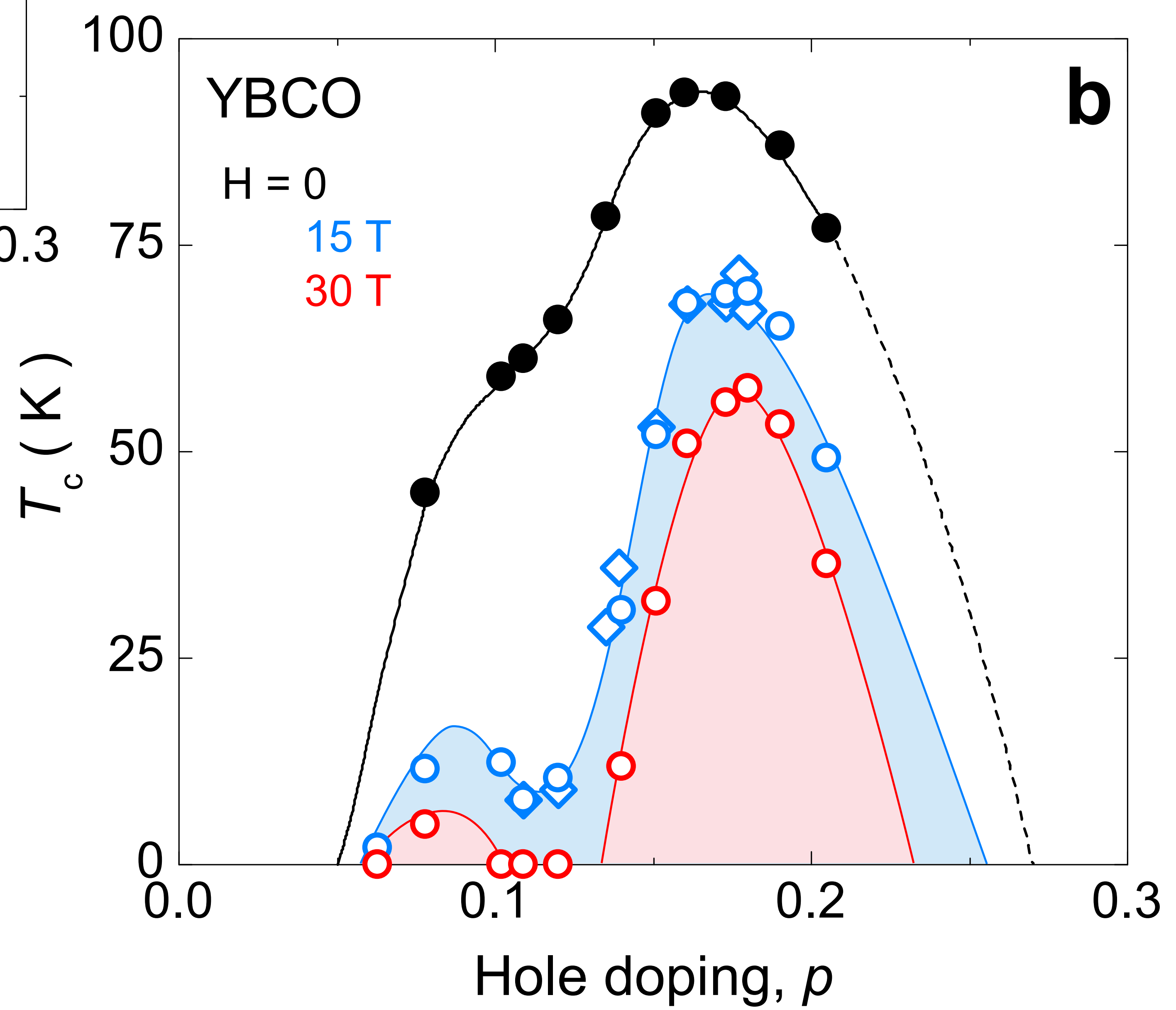




G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223.
Tuson Park, F. Ronning, H. Q. Yuan, M. B. Salamon, R. Movshovich, J. L. Sarrao, and J. D. Thompson, *Nature* **440**, 65 (2006)

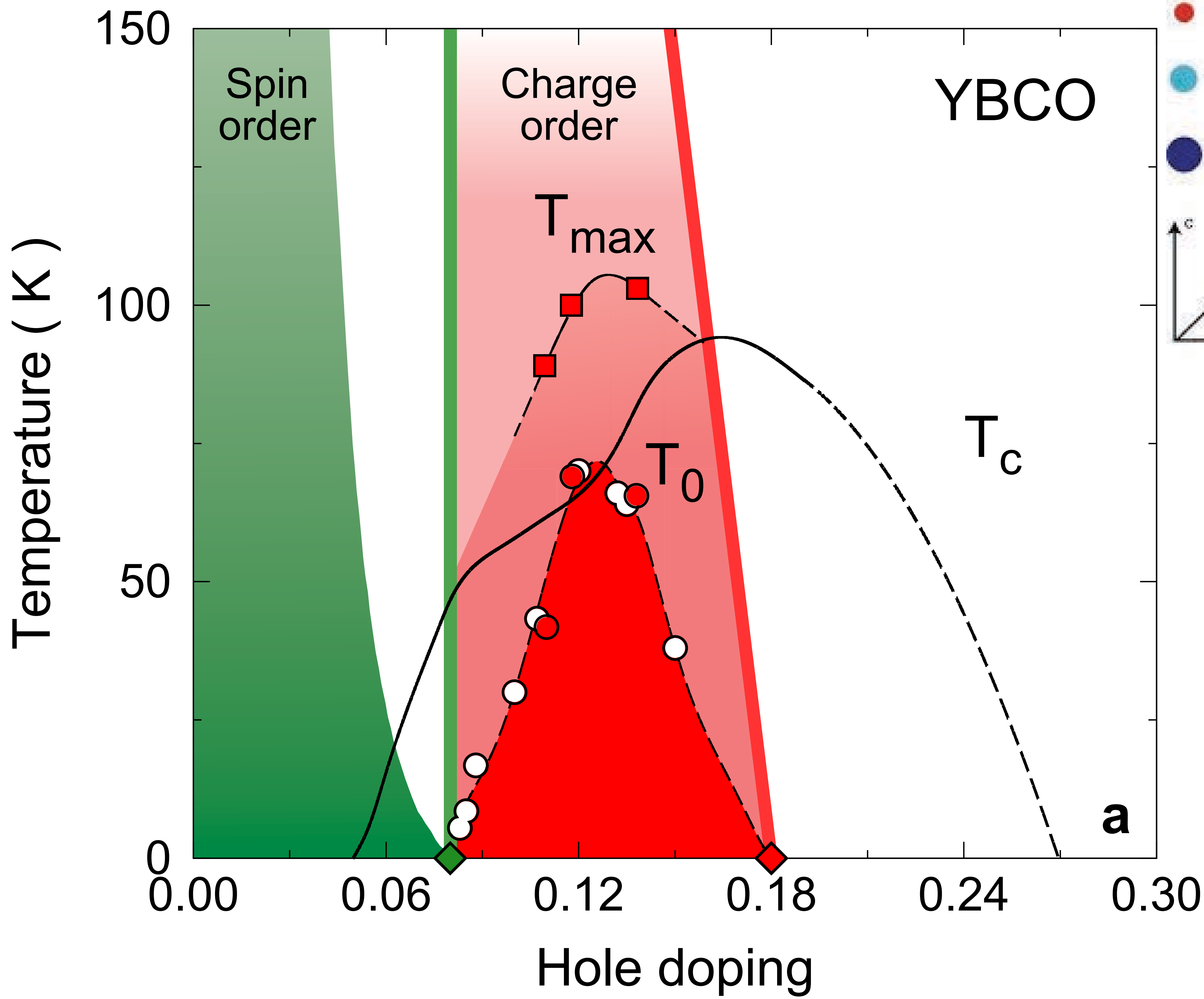


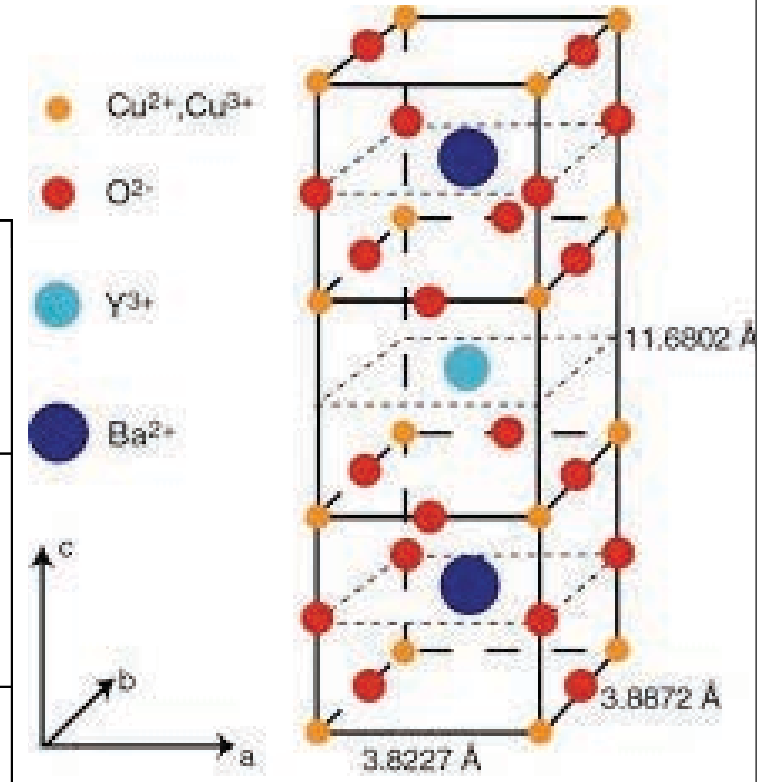




Hole-doped cuprates

G. Grissonnanche et al., preprint





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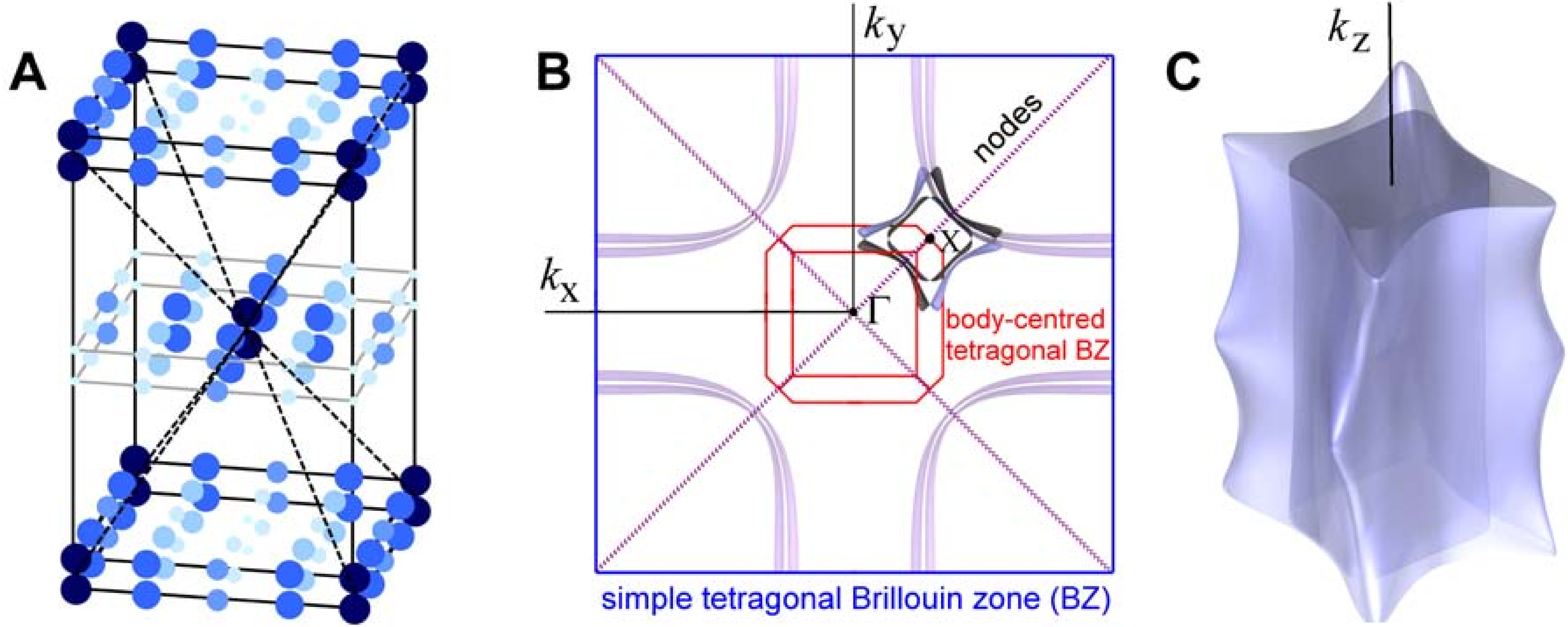
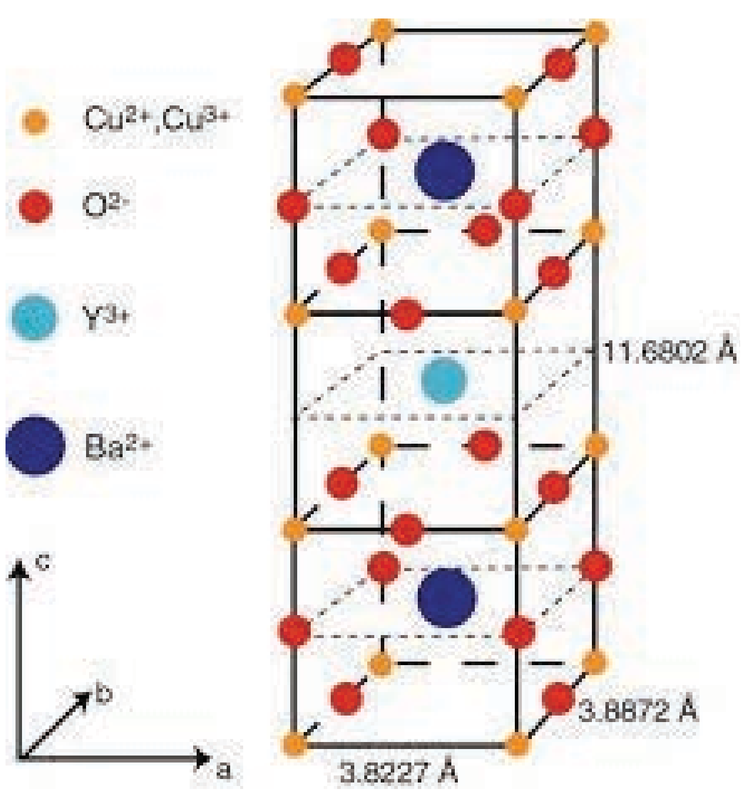
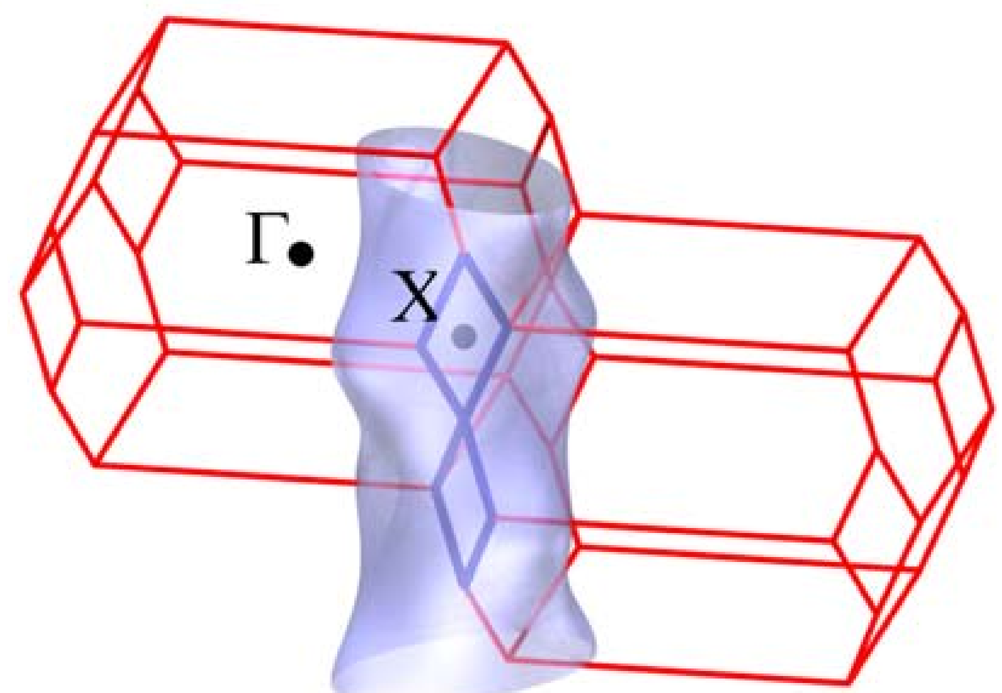






S. Sebastian et al., pr

Outline

1. Weak coupling theory of SDW ordering, and d-wave superconductivity

2. Universal critical theory of SDW ordering

3. Emergent pseudospin symmetry, and quadrupolar density wave

4. Quantum Monte Carlo without the sign problem

Outline

1. Weak coupling theory of SDW ordering,
   and d-wave superconductivity

2. Universal critical theory of SDW ordering

3. Emergent pseudospin symmetry, and
   quadrupolar density wave

4. Quantum Monte Carlo
   without the sign problem

The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c^\dagger_{i\alpha} c_{i\alpha}$$

$t_{ij} \to$ "hopping". $U \to$ local repulsion, $\mu \to$ chemical potential

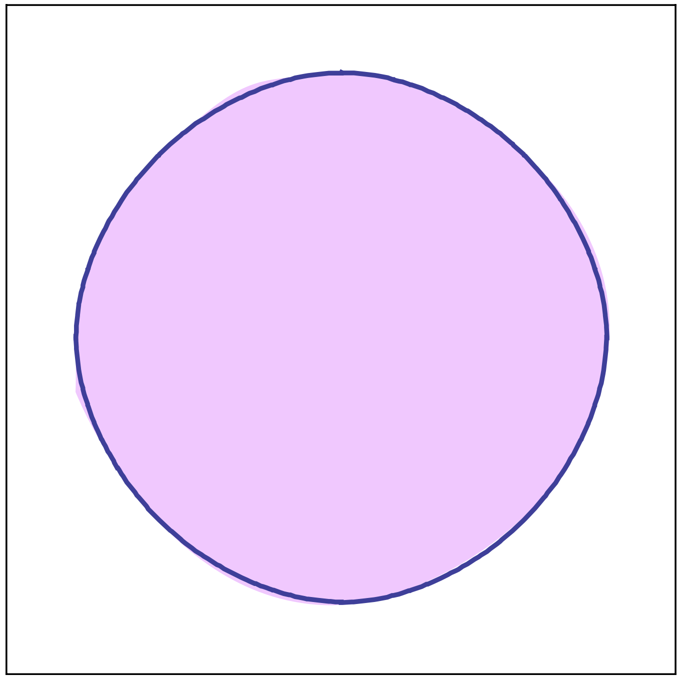
Spin index $\alpha = \uparrow, \downarrow$

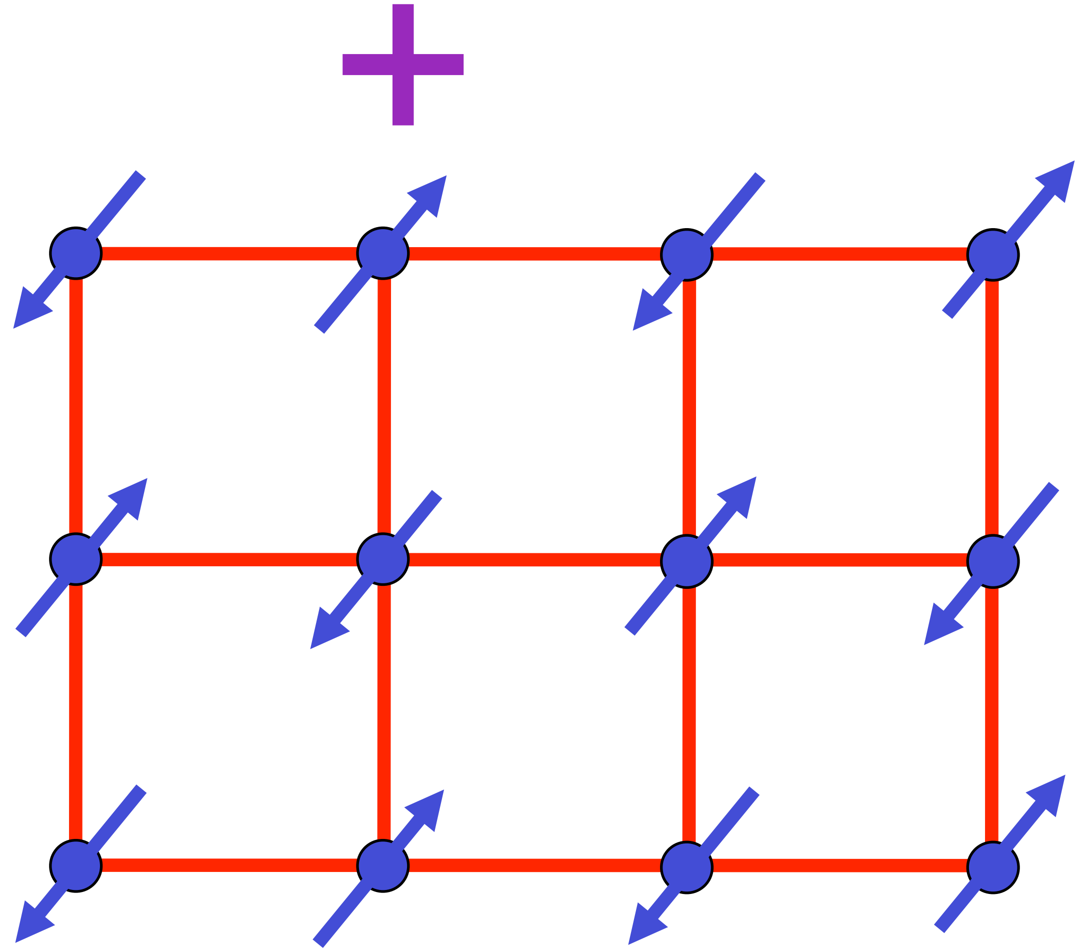
$$n_{i\alpha} = c^\dagger_{i\alpha} c_{i\alpha}$$

$$c^\dagger_{i\alpha} c_{j\beta} + c_{j\beta} c^\dagger_{i\alpha} = \delta_{ij}\delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface





The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

The Hubbard Model

Decouple *U* term by a Hubbard-Stratanovich transformation

$$\mathcal{S} = \int d^2 r d\tau \left[\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi}\right]$$

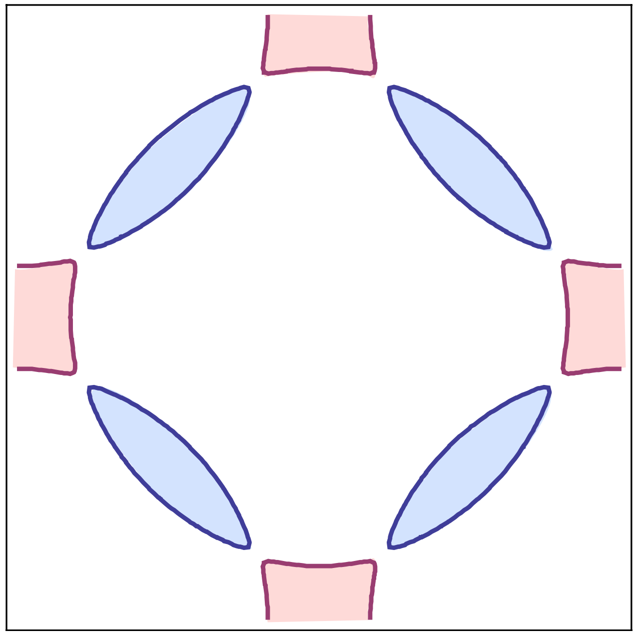
$$\mathcal{L}_c = c_a^\dagger \varepsilon(-i\boldsymbol{\nabla}) c_a$$

$$\mathcal{L}_\varphi = \frac{1}{2}(\boldsymbol{\nabla}\varphi_\alpha)^2 + \frac{r}{2}\varphi_\alpha^2 + \frac{u}{4}\left(\varphi_\alpha^2\right)^2$$

$$\mathcal{L}_{c\varphi} = \lambda\, \varphi_\alpha\, e^{i\mathbf{K}\cdot\mathbf{r}}\, c_a^\dagger\, \sigma_{ab}^\alpha\, c_b.$$

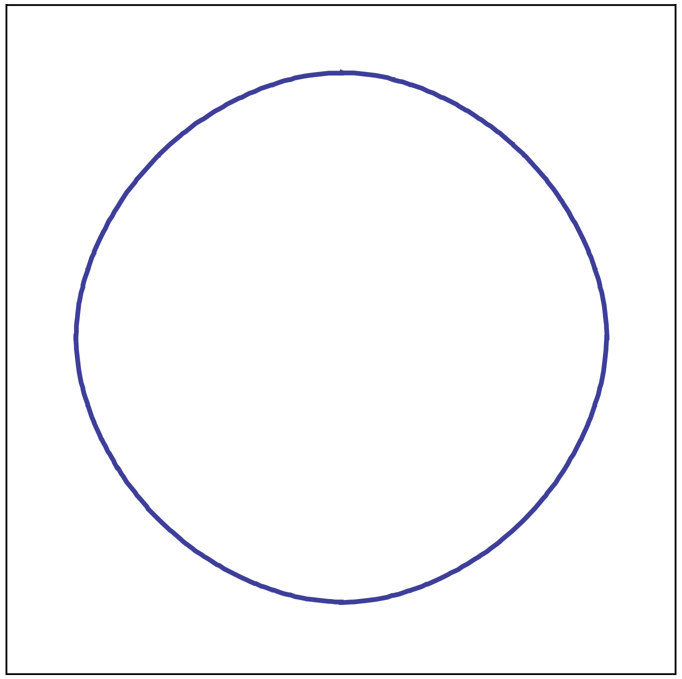
"Yukawa" coupling between fermions and antiferromagnetic order: $\lambda^2 \sim U$, the Hubbard repulsion

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron and hole pockets



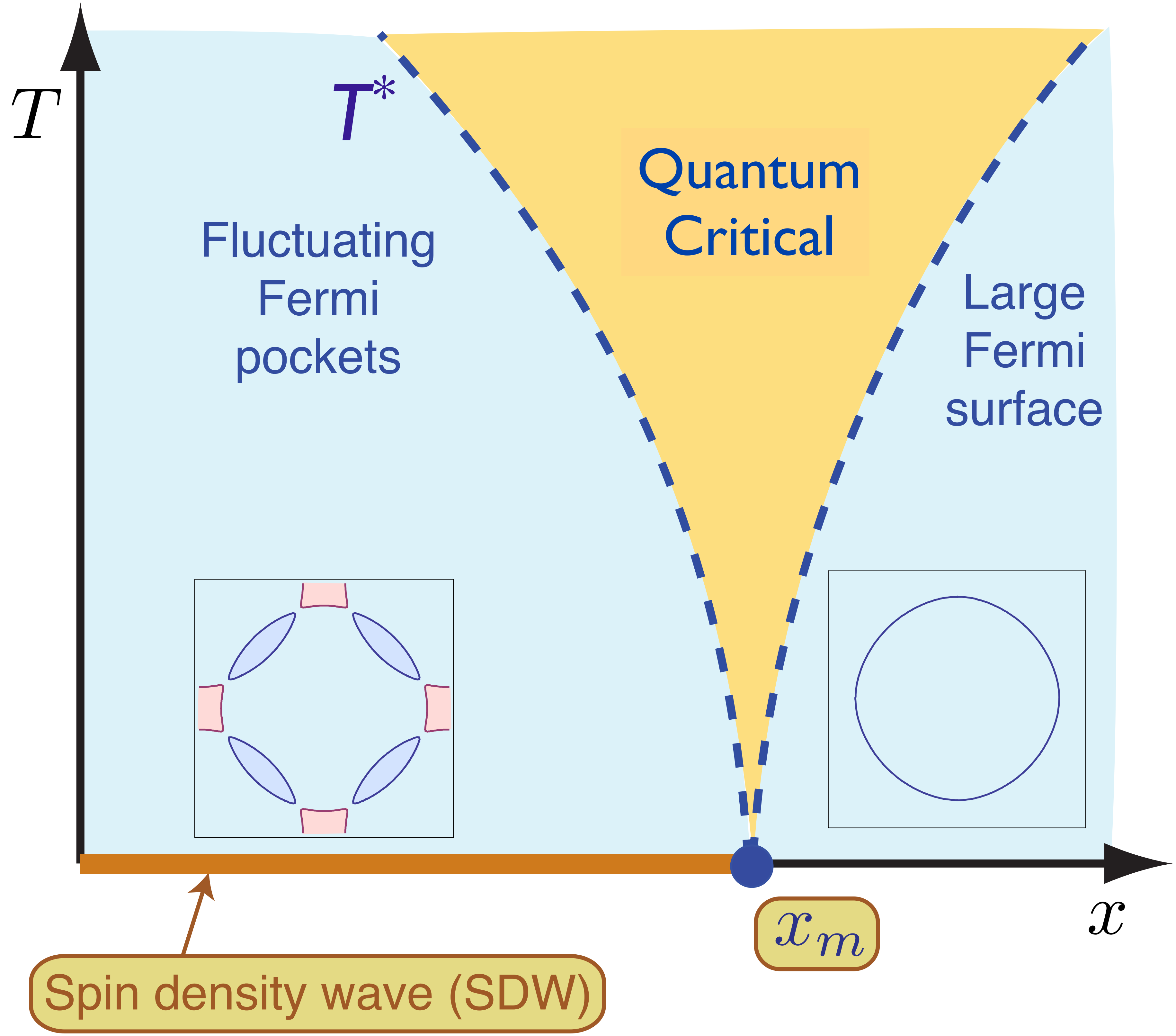
$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large" Fermi surface

Increasing interaction

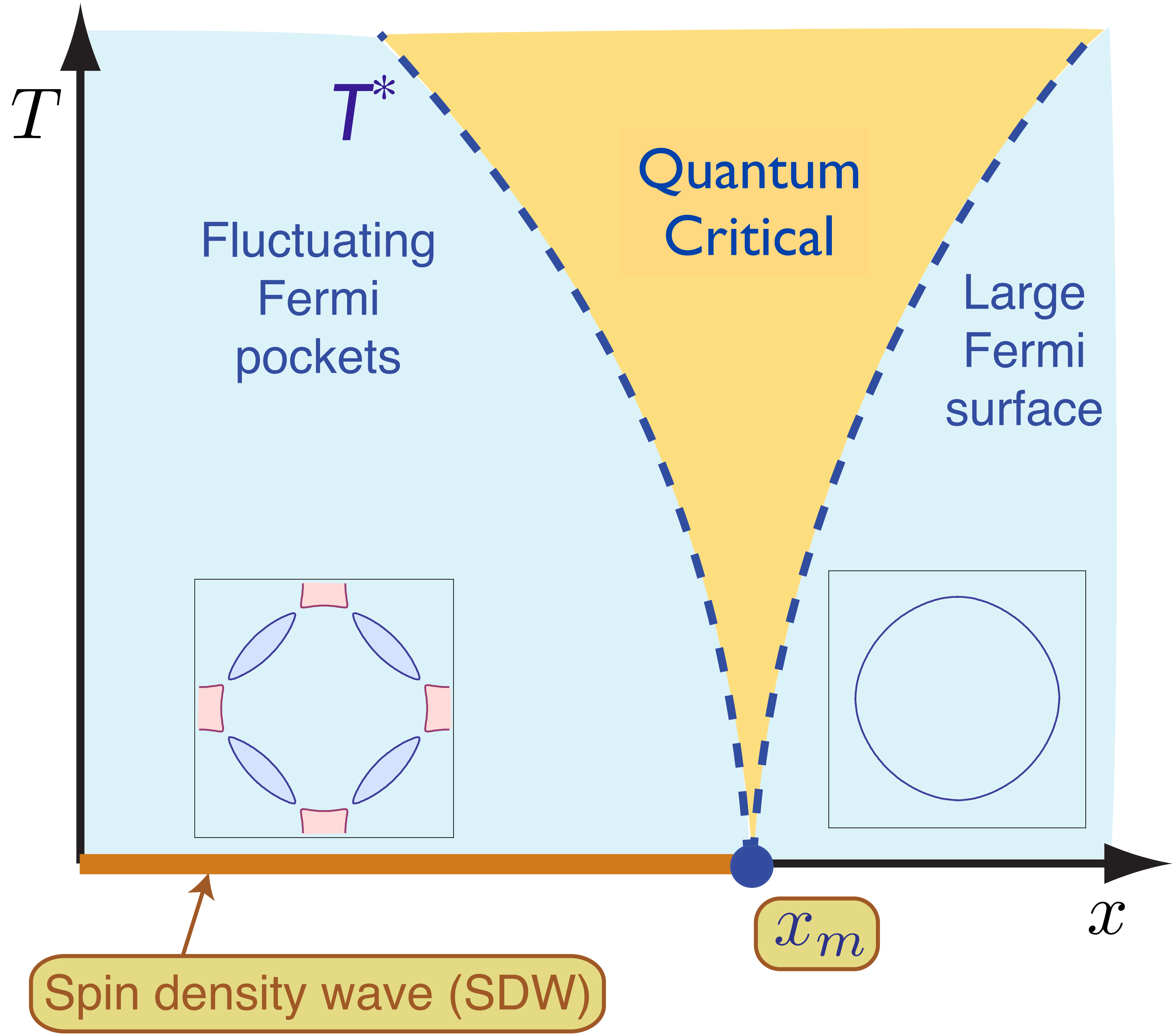
S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

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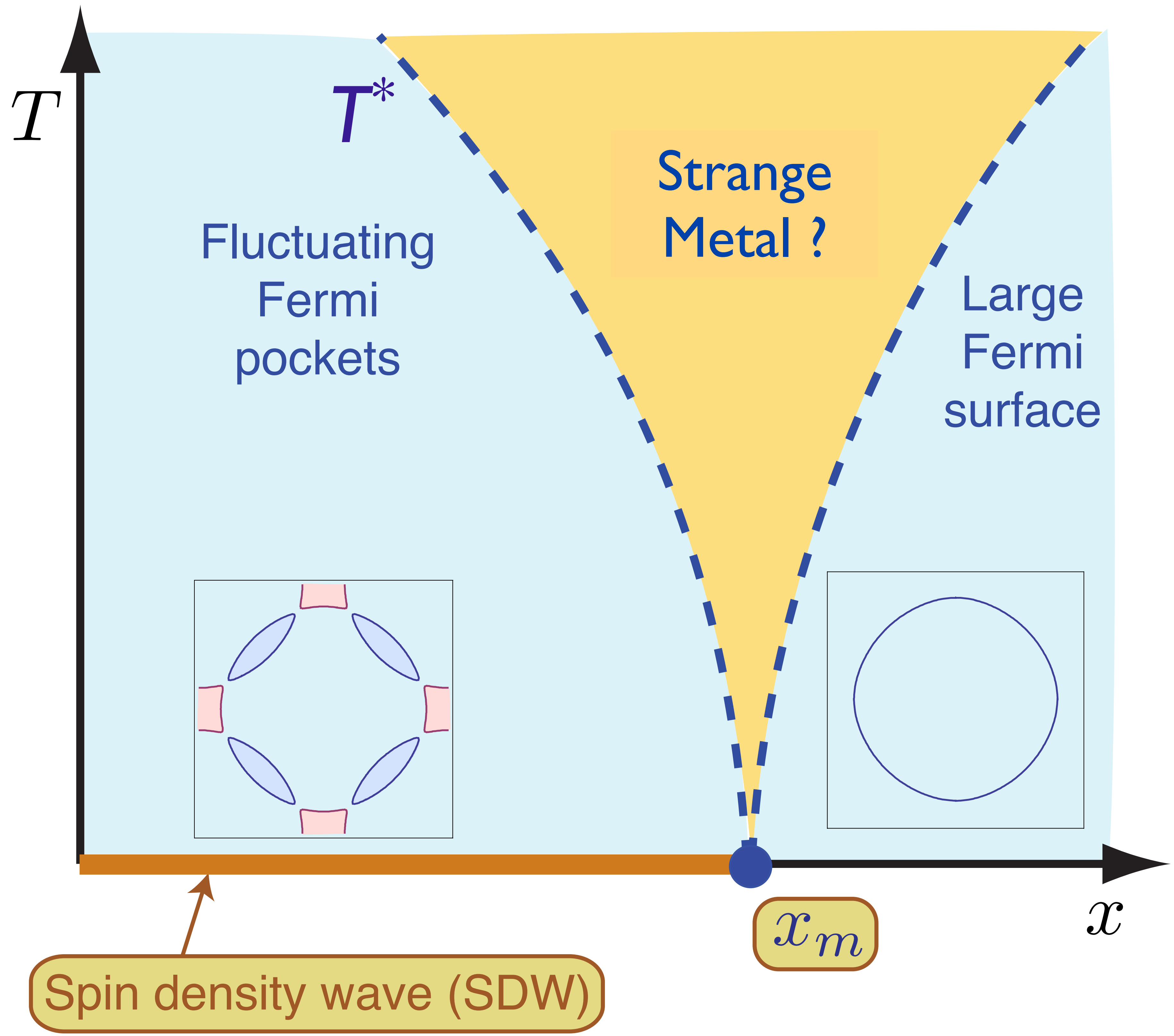


Relaxation and equilibration times $\sim \hbar / k_B T$ are robust properties of strongly-coupled quantum criticality

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = -\sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c_{\mathbf{k},\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

$$H_{ee} = -\frac{1}{2}\sum_{\mathbf{q}}\sum_{\mathbf{p},\gamma,\delta}\sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c_{\mathbf{k},\alpha}^{\dagger} c_{\mathbf{k}+\mathbf{q},\beta} c_{\mathbf{p},\gamma}^{\dagger} c_{\mathbf{p}-\mathbf{q},\delta},$$

where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\gamma\delta}\frac{\chi_0}{\xi^{-2}+(\mathbf{q}-\mathbf{K})^2},$$

with $\chi_0\xi^2$ the SDW susceptibility and ξ the SDW correlation length.

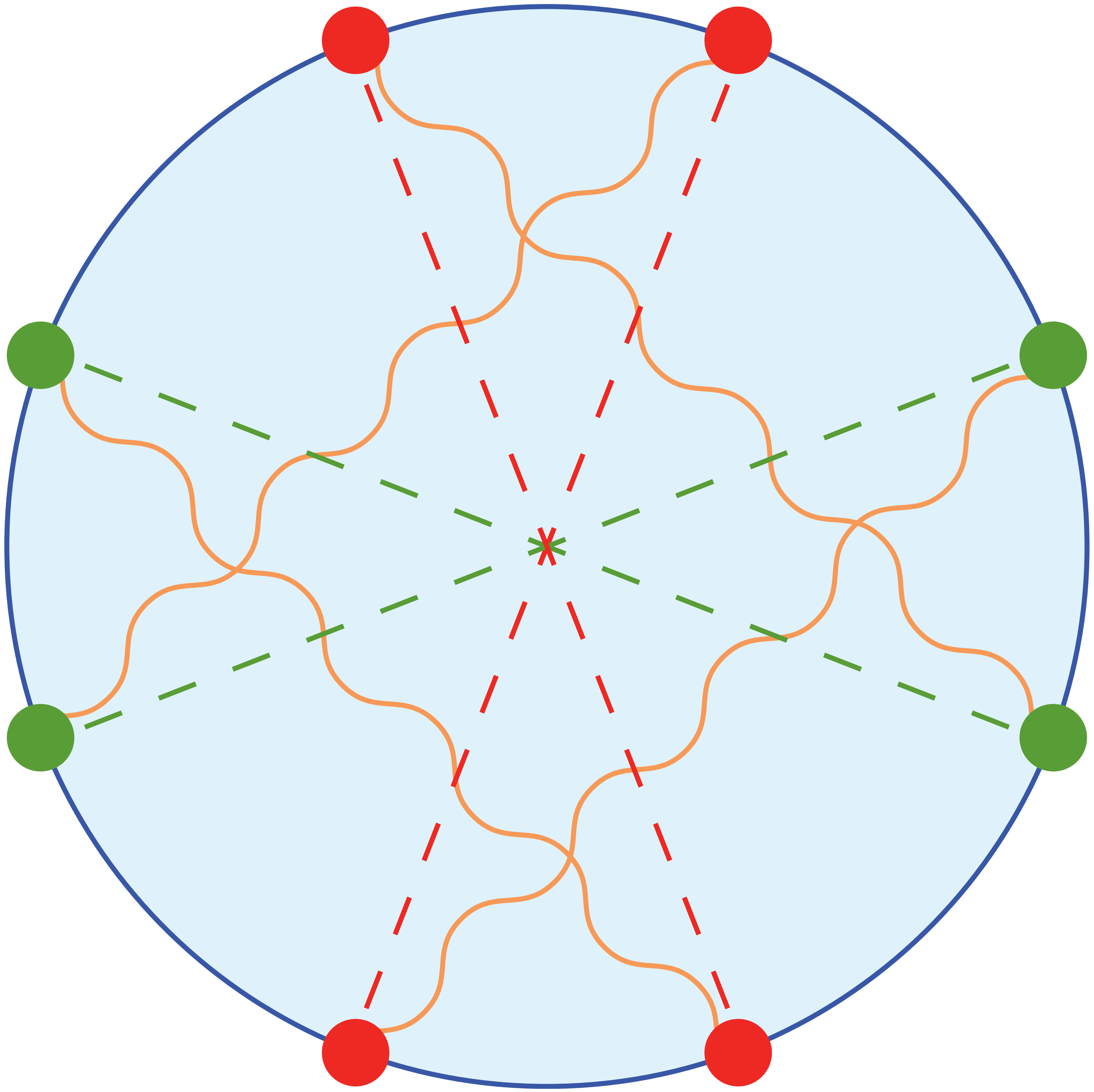
BCS Gap equation

In BCS theory, this interaction leads to the 'gap equation' for the pairing gap $\Delta_{\mathbf{k}} \propto \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$.

$$\Delta_{\mathbf{k}} = - \sum_{\mathbf{p}} \left(\frac{3\chi_0}{\xi^{-2} + (\mathbf{p} - \mathbf{k} - \mathbf{K})^2} \right) \frac{\Delta_{\mathbf{p}}}{2\sqrt{\varepsilon_{\mathbf{p}}^2 + \Delta_{\mathbf{p}}^2}}$$

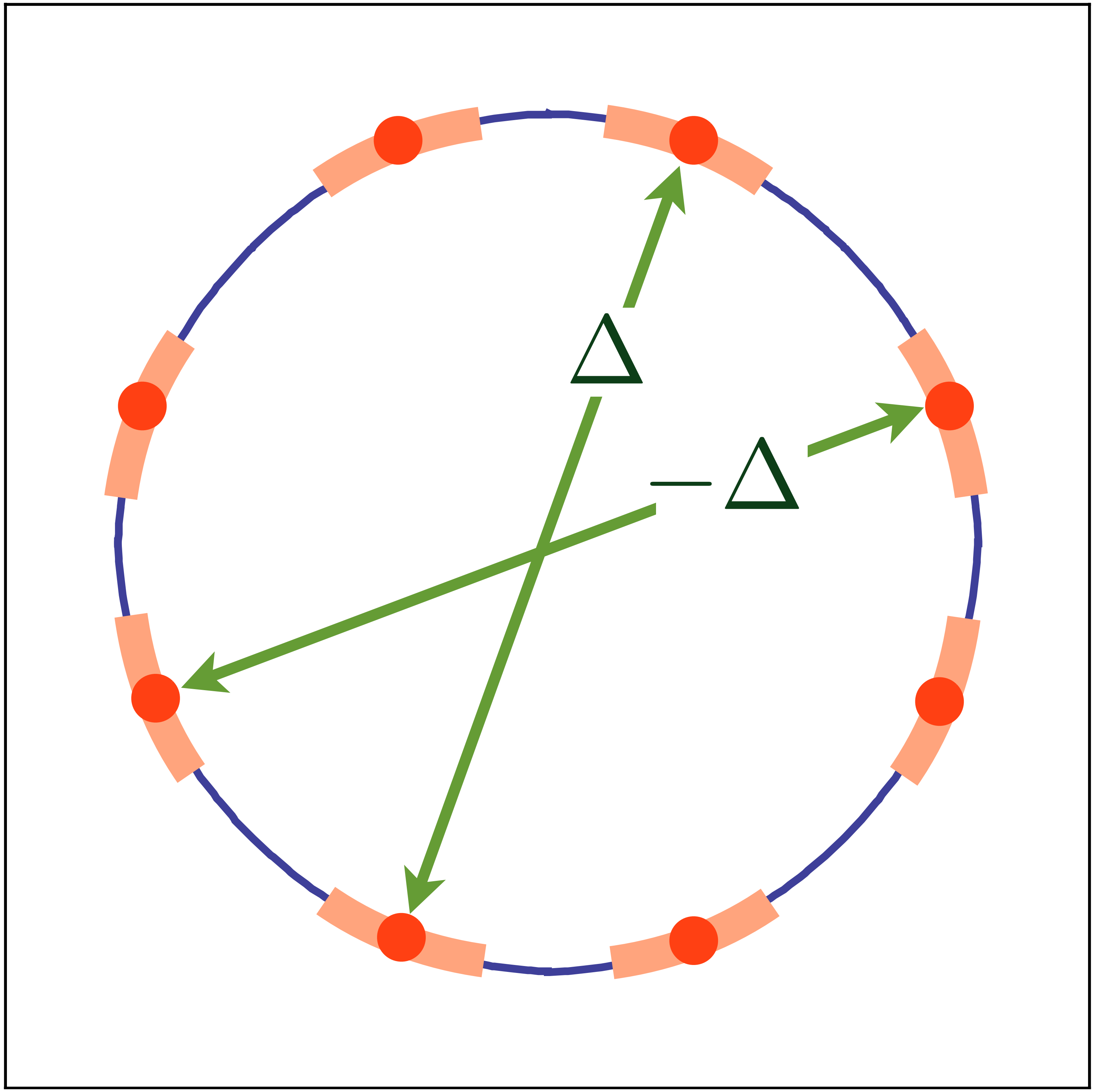
Non-zero solutions of this equation require that $\Delta_{\mathbf{k}}$ and $\Delta_{\mathbf{p}}$ have opposite signs when $\mathbf{p} - \mathbf{k} \approx \mathbf{K}$.

Pairing "glue" from antiferromagnetic fluctuations



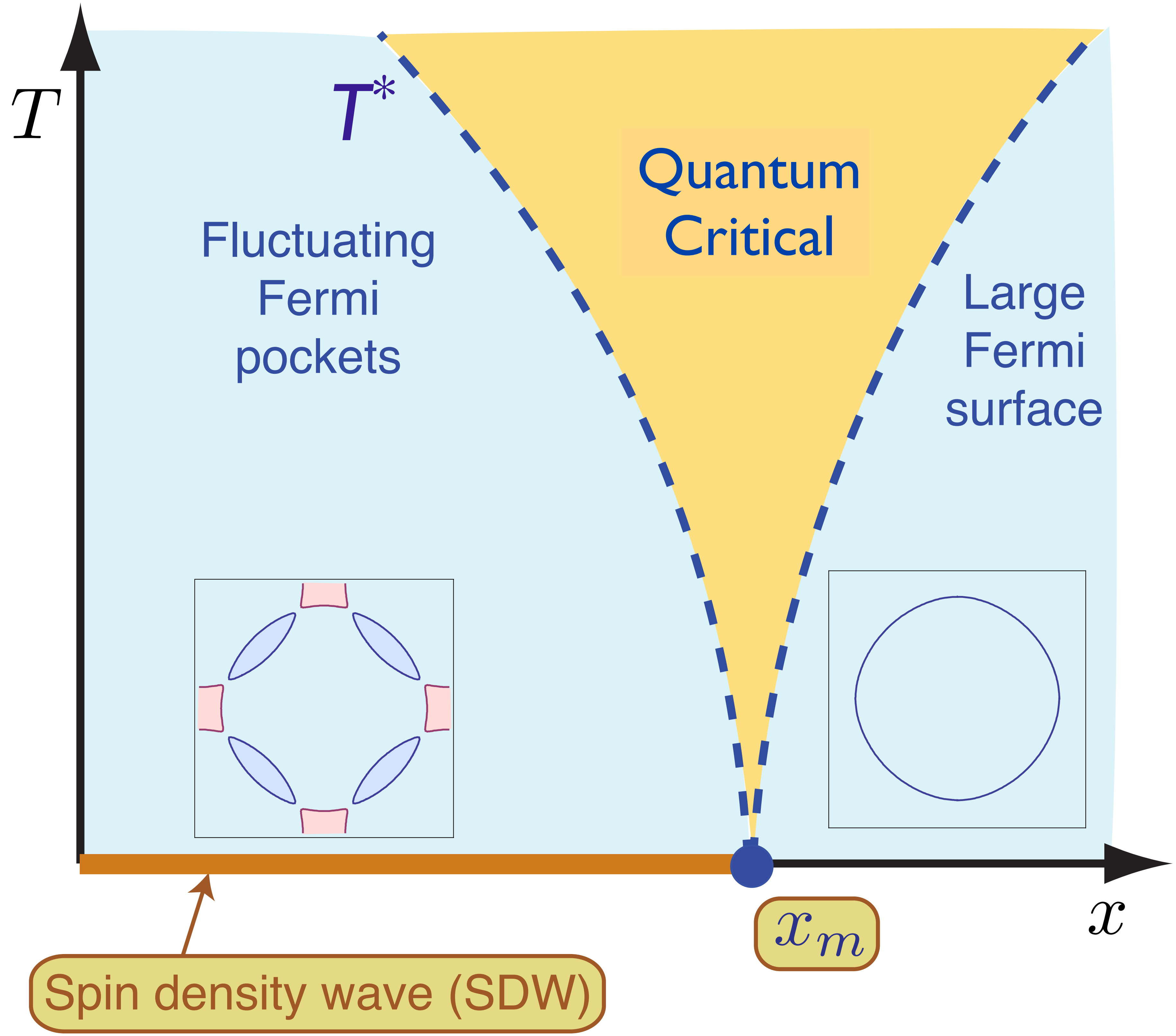
V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at *and near* hot spots

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

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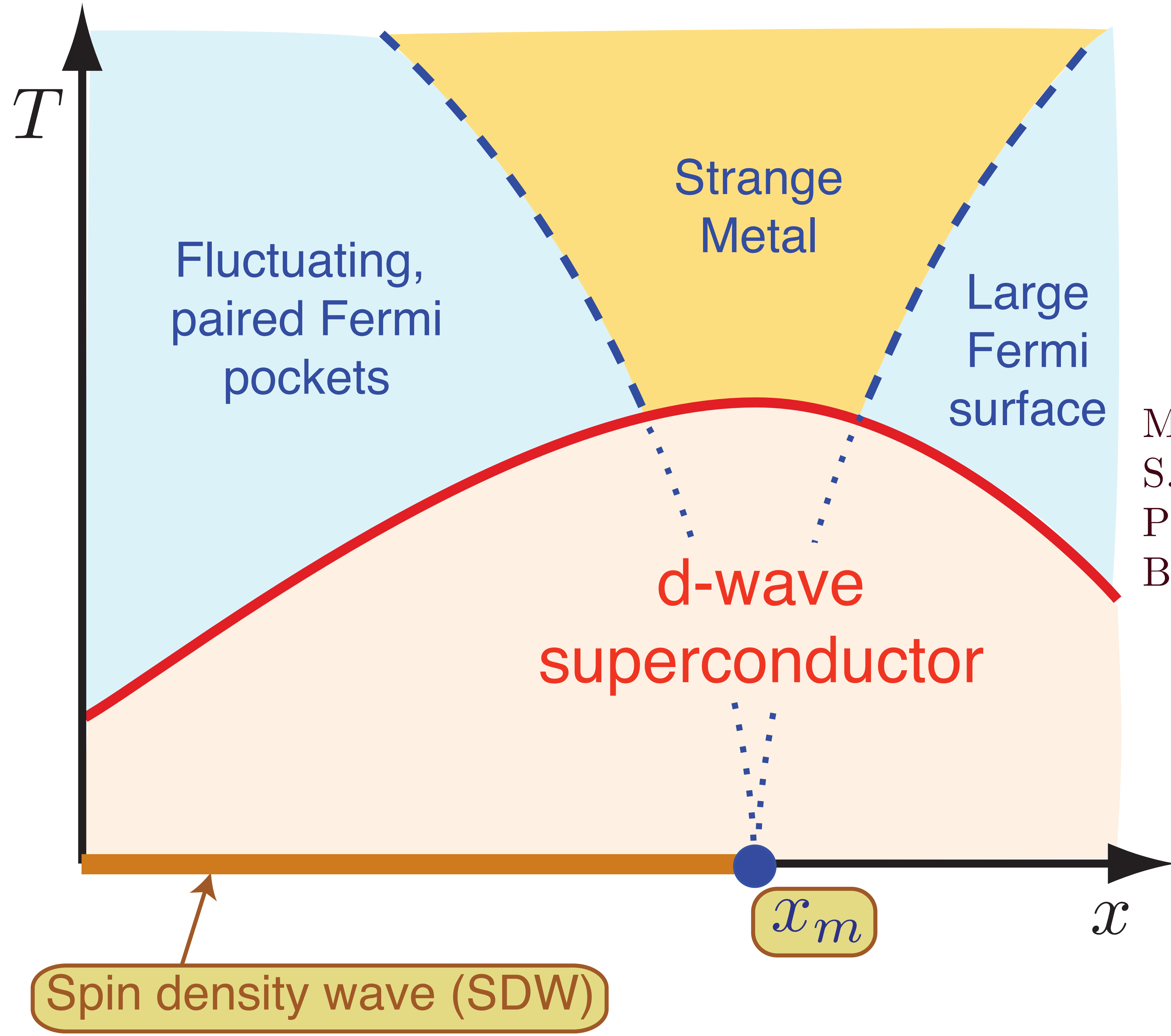
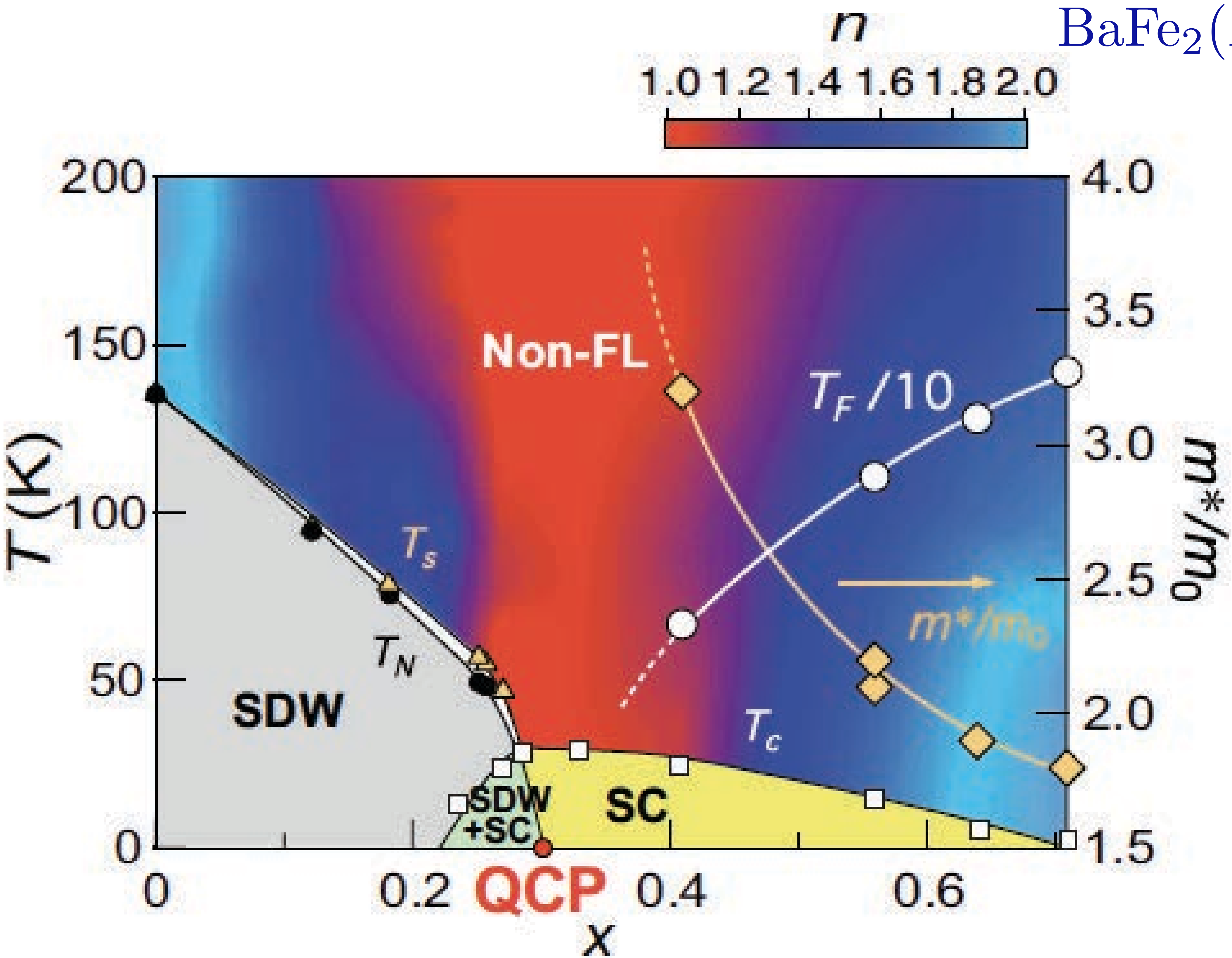


M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)

SDW quantum critical point is unstable to *d*-wave superconductivity
This instability is stronger than that in the BCS theory

$BaFe_2(As_{1-x}\ P_x)_2$



K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, Science in press

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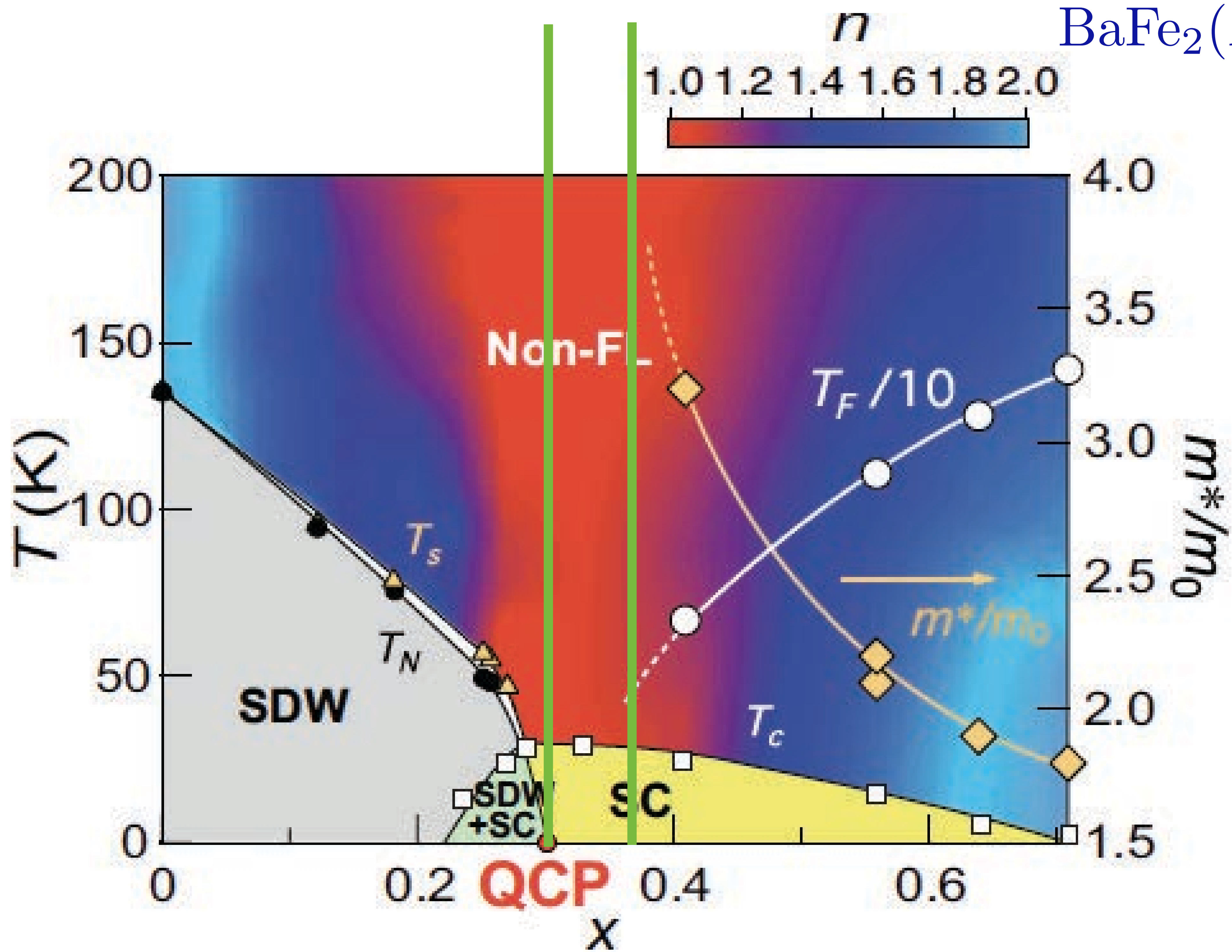
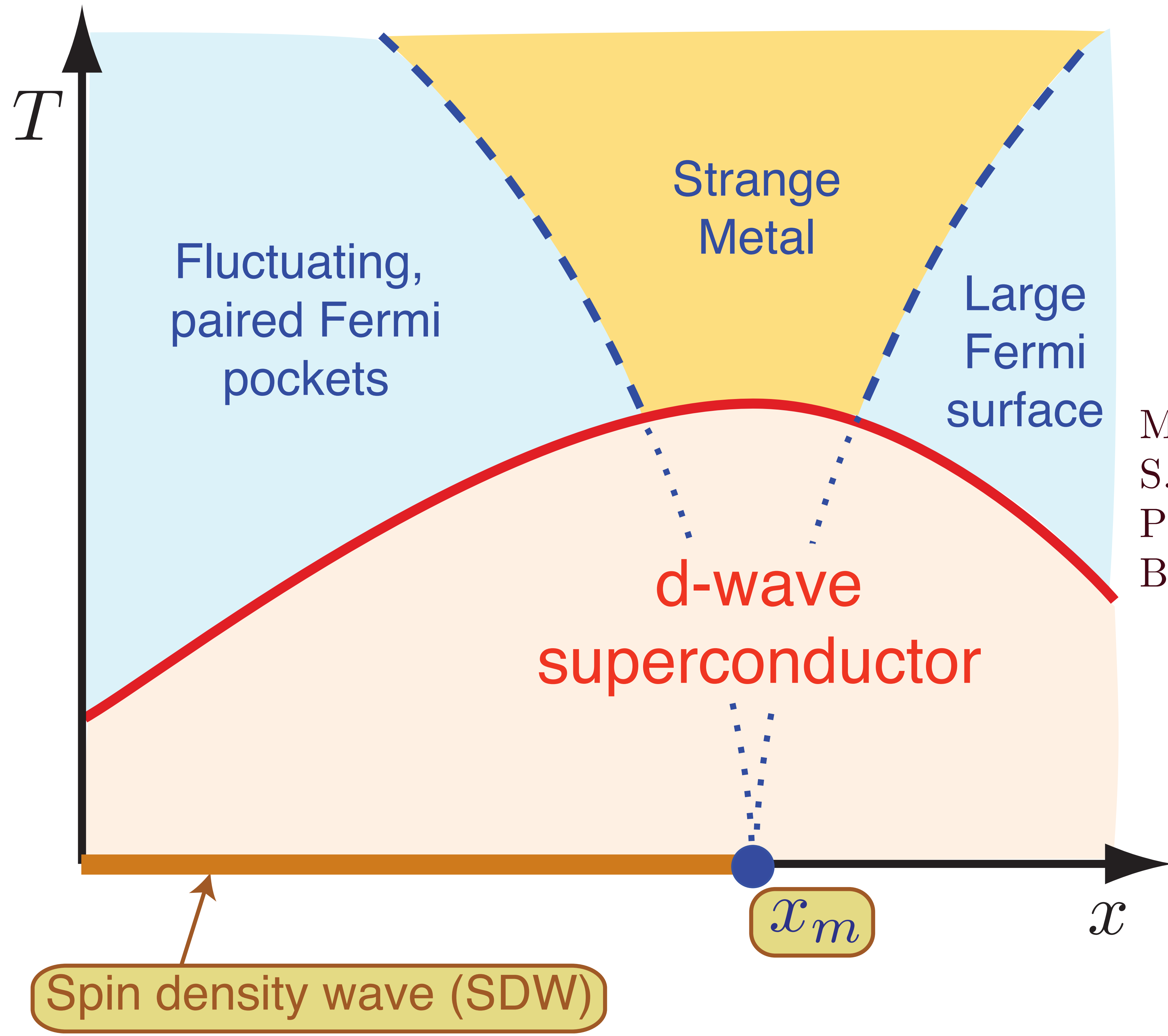


Notice shift between the position of the QCP in the superconductor, and the divergence in effective mass in the metal measured at high magnetic fields

Theory of quantum criticality in the cuprates



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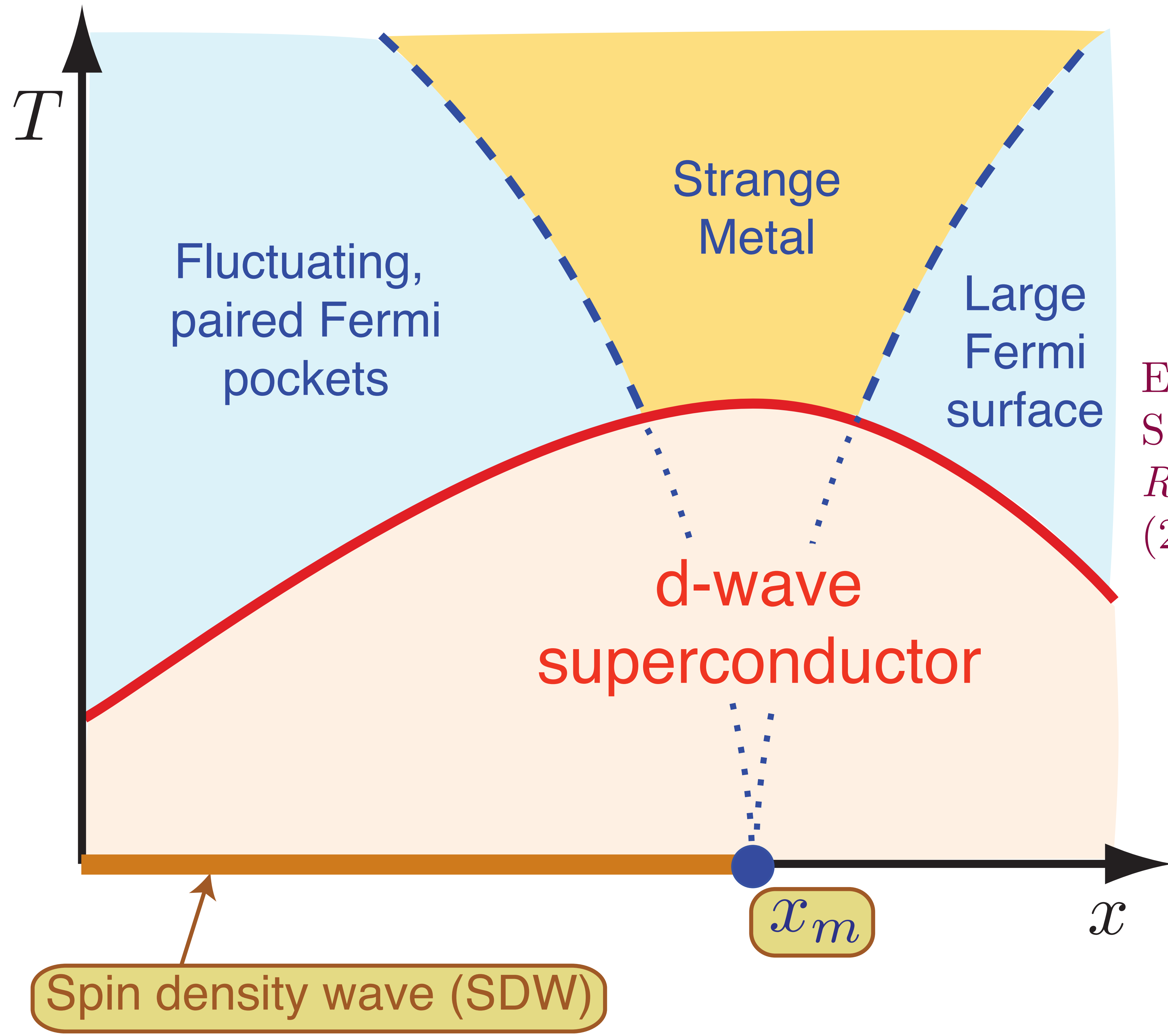
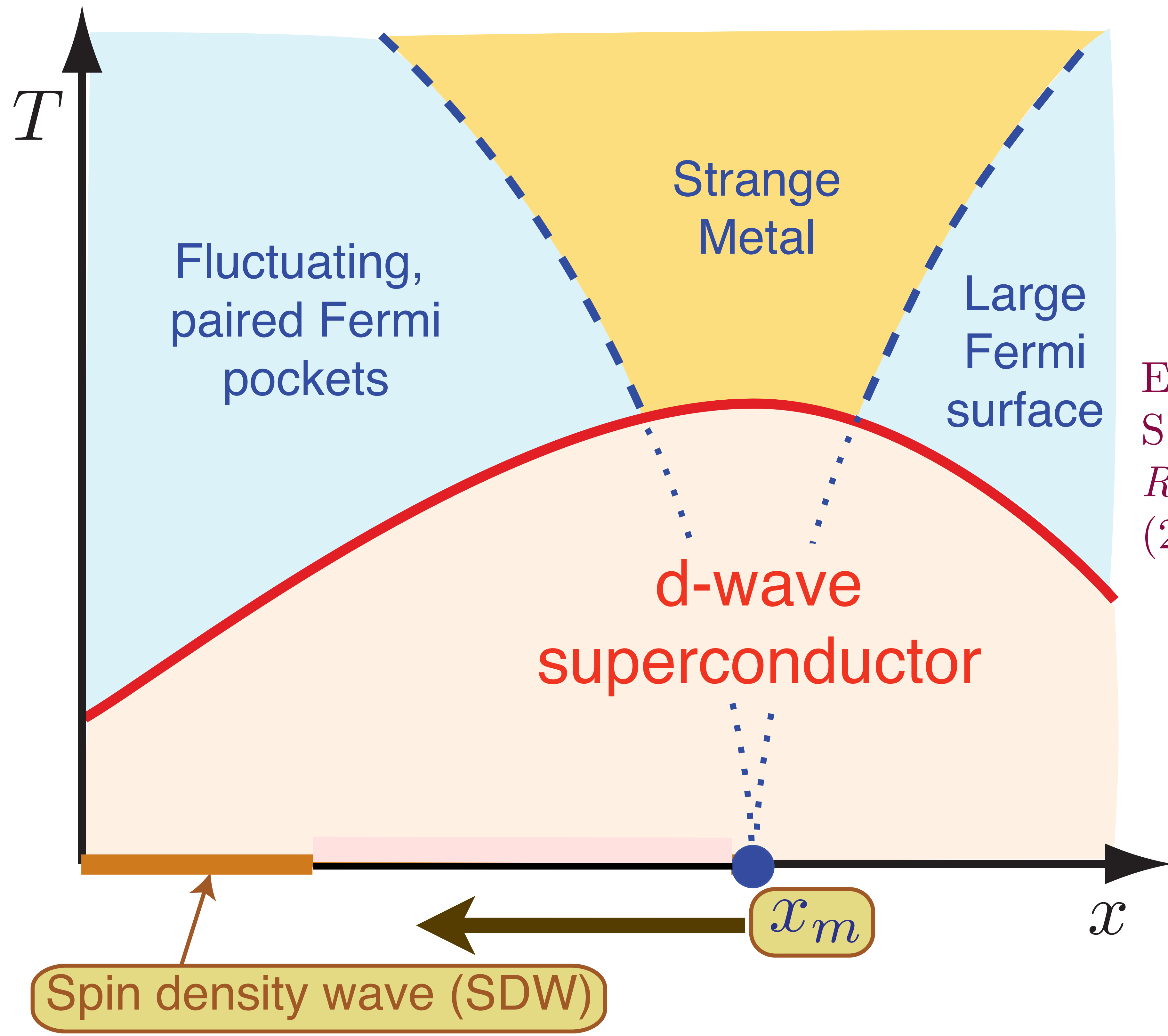


E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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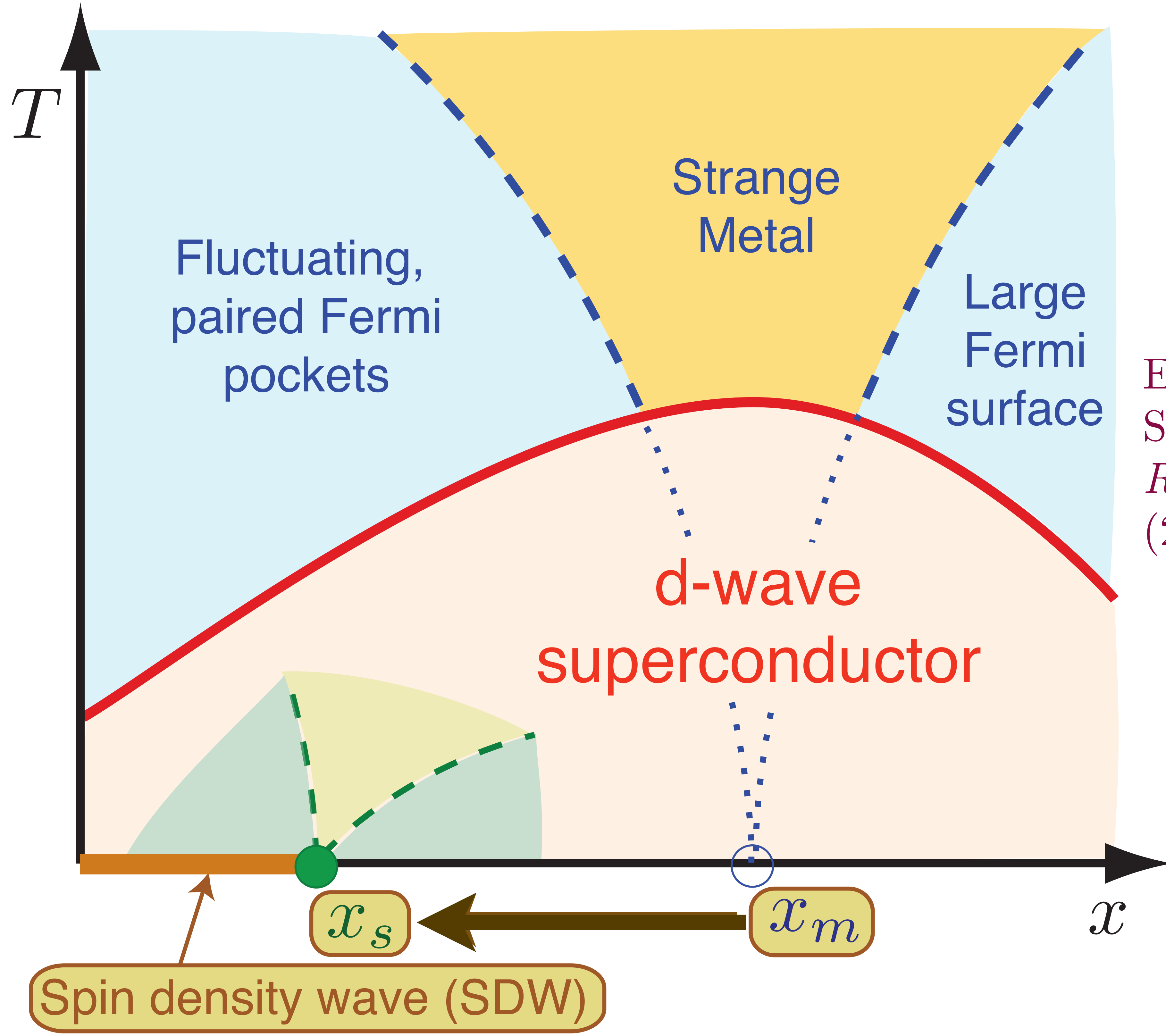

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

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different effects compete:

- Pairing glue becomes stronger.

- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.

- Other instabilities can appear *e.g.* to charge density waves/stripe order.

Outline

1. Weak coupling theory of SDW ordering, and d-wave superconductivity

2. Universal critical theory of SDW ordering

3. Emergent pseudospin symmetry, and quadrupolar density wave

4. Quantum Monte Carlo without the sign problem

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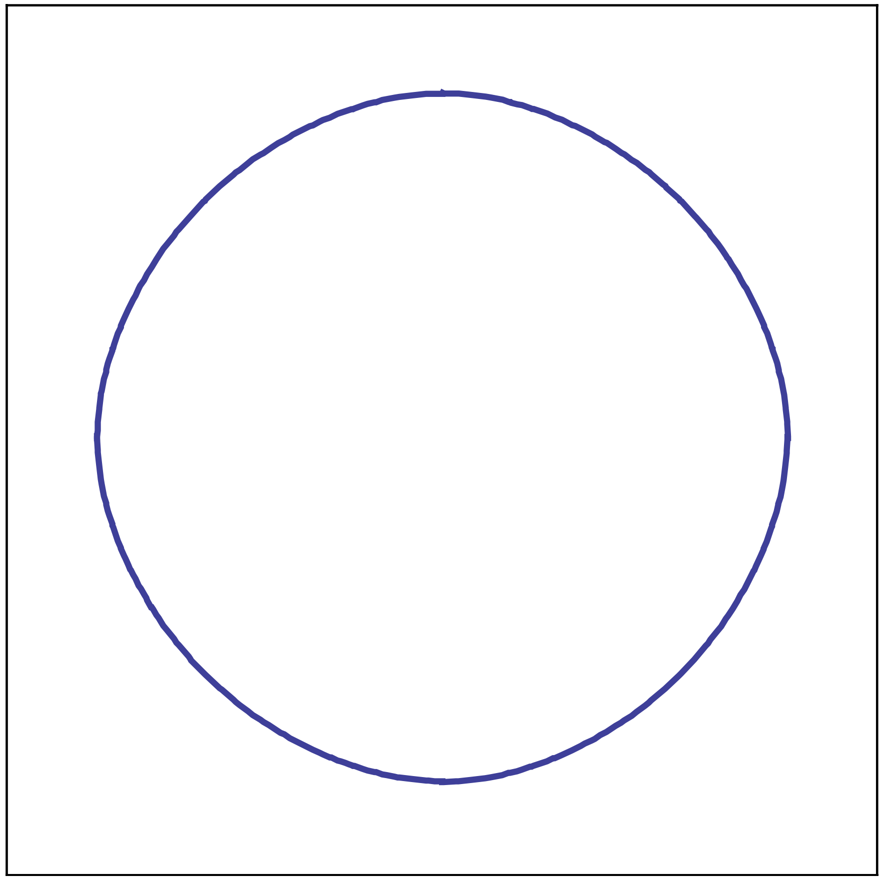
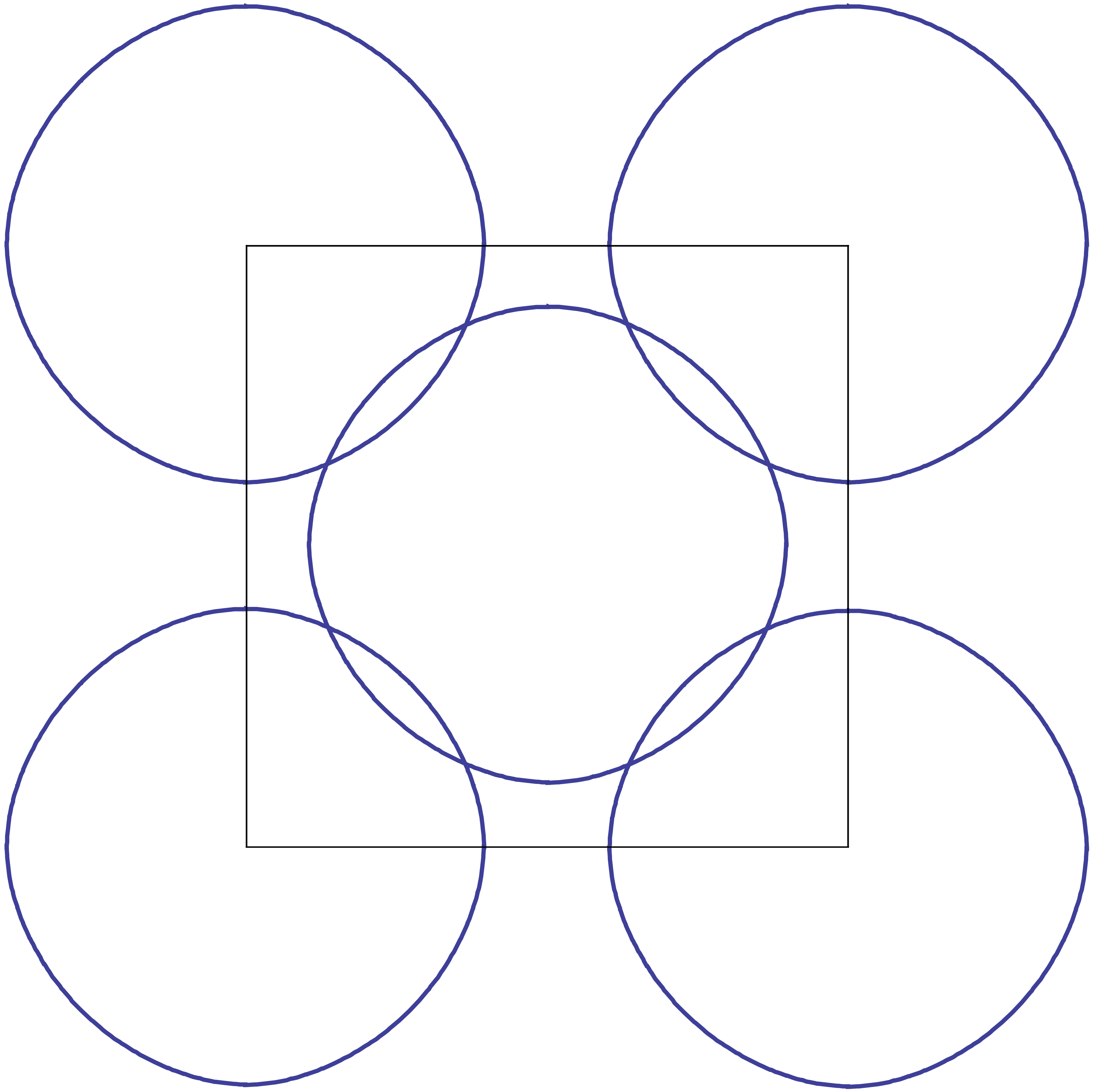
Fermi surface+antiferromagnetism



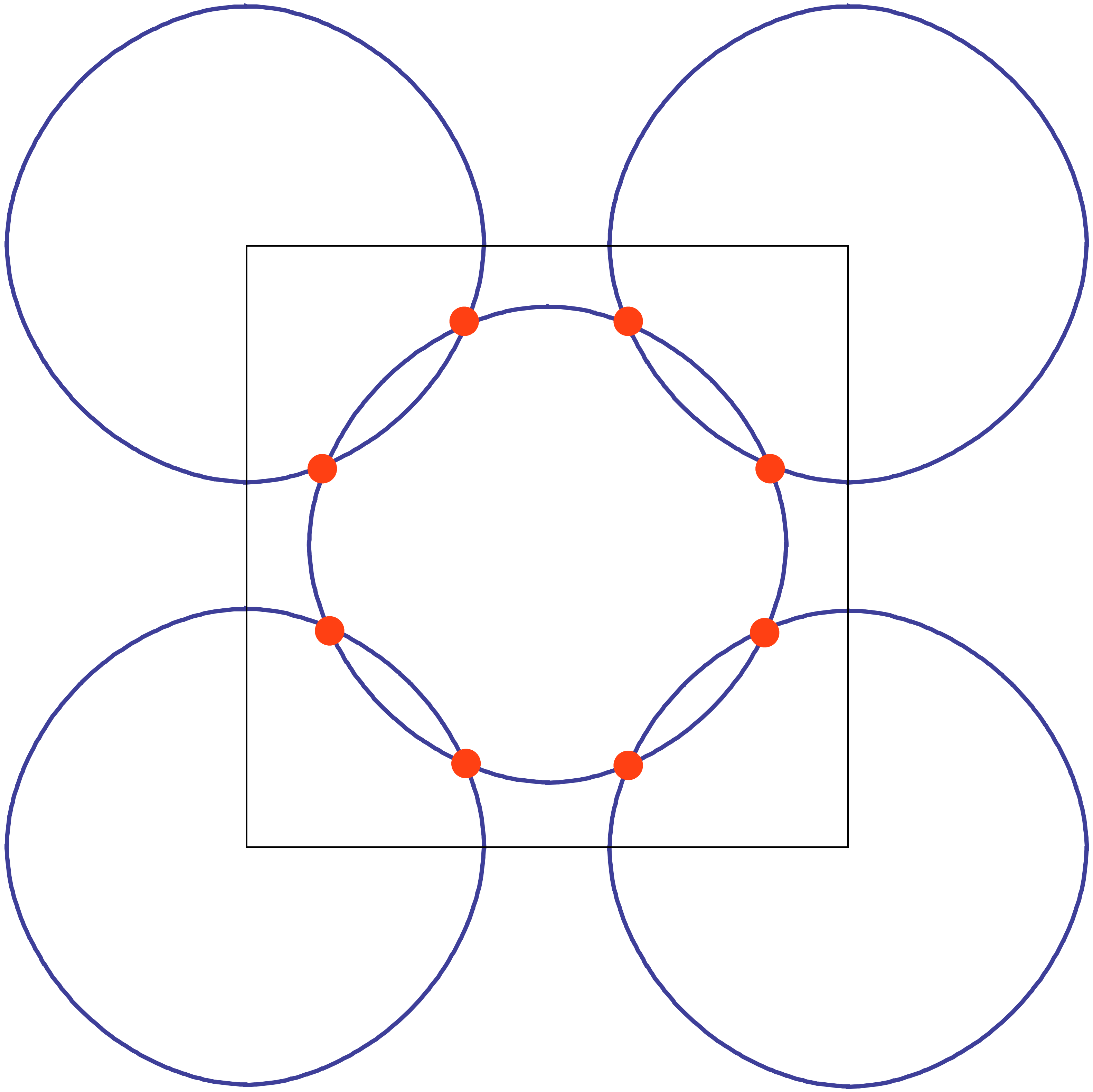
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



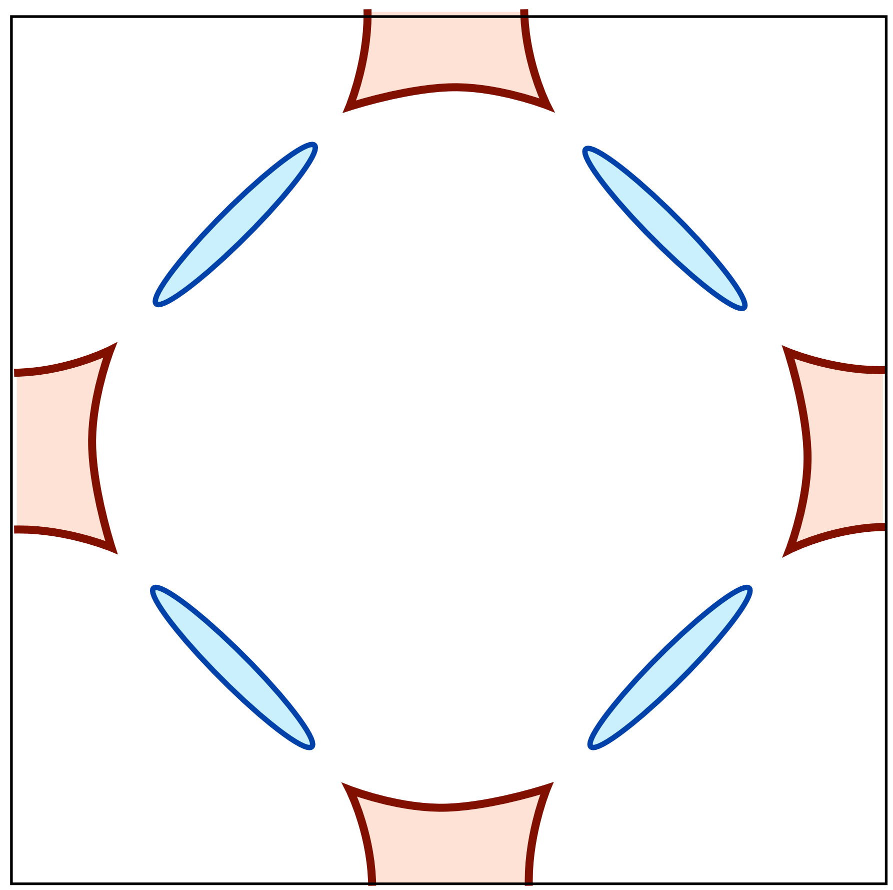
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism

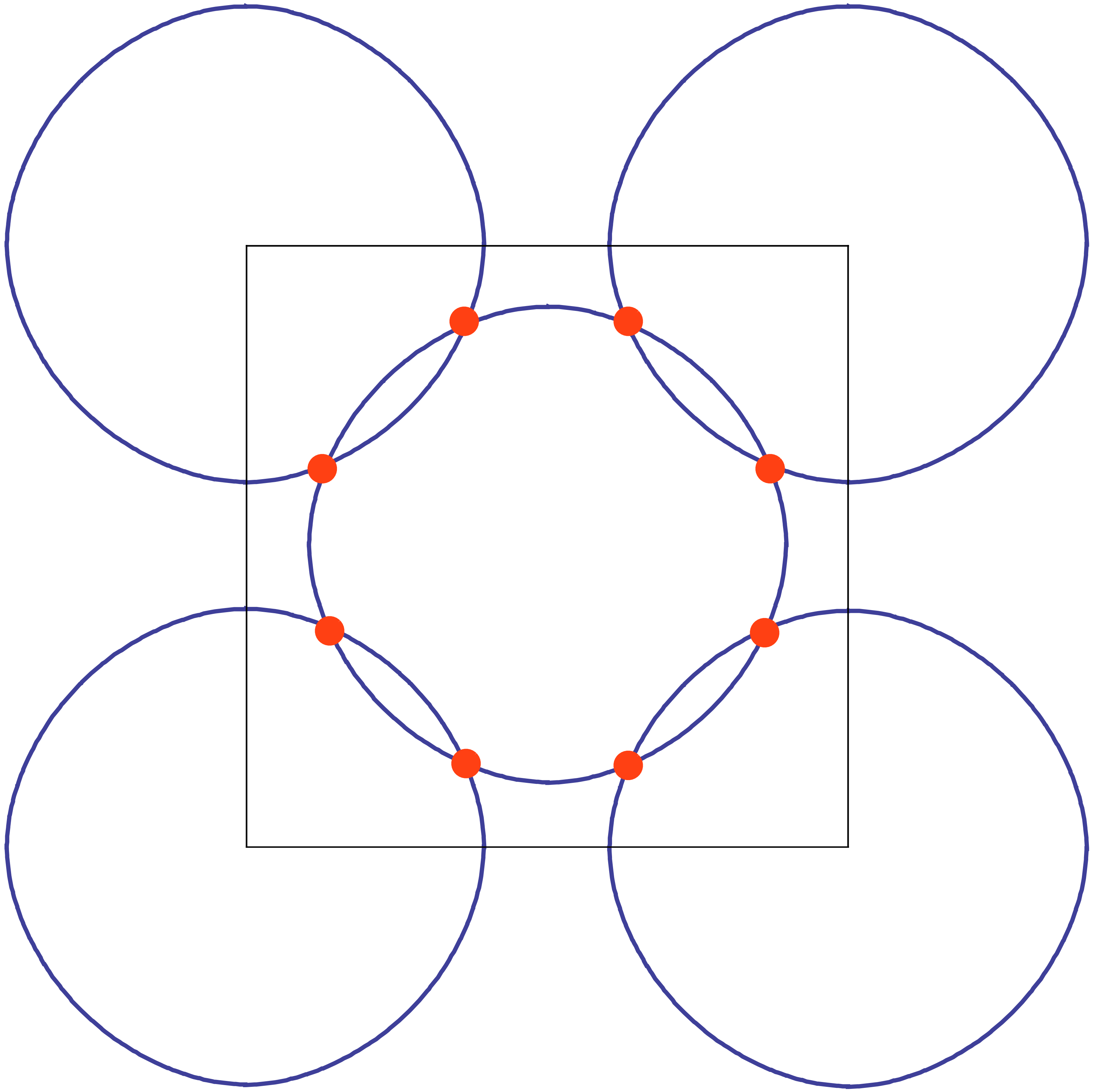


"Hot" spots

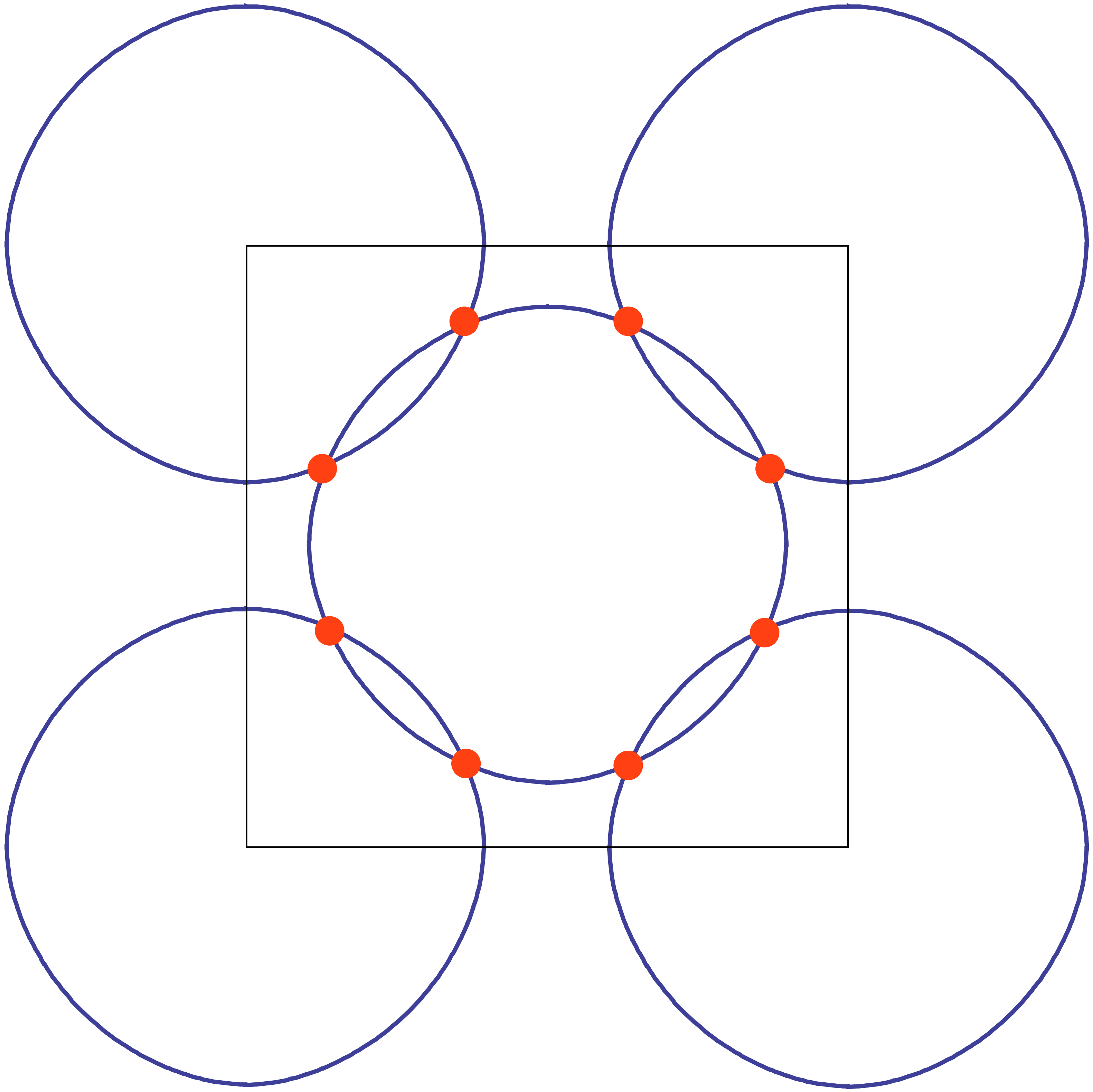
Fermi surface+antiferromagnetism



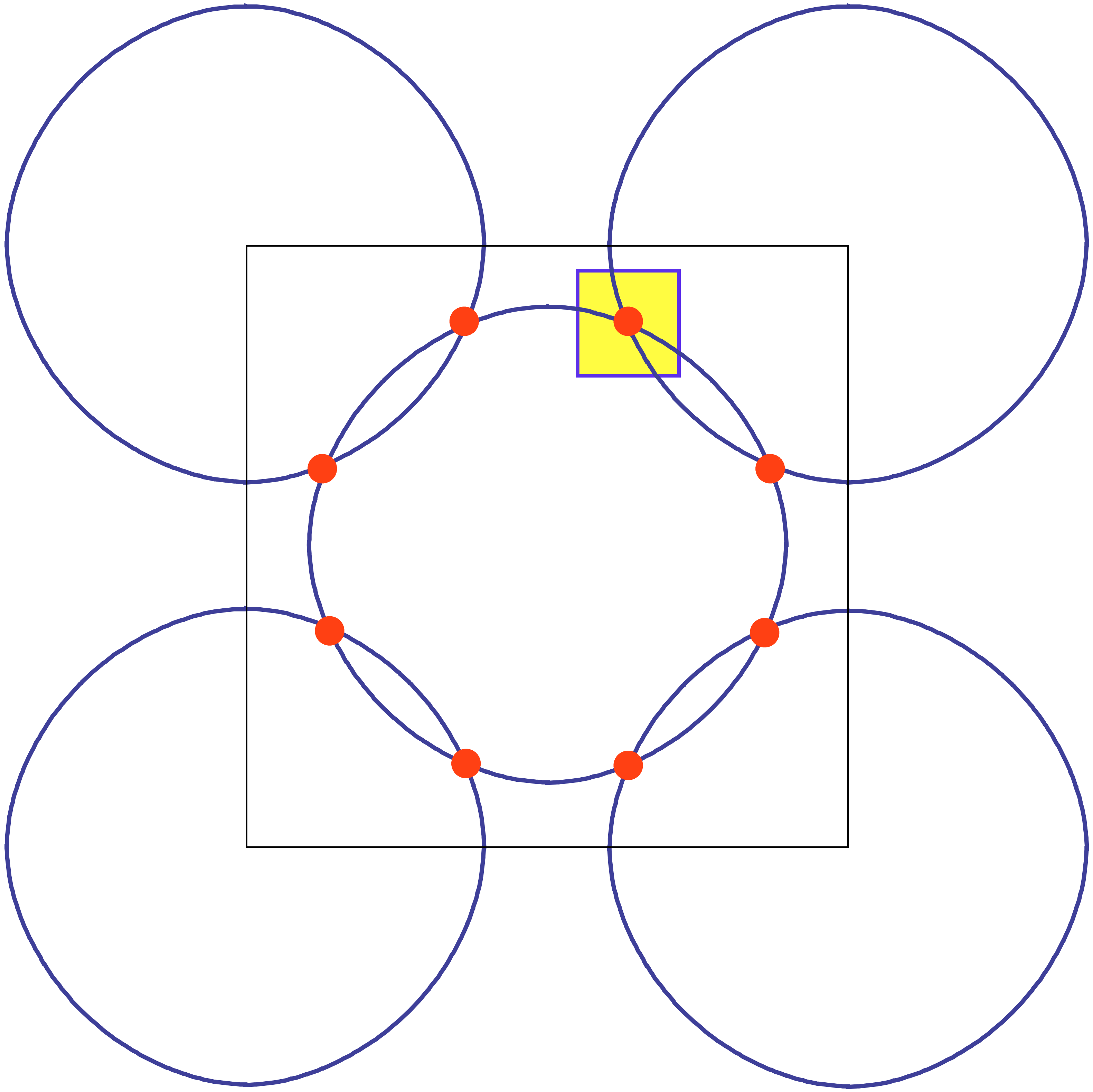
Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$



“Hot” spots

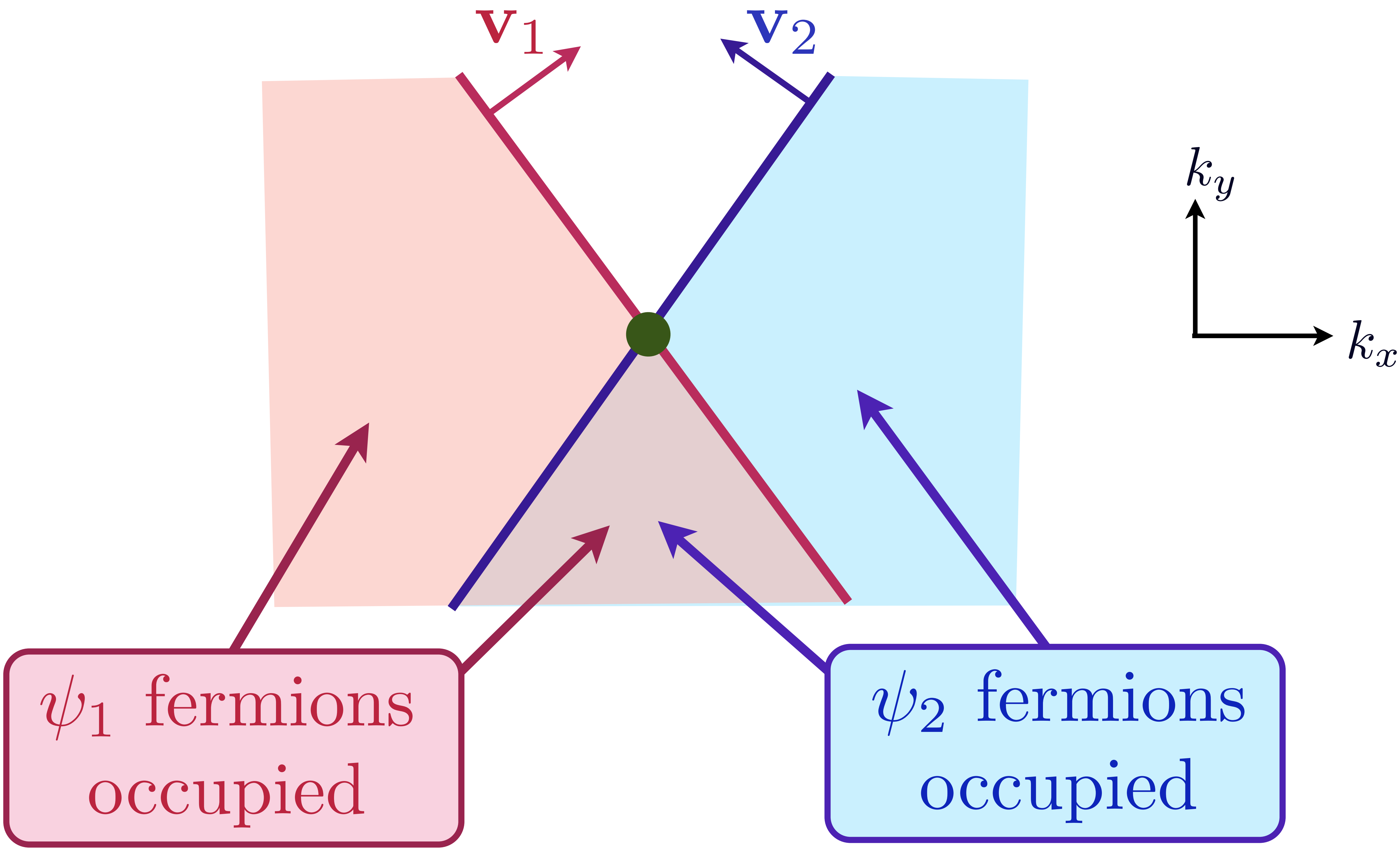


Low energy theory for critical point near hot spots

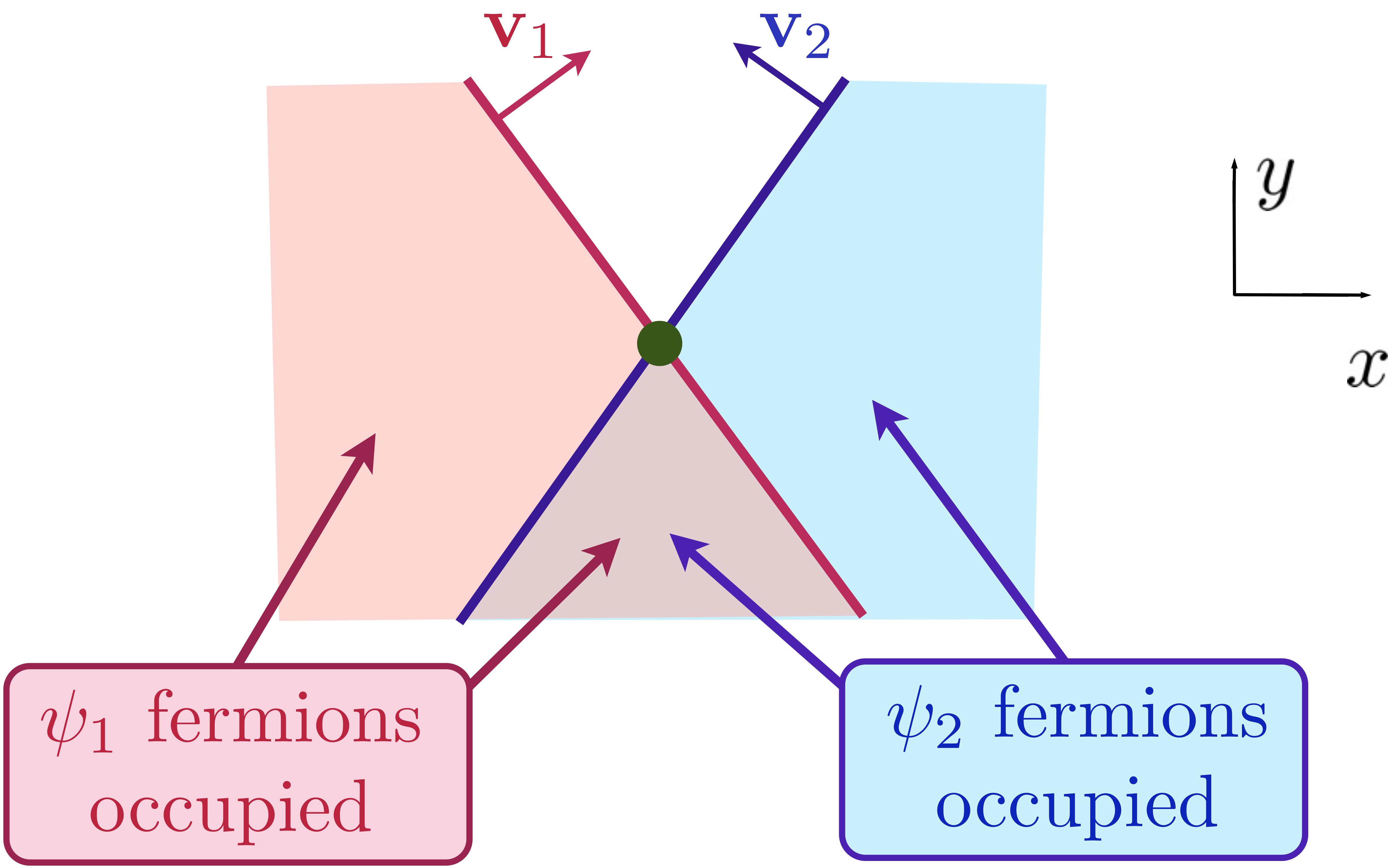


Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ



$$\mathcal{L}_f = \psi_{1\alpha}^\dagger \left(\zeta \partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\zeta \partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha}$$



Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

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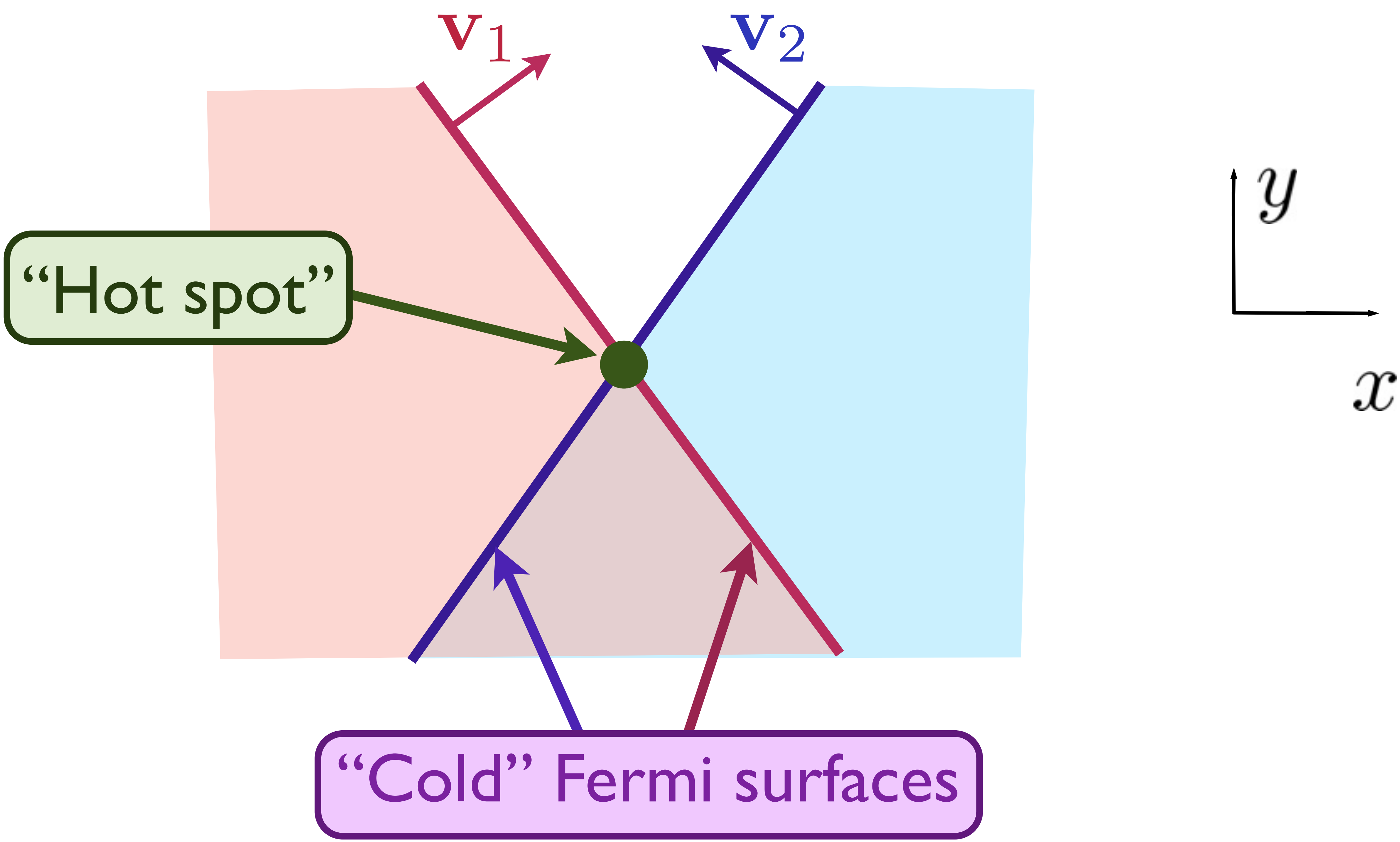

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Order parameter: $$\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$$

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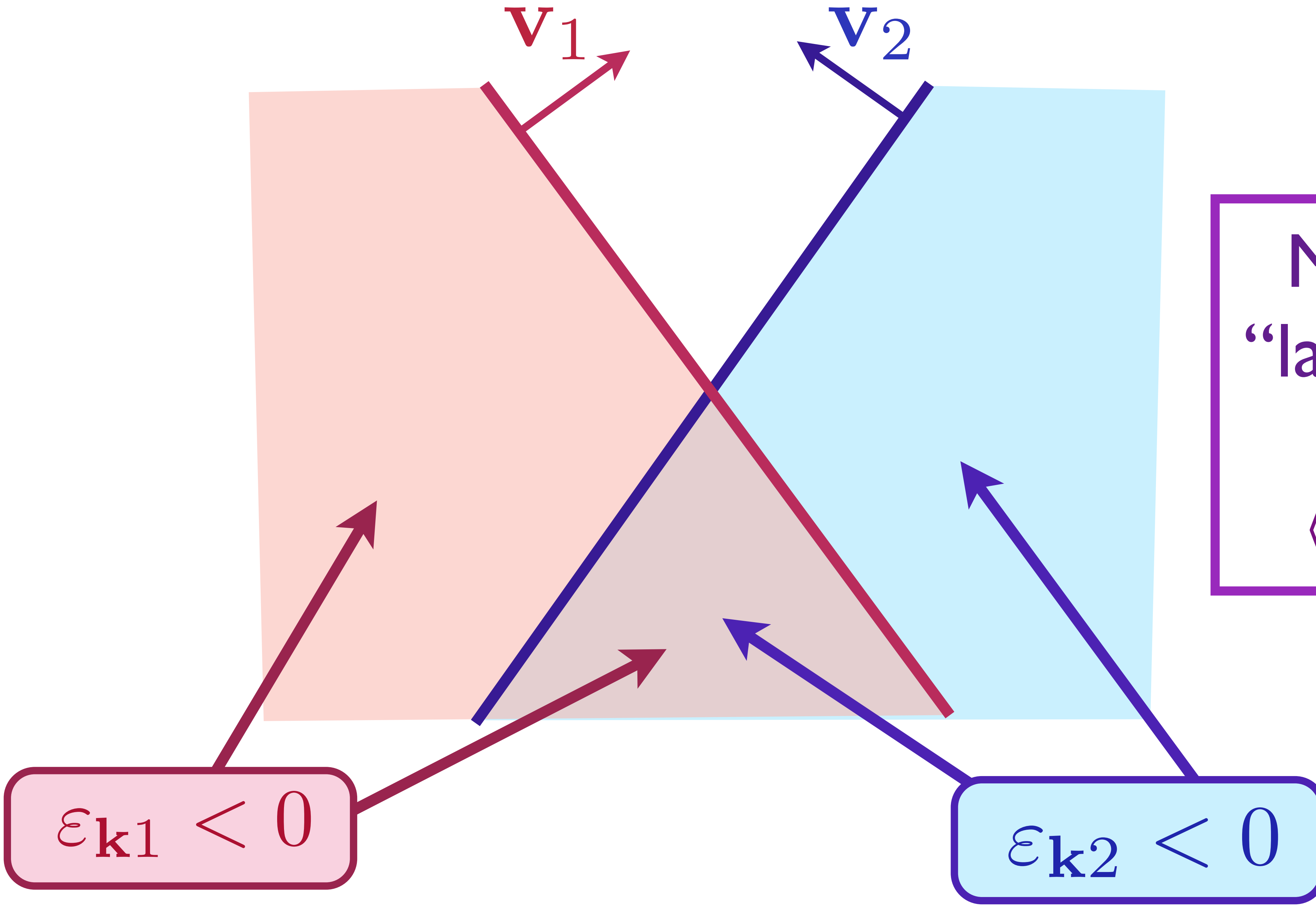
"Yukawa" coupling: $\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi^\dagger_{1\alpha}\vec{\sigma}_{\alpha\beta}\psi_{2\beta} + \psi^\dagger_{2\alpha}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}\right)$

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

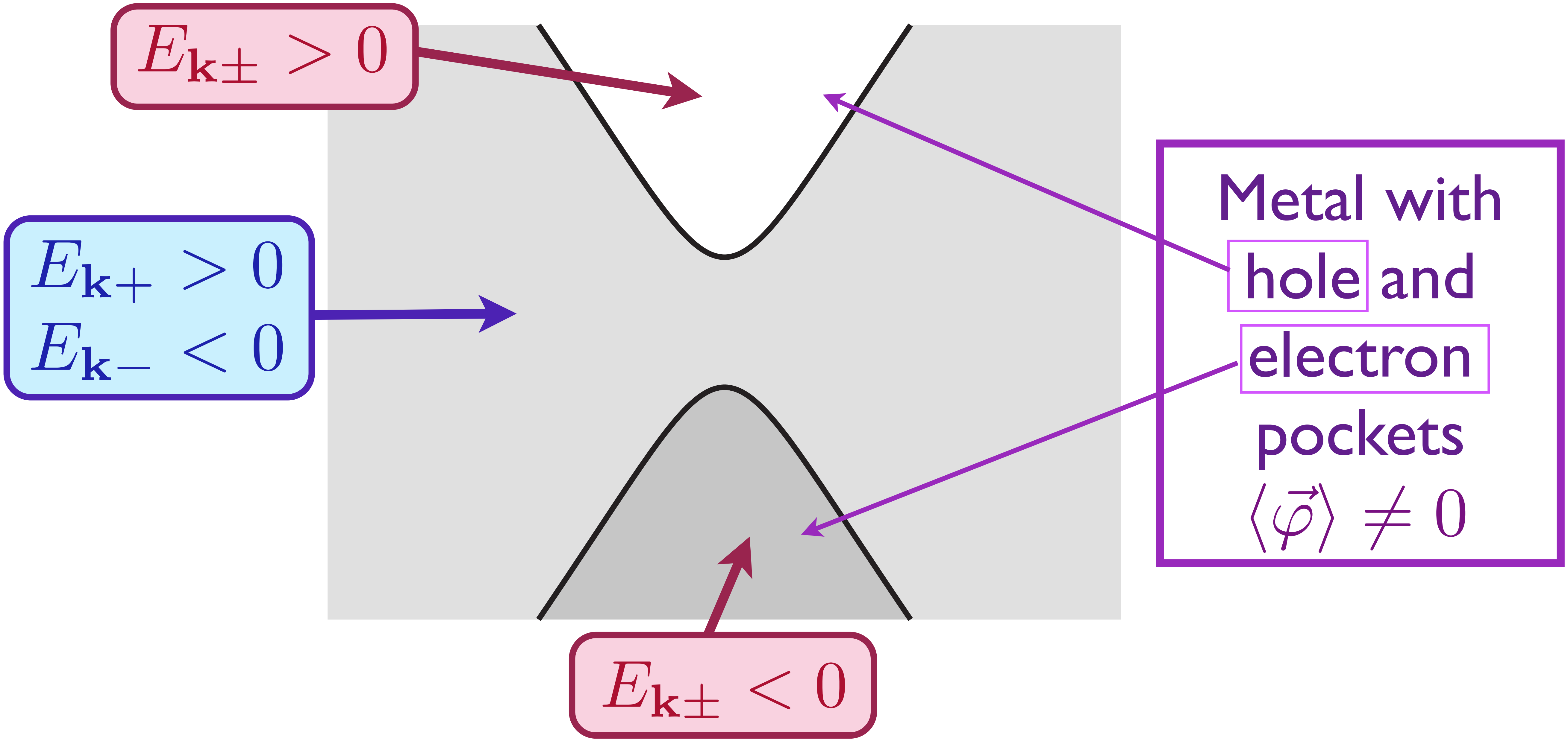
$$\mathcal{L}_f = \psi_{1\alpha}^\dagger \left(\zeta\partial_\tau - i\mathbf{v}_1 \cdot \mathbf{\nabla}_r\right)\psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\zeta\partial_\tau - i\mathbf{v}_2 \cdot \mathbf{\nabla}_r\right)\psi_{2\alpha}$$



Metal with "large" Fermi surface
$\langle \vec{\varphi} \rangle = 0$

Fermion dispersions: $\varepsilon_{\mathbf{k}1} = \mathbf{v}_1 \cdot \mathbf{k}$ and $\varepsilon_{\mathbf{k}2} = \mathbf{v}_2 \cdot \mathbf{k}$

$$\mathcal{L}_f = \psi_{1\alpha}^\dagger \left(\zeta \partial_\tau - i \mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\zeta \partial_\tau - i \mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha}$$
$$-\lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right)$$

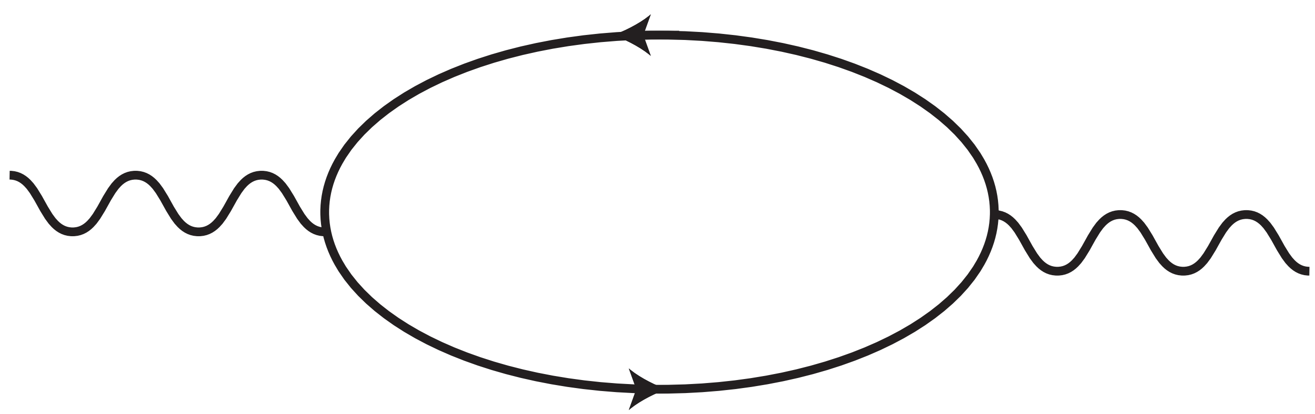


Fermion dispersions:

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}1} + \varepsilon_{\mathbf{k}2}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}1} - \varepsilon_{\mathbf{k}2}}{2}\right)^2 + \lambda^2 |\vec{\varphi}|^2}$$

Hertz action.

Upon integrating the fermions out, the leading term in the $\vec{\varphi}$ effective action is $-\Pi(q,\omega_n)|\vec{\varphi}(q,\omega_n)|^2$, where $\Pi(q,\omega_n)$ is the fermion polarizability. This is given by a simple fermion loop diagram



$$\Pi(q,\omega_n) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d\epsilon_n}{2\pi} \frac{1}{[-i\zeta(\epsilon_n+\omega_n)+\mathbf{v}_1\cdot(\mathbf{k}+\mathbf{q})][-i\zeta\epsilon_n+\mathbf{v}_2\cdot\mathbf{k}]} \tag{1}$$

We define oblique co-ordinates $p_1 = \mathbf{v}_1 \cdot \mathbf{k}$ and $p_2 = \mathbf{v}_2 \cdot \mathbf{k}$. It is then clear that the integrand in (1) is independent of the $(d-2)$ transverse momenta, whose integral yields an overall factor Λ^{d-2} (in $d=2$ this factor is precisely 1). Also, by shifting the integral over k_1 we note that the integral is independent of q. So we have

$$\Pi(q,\omega_n) = \frac{\Lambda^{d-2}}{|\mathbf{v}_1 \times \mathbf{v}_2|} \int \frac{dp_1 dp_2 d\epsilon_n}{8\pi^3} \frac{1}{[-i\zeta(\epsilon_n+\omega_n)+p_1][-i\zeta\epsilon_n+p_2]}. \tag{2}$$

Next, we evaluate the frequency integral to obtain

$$\begin{aligned} \Pi(q,\omega_n) &= \frac{\Lambda^{d-2}}{\zeta|\mathbf{v}_1 \times \mathbf{v}_2|} \int \frac{dp_1 dp_2}{4\pi^2} \frac{[\text{sgn}(p_2)-\text{sgn}(p_1)]}{-i\zeta\omega_n+p_1-p_2} \\ &= -\frac{|\omega_n|\Lambda^{d-2}}{4\pi|\mathbf{v}_1 \times \mathbf{v}_2|}. \end{aligned} \tag{3}$$

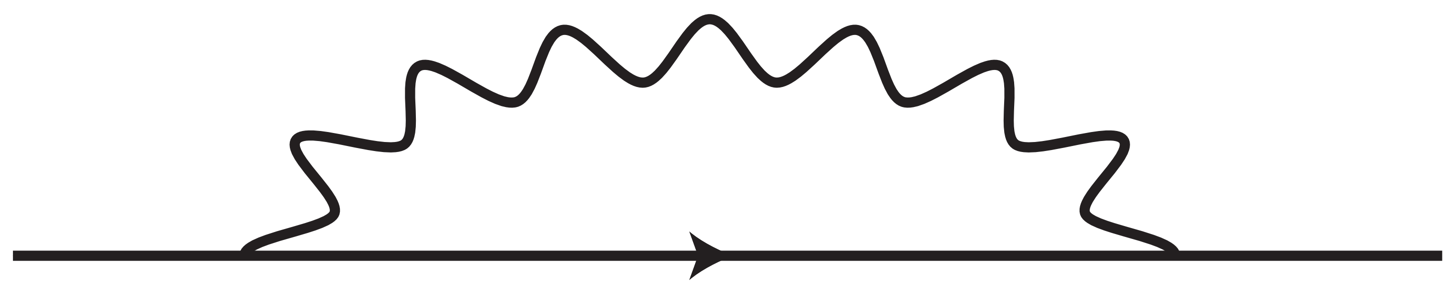
In the last step, we have dropped a frequency-independent, cutoff-dependent constant which can absorbed into a redefinition of r. Notice also that the factor of ζ has cancelled.
Inserting this fermion polarizability in the effective action for $\vec{\varphi}$, we obtain the Hertz action for the SDW transition:

$$\begin{aligned} \mathcal{S}_H = \int \frac{d^d k}{(2\pi)^d} T \sum_{\omega_n} \frac{1}{2} \left[k^2 + \gamma|\omega_n| + s\right] |\vec{\varphi}(k,\omega_n)|^2 \\ + \frac{u}{4} \int d^d x d\tau \left(\vec{\varphi}^2(x,\tau)\right)^2. \end{aligned} \tag{4}$$

Exercise: Perform a tree-level RG rescaling on $\mathcal{S}_H$. Now we rescale co-ordinates as $x' = xe^{-\ell}$ and $\tau' = \tau e^{-z\ell}$. Here z is the dynamic critical exponent. Show that the gradient and non-local terms become invariant for $z = 2$ (previous theories considered here had $z = 1$). Then show that the transformation of the quartic term is $u' = ue^{(2-d)\ell}$. This led Hertz to conclude that the SDW quantum critical point was described by a Gaussian theory for the SDW order parameter in $d \geq 2$.

Fate of the fermions.

Let us, for now, assume the validity of the Hertz Gaussian action, and compute the leading correction to the electronic Green's function. This is given by the following Feynman graph for the electron self energy, Σ. At zero momentum for the ψ_1 fermion we have



$$\Sigma_1(0,\omega_n) = \lambda^2 \int \frac{d^d q}{(2\pi)^d} \int \frac{d\epsilon_n}{2\pi} \frac{1}{\left[q^2 + \gamma|\epsilon_n|\right]\left[-i\zeta(\epsilon_n+\omega_n) + \mathbf{v}_2 \cdot \mathbf{q}\right]}. \tag{5}$$

We first perform the integral over the $\mathbf{q}$ direction parallel to $\mathbf{v}_2$, while ignoring the subdominant dependence on this momentum in the boson propagator. The dependence on ζ immediately disappears, and we have

$$\Sigma_1(0,\omega_n) = i\,\frac{\lambda^2}{|v_2|}\int \frac{d^{d-1}q}{(2\pi)^{d-1}}\int \frac{d\epsilon_n}{2\pi}\frac{\text{sgn}(\epsilon_n+\omega_n)}{|q|^2+\gamma|\epsilon_n|}$$
$$= i\,\frac{\lambda^2}{\pi|v_2|\gamma}\text{sgn}(\omega_n)\int \frac{d^{d-1}q}{(2\pi)^{d-1}}\ln\left(\frac{|q|^2+\gamma|\omega_n|}{|q|^2}\right). \quad (6)$$

Evaluation of the q integral shows that

$$\Sigma_1(0,\omega_n) \sim |\omega_n|^{(d-1)/2} \quad (7)$$

The most important case is $d = 2$, where we have

$$\Sigma_1(0,\omega_n) = i\,\frac{\lambda^2}{\pi|v_2|\sqrt{\gamma}}\text{sgn}(\omega_n)\sqrt{|\omega_n|} \quad , \quad d = 2. \quad (8)$$

Strong coupling physics in $d = 2$

The theory so far has the boson propagator

$$\sim \frac{1}{q^2 + \gamma|\omega|}$$

which scales with dynamic exponent $z_b = 2$, and now a fermion propagator

$$\sim \frac{1}{-i\zeta\omega + c_1|\omega|^{(d-1)/2} + \mathbf{v}\cdot\mathbf{q}}.$$

First note that for $d < 3$, the bare $-i\zeta\omega$ term is less important than the contribution from the self energy at low frequencies. This indicates that ζ is *irrelevant* in the critical theory, and we can set $\zeta \to 0$. Fortunately, all the loop diagrams evaluated so far are independent of ζ.

Setting $\zeta = 0$, we see that the fermion propagator scales with dynamic exponent $z_f = 2/(d-1)$. For $d > 2$, $z_f < z_b$, and so at small momenta the boson fluctuations have lower energy than the fermion fluctuations. Thus it seems reasonable to assume that the fermion fluctuations are not as singular, and we can focus on an effective theory of the SDW order parameter $\vec{\varphi}$ alone. In other words, the Hertz assumptions appear valid for $d > 2$.

However, in $d = 2$, we have $z_f = z_b = 2$. Thus fermionic and bosonic fluctuations are equally important, and it is not appropriate to integrate the fermions out at an initial stage. We have to return to the original theory of coupled bosons and fermions. This turns out to be strongly coupled, and exhibits complex critical behavior. For more details, see

M. A. Metlitski and S. Sachdev, arXiv:1005.1288 (Physical Review B **82**, 075127 (2010)).

$$\mathcal{L}_f = \psi_{1\alpha}^\dagger \left(\zeta\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\zeta\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha}$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

"Yukawa" coupling: $\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta}\right)$

Perform RG on both fermions and $\vec{\varphi}$, using a *local* field theory.

$$\mathcal{L}_f = \psi_{1\alpha}^\dagger \left(\zeta \partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\zeta \partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha}$$

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Under the rescaling $x' = xe^{-\ell}$, $\tau' = \tau e^{-z\ell}$, the spatial gradients are fixed if the fields transform as

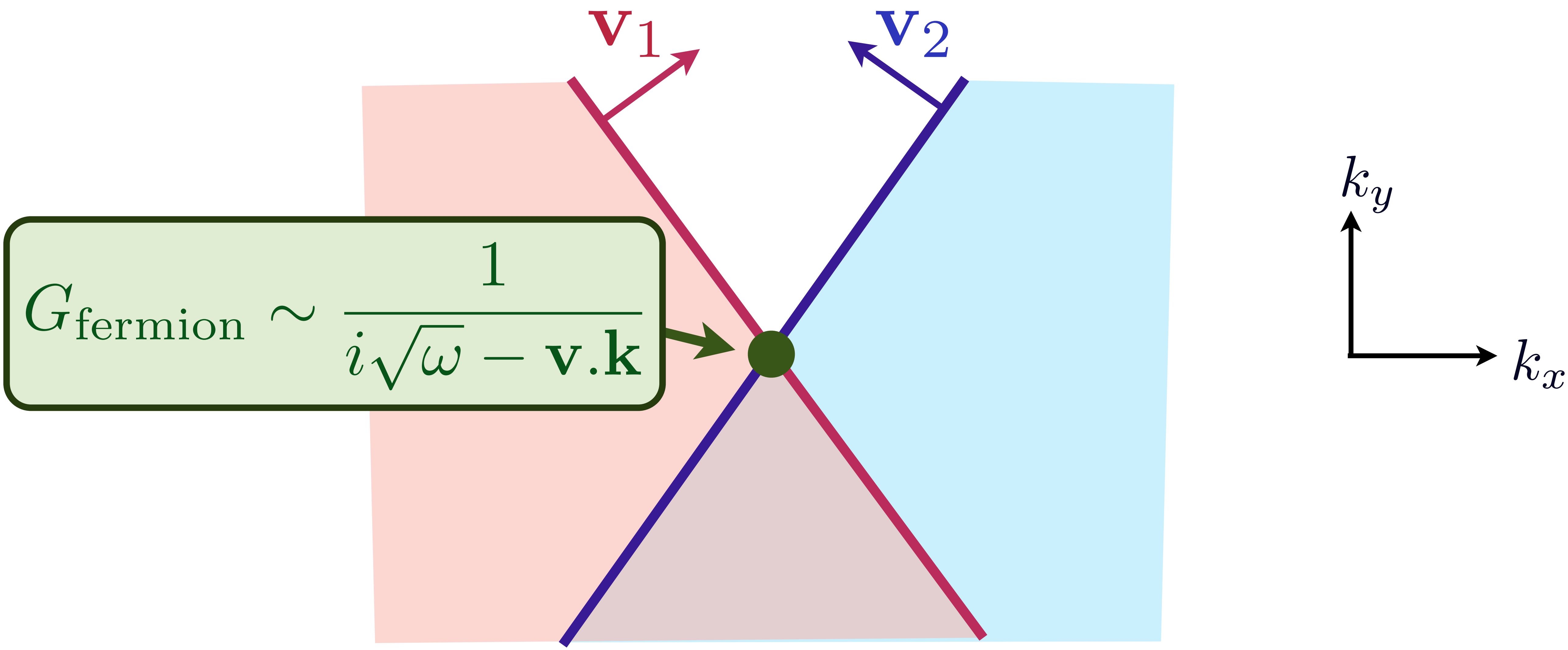
$$\vec{\varphi}' = e^{(d+z-2)\ell/2} \vec{\varphi} \quad ;\text{"} \quad \psi' = e^{(d+z-1)\ell/2} \psi.$$

Then the Yukawa coupling transforms as

$$\lambda' = e^{(4-d-z)\ell/2} \lambda$$

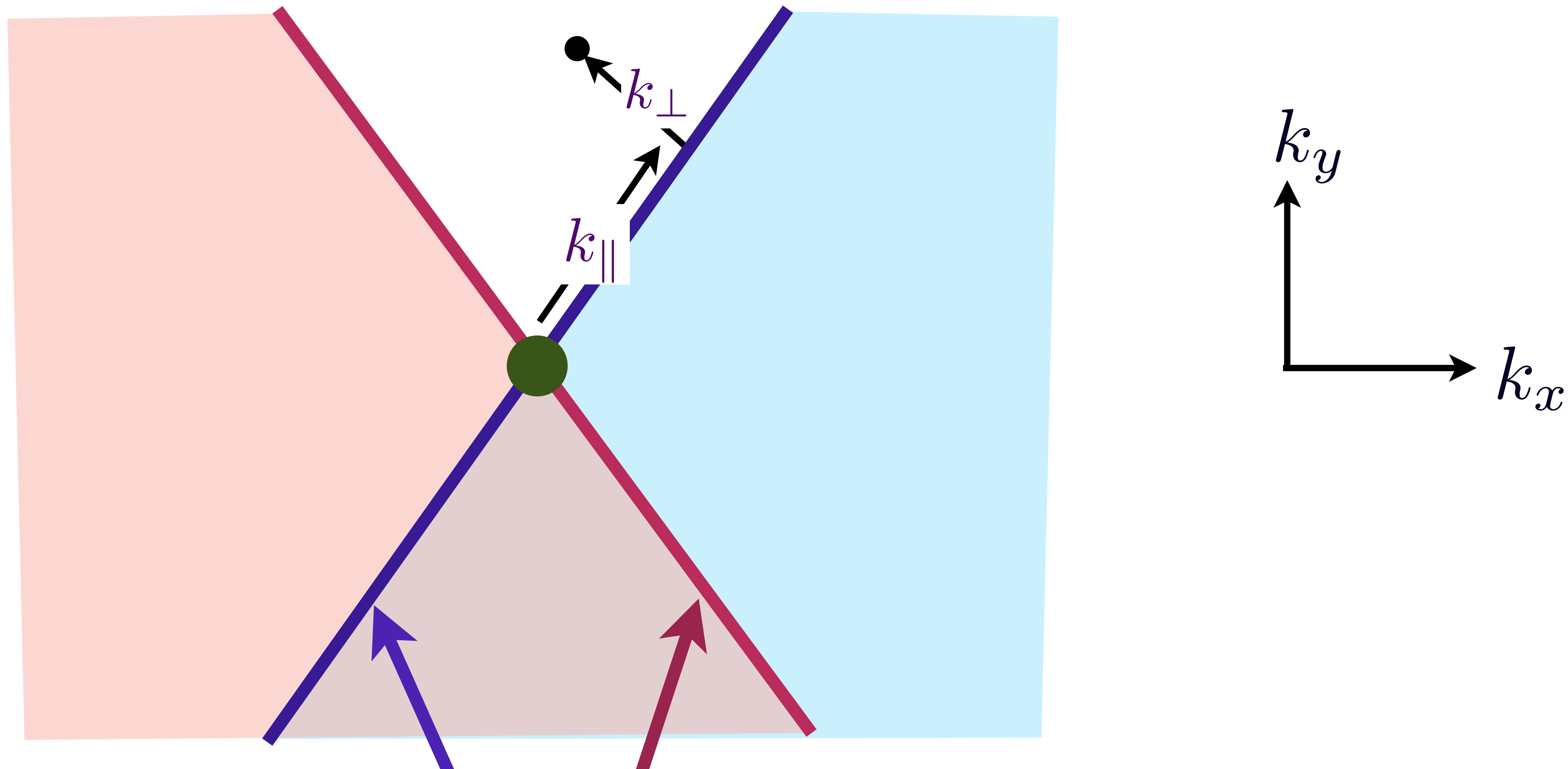
For $d = 2$, with $z = 2$ the bare time-derivative terms ζ, $\widetilde{\zeta}$ are irrelevant, but the Yukawa coupling is invariant. Thus we have to work at fixed $\lambda = 1$, and cannot expand in powers of λ: critical theory is *strongly coupled.*

Critical point theory is strongly coupled in $d = 2$
Results are *independent* of coupling λ



A. J. Millis, *Phys. Rev. B* **45**, 13047 (1992)
Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004)

Critical point theory is strongly coupled in $d = 2$
Results are *independent* of coupling λ



$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{i\omega - v_F(k_\parallel)k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Outline

1. Weak coupling theory of SDW ordering, and d-wave superconductivity

2. Universal critical theory of SDW ordering

3. Emergent pseudospin symmetry, and quadrupolar density wave

4. Quantum Monte Carlo without the sign problem

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4. Quantum Monte Carlo without the sign problem

Emergent $[SU(2)]^4$ pseudospin symmetry

$$\mathcal{L}_f = \psi^\dagger_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}_1\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha} + \psi^\dagger_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}_2\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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Introduce the spinors

$$\Psi_{1\alpha} = \begin{pmatrix} \psi_{1\alpha} \\ \epsilon_{\alpha\beta}\psi^\dagger_{1\beta} \end{pmatrix} \quad , \quad \Psi_{2\alpha} = \begin{pmatrix} \psi_{2\alpha} \\ \epsilon_{\alpha\beta}\psi^\dagger_{2\beta} \end{pmatrix}$$

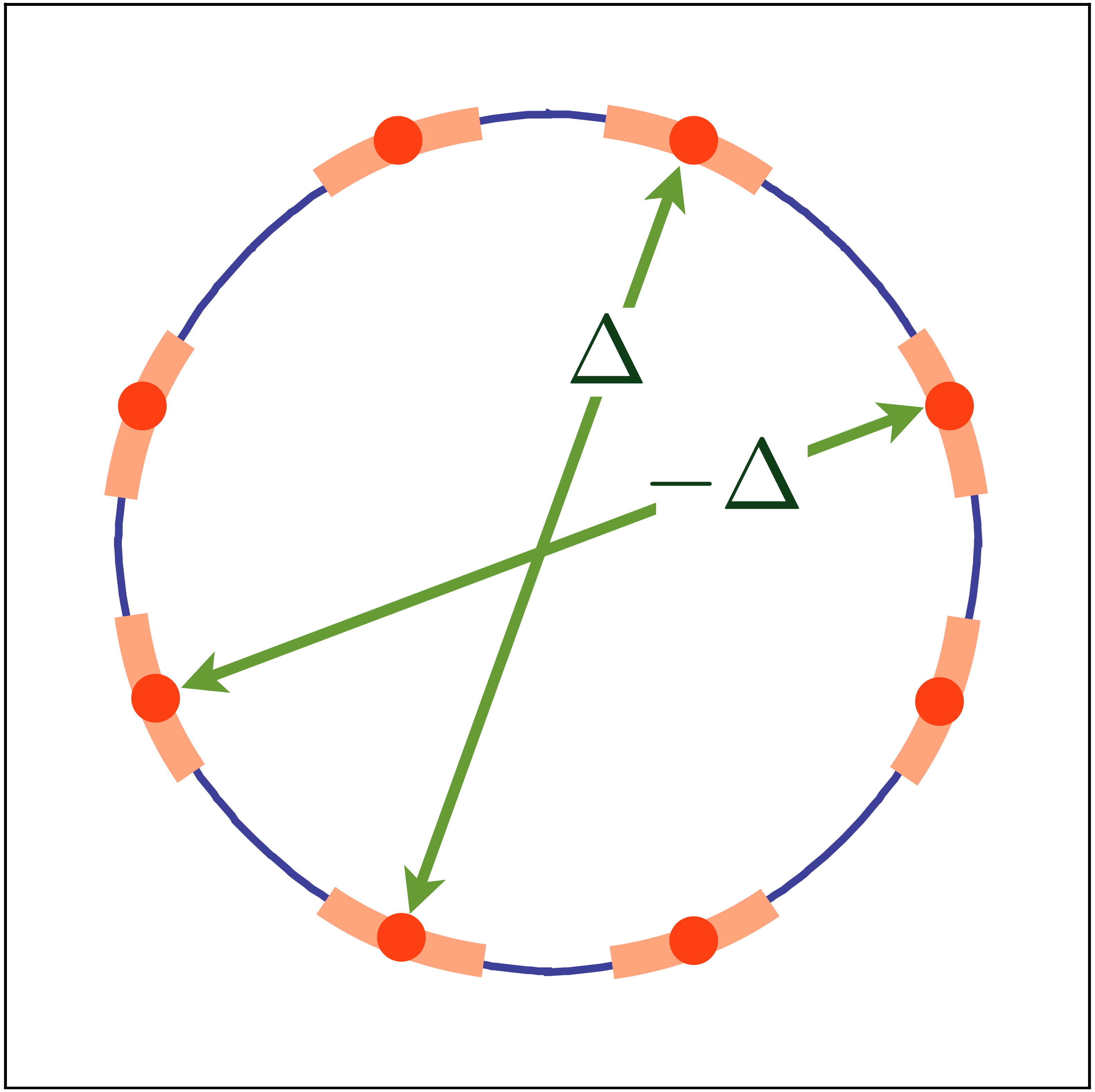
Then the Lagrangian is invariant under the SU(2) transformation U with

$$\Psi_1 \to U\Psi_1 \quad , \quad \Psi_2 \to U\Psi_2$$

Note that U can be chosen *independently* at the 4 pairs of hotspots.

This symmetry relies on the linearization of the fermion dispersion about the hot spots.

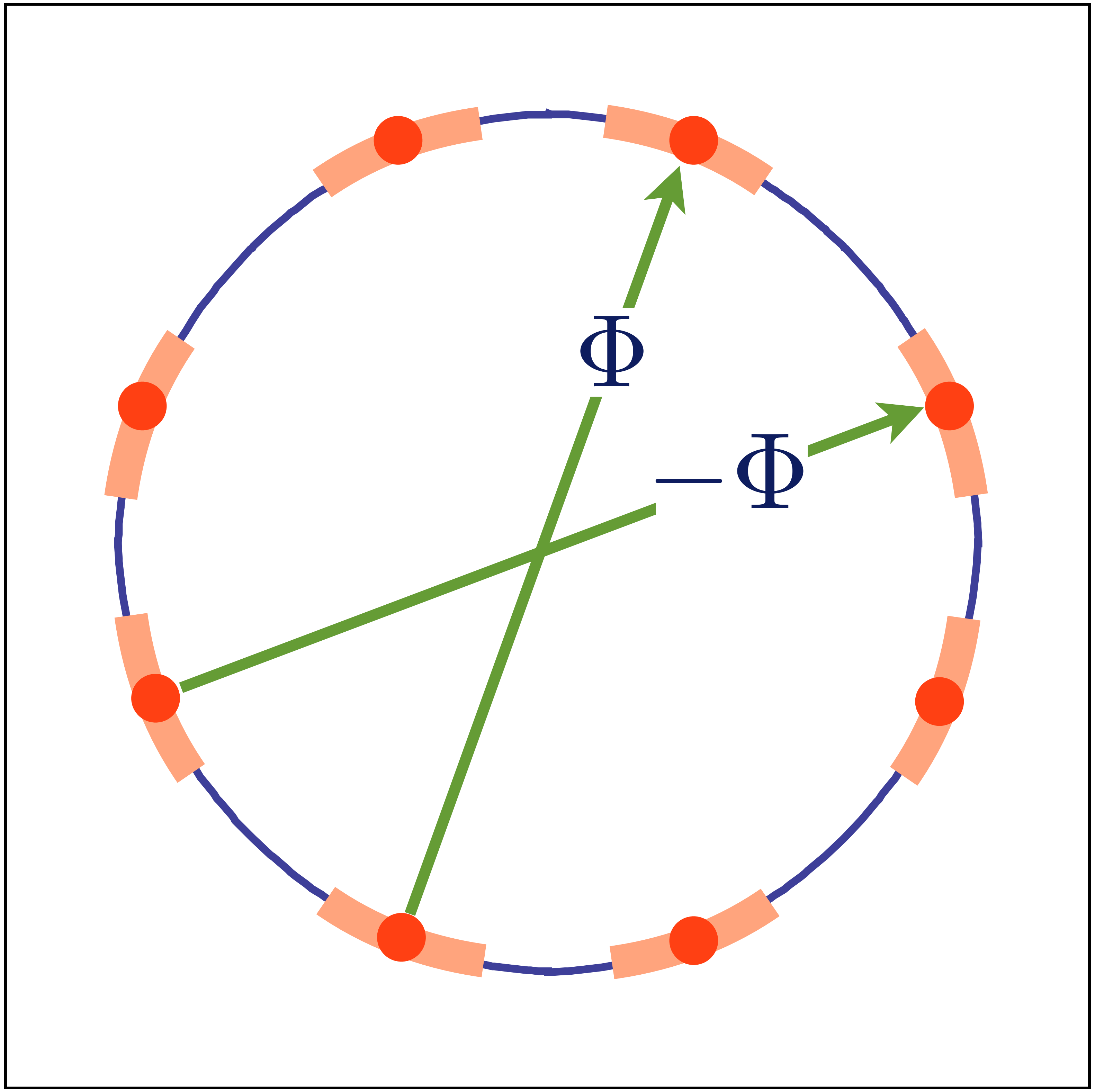
$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta(\cos k_x - \cos k_y)$$



Unconventional pairing at *and near* hot spots

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

After pseudospin rotation



$\mathbf{Q}$ is '$2k_F$' wavevector

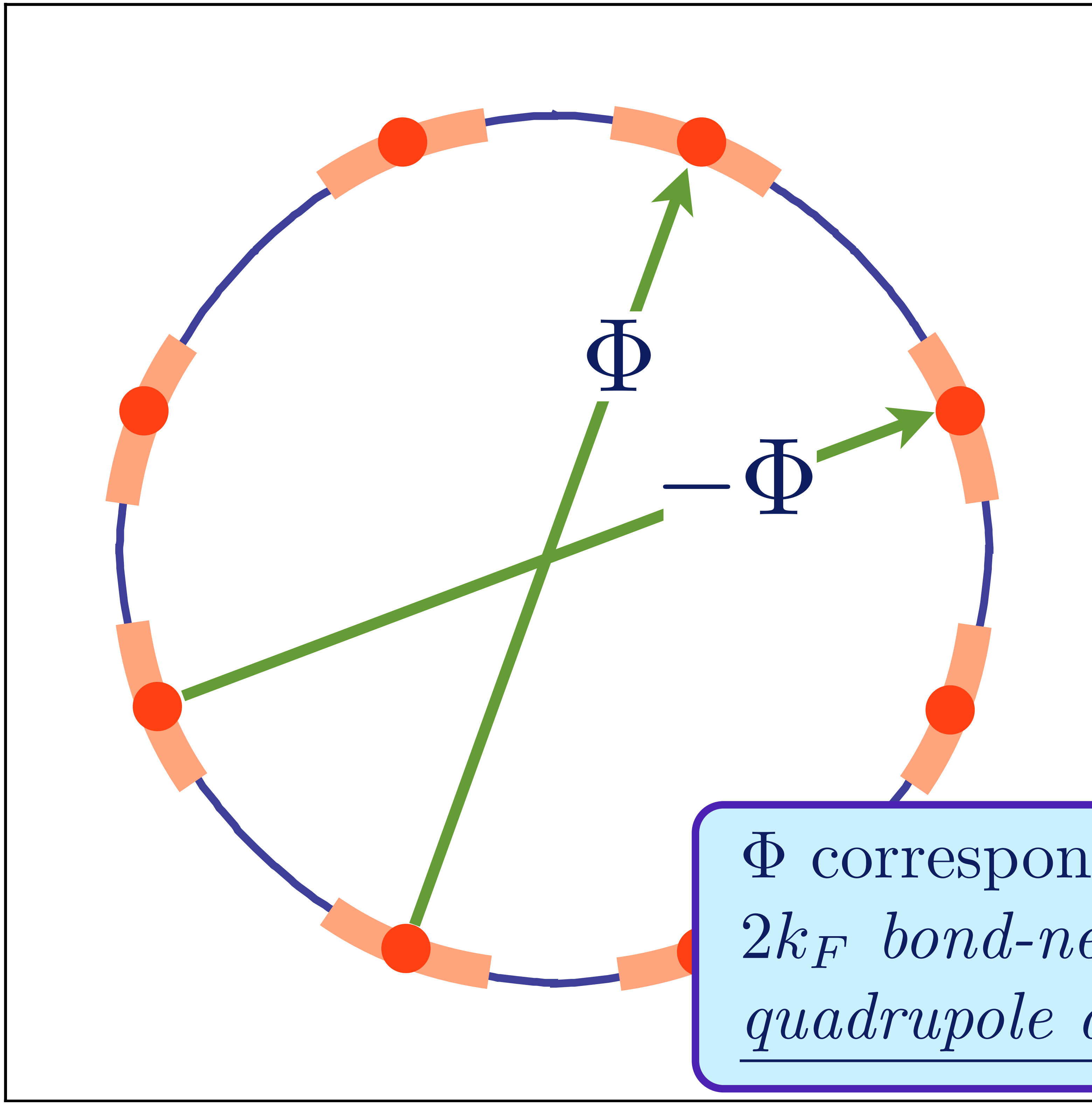
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

K. B. Efetov, H. Meier, and C. Pepin, arXiv:1210.3276

Unconventional particle-hole pairing at *and near* hot spots

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

After pseudospin rotation



$\mathbf{Q}$ is '$2k_F$' wavevector

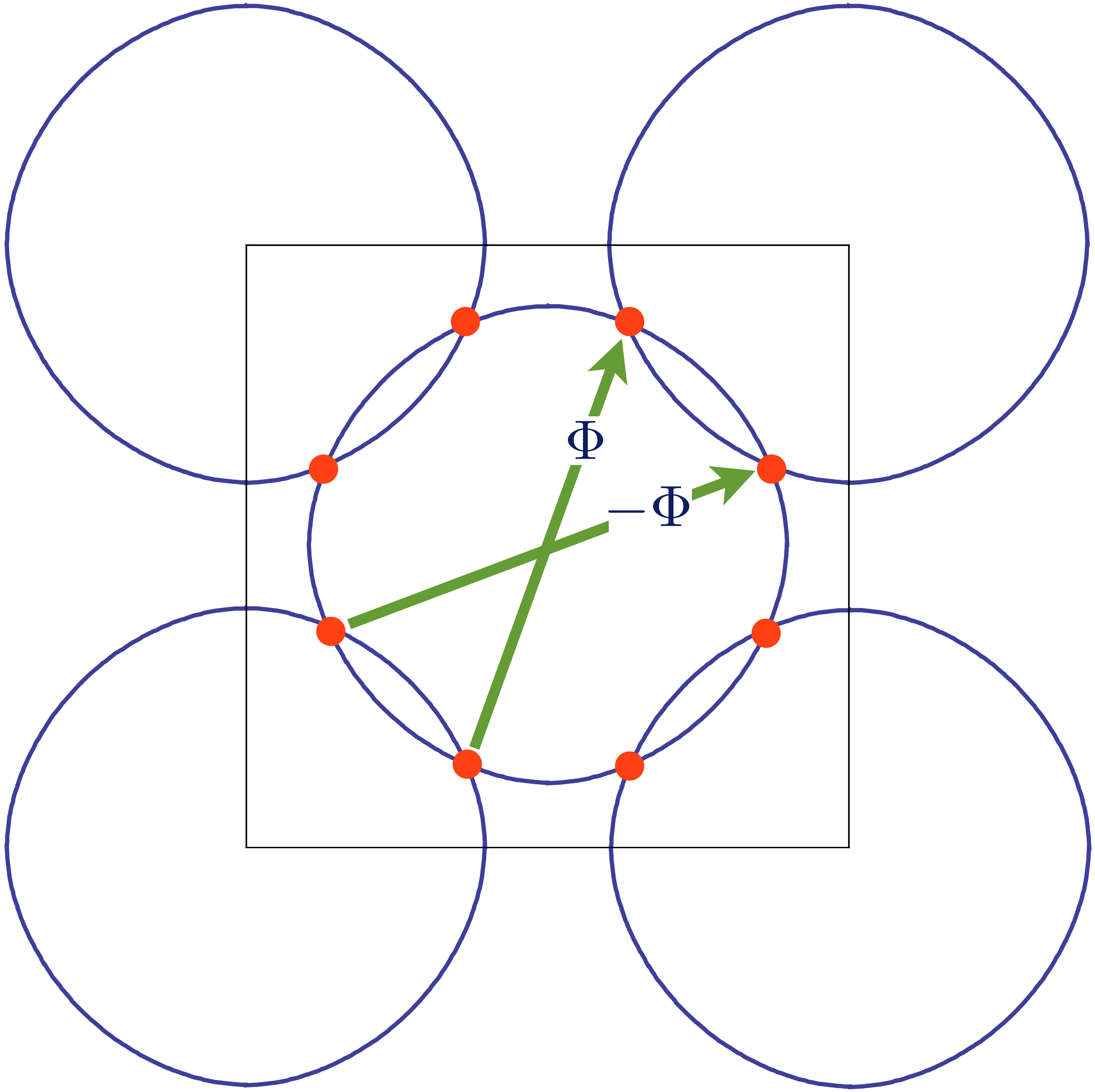
Φ corresponds to a $2k_F$ *bond-nematic* or a *quadrupole density wave*

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

K. B. Efetov, H. Meier, and C. Pepin, arXiv:1210.3276

Unconventional particle-hole pairing at *and near* hot spots

Quadrupole density wave



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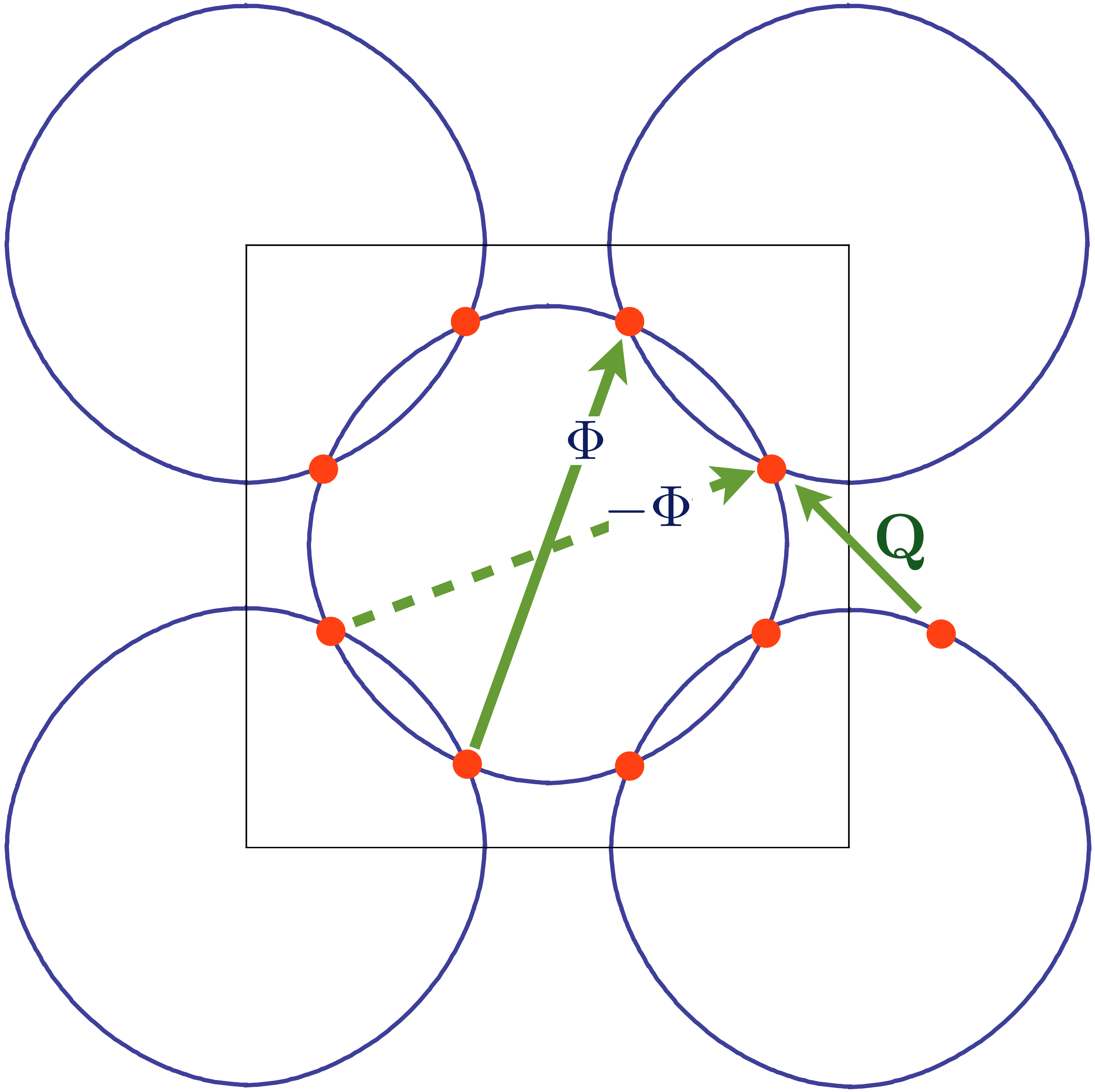

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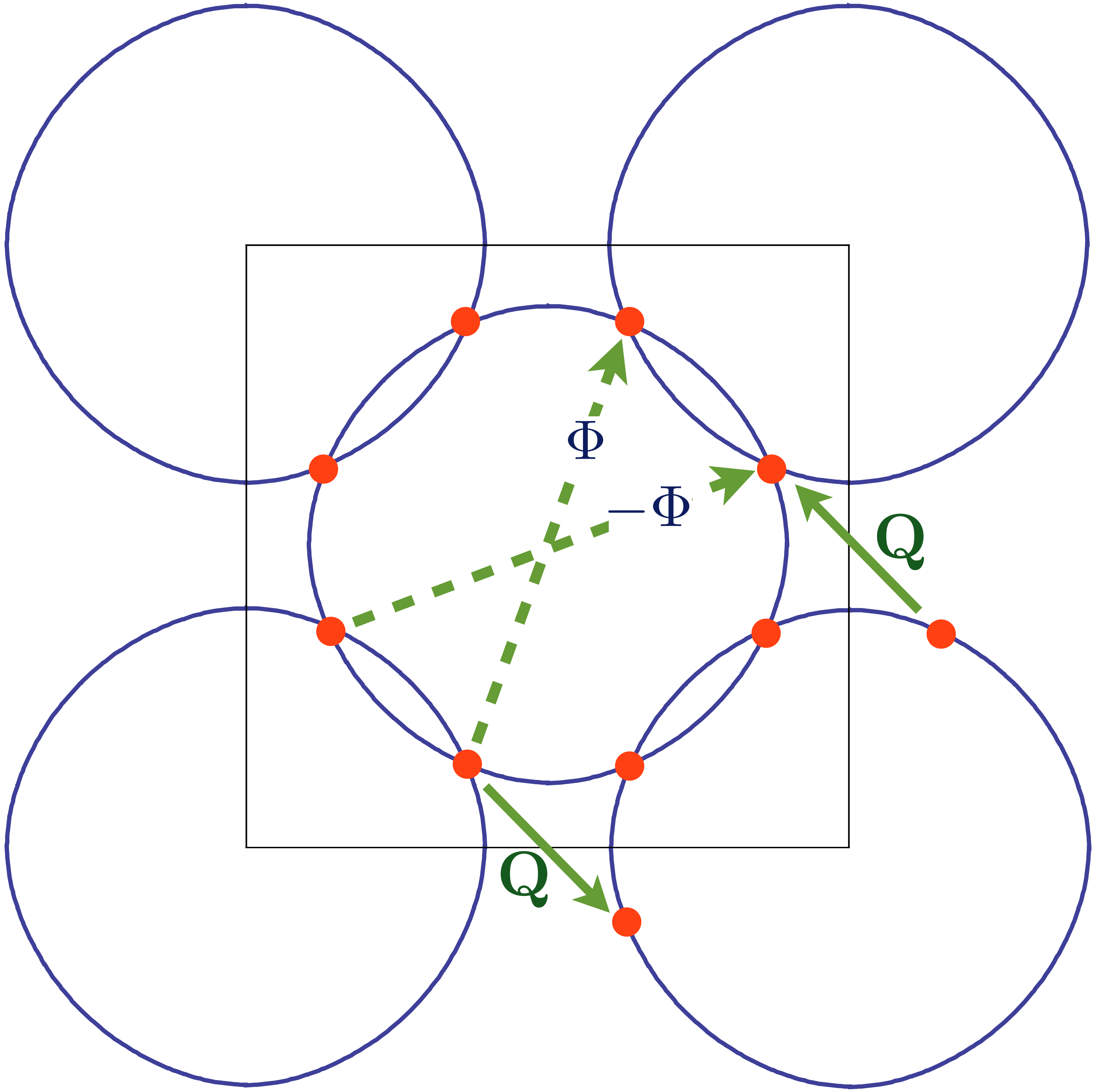

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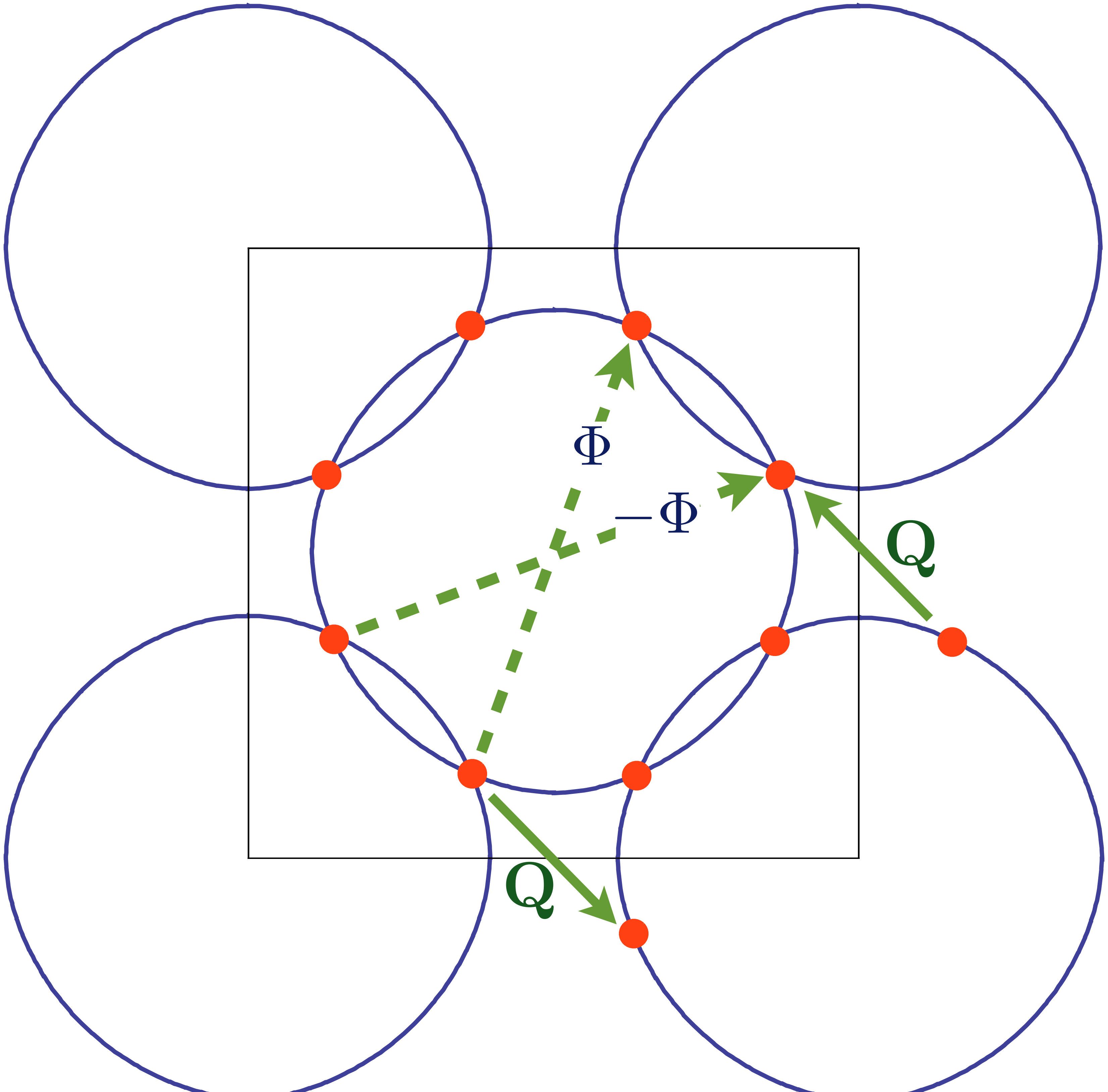
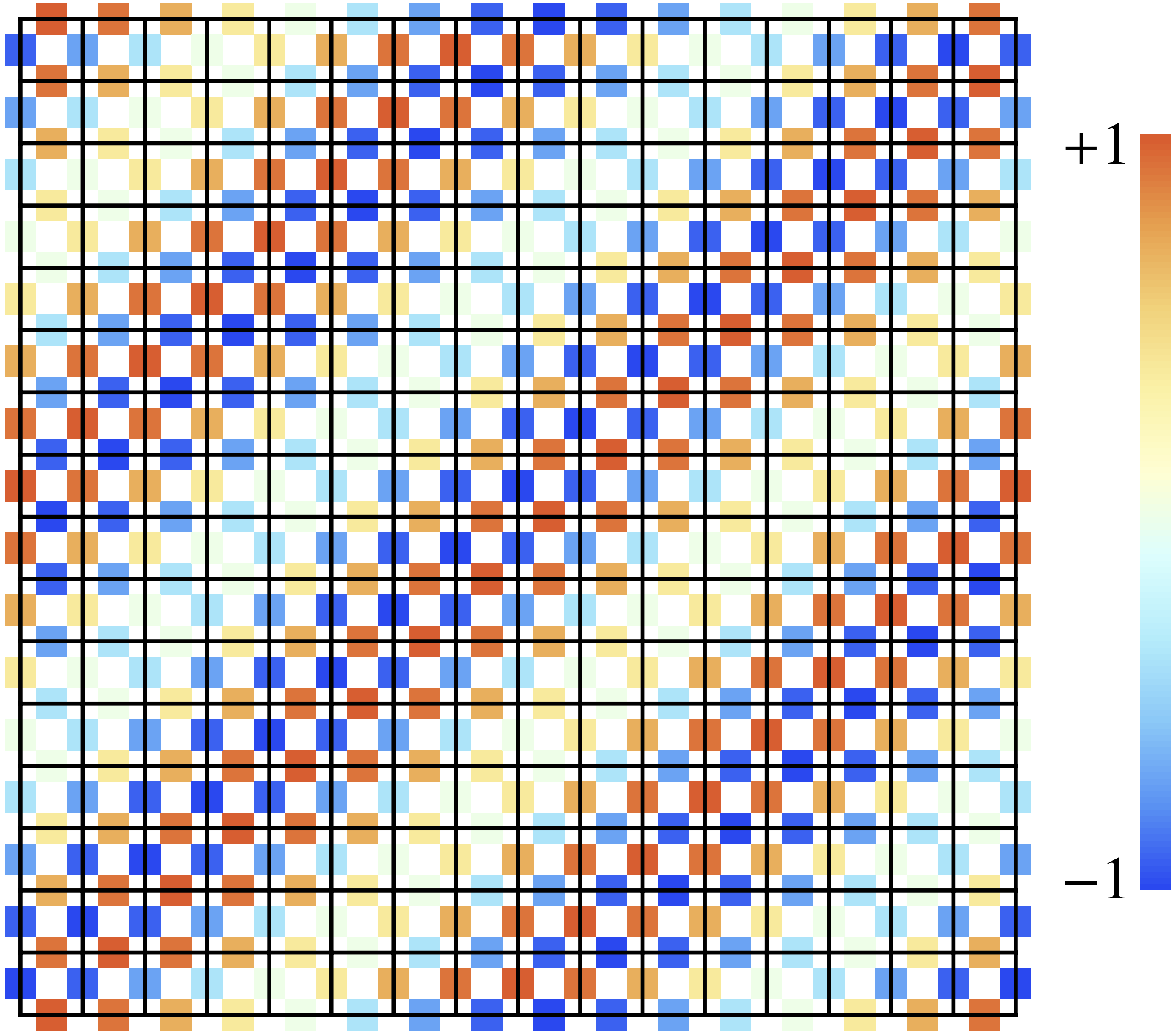


$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

Quadrupole density wave



"Bond density" measures amplitude for electrons to be in spin-singlet valence bond.

No modulations on sites, $\langle c^{\dagger}_{\mathbf{r}\alpha} c_{\mathbf{s}\alpha} \rangle$ is modulated only for $\mathbf{r} \neq \mathbf{s}$.

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

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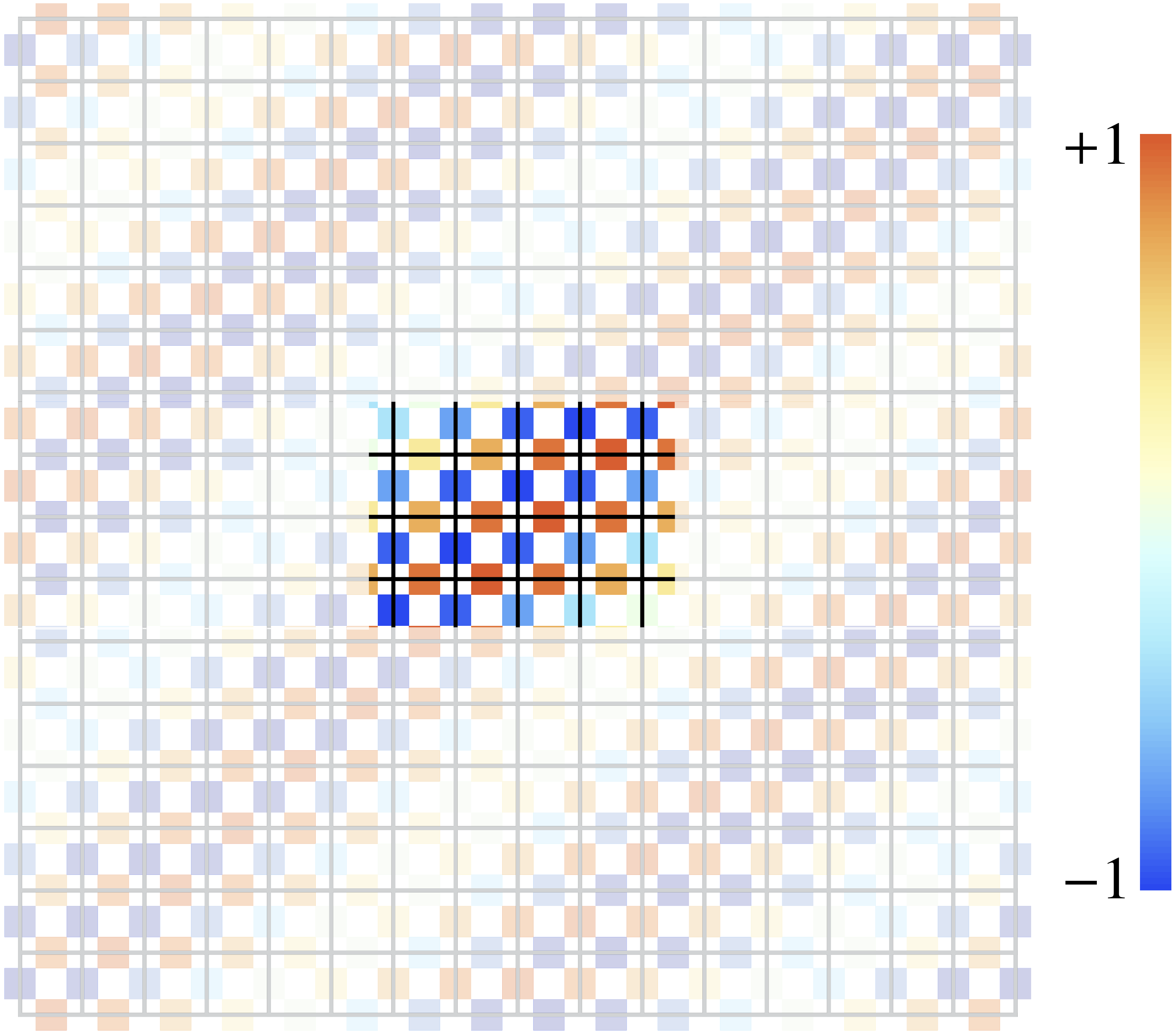

Local Ising nematic order with an envelope which oscillates

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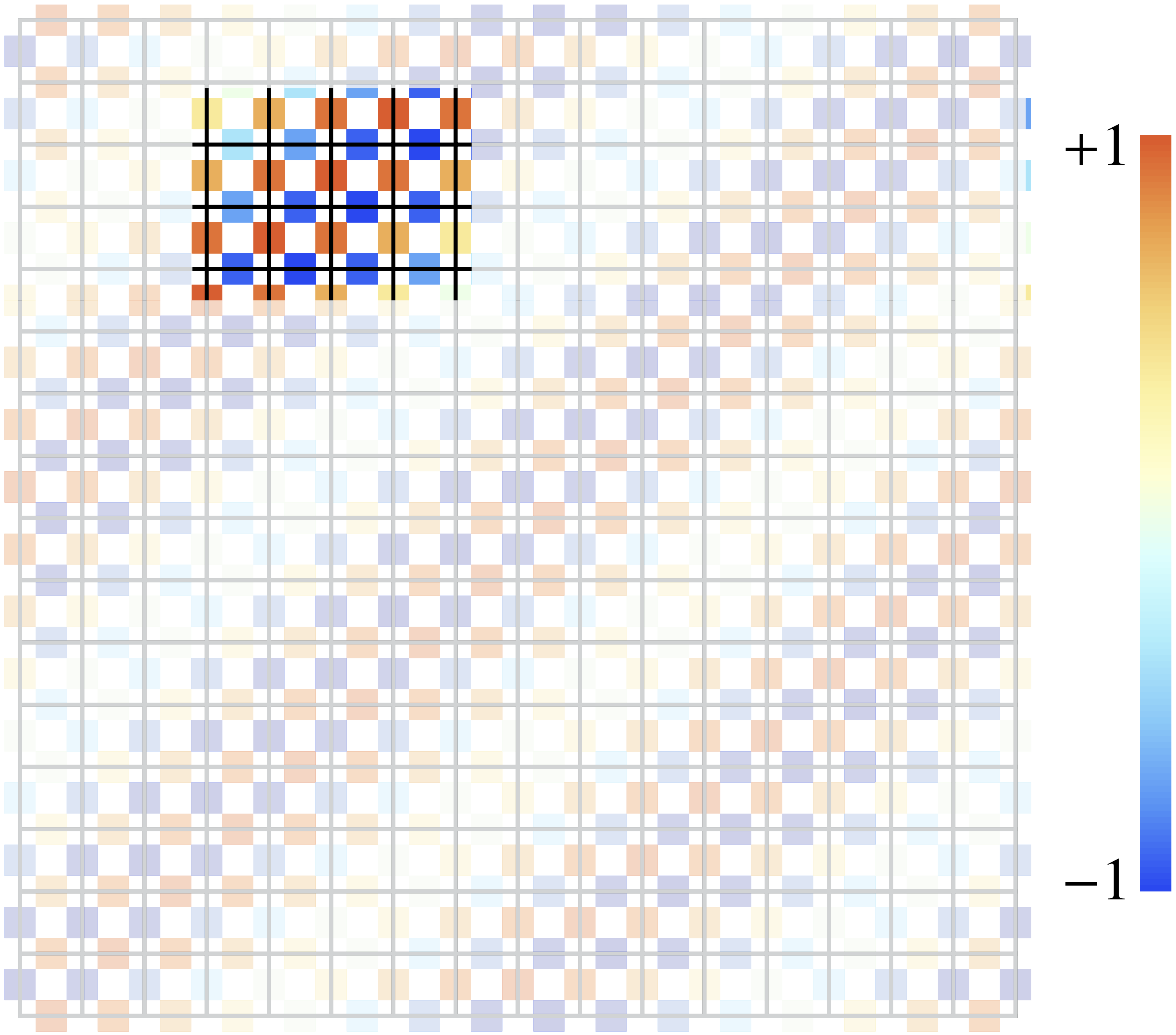

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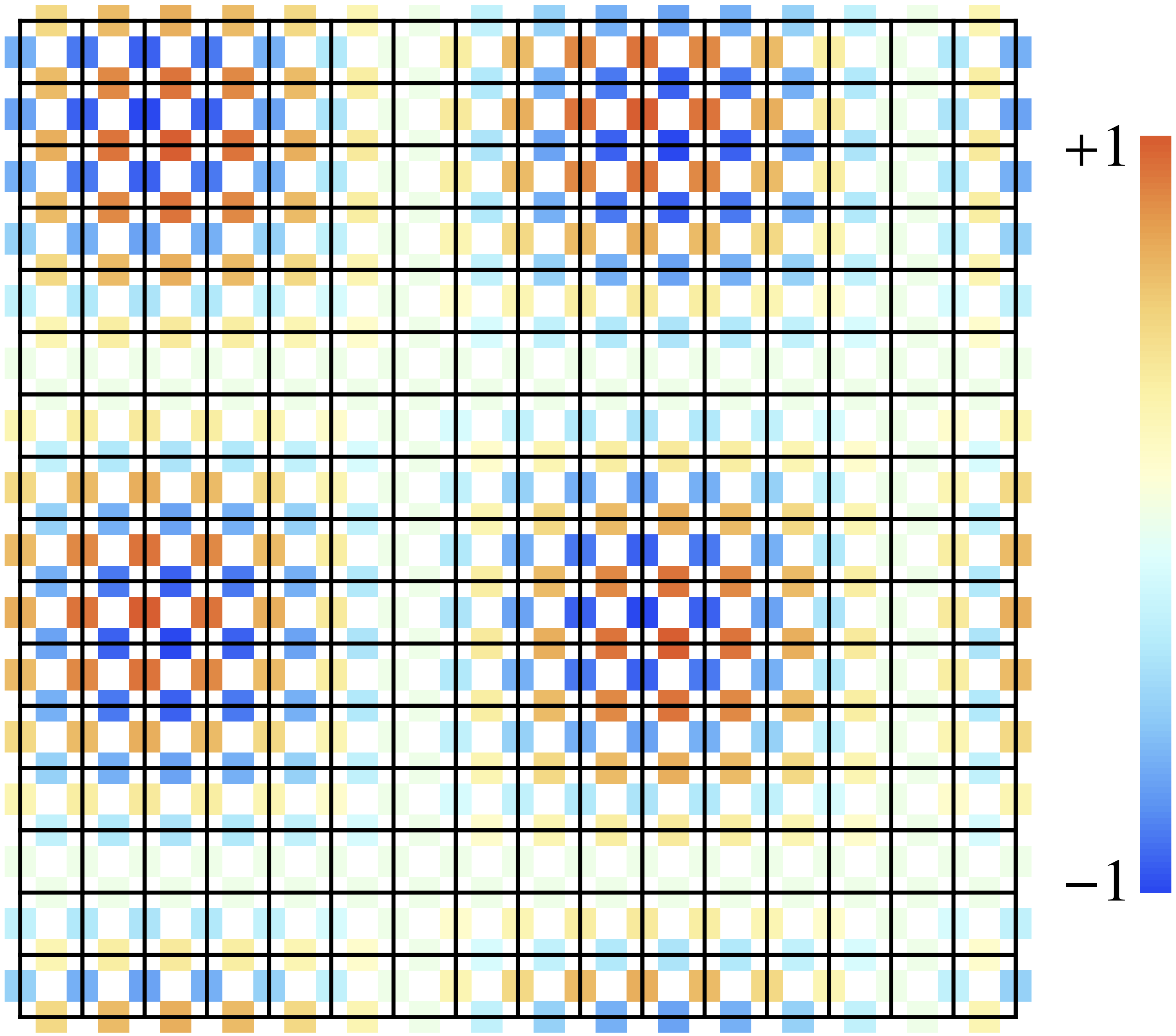

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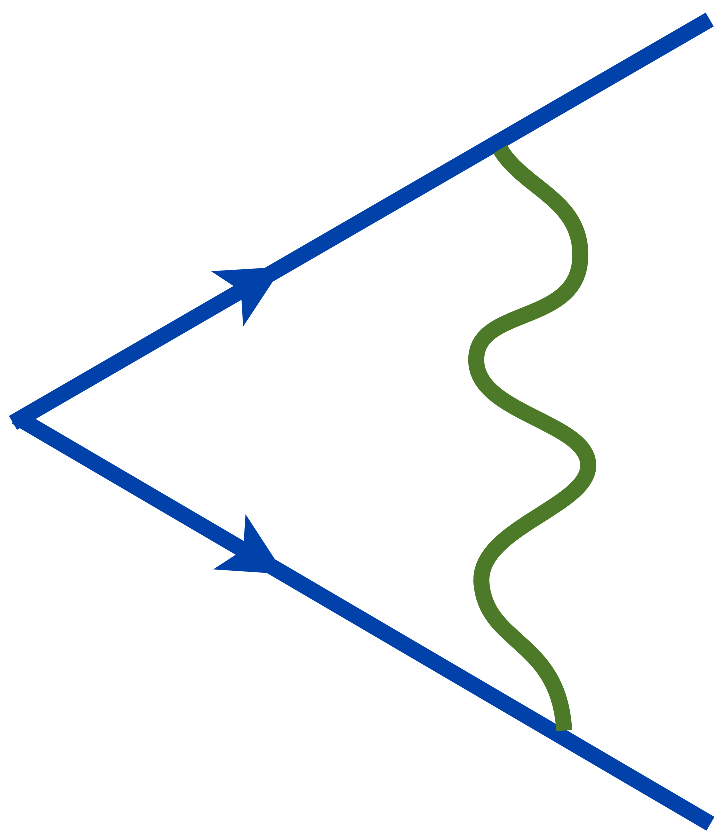
Strength of instability at quantum criticality

BCS theory

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Cooper logarithm



Strength of instability at quantum criticality

BCS theory

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Electron-phonon coupling

Debye frequency

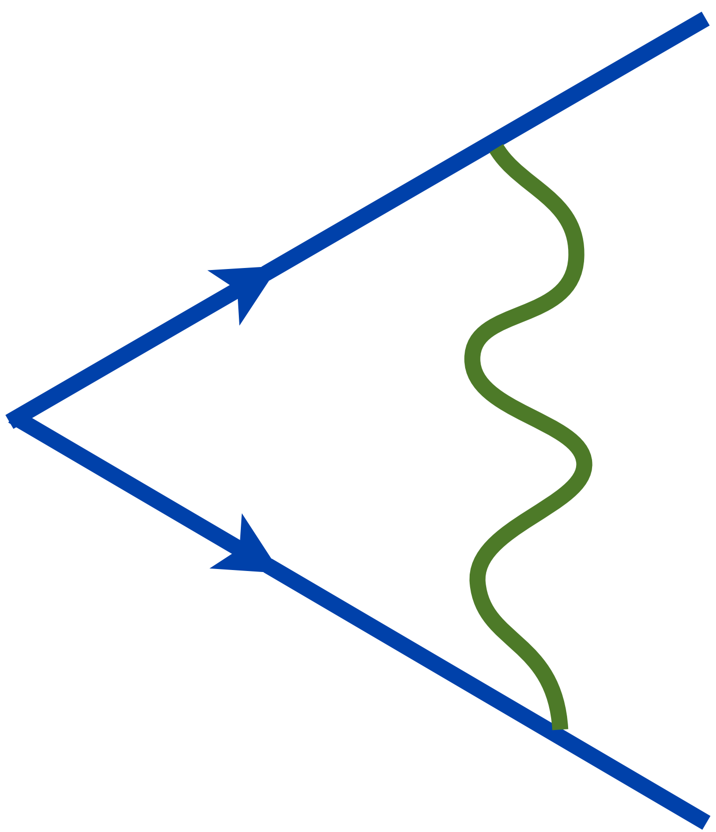
Implies

$$T_c \sim \omega_D \exp\left(-1/\lambda\right)$$

Strength of instability at quantum criticality

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)
Y. Wang and A. Chubukov, arXiv:1210.2408

Strength of instability at quantum criticality

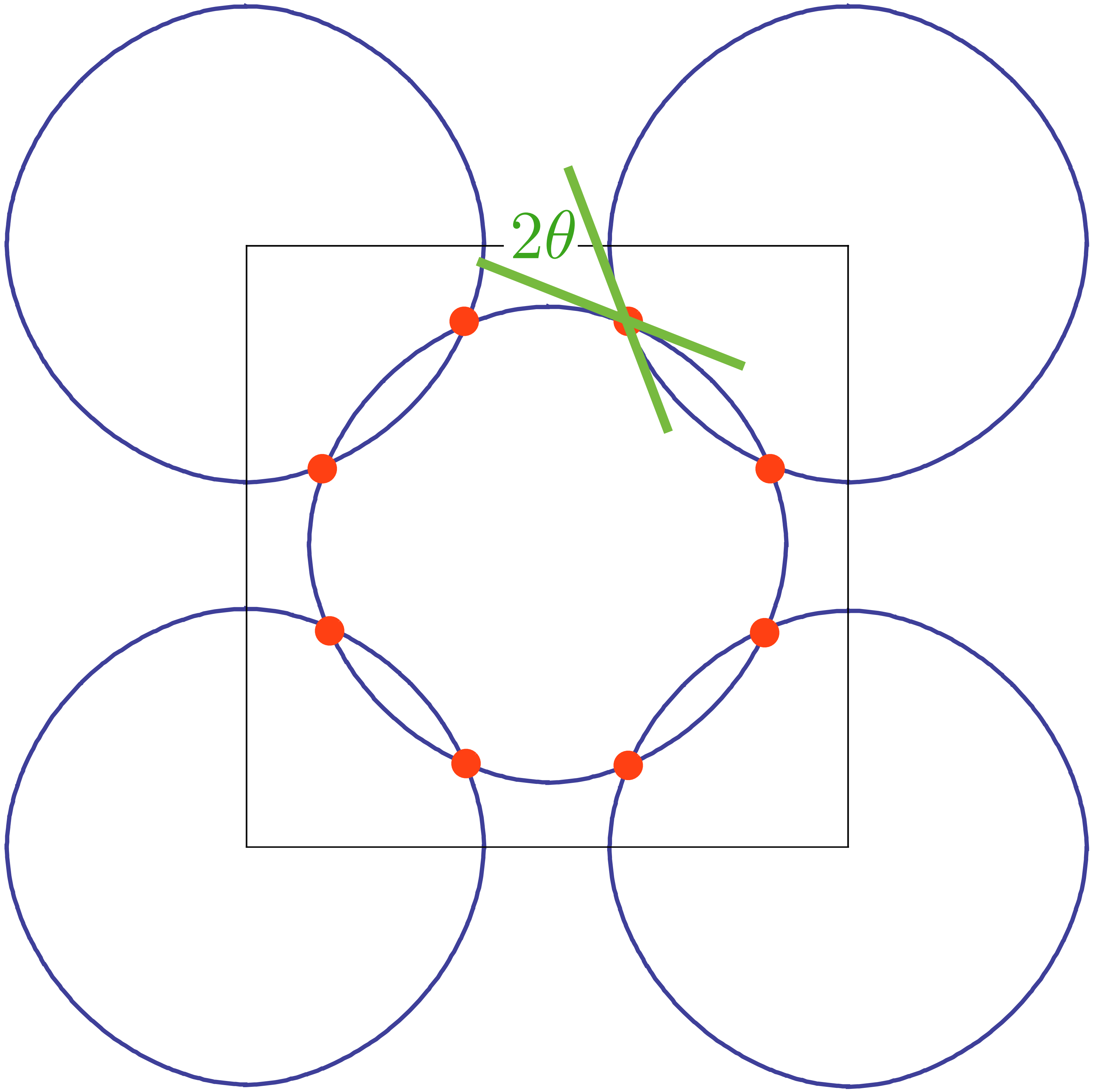
Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

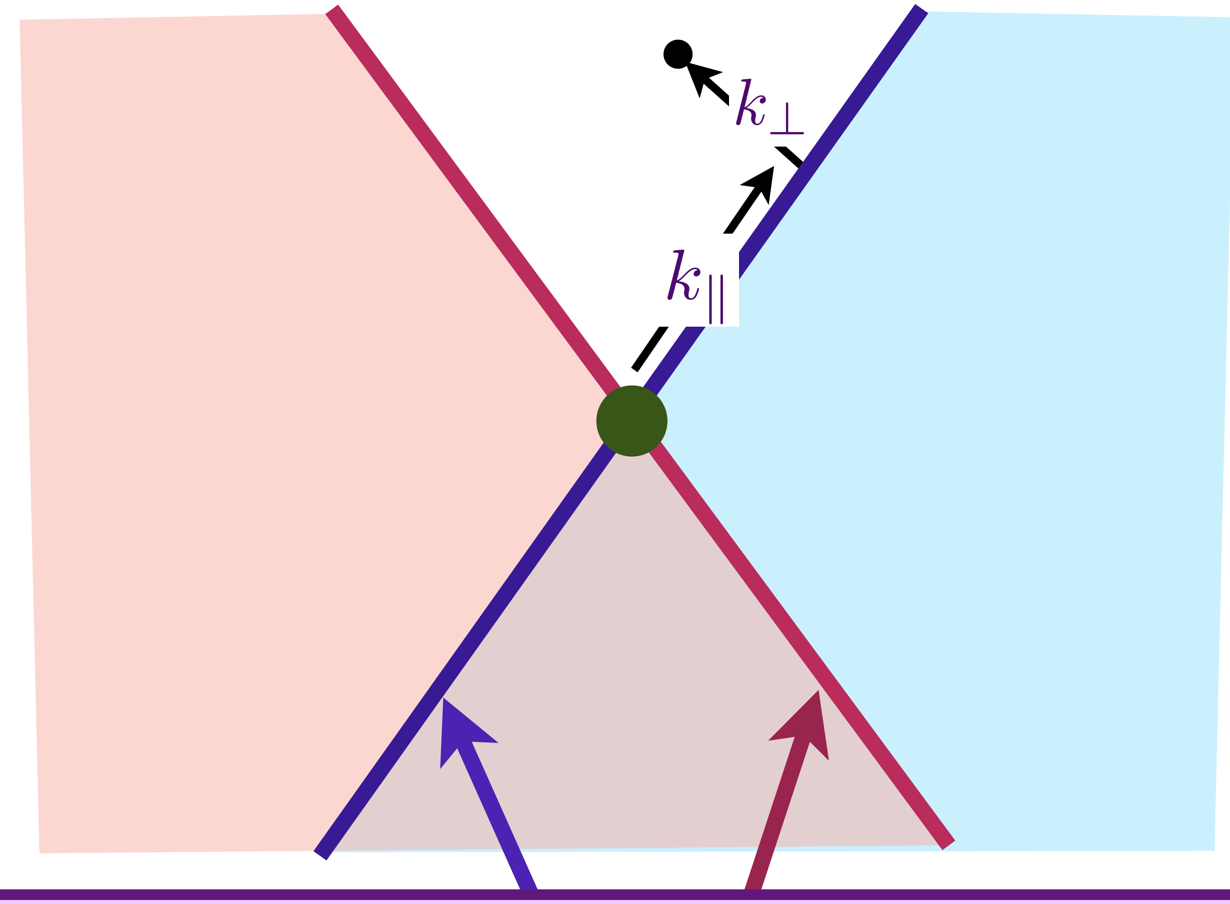
Fermi energy

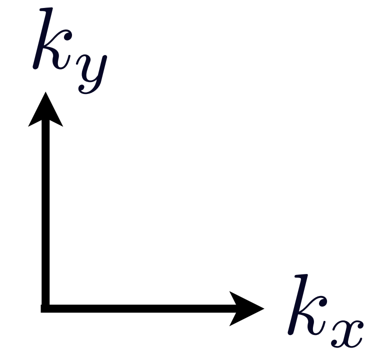
$\alpha = \tan\theta$, where 2θ is the angle between Fermi lines.
Independent of interaction strength U in 2 dimensions.

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)
Y. Wang and A. Chubukov, arXiv:1210.2408

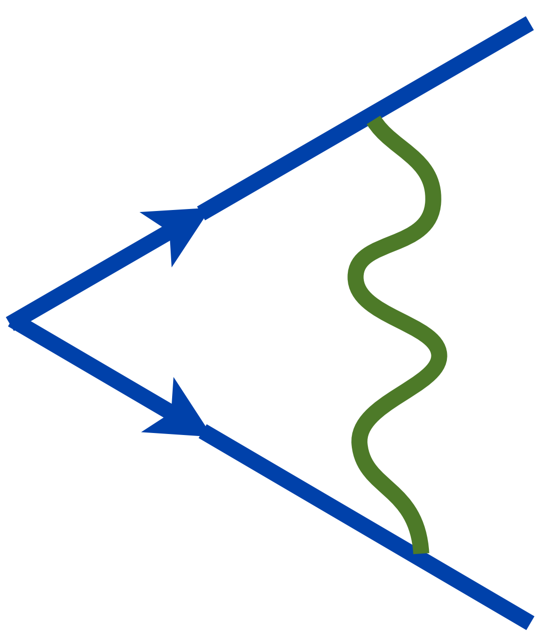

2θ

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



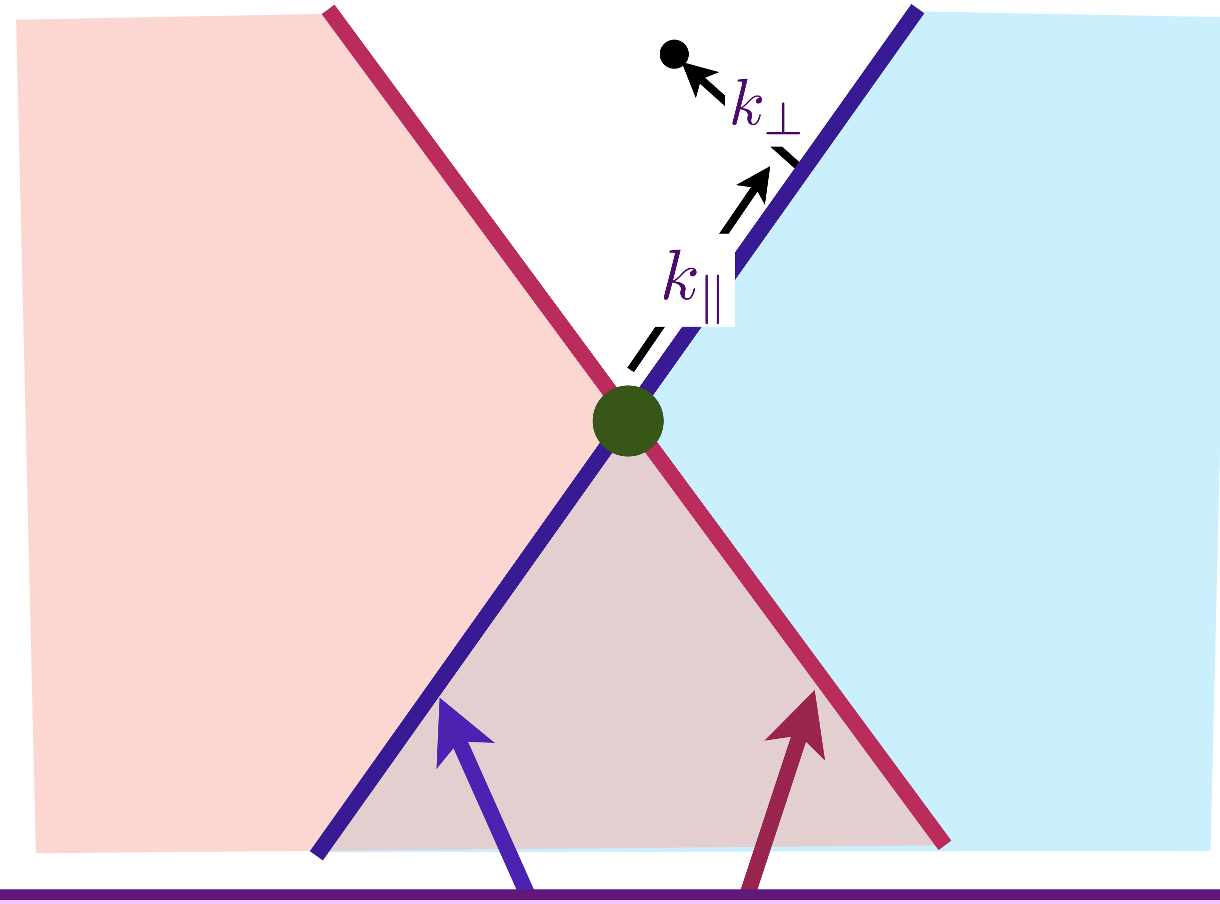


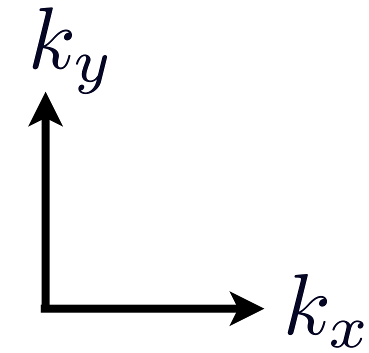
$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{i\omega - v_F(k_\parallel) k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$



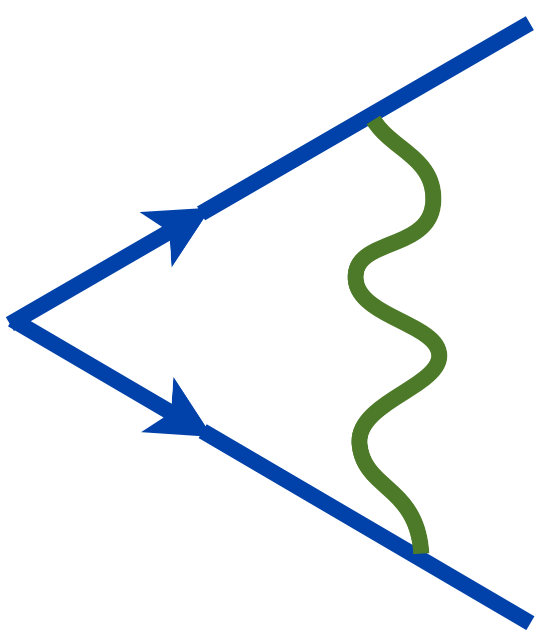
$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)





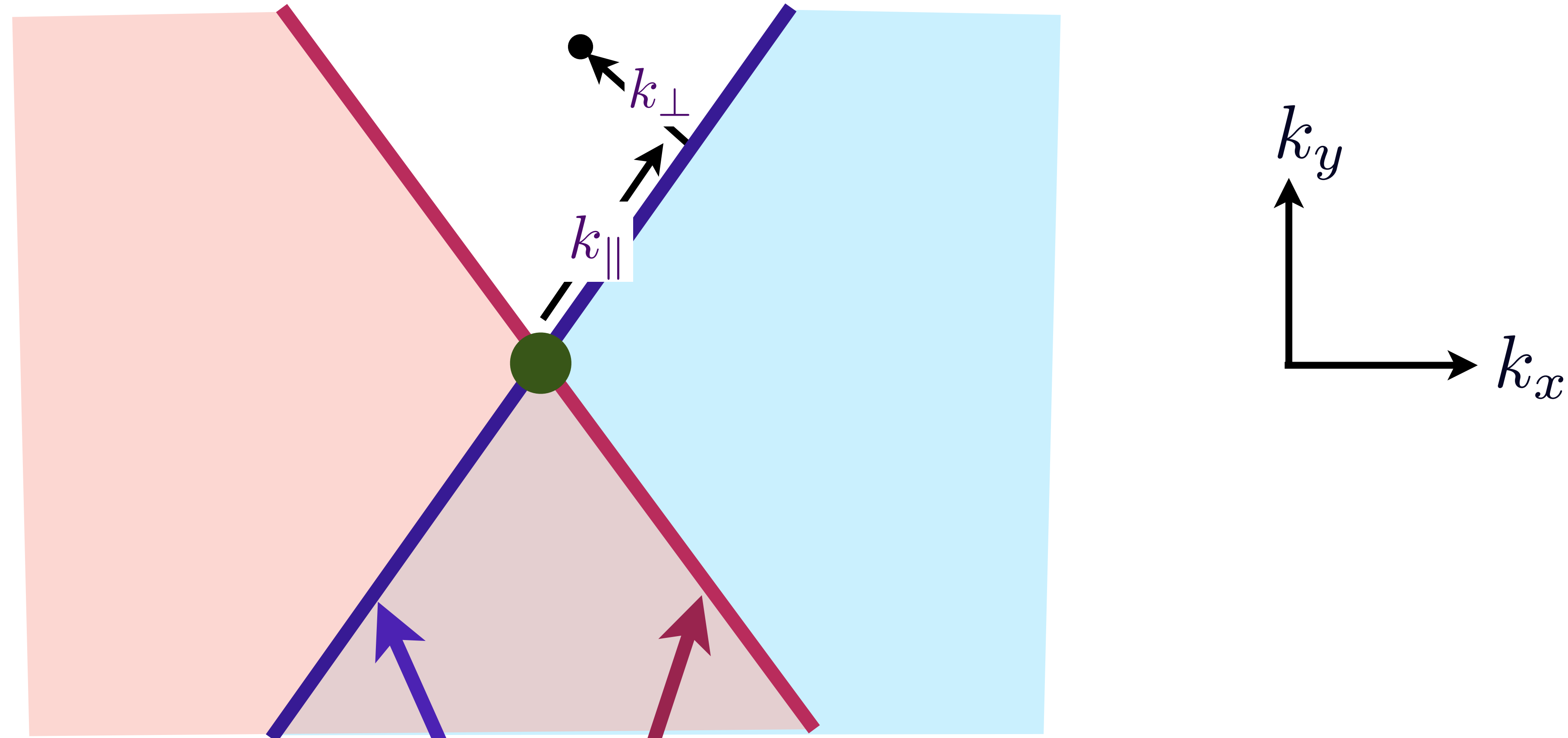
$$G_{\rm fermion} = \frac{Z(k_\parallel)}{i\omega - v_F(k_\parallel) k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$



$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \underbrace{\left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}}$$

Cooper logarithm

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



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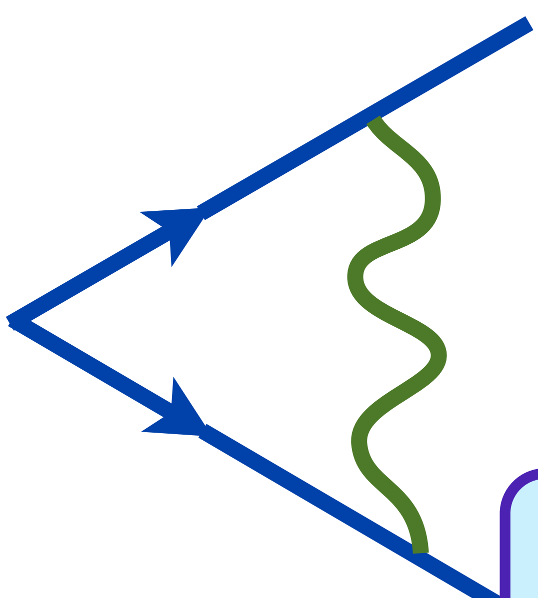

$$\int dk_\parallel \frac{1}{k_\parallel^2} \underbrace{\left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}}$$

Spin fluctuation propagator

Cooper logarithm

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2\left(\frac{E_F}{\omega}\right)$$

- $\log^2$ singularity arises from Fermi lines; singularity *at* hot spots is weaker.
- Interference between BCS and quantum-critical logs.
- Momentum dependence of self-energy is crucial.
- Not suppressed by $1/N$ factor in $1/N$ expansion.

Enhancement of Φ susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{3\pi(1+\alpha^2)} \log^2\left(\frac{E_F}{\omega}\right)$$

- Emergent pseudospin symmetry of low energy theory also induces $\log^2$ in a single "*d*-wave" particle-hole channel. Fermi-surface curvature reduces prefactor by 1/3.
- Φ corresponds to a $2k_F$ *bond-nematic* or a *quadrupole density wave*

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)
K. B. Efetov, H. Meier, and C. Pepin, arXiv:1210.3276

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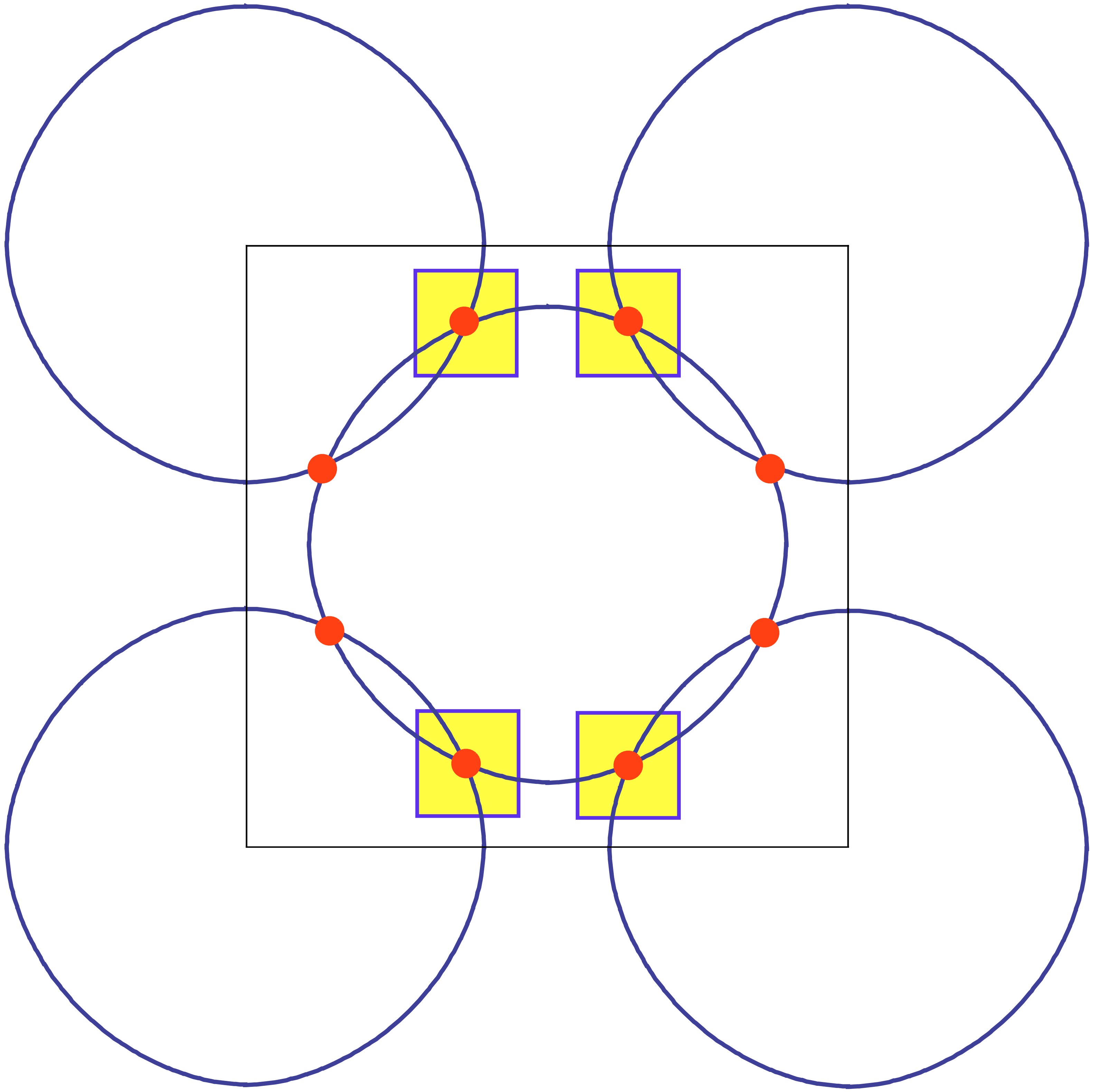
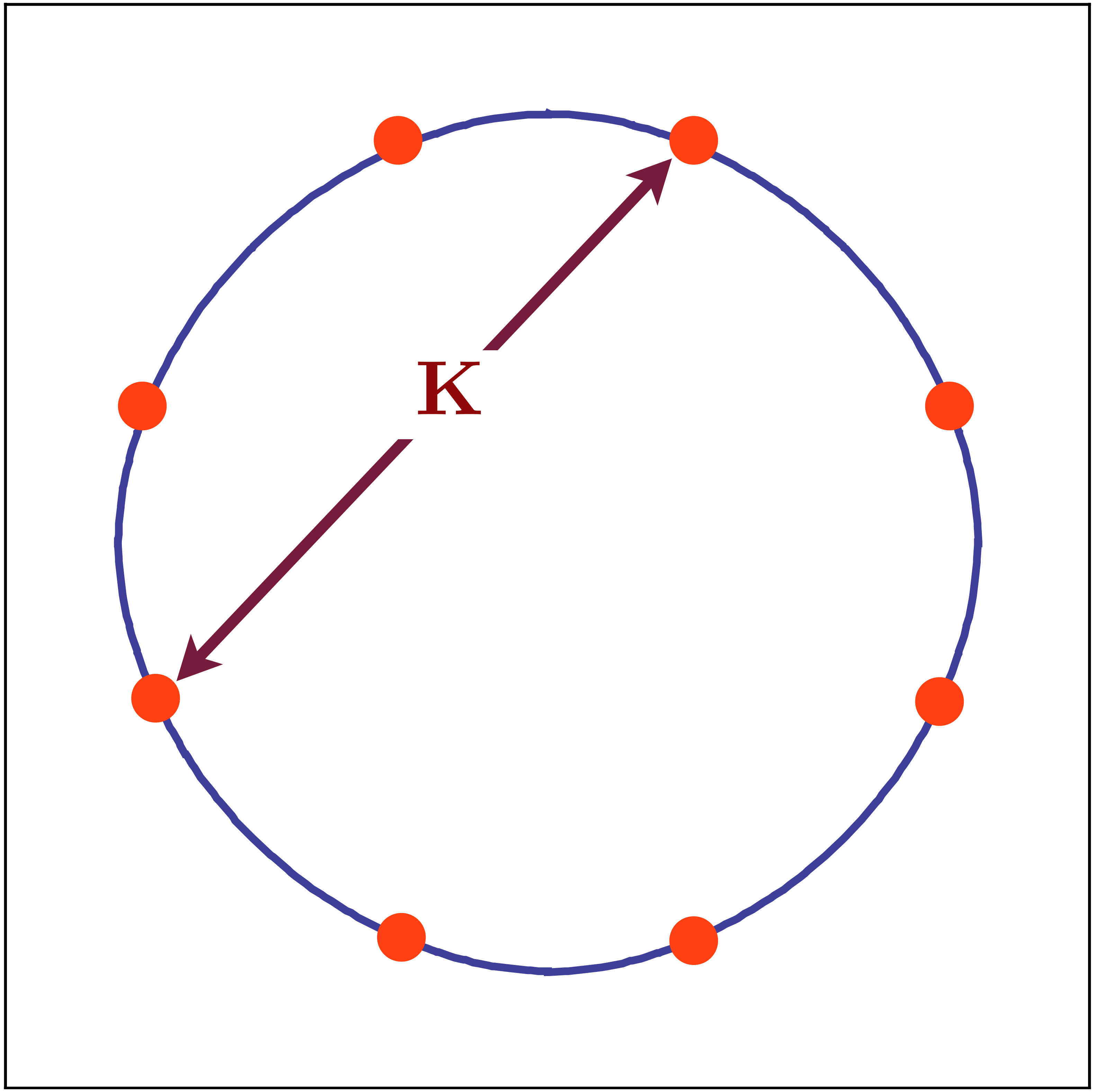


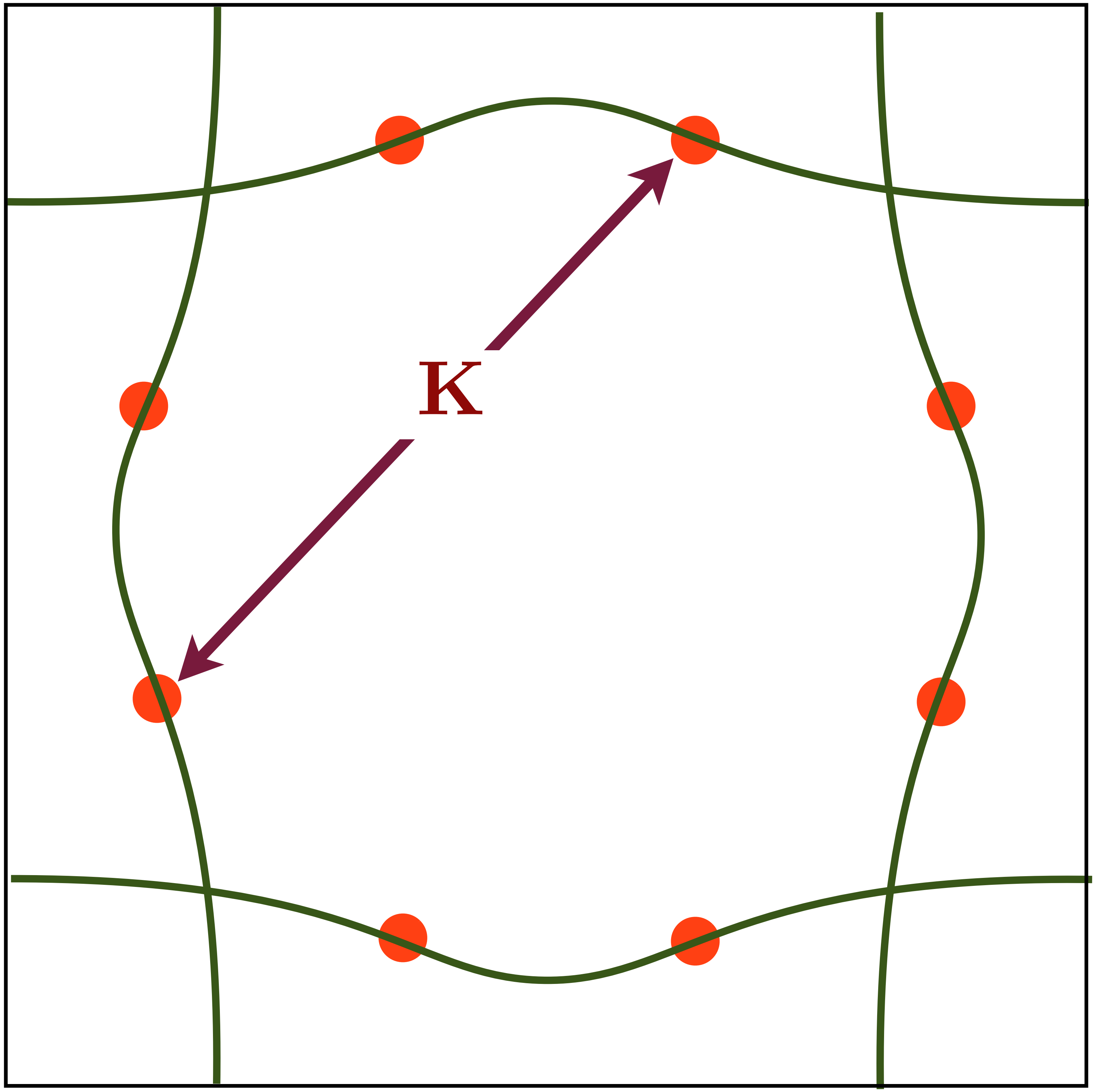
Low energy theory for critical point near hot spots

QMC for the onset of antiferromagnetism



Hot spots in a single band model

QMC for the onset of antiferromagnetism

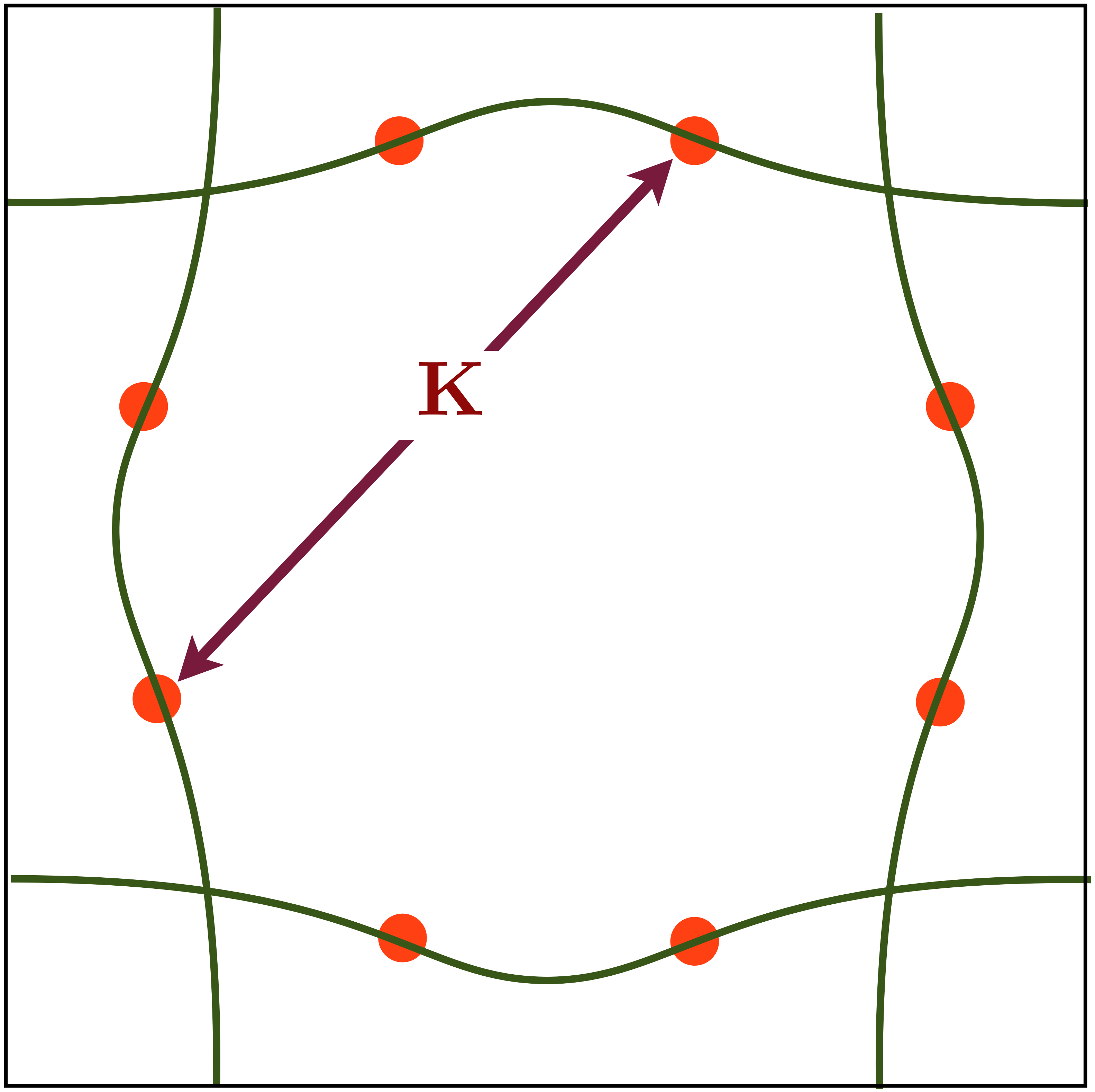


Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

QMC for the onset of antiferromagnetism

Faithful realization of the *generic* universal low energy theory for the onset of antiferromagnetism.



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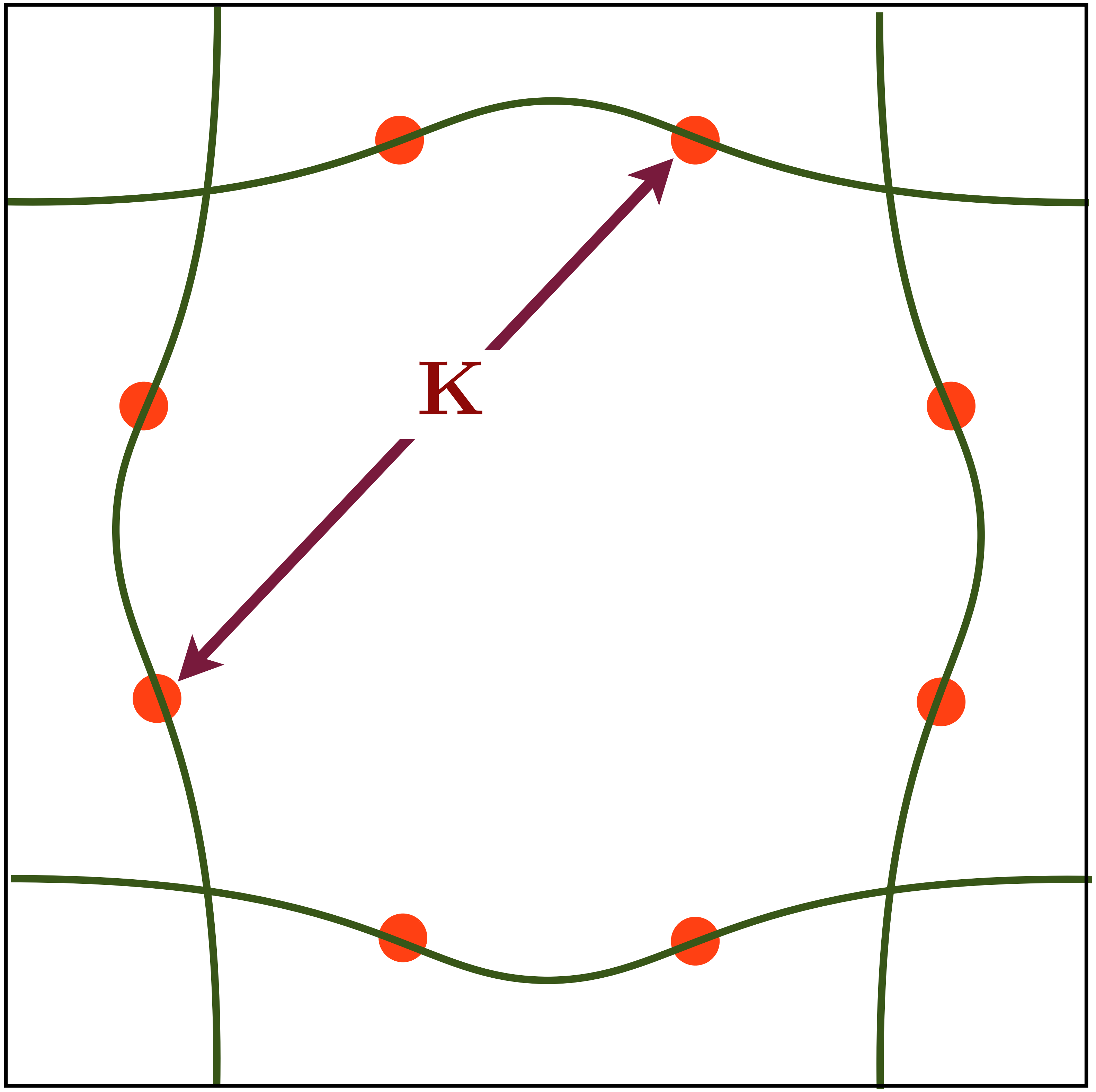

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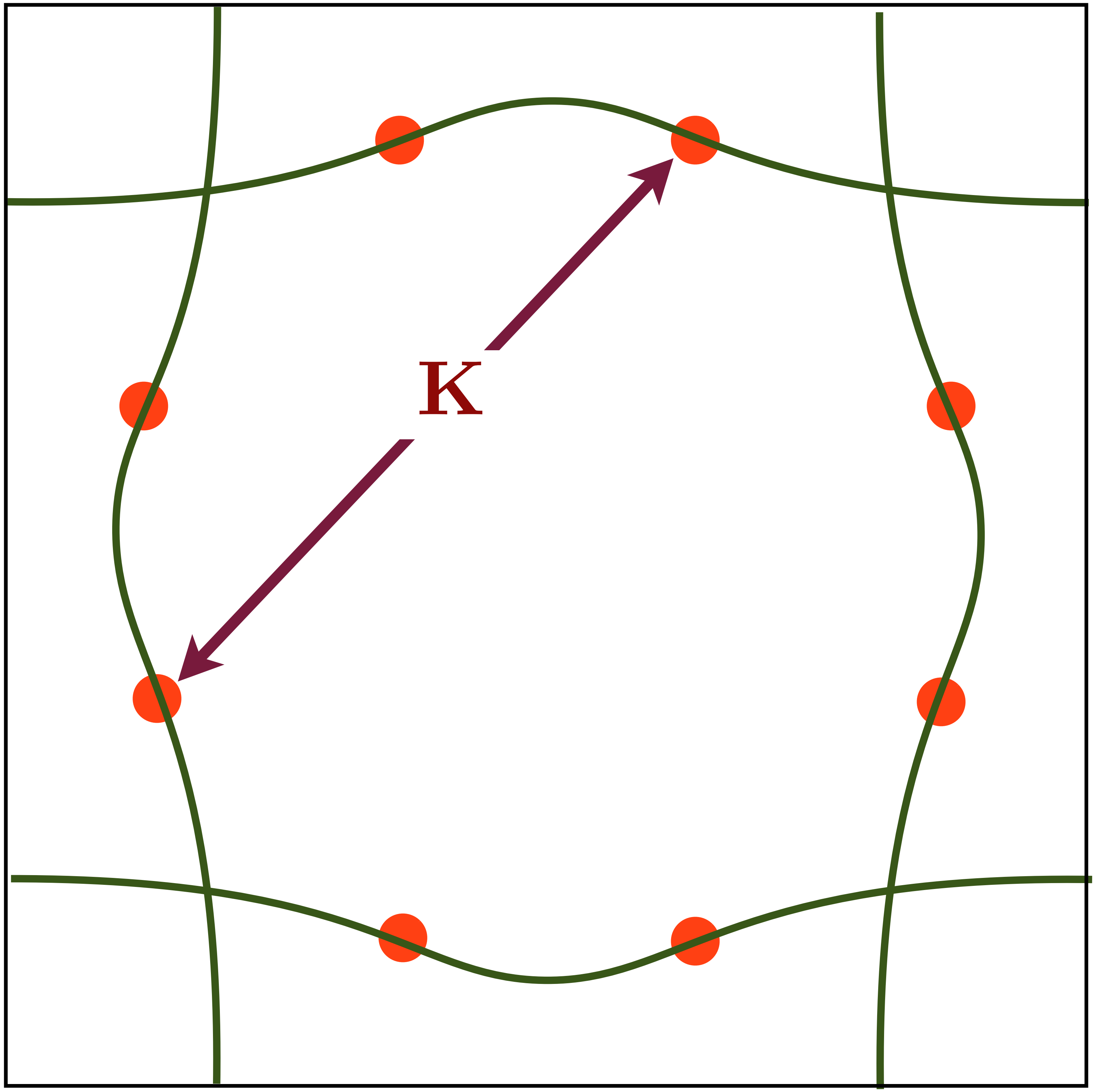
Sign problem is absent as long as ***K*** connects hotspots in distinct bands



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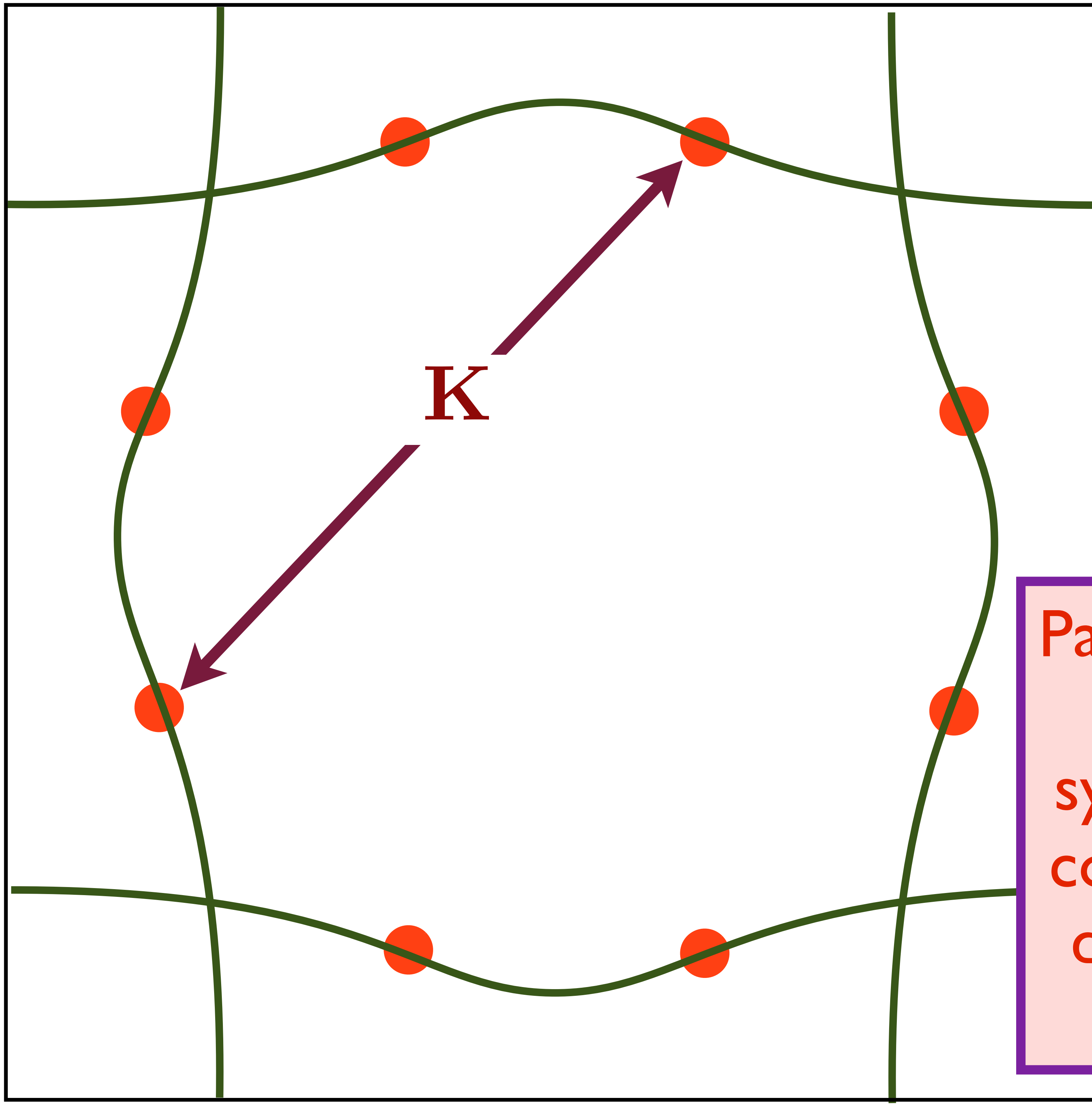

Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Particle-hole or point-group symmetries or commensurate densities ***not*** required !

QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha^{(x)} \mathcal{D}c_\alpha^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(x)} \right) c_{\mathbf{k}\alpha}^{(x)}$$

$$+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(y)} \right) c_{\mathbf{k}\alpha}^{(y)}$$

$$+ \int d\tau d^2x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}^{(y)} + \text{H.c.}$$

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_{\alpha}^{(x)} \mathcal{D}c_{\alpha}^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(x)} \right) c_{\mathbf{k}\alpha}^{(x)}$$

$$+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(y)} \right) c_{\mathbf{k}\alpha}^{(y)}$$

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No sign problem !

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E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Applies without changes to the microscopic band structure in the iron-based superconductors

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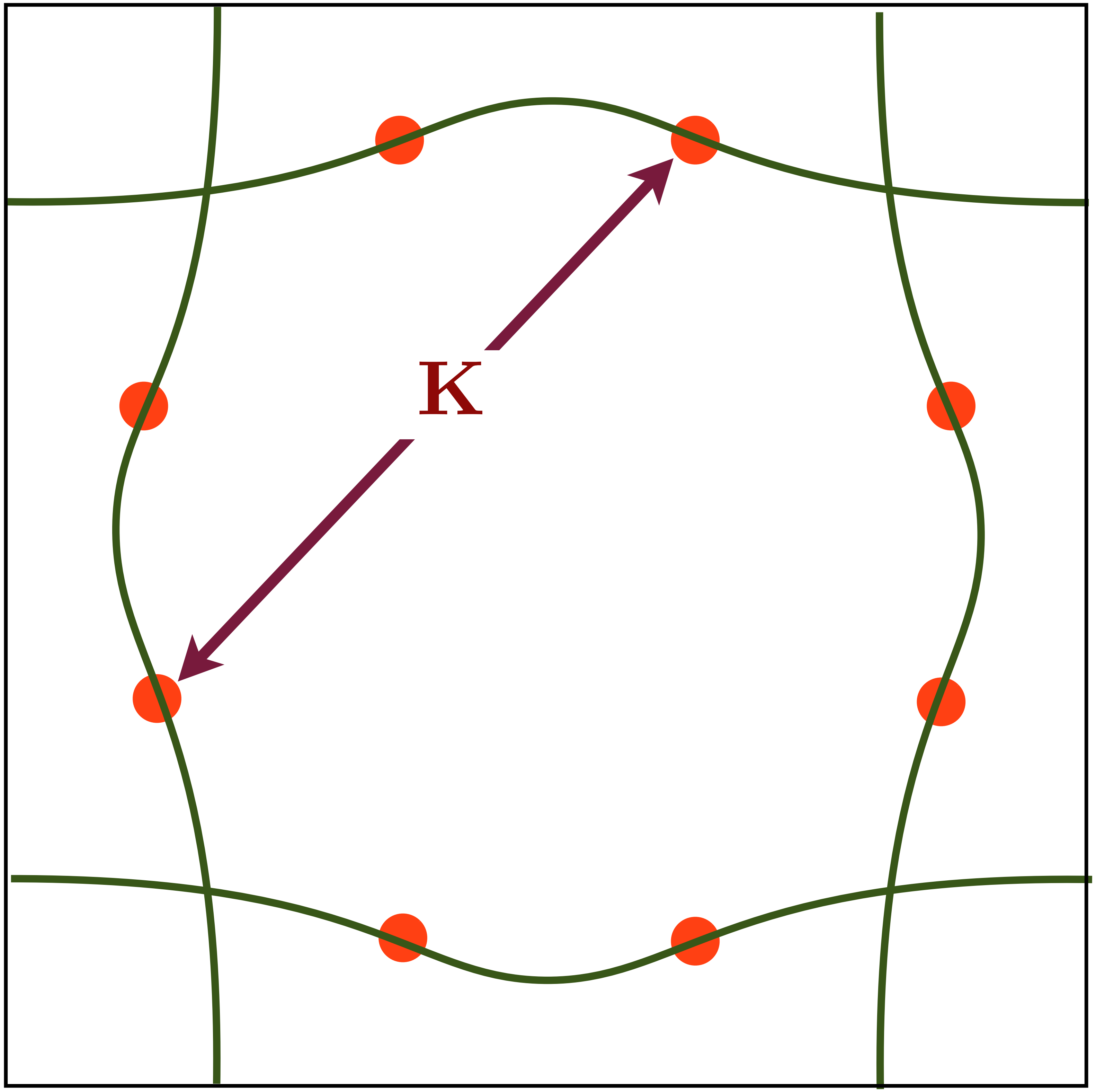
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Can integrate out $\vec{\varphi}$ to obtain an extended Hubbard model. The interactions in this model only couple electrons in separate bands.

Tuesday, January 8, 13

QMC for the onset of antiferromagnetism



Hot spots in a two band model

E. Berg,
M. Metlitski, and
S. Sachdev,
Science **338**, 1606
(2012).

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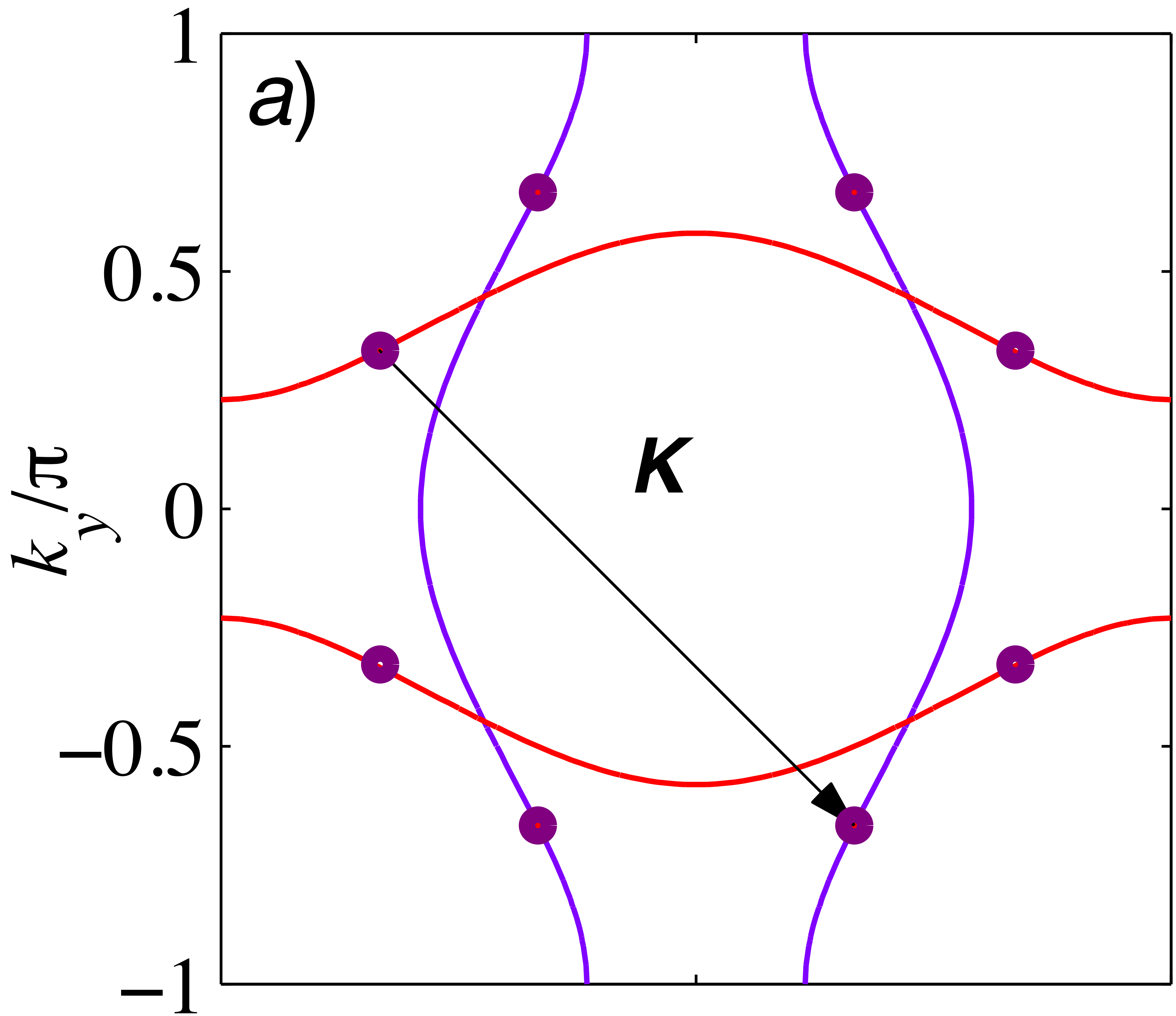

Center Brillouin zone at (π,π,)

E. Berg,
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S. Sachdev,
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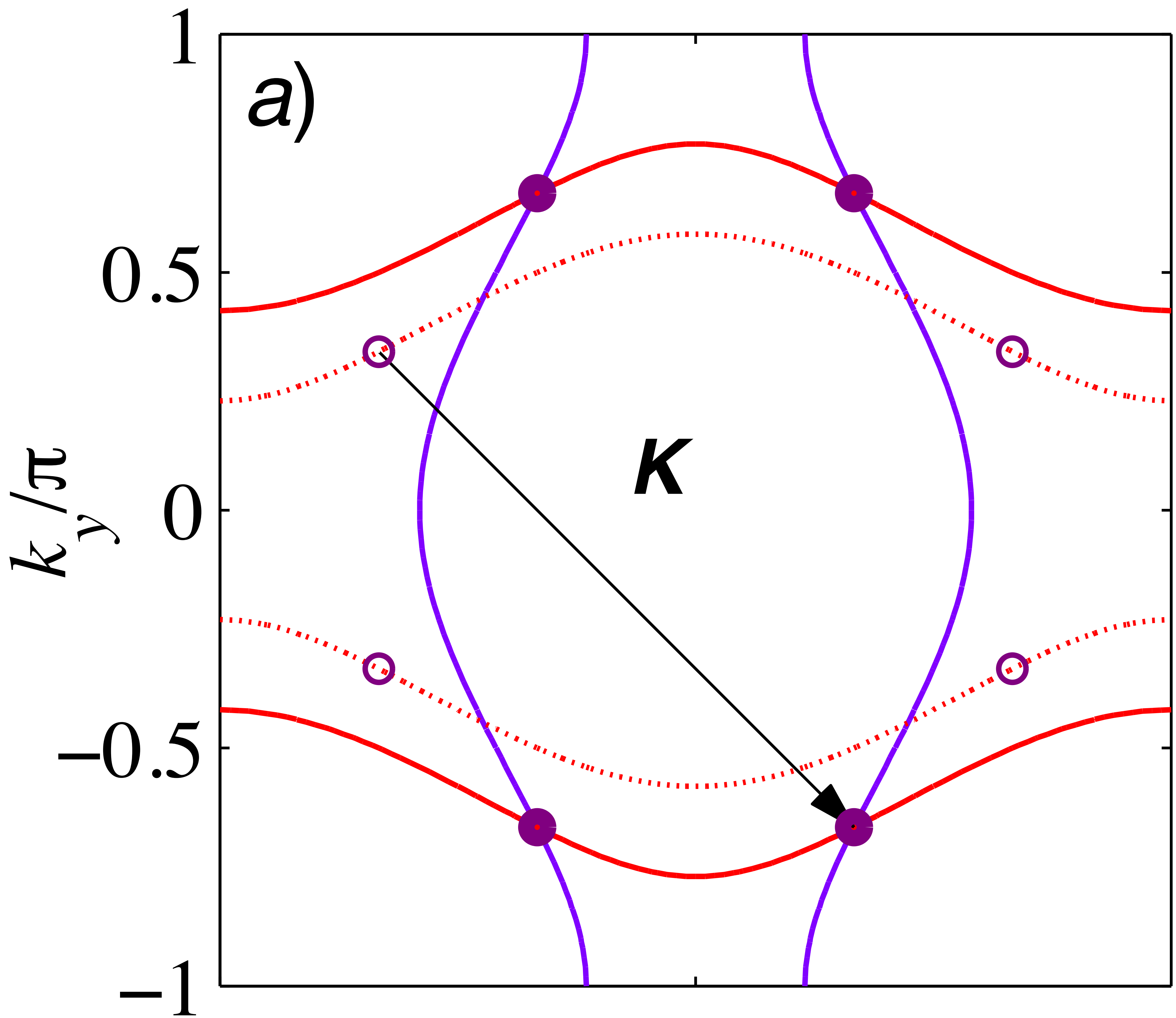

Move one of the Fermi surface by $(\pi,\pi,)$

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

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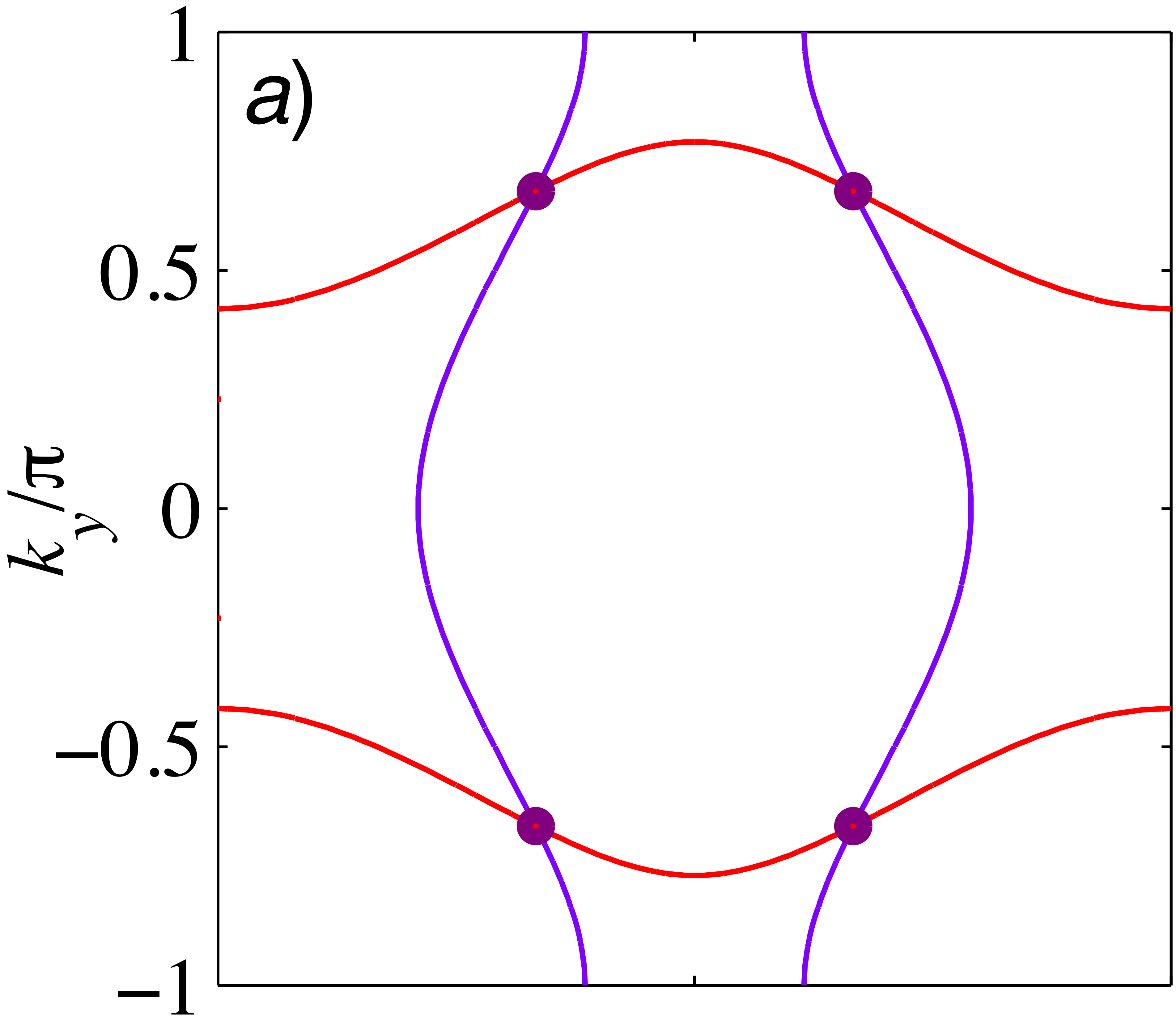

Now hot spots are at Fermi surface intersections

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S. Sachdev,
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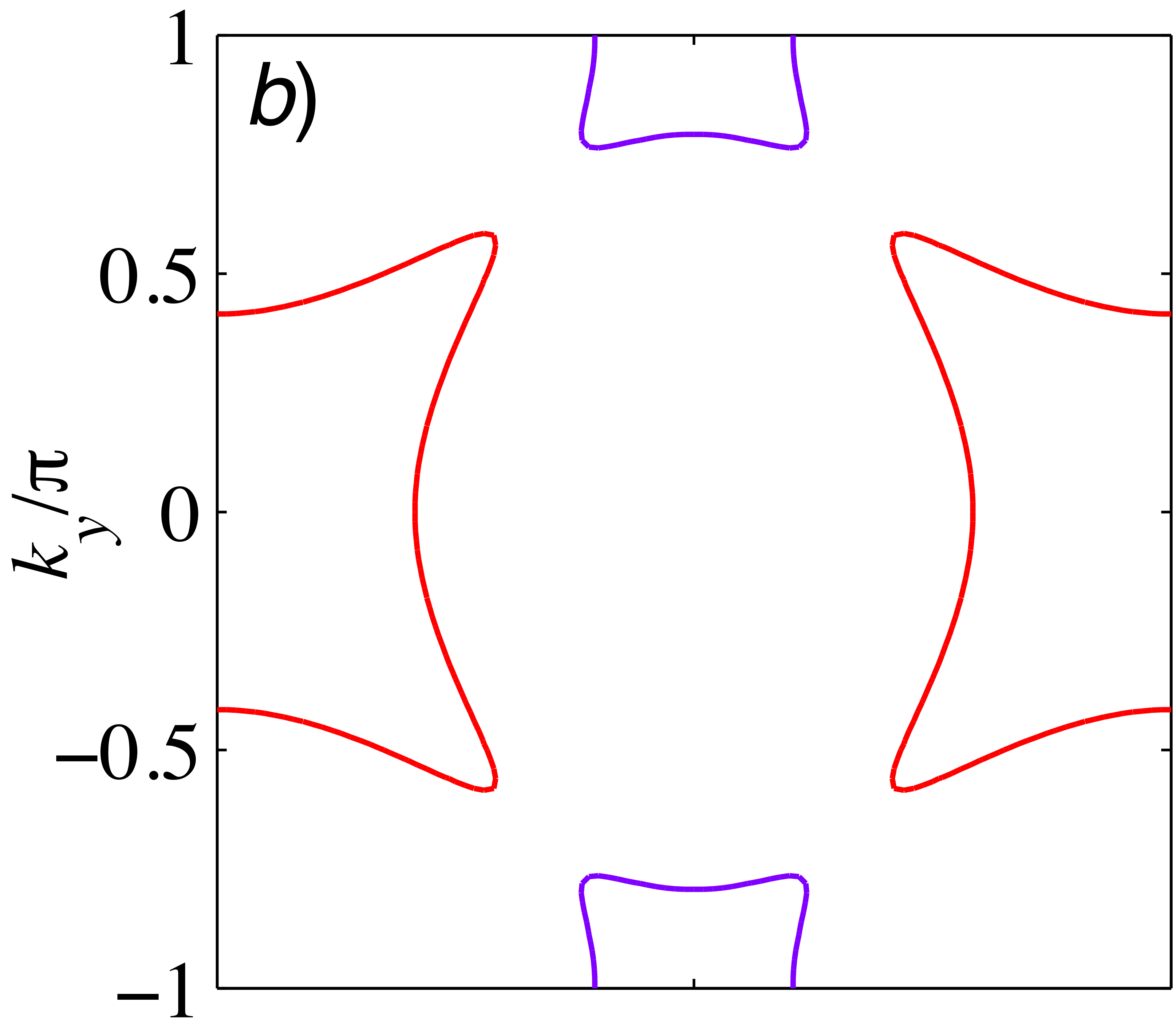

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Expected Fermi surfaces in the AFM ordered phase

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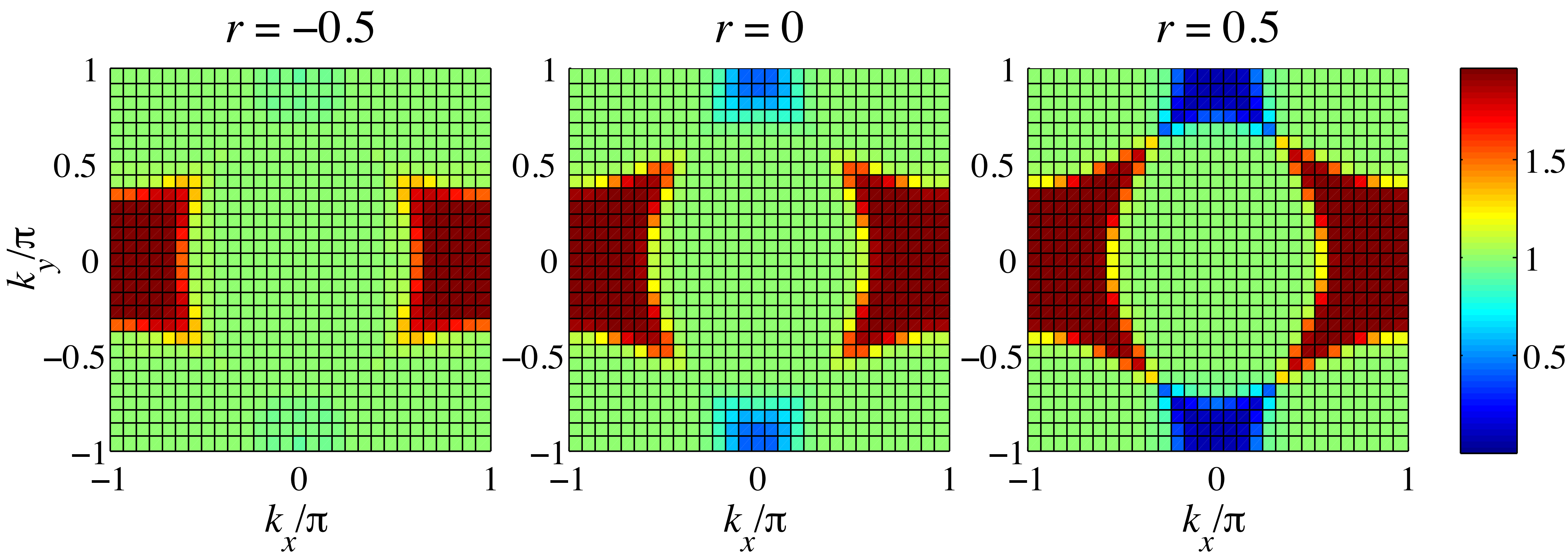
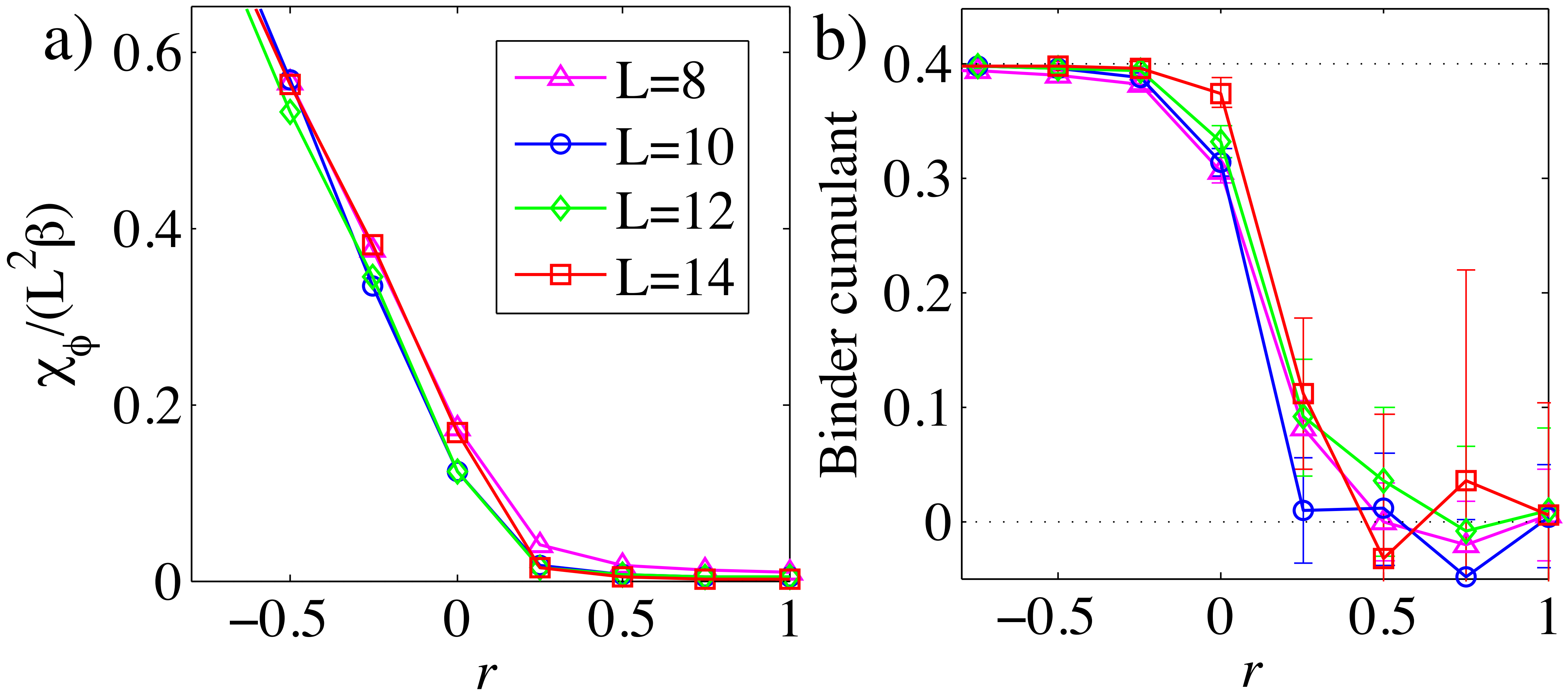


Electron occupation number $n_{\mathbf{k}}$
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

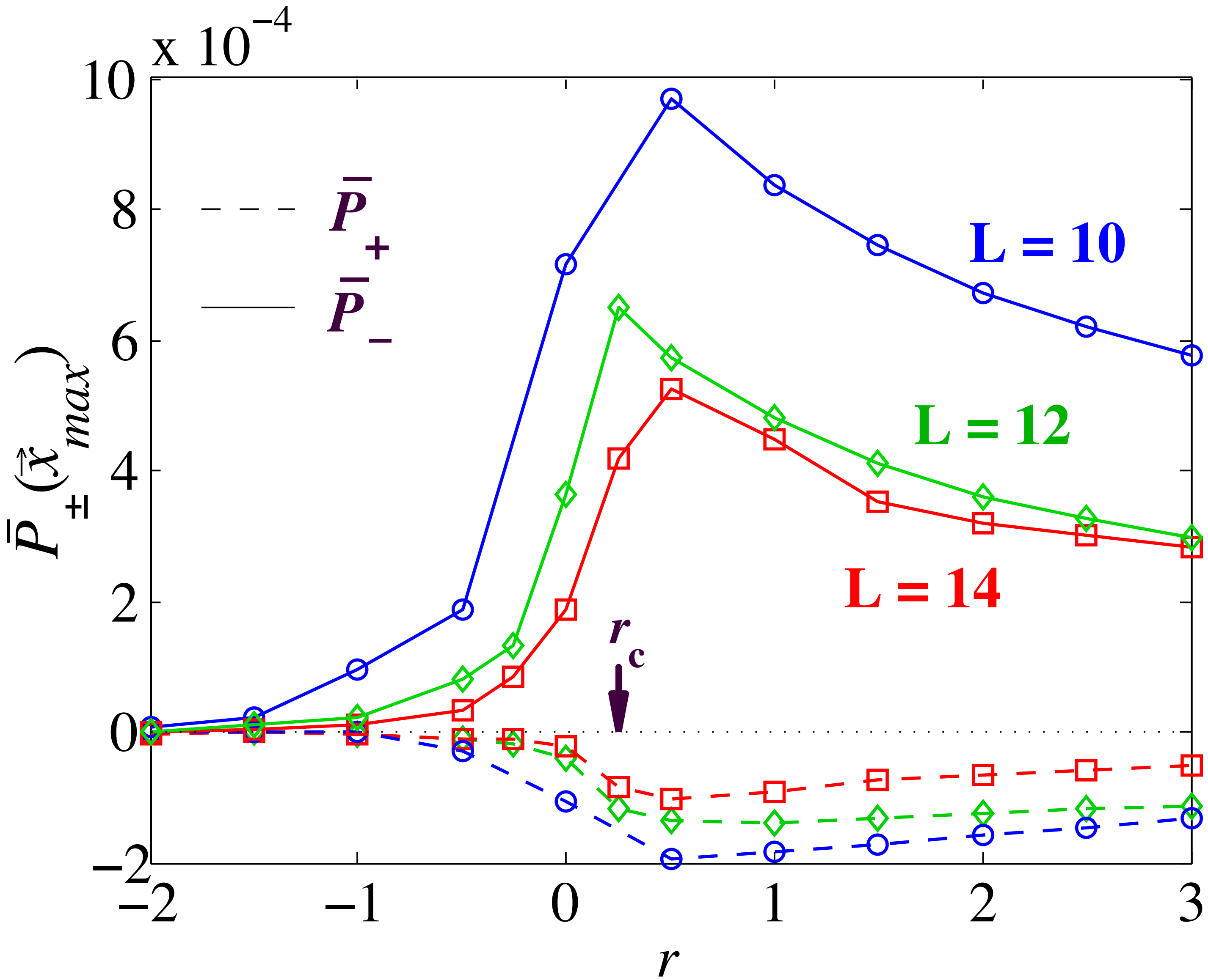
QMC for the onset of antiferromagnetism



AF susceptibility, χ_φ, and Binder cumulant as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

QMC for the onset of antiferromagnetism



s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Conclusions

- Solved sign-problem for generic universal theory for the onset of antiferromagnetism in two-dimensional metals.

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- Good prospects for studying competing charge orders, and non-Fermi liquid physics at non-zero temperature.