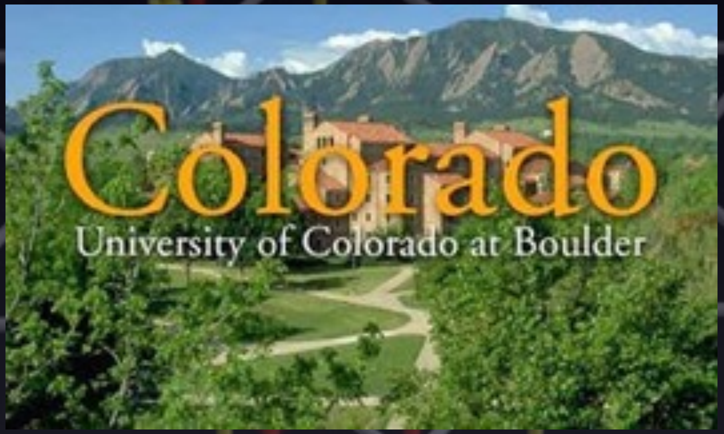
Theory of Quantum Spin Liquids IV

Michael Hermele



Center for Theory of Quantum Matter
CTQM

2015 Magnet Lab Theory Winter School:
New Trends in Frustrated Magnetism
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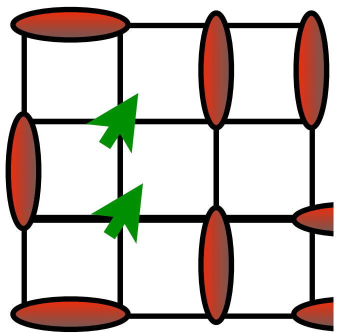
Goal of today's lecture

- Yesterday: physical picture of quantum spin liquids (QSLs) in terms of formation/condensation of strings
- Today: Parton gauge theory, and Z_2 QSLs via condensation of double vortices

How do we make string pictures into theories?

- Questions we might like to answer…
- Given a particular model, does it have a QSL ground state, and if so what kind? (Often hard! But there has been & continues to be exciting progress along these lines.)
- Given a *class* of models, what kinds of QSL phases can occur in principle? What are their physical properties that can be detected in experiments and in numerical simulations? (Parts of this question are manageable, parts of it are very hard. Very much an active topic of research!)
- What is meant by class of models? Specify degrees of freedom, symmetries and how they act on d.o.f., range of interactions (*e.g*. finite-range). These are *physical* requirements, appropriate for some material or class of materials, and thus appropriate to learn what is possible in realistic models.



Example: square lattice S=1/2 Heisenberg models…

$$H = J \sum_{\langle r r' \rangle} \vec{S}_r \cdot \vec{S}_{r'} + \cdots$$

Tools to generate effective theories of QSLs

- Given a class of models, there are well-developed tools to generate effective gauge theories of QSLs. These tools go under the general heading of parton theories.

- Parton theories do not provide reliable information about any particular model, except *maybe* as a means to construct trial wave functions.

Credit for this term goes to Senthil; blame for using it goes to me

- Instead, their purpose is to demonstrate *emergeability*. That is, a given effective (gauge) theory can emerge in a given class of microscopic models.

- Demonstrating *non-emergeability* is also interesting, but requires different methods.

Partons in practice

- Focus on S=1/2 Heisenberg antiferromagnet on square lattice

$$H = J \sum_{\langle r r' \rangle} \vec{S}_r \cdot \vec{S}_{r'} + \cdots$$

- Represent a single S=1/2 spin using S=1/2 fermions:

Spin operator: $\vec{S} = \frac{1}{2} f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta \quad (\alpha, \beta = \uparrow, \downarrow)$ Constraint: $f_\alpha^\dagger f_\alpha = 1$

- Functional integral: $Z = \int [df\, d\bar{f}\, d\lambda] \exp(-S)$

$$S = \int d\tau \sum_r \left[\bar{f}_{r\alpha} \partial_\tau f_{r\alpha} + i\lambda_r (\bar{f}_{r\alpha} f_{r\alpha} - 1) \right] + \int d\tau H[f, \bar{f}]$$

Partons in practice

- Nearest-neighbor exchange is quartic in partons. Decouple using complex Hubbard-Stratonovich field living on lattice links…

$$Z = \int [df\, d\bar{f}\, d\lambda\, d\chi] \exp(-S')$$

$$S' = \int d\tau \sum_r \left[\bar{f}_{r\alpha}\partial_\tau f_{r\alpha} + i\lambda_r(\bar{f}_{r\alpha} f_{r\alpha} - 1)\right] + \int d\tau \sum_{\langle rr'\rangle} \left[\frac{1}{J}|\chi_{rr'}|^2 + (\chi_{rr'}\bar{f}_{r\alpha} f_{r'\alpha} + \text{H.c.})\right] + \cdots$$

- U(1) gauge *redundancy*. (Often called gauge symmetry, but this is misleading terminology)

$$f_{r\alpha}(\tau) \to e^{i\phi_r(\tau)} f_{r\alpha}(\tau)$$

$$\chi_{rr'}(\tau) \to e^{i[\phi_r(\tau) - \phi_{r'}(\tau)]}\chi_{rr'}(\tau)$$

$$\lambda_r(\tau) \to \lambda_r(\tau) - \partial_\tau \phi_r(\tau)$$

Write ⇒

Time component of vector potential

$$\lambda_r(\tau) = a_0(r,\tau)$$

$$\chi_{rr'} = |\chi_{rr'}| \exp(i a_{rr'})$$

Spatial components of vector potential

Partons in practice

$$S' = \int d\tau \sum_r \left[\bar{f}_{r\alpha}\partial_\tau f_{r\alpha} + ia_0(\bar{f}_{r\alpha}f_{r\alpha} - 1)\right] + \int d\tau \sum_{\langle rr'\rangle} \left[\frac{1}{J}|\chi_{rr'}|^2 + (|\chi_{rr'}|\bar{f}_{r\alpha}e^{ia_{rr'}}f_{r'\alpha} + \text{H.c.})\right] + \cdots$$

- So far we have only discussed *formal* properties of a change of variables.
- In particular, the partons should not be thought of as low-energy degrees of freedom (*e.g*. quasiparticles) of a putative QSL phase:

partons ≠ spinons

- Now, imagine obtaining a low-energy effective theory, by integrating out high-frequency modes.
- Many terms will be generated, including terms that can pin $|\chi_{rr'}|$ to some value, and other terms that suppress fluctuations in magnetic flux of the vector potential.
- If such terms dominate, they drive the gauge theory into its deconfined phase (if this phase is stable).
- Effective theory for deconfined phase: constant $\chi_{rr'}$ and λ_r + fluctuations (not restricted to perturbative fluctuations)

Partons in practice

- Many different QSL effective theories can be generated this way (for same class of models), *e.g*. background magnetic flux can be different.
- Properties of these different QSL phases can then be studied (doing this can be highly non-trivial for gapless states; interesting/challenging quantum field theory problems)
- Issue 1: little information about specific Hamiltonians
- Issue 2: hard to connect low-energy degrees of freedom to microscopic model. For example, effective theories are gauge theories, but not obvious what the electric field (string) is microscopically … symmetry analysis usually the best we can do.

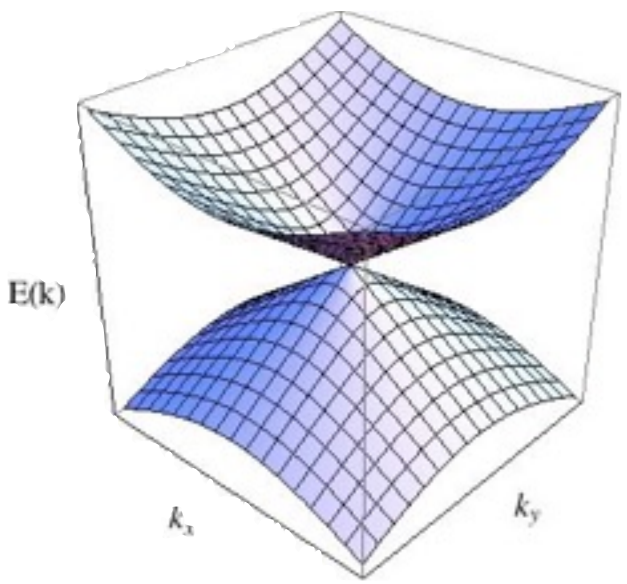
Example parton effective theory

- Example Hamiltonian effective theory:

$$H_{\text{gauge}} = h \sum_{\langle rr' \rangle} e^2_{rr'} - K \sum_{\square} \cos[(\nabla \times a)_\square] - t \sum_{\langle rr' \rangle} [f^\dagger_{r\alpha} e^{i\bar{a}_{rr'}} e^{i a_{rr'}} f_{r'\alpha} + \text{H.c.}] + \cdots$$

$$(\text{div}\, e)_r = f^\dagger_{r\alpha} f_{r\alpha} - 1$$

$\bar{a}_{rr'}$ gives background flux; let's say π flux / plaquette



Fermions have Dirac dispersion

π	π	π
π	π	π
π	π	π

- Deconfining limit: *K* large, expand cosine, derive continuum Dirac theory for fermions…

$$\mathcal{L} = \bar{\Psi}(\partial_\mu + i a_\mu)\Psi + \frac{1}{2e^2} \sum_\mu \Big(\sum_{\nu\lambda} \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda \Big)^2 + \cdots$$

Now the fermions are low-energy degrees of freedom, and I will call them spinons

Example parton effective theory

- Example Hamiltonian effective theory:

$$H_{\text{gauge}} = h \sum_{\langle rr' \rangle} e^2_{rr'} - K \sum_{\Box} \cos[(\nabla \times a)_{\Box}] - t \sum_{\langle rr' \rangle} [f^\dagger_{r\alpha} e^{i\bar{a}_{rr'}} e^{i a_{rr'}} f_{r'\alpha} + \text{H.c.}] + \cdots$$

$(\text{div}\, e)_r = f^\dagger_{r\alpha} f_{r\alpha} - 1$ $\qquad$ $\bar{a}_{rr'}$ gives background flux; let's say π flux / plaquette

- Confining limit: h large, $e \approx 0$. Degenerate perturbation theory in t/h recovers Heisenberg antiferromagnet at leading order, generates other local spin interactions at higher order

- Taking the confining limit basically "runs backward" the procedure we used to arrive at the effective theory in the first place.

- In practice, we often establish emergeability by writing down a gauge theory Hamiltonian like this one, then taking the confining limit.

Vortices and Z_2 spin liquids

- Different approach to QSLs: start with an ordered phase, and disorder it somehow
- Essentially the idea is to describe a QSL in terms of the excitations of some ordered phase.
- Advantage: excitations of ordered phases are easy to understand physically and often easy to connect to microscopic physics
- Disadvantage: usually have to disorder via mechanisms that are fine in principle, but might be hard to achieve in realistic models. On the other hand, we already know QSLs are not easy to find, so this should not discourage us too much.

- Today: basic intuition for one example, description of Z_2 spin liquids as condensates of double vortices

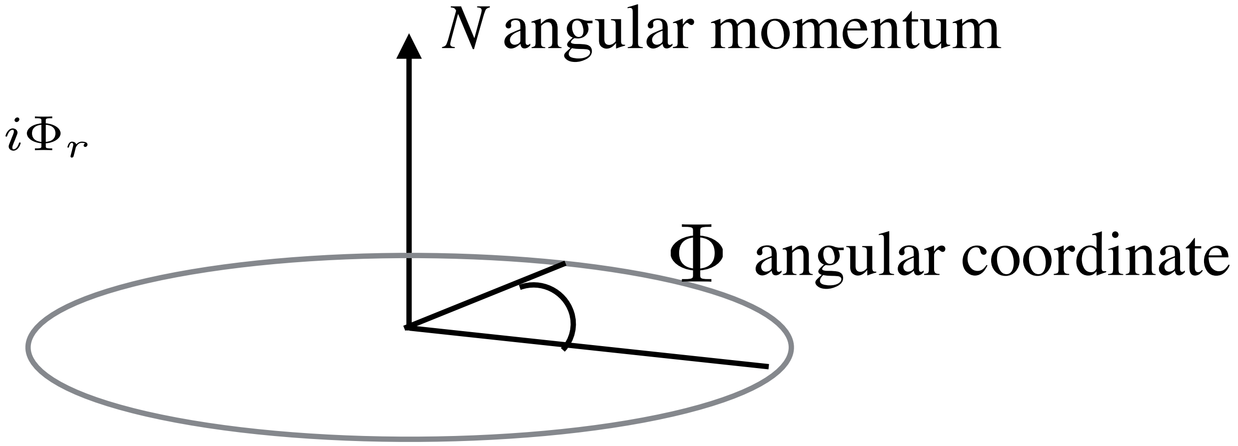
Bose Hubbard model

- On every site of the square lattice, put an O(2) quantum rotor:

$N_r \in \mathbb{Z}$ $\Phi_r \in [0, 2\pi)$

$[\Phi_r, N_{r'}] = i\delta_{rr'}$ $[N_r, e^{\pm i\Phi_r}] = \pm e^{\pm i\Phi_r}$

Conjugate number and phase

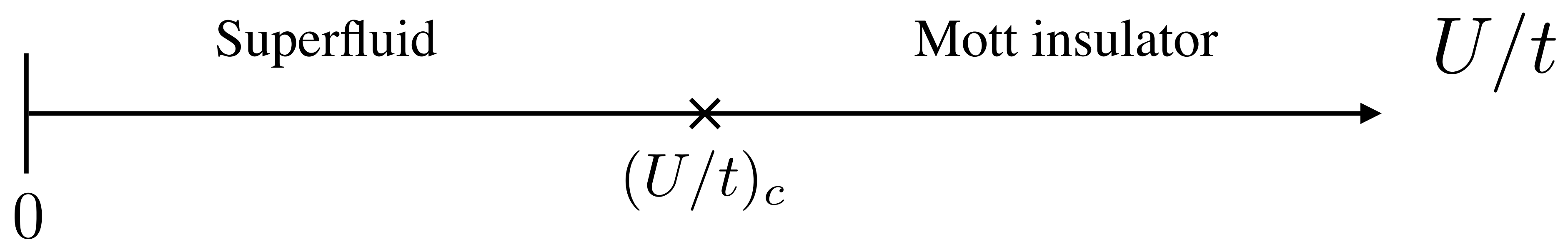


- Hamiltonian: $$H = U \sum_r N_r^2 - t \sum_{\langle rr' \rangle} \cos(\Phi_r - \Phi_{r'})$$

- Boson filling: $$n = \sum_r N_r / N_{sites} = 0$$

- Not exactly a model of bosons, but can view this is an effective model for bosons at integer filling, N_r gives deviations from the background density.

Bose Hubbard model: phase diagram



$$\langle e^{i\Phi} \rangle \neq 0$$

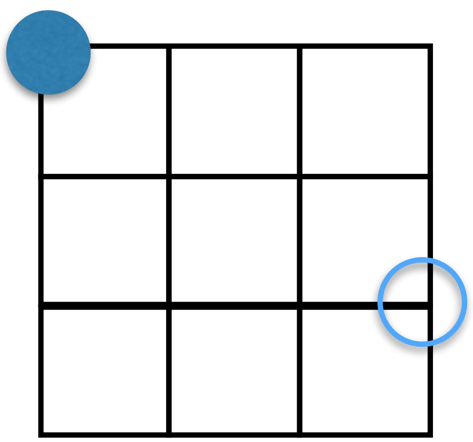
Gapless excitations
(Goldstone bosons)

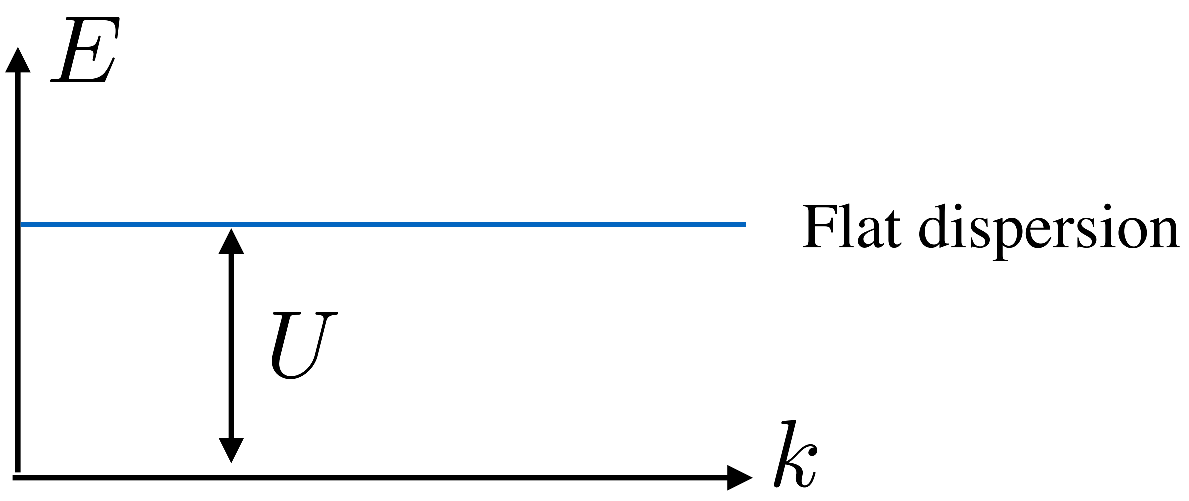
$$\langle e^{i\Phi} \rangle = 0$$

Energy gap to all
excitations

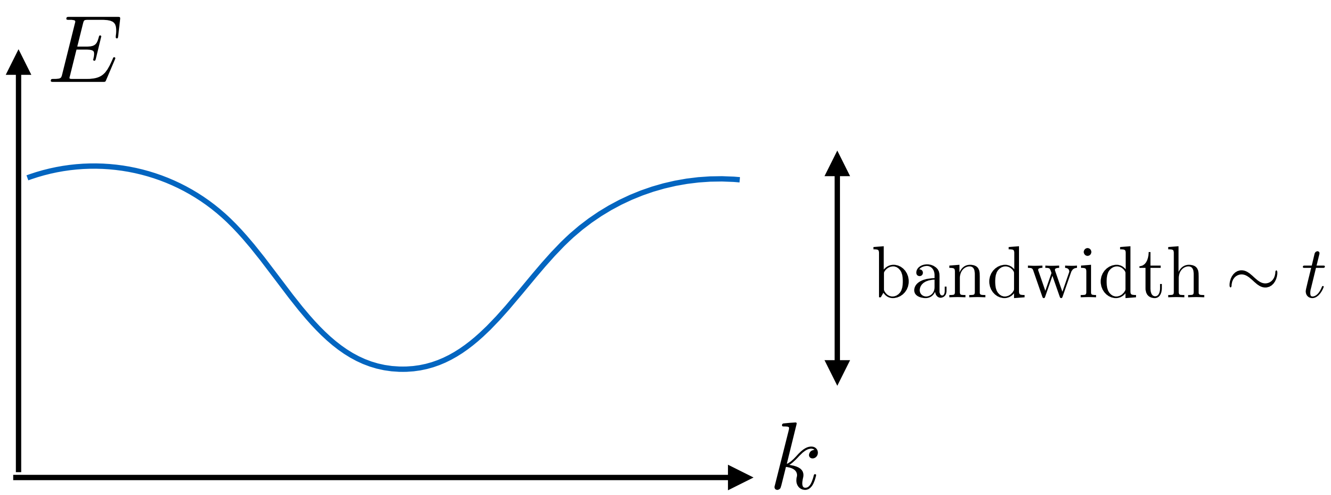
Mott insulator phase

- Very large-U: excitations are just adding/removing bosons from lattice sites

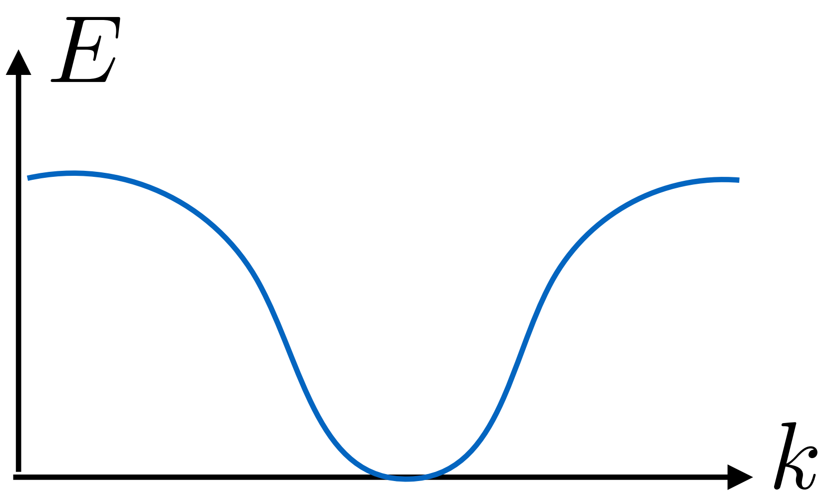




- Turn on small t, excitations begin to disperse



- Eventually, at critical t/U, excitations come down to E=0 and condense



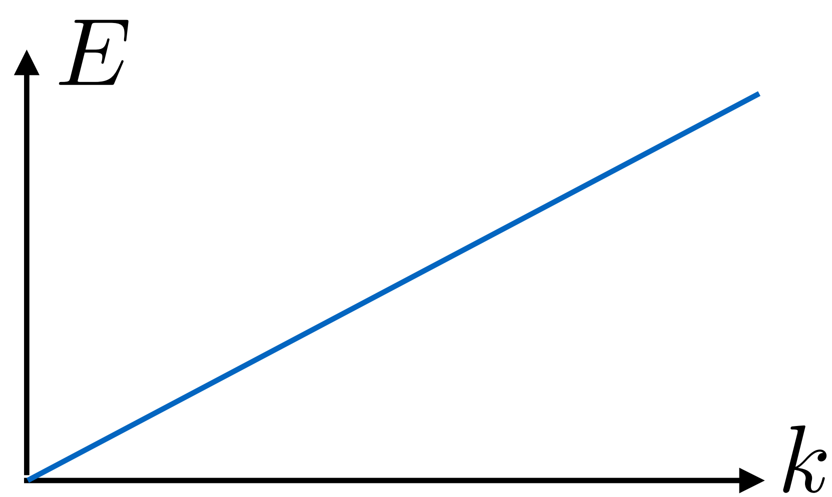
Superfluid phase

$$H = U\sum_r N_r^2 - t\sum_{\langle rr'\rangle}\cos(\Phi_r - \Phi_{r'})$$

- Large t/U, expand cosine to get a quadratic effective Hamiltonian

$$H_{eff} = U\sum_r N_r^2 + \frac{t}{2}\sum_{\langle rr'\rangle}(\Phi_r - \Phi_{r'})^2 \quad\Longrightarrow\quad H_{eff} = \int d^2r\left[\mathcal{U}\,N^2 + \frac{K}{2}(\nabla\Phi)^2\right]$$

Continuum limit



Linearly dispersing superfluid sound mode (Goldstone boson)

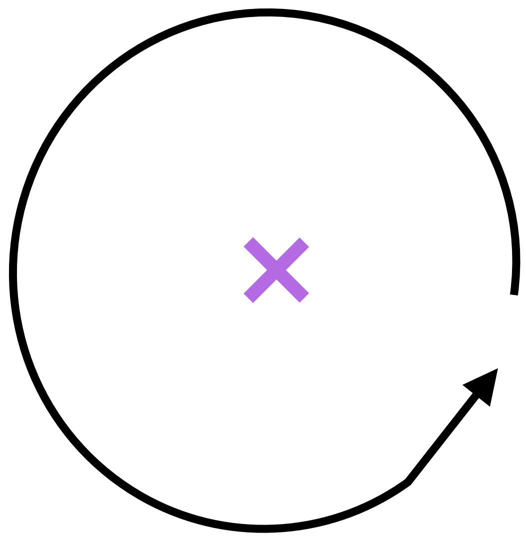
Can we get from this effective theory back to the Mott insulating state?

No! This theory does not know about the discreteness of charge, cannot have a Mott insulator phase.

Suggests that also we are missing some aspect of the superfluid phase…

Superfluid vortices

- Superfluid also has vortex excitations



$$\oint \nabla\Phi \cdot d\ell = 2\pi$$

$$\Phi \to \Phi + 2\pi$$

Boson phase winds by 2π going around vortex

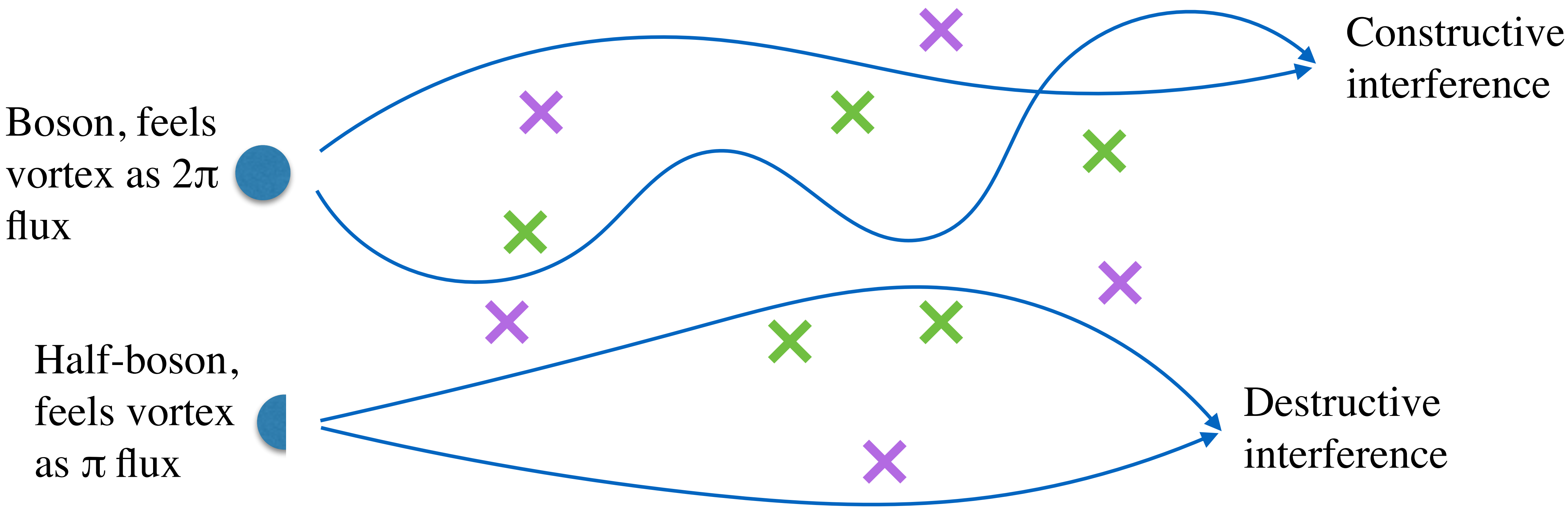
In general, vorticity is an integer; for *n*-fold vortex:

$$\oint \nabla\Phi \cdot d\ell = 2\pi n_v$$

Usually, single vortices (n_v=1) are lowest energy

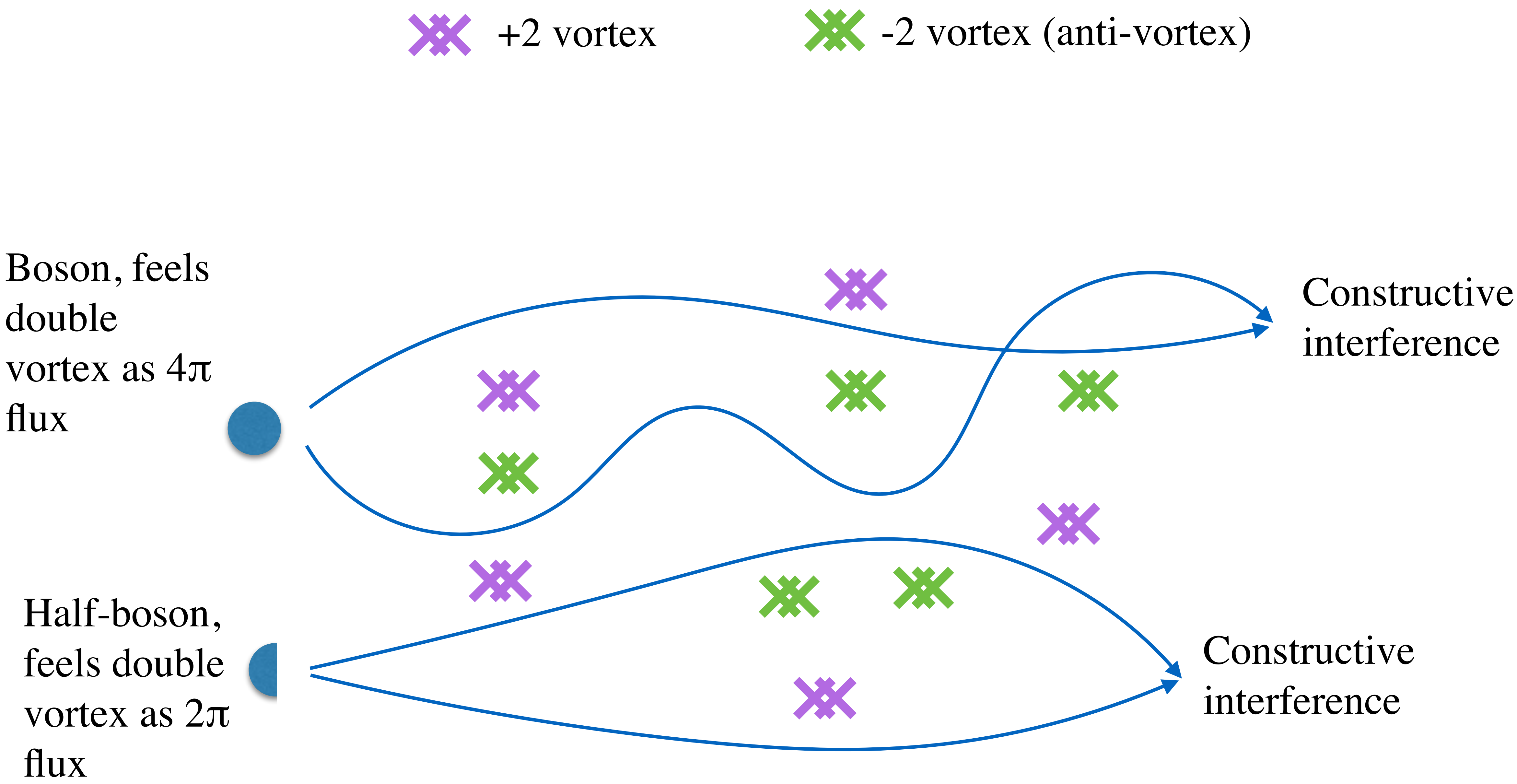
Mott insulator via vortex condensation

- When vortices come down to zero energy, they condense. This leads back to the Mott insulator



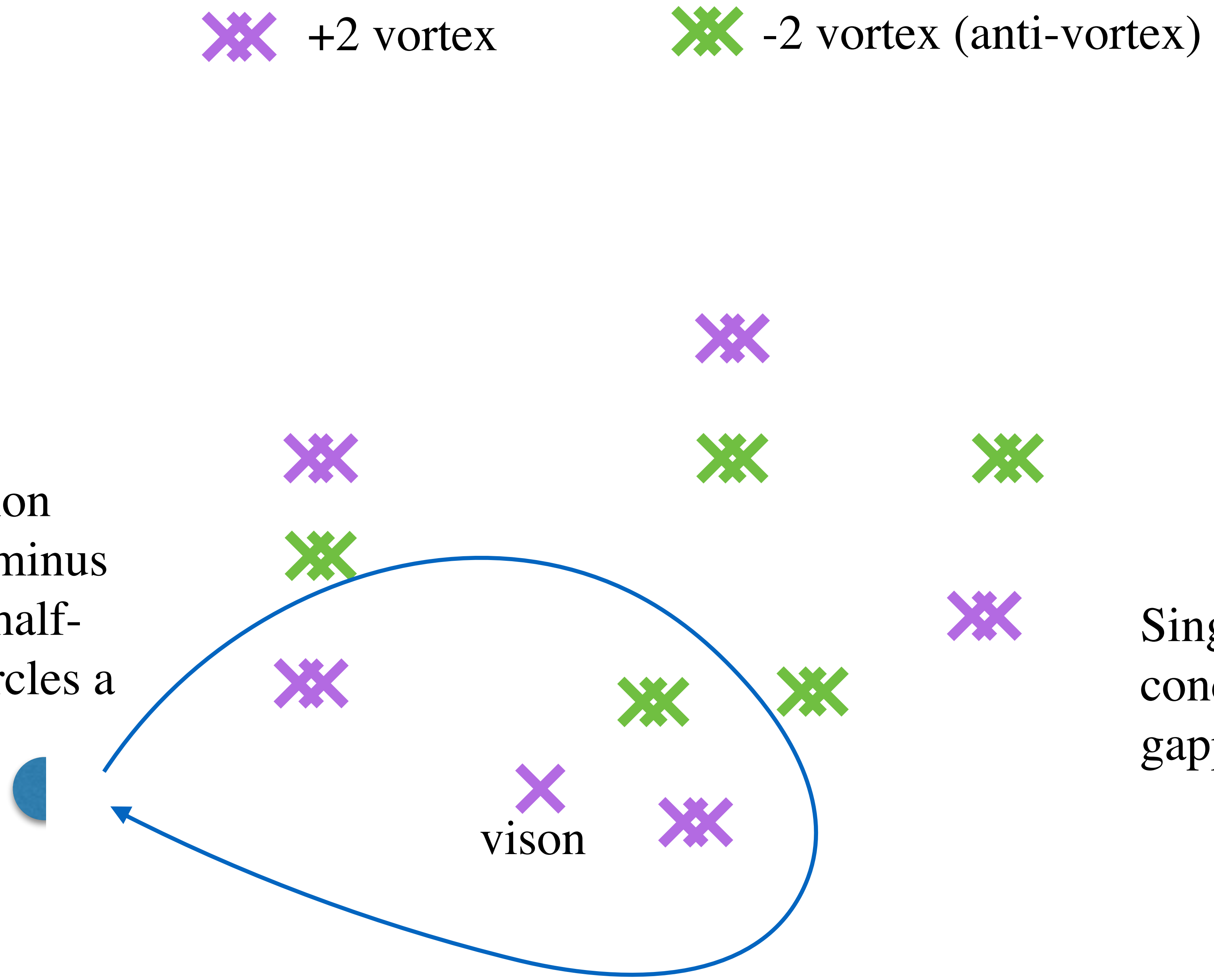
- Only integer charge excitations are free to propagate in the vortex condensate - discreteness of charge is restored.

Z_2 QSL via double vortex condensation



- If double-vortices condense, half-integer charge excitations can also propagate (but not other fractions of a boson).

Z_2 QSL via double vortex condensation



Wavefunction picks up a minus sign when half-boson encircles a vison

Single vortices are not condensed and become gapped visons.

In the language we used earlier, we identify the half-boson as the spinon, while the vison is a flux excitation