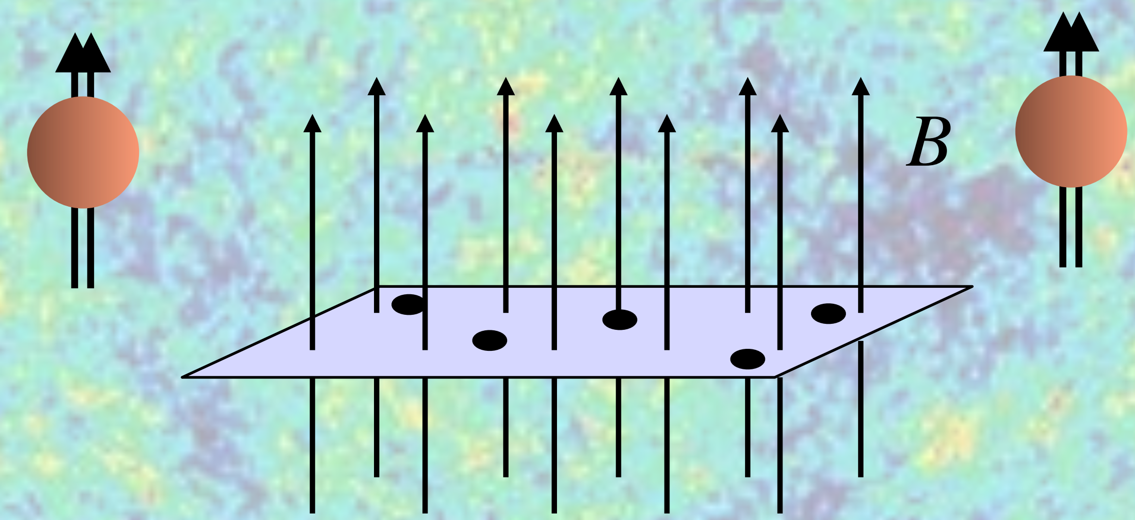
R2CdaQch3G/Om+rHOQVHYfhQSsLAxV9uXoJeuXOuNmo=

R2CdaQch3G/Om+rHOQVHYfhQSsLAxV9uXoJeuXOuNmo=

I have no clue what this means.

The Incredible World of 2D Electrons in a Magnetic Field



NTERNATIONAL YEAR OF
Quantum Science
and Technology

A story with many twists and turns

A story with many twists and turns

- Unfolding of the fractional quantum Hall effect mystery

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- Mission: Impossible?

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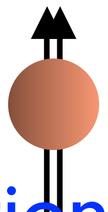
- The key:  Composite fermions - motivation, confirmation

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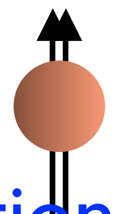
- The key:  Composite fermions - motivation, confirmation
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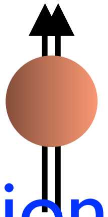
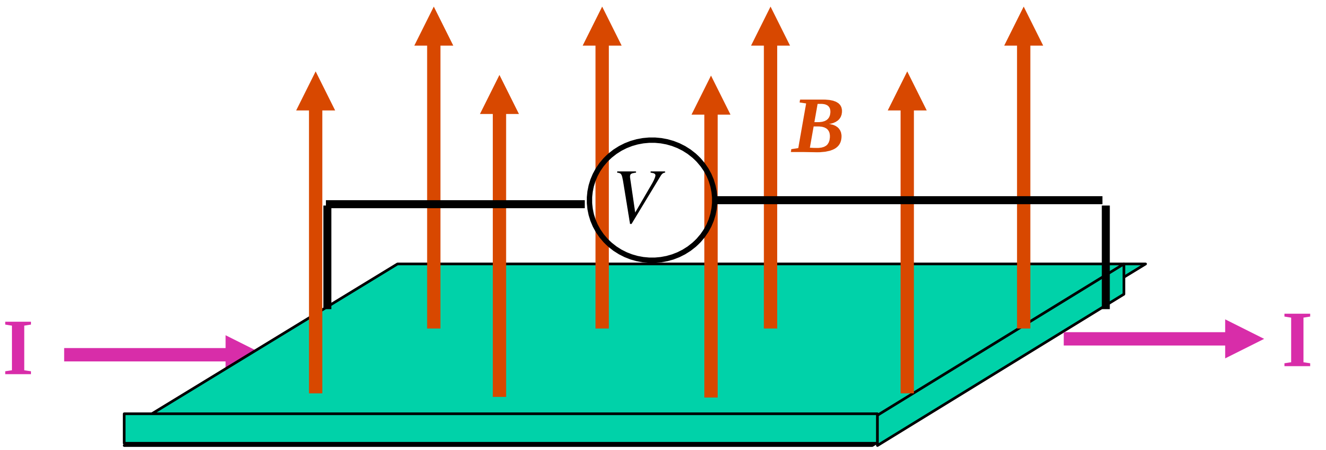
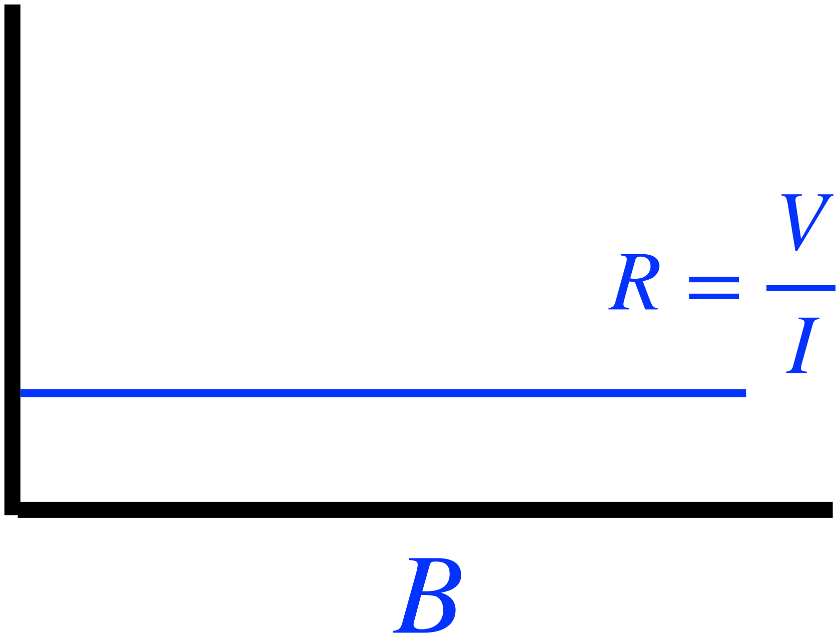
- The key:  Composite fermions - motivation, confirmation
- Anyons, Majoranas
- More tomorrow

The Hall effect and the Hall resistance (1879)



Edwin Hall
1979



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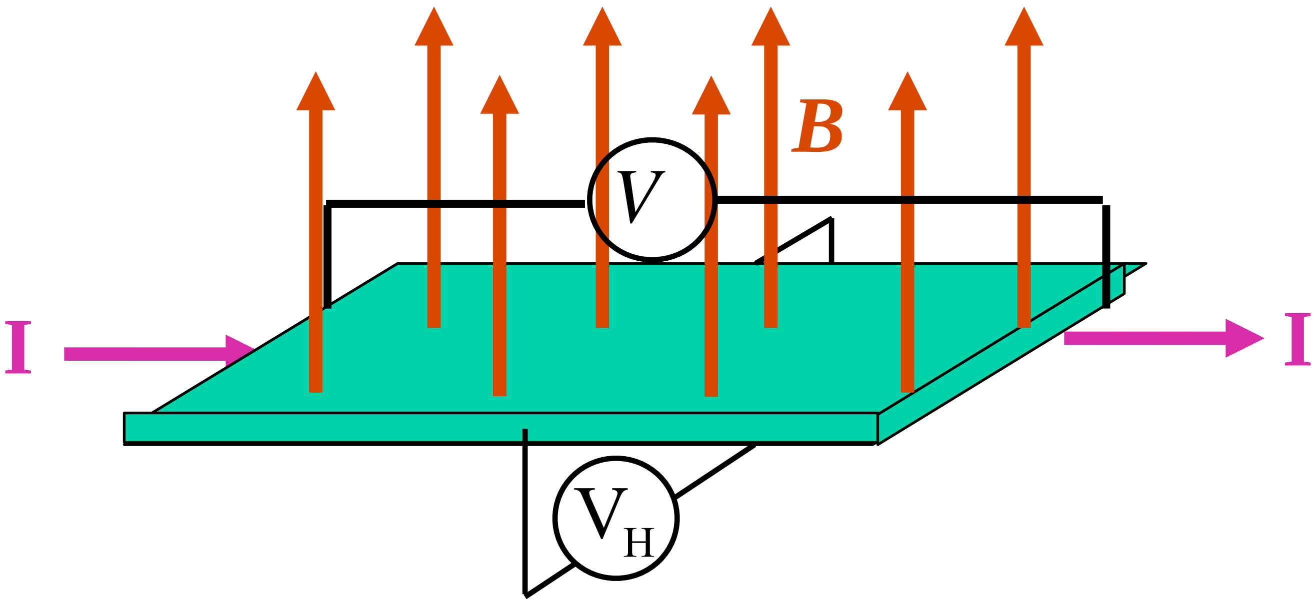

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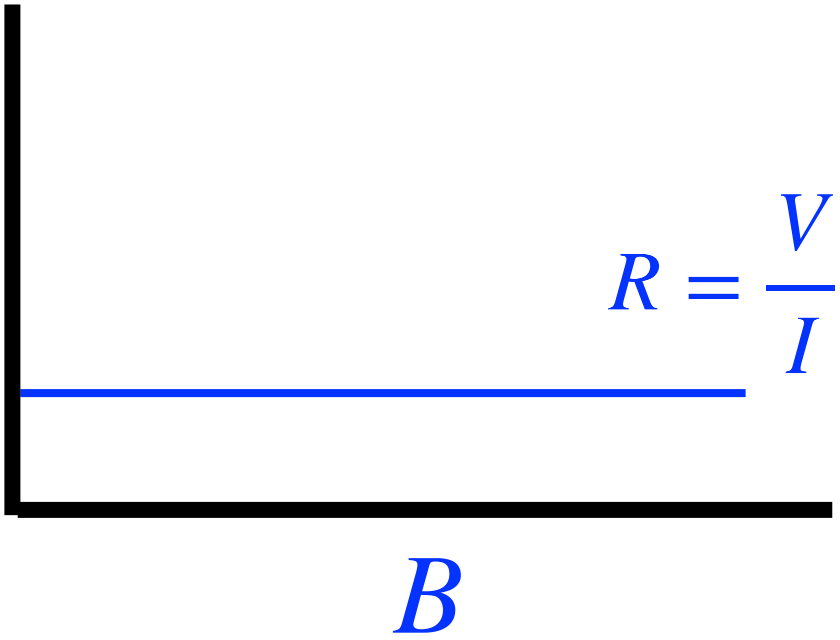

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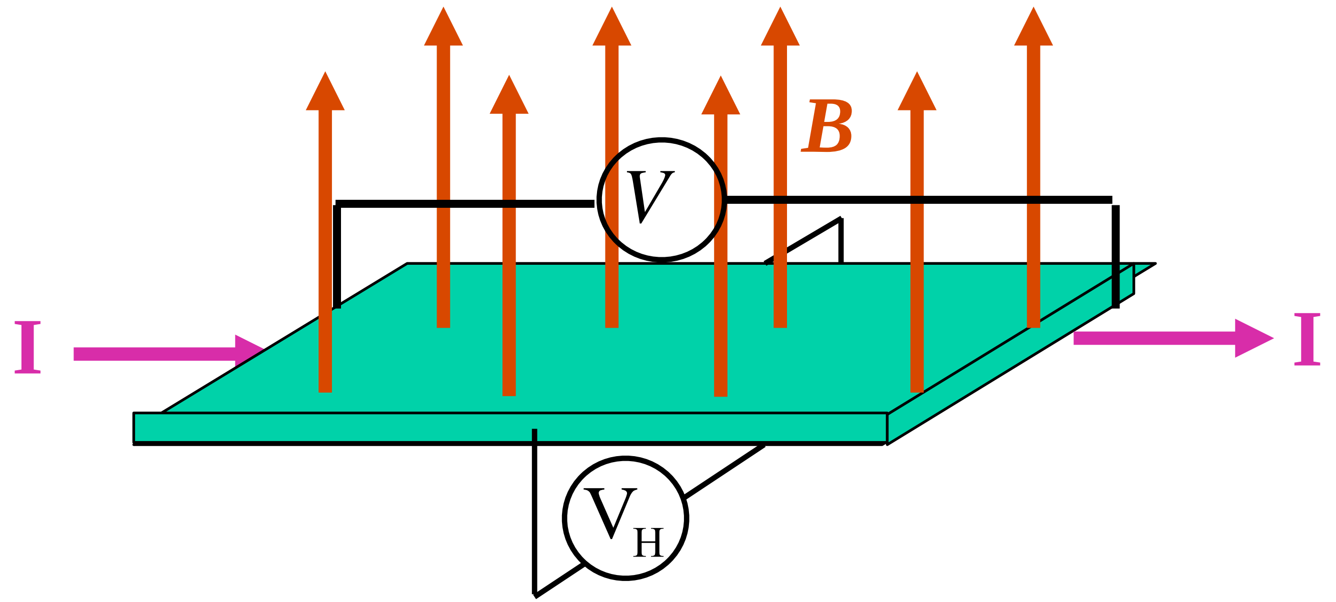

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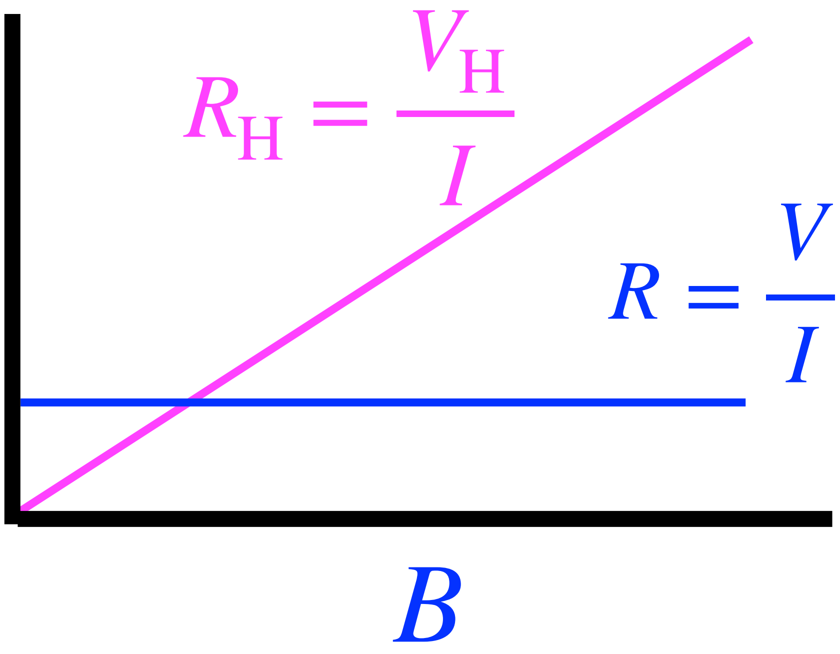

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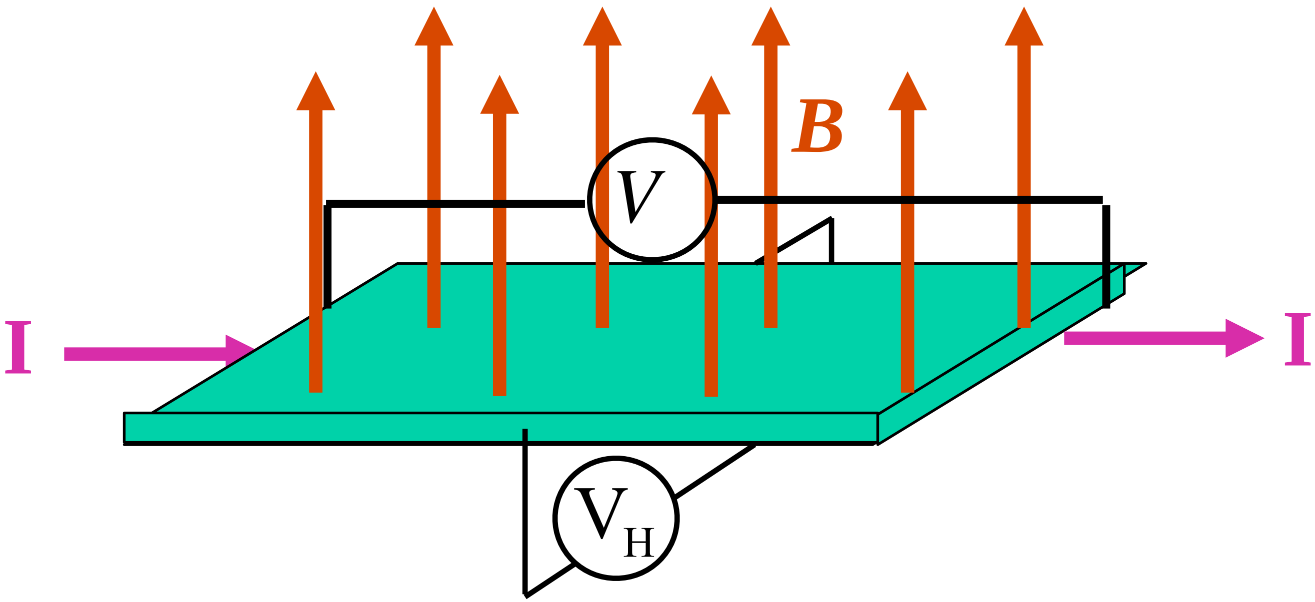

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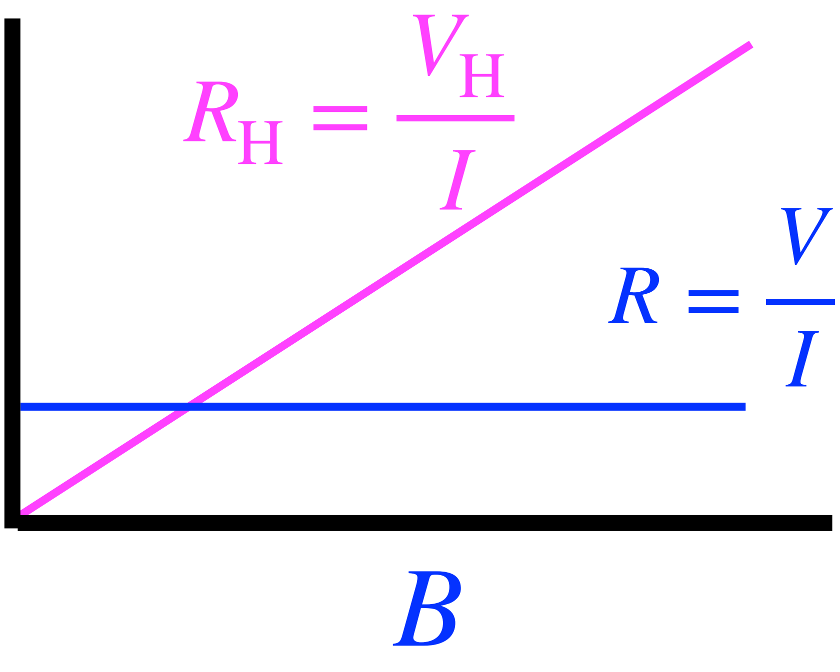
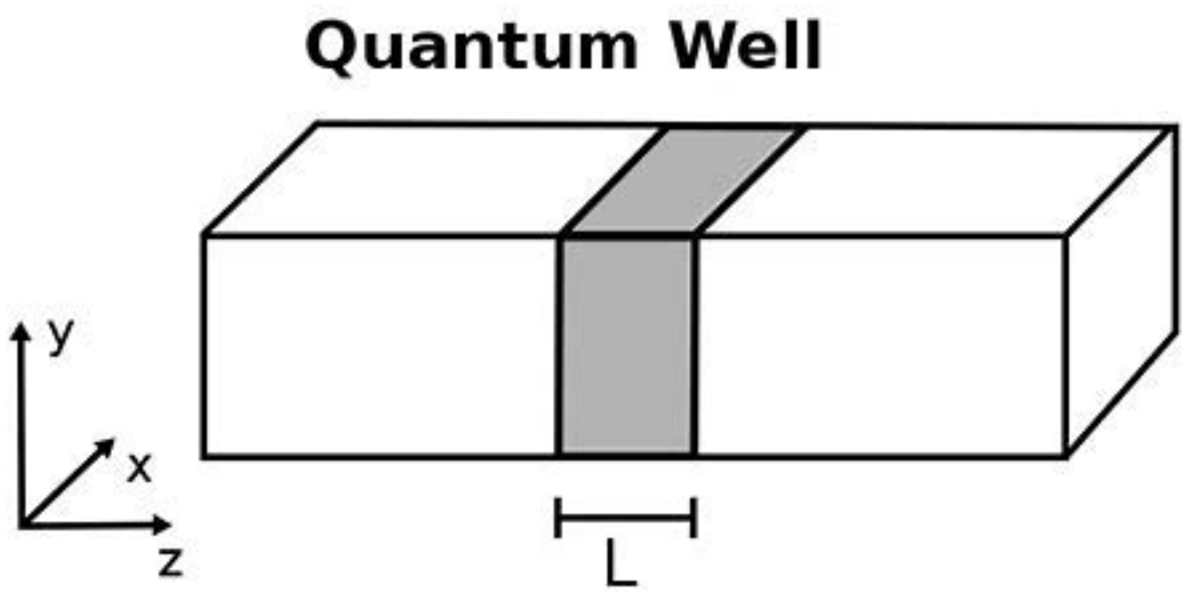


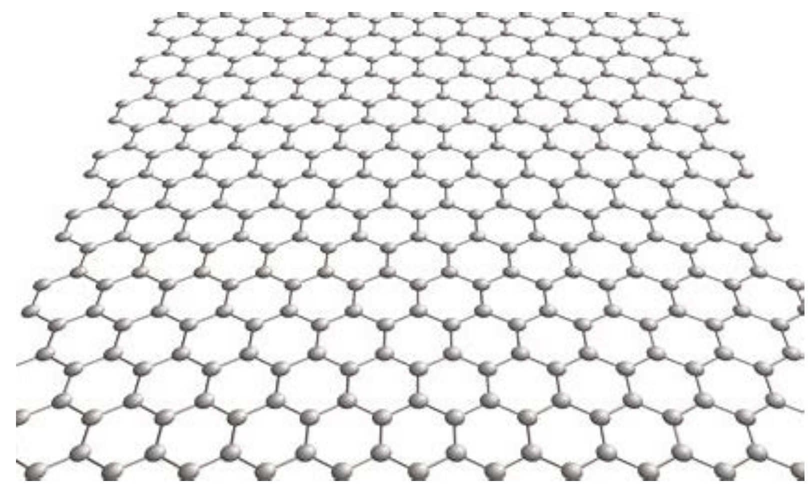
So it remained for a century.

Enter: Two-dimensional electron systems



MOSFET

Graphene



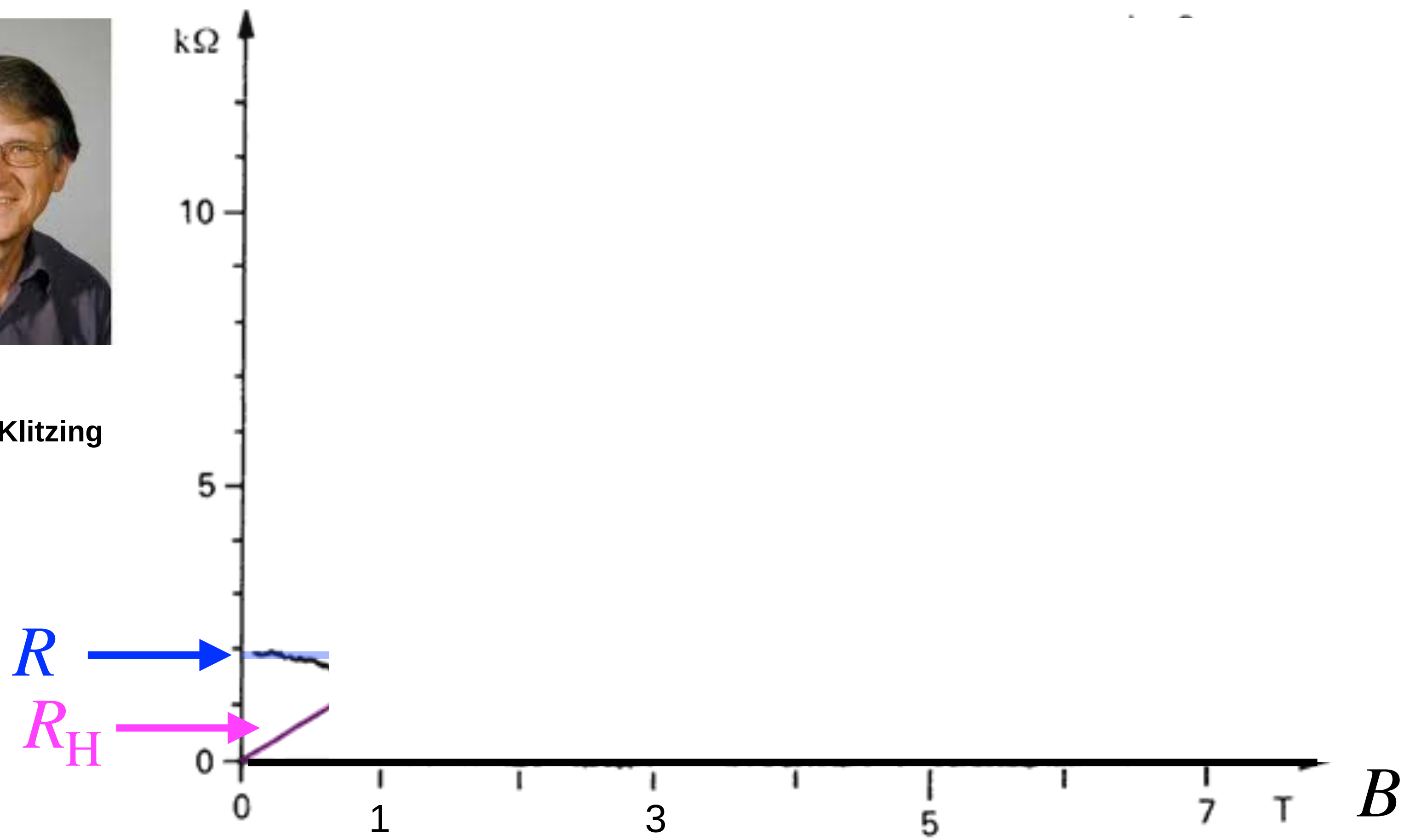
The Hall effect in two dimensions

VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS 11 AUGUST 1980

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing
Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France

Klaus von Klitzing



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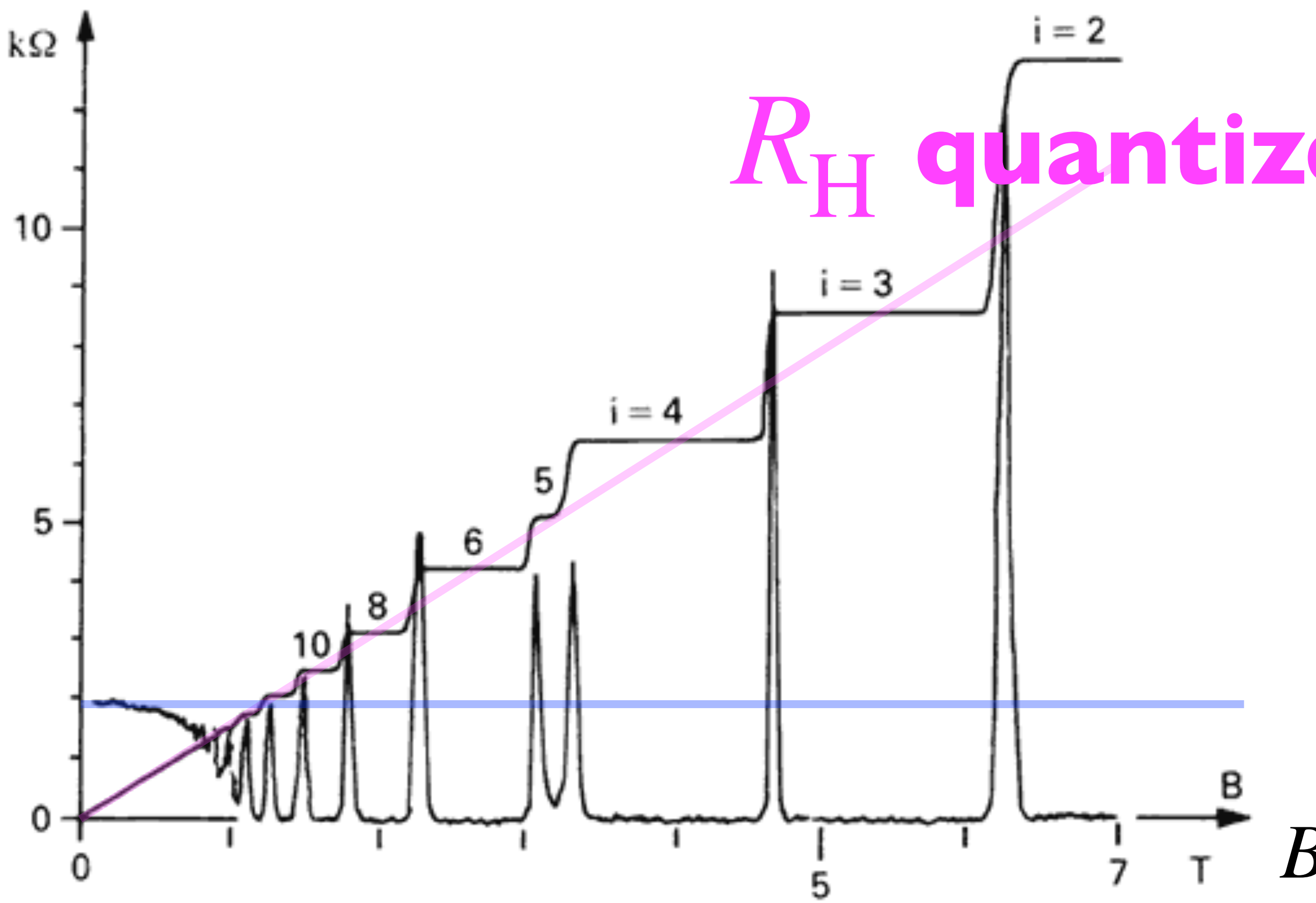

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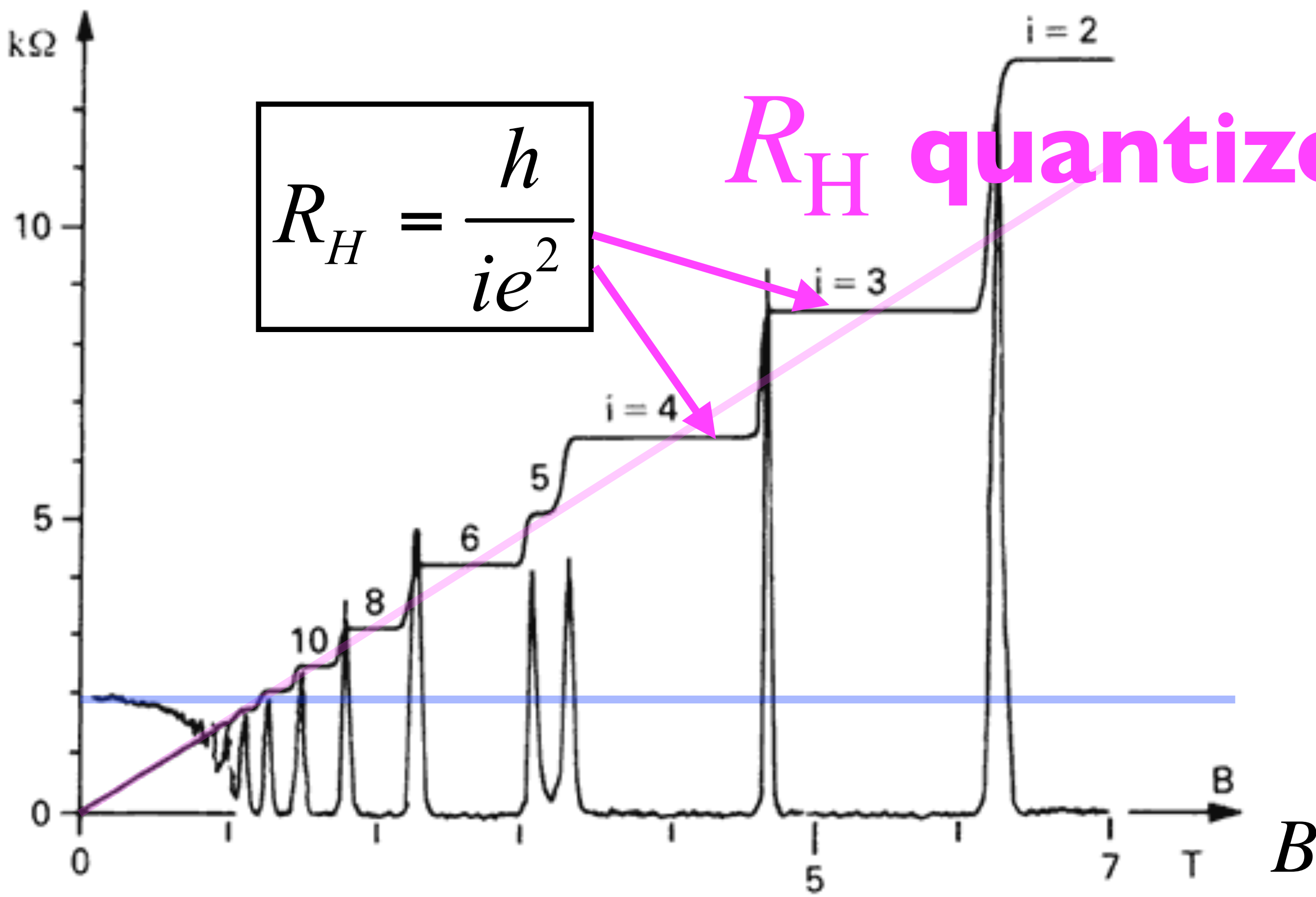

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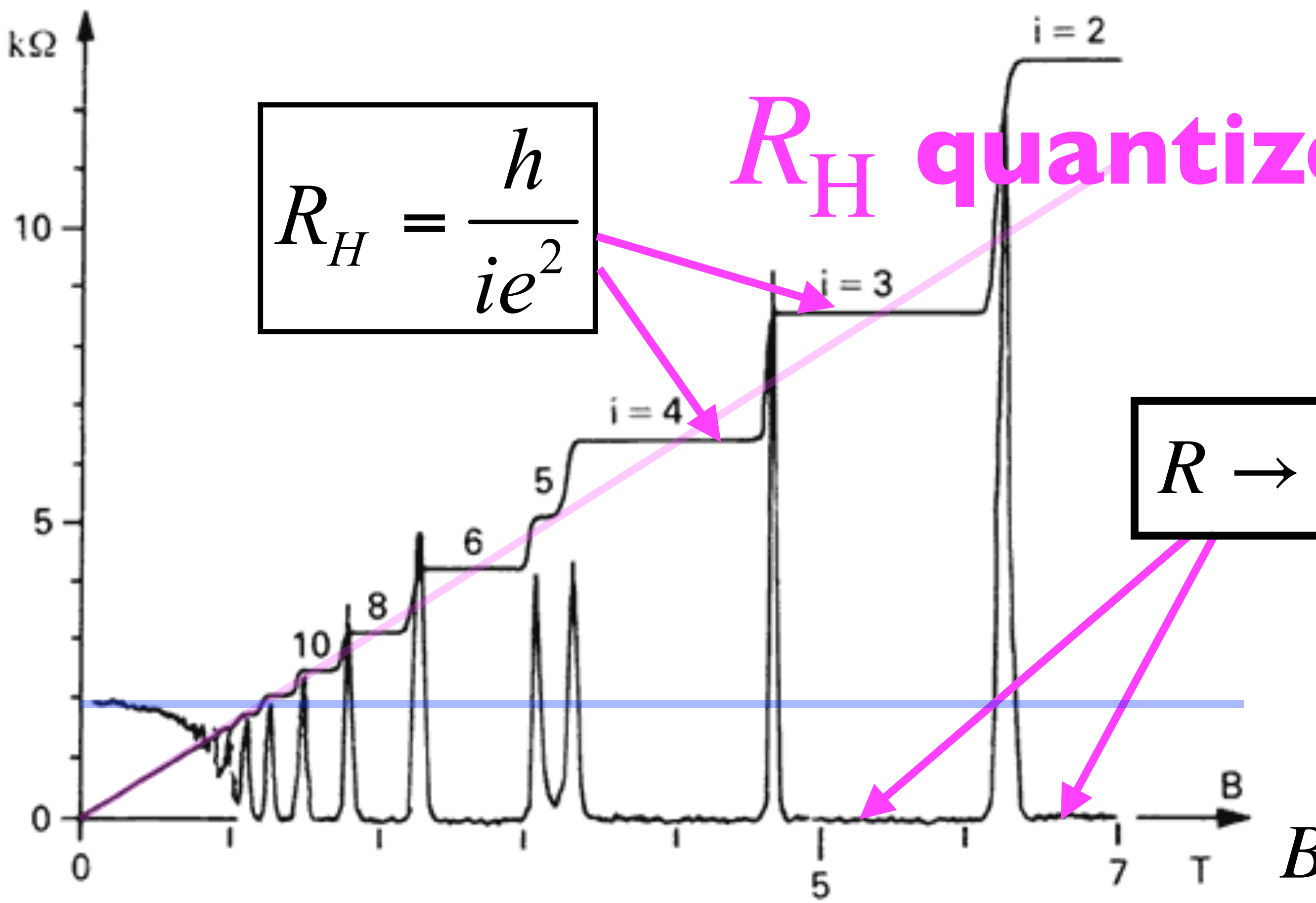

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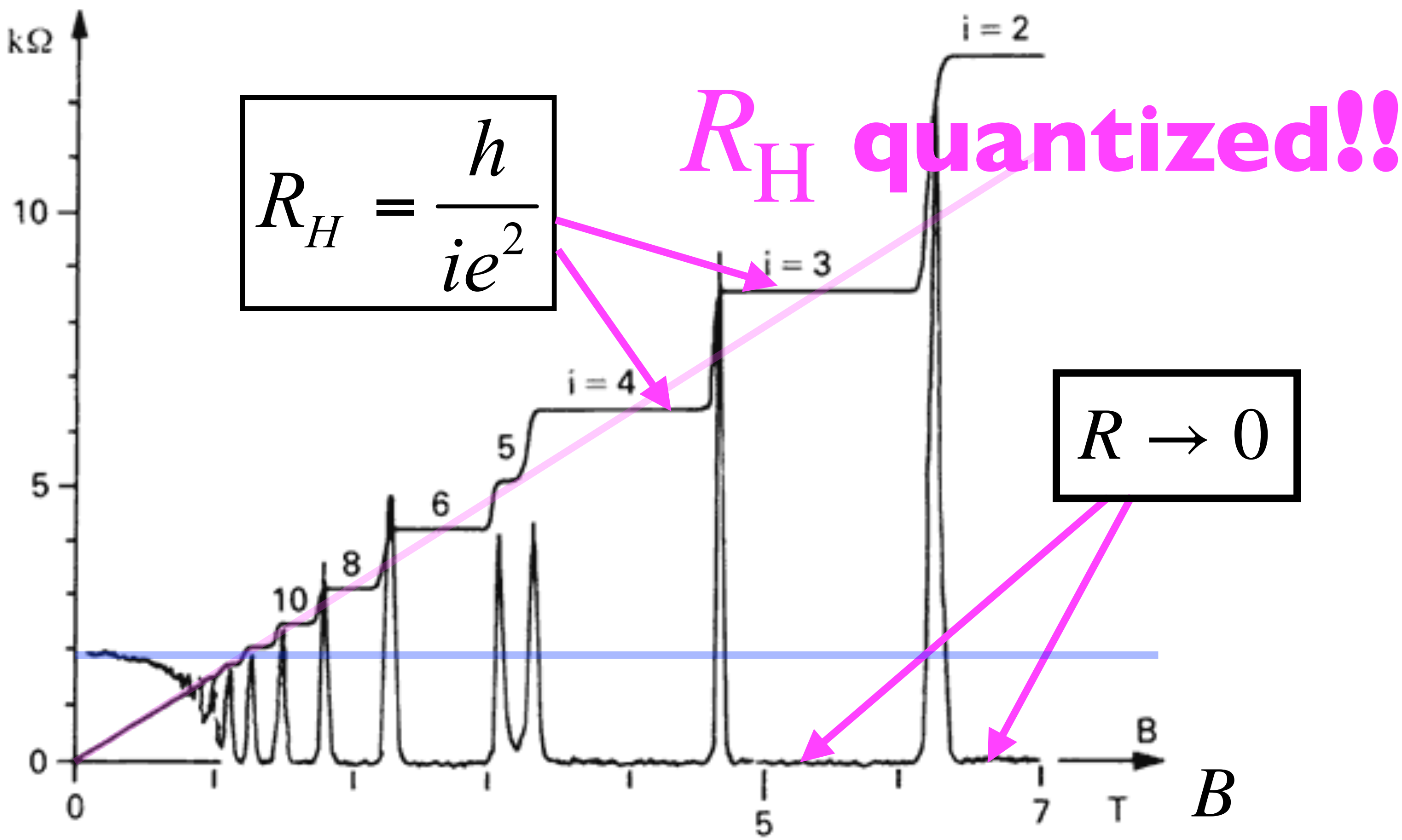

Integer quantum Hall effect

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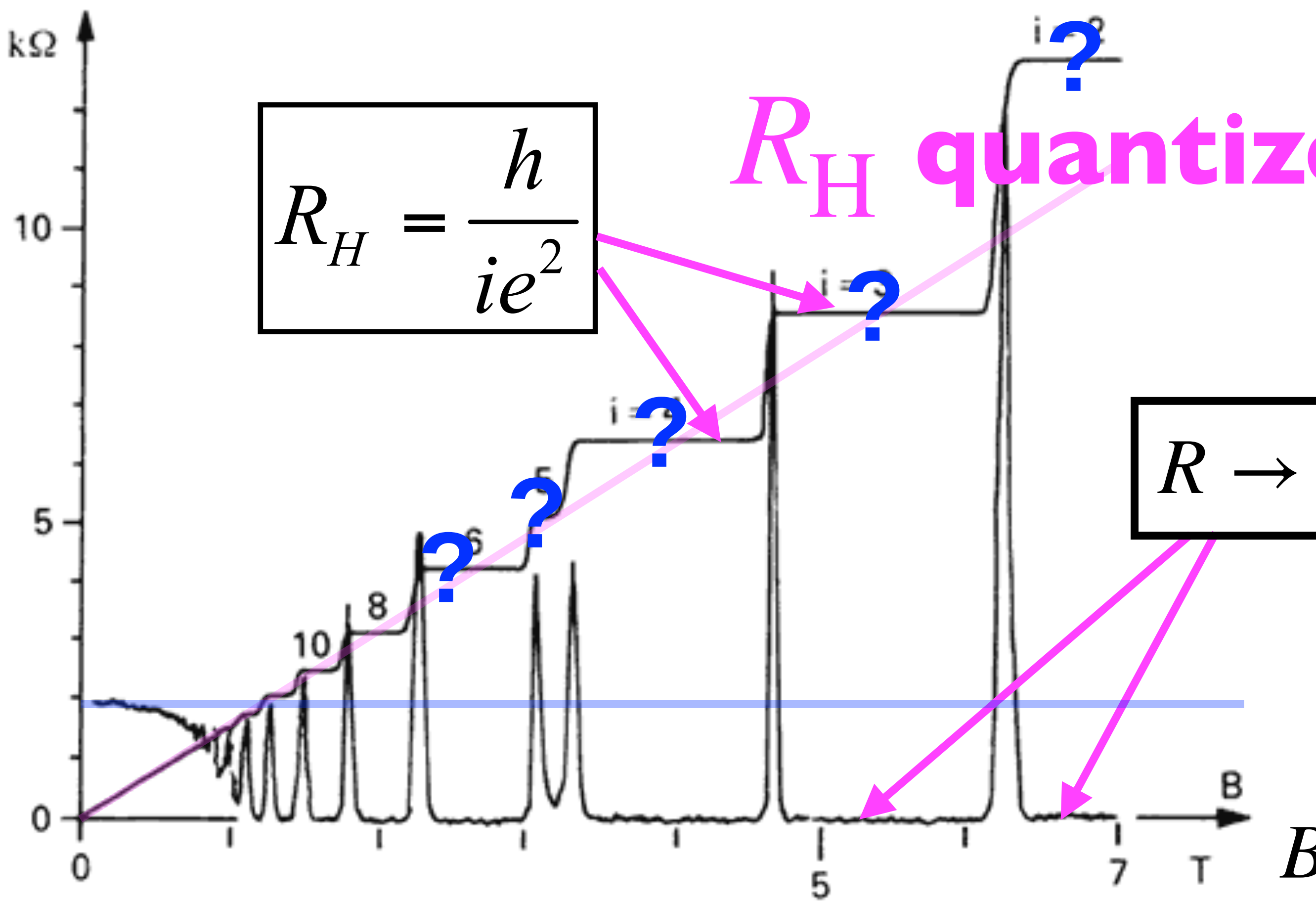

Integer quantum Hall effect

Minimize energy

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- Two types of energies: kinetic and interaction.

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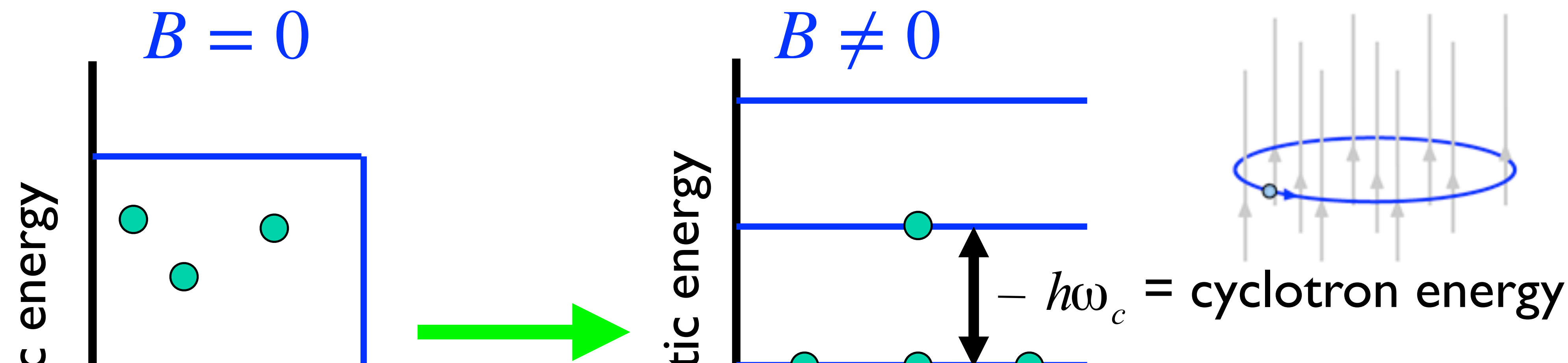
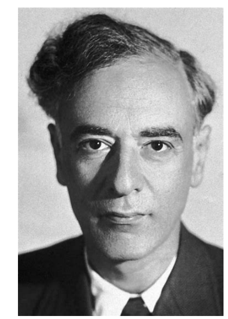
Minimize energy

- Two types of energies: kinetic and interaction.
- Magic of quantum mechanics: In a magnetic field, the kinetic energy takes certain special values, that is, it becomes quantized! (Landau levels.)
- First forget about the interaction and minimize the kinetic energy.

Landau levels (2D)





$$N_\phi = \frac{BA}{\phi_0}$$ = number of available spaces in each LL (one per ϕ_0)

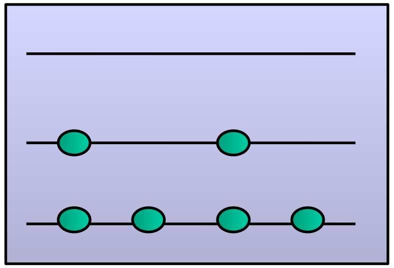
"flux quantum" $\phi_0 = \frac{hc}{e}$, $\rho = \frac{N}{A}$ = density,

Filling factor $\nu = \frac{N}{N_\phi} = \frac{N\phi_0}{BA} = \frac{\rho\phi_0}{B}$ = # of occupied Landau levels

Origin of the IQHE

Origin of the IQHE

- At general filling factors, we have many possibilities:



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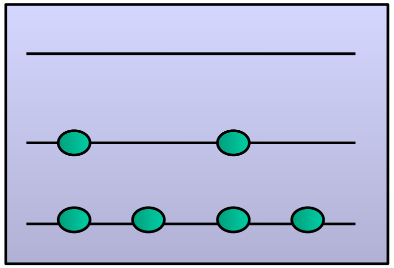
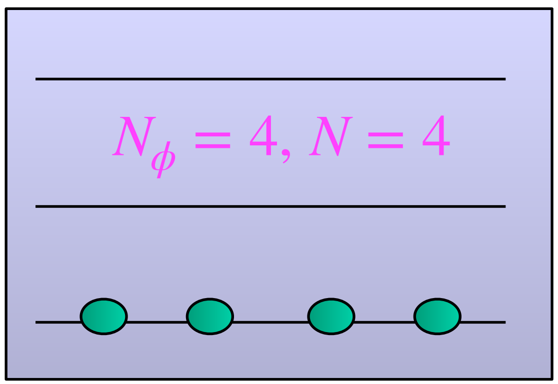


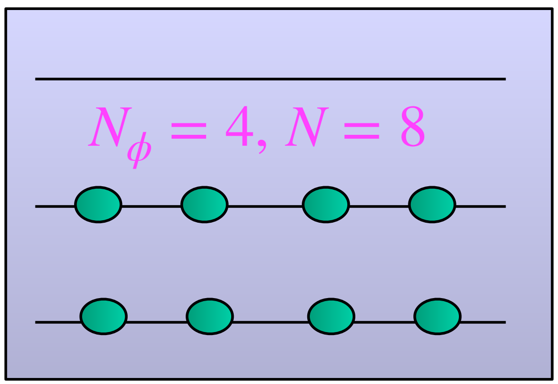
- At integer filling factors, however, **unique gapped** states are obtained.



Φ_1

$\nu = 1$

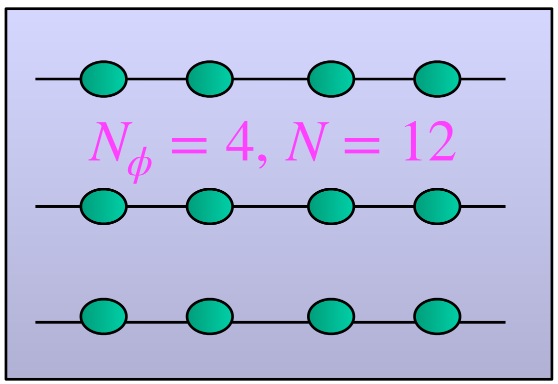
$R_{\mathrm{H}} = \frac{h}{1e^2}$



Φ_2

$\nu = 2$

$R_{\mathrm{H}} = \frac{h}{2e^2}$



Φ_3

$\nu = 3$

$R_{\mathrm{H}} = \frac{h}{3e^2}$

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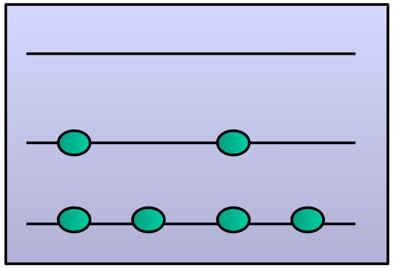

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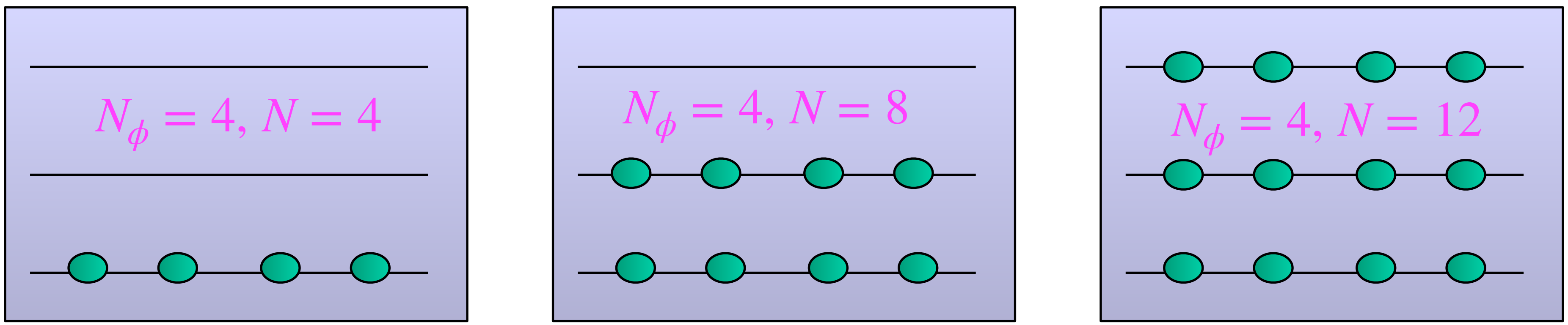
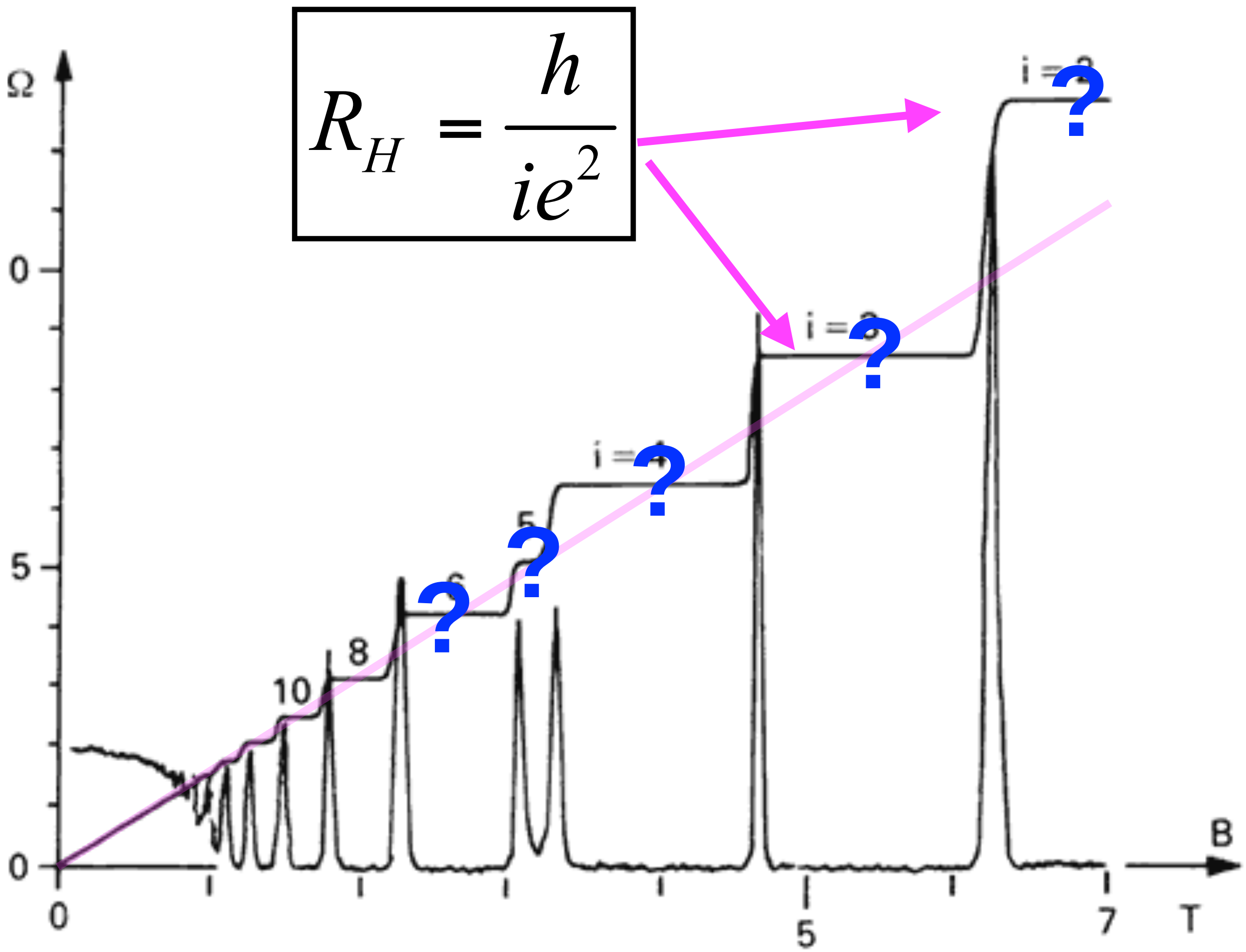


Φ_1	Φ_2	Φ_3
$\nu = 1$	$\nu = 2$	$\nu = 3$
$R_\text{H} = \dfrac{h}{1e^2}$	$R_\text{H} = \dfrac{h}{2e^2}$	$R_\text{H} = \dfrac{h}{3e^2}$

- The IQHE is well understood. (Disorder also plays a crucial role, but that is not directly relevant to this talk.)

The IQHE mystery



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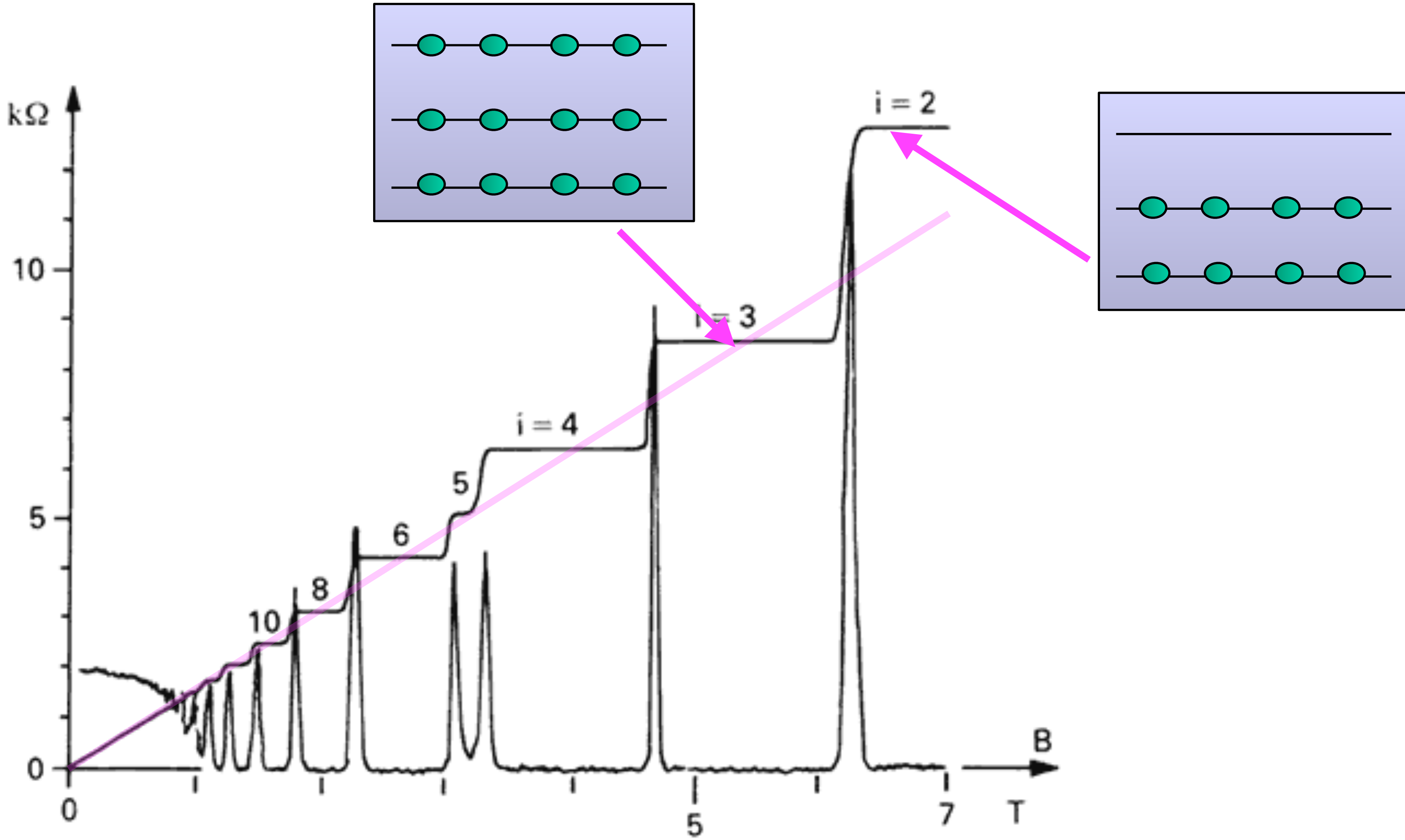

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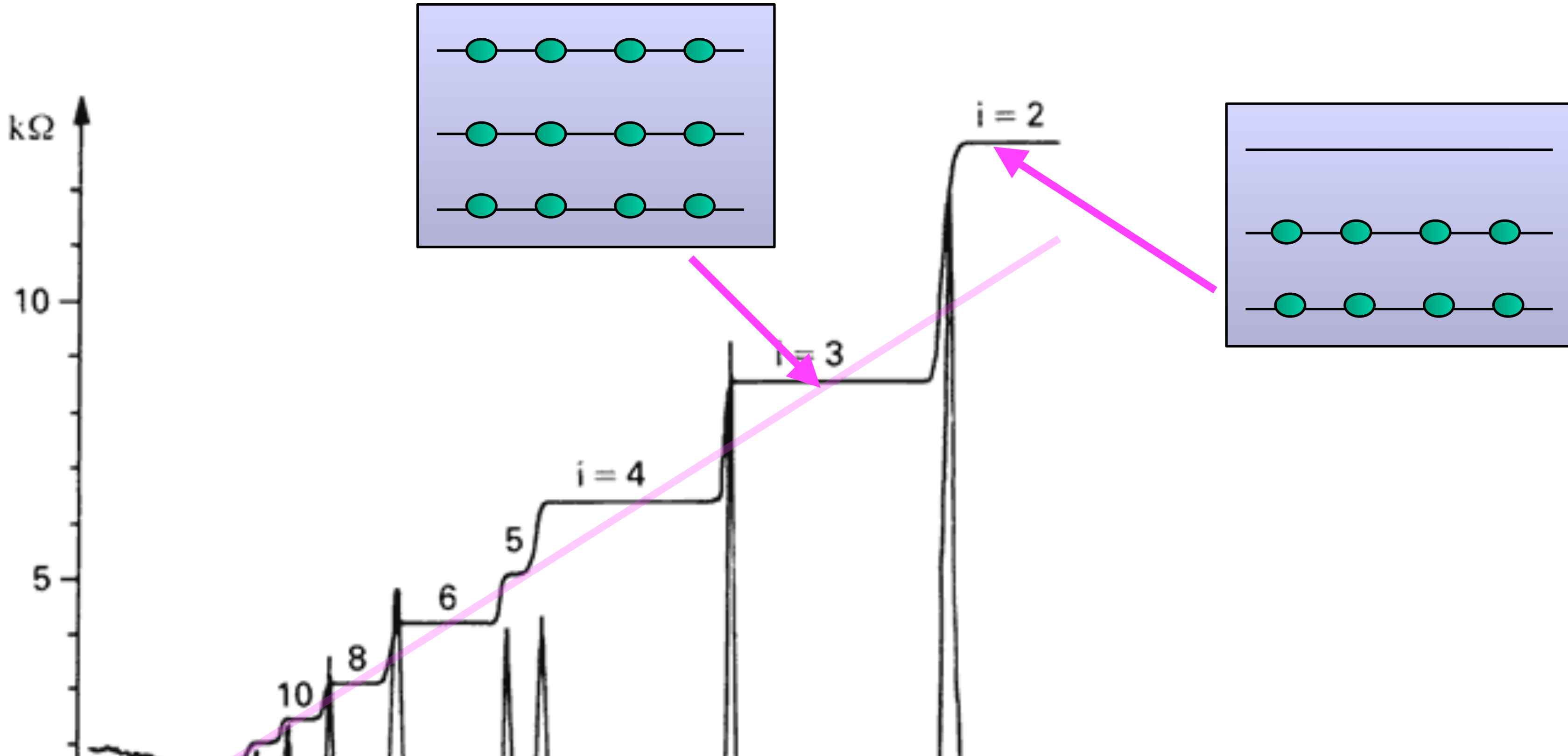

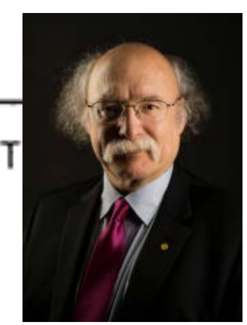

Haldane

Thouless

Topological interpretation

The 1/3 effect

VOLUME 48, NUMBER 22 PHYSICAL REVIEW LETTERS 31 MAY 1982

Two-Dimensional Magnetotransport in the Extreme Quantum Limit

D. C. Tsui,[a),(b)] H. L. Stormer,[(a)] and A. C. Gossard
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 5 March 1982)

$$R_H = \frac{h}{\frac{1}{3}e^2}$$

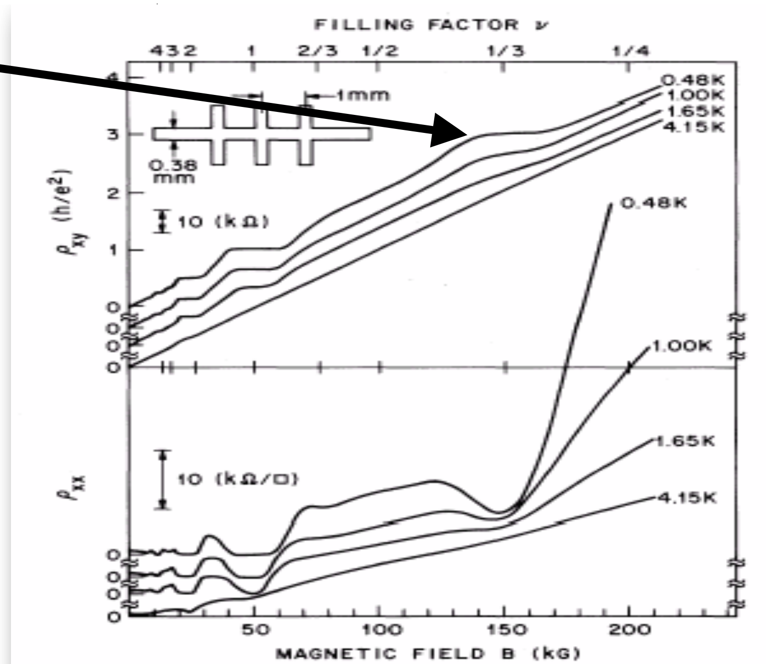

FIG. 1. ρ_{xy} and ρ_{xx} vs B, taken from a GaAs-$Al_{0.3}$-$Ga_{0.7}$As sample with $n = 1.23 \times 10^{11}/cm^2$, $\mu = 90\,000\ cm^2/V\ sec$, using $I = 1\ \mu A$. The Landau level filling factor is defined by $\nu = nh/eB$.

Yet another surprise!

The 1/3 effect

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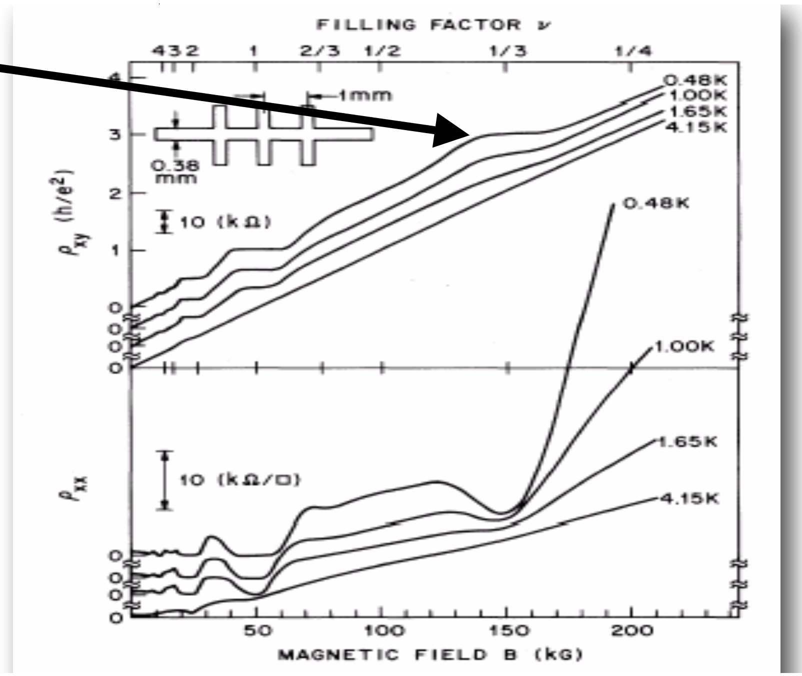


Yet another surprise!

VOLUME 50, NUMBER 18 PHYSICAL REVIEW LETTERS 2 MAY 1983

Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations

R. B. Laughlin
Lawrence Livermore National Laboratory, University of California, Livermore, California 94550
(Received 22 February 1983)

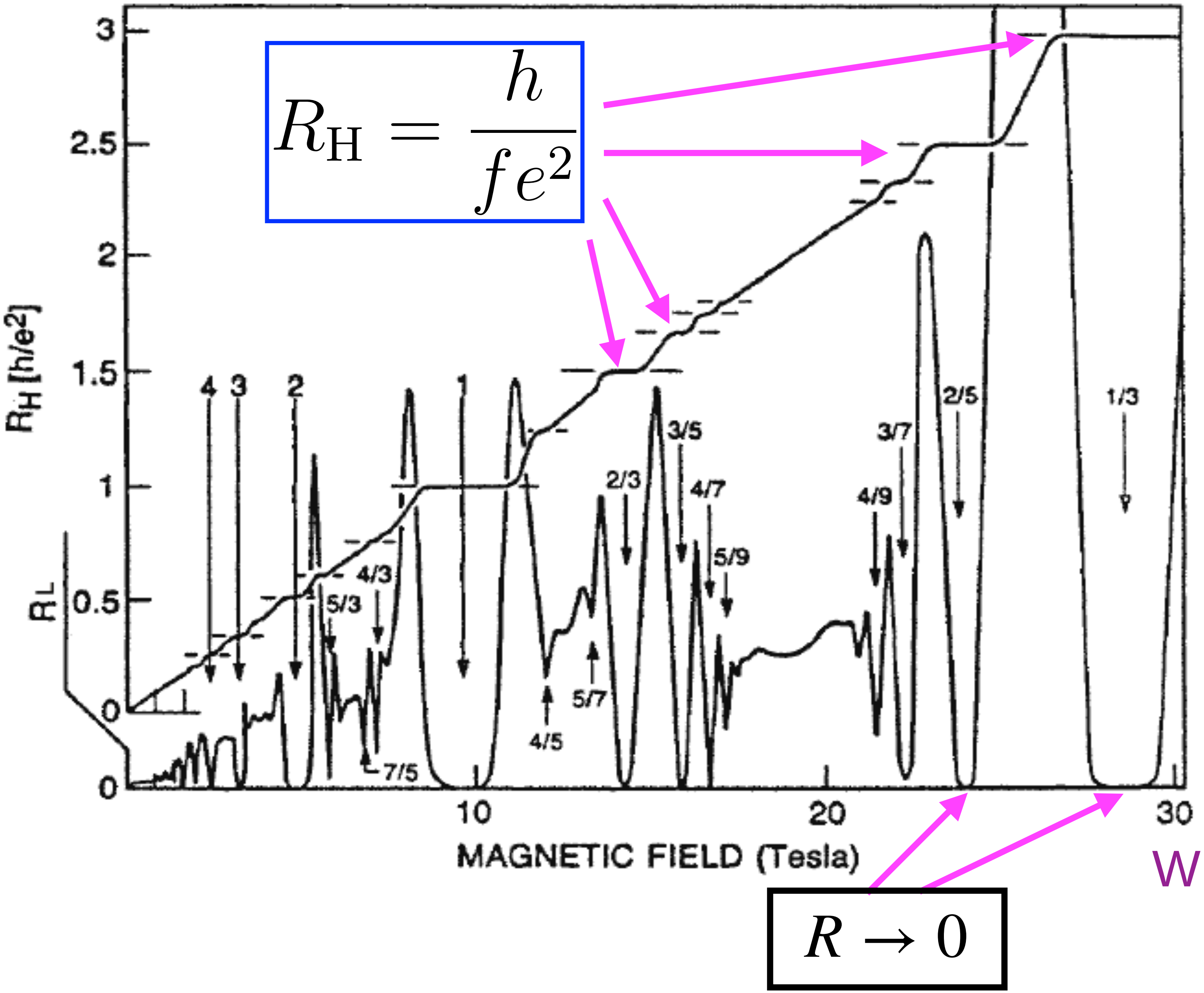
$$\Psi_{1/3} = \prod_{j<k} (z_j - z_k)^3 e^{-\sum_j |z_j|^2/4} \qquad z_j = x_j - iy_j$$

Not over yet!

This was still only the beginning!

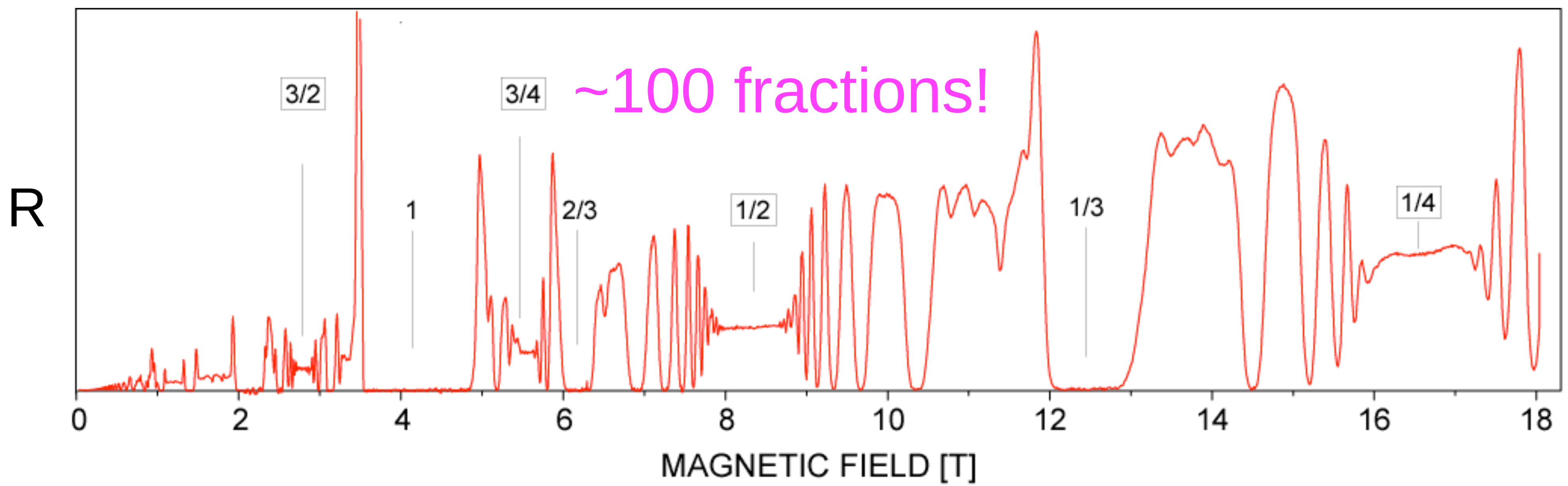
Improved experiments revealed an incredibly rich structure.

The fractional quantum Hall effect (FQHE)



Willett et al.

The fractional quantum Hall effects

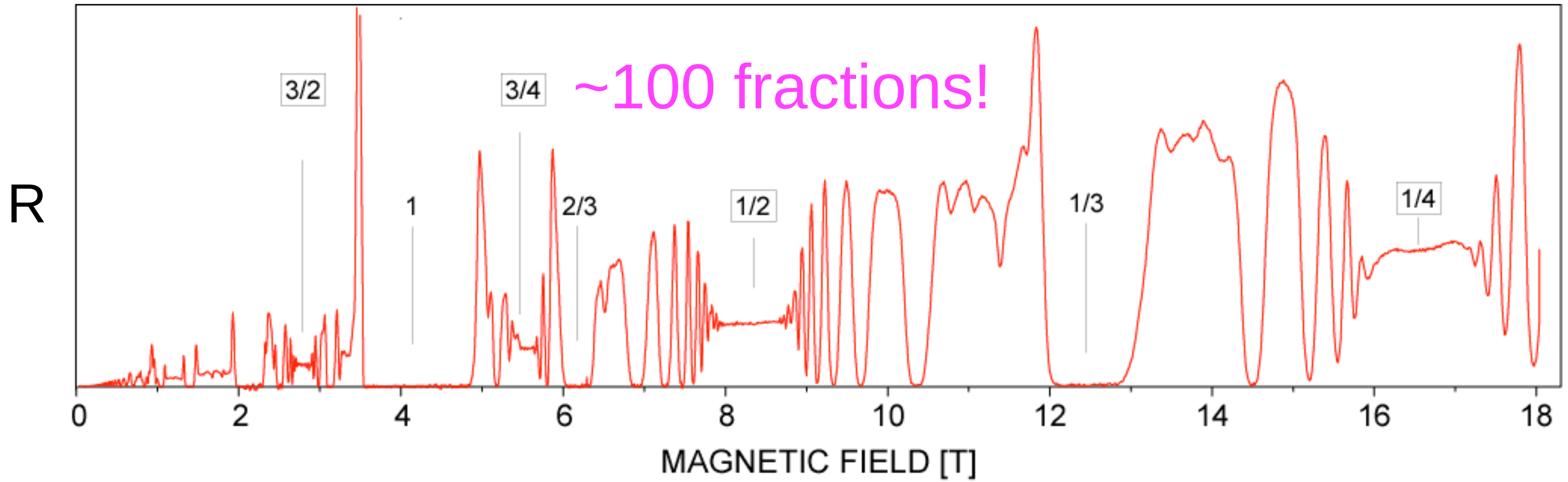


Pan, Stormer et al.

- The FQHE is among the most stunning manifestations of quantum mechanics at the macroscopic scale. It is one of the most striking mysteries nature has presented in quantum condensed matter.

The fractional quantum Hall effects

The most beautiful single trace in physics



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High temperature superconductivity 1987

The minimal model:
2D electrons in the lowest Landau level

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$$H\Psi = E\Psi$$

$$H = \sum_j \frac{1}{2m}(\vec{p}_j + e\overrightarrow{A}(\vec{r}))^2 + \sum_{j<k} \frac{e^2}{|\vec{r}_j - \vec{r}_k|} + E_{\text{Zeeman}} + \sum_j V_{\text{disorder}}(\vec{r}_j)$$

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$B \to \infty$ no disorder

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No parameters. No mass. No kinetic energy.

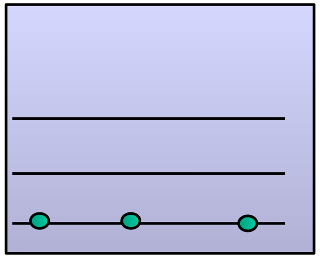
Objective:

- solve this problem as a function of the filling factor
- identify the underlying physics
- predict, calculate

Impossible ?!

Impossible ?!

- Interaction essential.



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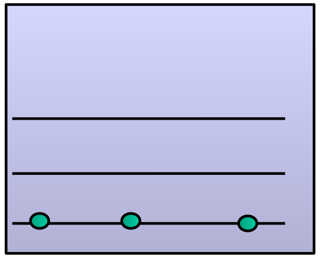

- The most srongly correlated system in the world.

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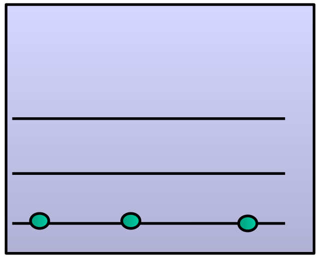

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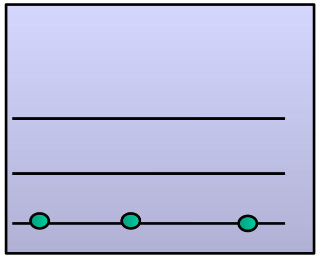

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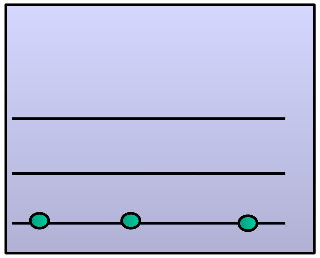
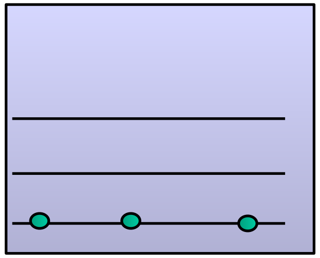


- The most srongly correlated system in the world.
- Exact solution not known.
- No small parameter.
- Infinite possibilities. At $\nu = 2/5$, a system of 100 electrons has 10^{69} distinct configurations, and a system of a billion electrons has $10^{8\times10^8}$.

Impossible ?!

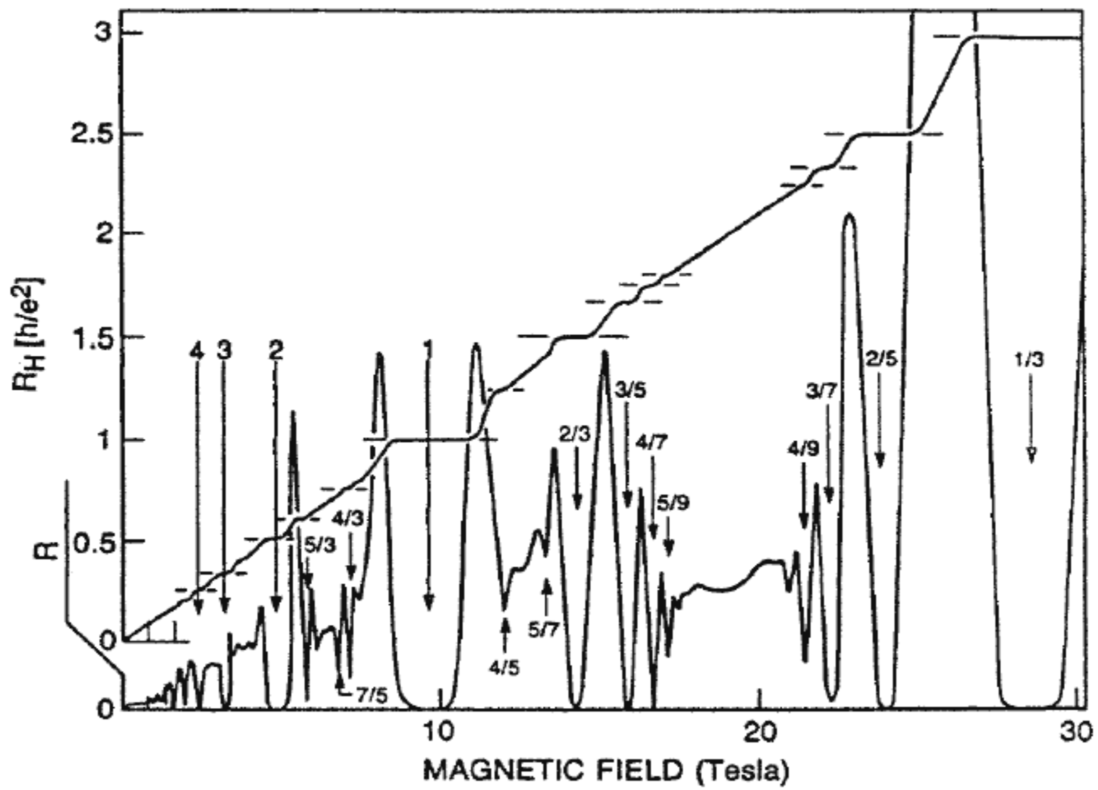
- Interaction essential.



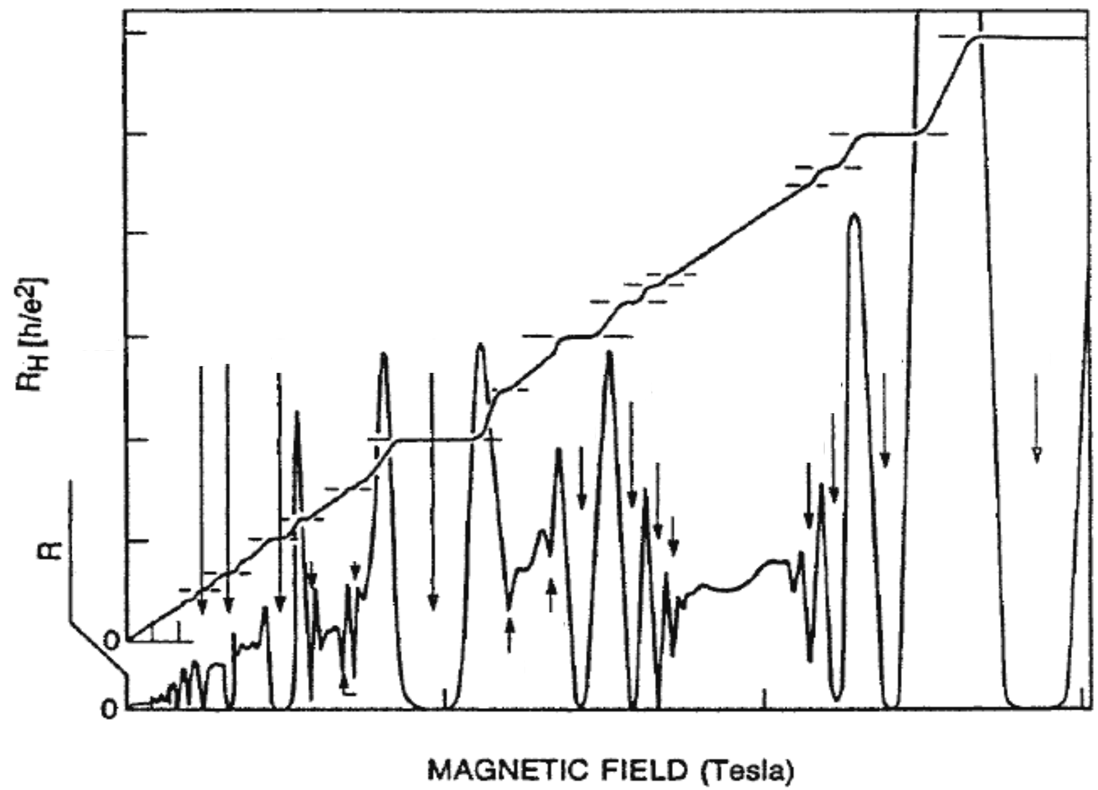
- The most srongly correlated system in the world.
- Exact solution not known.
- No small parameter.
- Infinite possibilities. At $\nu = 2/5$, a system of 100 electrons has10^{69} distinct configurations, and a system of a billion electrons has $10^{8\times10^8}$.
- Don't know where to begin. No hope.

Composite Fermions: Inspiration

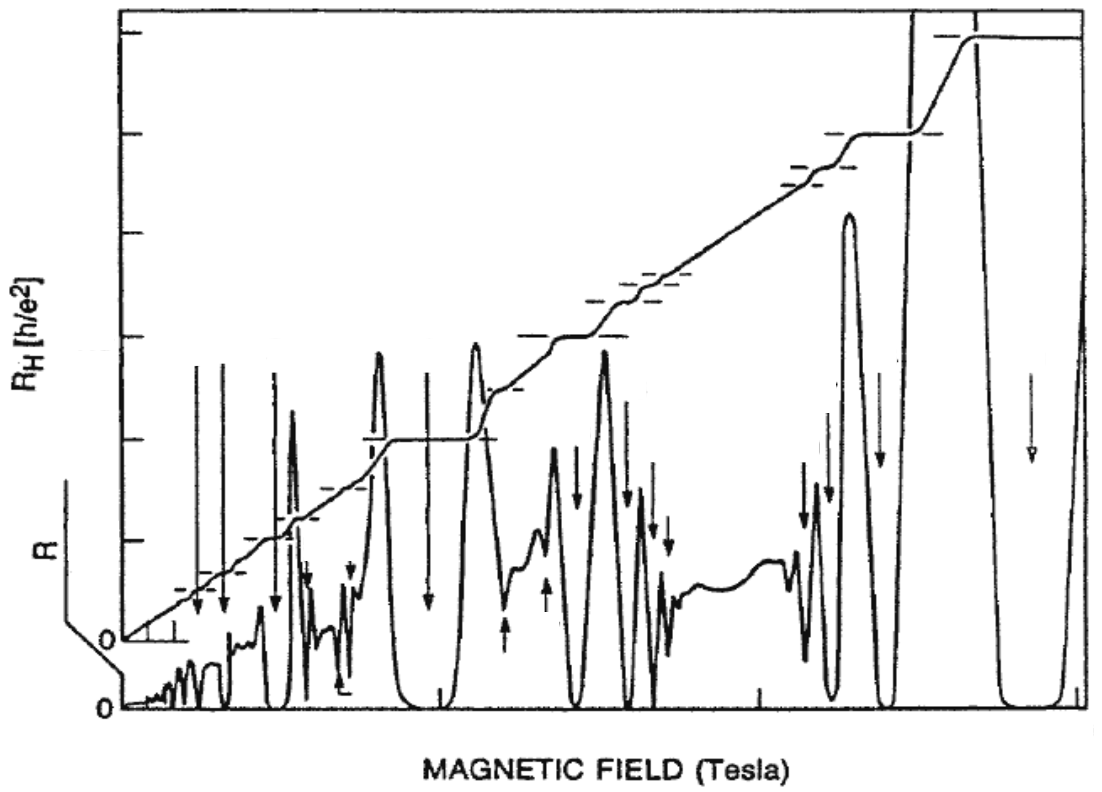
The power of the right question at the right time



The power of the right question at the right time

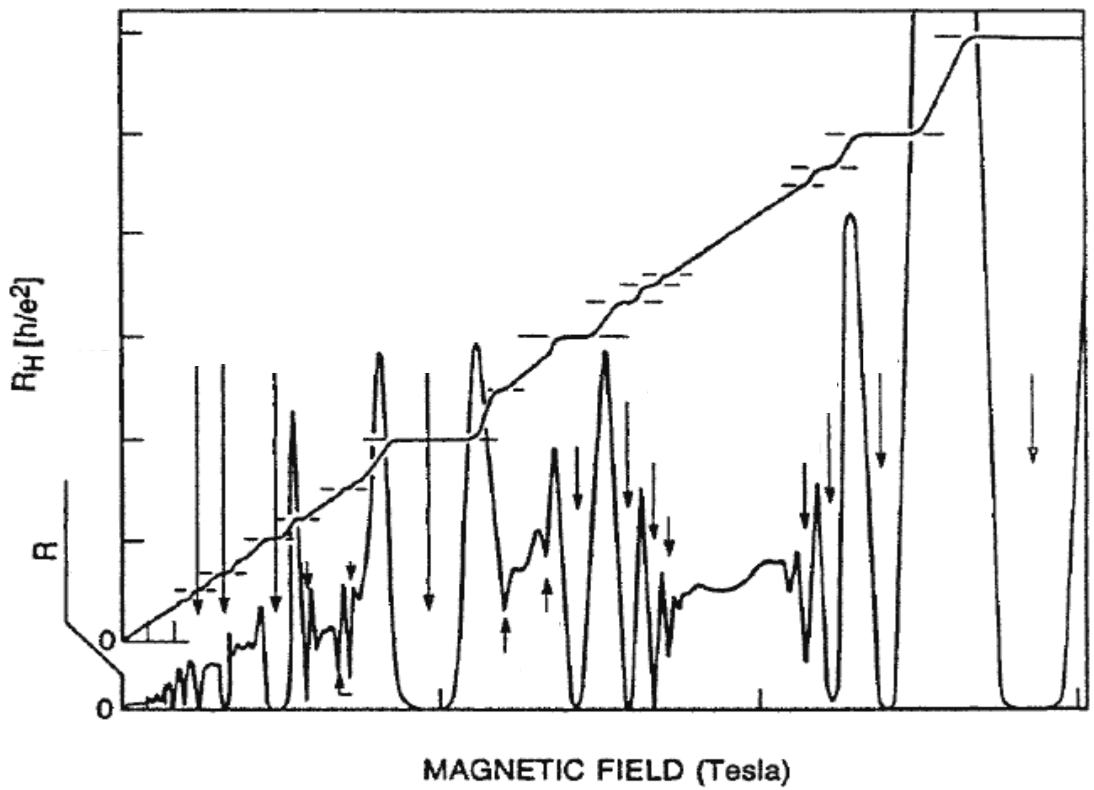


The power of the right question at the right time



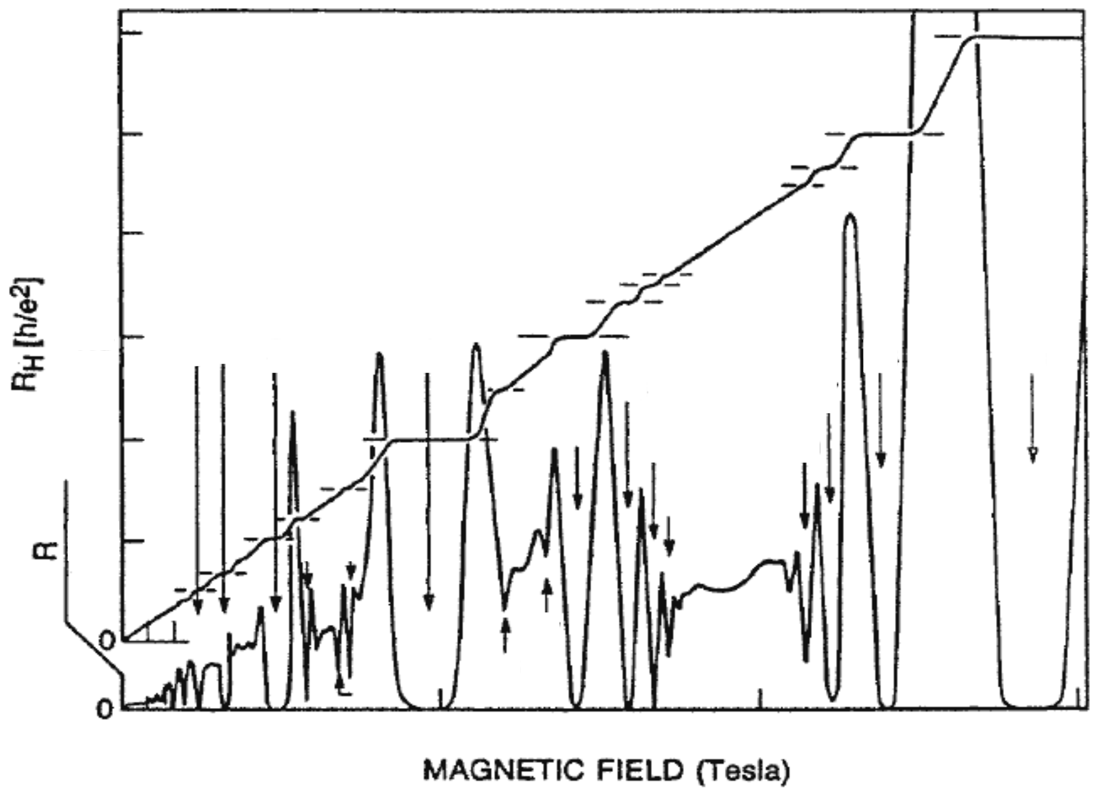
- Observation: The fractional and the integer quantum Hall effects are qualitatively identical.

The power of the right question at the right time



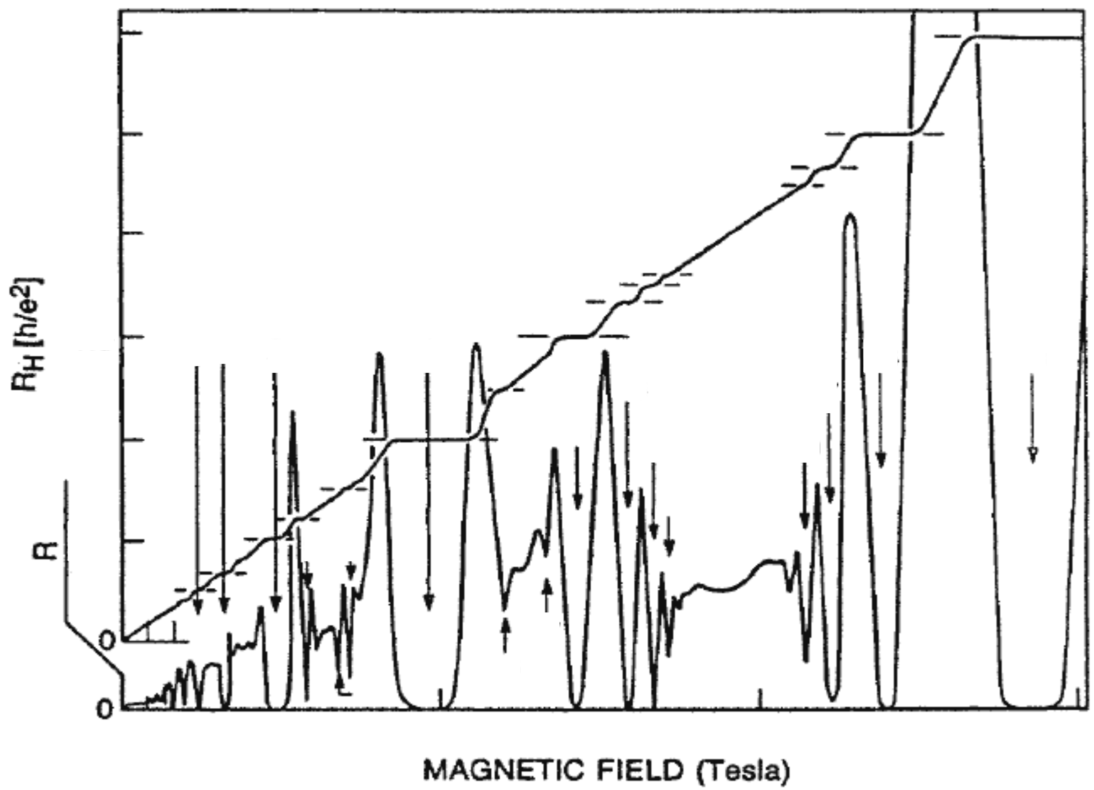
- Observation: The fractional and the integer quantum Hall effects are qualitatively identical.
- Question 1: Can we unify the two?

The power of the right question at the right time



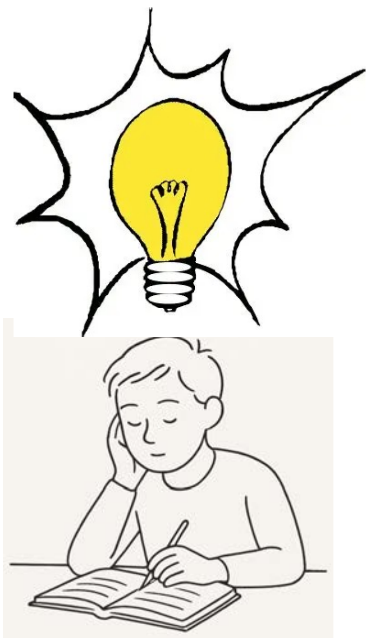
- Observation: The fractional and the integer quantum Hall effects are qualitatively identical.
- Question 1: Can we unify the two?
- Question 2: Can we understand the FQHE as some kind of IQHE?

The power of the right question at the right time



- Observation: The fractional and the integer quantum Hall effects are qualitatively identical.
- Question 1: Can we unify the two?
- Question 2: Can we understand the FQHE as some kind of IQHE?
- Question 3: What are the weakly interacting emergent fermions whose IQHE produces the FQHE of electrons?

A sudden insight

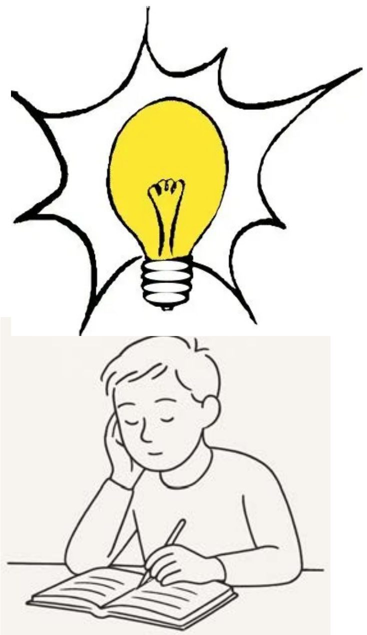


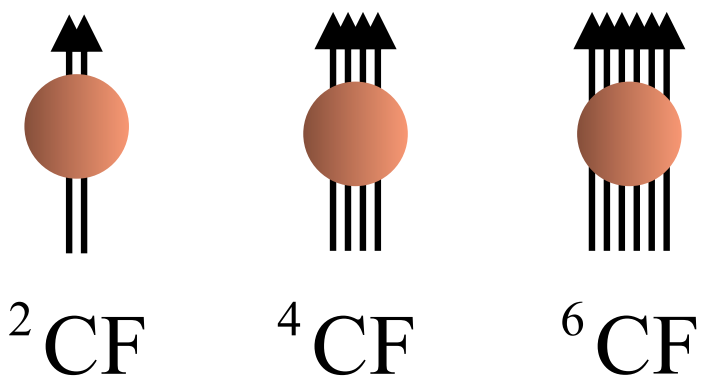
A sudden insight

VOLUME 63, NUMBER 2 PHYSICAL REVIEW LETTERS 10 JULY 1989

Composite-Fermion Approach for the Fractional Quantum Hall Effect

J. K. Jain
Section of Applied Physics, Yale University, P.O. Box 2157 Yale Station, New Haven, Connecticut 06520
(Received 24 January 1989)





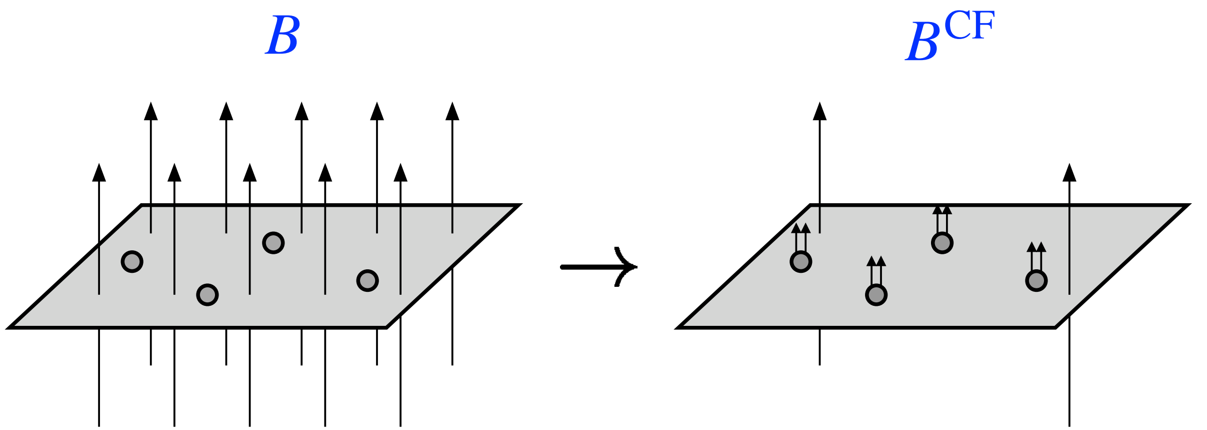
$$\phi_0 = \frac{hc}{e}$$

composite fermion = electron + $2m$ quantized vortices

often pictorially viewed as

composite fermion = electron + $2m$ magnetic flux quanta

The composite fermion: pictorial view

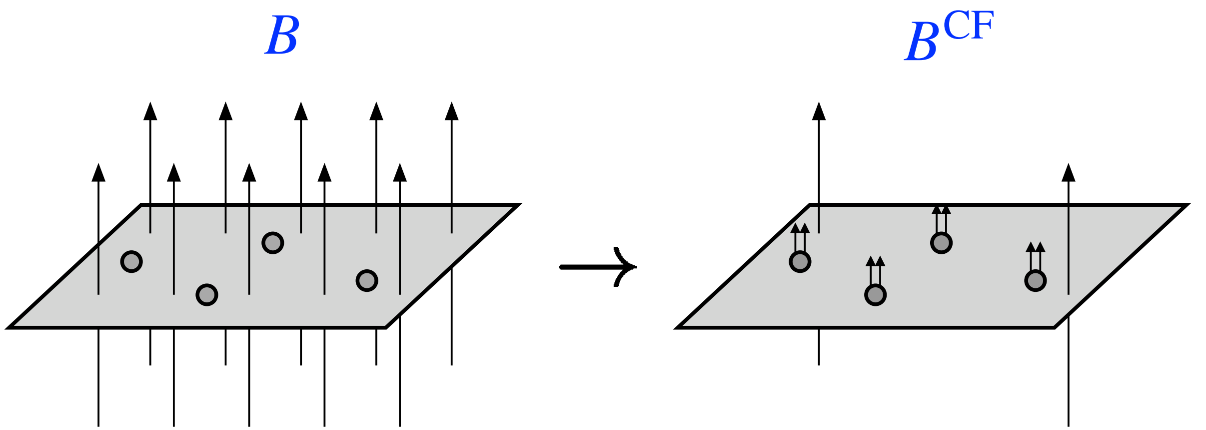


$$B^{\mathrm{CF}} = B - 2m\rho\phi_0 \qquad \nu^{\mathrm{CF}} = \frac{\nu}{1-2m\nu} \qquad \nu = \frac{\nu^{\mathrm{CF}}}{2m\nu^{\mathrm{CF}} \pm 1}$$

$\phi_0 = hc/e$, ρ = density $\quad$ $\nu^{\mathrm{CF}} = \rho\phi_0/|B^{\mathrm{CF}}| =$ CF filling factor

- Postulate: Strongly interacting electrons at B transform into weakly interacting composite fermions at B^{CF}. The CFs form their own Landau-like levels called "Λ levels," and have a filling factor ν^{CF}.

The composite fermion: pictorial view



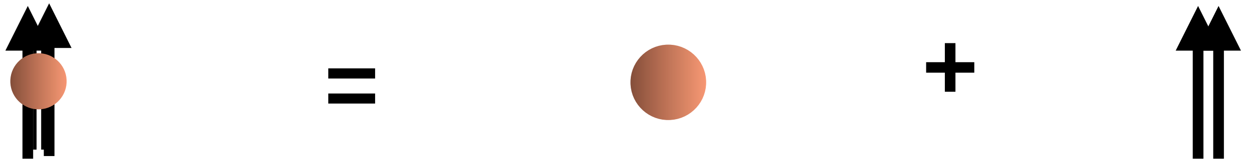
$$B^{\rm CF} = B - 2m\rho\phi_0 \qquad \nu^{\rm CF} = \frac{\nu}{1-2m\nu} \qquad \nu = \frac{\nu^{\rm CF}}{2m\nu^{\rm CF}\pm 1}$$

$\phi_0 = hc/e$, ρ = density $\quad \nu^{\rm CF} = \rho\phi_0/|B^{\rm CF}|$ = CF filling factor

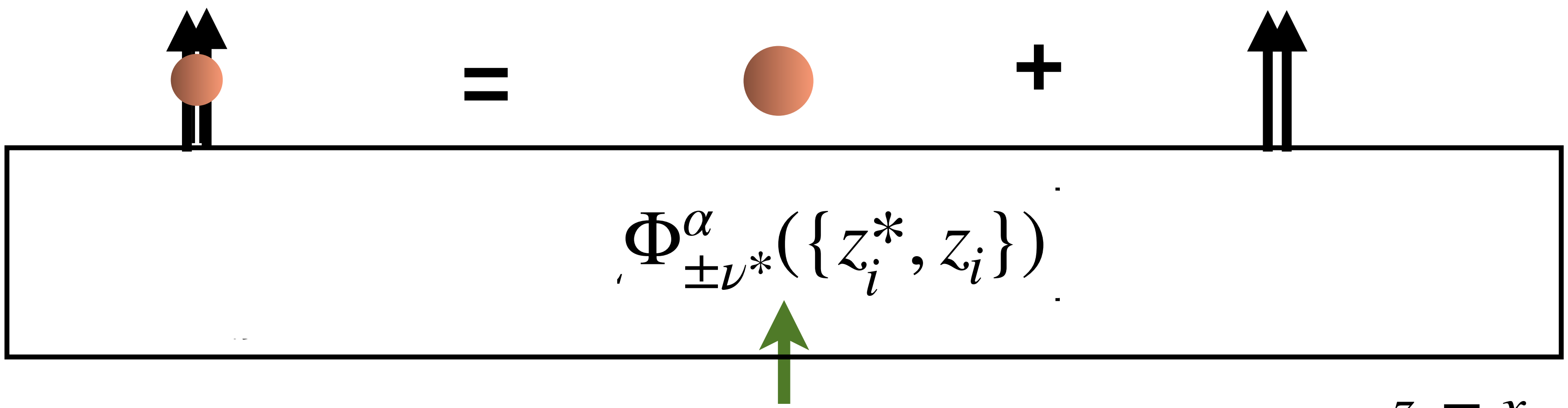
- Postulate: Strongly interacting electrons at B transform into weakly interacting composite fermions at $B^{\rm CF}$. The CFs form their own Landau-like levels called "Λ levels," and have a filling factor $\nu^{\rm CF}$.

In particular: $\nu^{\rm CF} = p \;\Leftrightarrow\; \nu = \dfrac{p}{2mp \pm 1}$

Microscopic theory: composite-fermionization



Microscopic theory: composite-fermionization



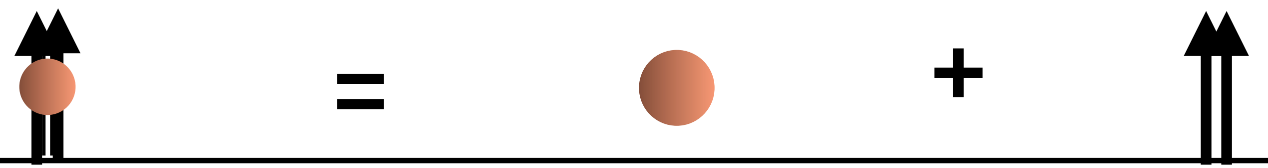
$$\Phi^{\alpha}_{\pm\nu^*}(\{z_i^*, z_i\})$$

$$z_j = x_j - iy_j$$

wave function of noninteracting electrons

$$\Phi_{-\nu^*} = [\Phi_{\nu^*}]^*$$

Microscopic theory: composite-fermionization



$$\Phi^{\alpha}_{\pm\nu^*}(\{z_i^*, z_i\}) \prod_{j<k} (z_j - z_k)^{2m}$$

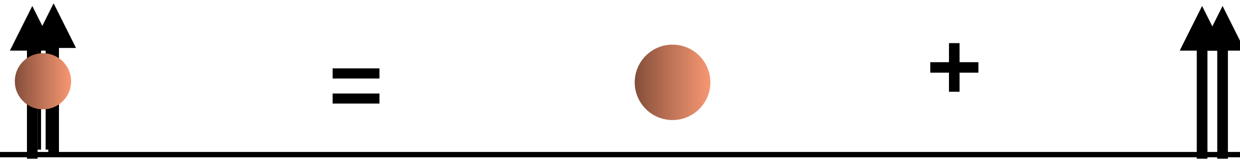
$z_j = x_j - iy_j$

wave function of noninteracting electrons

$\Phi_{-\nu^*} = [\Phi_{\nu^*}]^*$

the Laughlin-Jastrow factor attaches 2m quantized vortices to each electron

Microscopic theory: composite-fermionization



$$\mathscr{P}_{\rm LLL}\Phi^{\alpha}_{\pm\nu^*}(\{z_i^*, z_i\})\prod_{j<k}(z_j - z_k)^{2m}$$

$z_j = x_j - iy_j$

projects into the lowest Landau level

wave function of noninteracting electrons

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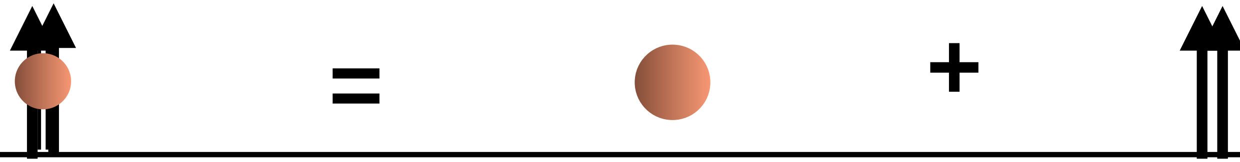
Microscopic theory: composite-fermionization



$$\Psi^{\alpha}_{\nu=\frac{\nu^*}{2m\nu^*\pm 1}} = \mathscr{P}_{\rm LLL}\Phi^{\alpha}_{\pm\nu^*}(\{z_i^*, z_i\})\prod_{j<k}(z_j - z_k)^{2m}$$

$z_j = x_j - iy_j$

wave function of interacting electrons in the lowest Landau level

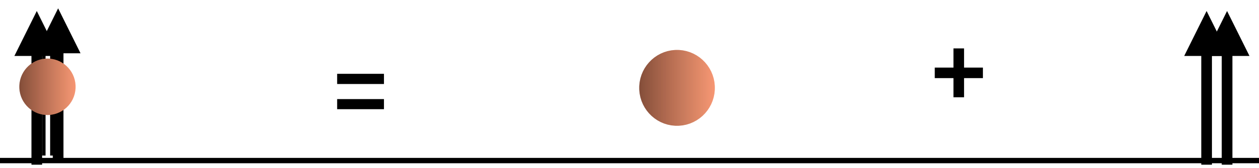
projects into the lowest Landau level

wave function of noninteracting electrons

$\Phi_{-\nu^*} = [\Phi_{\nu^*}]^*$

the Jastrow factor attaches 2m quantized vortices to each electron

Microscopic theory: composite-fermionization



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wave function of interacting electrons in the lowest Landau level

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wave function of noninteracting electrons

$\Phi_{-\nu^*} = [\Phi_{\nu^*}]^*$

the Jastrow factor attaches 2m quantized vortices to each electron

$z_j = x_j - iy_j$

- A single equation provides *ansatz* wave functions for all eigenstates (from which eigenenergies may be obtained) at all filling factors.
- Unique, parameter-free wave functions for the ground states and their low-energy charged and neutral excitations at $\nu^{\rm CF} = p$, i.e. at $\nu = p/(2mp \pm 1)$.

Effective (Chern-Simons) field theory of CFs

Zero temperature partition function

$$\mathcal{Z} = \int \mathcal{D}\psi \mathcal{D}\psi^* \mathcal{D}\vec{a} \exp\left(\frac{i}{\hbar}\mathcal{S}\right)$$

$$\mathcal{S} = \int d^2\vec{r} \int dt\ \mathcal{L}$$

Lopez and Fradkin
Halperin, Lee, Read
D. T. Son

$$\mathcal{L} = \psi^*(i\partial_t - a_0)\psi + \frac{1}{2m_b}\left|\left(-i\hbar\vec{\nabla} + \frac{e}{c}\vec{A} - \frac{e}{c}\vec{a}\right)\psi\right|^2 + \frac{1}{2p\phi_0}a_0\vec{\nabla}\times\vec{a} + \int d^2\vec{r}'\rho(\vec{r})V(\vec{r}-\vec{r}')\rho(\vec{r}')$$

The action is written in terms of Grassmann variables. The flux attachment is introduced through a Lagrange multiplier, which can be integrated to produce

$$\vec{\nabla}\times\vec{a}(\vec{r}) = 2p\phi_0\rho(\vec{r}) = 2p\phi_0\psi^*(\vec{r})\psi(\vec{r})$$

$$\mathcal{L}_{CS} \sim \epsilon^{\mu\nu\lambda}A_\mu F_{\nu\lambda} = 2\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

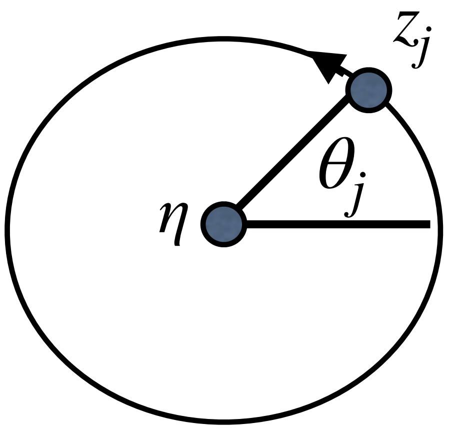
$a_0\vec{\nabla}\times\vec{a}$ is precisely the CS Lagrangian in the Coulomb gauge.

$$\mathcal{L}_{CS} = \frac{1}{4p\phi_0}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda = \frac{1}{2p\phi_0}\epsilon^{ij}a_0\partial_i a_j - \frac{1}{4p\phi_0}\epsilon^{ij}a_i\partial_0 a_j$$

The composite fermion is inherently quantum

INTERNATIONAL YEAR OF
Quantum Science
and Technology

$$\Psi_{\text{vortex}}(\eta) = \prod_{j=1}^{N} (z_j - \eta) = \prod_{j=1}^{N} |z_j - \eta| e^{i\theta_j}$$



- A vortex is an inherently quantum mechanical (carries quantum mechanical phases), topological, and collective entity.
- Hence the CF is also a quantum mechanical, topological, and collective particle. The Berry phases due to the vortices partly cancel the AB phase from the external magnetic field to produce B^{CF}.

How real are composite fermions?

23 PHYSICAL REVIEW LETTERS 6 December 1993

How Real Are Composite Fermions?

W. Kang, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 14 September 1993)

Do composite fermions exist? If so, in what sense do they behave as particles?

How real are composite fermions?

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Of course, experiments have the final word. The theory needs to make (nontrivial) predictions and explain experimental facts.

We shall see that the CF theory successfully predicts thousands of nontrivial facts in a unified, natural and unambiguous fashion. Observations that would appear bewildering are seen as trivial and unavoidable consequences of CFs.

IQHE of CFs

IQHE of CFs = $p/(2mp \pm 1)$ FQHE of electrons

Consider $\nu = \dfrac{p}{2mp \pm 1}$

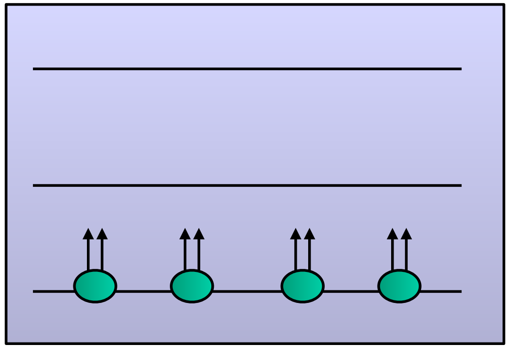
IQHE of CFs = $p/(2mp \pm 1)$ FQHE of electrons

Consider $\nu = \dfrac{p}{2mp \pm 1} \Rightarrow \nu^{\mathrm{CF}} = \dfrac{\nu}{1 - 2m\nu} = \pm\, p$

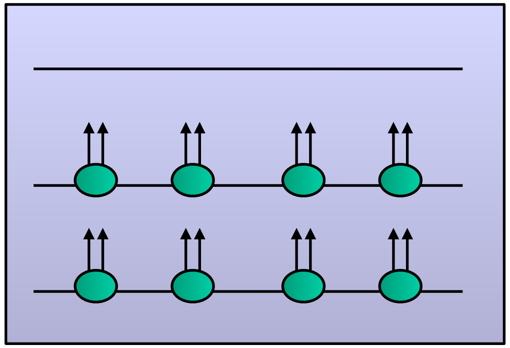
IQHE of CFs = $p/(2mp \pm 1)$ FQHE of electrons

Consider $\nu = \frac{p}{2mp \pm 1} \Rightarrow \nu^{\mathrm{CF}} = \frac{\nu}{1 - 2m\nu} = \pm p$

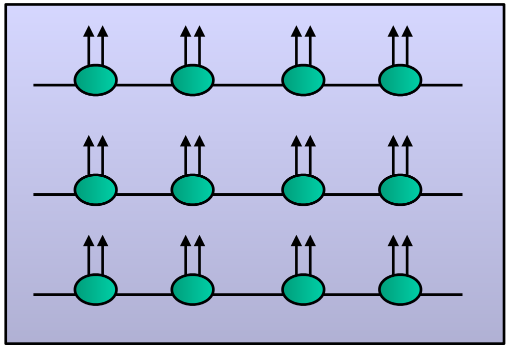
At these filling factors, the infinite choices of the electron problem disappear when we view the problem in terms of non-interacting composite fermions, and unique, gapped states are obtained !!



$\nu = \frac{1}{3} \Rightarrow \nu^{\mathrm{CF}} = 1$



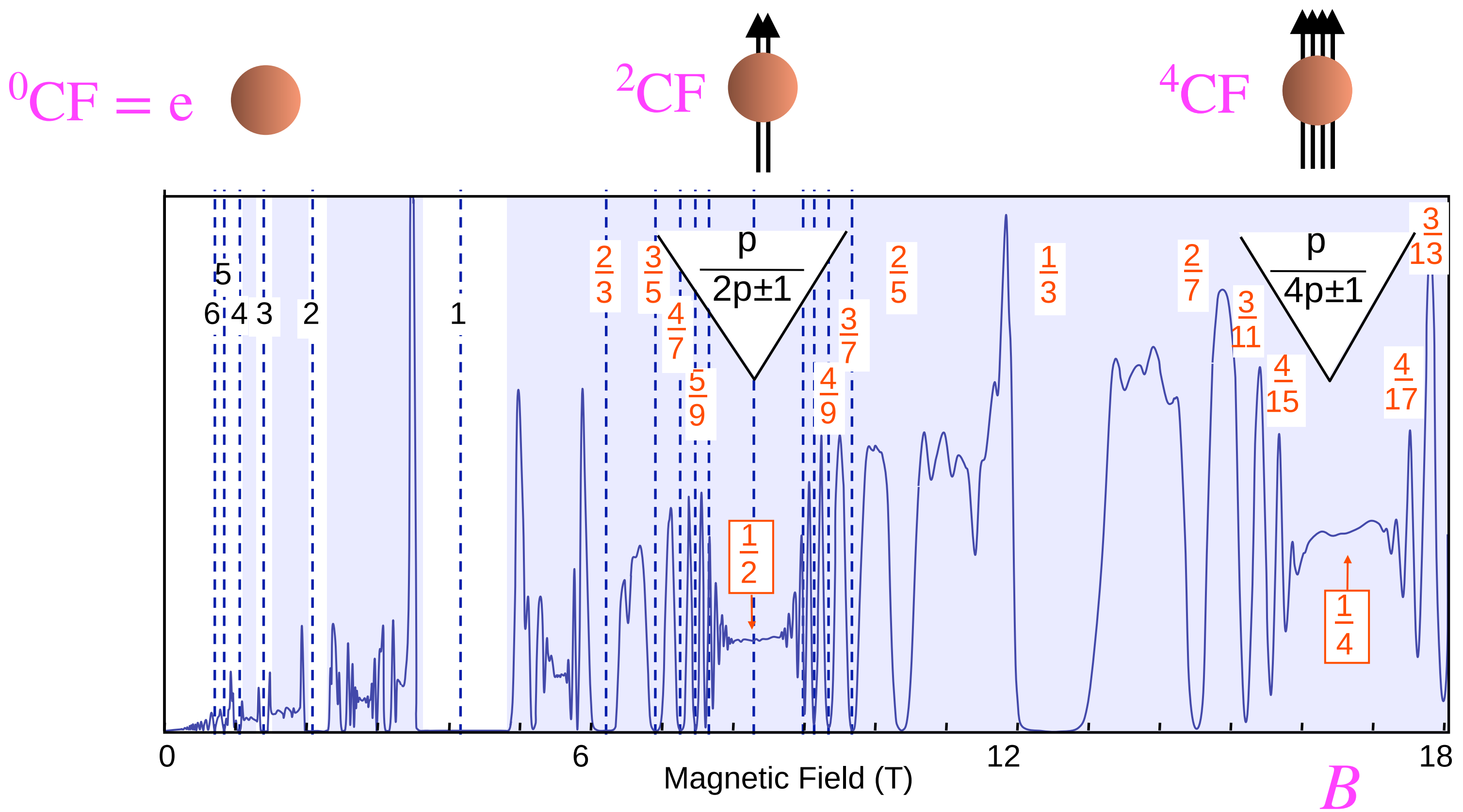
$\nu = \frac{2}{5} \Rightarrow \nu^{\mathrm{CF}} = 2$



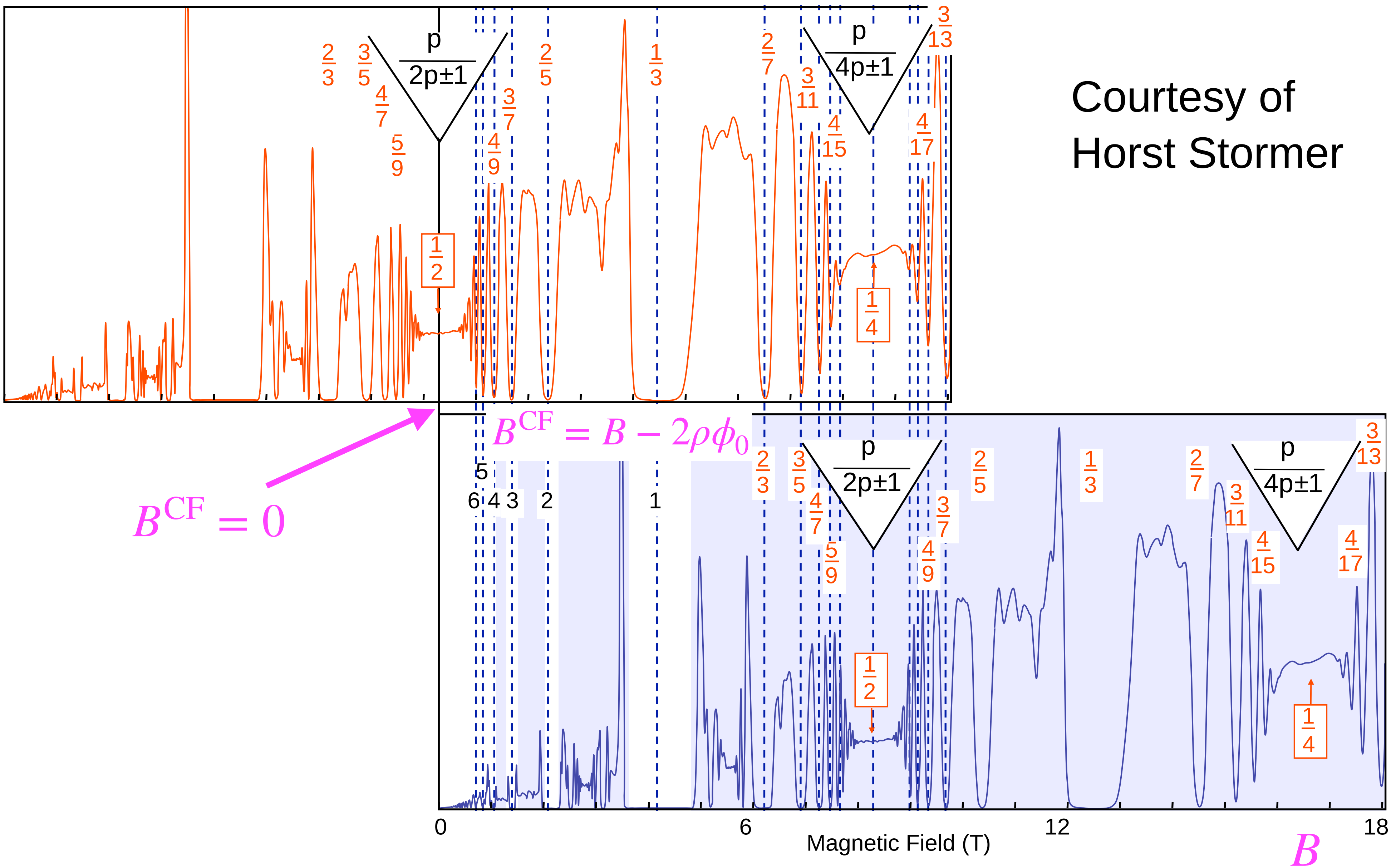
$\nu = \frac{3}{7} \Rightarrow \nu^{\mathrm{CF}} = 3$

Correctly predicts almost all observed fractions

Courtesy of
Horst Stormer



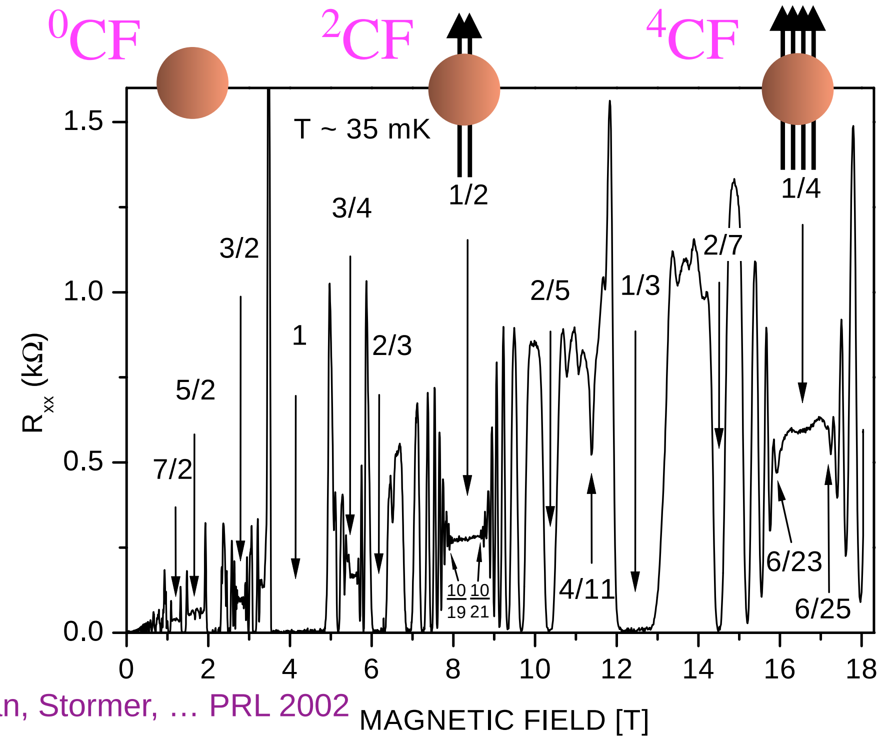
Correctly predicts almost all observed fractions



$$\nu^{CF} = p \ \textbf{ IQHE of CFs} \Rightarrow \textbf{FQHE at } \nu = \frac{p}{2mp \pm 1}$$

Correctly predicts almost all observed fractions

Correctly predicts almost all observed fractions



Pan, Stormer, … PRL 2002

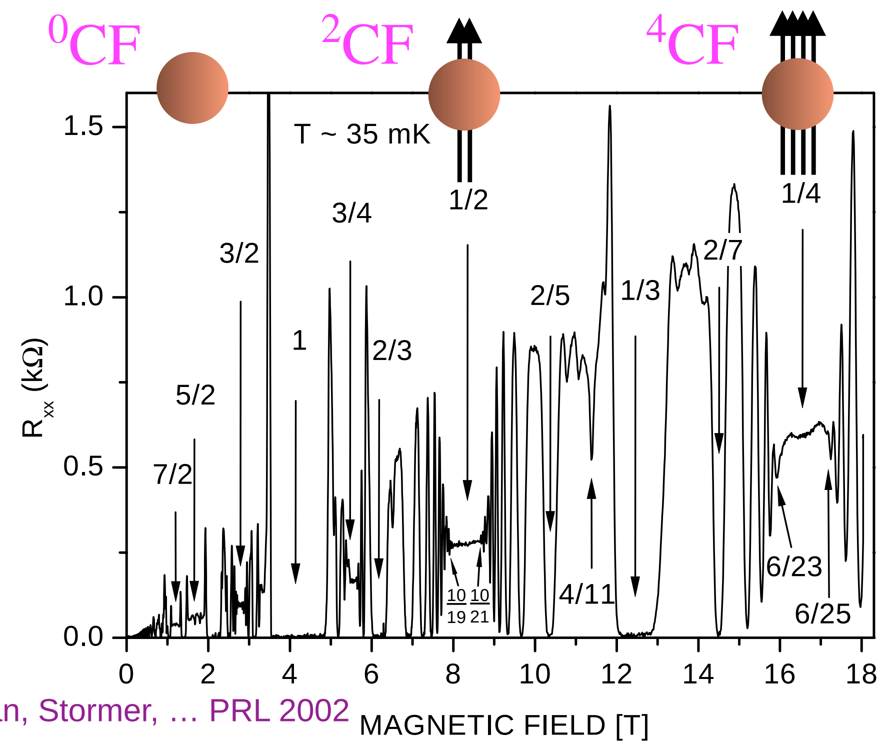
$$\frac{p}{2p+1} = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \frac{8}{17}, \frac{9}{19}, \frac{10}{21}$$

$$\frac{p}{2p-1} = \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{6}{11}, \frac{7}{13}, \frac{8}{15}, \frac{9}{17}, \frac{10}{19}$$

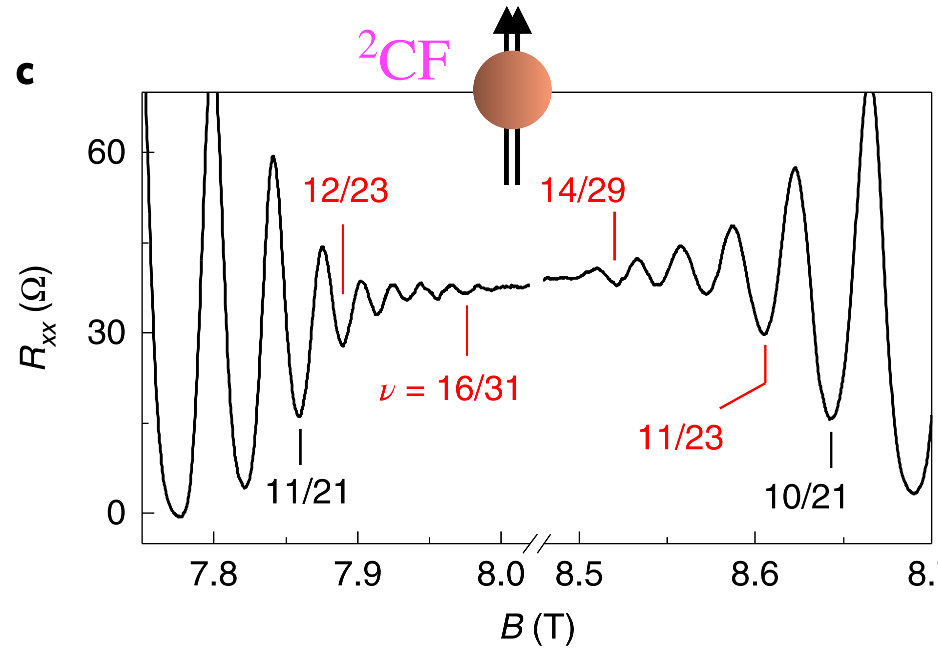
$$\frac{p}{4p+1} = \frac{1}{5}, \frac{2}{9}, \frac{3}{13}, \frac{4}{17}, \frac{5}{21}, \frac{6}{25}$$

$$\frac{p}{4p-1} = \frac{2}{7}, \frac{3}{11}, \frac{4}{15}, \frac{5}{19}, \frac{6}{23}$$

Correctly predicts almost all observed fractions



Pan, Stormer, … PRL 2002



Chung, … Shayegan, Pfeiffer, Nat. Mat. 2021

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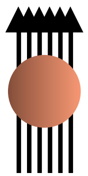
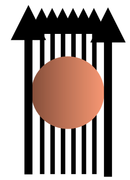
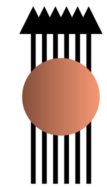
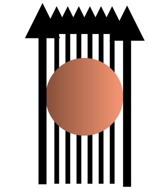
$$\frac{p}{2p+1} = \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{6}{11}, \frac{7}{13}, \frac{8}{15}, \frac{9}{17}, \frac{10}{19}, \frac{11}{21}, \frac{12}{23}, \frac{13}{25}, \frac{14}{27}, \frac{15}{29}, \frac{16}{31}$$

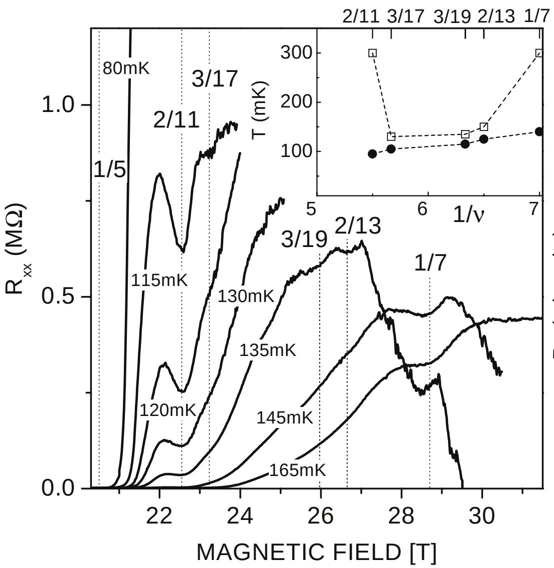
^{2}CFs, ^{4}CFs, ^{6}CFs and ^{8}CFs observed

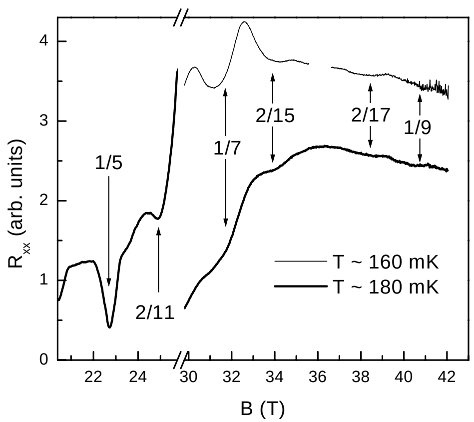
^{6}CF 

^{8}CF 

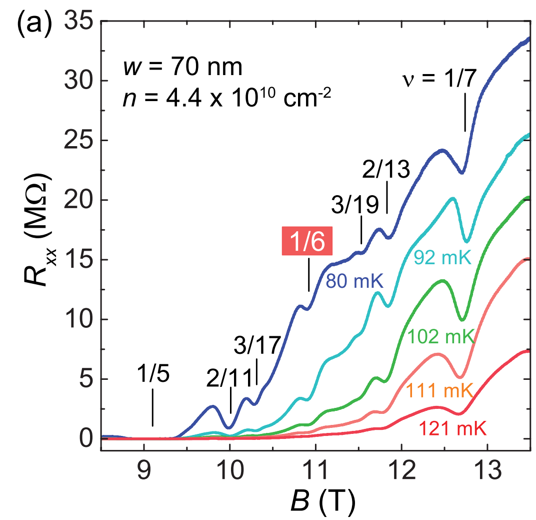
^{6}CF 

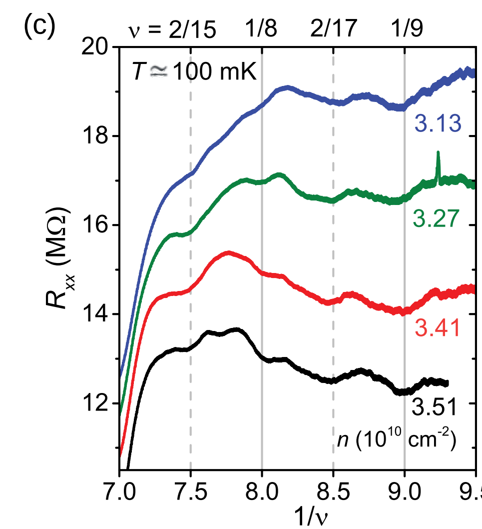
^{8}CF 





Pan, Stormer, … PRL 2002

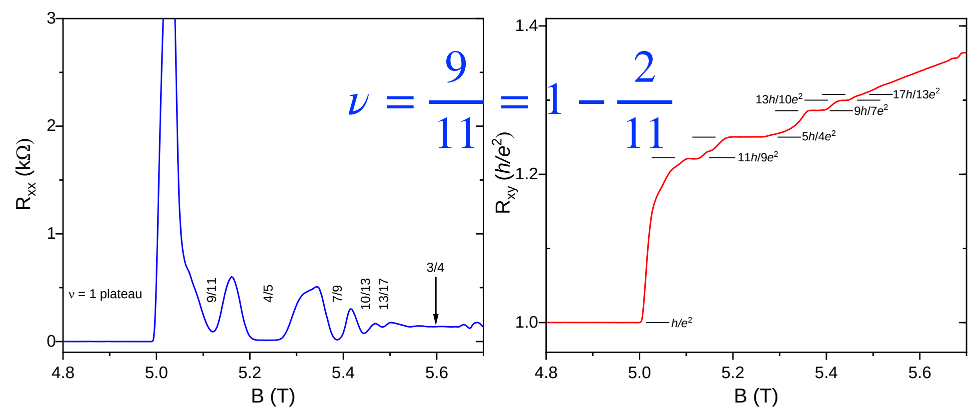




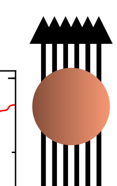
Wang, … Shayegan, PRL 2025

$$\frac{p}{6p+1} = \frac{1}{7}, \frac{2}{13}, \frac{3}{19} \qquad \frac{p}{6p-1} = \frac{2}{11}, \frac{3}{17}$$

$$\frac{p}{8p+1} = \frac{1}{9}, \frac{2}{17} \qquad \frac{p}{8p-1} = \frac{2}{15}$$



$$\nu = \frac{9}{11} = 1 - \frac{2}{11}$$

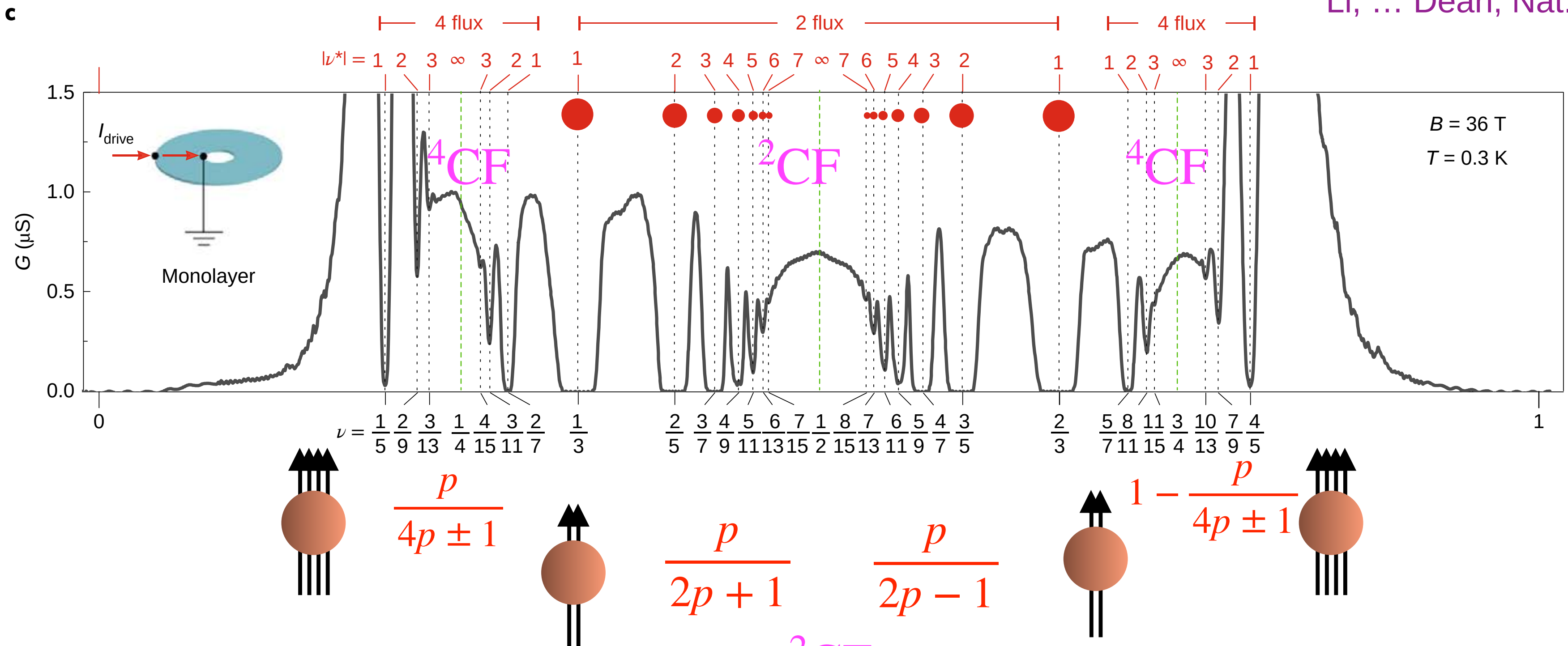


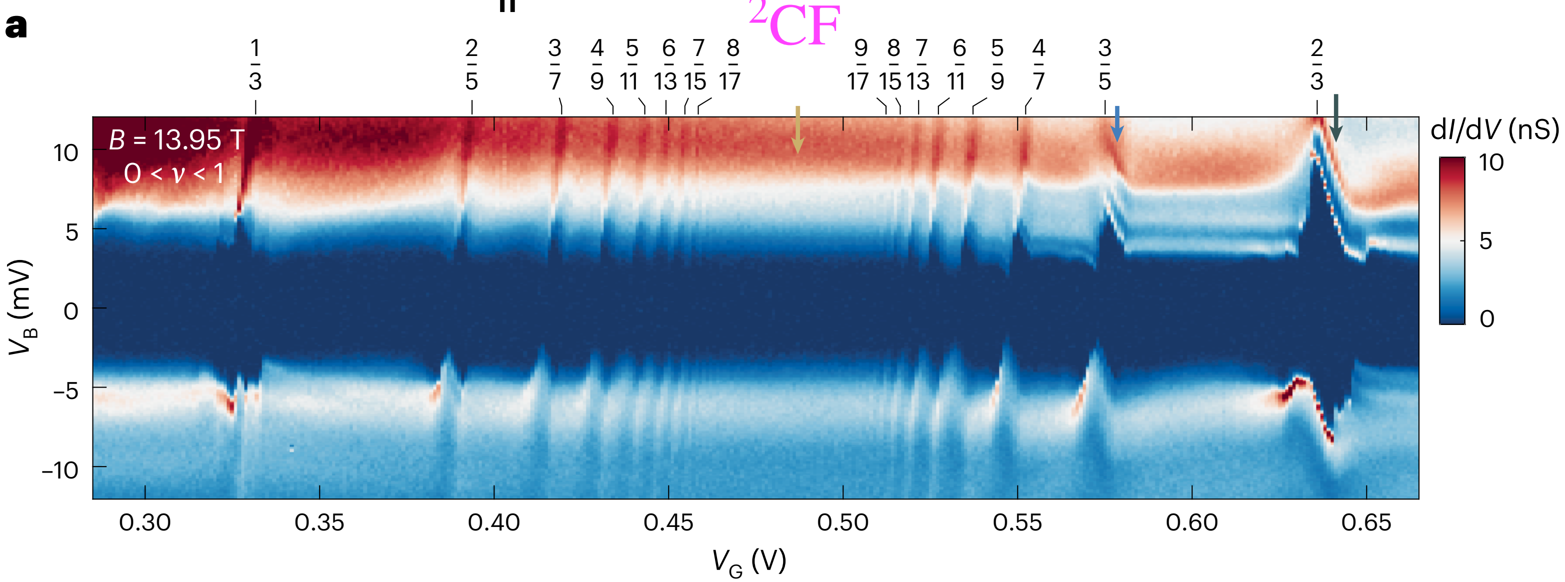
^{6}CF

Huang, … Csathy, Nat. Comm. 2024

CFs in monolayer and bilayer graphene

Li, … Dean, Nat. Phys. 2019





High resolution tunneling spectroscopy of bilayer graphene, Hu, … Yazdani, Nature 2025

A new phase of CFs: Composite-fermion metal

Half-filled Landau level = CF metal

PHYSICAL REVIEW B VOLUME 47, NUMBER 12 15 MARCH 1993-II

Theory of the half-filled Landau level

B. I. Halperin Patrick A. Lee Nicholas Read

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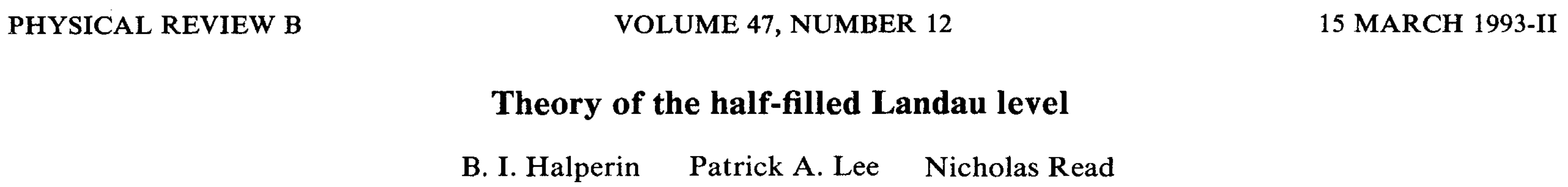
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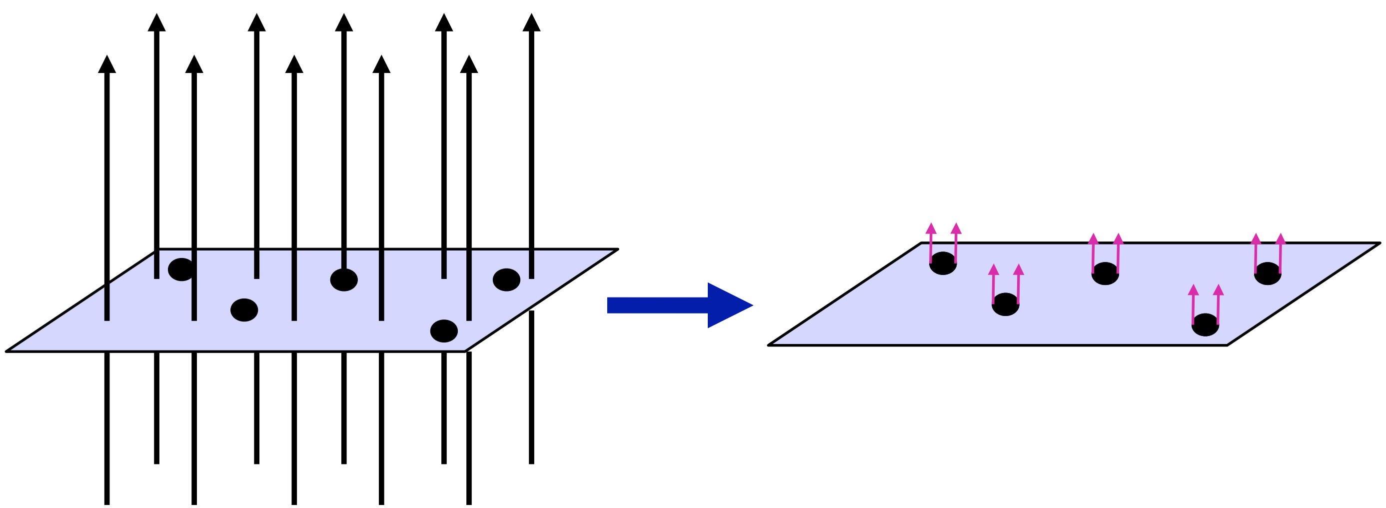


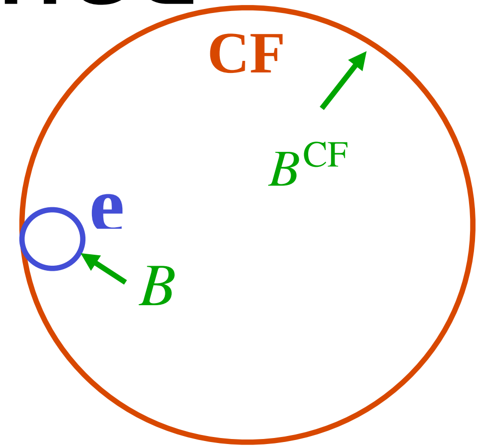
CF metal is fully confirmed

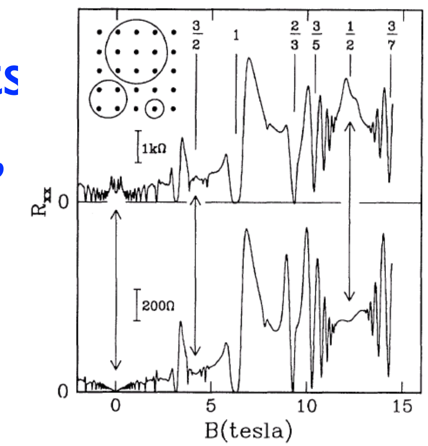
- No gap ⇒ no FQHE.
- Shubnikov-de Haas oscillations.
- Fermi wave vector k_F.
- Semiclassical cyclotron orbits
Direct measurement of B^{CF}, which can be negative.
- Luttinger area rule.
- Commensurability / Weiss oscillations.
- Spin polarization
- Mass anisotropy

CF Cyclotron radius

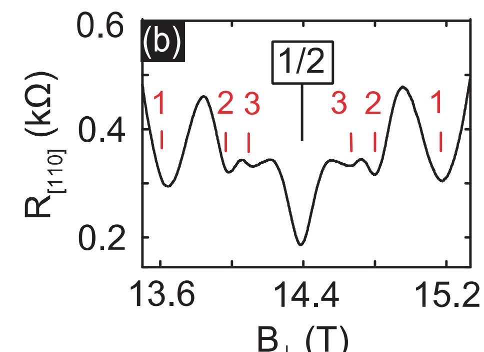
$$R^{CF} = \frac{\hbar k_F}{eB^{CF}}$$

$$k_F = \sqrt{4\pi\rho}$$

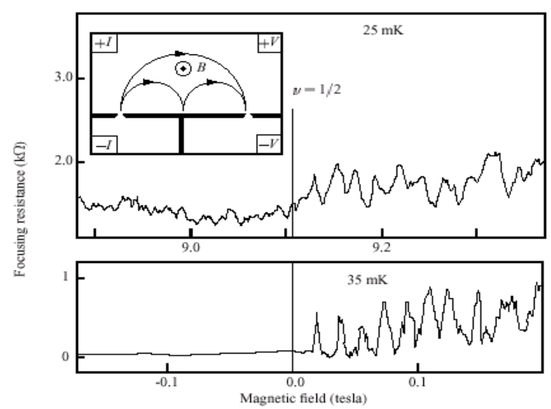




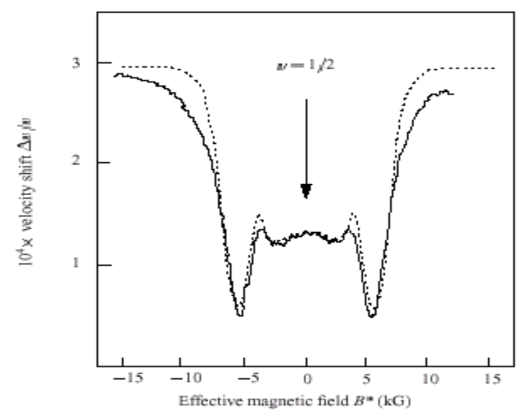
antidot resonances (Kang)



geometric resonances in a periodic potential (Kamburov, Shayegan et al.)



magnetic focusing (Goldman; Smet)

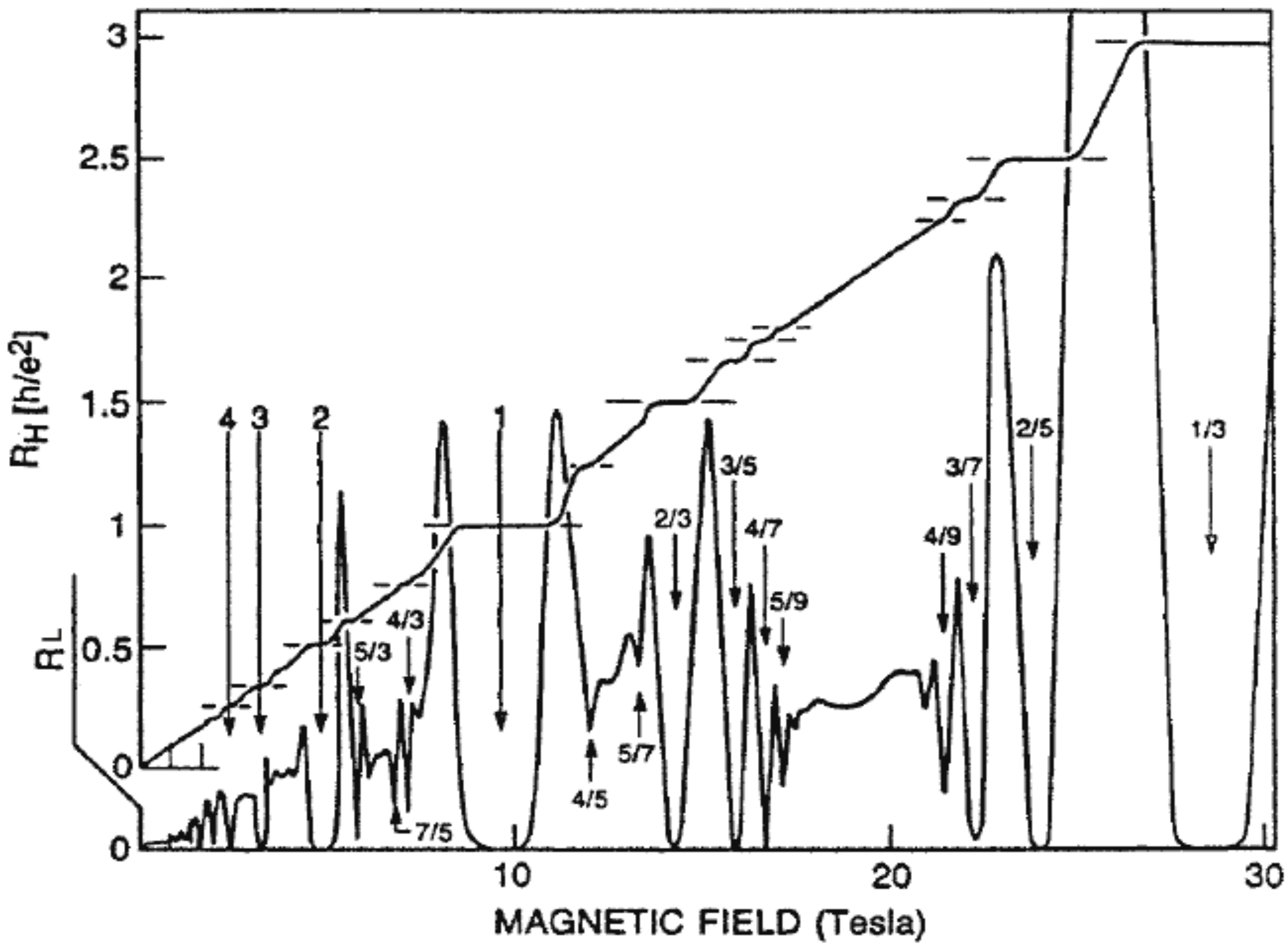


surface acoustic wave resonances (Willett, 1993)

Just as electrons exist even without QHE, CFs exist even without their QHE.

The CFs are thus more fundamental than the FQHE. The FQHE results from the existence of CFs and not *vice versa*. (In contrast, fractional charge and fractional statistics follow from the FQHE, and thus ultimately from composite fermions.)

Summary of the physics so far



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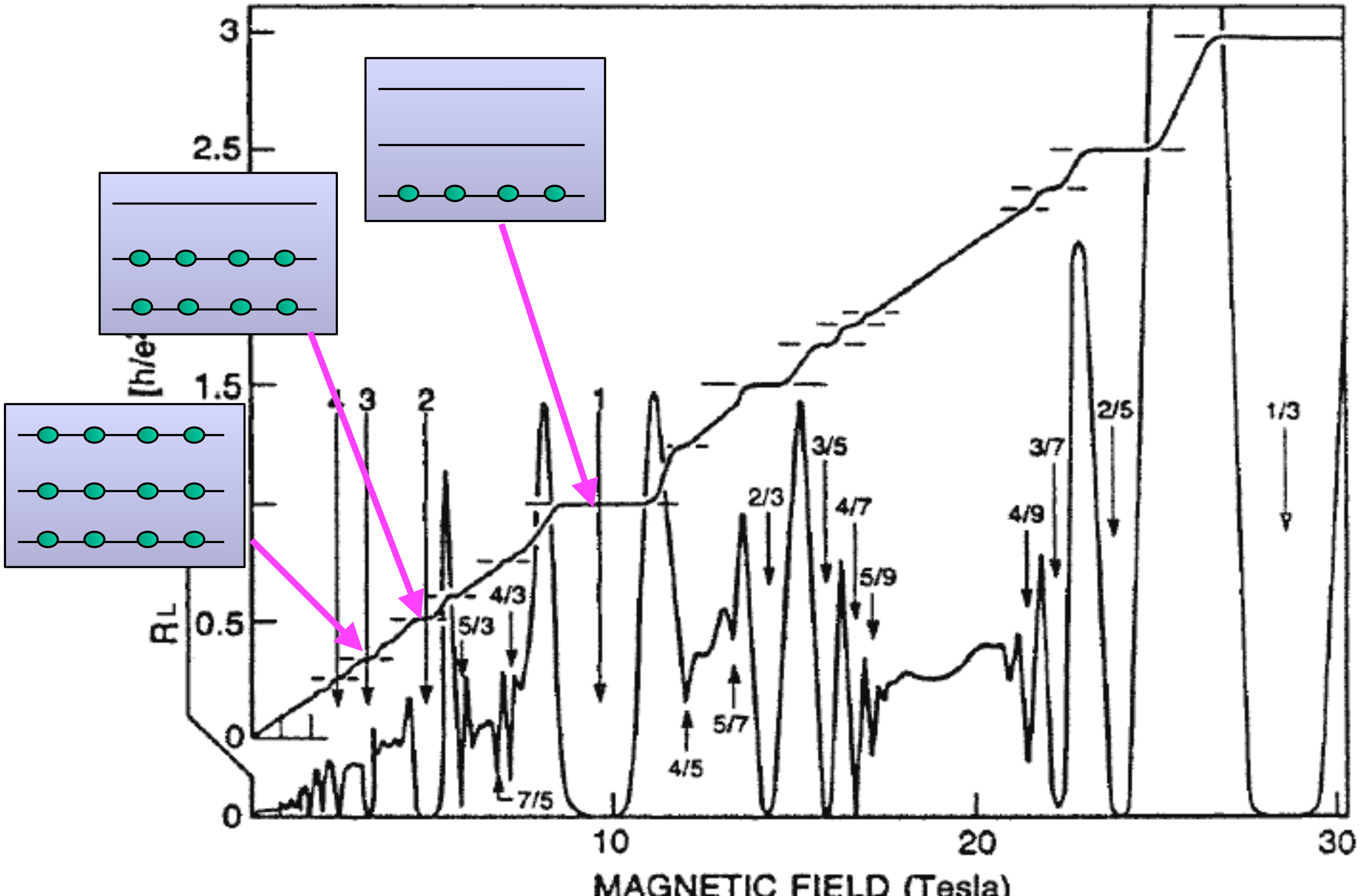

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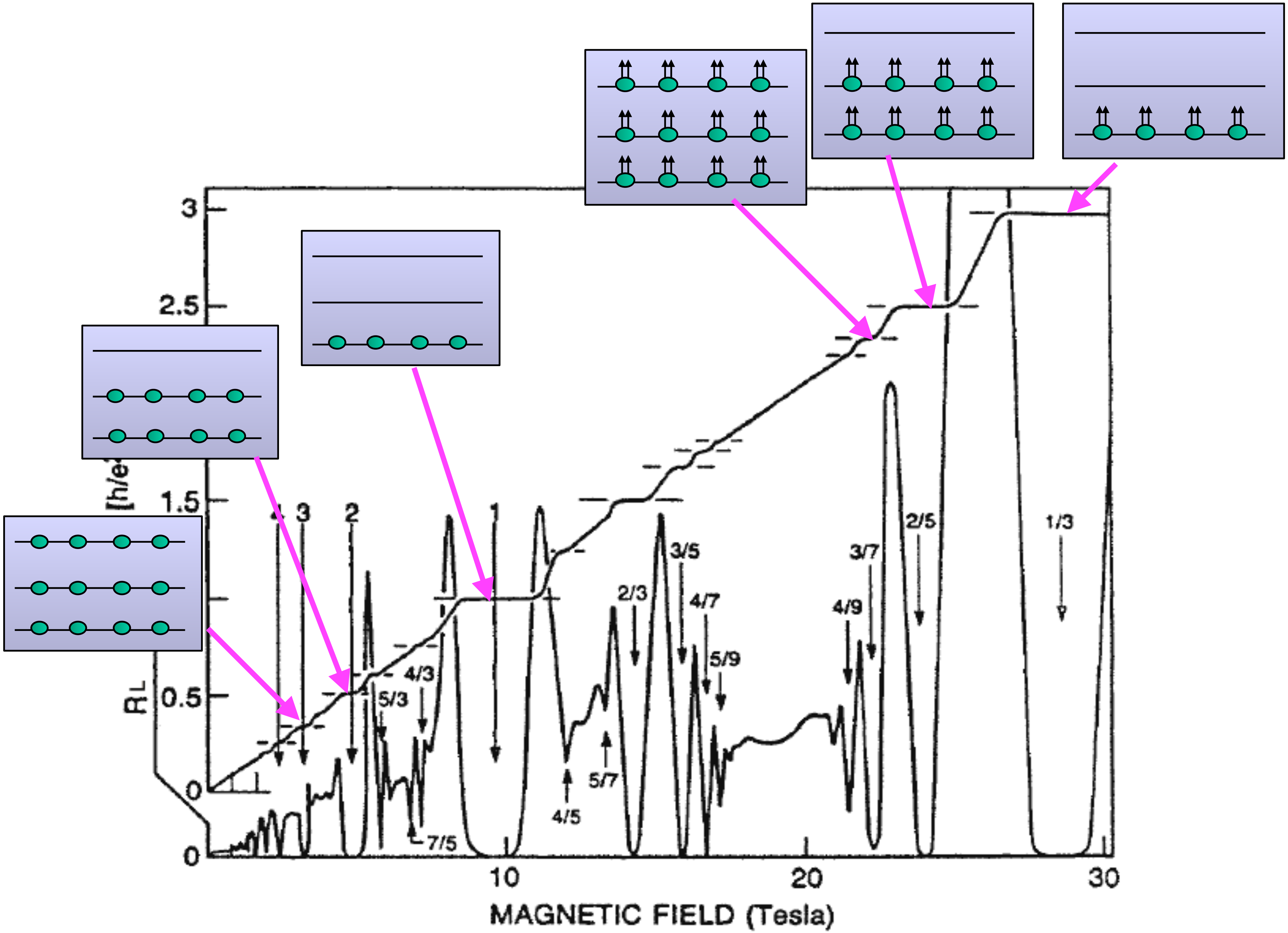

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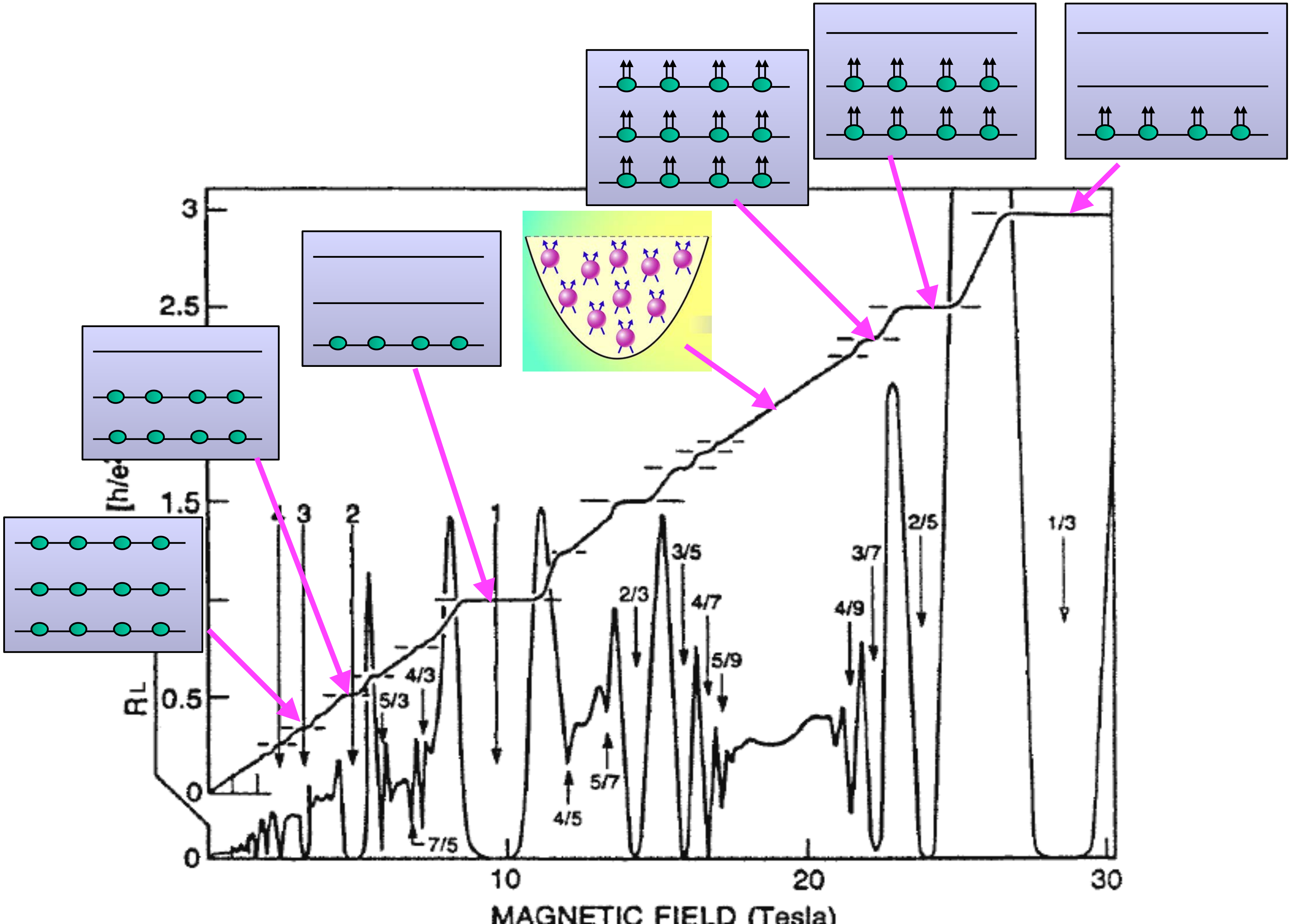
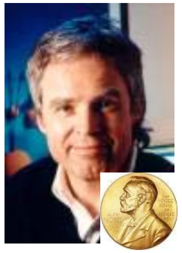


How real are composite fermions?

R 23 PHYSICAL REVIEW LETTERS 6 DECEMBER 1993

How Real Are Composite Fermions?



W. Kang, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 14 September 1993)

"Composite fermions are as real as Cooper pairs."
-Horst Stormer

KIAS 고등과학원 KPS 한국물리학회

http://home.kias.re.kr/MKG/h/APR2017KPS/

2017. 04. 20.

2017 KPS - KIAS PLENARY TALK

2017 한국물리학회
봄 학술논문발표회
기조강연

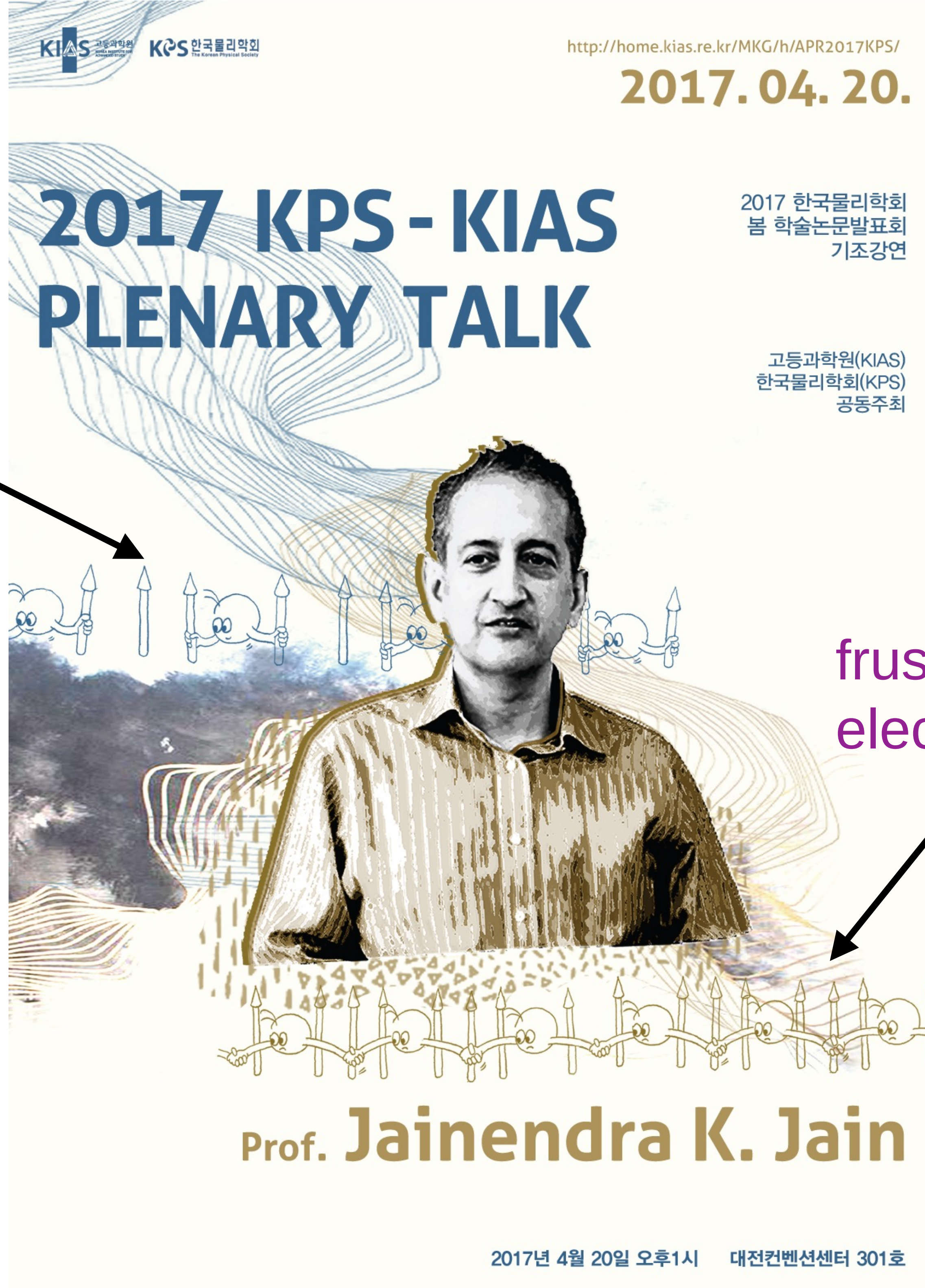
고등과학원(KIAS)
한국물리학회(KPS)
공동주최

Prof. Jainendra K. Jain

2017년 4월 20일 오후1시 대전컨벤션센터 301호

frustrated electrons 😞

happy composite fermions 😀



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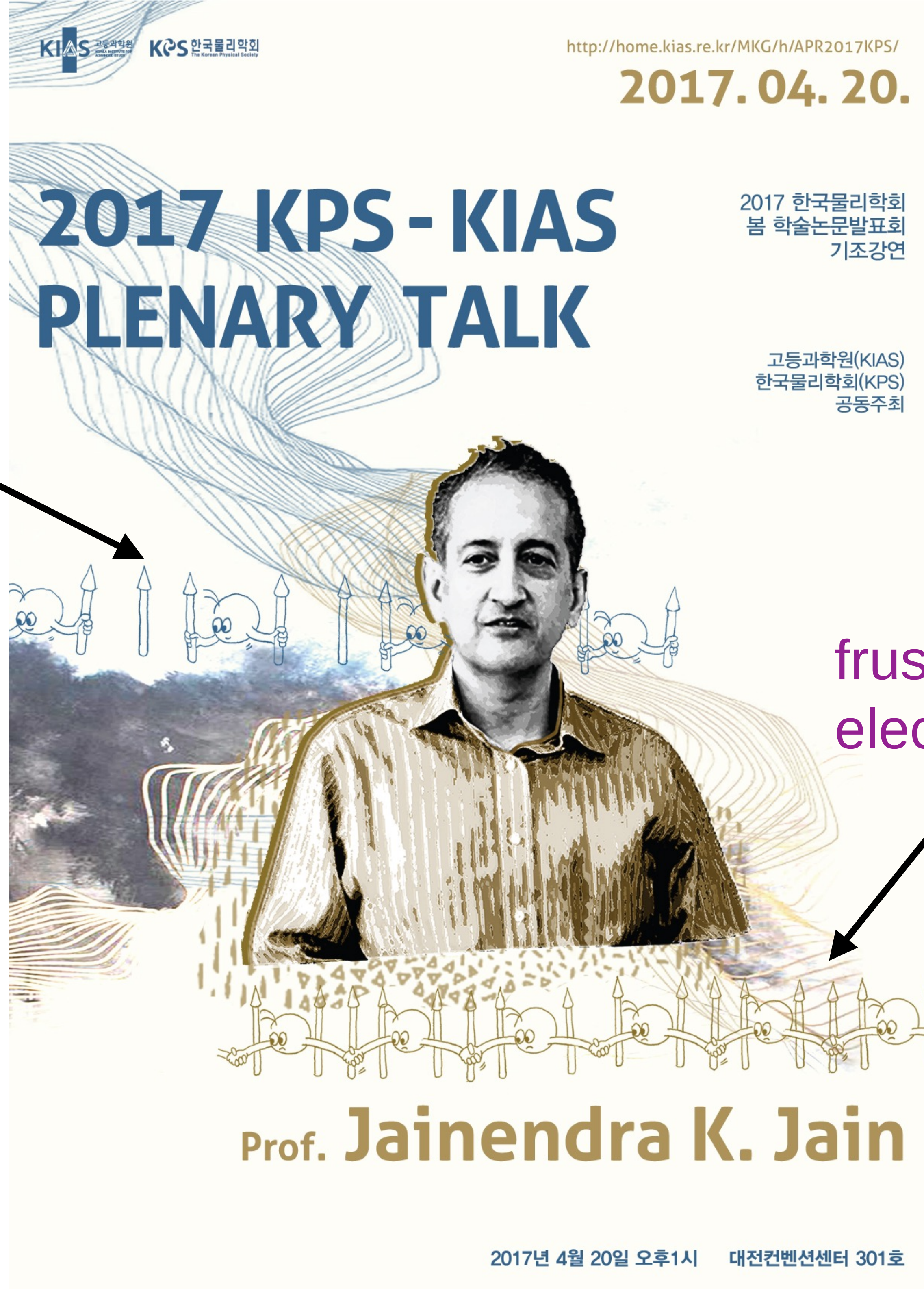
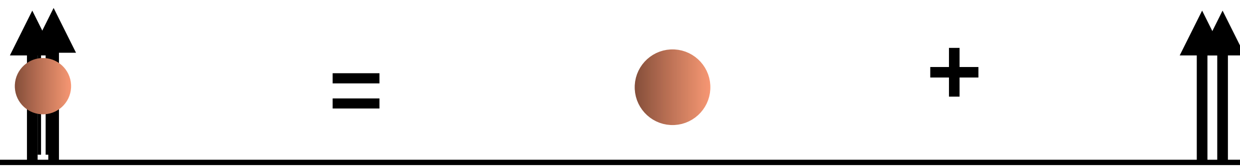


frustrated electrons 😞

Kwon Park

Deep dive into the heart of the FQHE: Microscopic validation

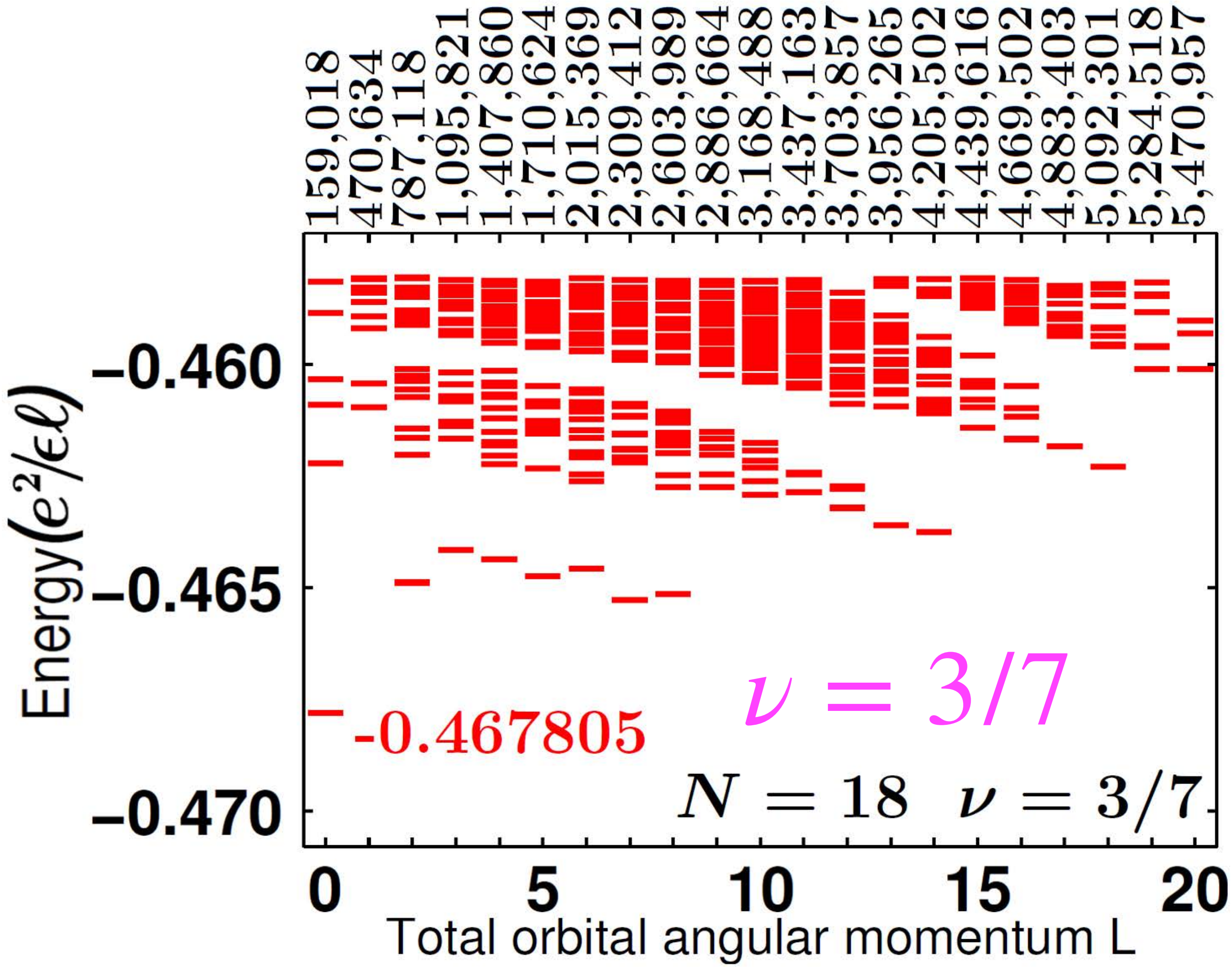
Rigorous, unbiased tests against exact results

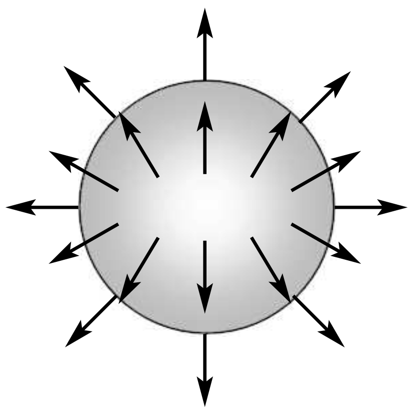


$$\Psi^{\alpha}_{\nu=\frac{\nu^{\mathrm{CF}}}{2m\nu^{\mathrm{CF}}\pm 1}} = \mathscr{P}_{\mathrm{LLL}}\Phi^{\alpha}_{\pm\nu^{\mathrm{CF}}}(\{z_i^*, z_i\})\prod_{j<k}(z_j - z_k)^{2m}$$

- Obtain the exact eigenstates / eigenenergies by a brute force diagonalization of the Coulomb interaction.
- Obtain the eigenstates / eigenenergies from the CF theory, without making any approximations.
- Compare the results from these two independent calculations with no parameters.

$\nu = 3/7$: An example





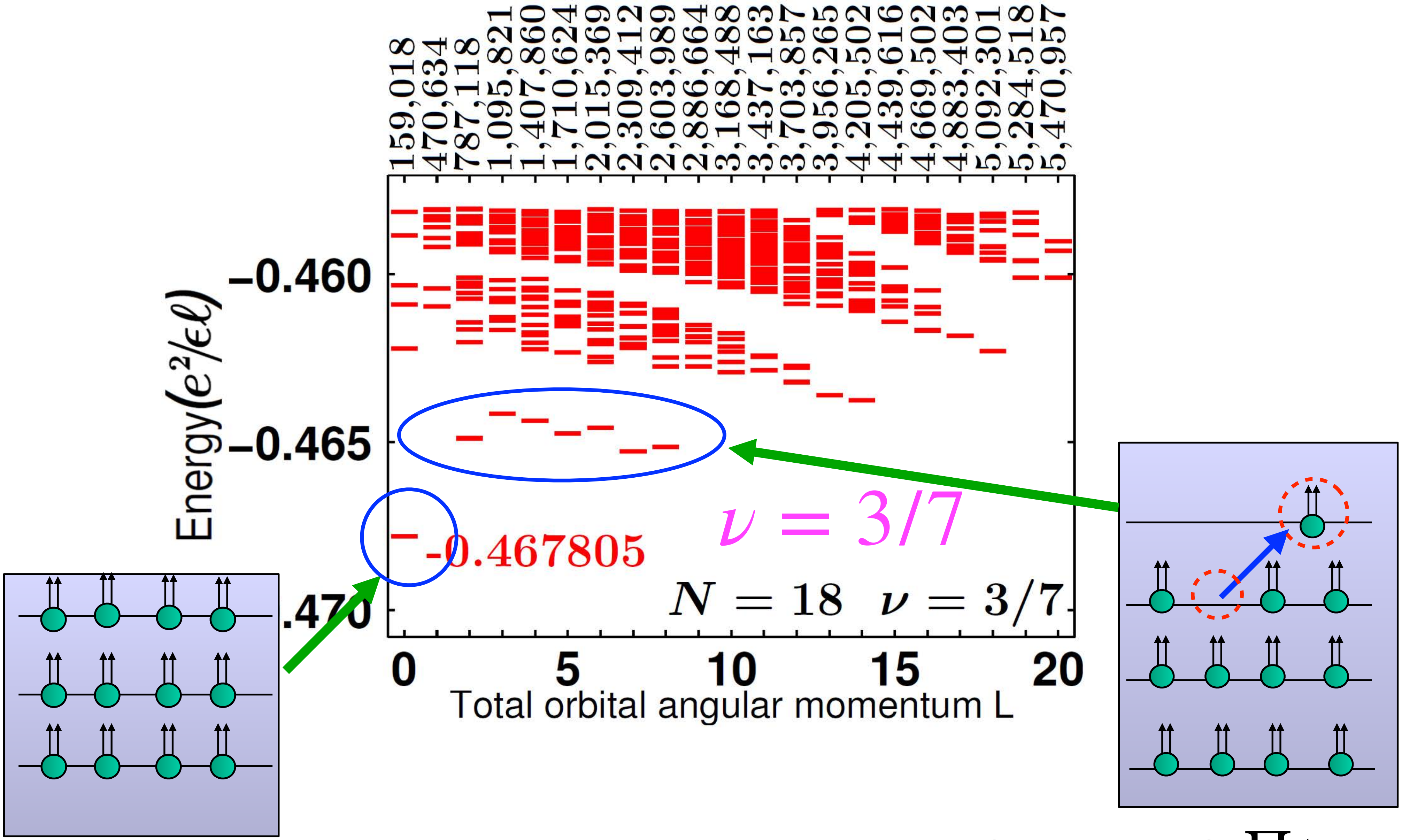
N electrons
2Q flux quanta

Haldane 1983

Fairly large system
Only the very low-energy spectrum shown
All structure due to interaction
Huge amount of information

Balram, Wójs, Jain 2013

$\nu = 3/7$: ground state + neutral excitations

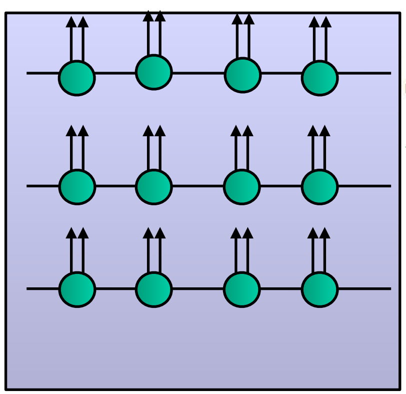


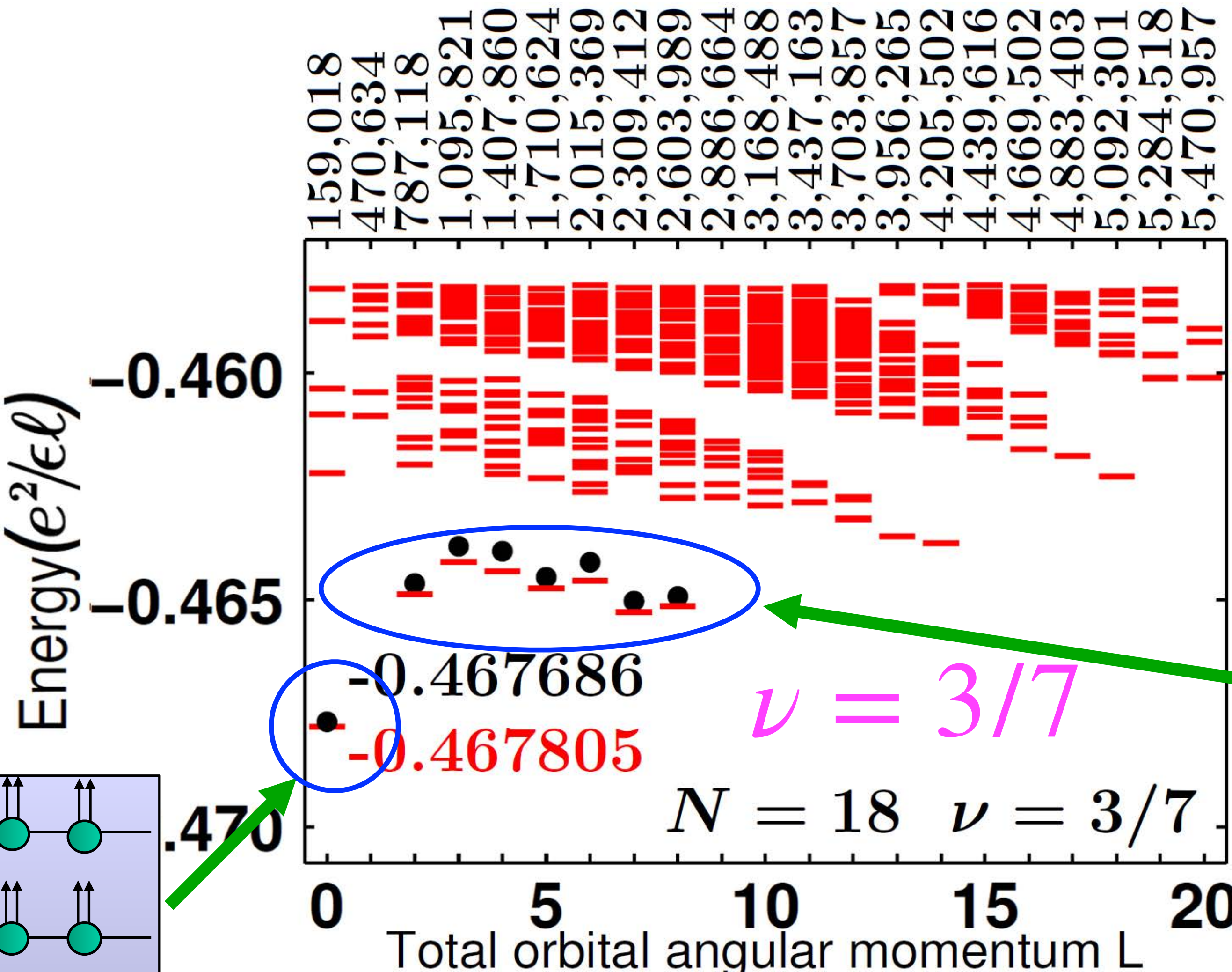
$$\Psi_{3/7} = \mathcal{P}_{\rm LLL} \Phi_3 \prod_{j<k} (z_j - z_k)^2$$

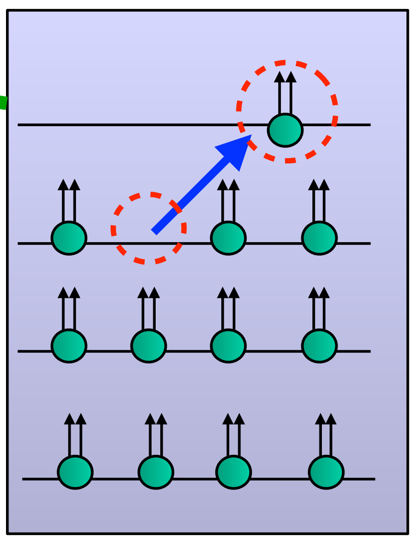
$$\Psi^{\rm ex}_{3/7} = \mathcal{P}_{\rm LLL} \Phi^{\rm ex}_3 \prod_{j<k} (z_j - z_k)^2$$

Balram, Wójs, Jain 2013

$\nu = 3/7$: ground state + neutral excitations







Almost exact agreement with no parameters!

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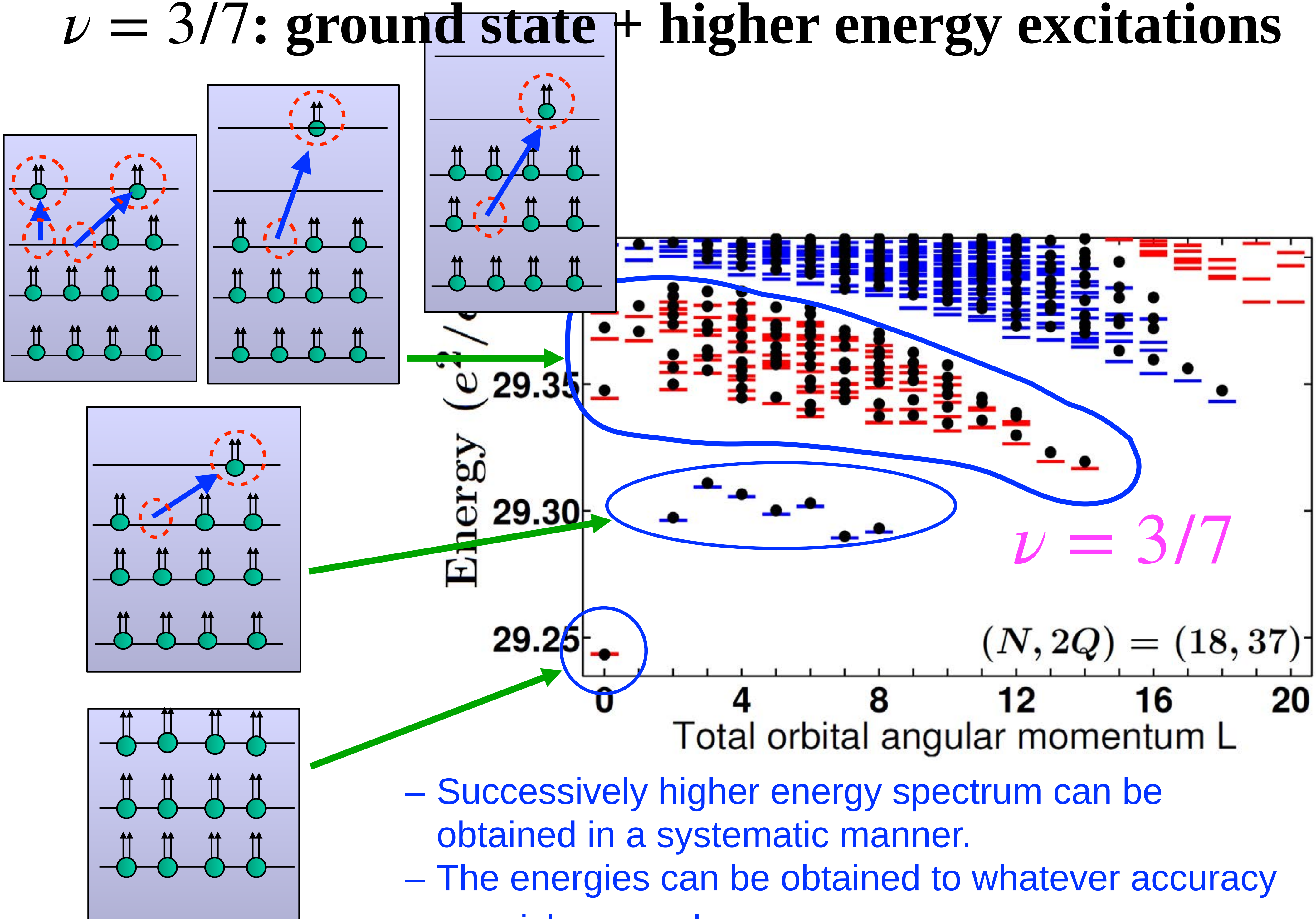
$$\Psi^{\rm ex}_{3/7} = \mathcal{P}_{\rm LLL}\Phi^{\rm ex}_3 \prod_{j<k}(z_j - z_k)^2$$

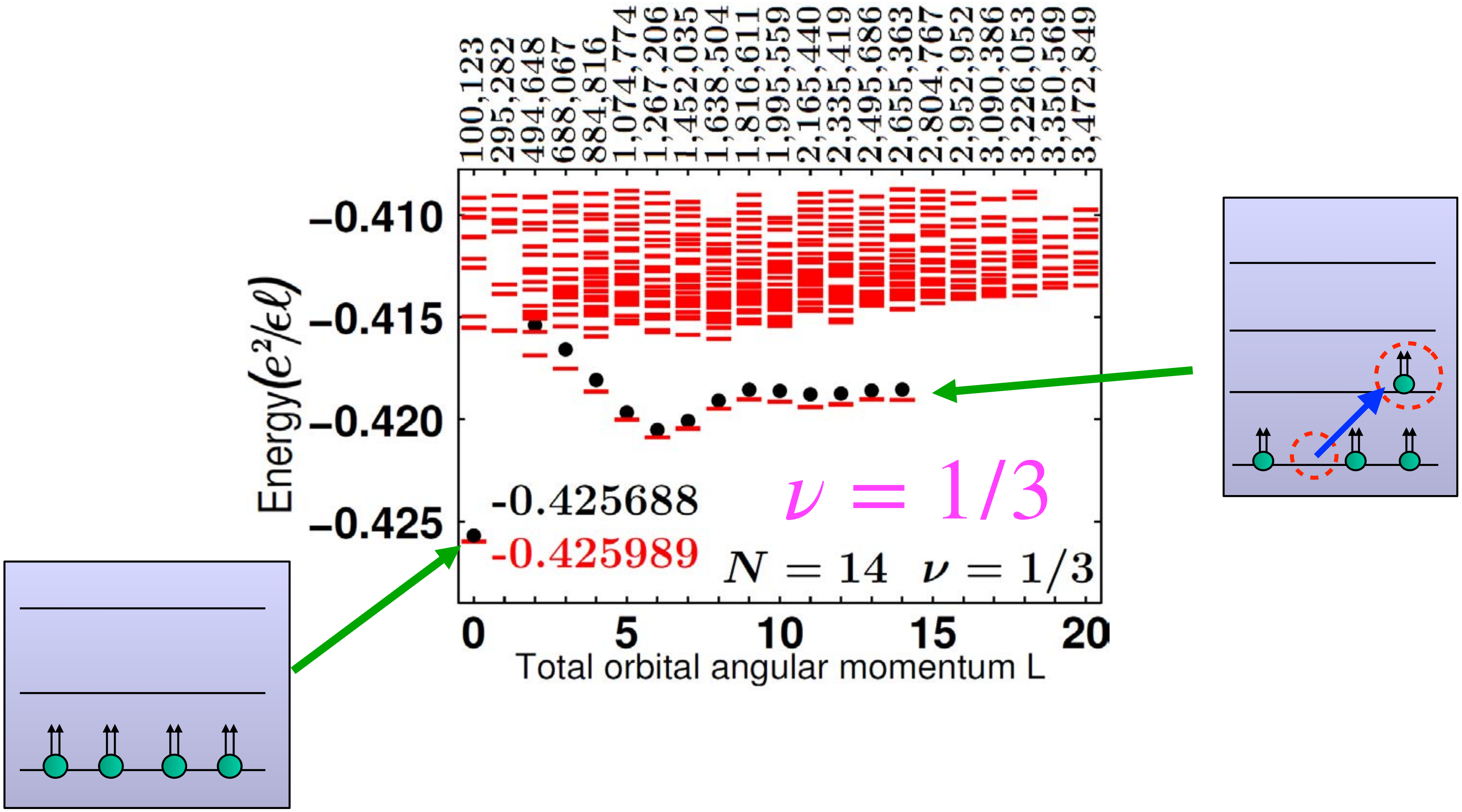
Balram, Wójs, Jain 2013

$\nu = 3/7$: ground state + higher energy excitations



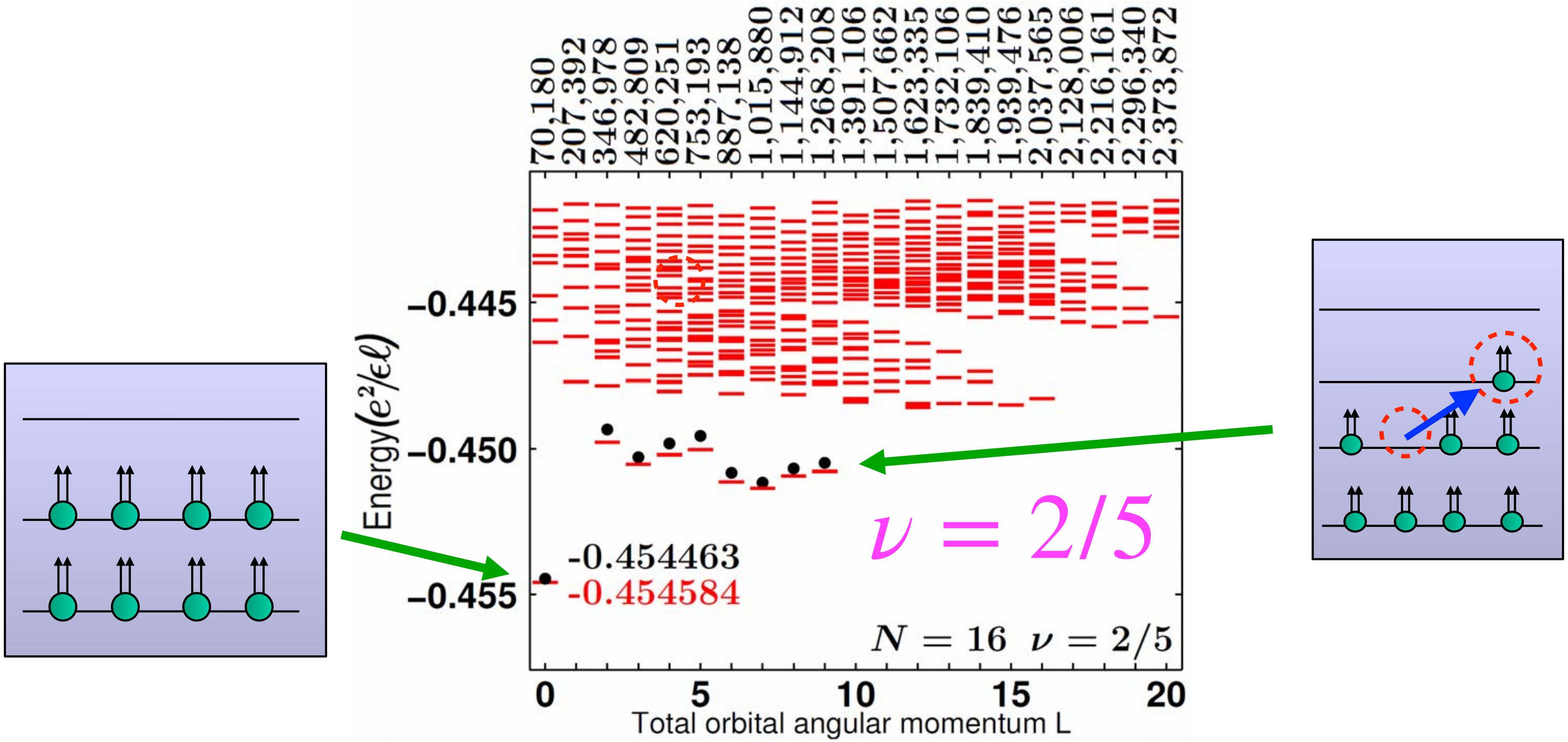
– Successively higher energy spectrum can be obtained in a systematic manner.
– The energies can be obtained to whatever accuracy we wish or need.

Balram, Wójs, Jain 2013



Similar agreement at other fractions

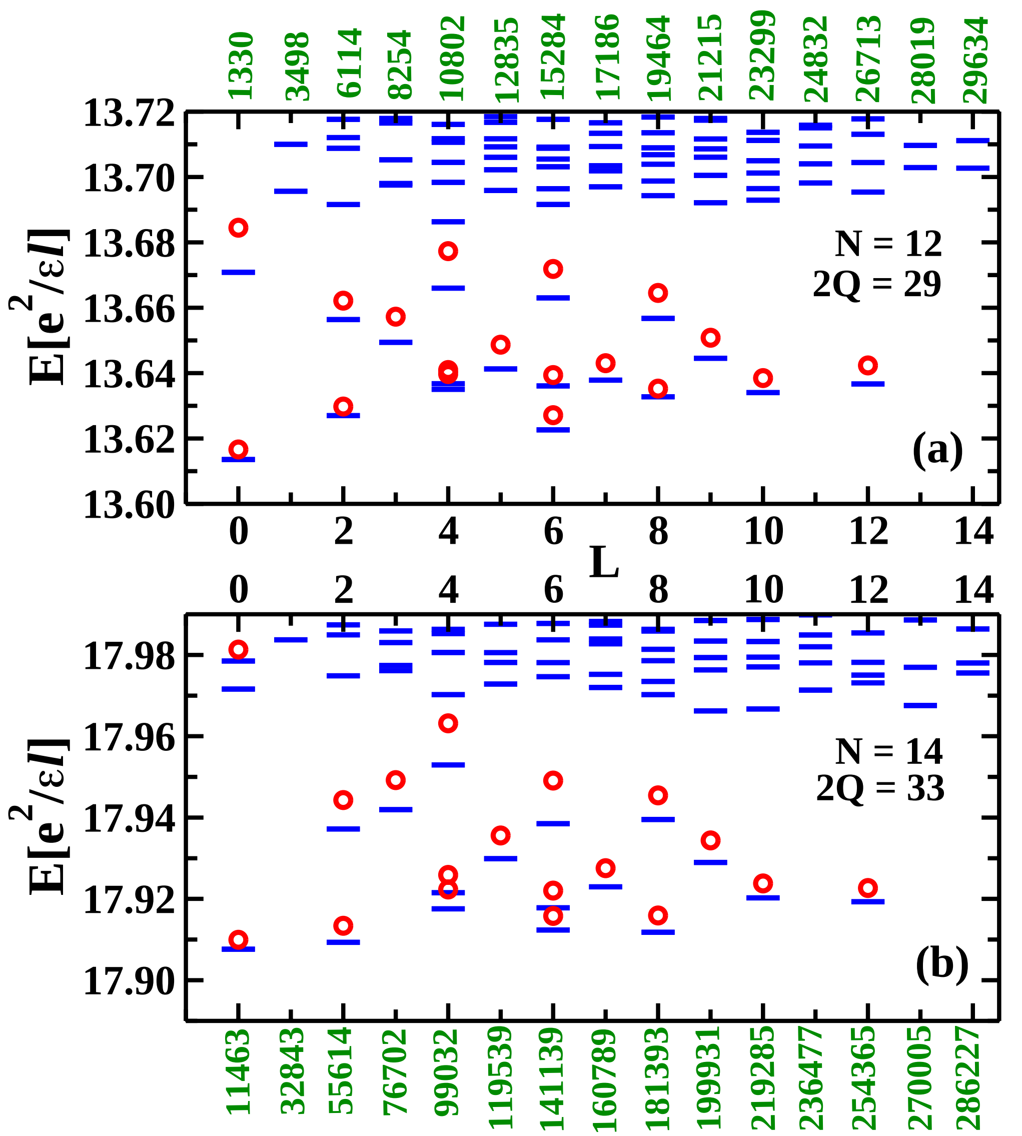
Balram, Wójs, Jain 2013



Similar agreement at other fractions

Balram, Wójs, Jain 2013

Many quasiparticles / quasiholes



$1/3 < \nu < 2/5$

4 quasiparticles of $\nu = 1/3$

6 quasiparticles of $\nu = 1/3$

No free parameters

Mukherjee, Mandal, Wójs, Jain 2012

Balram, Wójs, Jain 2013

Overlaps are close to perfect

$$\langle \Psi_{4/9} = P_{\rm LLL}\Phi_4 \prod_{j<k}(z_j - z_k)^2 \,|\, \Psi_{4/9}^{\rm Exact-Coulomb}\rangle = 0.9951 \;(\nu = 4/9;\; N = 16)$$

—A.C. Balram, unpublished

The comparisons thus demonstrate that the CF theory accurately predicts essentially all observables.

Do the CFs do anything else?

The Expanding Universe of CFs

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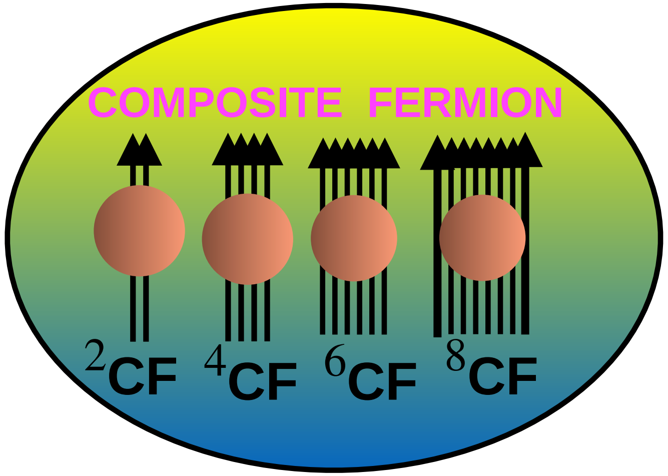

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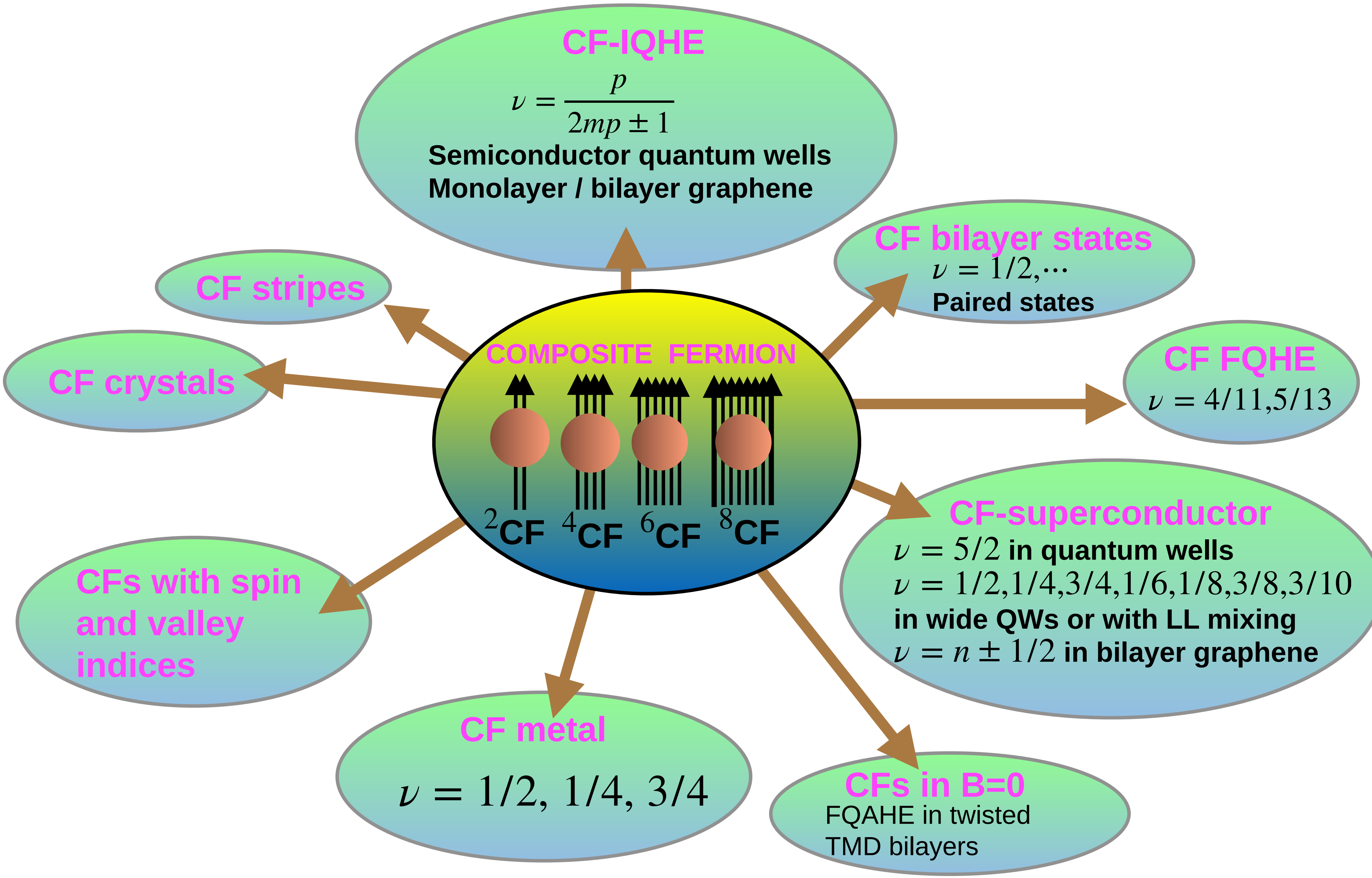
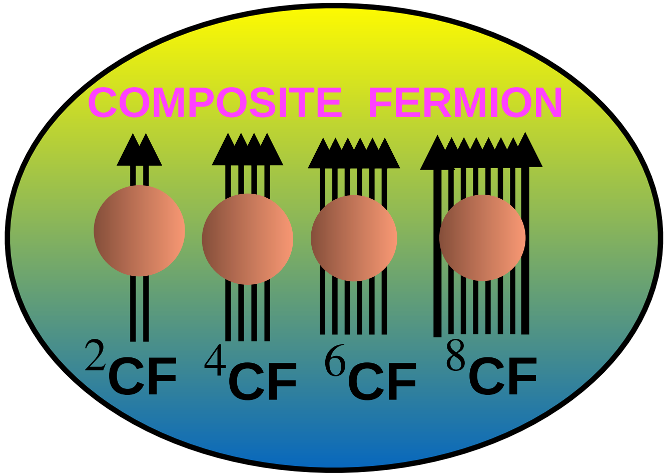


The Expanding Universe of CFs

- Transport gaps
- CF mass
- Charged excitations
- Neutral excitations
- Fractional charge
- Fractional statistics
- Edge modes
- Plateau transition
- Scaling exponents
- Electron spectral function
- Spin polarizations
- Spin wave excitations
- Spin rotons
- Skyrmions



- CF mass
- CF g-factor
- CF Fermi wave vector
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- Surface acoustic wave absorption
- Semiclassical cyclotron orbits
- Magnetic focusing
- Berry phase
- S-dH quantum oscillations
- Thermopower
- Bilayer drag

- Fractional charge
- Majorana
- Braid statistics

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- Grand unification:
  -unification of all fractions $\nu = p/(2mp \pm 1)$
  -unification of the FQHE and IQHE
  -unification of the FQHE and non-FQHE metallic states
  -many other states of CFs such as superconductors, stripes, crystals, spin unpolarized states, etc.

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- Microscopic theory: Surprisingly accurate.
- Simplicity: Much phenomenology explained without any detailed theory.
- Nontriviality: The emergence of Fermi sea and Landau-like levels within the lowest electron Landau level would be utterly unthinkable without composite fermions. There would be no reason to expect any Fermi sea in terms of electrons.

Cracking the code

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$$\Psi^{\alpha}_{\nu=\frac{\nu^{\rm CF}}{2m\nu^{\rm CF}\pm 1}} = \mathscr{P}_{\rm LLL}\, \Phi^{\alpha}_{\pm\nu^{\rm CF}} \prod_{j<k} (z_j - z_k)^{2m}$$

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What at first had seemed an impossibility has become perhaps the best understood strongly correlated state in nature!!

The most recent twist: Composite fermions in $B = 0$

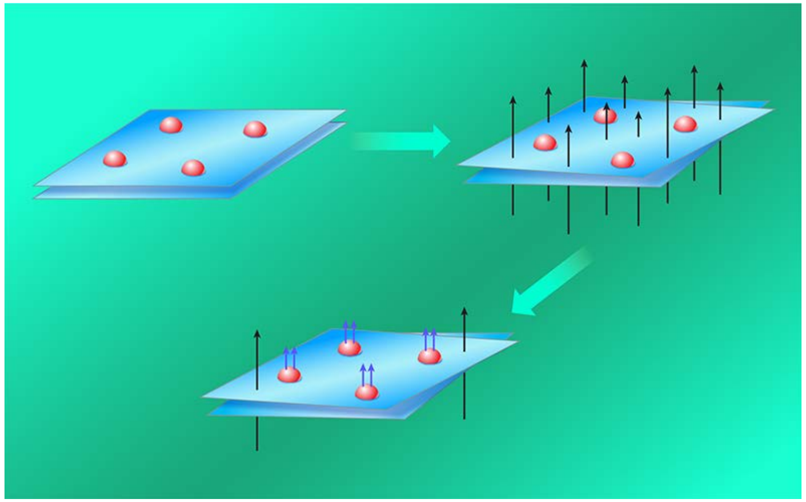
CFs at zero B: fractional quantum anomalous Hall effect

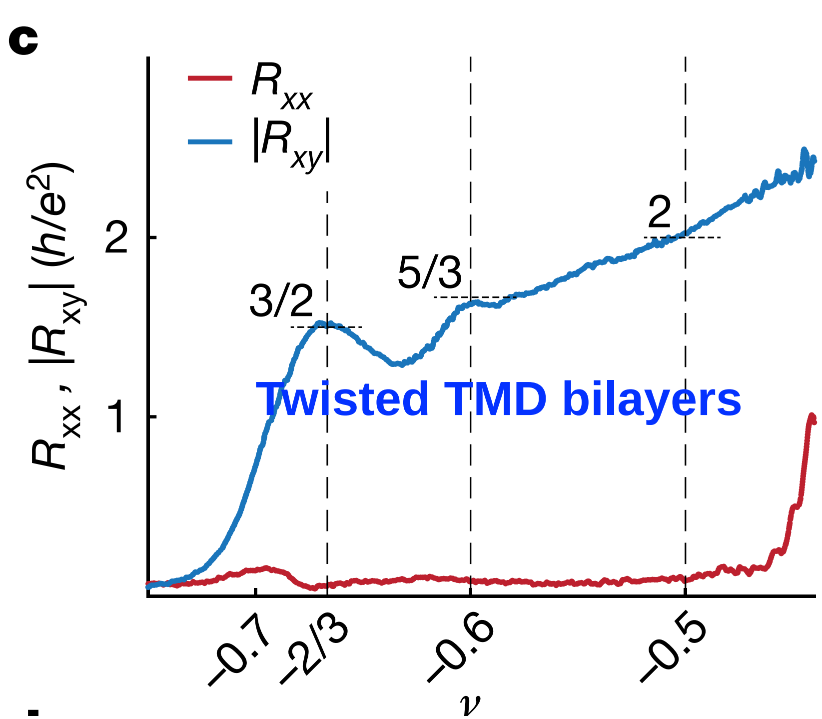
Physics — VIEWPOINT

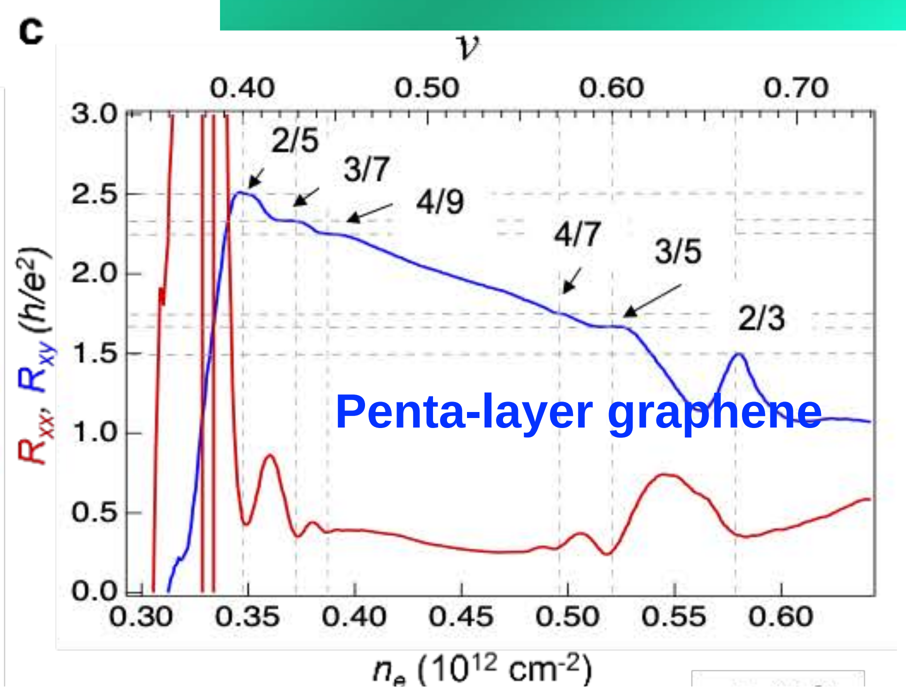
In a Twist, Composite Fermions Form and Flow without a Magnetic Field

Certain twisted semiconductor bilayers are predicted to host a Fermi liquid of composite fermions—remarkably, without an applied magnetic field.

By Jainendra Jain







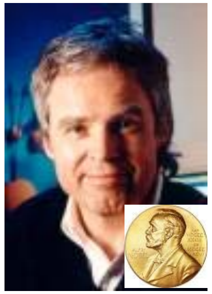
Park, …, Xu, Nature 2023

$$\frac{p}{2p+1} = \frac{2}{5}, \frac{3}{7}, \frac{4}{9} \qquad \frac{p}{2p-1} = \frac{2}{3}, \frac{3}{5}, \frac{4}{7}$$

Lu, … Ju, Nature 2023

"absolutely mindboggling! weirder than we ever thought."

-Horst Stormer

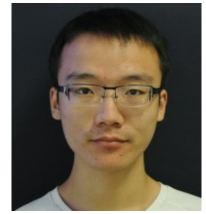


The Real Heroes

Mytraya Gattu
Penn State

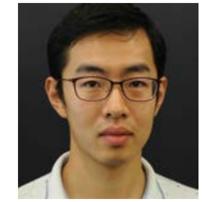
Aamir Makki
Penn State

Uddalok Nag
Penn State



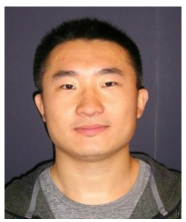
Tongzhou Zhao
IOP-CAS Beijing

Anirban Sharma
Industry



Jianyun Zhao

Jonathan Schirmer



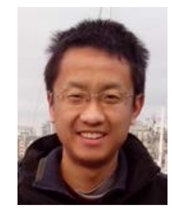
Yayun Hu
China

G. J. Sreejith
IISER Pune

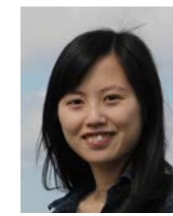
Ajit Balram
IMSc, Chennai

Yinghai Wu
Huazhong U of Sci
and Tech, Wuhan

Bill Faugno
Collage de
France



Songyang Pu
WUSTL



Yuhe Zhang

Shivakumar Jolad
Flame U, Pune

Mike Peterson
CalState Long Beach

Vito Scarola
Virginia Tech

Kwon Park
KIAS, S Korea

Rajiv Kamilla
Goldman Sachs

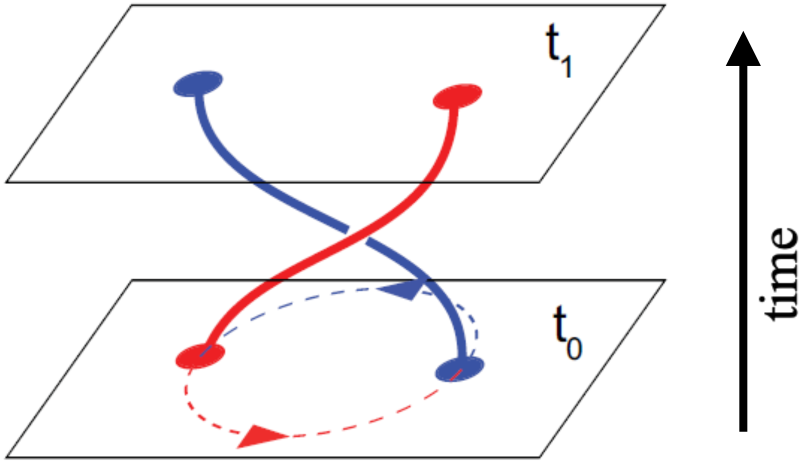
Kevin Wu
Verizon

Gautam Dev
software engineer

Thank you!

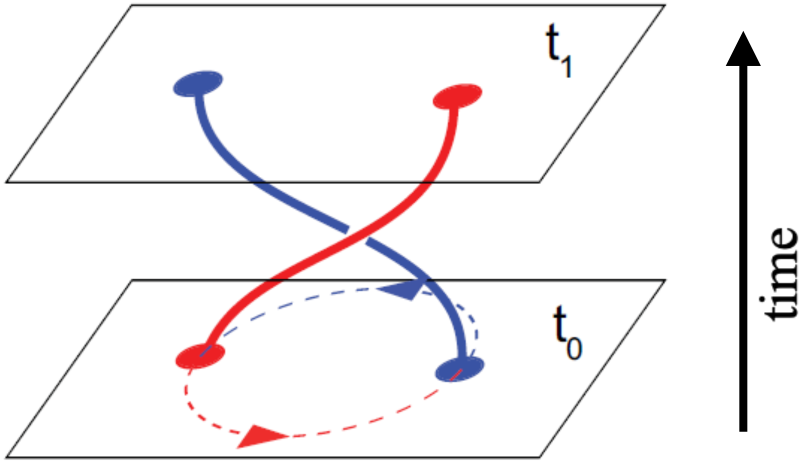
Anyons

Anyons



An exchange of two anyons produces a phase factor of $e^{i\pi\theta*}$.

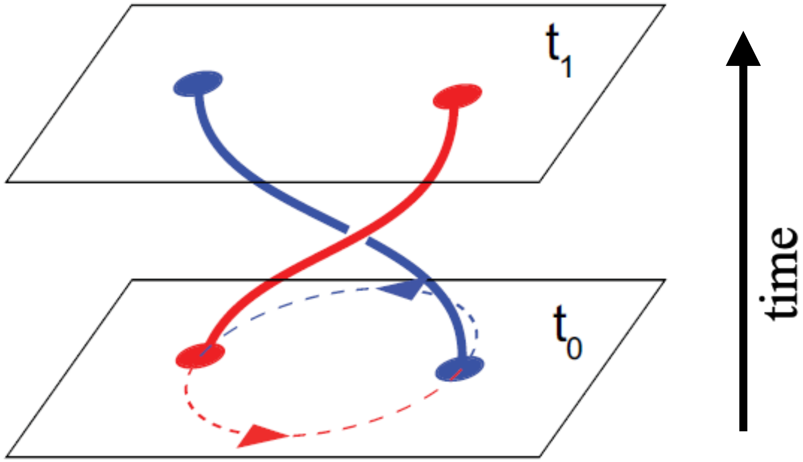
Anyons



An exchange of two anyons produces a phase factor of $e^{i\pi\theta^*}$.

They are generalizations of bosons ($\theta^* = 0$) and fermions ($\theta^* = 1$).

Anyons



An exchange of two anyons produces a phase factor of $e^{i\pi\theta^*}$.

They are generalizations of bosons ($\theta^* = 0$) and fermions ($\theta^* = 1$).

The quasiparticles of the FQHE are fractionally charged anyons (Laughlin 83, Halperin 84).
This follows from general topological arguments and has experimental support.

The CF theory gives an account of the FQHE without appealing to fractional charge and fractional statistics.

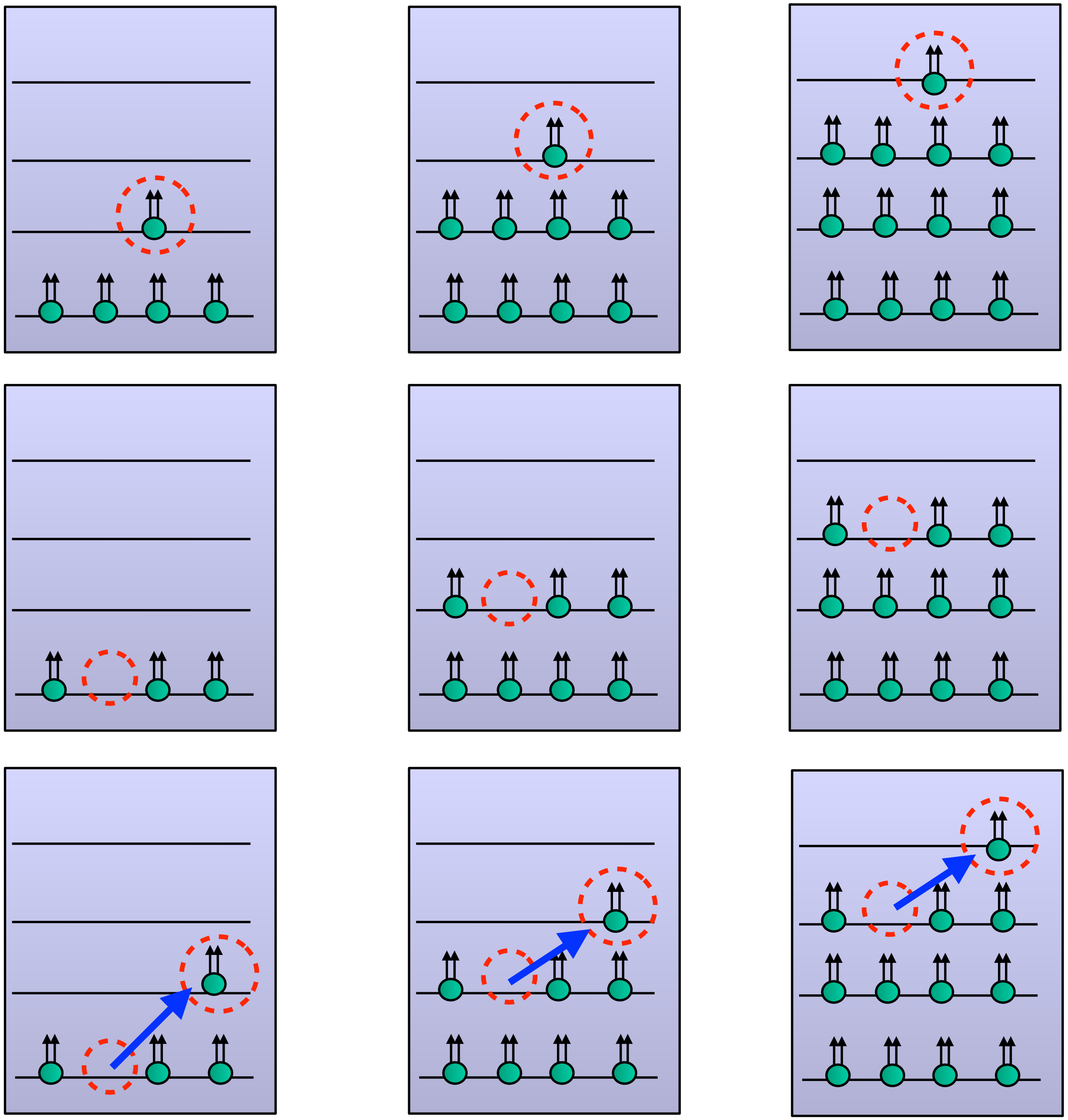
How about quasiparticles and quasiholes in the CF theory?

Quasiparticle = an isolated CF in a Λ level

quasiparticle
= isolated CF

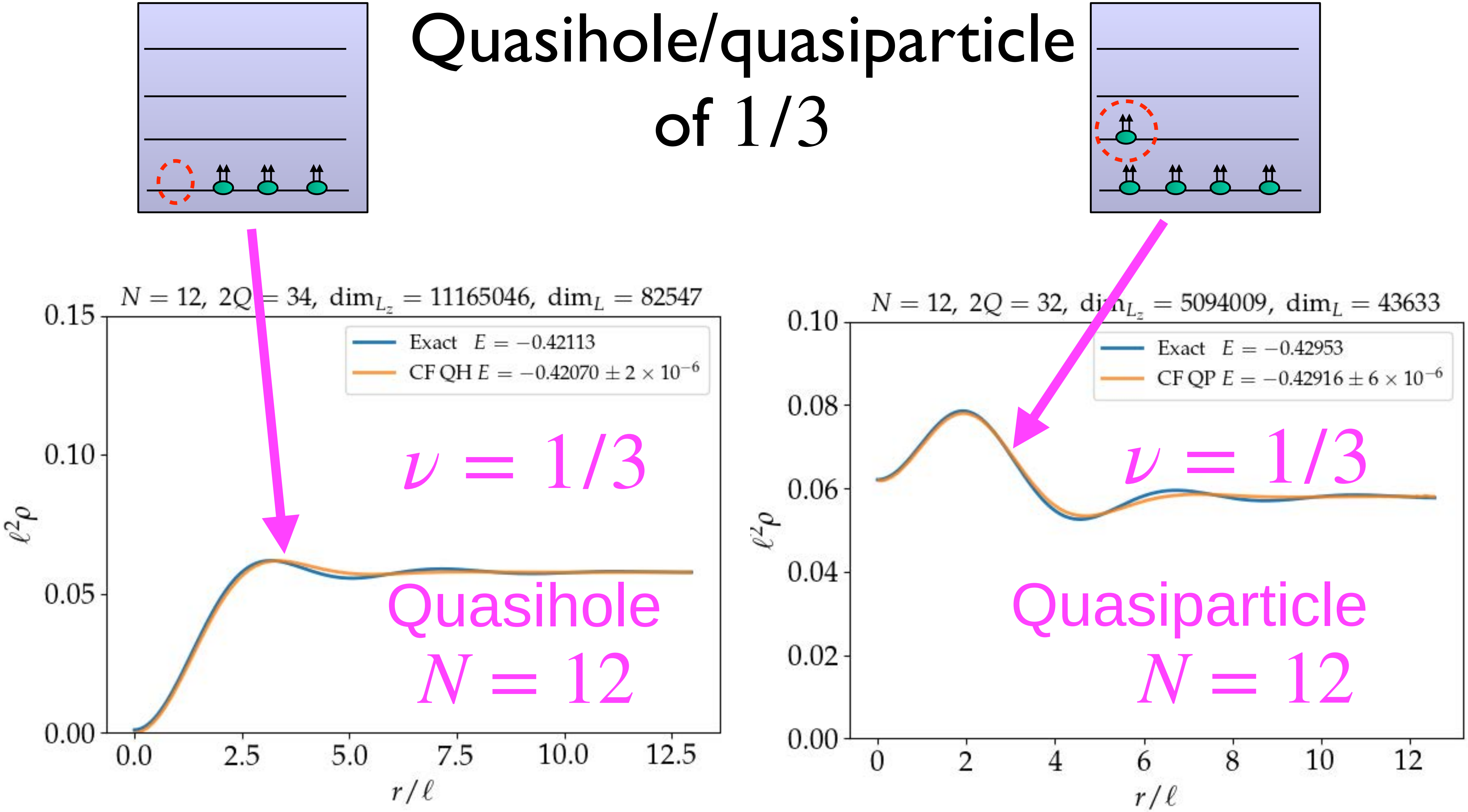
quasihole
= missing CF

neutral excitation
= CF exciton



Unified description of all excitations

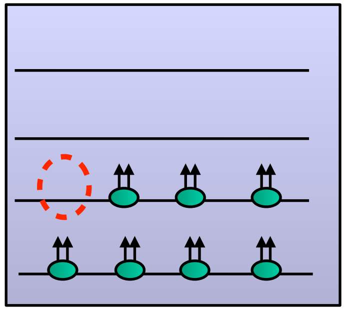
Quasihole/quasiparticle of 1/3

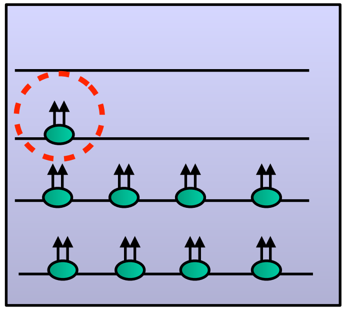


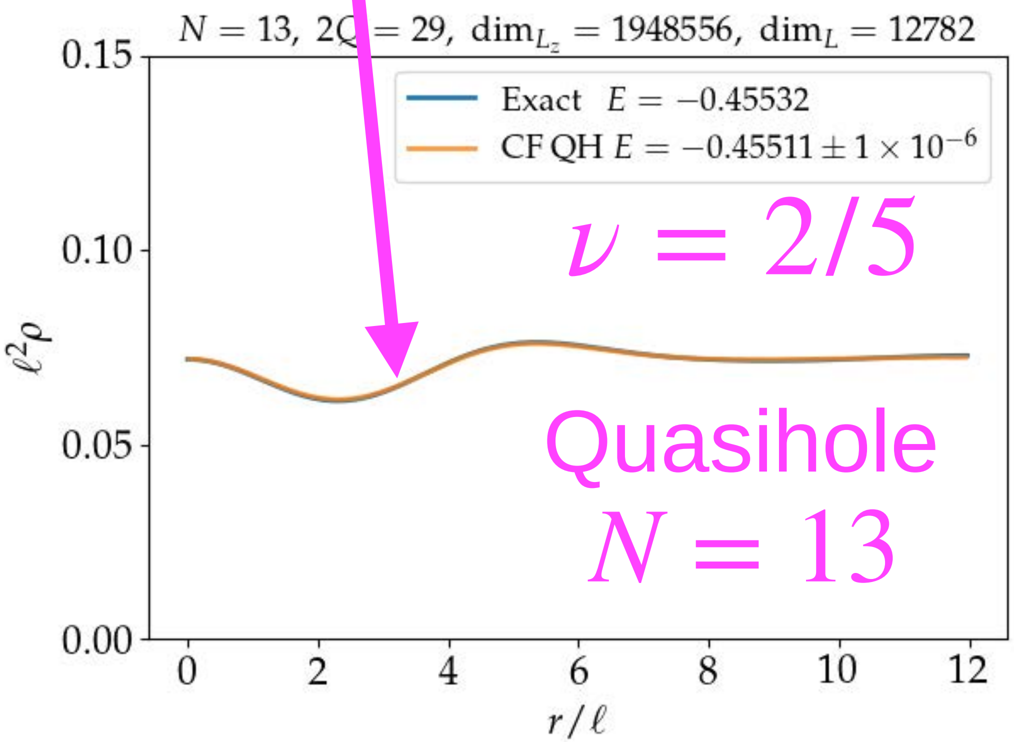
- There are ~ 6 electrons in a disk of radius 6.

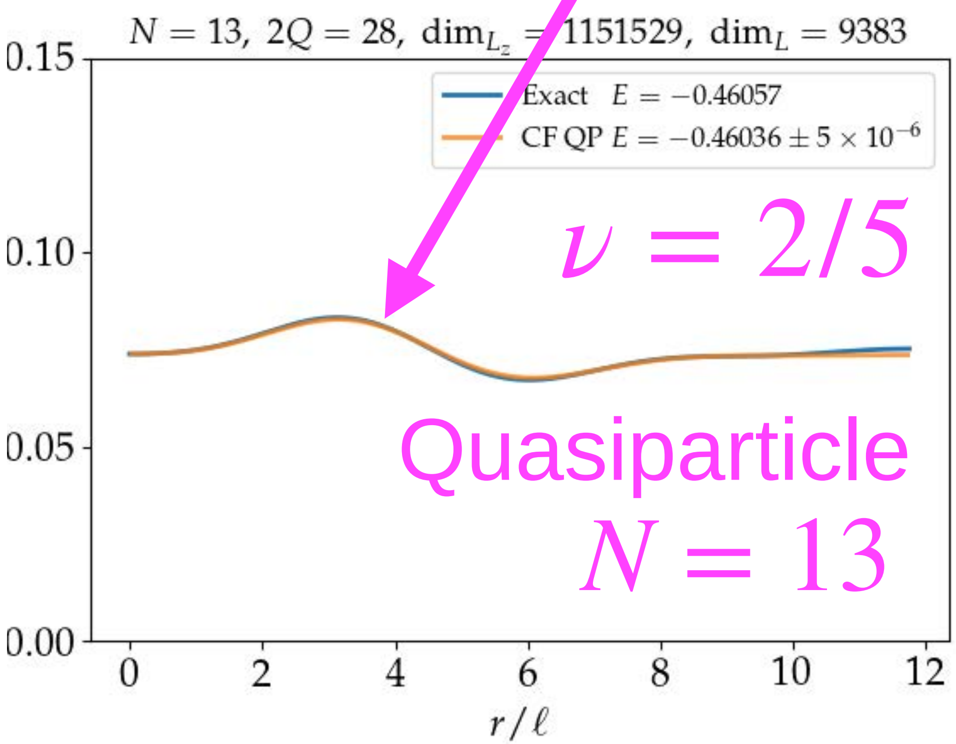
Gattu, Sreejith, JKJ, 2023

Quasihole/quasiparticle of $2/5$



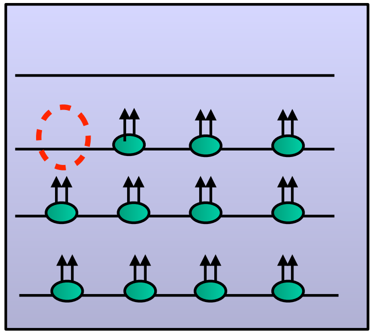




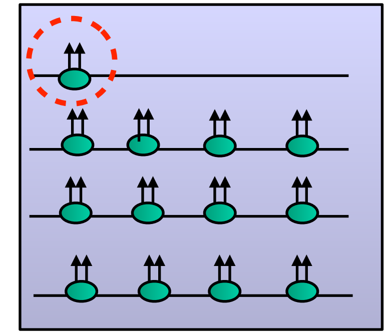


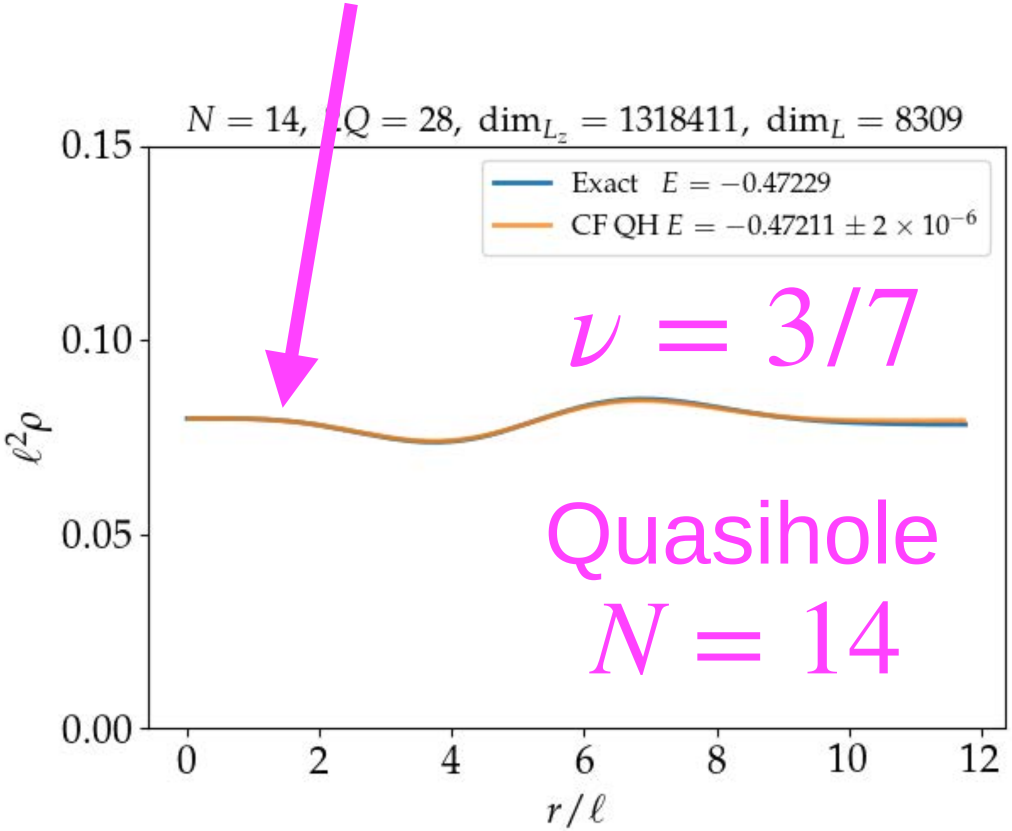
- The radius is $\sim 7-8$ magnetic lengths. A single quasiparticle of $2/5$ spreads over approximately $7-9$ electrons.

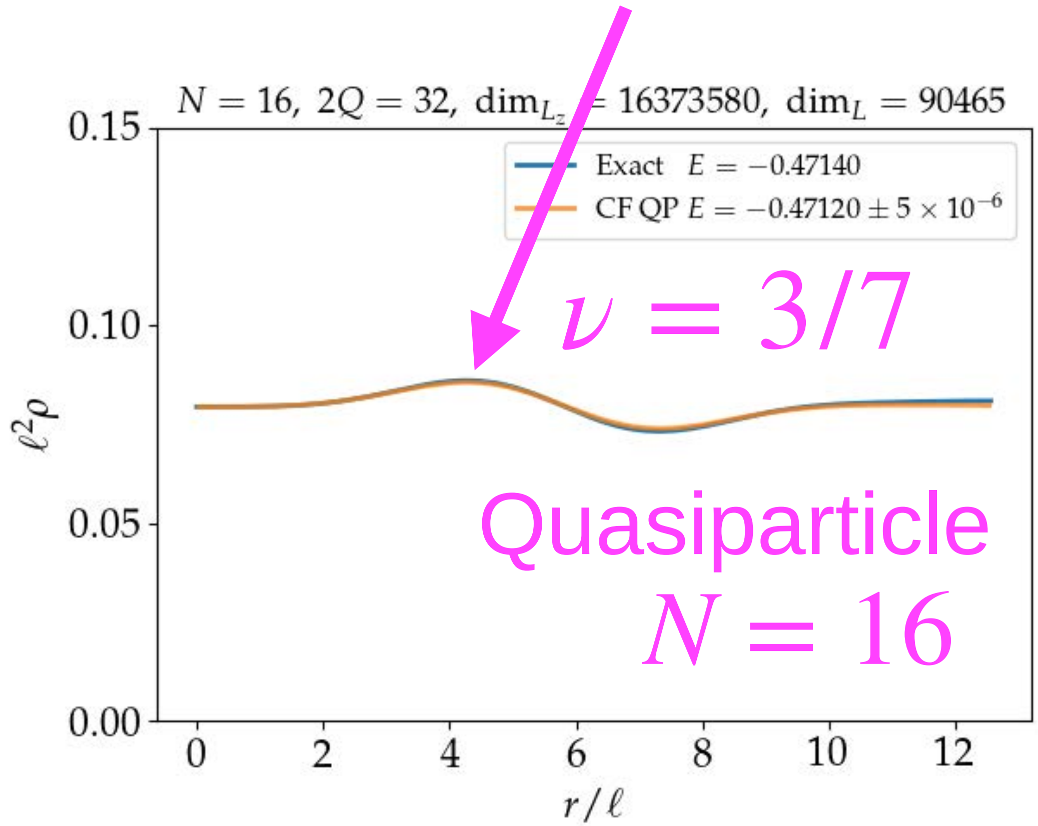
Gattu, Sreejith, JKJ, 2023



Quasihole/quasiparticle of $3/7$







- Even a single quasiparticle / quasihole is a very complex collective state. For $3/7$, it has a radius $\sim 8\ell$ and spreads over a region containing $13-14$ electrons.

Gattu, Sreejith, JKJ, 2023

A paradox?

A paradox?

Quasiparticle = an excited CF

A paradox?

Quasiparticle = an excited CF

Is it a charge-one fermion or a fractionally charged anyon?

A paradox?

Quasiparticle = an excited CF

Is it a charge-one fermion or a fractionally charged anyon?

No paradox really. It’s a question of what’s the reference state — the state with no particles, or the background FQH state — and what’s the measurement.

A paradox?

Quasiparticle = an excited CF

Is it a charge-one fermion or a fractionally charged anyon?

No paradox really. It's a question of what's the reference state — the state with no particles, or the background FQH state — and what's the measurement.

The fractional charge and braid statistics can be derived straightforwardly with the CF theory.

Fractional charge

- When we add an electron to a uniform density FQH system, we add a unit charge overall.
- However, as it gets dressed by vortices to become a CF, the unit charge is screened into a fractional charge, with the remainder leaking out to the edge.

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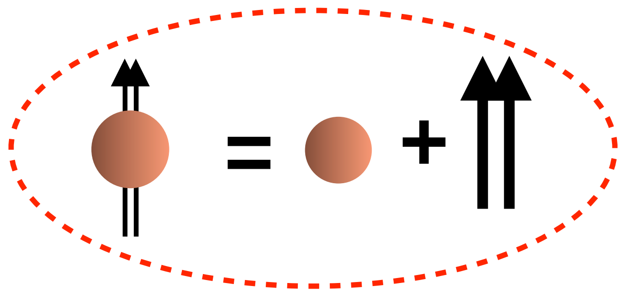
- However, as it gets dressed by vortices to become a CF, the unit charge is screened into a fractional charge, with the remainder leaking out to the edge.



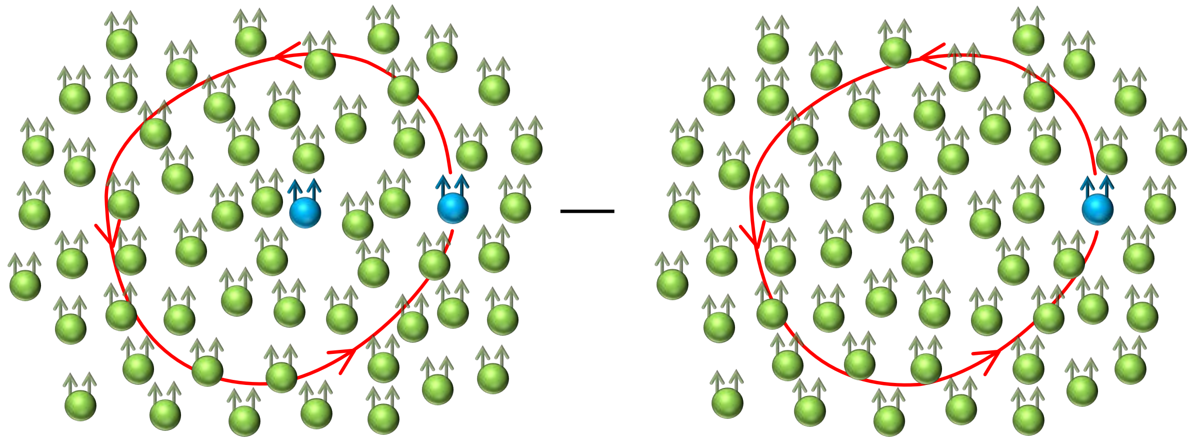
$$q^* = -1 + 2m\nu = -1 + 2m\frac{p}{2mp \pm 1} = \mp\frac{1}{2mp \pm 1}$$

Charge of an electron

Charge of $2m$ vortices

q^* can also be obtained by integrating the density.

Fractional statistics



- Because of fractional charge, the excess flux associated with each quasiparticle is a fractional: $2mq^* = 2m/(2mp \pm 1)$ thereby producing fractional statistics .

- Berry phase for a closed loop of a CF: $\Phi^* = -2\pi\left(\frac{BA}{\phi_0} - 2mN_\mathrm{e}\right)$

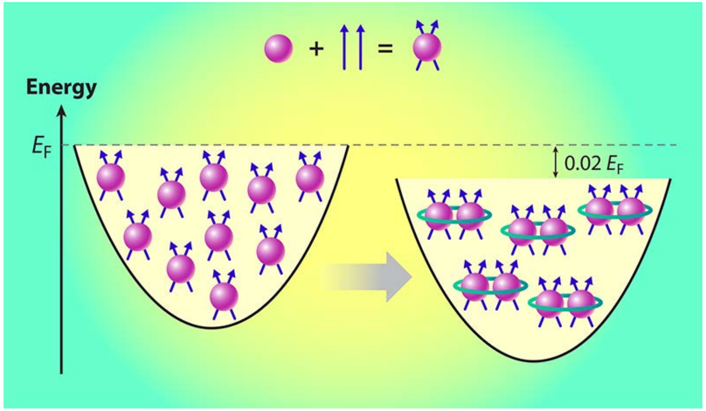
- The change in the Berry phase when another quasiparticle is inserted inside the loop (confirmed by direct evaluation):

- $\Delta\Phi^* = 2\pi \times 2m \times \Delta N_\mathrm{e} = 2\pi \times 2m \times q^* = 2\pi\frac{2m}{2mp \pm 1} \equiv 2\pi\theta^*$

CF “superconductivity”: Second mechanism of FQHE

CF pairing

- FQHE has been observed at many even-denominator fractions. These cannot be understood as IQHE of noninteracting CFs. $\nu = 5/2$ in quantum wells
- These FQHE states are understood in terms of pairing of CFs. This provides a second mechanism for FQHE.
- Pairing from purely repulsive interactions?!



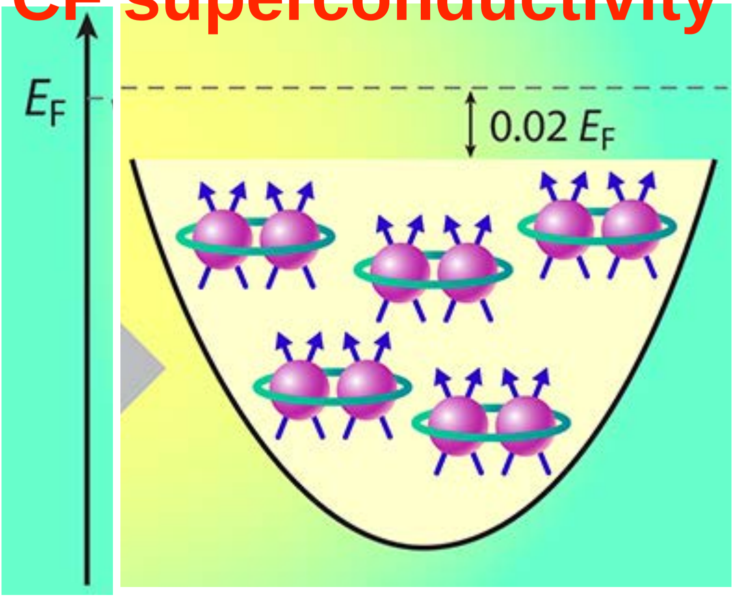
Majorana: non-Abelian anyons

From fundamental physics to technology?

*Moore Read

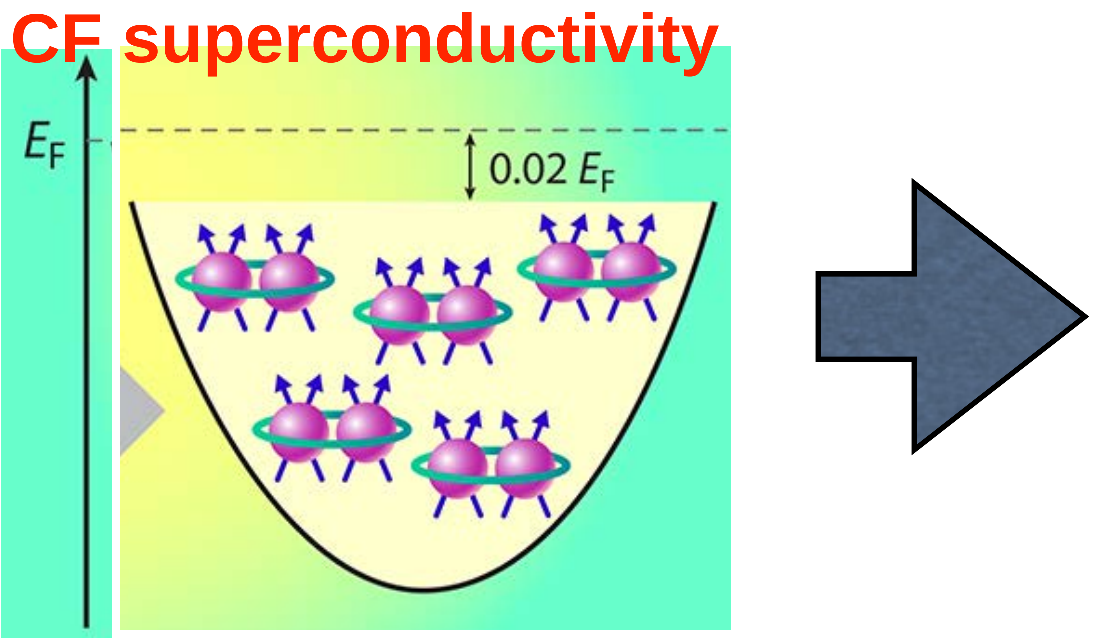
From fundamental physics to technology?

CF superconductivity



*Moore Read

From fundamental physics to technology?

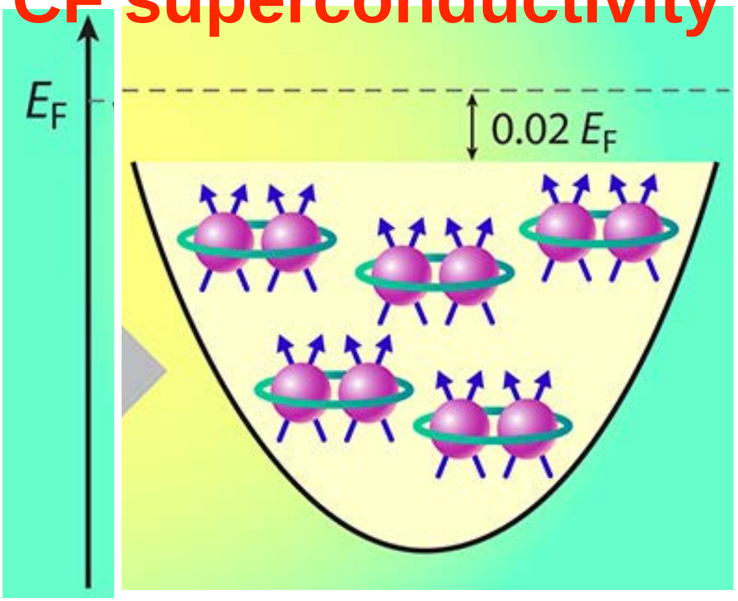


Majorana particle*
(even stranger!)

*Moore Read

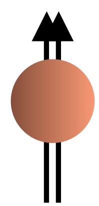
From fundamental physics to technology?

CF superconductivity



Majorana particle*
(even stranger!)

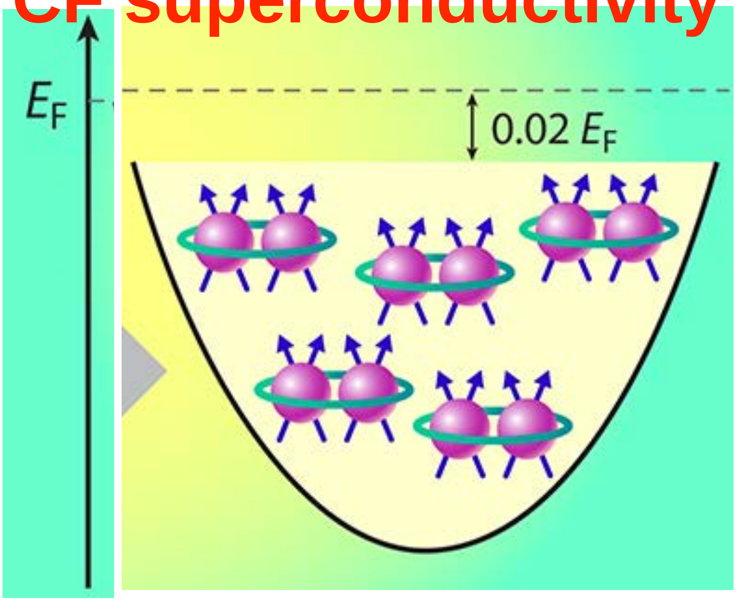
$$\mathscr{M} \times \mathscr{M} = \alpha | \quad \rangle + \beta | \quad \rangle$$



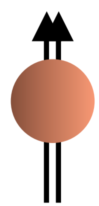
*Moore Read

From fundamental physics to technology?

CF superconductivity



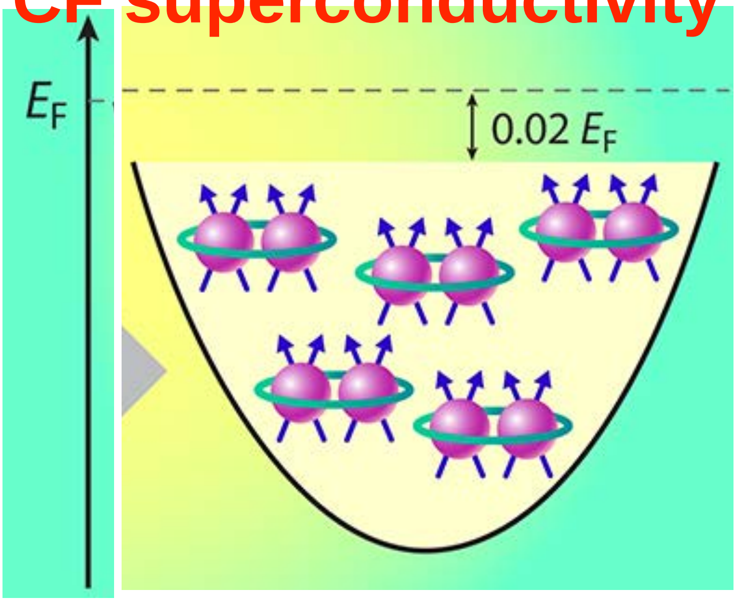
Majorana particle*
(even stranger!)

$$\mathscr{M} \times \mathscr{M} = \alpha \,|$$  $$\rangle + \beta \,| \quad \rangle$$

*Moore Read

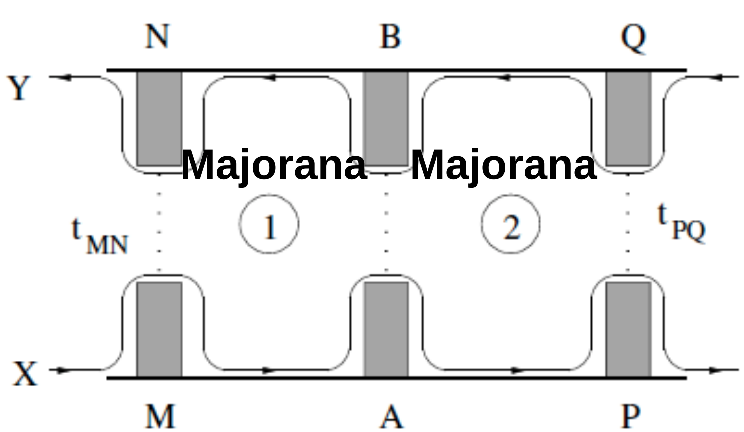
From fundamental physics to technology?

CF superconductivity



Majorana particle*
(even stranger!)

$$\mathscr{M} \times \mathscr{M} = \alpha \,|\, \Uparrow \,\rangle + \beta \,|\quad\rangle$$



Das Sarma, Freedman, Nayak, 2005; Nayak et al. 2008

A proposed future qubit (quantum bit) for fault tolerant quantum computation

*Moore Read

One can always dream!

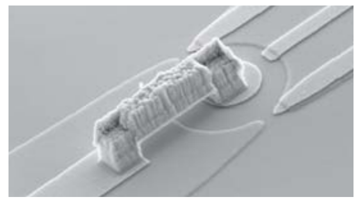
Editorial

https://doi.org/10.1038/s41567-025-03012-6

Fractional computing

We highlight how an abstract piece of condensed-matter physics – the fractional quantum Hall effect – may be ideally placed to implement quantum computers.

One of the few subfields of physics that routinely breaks though into public consciousness is quantum computing. Quantum computation offers either tan-



superconductor. The most prominent implementation of this are one-dimensional semiconductor nanowires that become super-

happens because electrons in the partially filled level can group themselves with the quanta of magnetic flux to create composite particles. This flux attachment mechanism gives states that fill the resulting Landau levels, providing an analogue of the integer filling. For example, when a level is one-third full of electrons, each electron can team up with three fluxes to account for all of the magnetic field and mimic a full level. The fractionalization of the electrons associated with these states indicates that the composite particles

"perhaps another platform might win the race to perform the first topologically protected quantum computation, but for now we would not bet against the dark horse of quantum Hall systems getting there first."

by external perturbations and therefore free from errors. There are many potential platforms for doing this, but all require the presence of exotic entities known as anyons: emergent quasiparticles that are neither fermions nor bosons. Additionally, the anyons must also be non-Abelian, meaning that exchanging particles in a different order will result in a different ground state of the overall system. The existence of such quasiparticles may seem counterintuitive, but evidence suggests they exist.

In addition to proving the existence of these non-Abelian anyons, one must work out how to control them to carry out the basic computing operations. This means being able to create the anyons as required, implement protocols to do particle exchange – two such exchanges are called a braid – and bring them back together to measure them.

One option is to use Majorana modes associated with the edge states of a topological

three-dimensional materials, and moving vortices around is difficult to scale to many qubits.

Enter the quantum Hall effect. This is one of the more abstract areas of condensed-matter physics, but on the positive side it is largely accepted that non-Abelian anyons exist in this setting[4] and that they can be braided[5] in interferometer devices like the one pictured[6].

When a strong magnetic field is applied perpendicular to a two-dimensional system, the allowed energy states for the electrons or holes in that system are highly degenerate bands called Landau levels. When a Landau level is completely filled, current can only flow via the topological edge states (producing a transverse response) and the bulk of the sample is insulating, meaning that the longitudinal conductivity goes to zero. This is known as the integer quantum Hall effect.

When a Landau level is fractionally filled, a similar transport response can occur. This

qubit from fractional quantum Hall anyons, as there are with all of the platforms we have discussed. In particular, the fractional states are rather fragile (although perhaps less so in graphene than in semiconductors[7]) and the degree of control needed to isolate and manipulate them will require exquisitely engineered devices. So, perhaps another platform might win the race to perform the first topologically protected quantum computations, but for now we would not bet against the dark horse of quantum Hall systems getting there first.

Published online: 11 August 2025

References

1. Das Sarma, S. *Nat. Phys.* **19**, 165–170 (2023).
2. Gu, Q. et al. *Science* **388**, 938–944 (2025).
3. Zhang, P. et al. *Nat. Phys.* **15**, 41–47 (2019).
4. Banerjee, M. et al. *Nature* **559**, 205–210 (2018).
5. Nakamura, J. et al. *Nat. Phys.* **16**, 931–936 (2020).
6. Ghosh, B. et al. *Nat. Phys.* https://doi.org/10.1038/s41567-025-02960-3 (2025).
7. Hu, Y. et al. *Nat. Phys.* **21**, 716–723 (2025).

OTY HEIBLUM AND BIKASH GHOSH

Open problems / Future prospects

- **Certain open problem / future directions**:
  -Puzzles remain regarding the nature of pairing of the $\nu = 5/2$ state. The nature of pairing of other even-denominator states also needs to be verified.
  -Need better understanding of composite fermions in FQAHE / periodic potentials.
  -Dream 1: Application to future technology?
  -Dream 2: The structures revealed in the study of CFs / FQHEs provide a clue for unraveling some other profound mysteries of nature.
  -More surprises??

Thank you!

The BCS wave function of CFs (fully polarized)

Sharma, Pu, Jain, PRB 2021

$$\Psi^{\mathrm{el-BCS}}(\{\vec{r}_j\}) = A[g^{(l)}(\vec{r}_1 - \vec{r}_2)g^{(l)}(\vec{r}_3 - \vec{r}_4)\cdots]$$

$$\Psi_{1/2}^{\mathrm{CF-BCS}} = P_{\mathrm{LLL}} \ \Psi^{\mathrm{el-BCS}}(\{\vec{r}_j\}) \prod_{j<k} (z_j - z_k)^2$$

$$g^{(l)}(\vec{r}_i - \vec{r}_j) = \sum_{\vec{k}}^{|\vec{k}| \le k_{\mathrm{cutoff}}} g_{\vec{k}}^{(l)} e^{i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)} \qquad g_{\vec{k}}^{(l)} \equiv \frac{v_{\vec{k}}}{u_{\vec{k}}} = \frac{\epsilon_{\vec{k}} - \sqrt{\epsilon_{\vec{k}}^2 + |\Delta_{\vec{k}}^{(l)}|^2}}{\Delta_{\vec{k}}^{(l)*}} = -g_{-\vec{k}}^{(l)}$$

$$\Delta_{\vec{k}}^{(l)} = \Delta \, |\vec{k}|^l e^{-il\theta}$$

$l = 1$: **p-wave**
$l = 3$: **f-wave**

- Two variational parameters: Δ and $k_{\mathrm{cutoff}}(\geq k_{\mathrm{F}})$.
- The CF-BCS wave function reduces to the CF Fermi sea for $\Delta = 0$ or $k_{\mathrm{cutoff}} = k_{\mathrm{F}}$.

Pairing from purely repulsive interaction?!

Empirically: The inter-CF interaction becomes attractive as the strength of the short range repulsion between the electrons is reduced. This may be done in three ways:

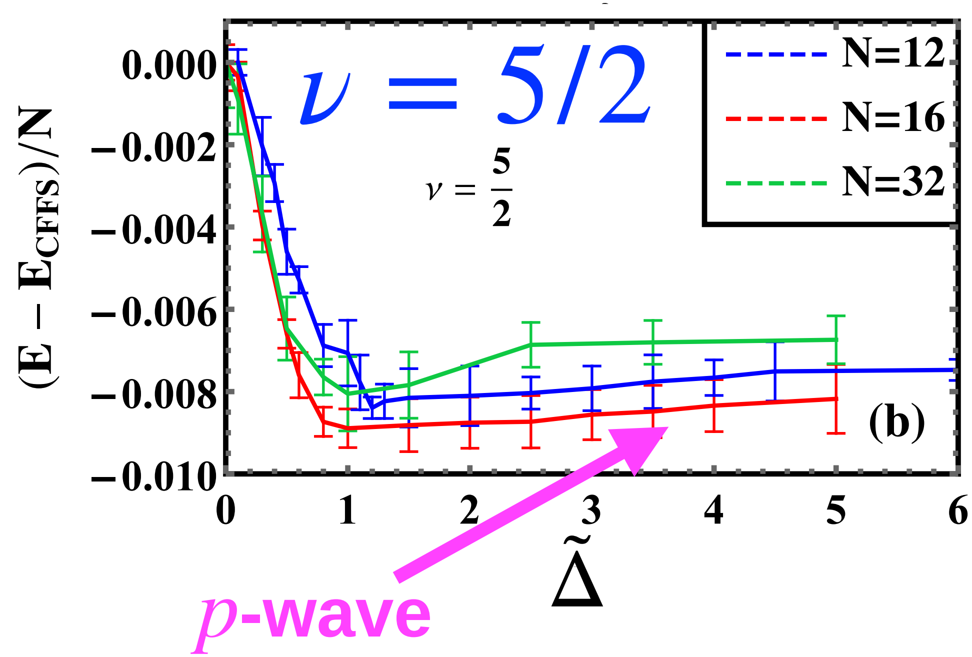
- By going to a higher LL
- By increasing the quantum well width / density
- By enhancing LL mixing

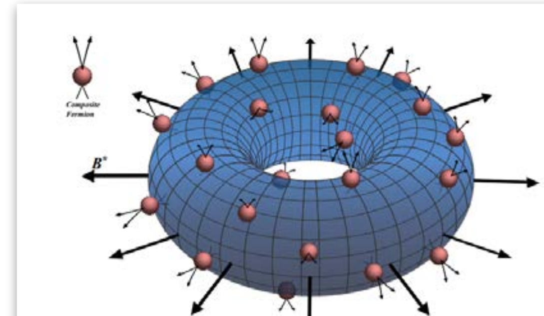
CF pairing at $\nu = 2 + 1/2 = 5/2$

PHYSICAL REVIEW B **104**, 205303 (2021)

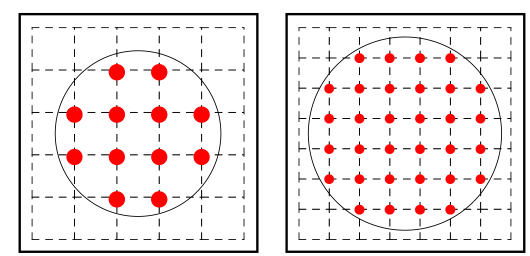
Bardeen-Cooper-Schrieffer pairing of composite fermions

Anirban Sharma, Songyang Pu, and J. K. Jain
Department of Physics, 104 Davey Lab, Pennsylvania State University, University Park, Pennsylvania 16802, USA





Pu, Wu, Jain, PRB 96, 195302 (2017)



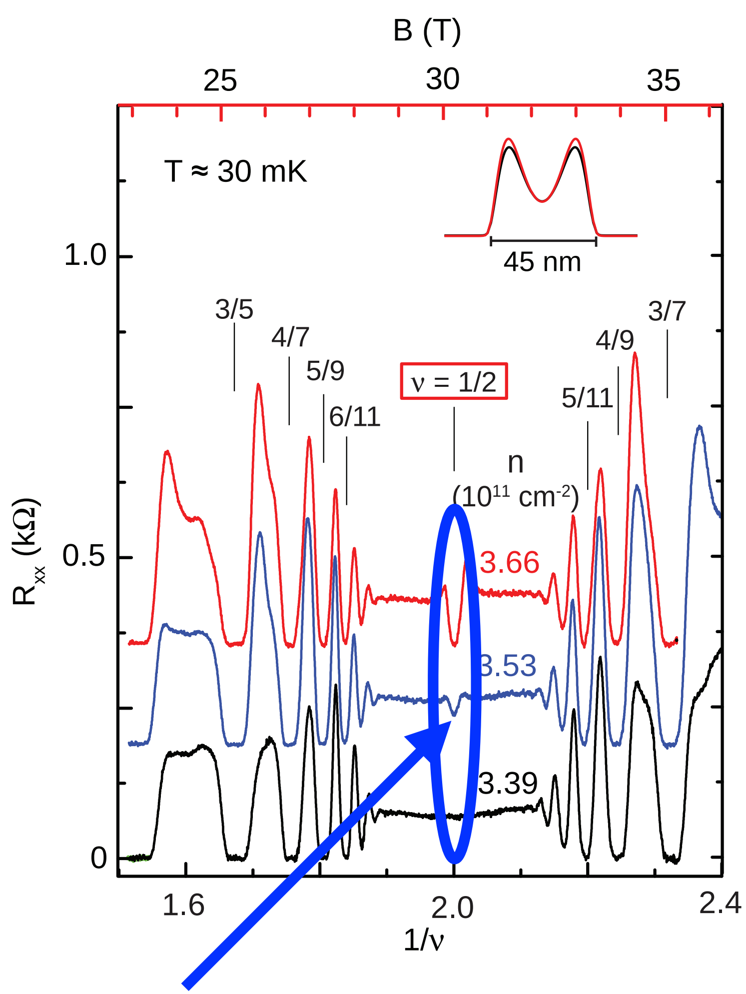
- A p-wave pairing instability occurs at $\nu = 5/2$.
- No instability at $\nu = 1/2$.

FQHE at $\nu = 1/2$ in wide quantum wells

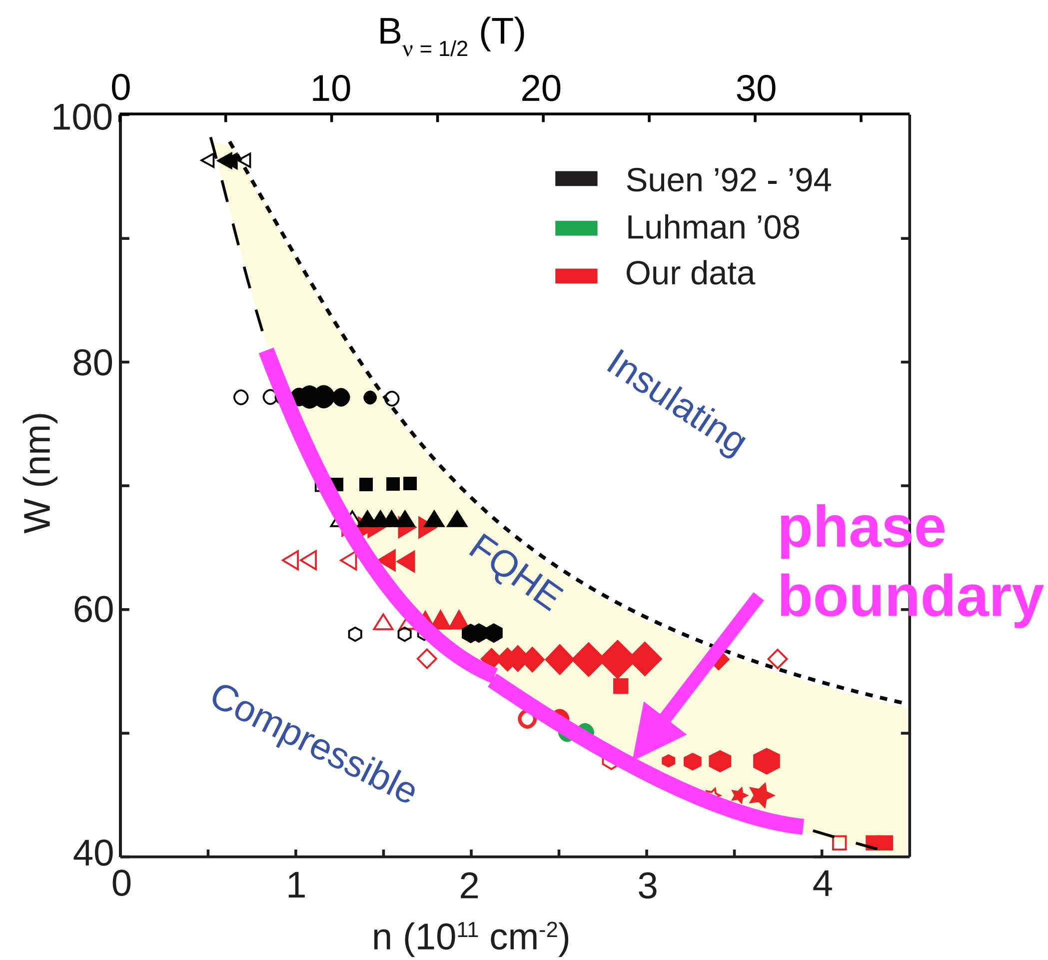
PHYSICAL REVIEW B **88**, 245413 (2013)

Phase diagrams for the stability of the $\nu = \frac{1}{2}$ fractional quantum Hall effect in electron systems confined to symmetric, wide GaAs quantum wells

J. Shabani, Yang Liu, M. Shayegan, L. N. Pfeiffer, K. W. West, and K. W. Baldwin



$\nu = 1/2$ FQHE



Suen, Engel, Santos, Shayegan, Tsui, PRL (1992)

FQHE at $\nu = 1/2$ in wide quantum wells

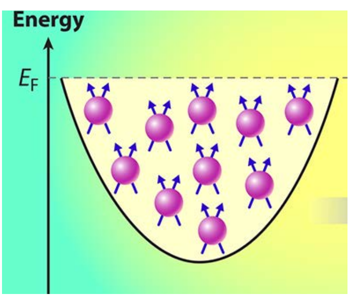
PHYSICAL REVIEW B **109**, 035306 (2024)

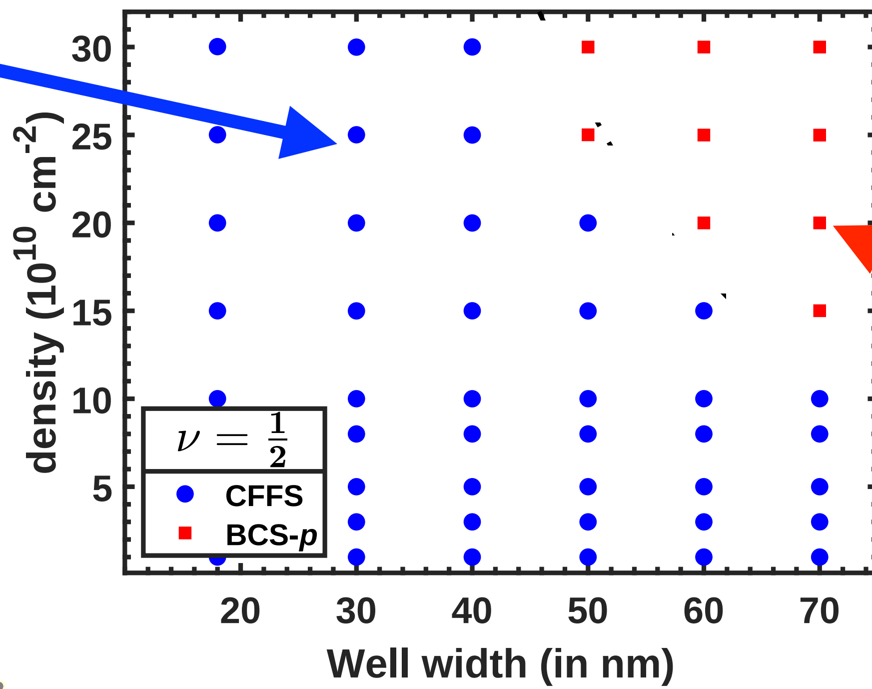
Editors' Suggestion | Featured in Physics

Composite-fermion pairing at half-filled and quarter-filled lowest Landau level

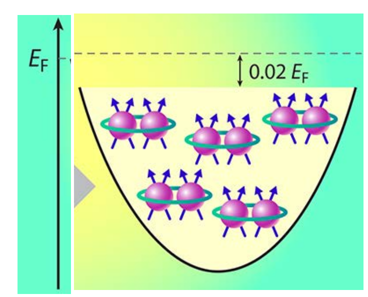
Anirban Sharma,[1] Ajit C. Balram,[2,3] and J. K. Jain[1]

CF metal





CF p-wave pairing



FQHE at $\nu = 1/2$ in wide quantum wells

PHYSICAL REVIEW B **109**, 035306 (2024)

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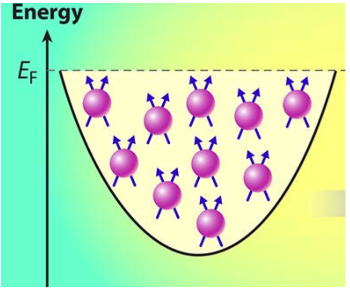
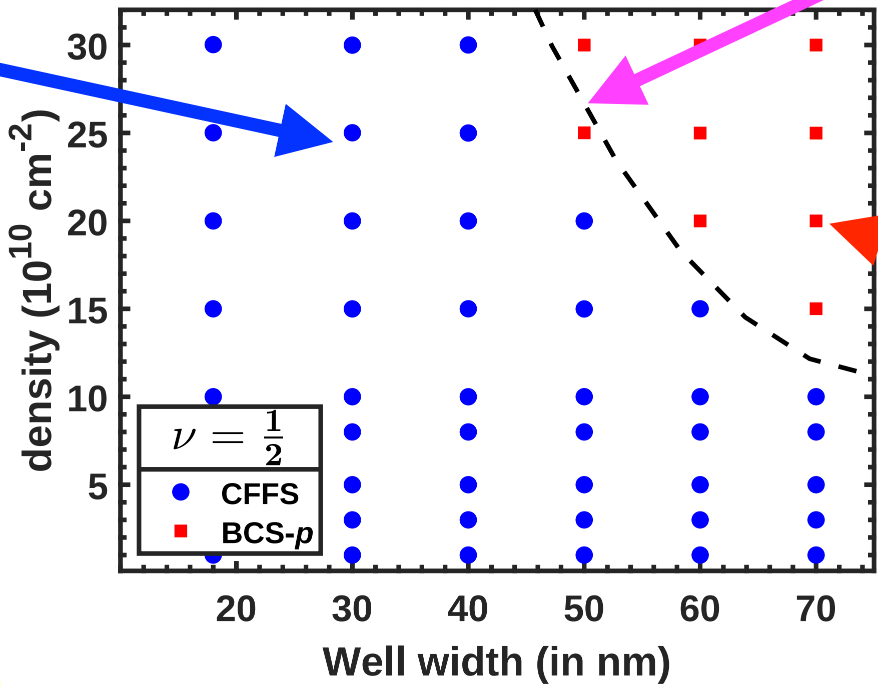
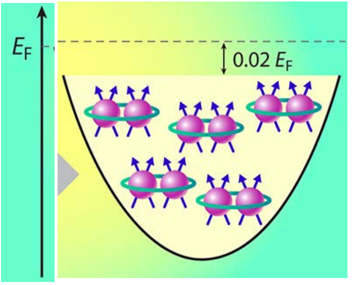




experimental phase boundary

Shabani, Liu Shayegan, et al. Phys. Rev. B (2013)

CF superconductor
p-wave pairing

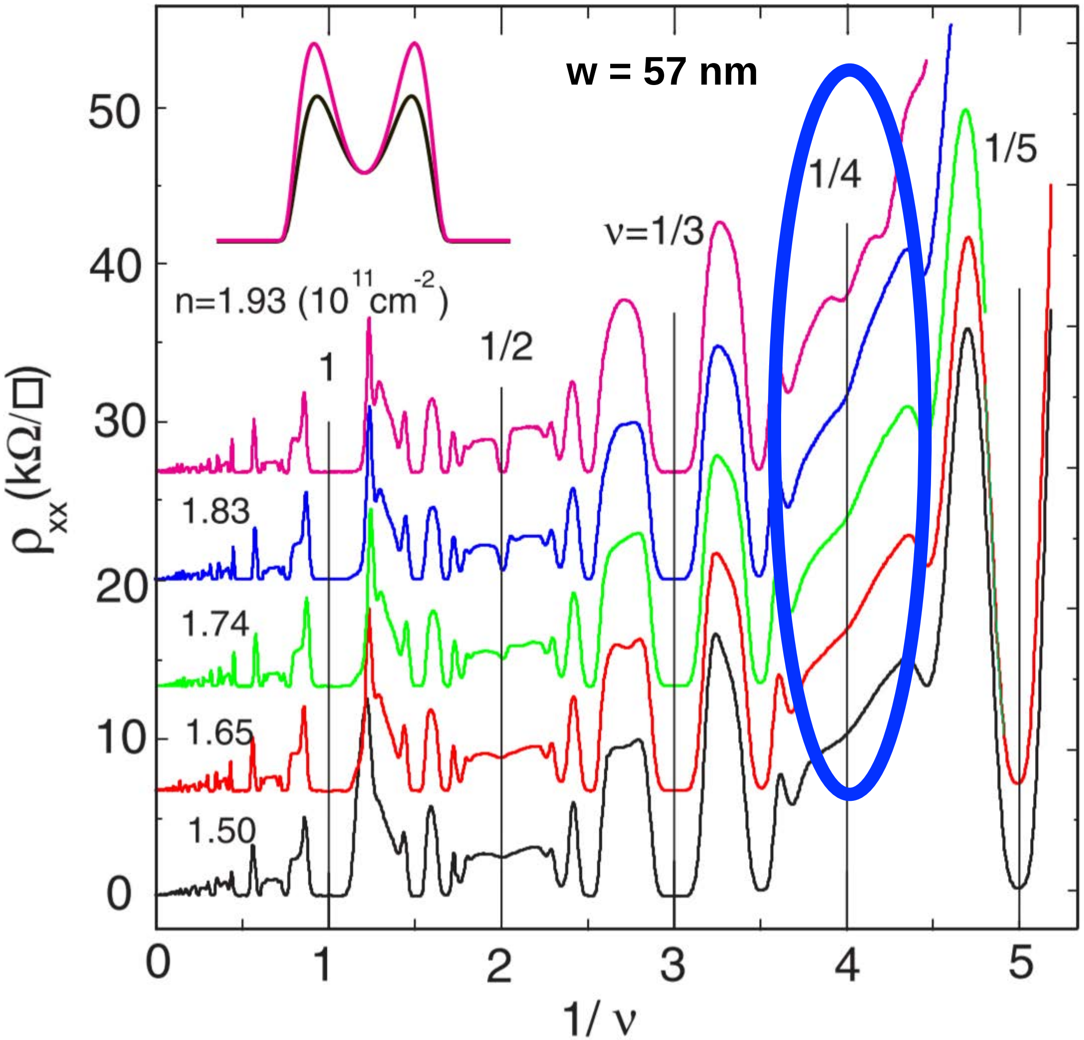


1/4 FQHE in wide quantum wells

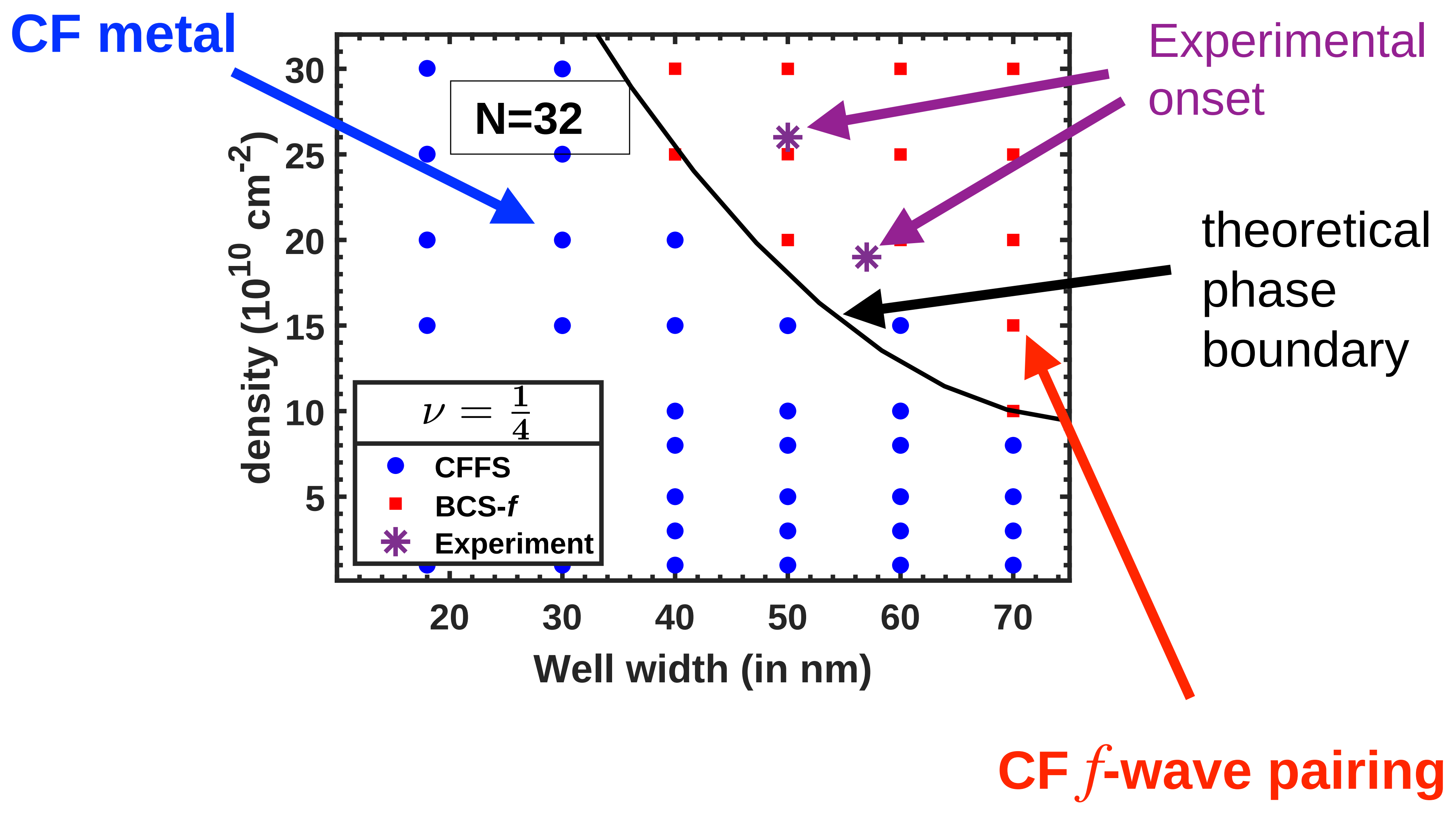
PRL 103, 046805 (2009) PHYSICAL REVIEW LETTERS week ending 24 JULY 2009

Correlated States of Electrons in Wide Quantum Wells at Low Fillings: The Role of Charge Distribution Symmetry

J. Shabani, T. Gokmen, and M. Shayegan
Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA
(Received 24 April 2009; published 22 July 2009)



CF pairing at $\nu = 1/4$ in wide quantum wells

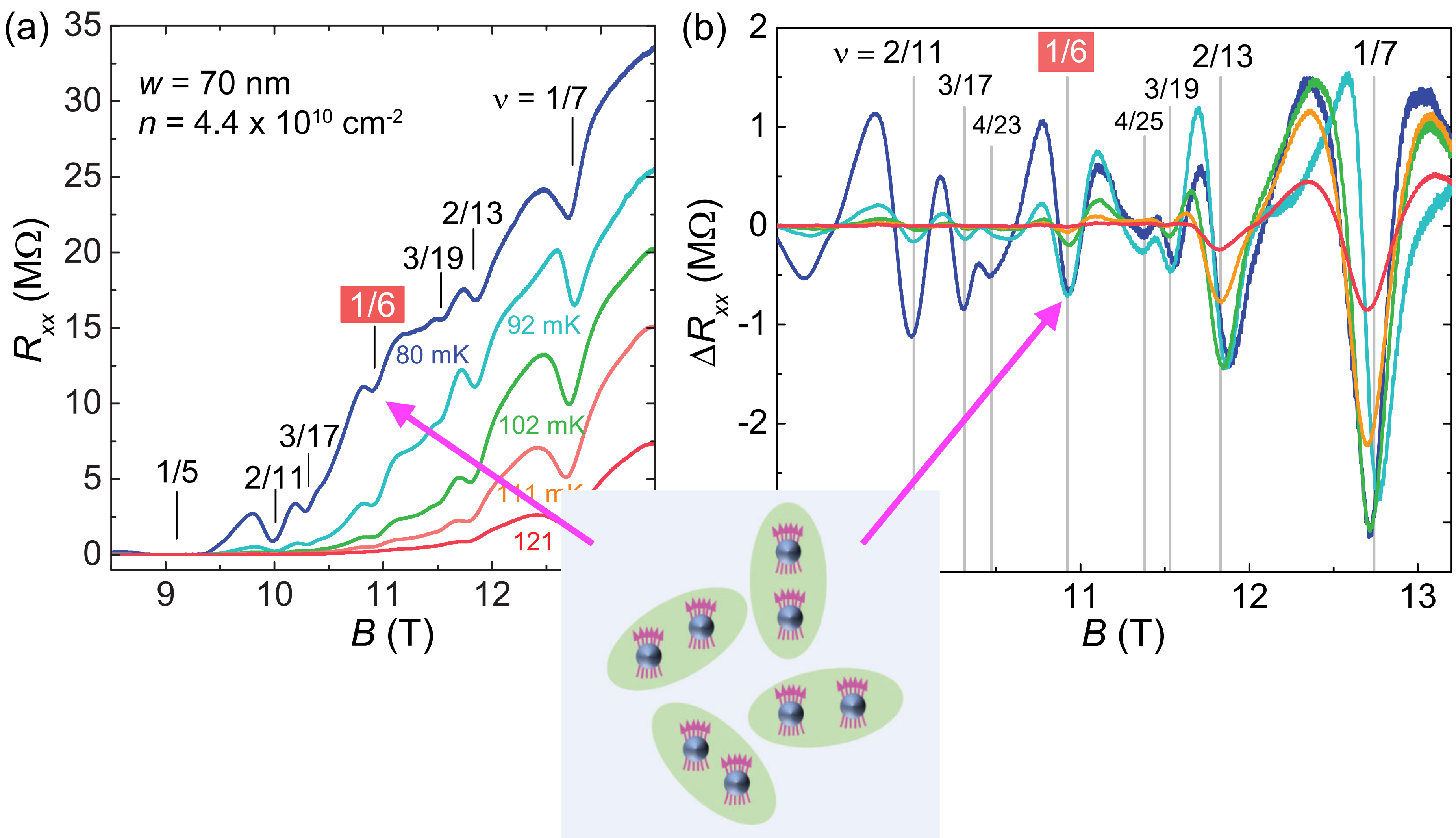


1/6 FQHE in wide quantum wells

PHYSICAL REVIEW LETTERS **134**, 046502 (2025)

Developing Fractional Quantum Hall States at Even-Denominator Fillings 1/6 and 1/8

Chengyu Wang, P. T. Madathil, S. K. Singh, A. Gupta, Y. J. Chung, L. N. Pfeiffer, K. W. Baldwin, and M. Shayegan
Department of Electrical and Computer Engineering, Princeton University, Princeton, New Jersey 08544, USA

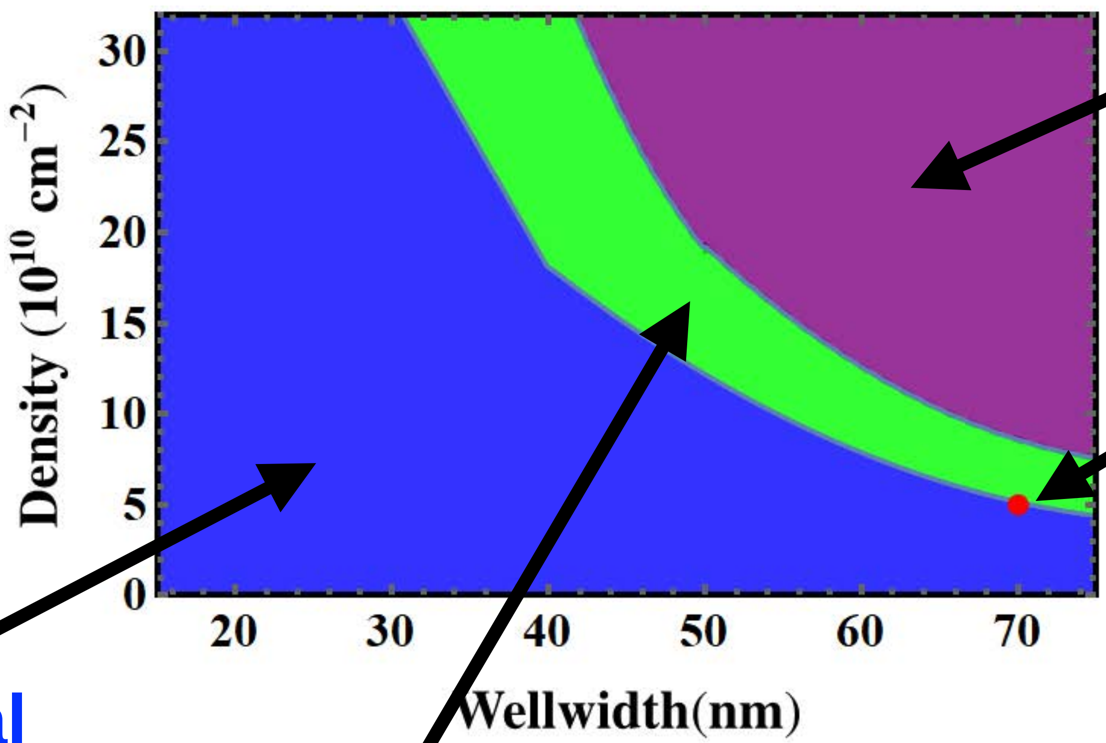


CF pairing at $\nu = 1/6$ in wide quantum wells

PHYSICAL REVIEW B **112**, 035118 (2025)

Interplay of superconducting, metallic, and crystalline states of composite fermions at $\nu = \frac{1}{6}$ in wide quantum wells

Ajit C. Balram,[1,2] Anirban Sharma,[3] and J. K. Jain[3]

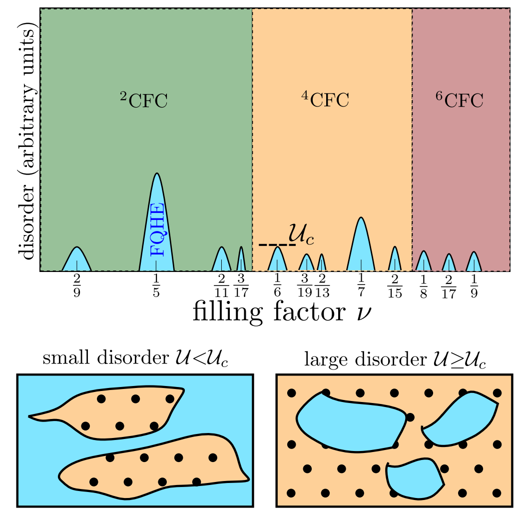


^{4}CF crystal / CF metal / f-wave pairing

Experiment

^{4}CF crystal

^{4}CF crystal / CF Fermi sea



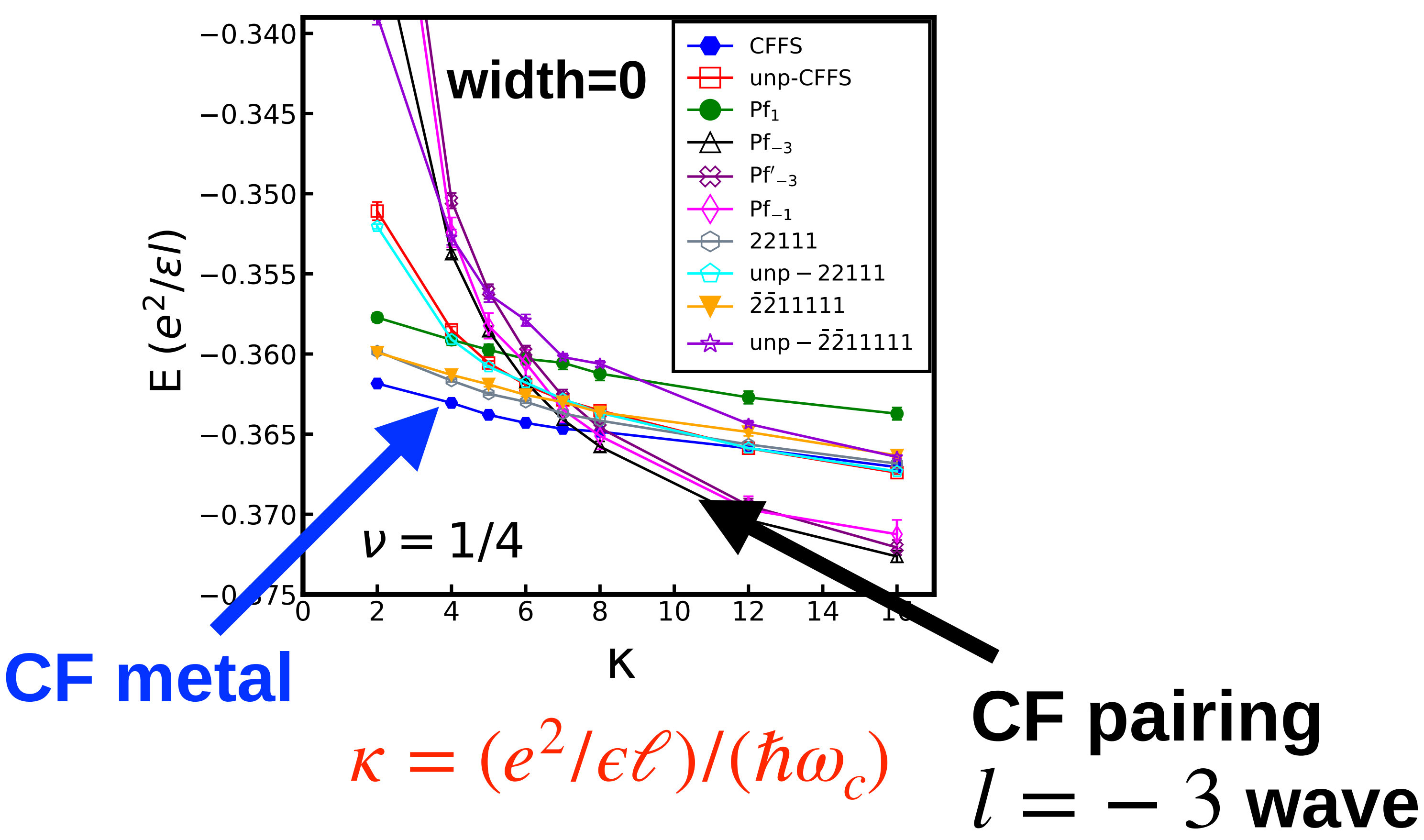
CF pairing at $\nu = 1/4$ induced by LL mixing

PHYSICAL REVIEW LETTERS **130**, 186302 (2023)

Composite Fermion Pairing Induced by Landau Level Mixing

Tongzhou Zhao,[1] Ajit C. Balram,[2,3] and J. K. Jain[4]

(Fixed phase diffusion Monte Carlo)



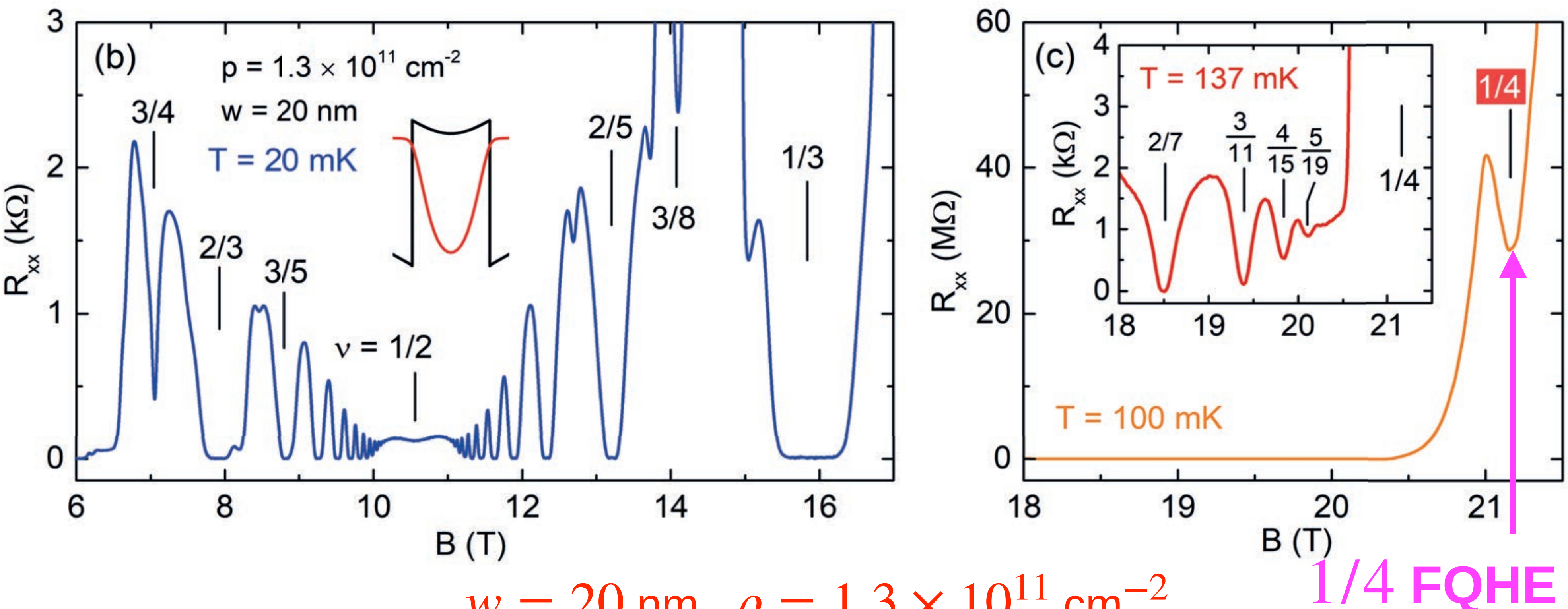
$\kappa = (e^2/\epsilon\ell)/(\hbar\omega_c)$

FQHE at $\nu = 1/4$ in high LL mixing

PHYSICAL REVIEW LETTERS **131**, 266502 (2023)

Fractional Quantum Hall State at Filling Factor $\nu = 1/4$ in Ultra-High-Quality GaAs Two-Dimensional Hole Systems

Chengyu Wang,[1] A. Gupta,[1] S. K. Singh,[1] P. T. Madathil,[1] Y. J. Chung,[1] L. N. Pfeiffer,[1] K. W. Baldwin,[1] R. Winkler,[2] and M. Shayegan[1]



$w = 20$ nm, $\rho = 1.3 \times 10^{11}$ cm^{-2}

- Evidence for FQHE at $\nu = 1/4$ is seen in high quality hole-type samples with $\kappa = 3 - 6$, riding on an insulating background.

Predicted angular momenta of CF Cooper pairs

- $\nu = 5/2$: p-wave pairing

  Moore, Read, Nucl. Phys. (1991);
  Read, Green, PRB (2000);
  Moller, Simon, PRB (2008);
  Sharma, Pu, Jain , PRB (2021)

- $\nu = 1/2$ in wide quantum wells: p-wave pairing

  Sharma, Balram, Jain, PRB (2024)

- $\nu = 1/4$ in wide quantum wells: f-wave pairing

  Faugno, Balram, Barkeshli, Jain, PRL (2019);
  Sharma, Balram, Jain, PRB (2024)

- $\nu = 1/6$ in wide quantum wells: f-wave pairing

  Balram, Sharma, Jain, PRB (2025)

- $\nu = 1/4$ with high Landau level mixing: $l = -3$-wave pairing

  Zhao, Balram, Jain, PRL (2023)

- $\nu = 1/2$ in $N = 3$ graphene Landau level: f-wave pairing

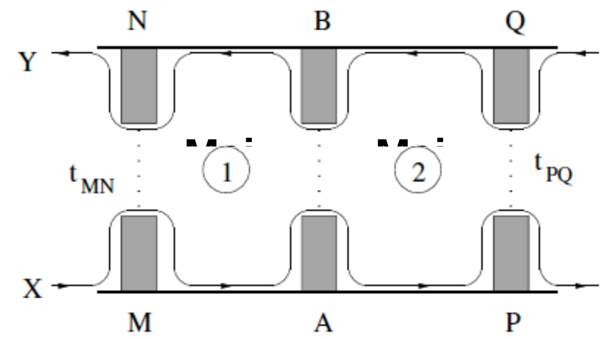
  Sharma, Pu, Balram, Jain, PRL (2023)

The chiral central charge is given by $c = 1 + l/2$, which can be determined from thermal Hall conductance.

From fundamental physics to technology?

Majorana

- Composite-fermion superconductors are predicted to harbor "Majorana particles," which are zero modes trapped inside the Abrikosov vortices. These are "non-Abelian anyons."
- Majoranas may be useful for making topological fault-tolerant qubits.



PHYSICAL REVIEW B VOLUME 61, NUMBER 15 15 APRIL 2000-I

Paired states of fermions in two dimensions with breaking of parity and time-reversal symmetries and the fractional quantum Hall effect

N. Read and Dmitry Green

PRL **94,** 166802 (2005) PHYSICAL REVIEW LETTERS week ending 29 APRIL 2005

Topologically Protected Qubits from a Possible Non-Abelian Fractional Quantum Hall State

Sankar Das Sarma,[1] Michael Freedman,[2] and Chetan Nayak[2,3]

Nuclear Physics B

Volume 360, Issues 2–3, 19 August 1991, Pages 362-396

Nonabelions in the fractional quantum hall effect

Gregory Moore, Nicholas Read

REVIEWS OF MODERN PHYSICS, VOLUME 80, JULY–SEPTEMBER 2

Non-Abelian anyons and topological quantum computation

Nayak et al.

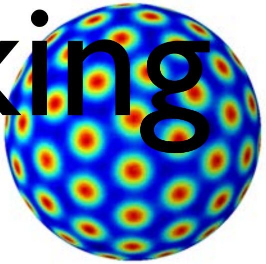
Observation of half-integer thermal Hall conductance

12 JULY 2018 | VOL 559 | NATURE | 205

Mitali Banerjee[1], Moty Heiblum[1]*, Vladimir Umansky[1], Dima E. Feldman[2], Yuval Oreg[1] & Ady Stern[1]

Crystal / CF liquid phase diagram

Crystal induced by LL mixing



PHYSICAL REVIEW LETTERS **121,** 116802 (2018)

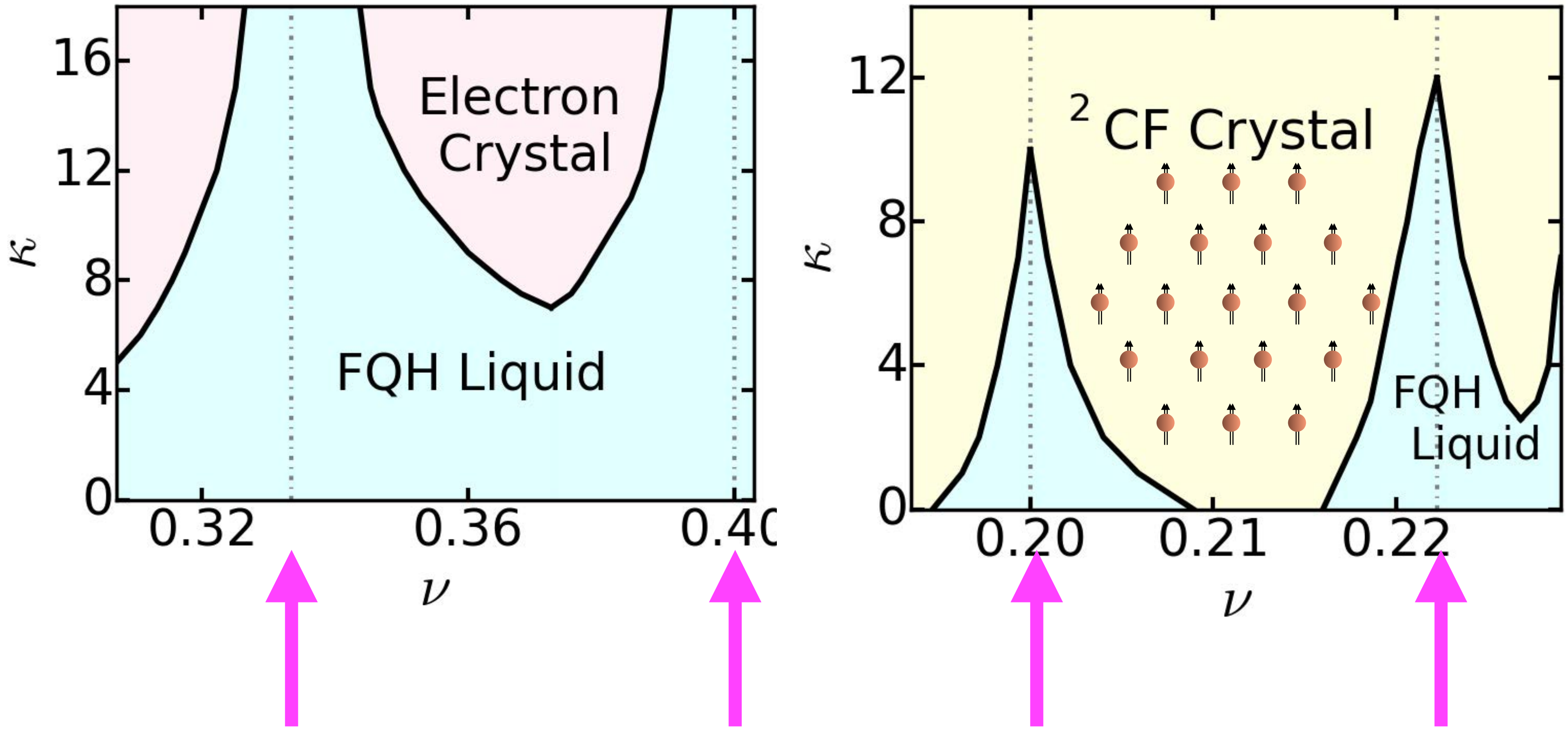
LL mixing parameter

$$\kappa = \frac{e^2/\epsilon\ell}{\hbar\omega_c}$$

Crystallization in the Fractional Quantum Hall Regime Induced by Landau-Level Mixing

Jianyun Zhao, Yuhe Zhang, and J. K. Jain
Department of Physics, 104 Davey Laboratory, The Pennsylvania State University, University Park, Pennsylvania 16802, USA

(Fixed phase diffusion Monte Carlo)



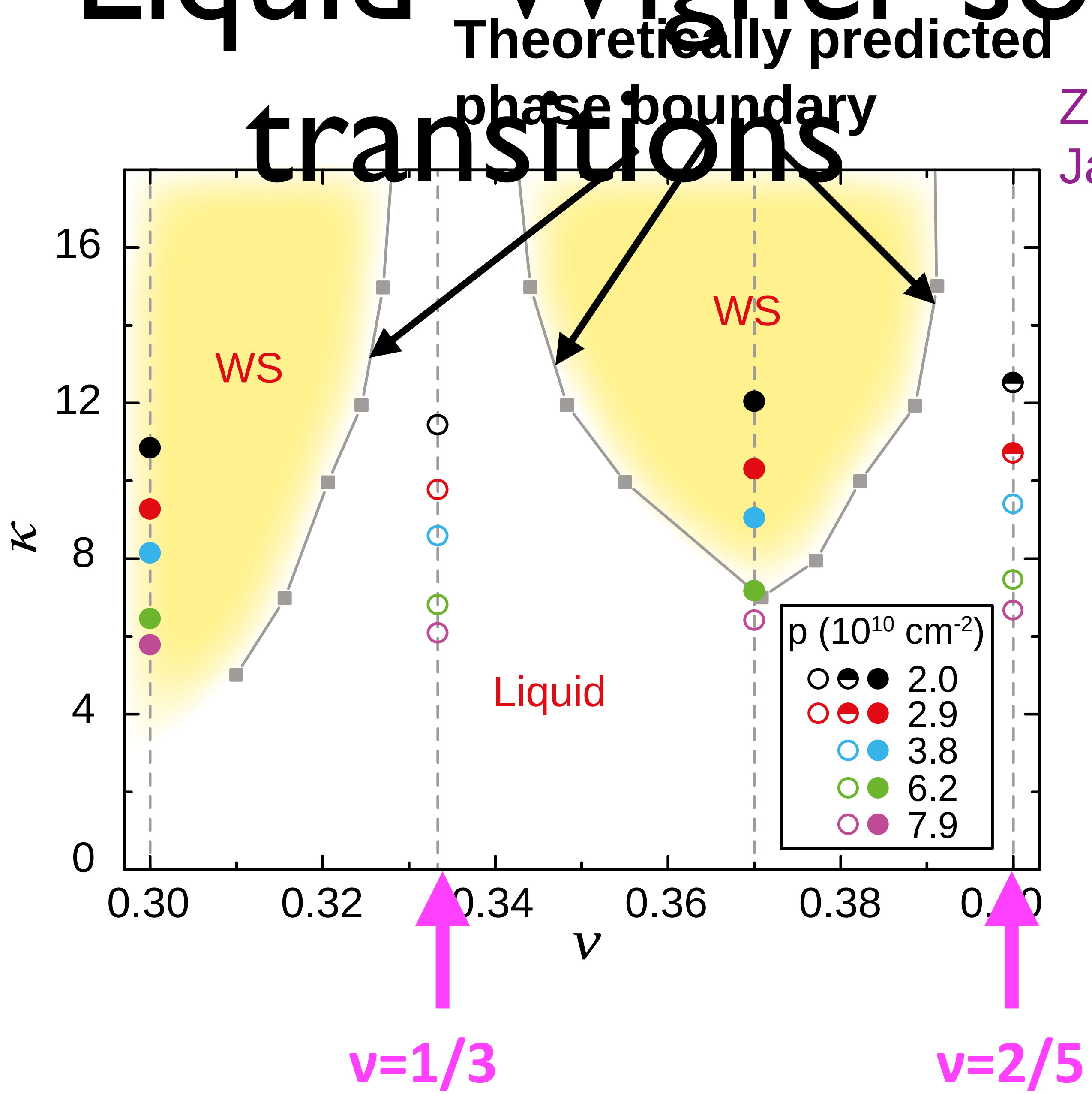
ν=1/3 ν=2/5 ν=1/5 ν=2/9

CF Liquid-Wigner solid transitions

LL mixing parameter

$$\kappa = \frac{e^2/\epsilon\ell}{\hbar\omega_c}$$

Zhao, Zhang, Jain (PRL 2018)



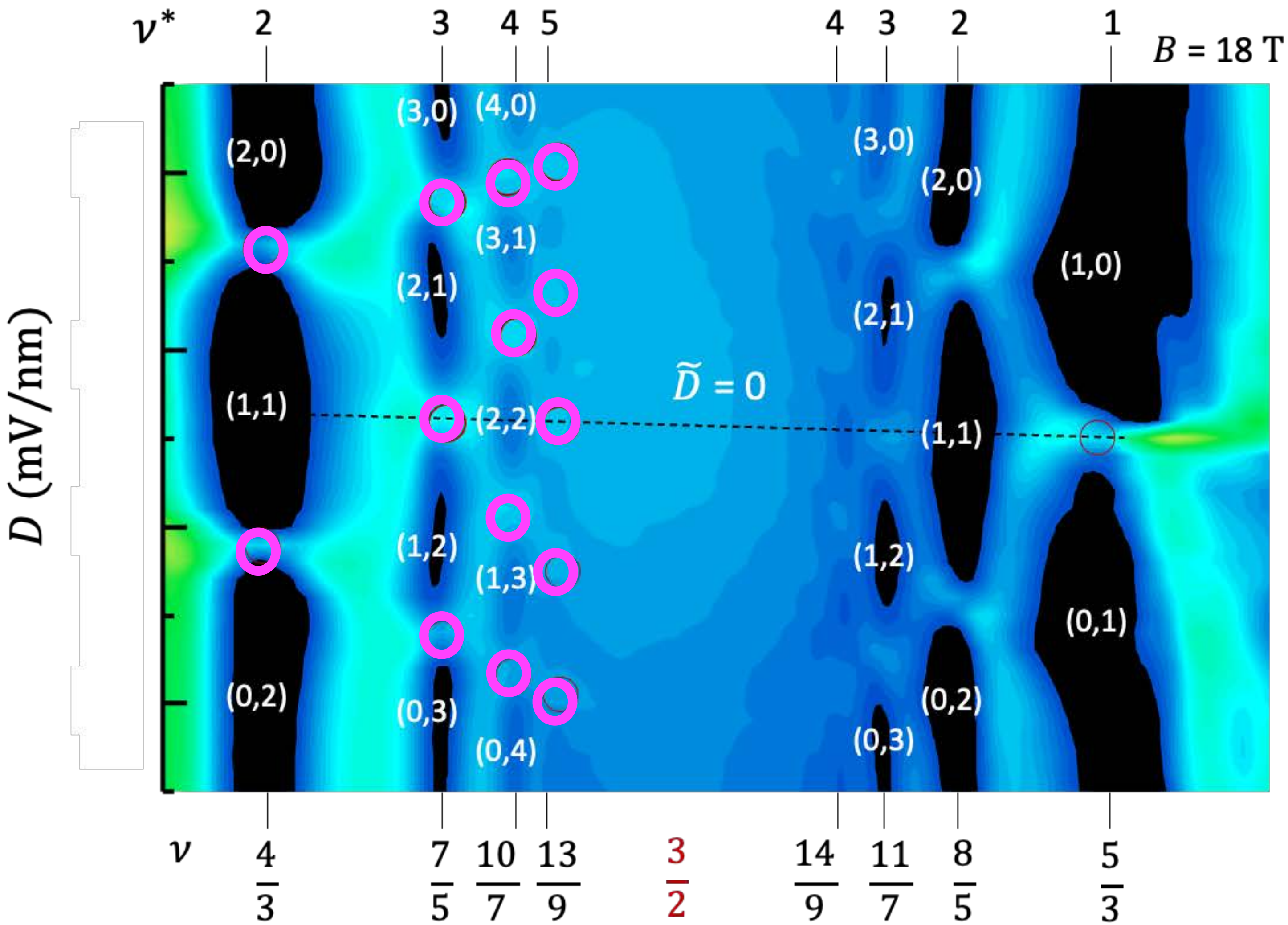
Ma, Shayegan, et al. (PRL 2020)

Two component (spin / valley) FQHE

Qualitative confirmation: valley polarization in BLG

Valley isospin polarizations of the N=0 Jain states in BLG

- Two valley isospin components: |+ 0> and |- 0> behave like spin



- All predicted states seen.
- Excellent account of the phase diagram with a single fitting parameter: CF mass.
- Mass off by a factor of 4 from its zeroth order value.

Du et al, PRL 75, 3926 (1995), Padamanabhan et al, PRB 80, 035423 (2009), Feldman et al, PRL 11, 076802 (2013)

Huang… JZ, PRX 12, 031019 (2022)

Slide from Jun Zhu

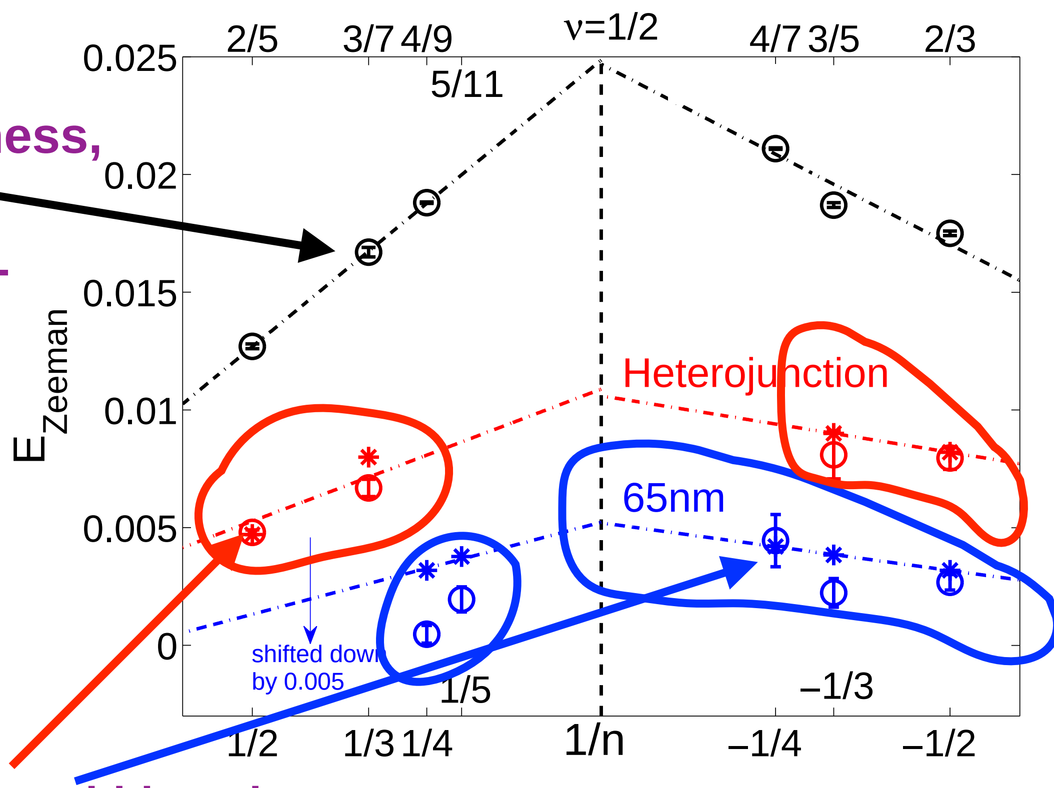
Quantitative confirmation

PRL 117, 116803 (2016) PHYSICAL REVIEW LETTERS week ending 9 SEPTEMBER 2016

Landau-Level Mixing and Particle-Hole Symmetry Breaking for Spin Transitions in the Fractional Quantum Hall Effect

Yuhe Zhang,[1] A. Wójs,[2] and J. K. Jain[1,3]

theory: zero thickness, no LL mixing
Park and Jain, PRL (1998)



L. W. Engel et al. PRB 45, 3418 (1992)
W. Kang et al. PRB 56, R12776 (1997)
Y. Liu et al. PRB 90, 085301 (2014)

Theory including finite width and Landau level mixing in a fixed phase diffusion Monte Carlo study
Zhang, Wojs, Jain, PRL (2016)

The CF theory obtains the ~ 1 % Coulomb energy difference between the competing states to within a few %.