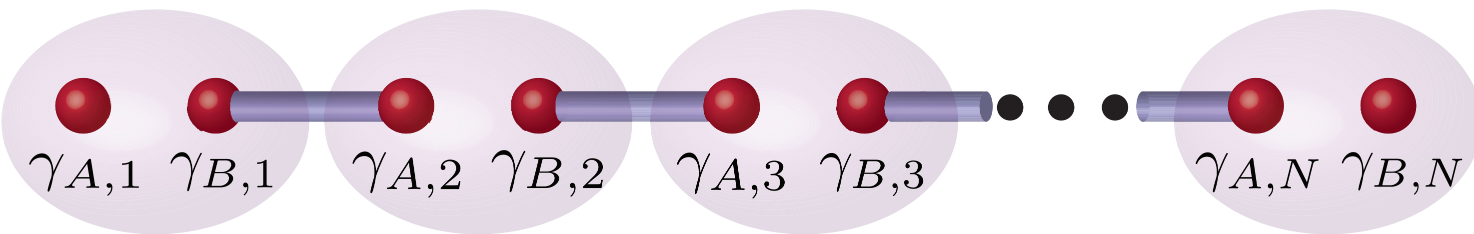
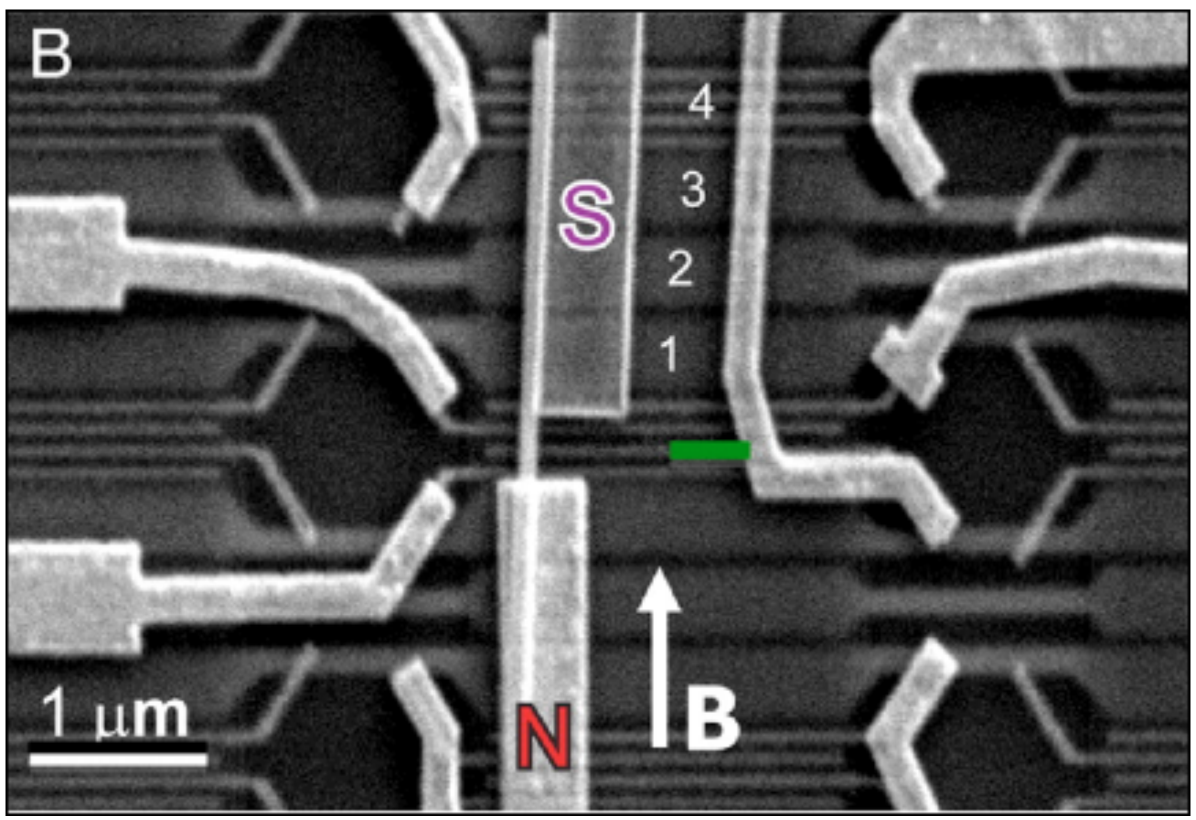
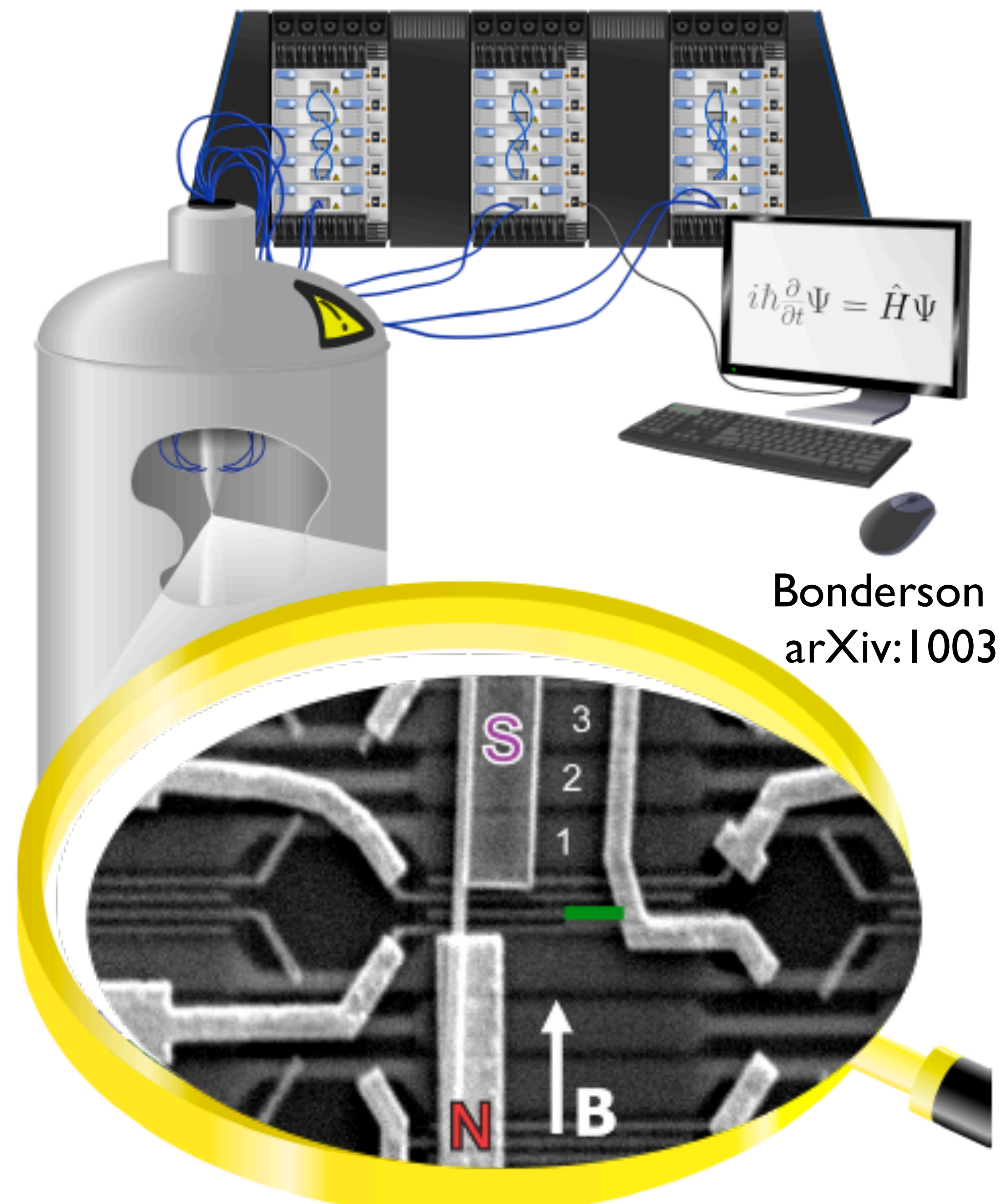
The quest for Majorana II





Mourik et al., Science 2012



Bonderson et al., arXiv:1003.2856

Jason Alicea (Caltech)

Summary so far

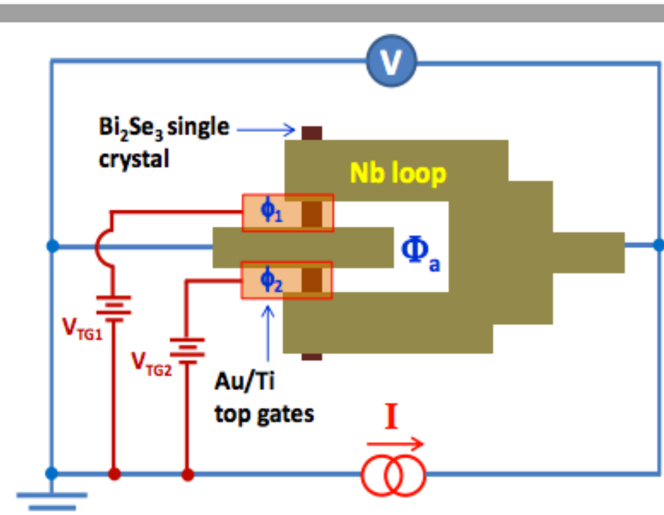
Theoretical toy models to experimental blueprints

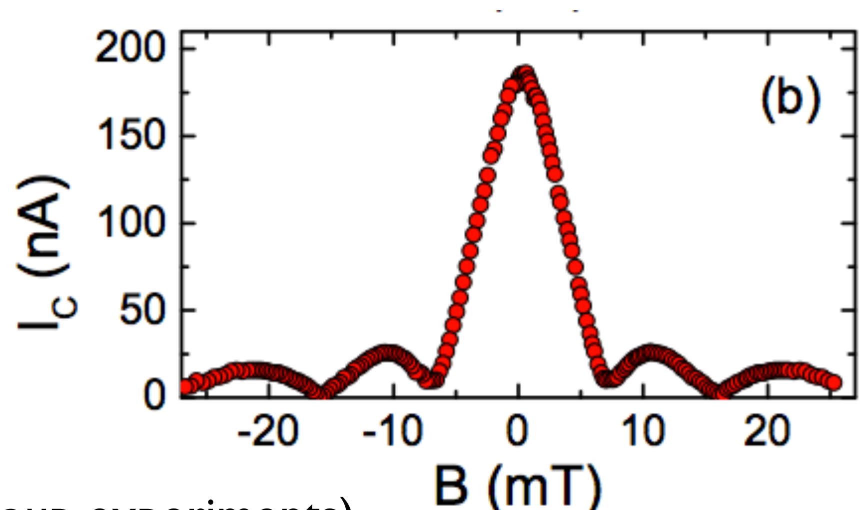
Kitaev chain	➔	Realistic proposals in (i) 2D topological insulator edges, (ii) 1D wires
2D p+ip superconductor	➔	Realistic proposals in (i) 3D topological insulator surfaces, (ii) 2D semiconductor structures

Majorana detection schemes

(i) Fractional Josephson effect
(ii) "Teleportation" experiments

Experiments on 3D topological insulators





(Images from van Harlingen group experiments)

Outline for final lecture

- Majorana detection via transport

- Experimental progress
  - 1D wires
  - 2D topological insulators

- Outlook: where are we going?

Outline for final lecture

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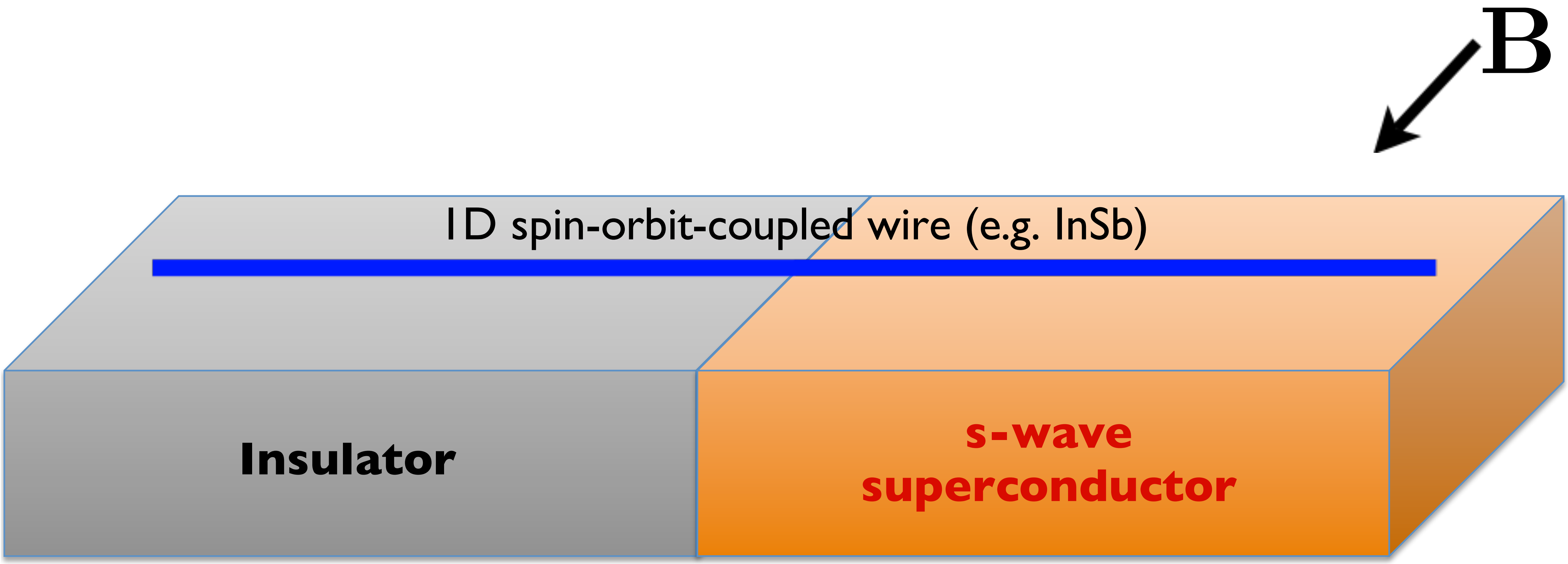
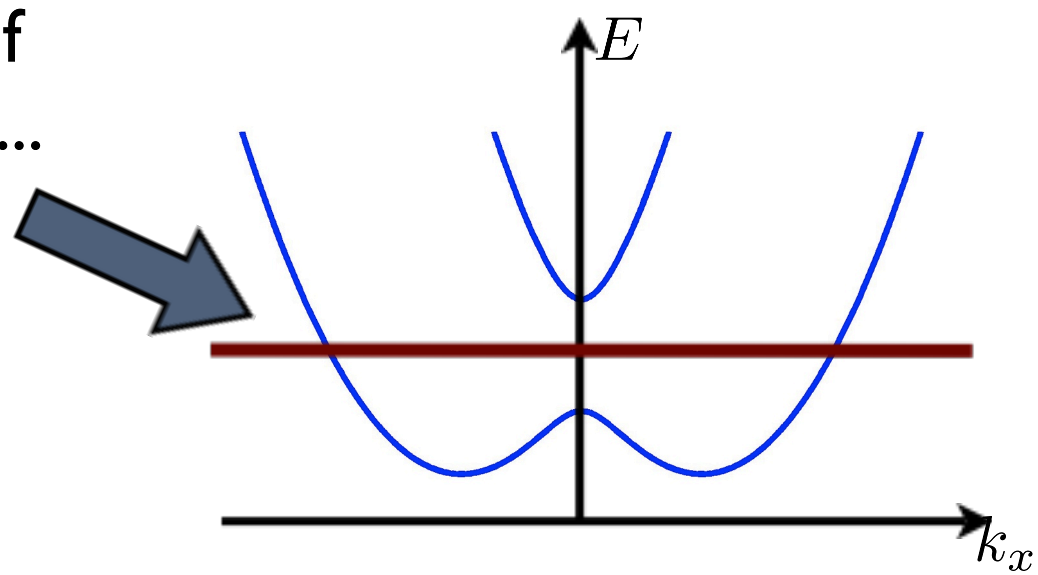
Detection via transport



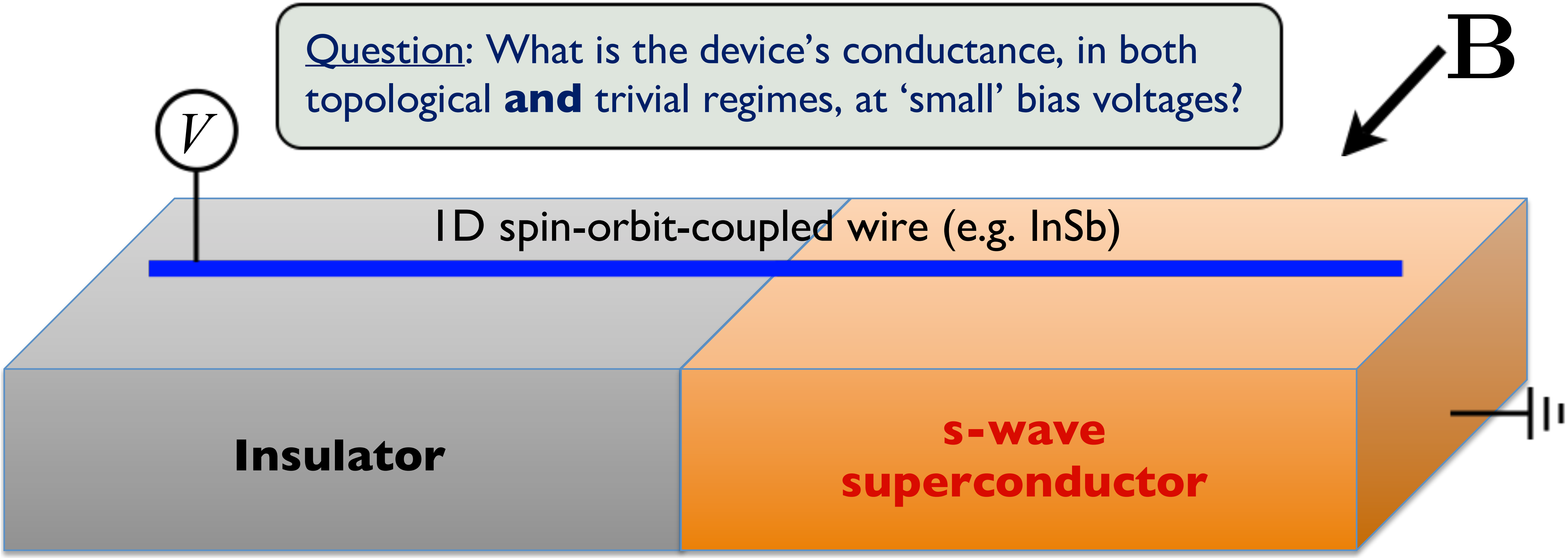
Right half is **topological** if chemical potential sits here...



...otherwise right half is **trivial**, and does not support Majoranas

Detection via transport

Question: What is the device's conductance, in both topological **and** trivial regimes, at 'small' bias voltages?



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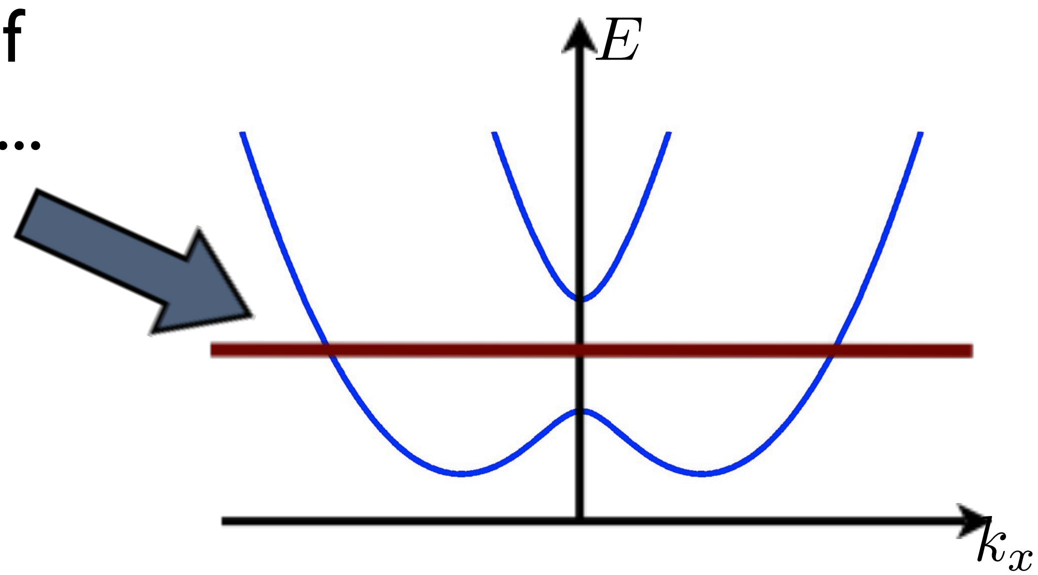
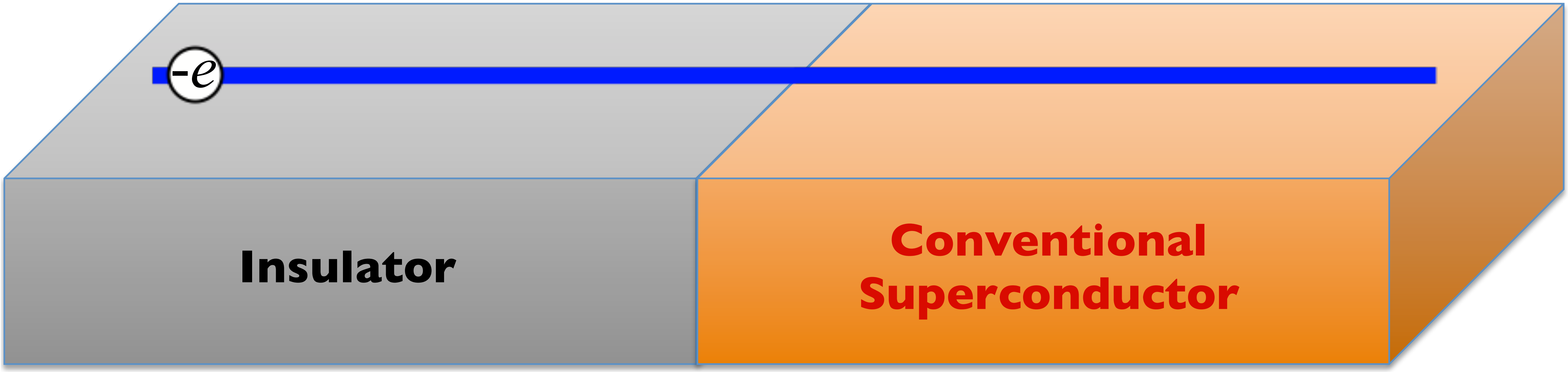


...otherwise right half is **trivial**, and does not support Majoranas

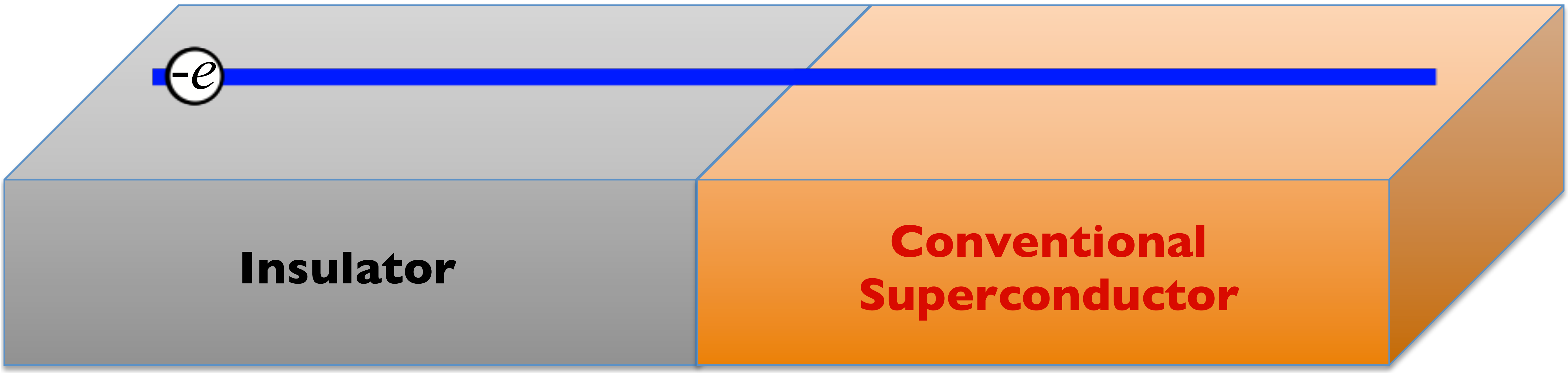
Detection via transport

Normal reflection

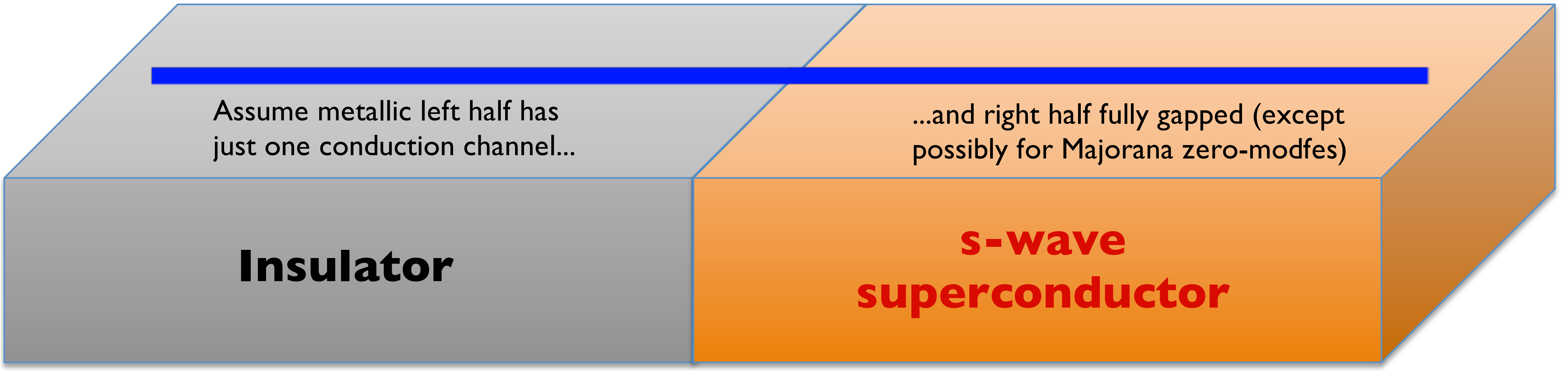


Detection via transport

Andreev reflection



Transport analysis



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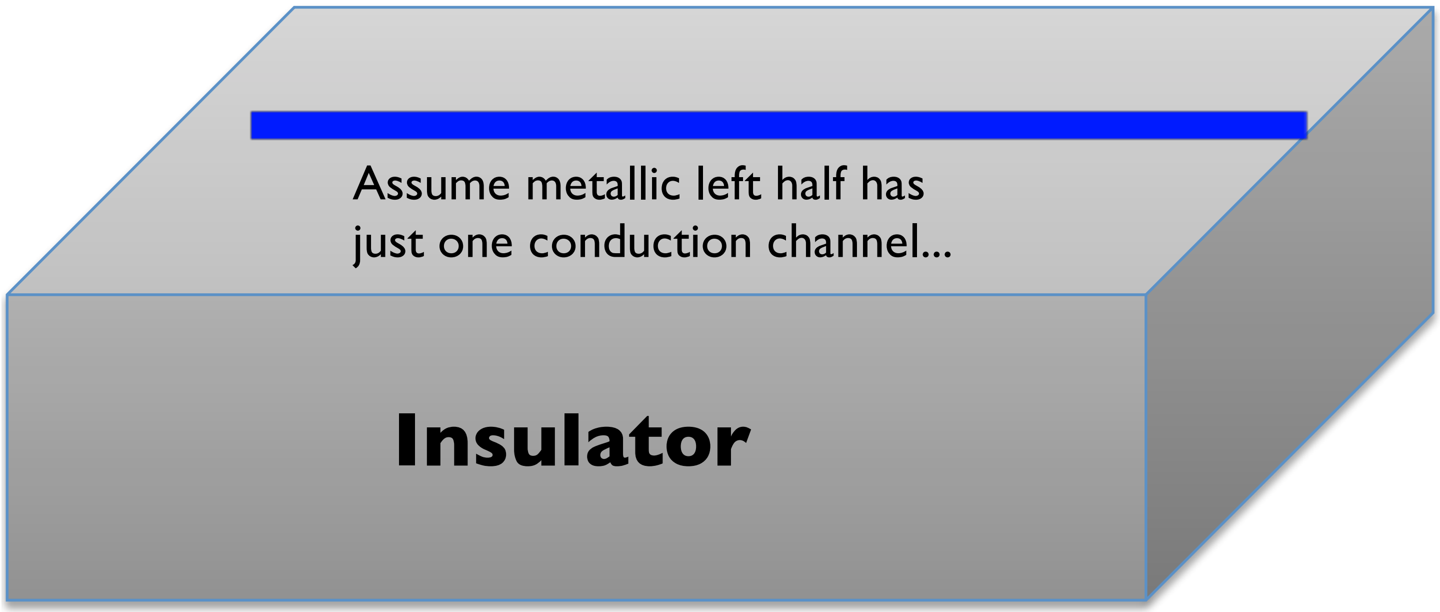
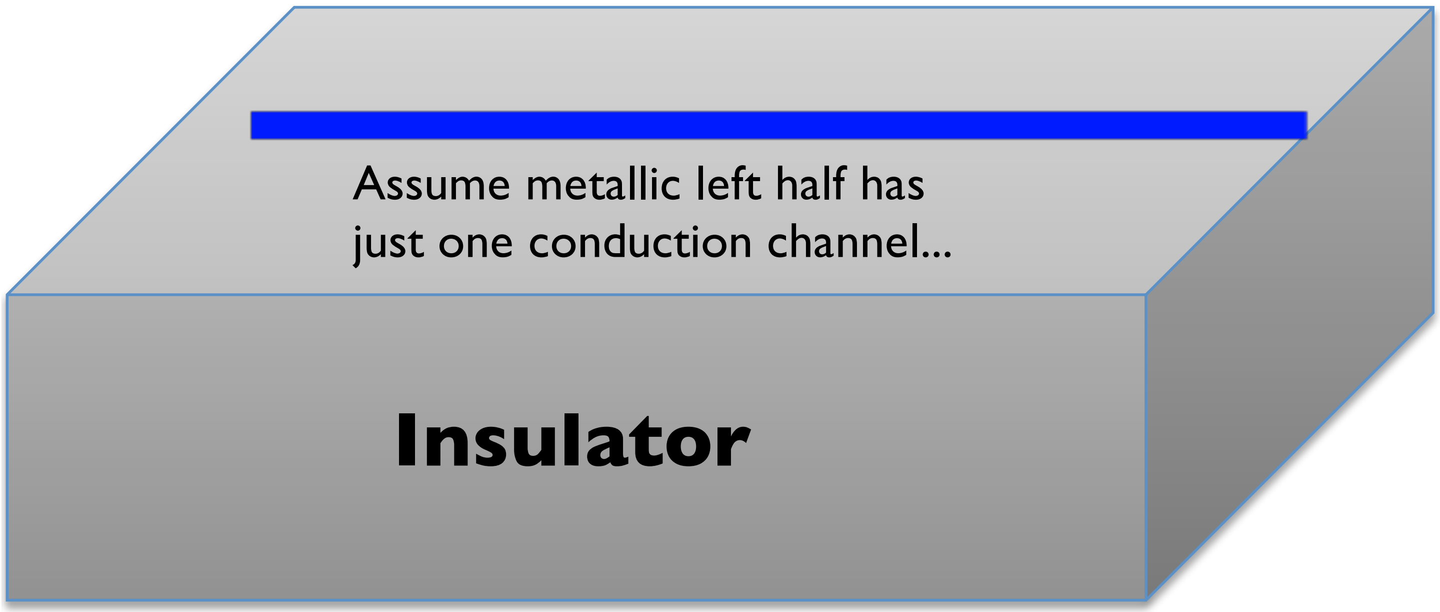
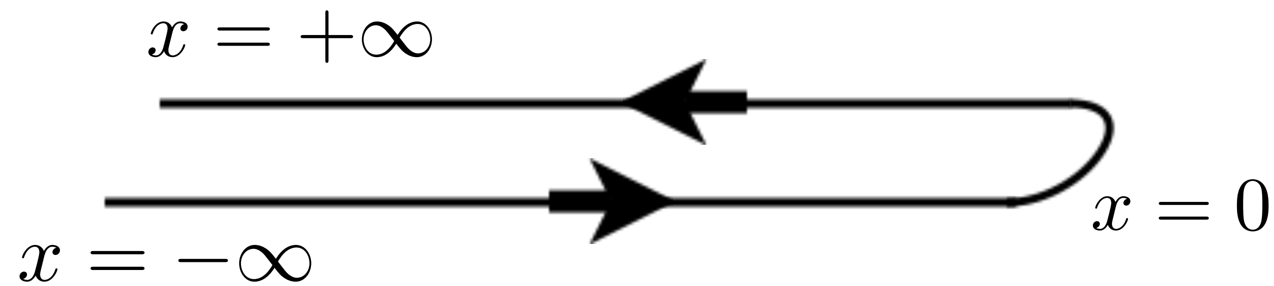


$$H_{\text{metal}} = \int_{-\infty}^{0} dx \left(-i v_F \psi_R^\dagger \partial_x \psi_R + i v_F \psi_L^\dagger \partial_x \psi_L \right)$$

Transport analysis

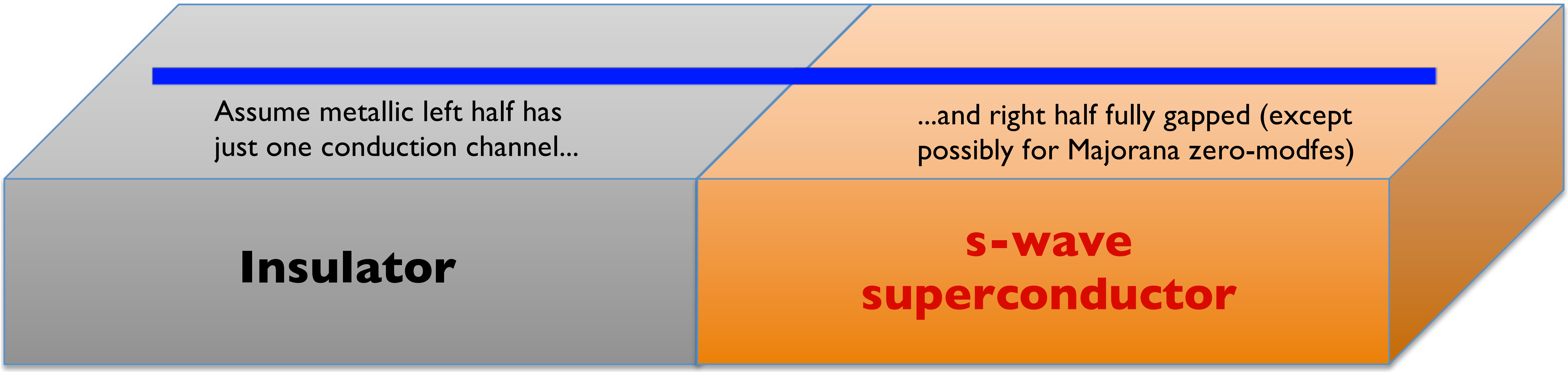




$$H_{\text{metal}} = \int_{-\infty}^{0} dx \left(-iv_F \psi_R^\dagger \partial_x \psi_R + iv_F \psi_L^\dagger \partial_x \psi_L \right)$$

$$\begin{cases} \psi(x > 0) \equiv \psi_L(-x) \\ \psi(x < 0) \equiv \psi_R(x) \end{cases} \Rightarrow \boxed{H_{\text{metal}} = \int_{-\infty}^{\infty} dx \left(-iv_F \psi^\dagger \partial_x \psi \right)}$$

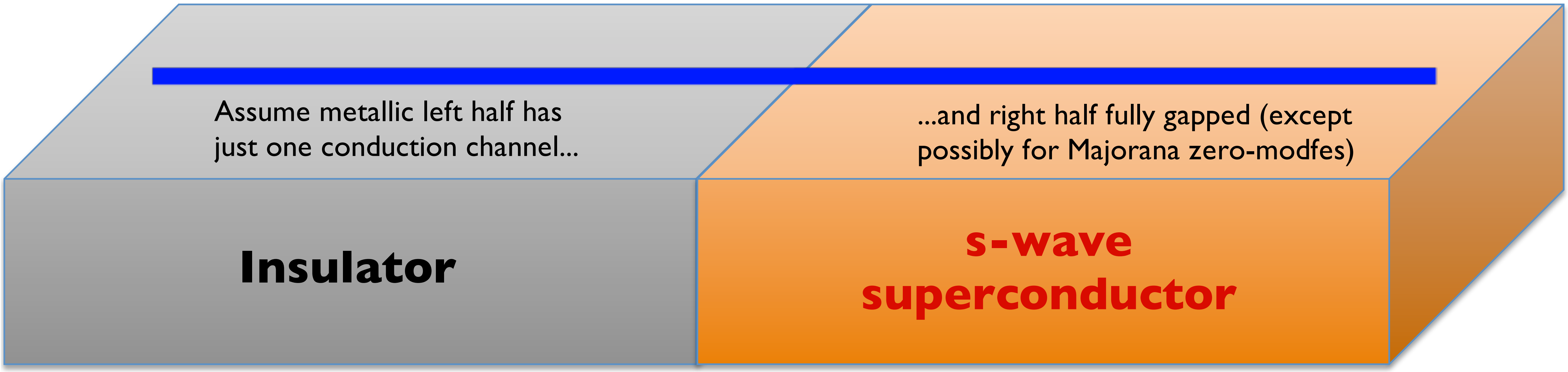
Transport analysis



$$H = H_{\text{metal}} + H_{\text{junction}}$$

Terms generated by superconducting half; depend on whether topological or trivial

Transport analysis



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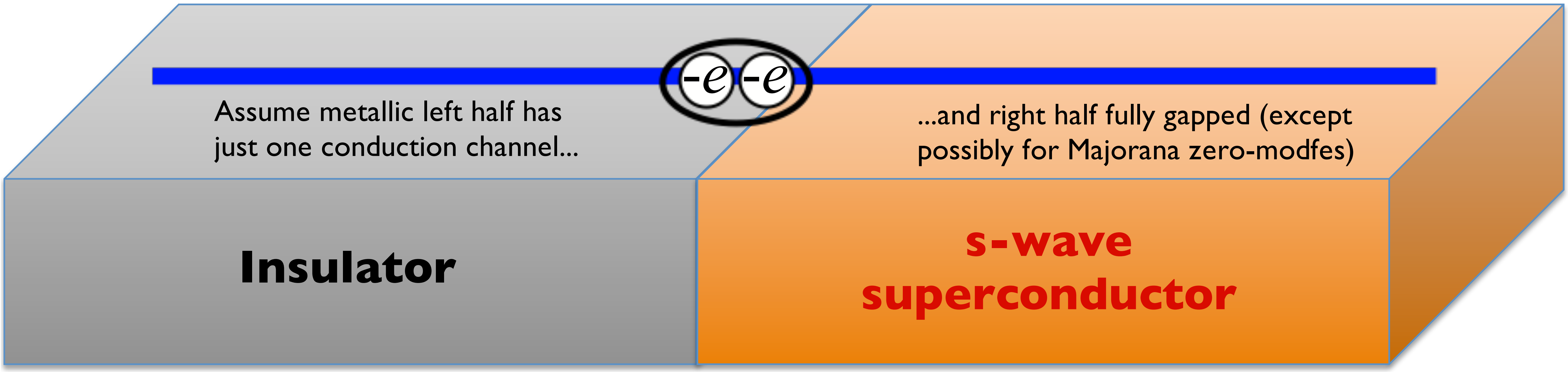
Terms generated by superconducting half; depend on whether topological or trivial

Trivial case:

$$H_{\text{junction}} = \int_{-\infty}^{\infty} dx \left[\Delta(\psi \partial_x \psi + H.c.) \right] \delta(x)$$

Transport analysis



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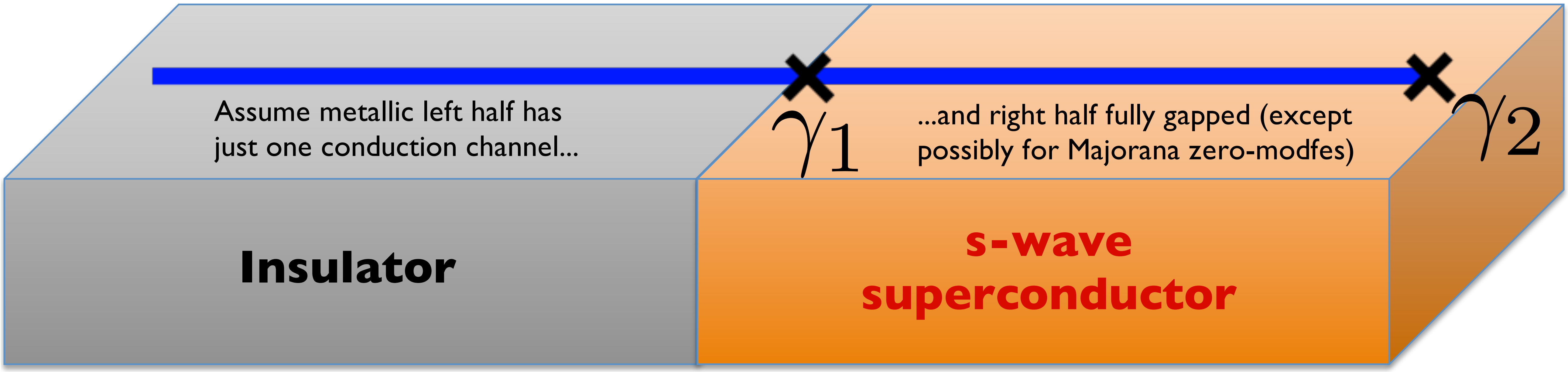

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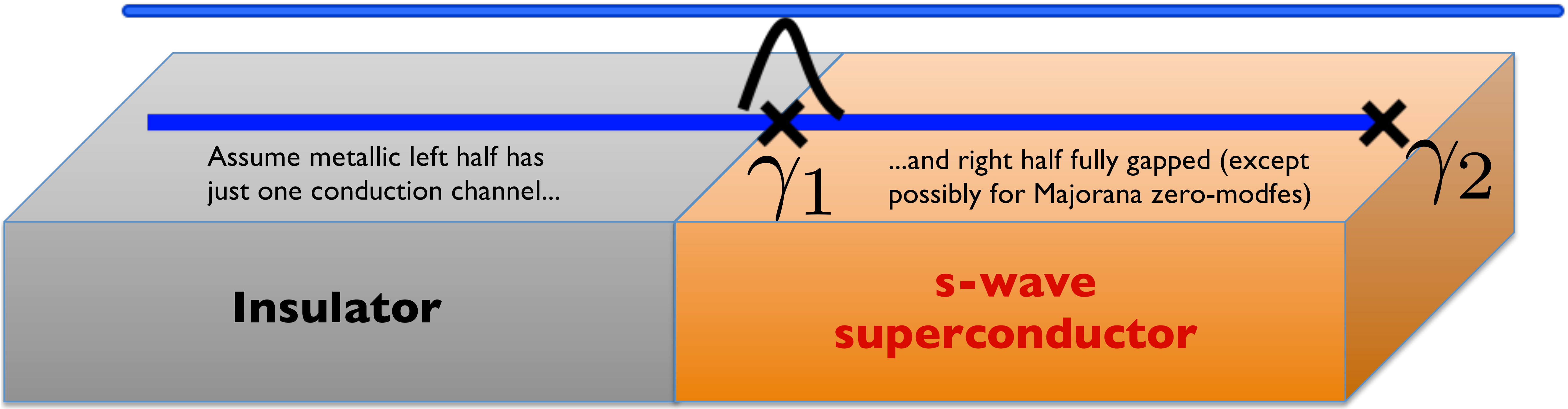
Trivial case: $$H_{\text{junction}} = \int_{-\infty}^{\infty} dx \left[\Delta(\psi \partial_x \psi + H.c.)\right] \delta(x)$$

Topological case: $$H_{\text{junction}} = t \int_{-\infty}^{\infty} dx \gamma_1 (\psi^\dagger - \psi) \delta(x)$$

Transport analysis



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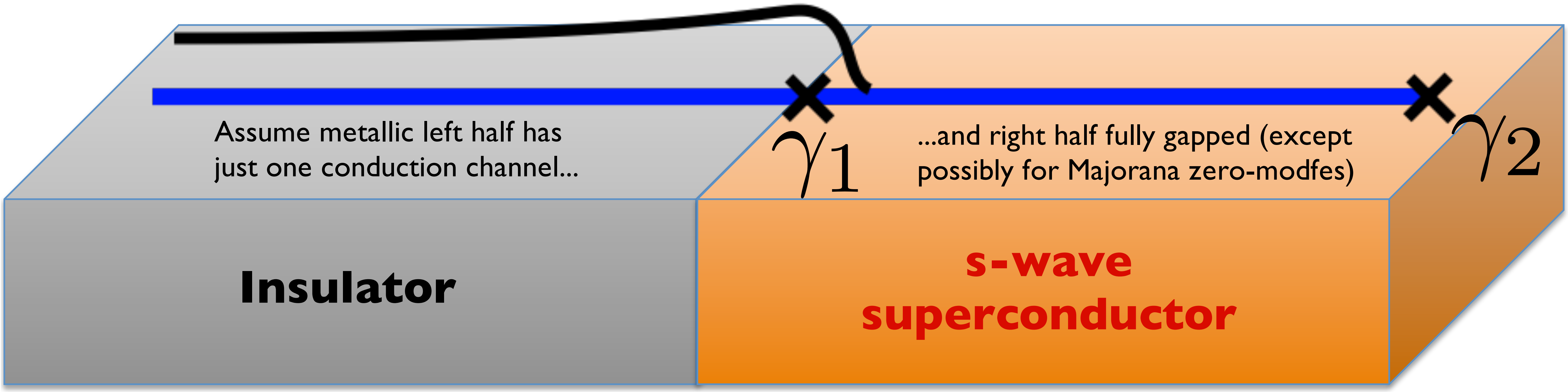

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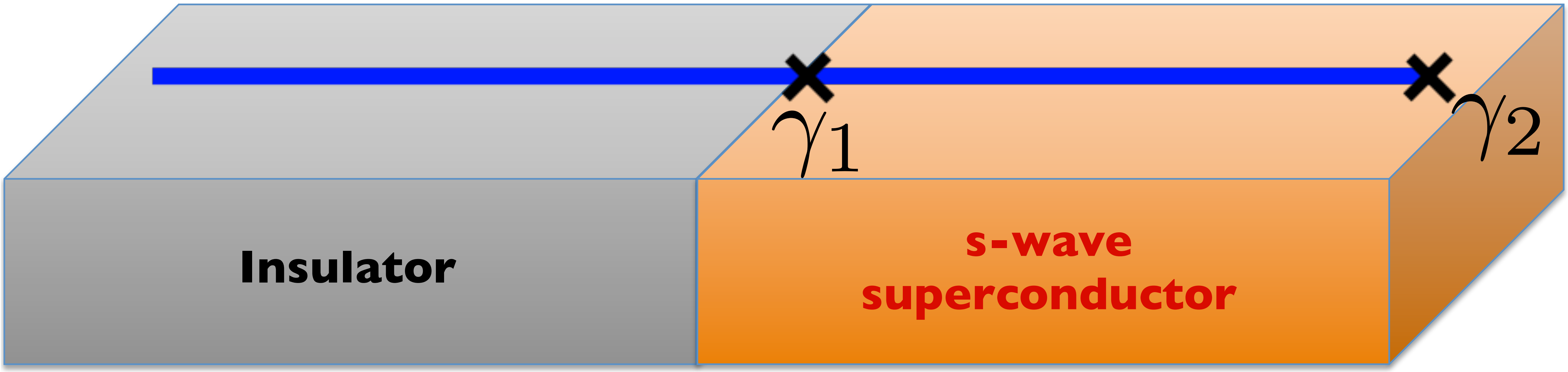
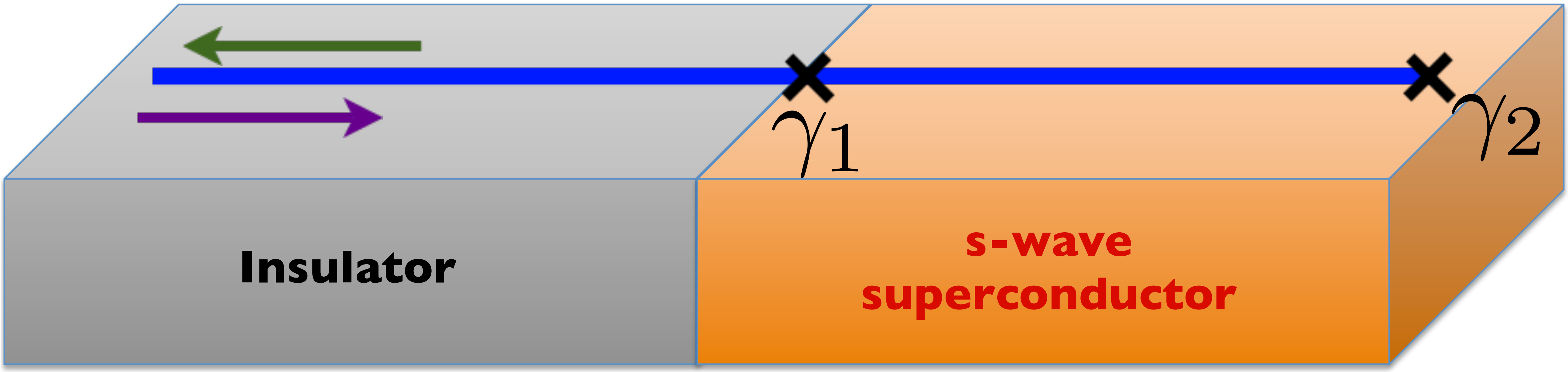


$$H = H_{\text{metal}} + H_{\text{junction}}$$

$$\Gamma_E = \int_{-\infty}^{\infty} dx e^{-i\frac{Ex}{v_F}} \left[P_E(x)\psi(x) + H_E(x)\psi^\dagger(x) \right]$$

Diagonalizes Hamiltonian (in either topological or trivial case)

Transport analysis



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Diagonalizes Hamiltonian (in either topological or trivial case)

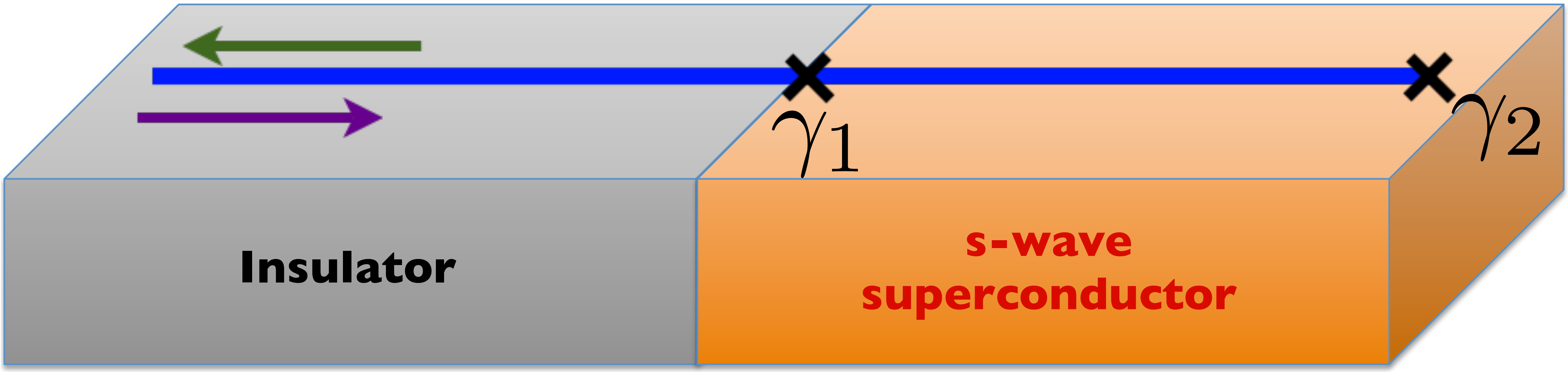
$$\begin{bmatrix} P_E(\infty) \\ H_E(\infty) \end{bmatrix} = \begin{bmatrix} S_{PP}(E) & S_{PH}(E) \\ S_{HP}(E) & S_{HH}(E) \end{bmatrix} \begin{bmatrix} P_E(-\infty) \\ H_E(-\infty) \end{bmatrix}$$

Outgoing amplitudes

Scattering matrix

Incoming amplitudes

Transport analysis



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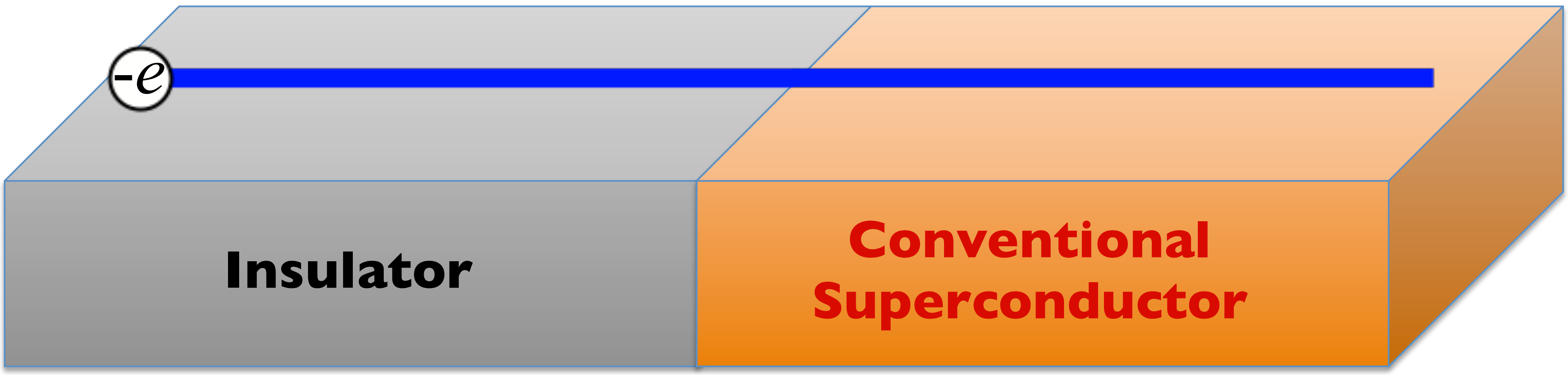
Outgoing amplitudes — Scattering matrix — Incoming amplitudes

$$G(V) = \frac{2e^2}{h}|S_{PH}(eV)|^2$$

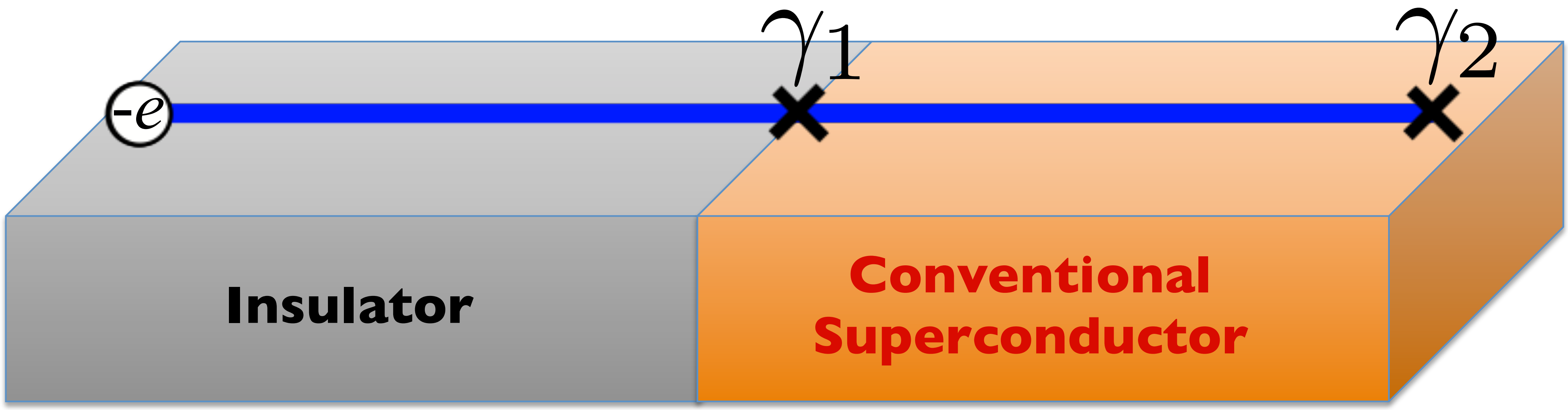
Universal in limit $V \to 0$!!

Detection via transport

No Majoranas → Perfect normal reflection → $G = 0$



With Majoranas → Perfect Andreev reflection → $G = 2e^2/h$



Sengupta et al. (2001); Bolech, Demler (2007); Law, Lee, Ng (2009); Fidkowski, JA, Lindner, Lutchyn, Fisher (2012)

Outline for final lecture

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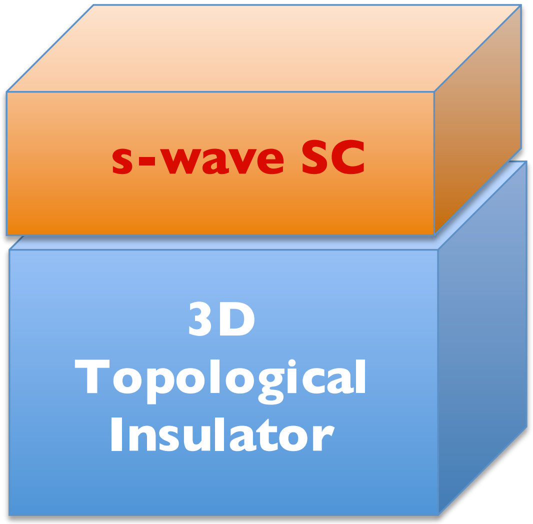

Signatures of Majorana Fermions in Hybrid Superconductor-Topological Insulator Devices

J. R. Williams,[1] A. J. Bestwick,[1] P. Gallagher,[1] Seung Sae Hong,[2] Y. Cui,[3,4] Andrew S. Bleich,[5] J. G. Analytis,[2,4] I. R. Fisher,[2,4] and D. Goldhaber-Gordon[1]

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arXiv:1312.3713 [pdf]

Topological Superconductor Bi2Te3/NbSe2 heterostructures

Jin-Peng Xu, Canhua Liu, Mei-Xiao Wang, Jianfeng Ge, Zhi-Long Liu, Xiaojun Yang, Yan Chen, Ying Liu, Zhu-An Xu, Chun-Lei Gao, Dong Qian, Fu-Chun Zhang, Qi-Kun Xue, Jin-Feng Jia

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arXiv:1309.6040 [pdf]

Two-dimensional superconductivity at the interface of a Bi2Te3/FeTe heterostructure

Qing Lin He, Hongchao Liu, Mingquan He, Ying Hoi Lai, Hongtao He, Gan Wang, Kam Tuen Law, Rolf Lortz, Jiannong Wang, Iam Keong Sou

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arXiv:1307.7764 [pdf]

Evidence for an anomalous current-phase relation of a dc SQUID with tunable topological junctions

Cihan Kurter, Aaron D. K. Finck, Yew San Hor, Dale J. Van Harlingen

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arXiv:1309.0163 [pdf, other]

Abstract

Signature of a topological phase transition in the Josephson supercurrent through a topological insulator

Vladimir Orlyanchik, Martin P. Stehno, Christopher D. Nugroho, Pouyan Ghaemi, Matthew Brahlek, Nikesh Koirala, Seongshik Oh, Dale J. Van Harlingen

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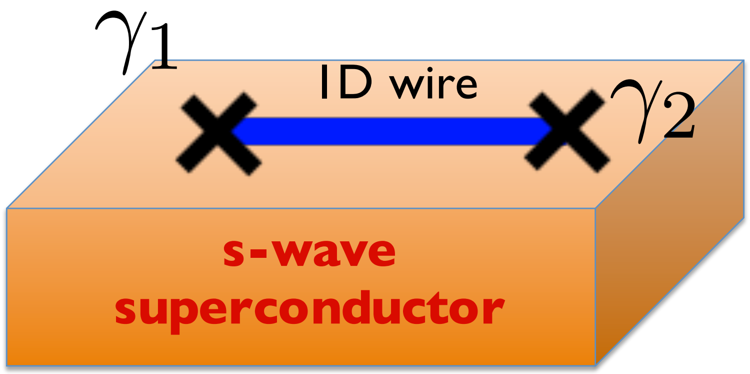
PHYSICAL REVIEW X **3**, 021007 (2013)

Josephson Supercurrent through the Topological Surface States of Strained Bulk HgTe

Jeroen B. Oostinga,[1] Luis Maier,[1] Peter Schüffelgen,[1] Daniel Knott,[1] Christopher Ames,[1] Christoph Brüne,[1] Grigory Tkachov,[2] Hartmut Buhmann,[1] and Laurens W. Molenkamp[1]

...and many others!



Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1]†

Evidence of Majorana fermions in an Al – InAs nanowire topological superconductor

Anindya Das*, Yuval Ronen*, Yonatan Most, Yuval Oreg, Moty Heiblum#, and Hadas Shtrikman

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Observation of Majorana Fermions in a Nb-InSb Nanowire-Nb Hybrid Quantum Device

M. T. Deng,[1] C. L. Yu,[1] G. Y. Huang,[1] M. Larsson,[1] P. Caroff,[2] and H. Q. Xu[1,3,*]

Observation of the fractional ac Josephson effect: the signature of Majorana particles

Leonid P. Rokhinson,[1,2,*] Xinyu Liu,[3] and Jacek K. Furdyna[3]

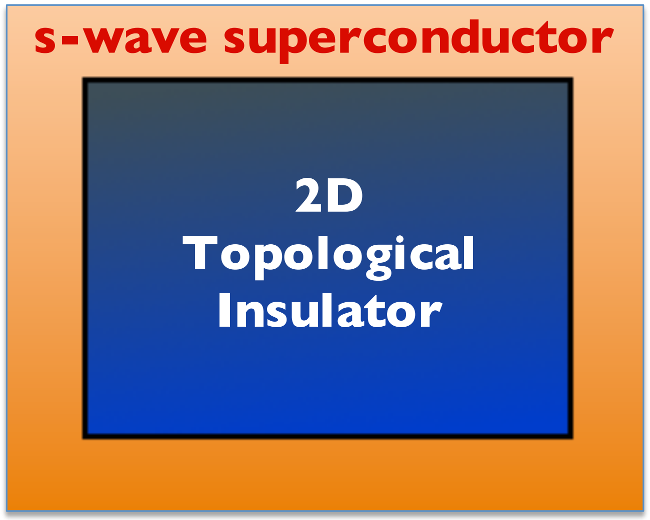
Anomalous Modulation of a Zero-Bias Peak in a Hybrid Nanowire-Superconductor Device

A. D. K. Finck, D. J. Van Harlingen, P. K. Mohseni, K. Jung, and X. Li

Phys. Rev. Lett. **110**, 126406 (2013)

Superconductor-Nanowire Devices from Tunneling to the Multichannel Regime: Zero-Bias Oscillations and Magnetoconductance Crossover

H. O. H. Churchill,[1,2] V. Fatemi,[2] K. Grove-Rasmussen,[3] M. T. Deng,[4] P. Caroff,[4] H. Q. Xu,[4,5] and C. M. Marcus



PRL 109, 186603 (2012) PHYSICAL REVIEW LETTERS week ending 2 NOVEMBER 2012

Andreev Reflection of Helical Edge Modes in InAs/GaSb Quantum Spin Hall Insulator

Ivan Knez* and Rui-Rui Du†
Department of Physics and Astronomy, Rice University, Houston, Texas 77251-1892, USA
Gerard Sullivan
Teledyne Scientific and Imaging, Thousand Oaks, California 91630, USA

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arXiv:1312.2559 [pdf, other]

Induced Superconductivity in the Quantum Spin Hall Edge

Sean Hart, Hechen Ren, Timo Wagner, Philipp Leubner, Mathias Mühlbauer, Christoph Brüne, Hartmut Buhmann, Laurens W. Molenkamp, Amir Yacoby

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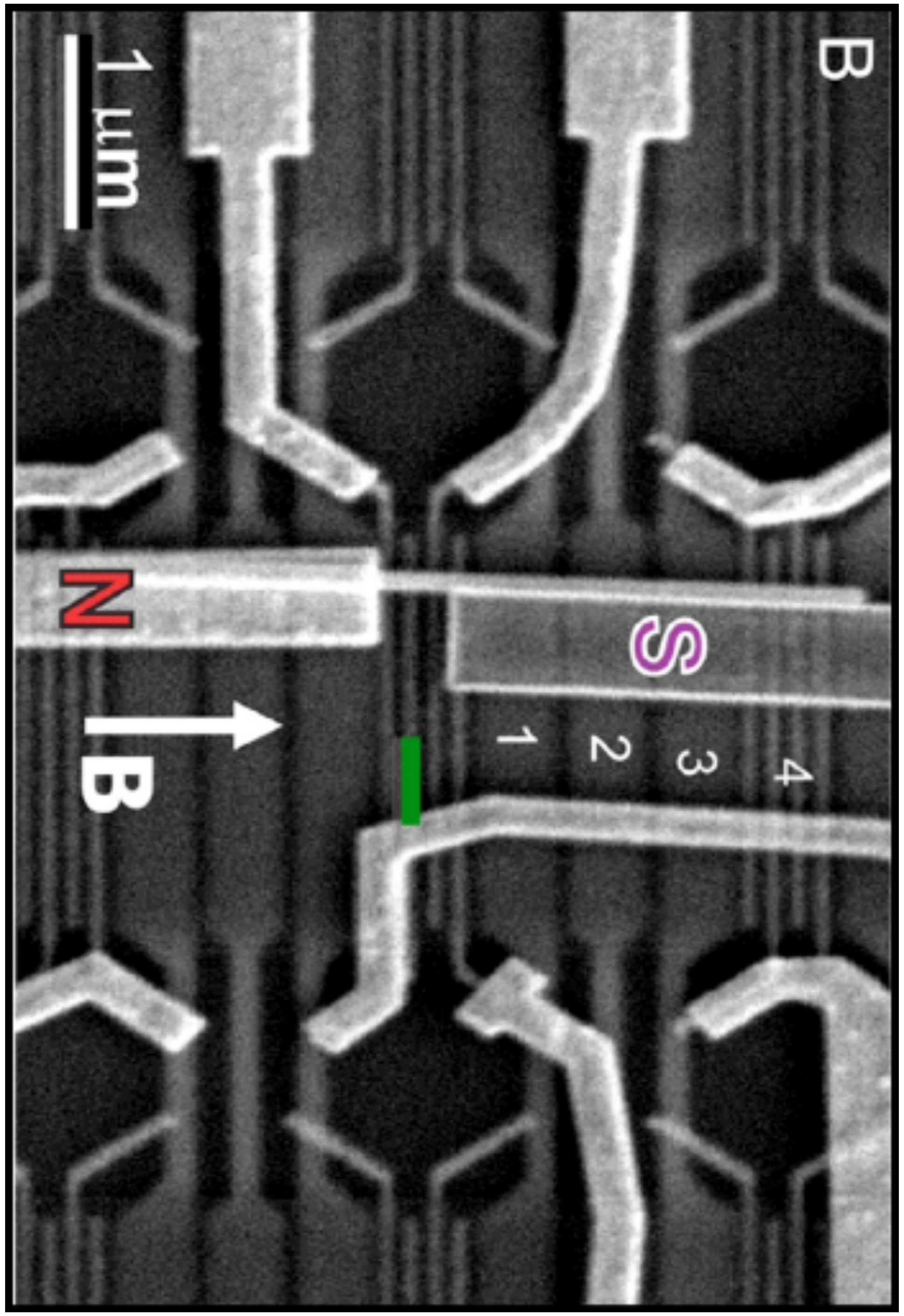
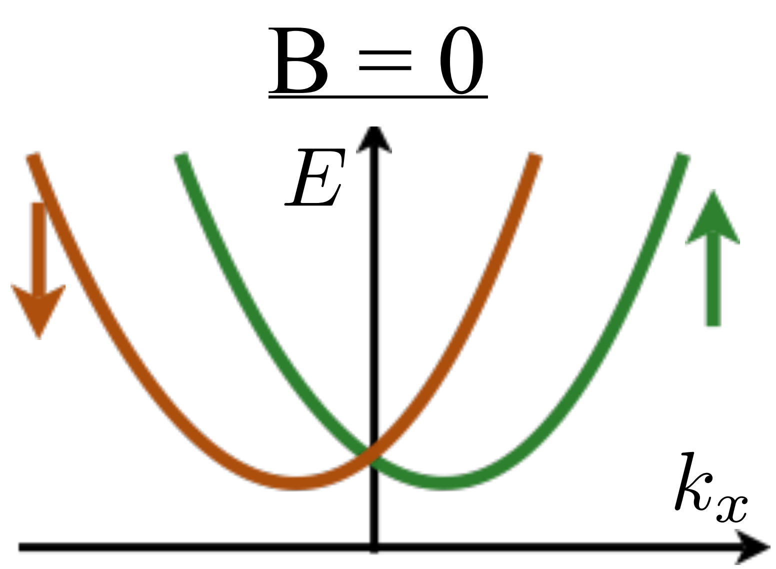
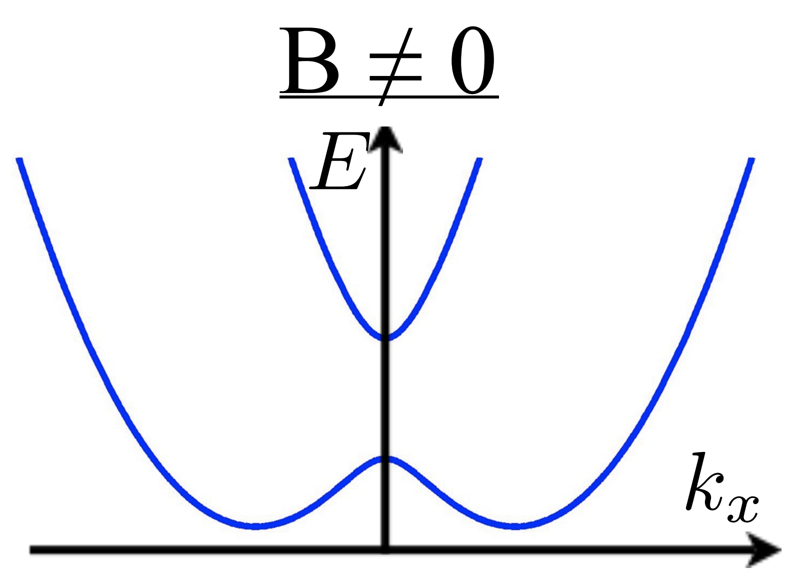
Despite fewer experiments to date, there is reason to be excited about the near-term prospects of this route to Majorana.

The Kouwenhoven experiment



Mourik et al., Science 2012





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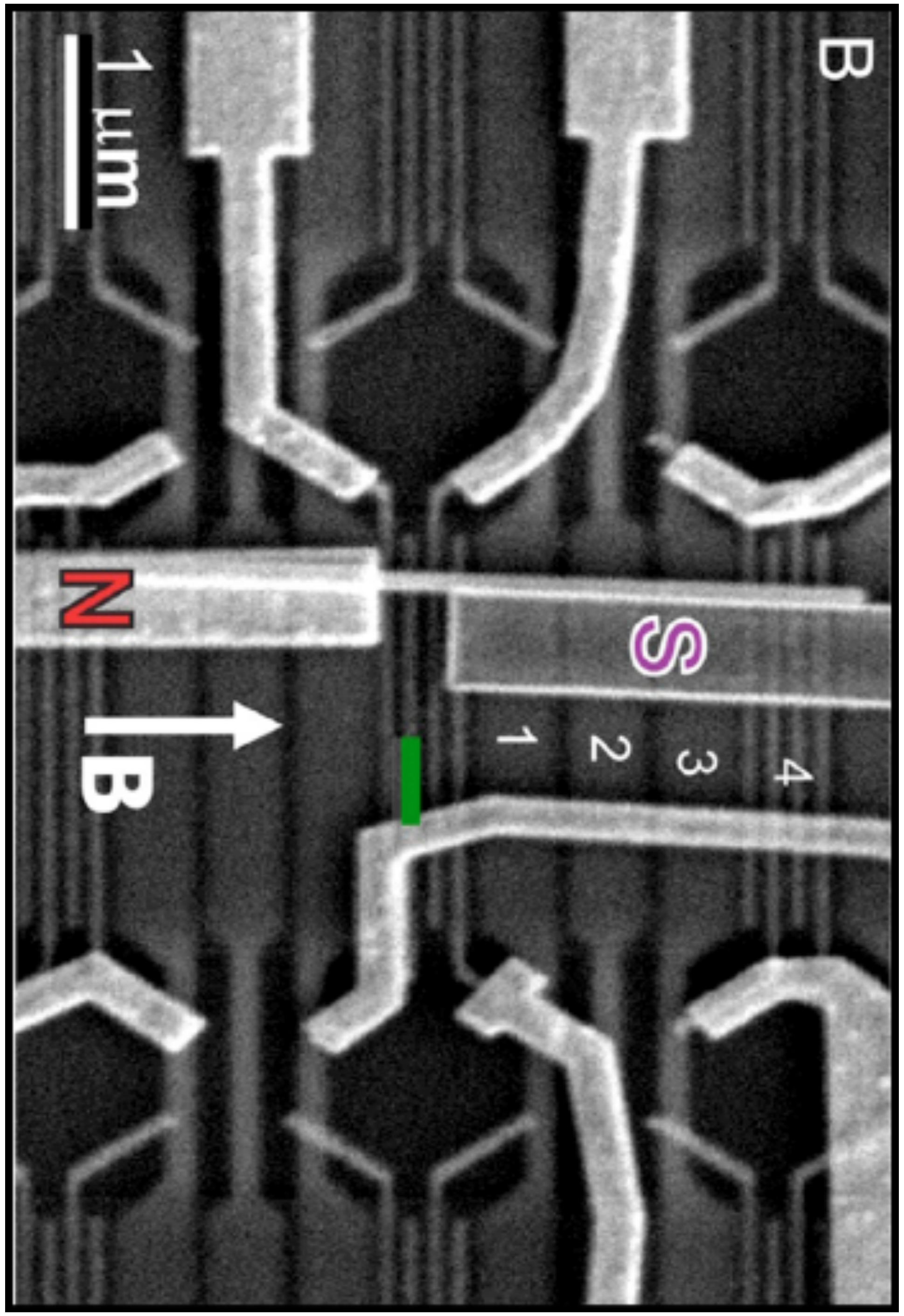
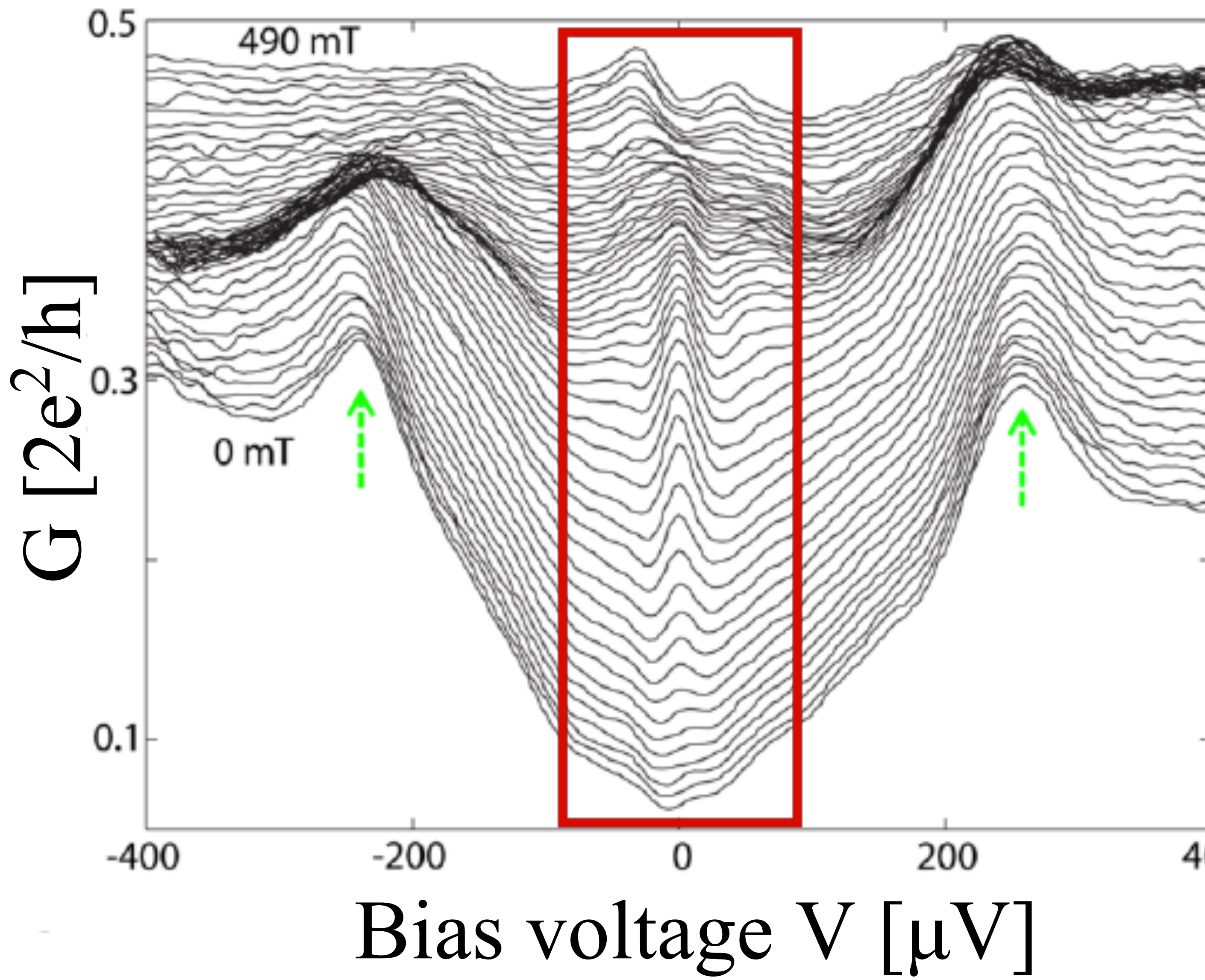
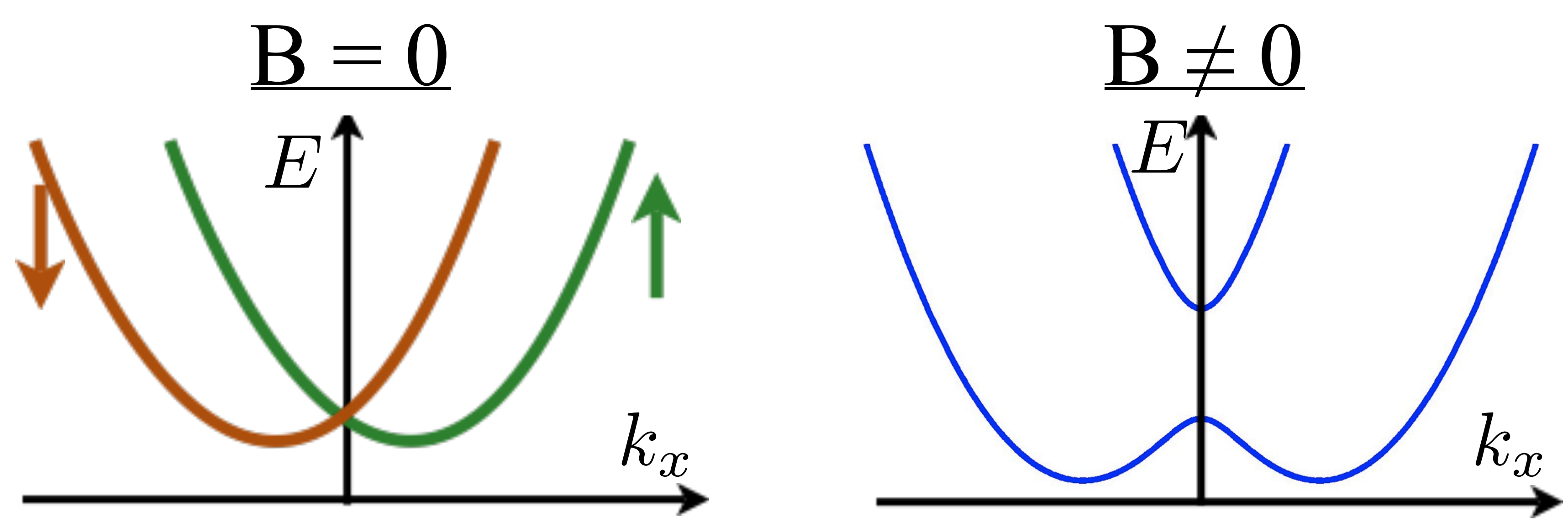






Mourik et al., Science 2012

So has a Majorana mode now been seen?

My answer: Maybe, but experiment falls short of "smoking gun".

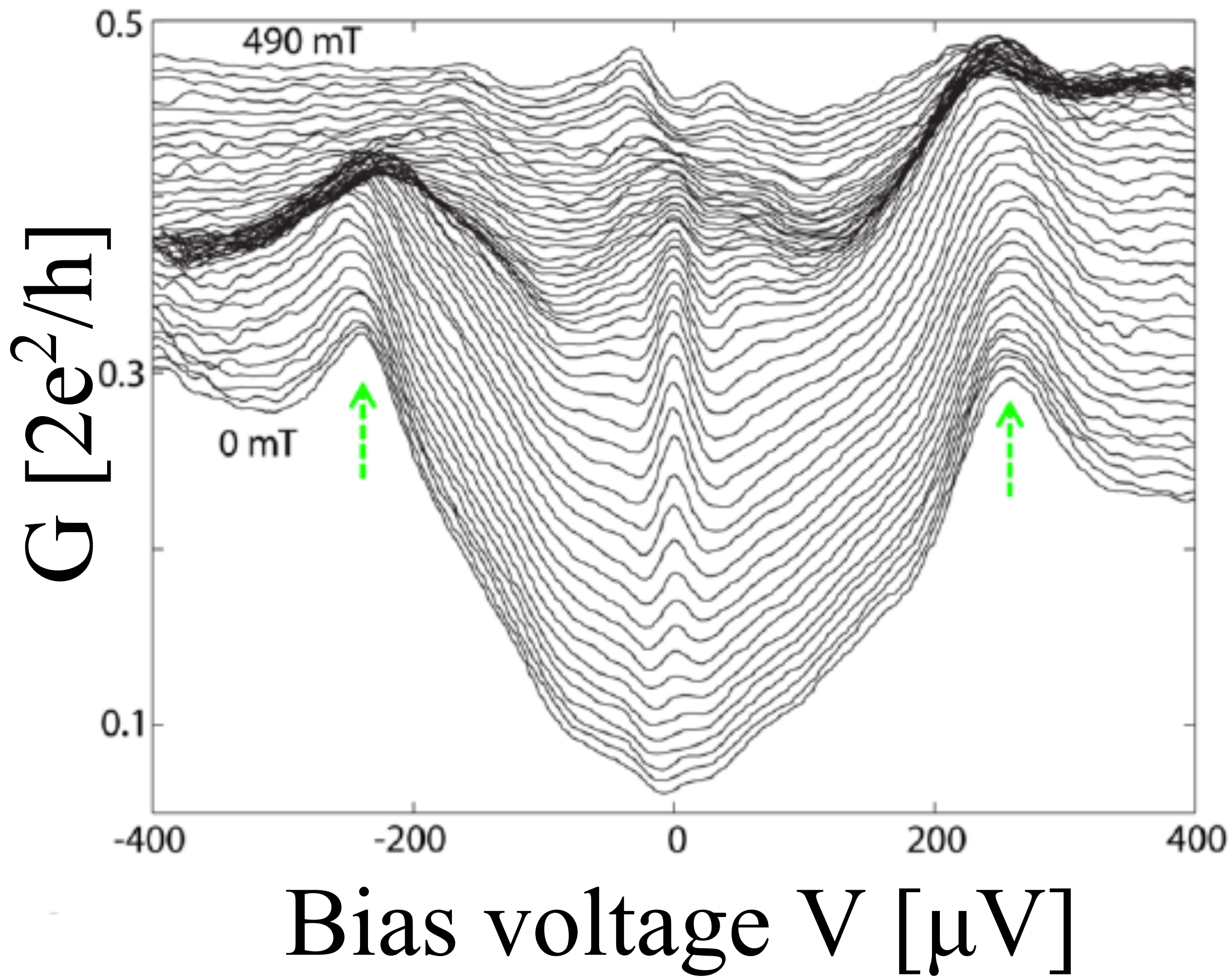
-Agrees qualitatively but not quantitatively with theory (peak height far too small)

-Disorder may lead to similar peaks ***even in a trivial superconductor***

-Gap is "soft", and suggests system is far from clean limit

-No signature of bulk phase transition from trivial to topological phase as magnetic field increases

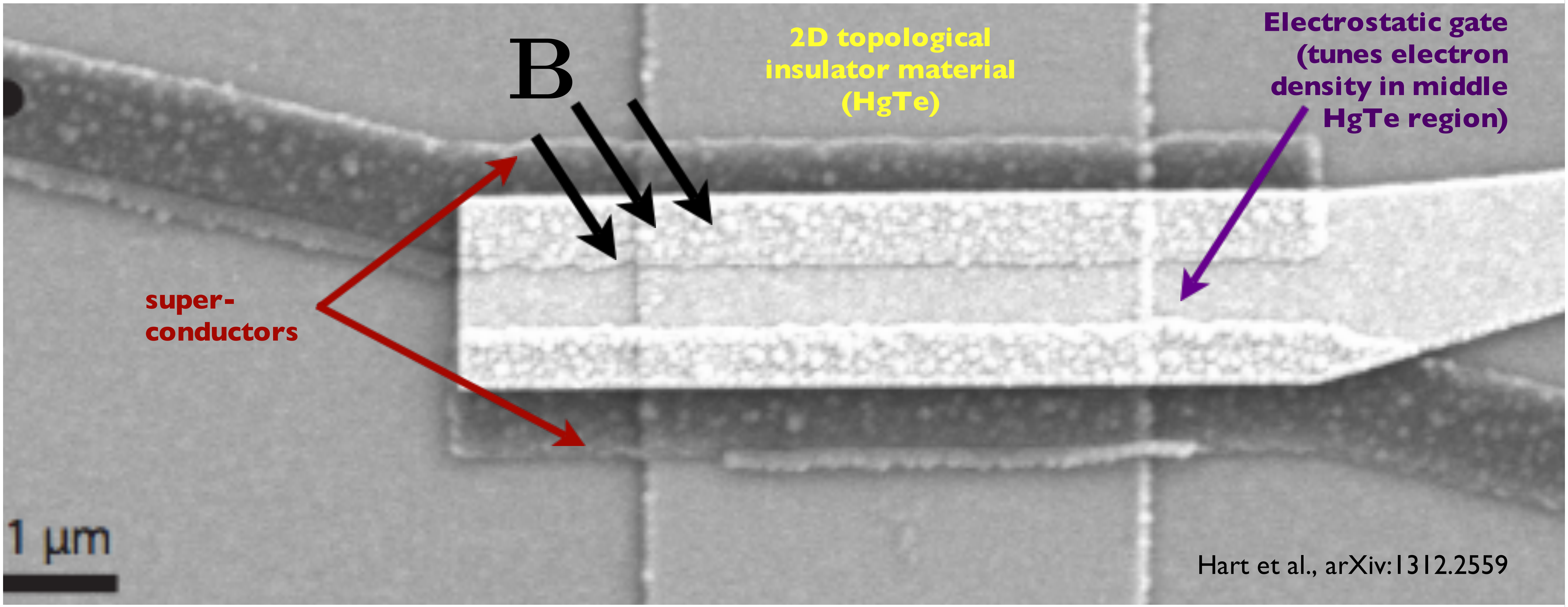
-Wires are quite small, so finite-size effects may be an issue



Mourik et al., Science 2012

Good news: New generation of experiments is well underway. Situation likely to be clarified within 1-2 years.

New experiments on 2D topo. insulator junctions



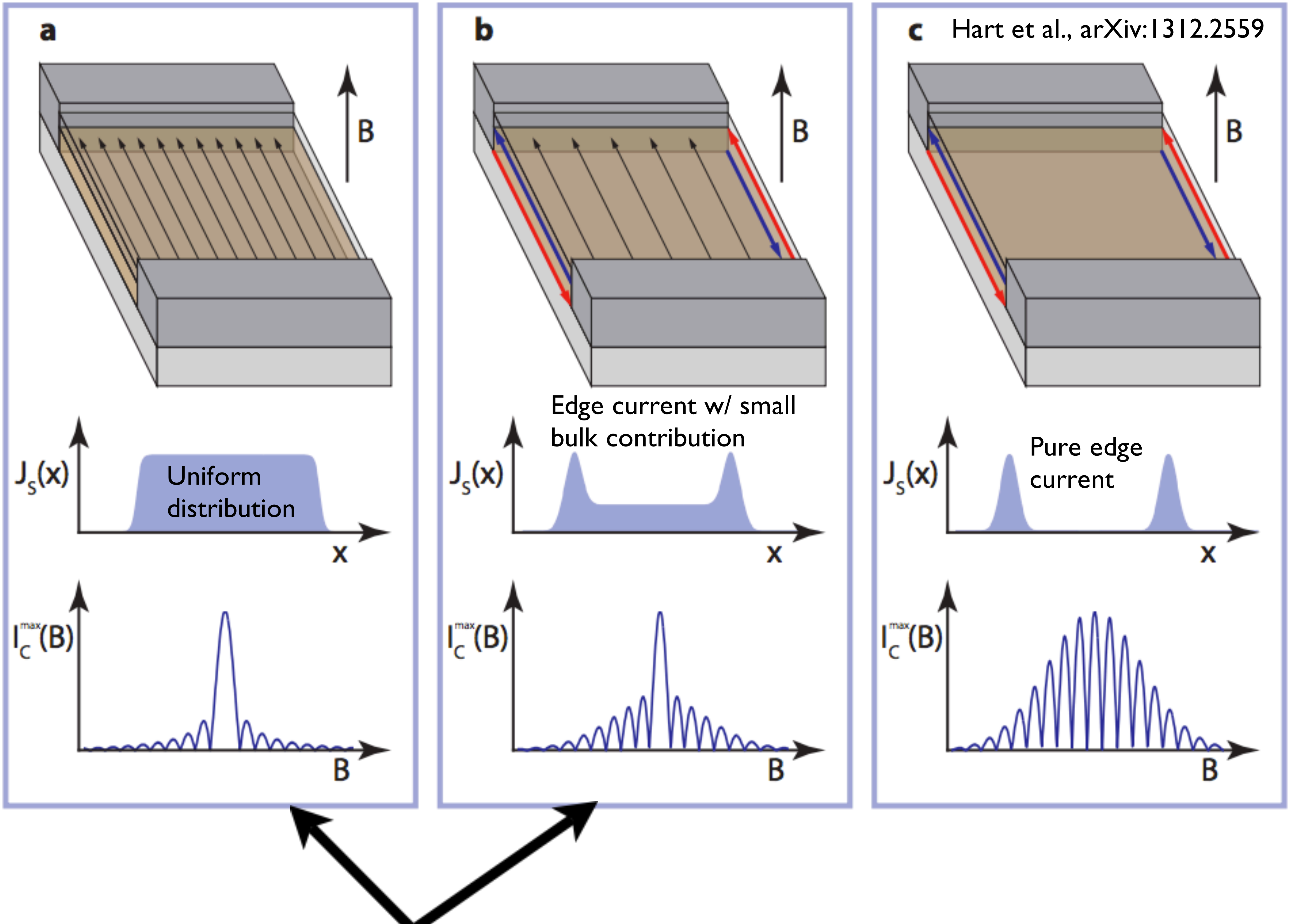
Outline of experiment:
(i) Apply a magnetic field through Josephson junction
(ii) Drive current between superconductors, measure voltage across junction
(iii) Extract "critical current" at which a finite voltage drop first develops
(iv) Repeat for many magnetic fields

From critical current versus field data, can extract the <u>spatial distribution of current</u> through junction!

Junction geometry

Current density

Resulting critical current

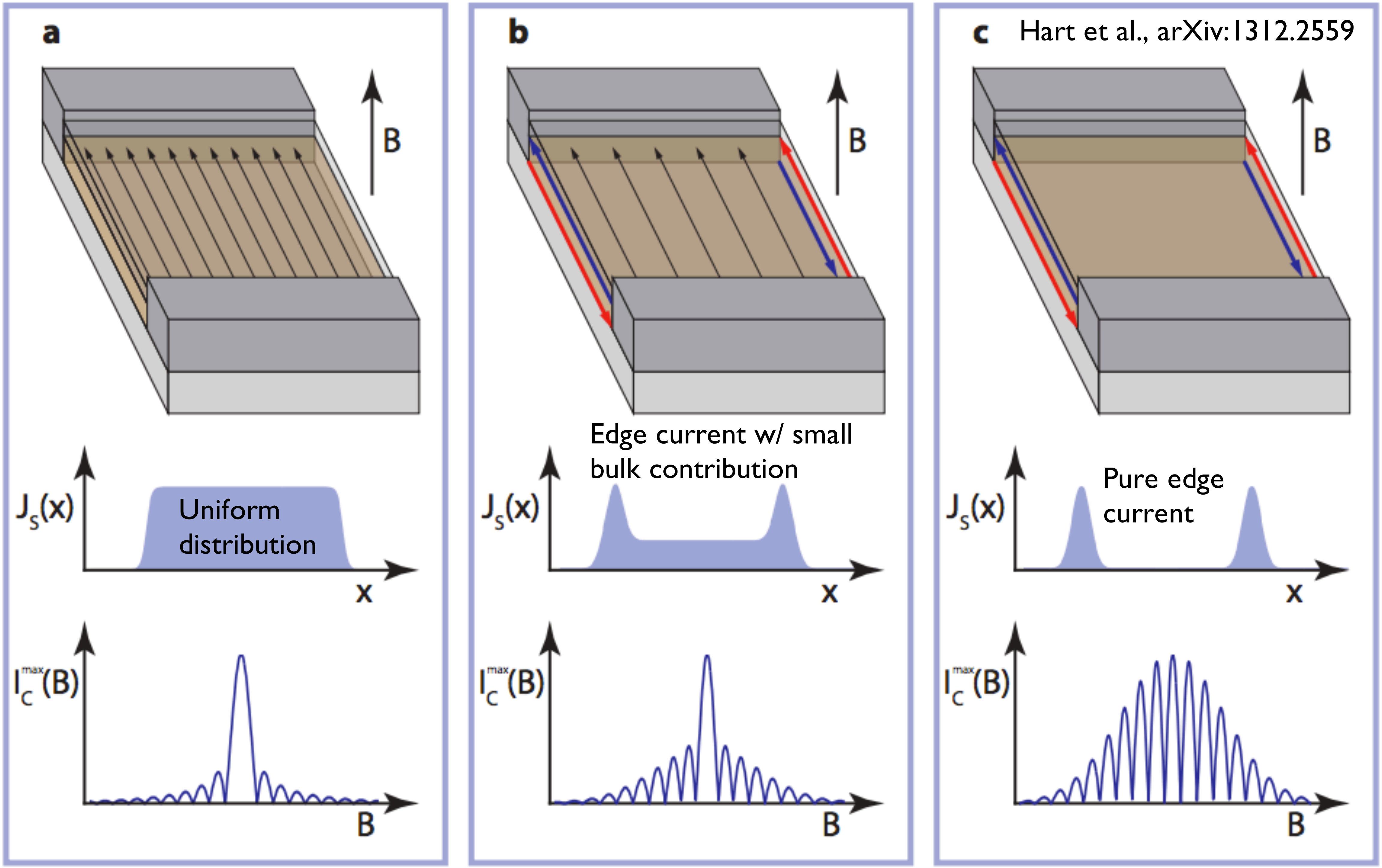


Can tune between these regimes by gating!

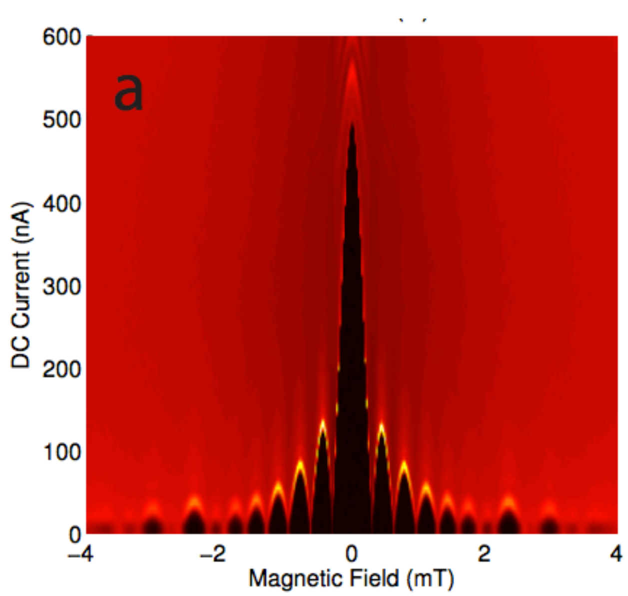
Junction geometry

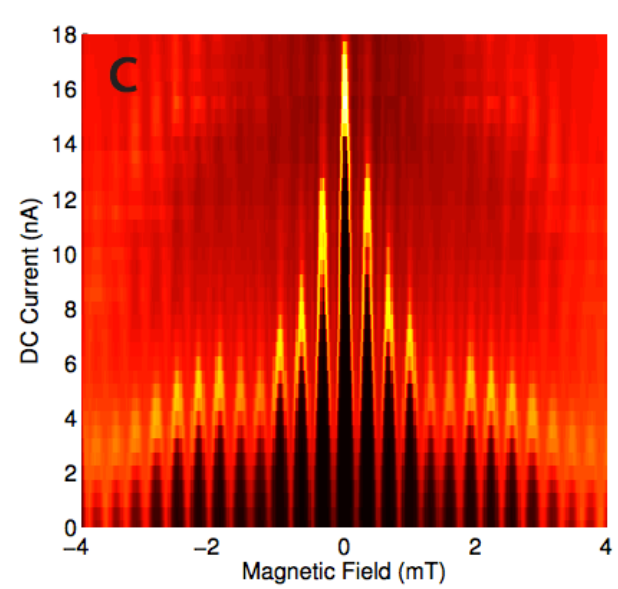
Current density

Resulting critical current



Experimental data

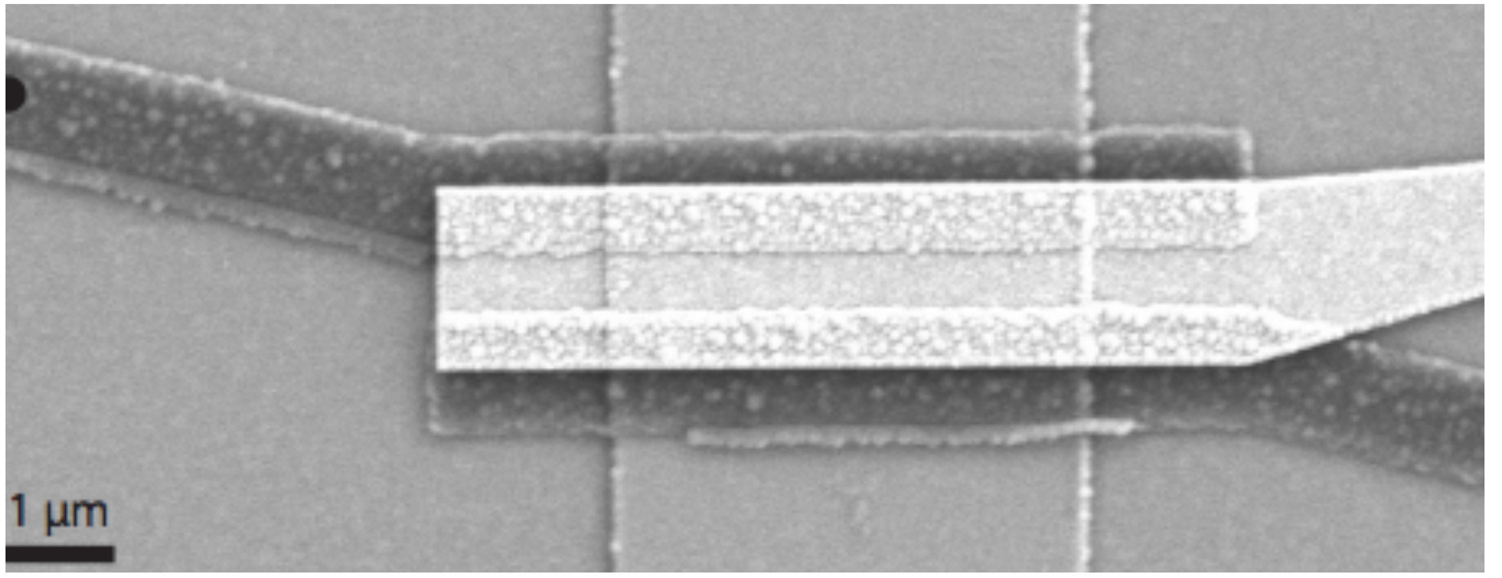




Reasons for enthusiasm

-Edge transport confirmed by new means

-Superconducting proximity effect clearly induced in topological insulator regime



Hart et al., arXiv:1312.2559

-Once this happens, topological superconductivity is almost guaranteed! (Not easy to find alternatives.)

Challenge to theory/experiment: find ways of conclusively revealing topological superconductivity, Majorana fermions

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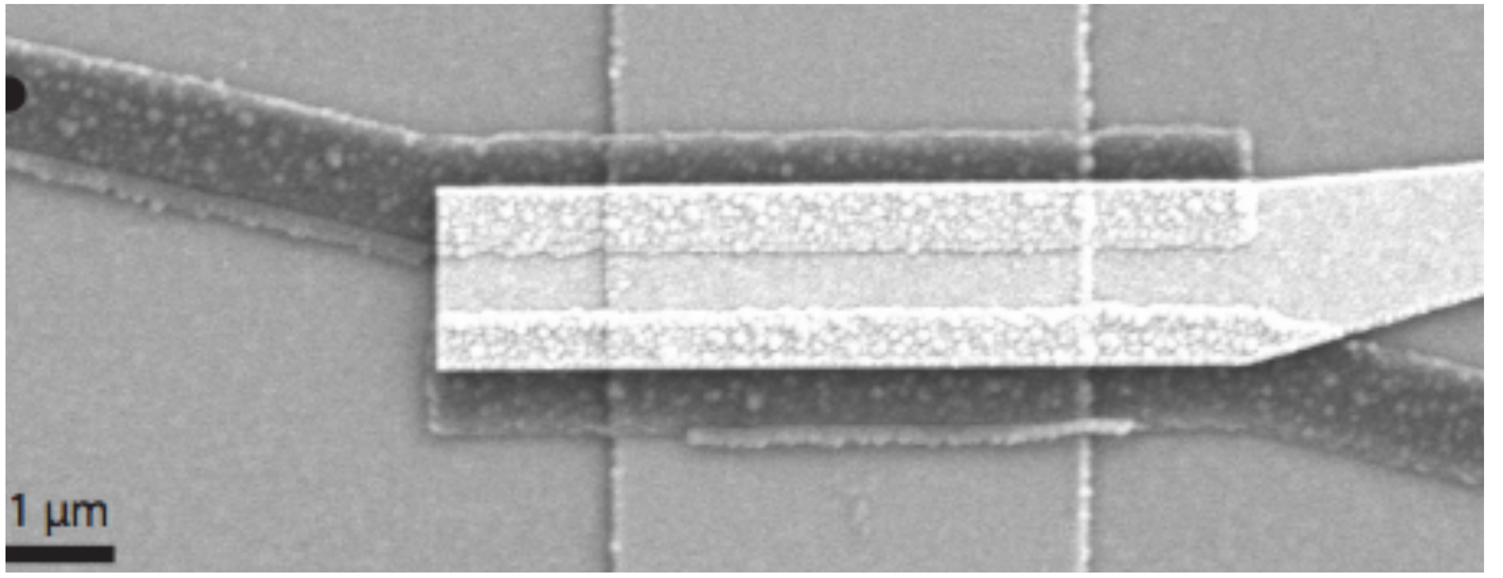

Hart et al., arXiv:1312.2559

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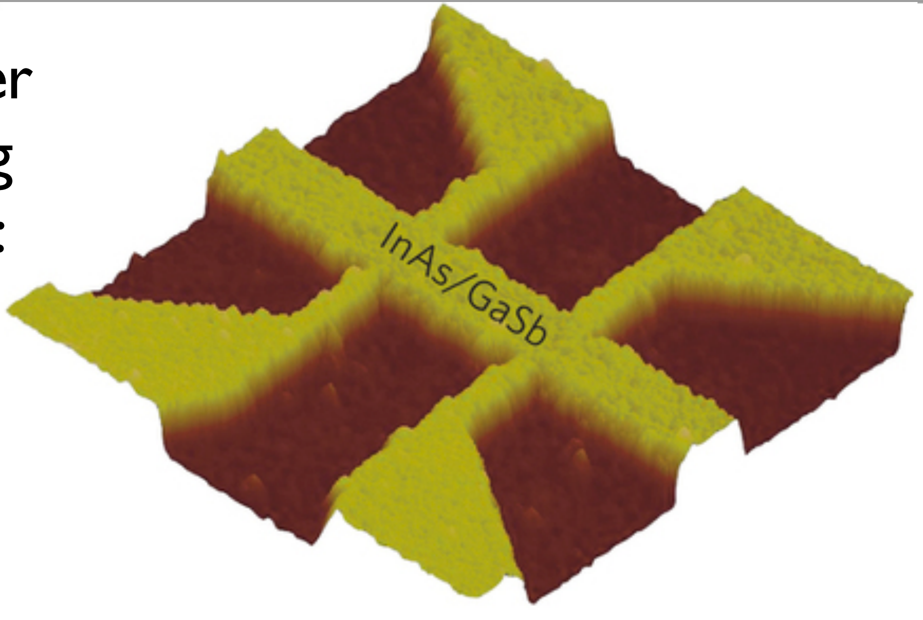
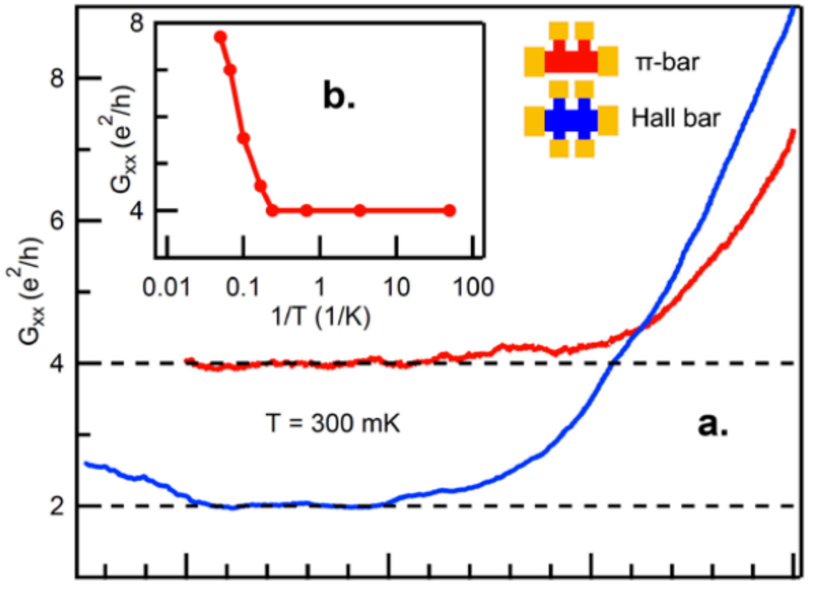
Another exciting system:





-Very clean transport data

-Couples well to superconductors

-But magnetic field dependence is strange...

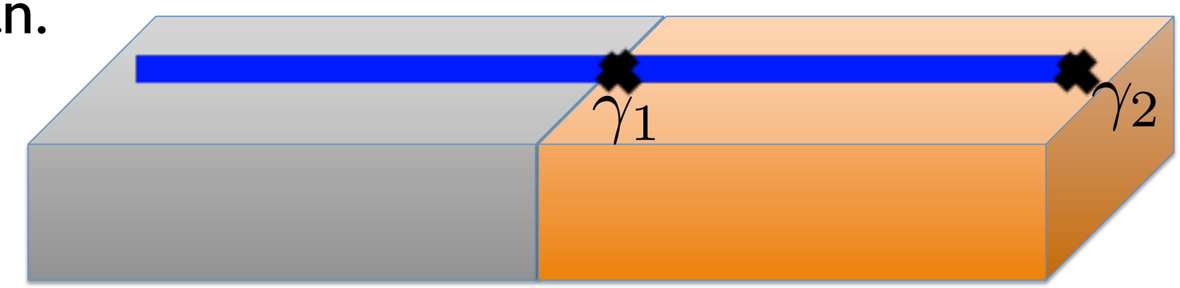
Liu, Hughes, et al. (2008); Du et al. (2013)

Homework Set 3

1. Consider the 1D wire transport setup we analyzed earlier. Show that (independent of any details of the Hamiltonians) the scattering matrix MUST be either purely diagonal or purely off-diagonal in the limit E = 0.

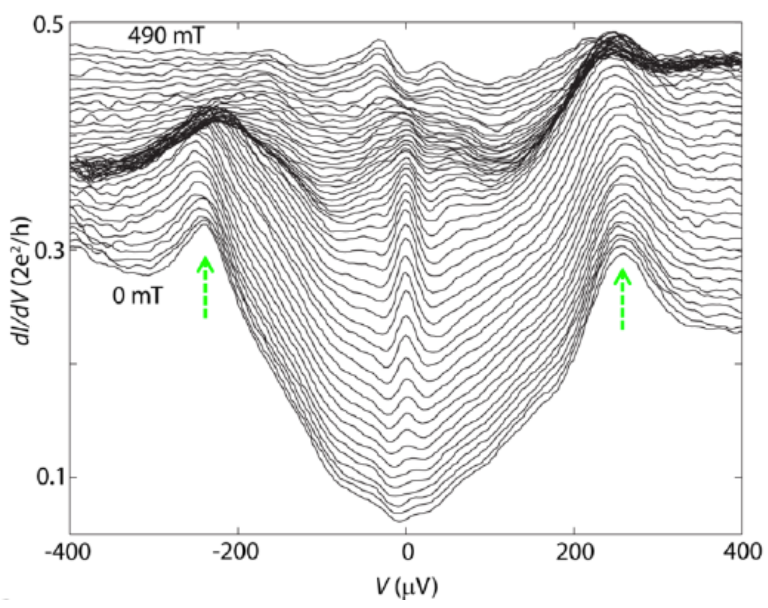
$$\begin{bmatrix} P_E(\infty) \\ H_E(\infty) \end{bmatrix} = \begin{bmatrix} S_{PP}(E) & S_{PH}(E) \\ S_{HP}(E) & S_{HH}(E) \end{bmatrix} \begin{bmatrix} P_E(-\infty) \\ H_E(-\infty) \end{bmatrix}$$

2. In the topological case, compute the conductance as a function of bias voltage and show that it is a Lorentzian.



$$G(V) = \frac{2e^2}{h}|S_{PH}(eV)|^2$$

3. Within a single theoretical framework, capture all of the major features of the conductance measured by Kouwenhoven et al., including the “soft gap”, non-quantized zero-bias peak, etc. Submit your result to Physical Review Letters.



Outline for final lecture

- Majorana detection via transport

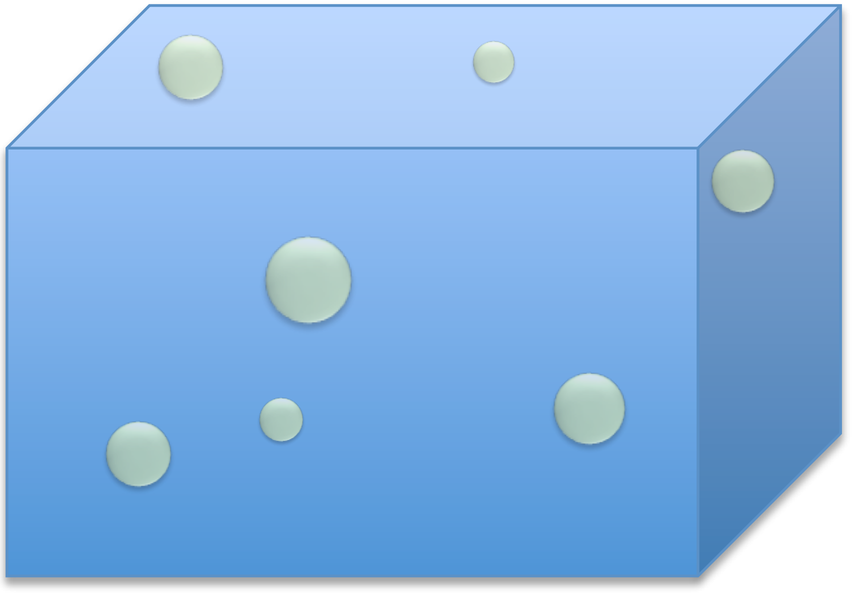
- Experimental progress
  - 1D wires
  - 2D topological insulators

- Outlook: where are we going?

Exchange statistics

Describes how wavefunctions transform when indistinguishable particles exchange positions

$$\psi(\mathbf{r_1}, \ldots, \mathbf{r_N})$$



Exchange statistics

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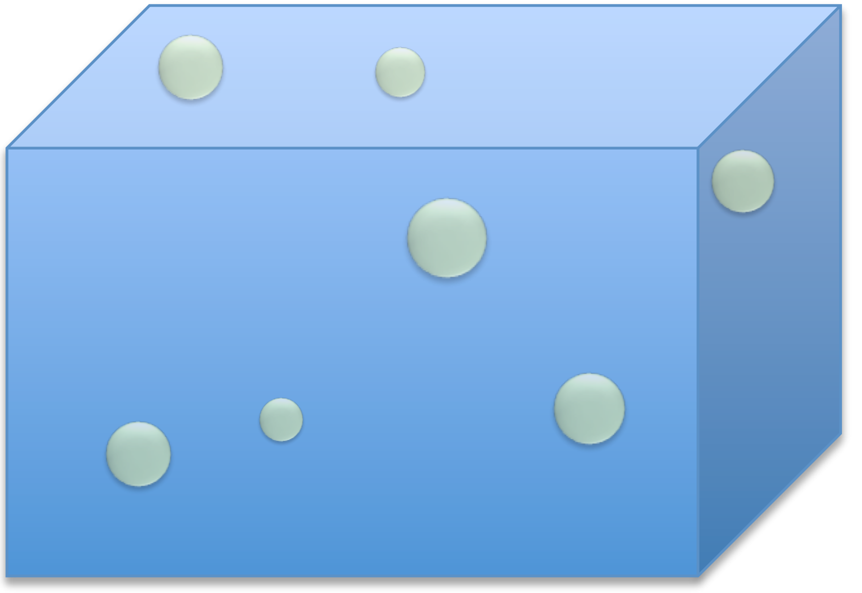

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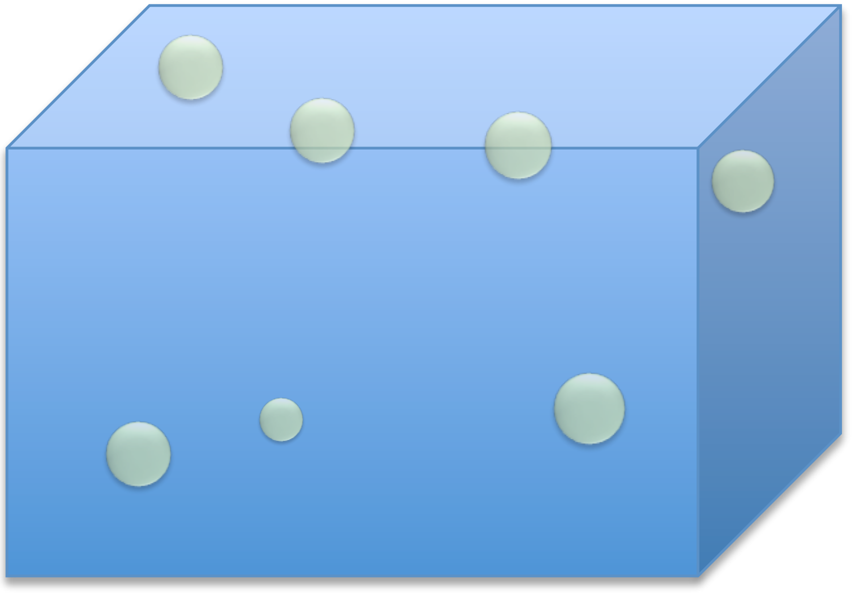

Exchange statistics

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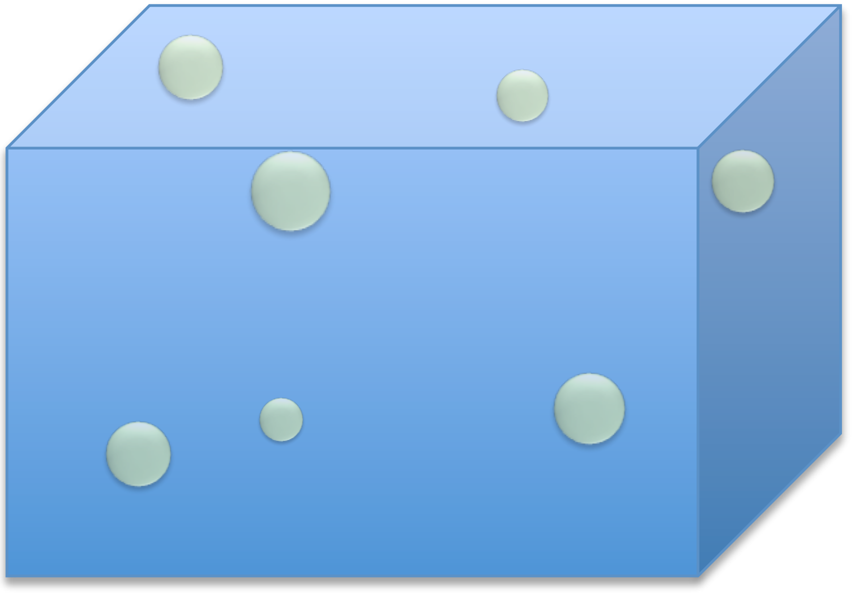
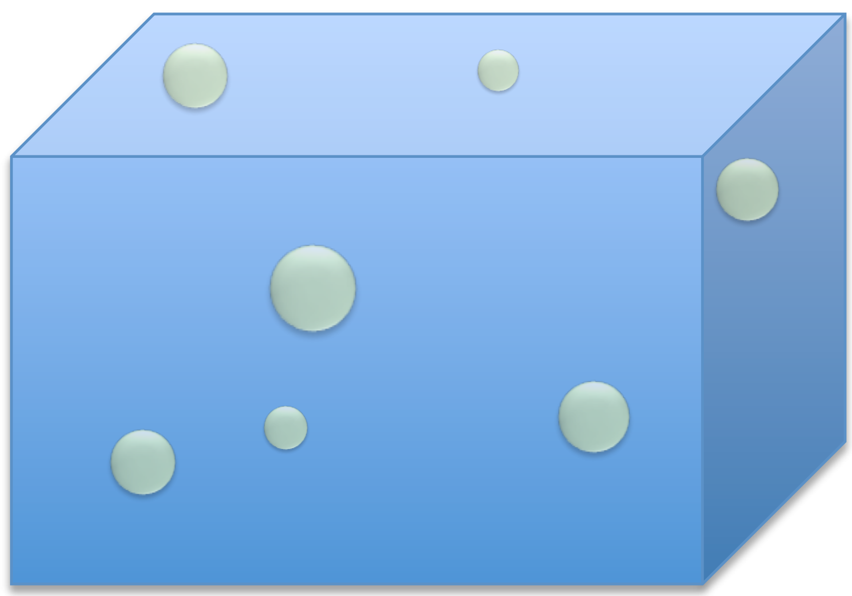


Exchange statistics

Describes how wavefunctions transform when indistinguishable particles exchange positions

$$\psi(\mathbf{r_1}, \ldots, \mathbf{r_N}) \longrightarrow \psi'(\mathbf{r_1}, \ldots, \mathbf{r_N})$$



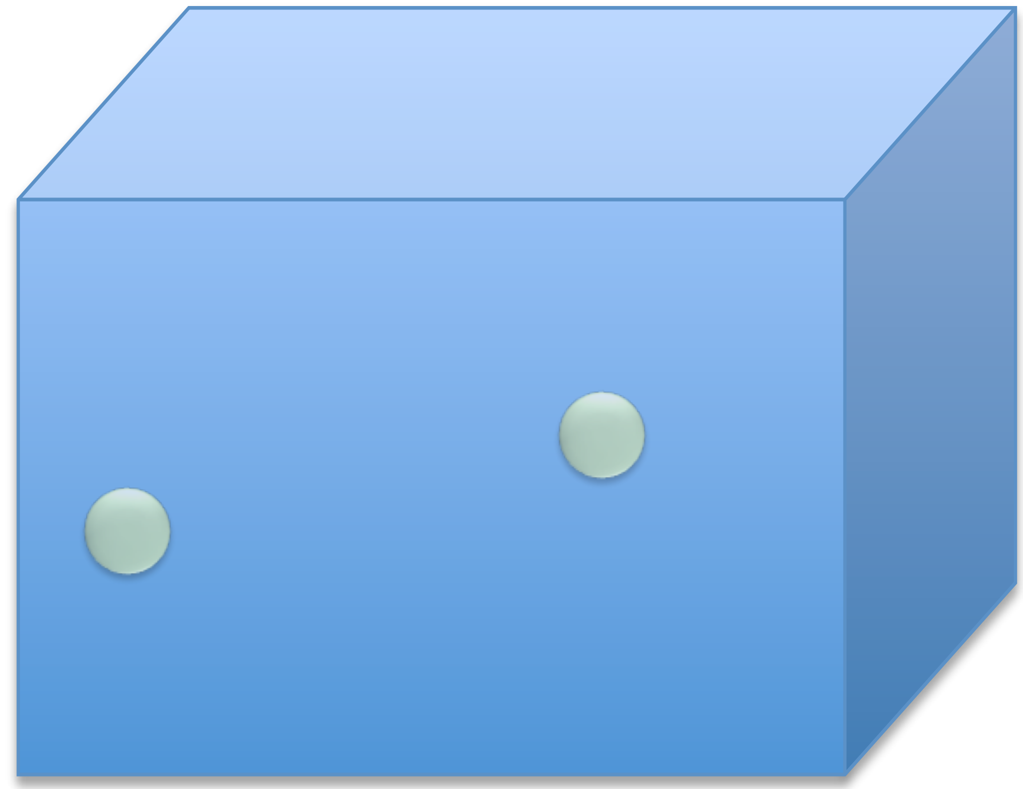
Extraordinarily fundamental!

Underlies most condensed matter phenomena.

Role of dimensionality

d = 3

Only bosons & fermions



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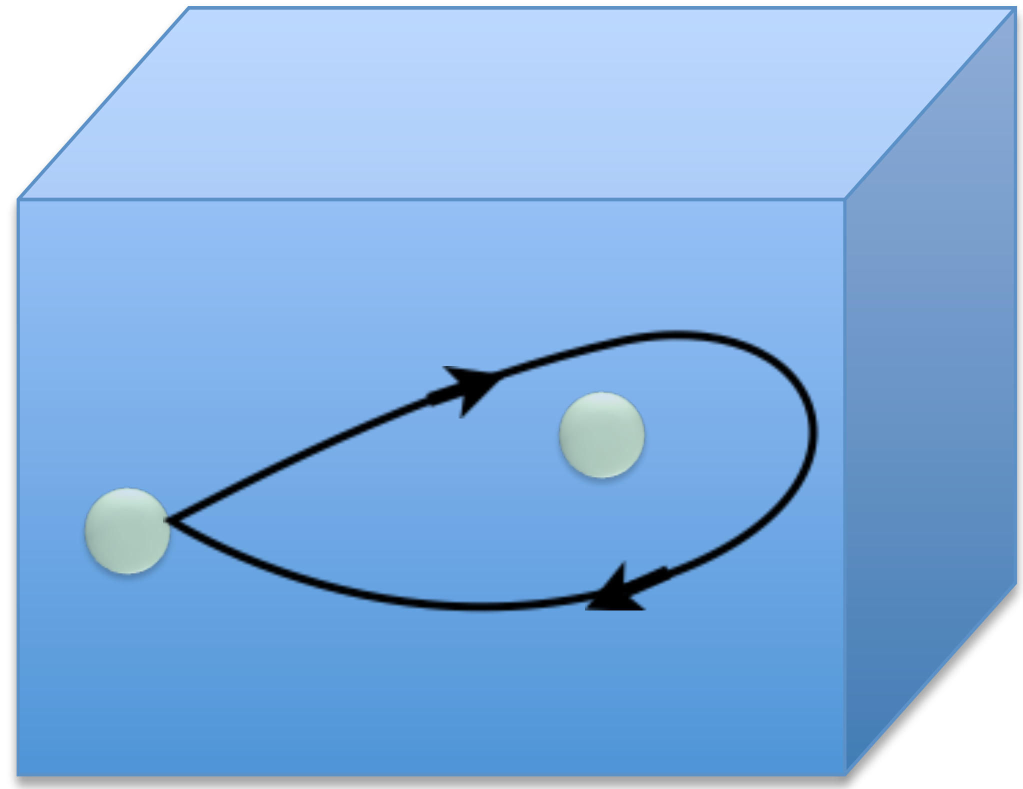

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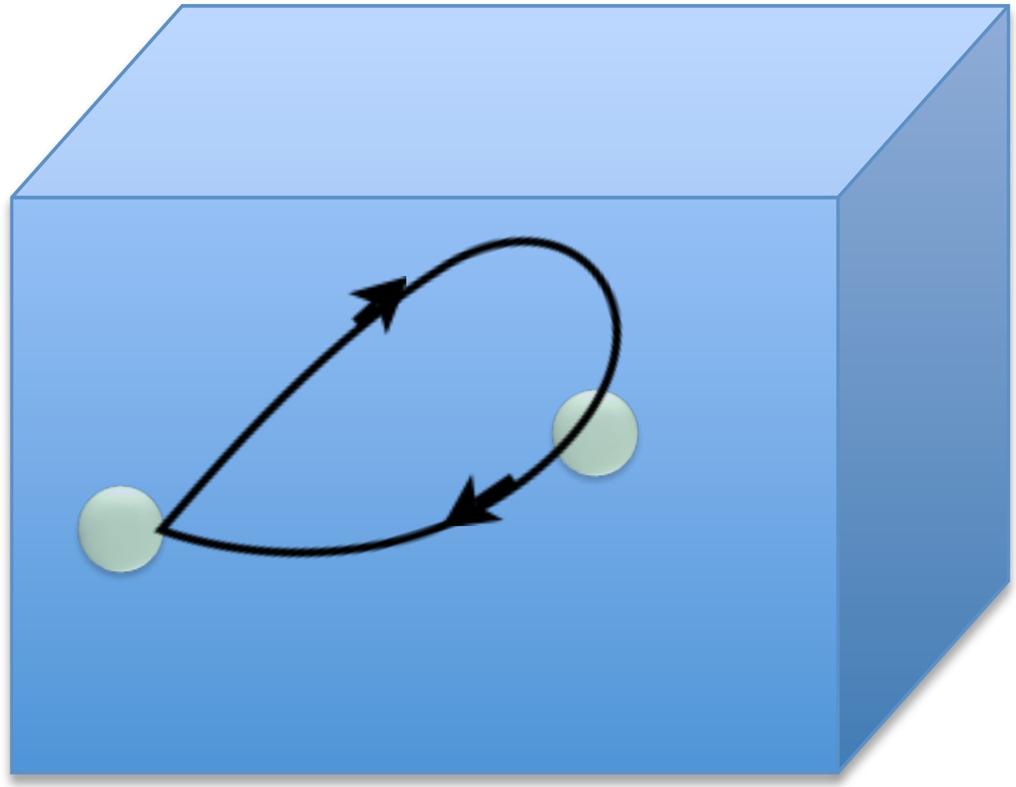

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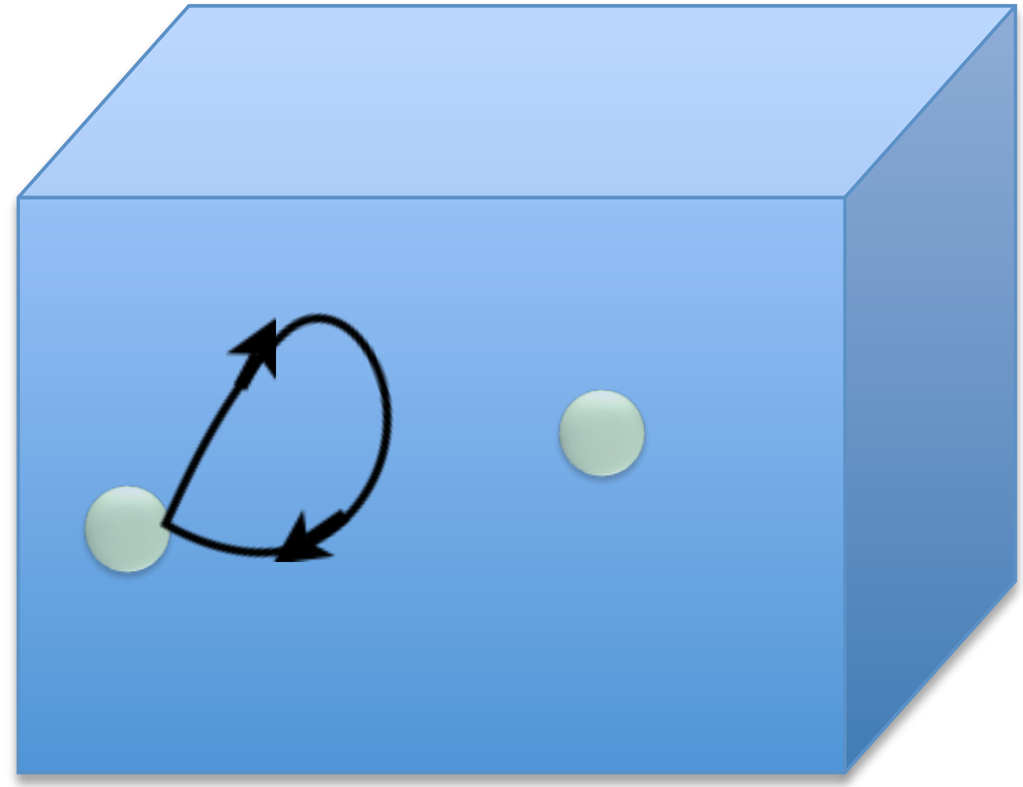

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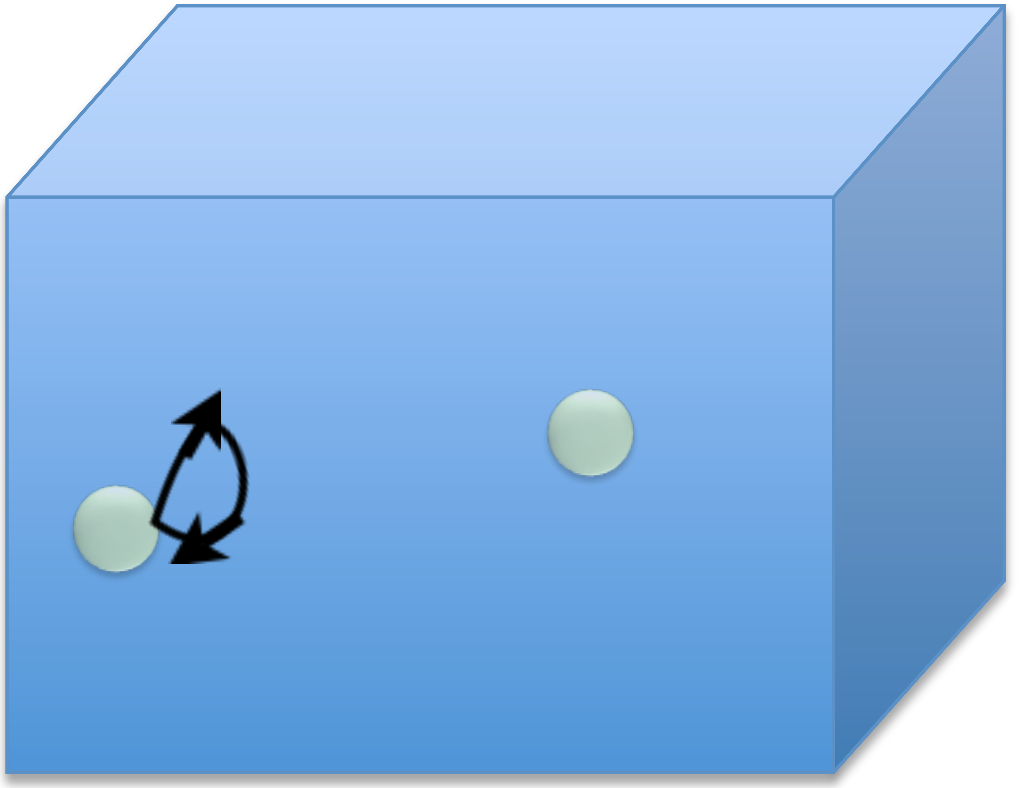

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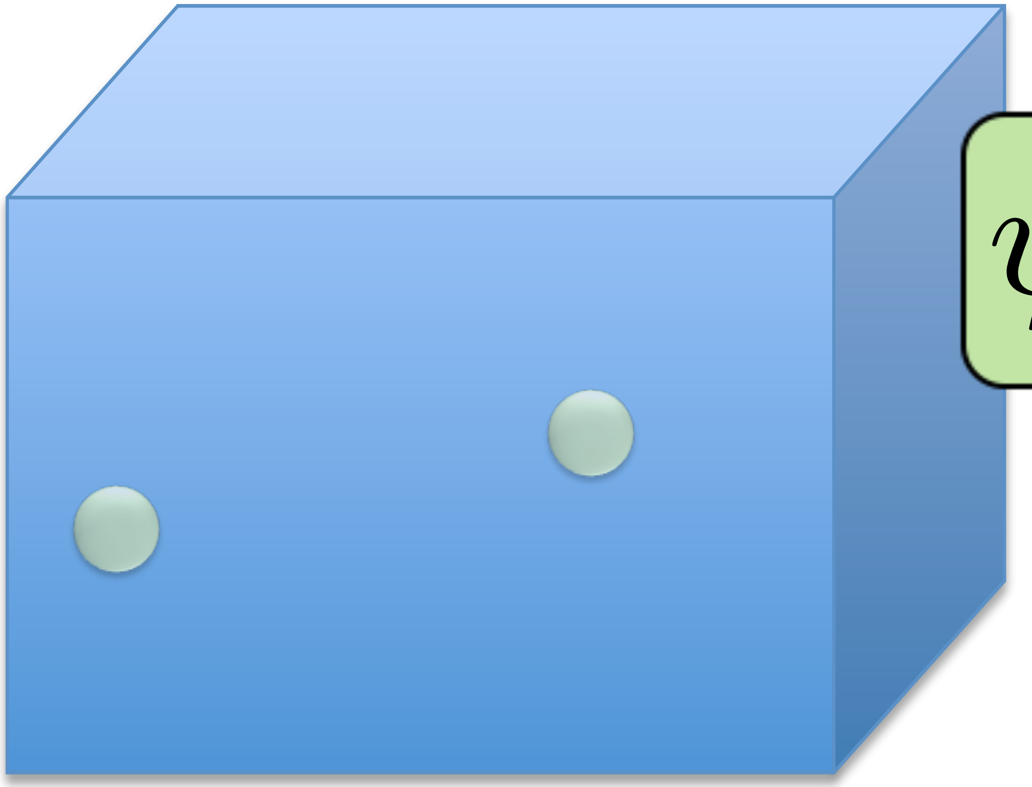
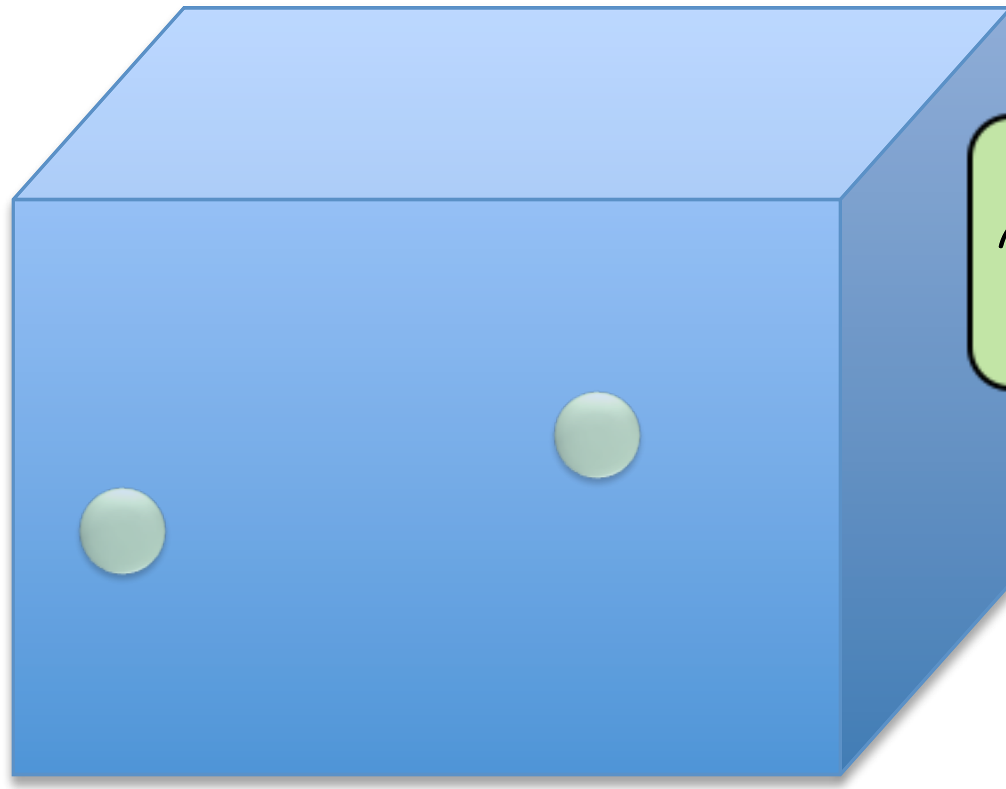


$$\psi \longrightarrow \pm\psi$$

Role of dimensionality

d = 3

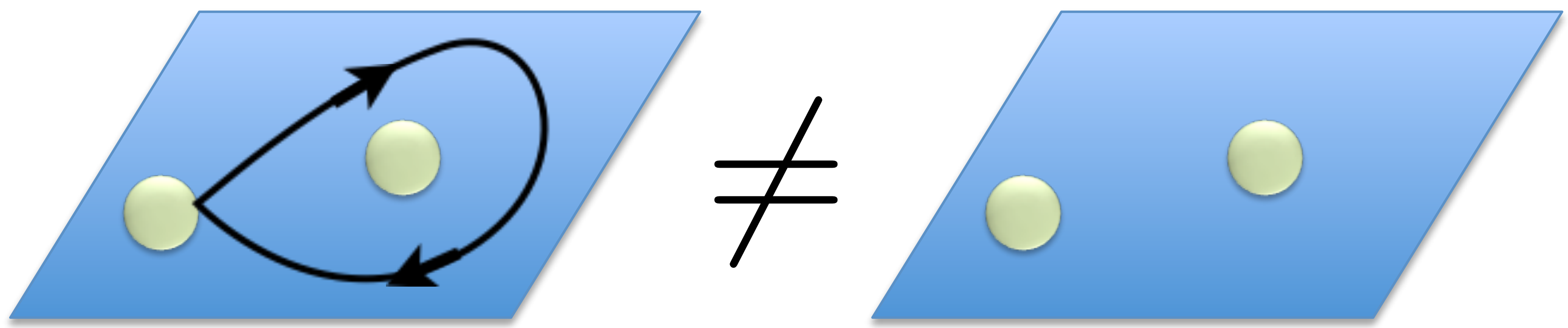
Only bosons & fermions



$$\psi \longrightarrow \pm\psi$$

d = 2

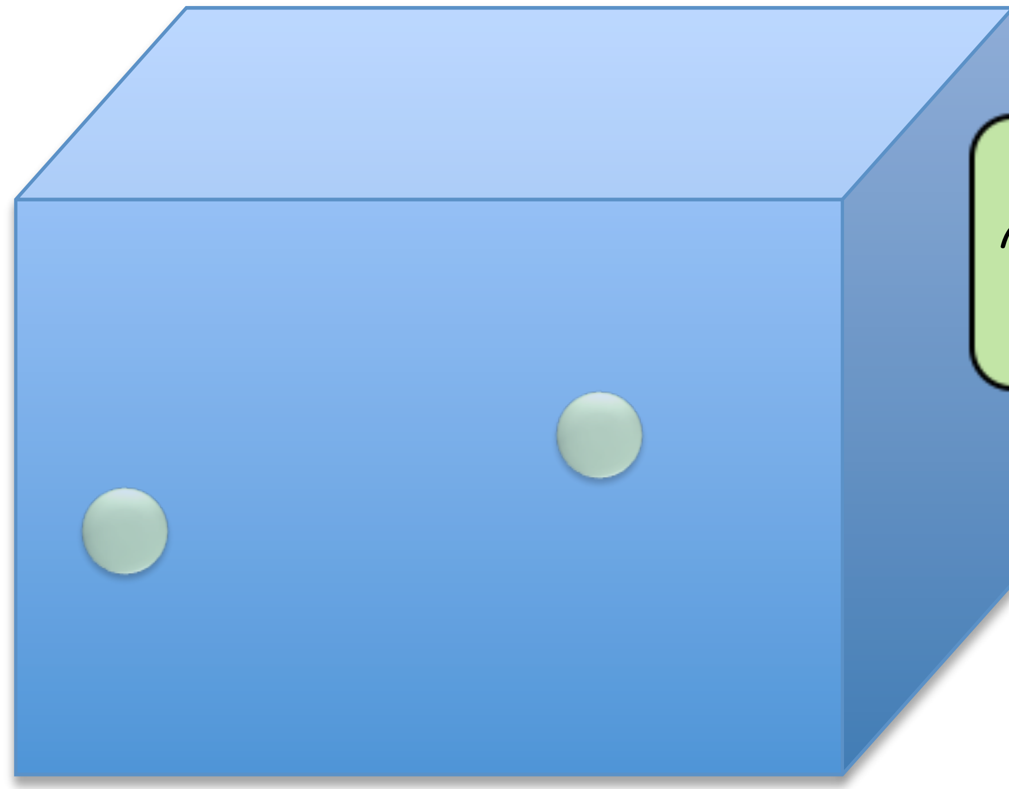
Anyons are now possible!



Role of dimensionality

d = 3

Only bosons & fermions



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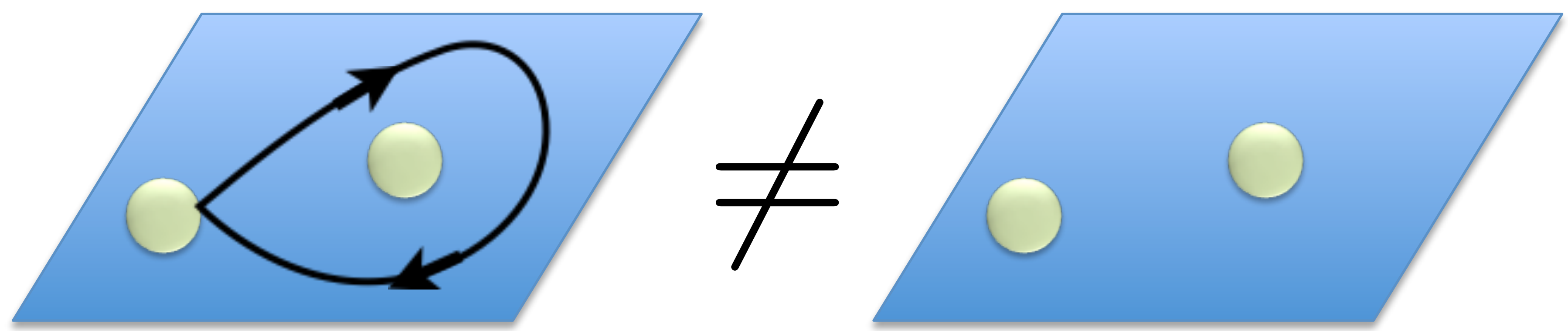


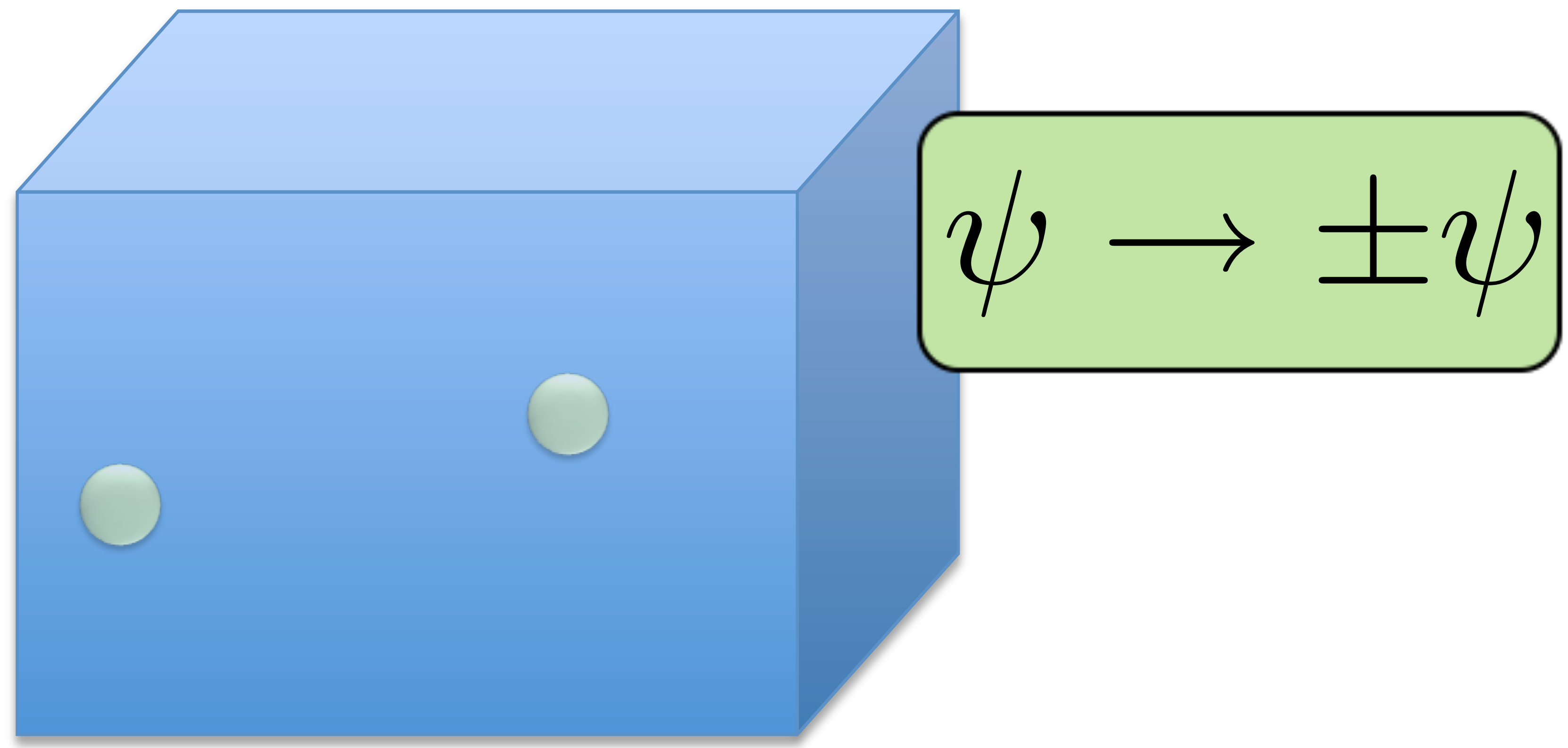
d = 1

Exchange not well defined...

Role of dimensionality

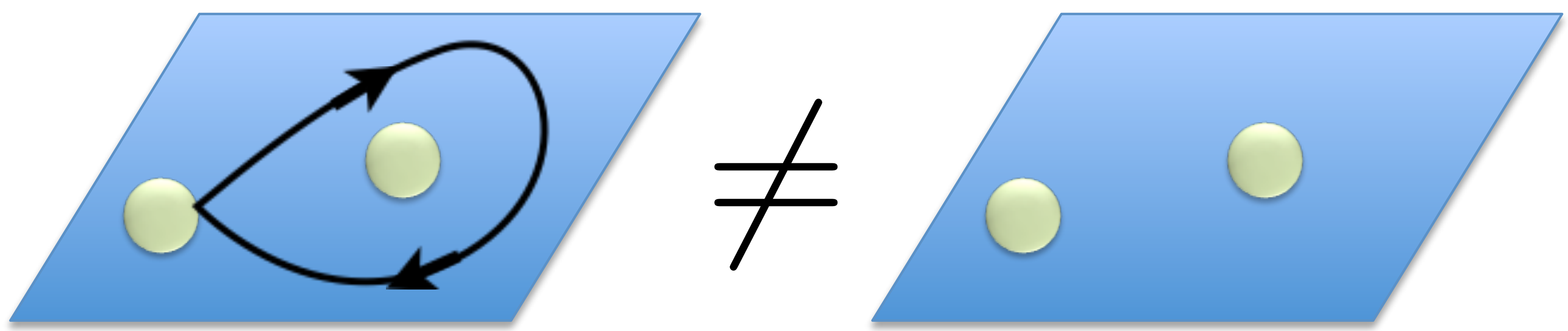
d = 3

Only bosons & fermions



d = 2

Anyons are now possible!



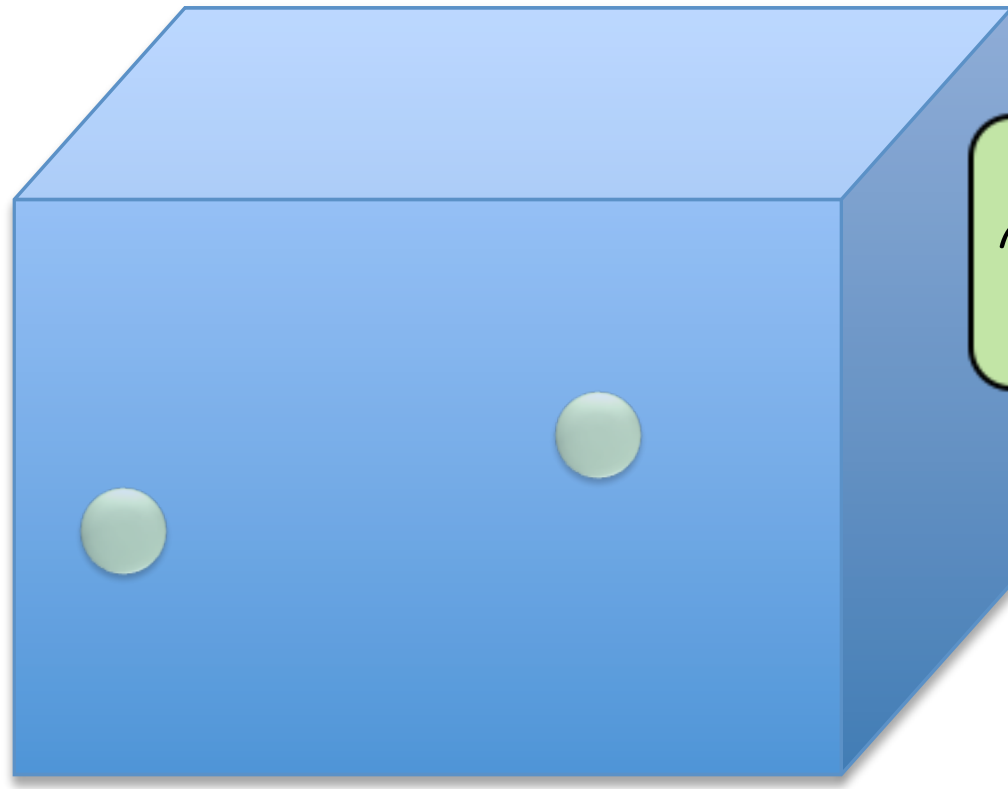
d = 1

Exchange not well defined...

Role of dimensionality

d = 3

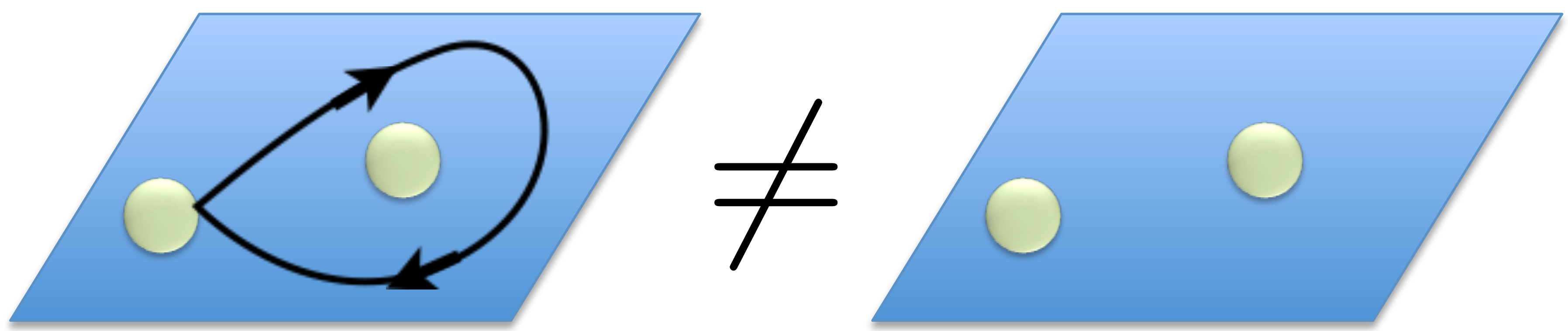
Only bosons & fermions



$$\psi \longrightarrow \pm\psi$$

d = 2

Anyons are now possible!



d = 1

Exchange not well defined...

Role of dimensionality

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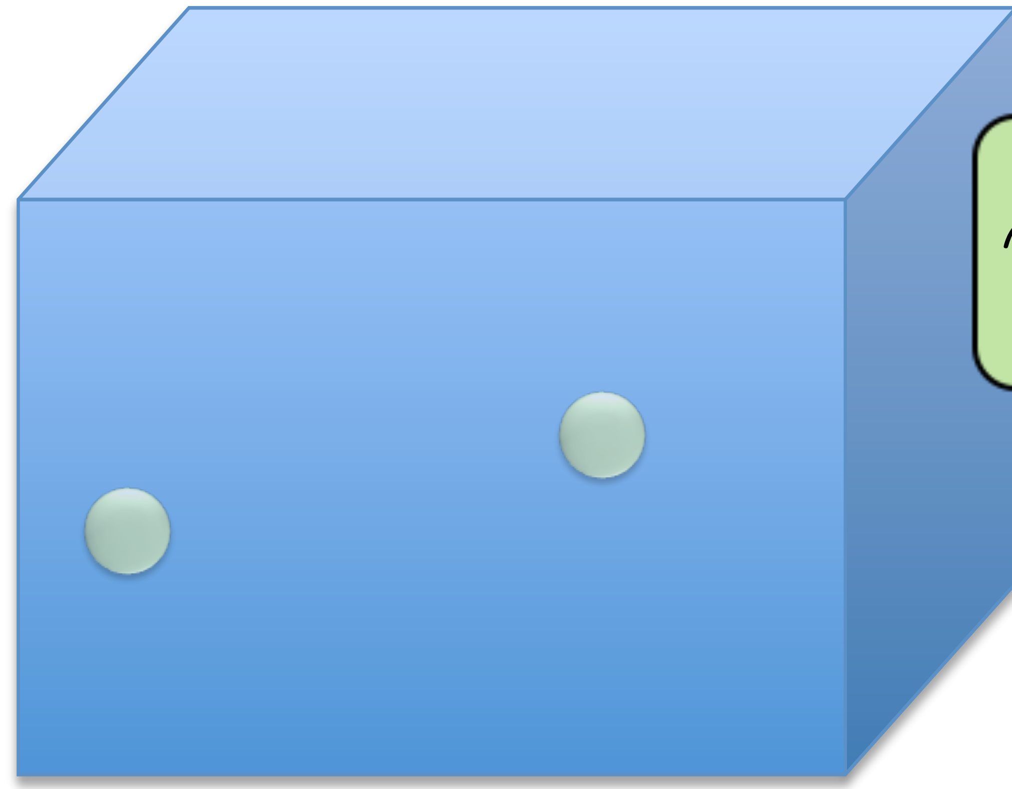
Only bosons & fermions



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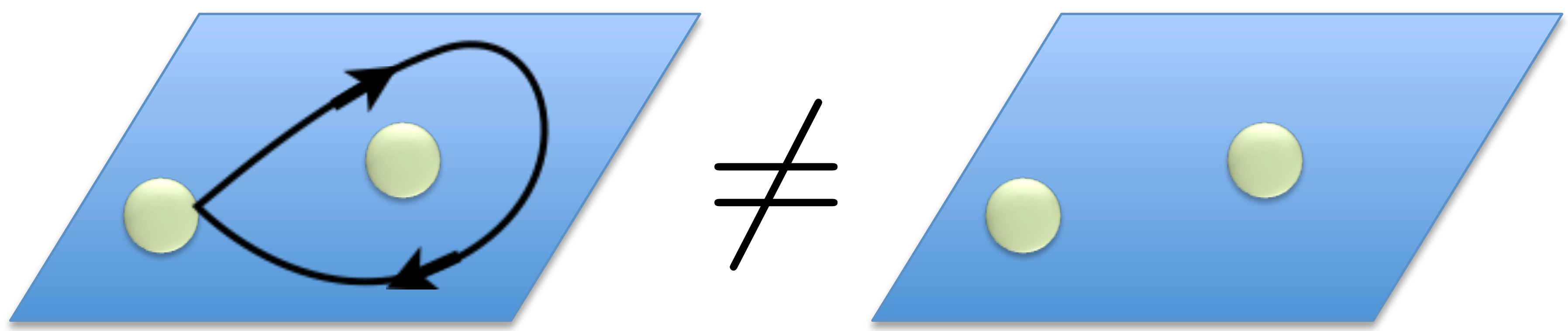
d = 2

Anyons are now possible!

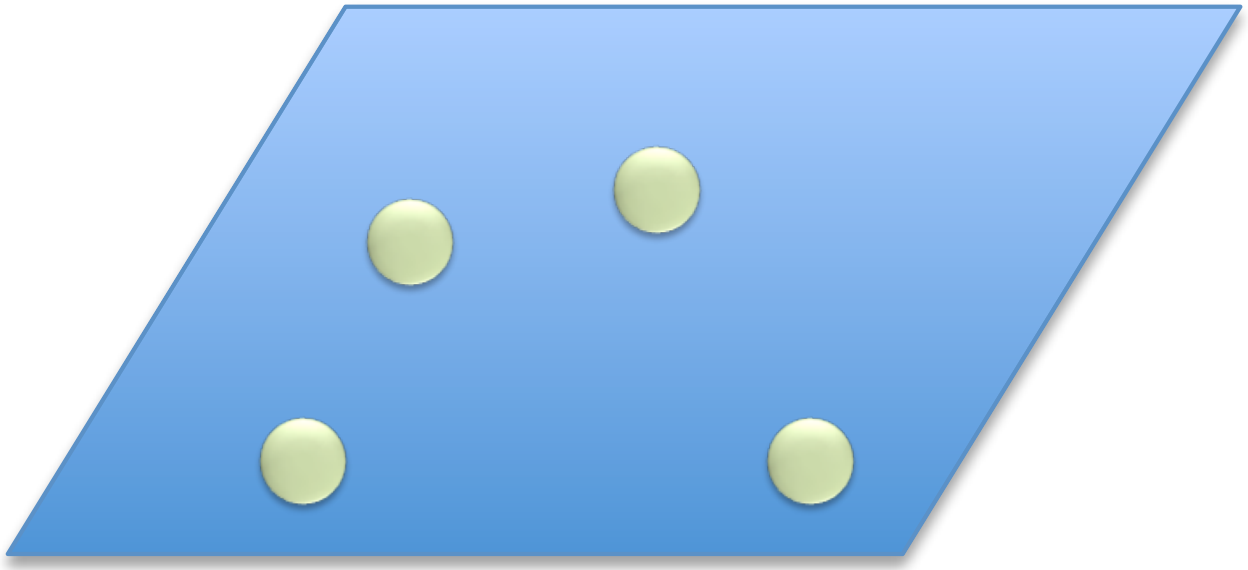


d = 1

Exchange not well defined...

...because particles inevitably "collide"

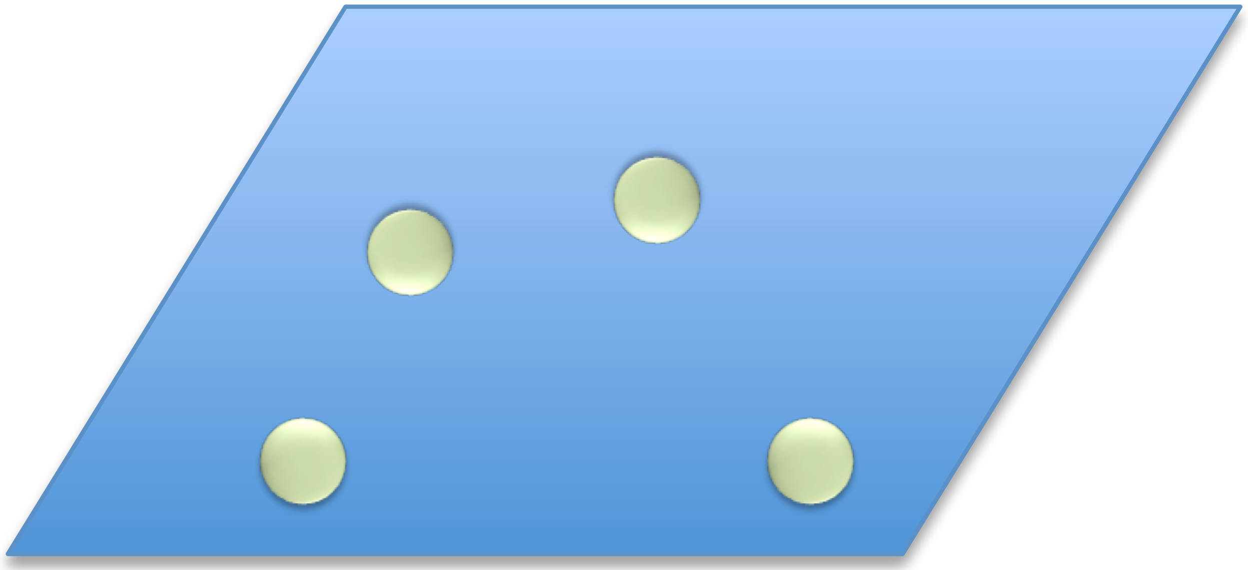
Non-Abelian anyons



(e.g., vortices in a p+ip superconductor)

$$\psi_a$$

Non-Abelian anyons

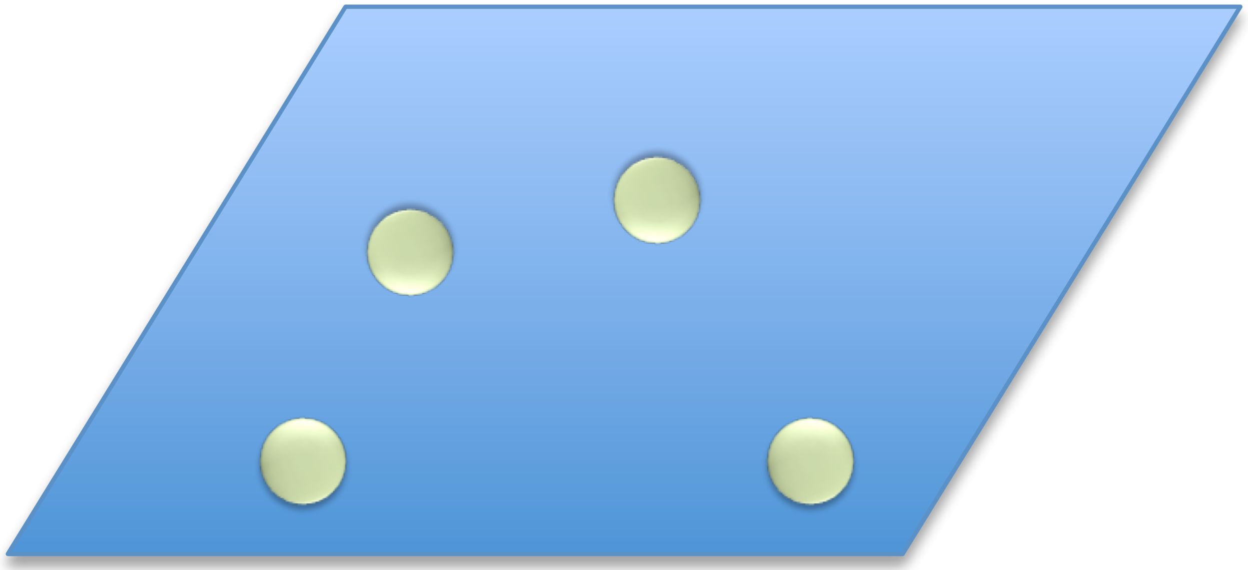


$$\psi_a \rightarrow U_{ab}\psi_b$$

Interesting for 2 reasons:

- Fundamental physics

Non-Abelian anyons



$$\psi_a \rightarrow U_{ab}\psi_b$$

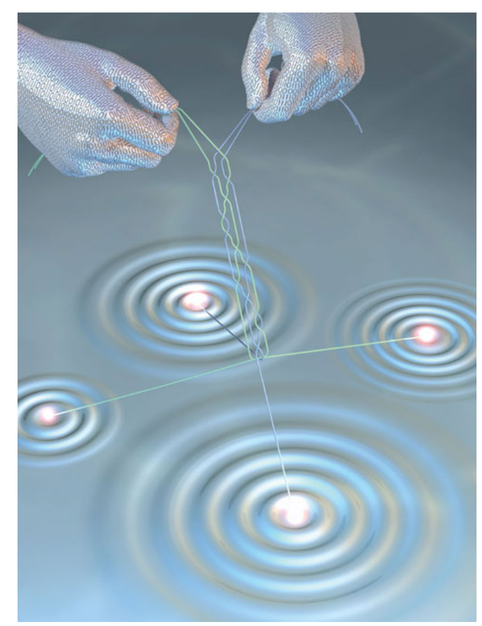
Qubits **Quantum gates**

Need sufficiently dense braid matrices for computational universality!

Interesting for 2 reasons:

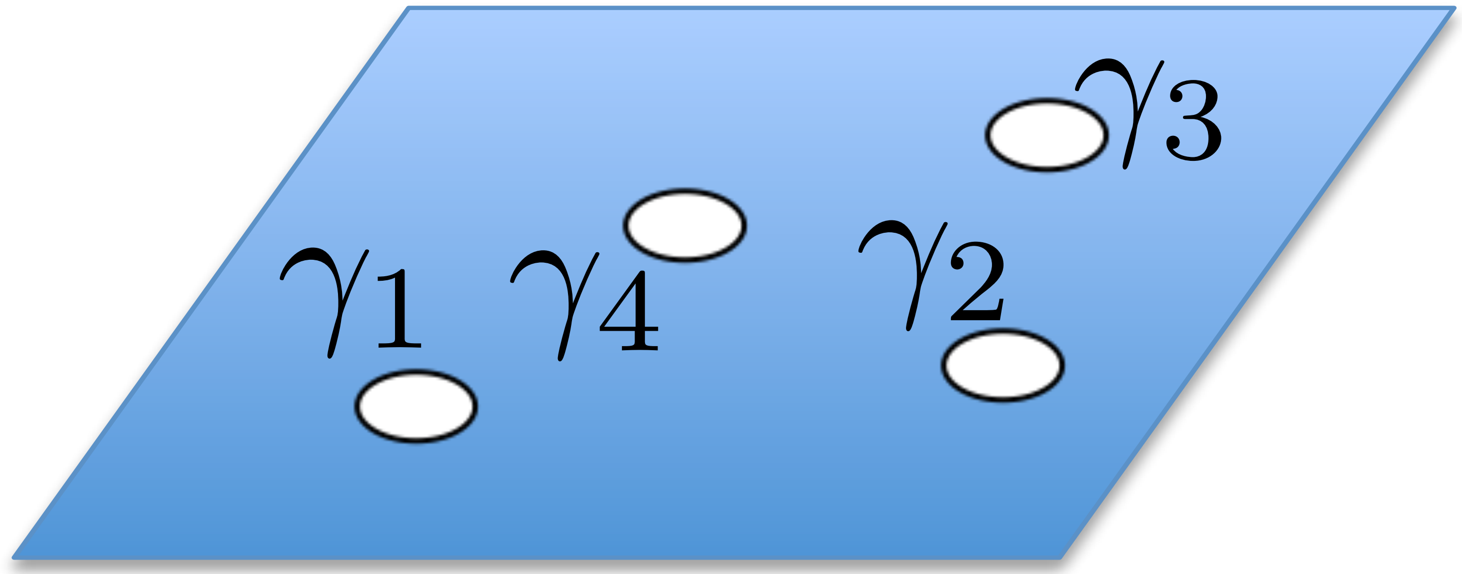
- Fundamental physics
- Decoherence-free quantum computation

Kitaev; Freedman; etc.
Nayak, Simon, Stern, Freedman, & Das Sarma, RMP **80**, 1083 (2008)



A conundrum

Majorana zero-modes in 2D topological superconductors are clearly interesting in this regard.

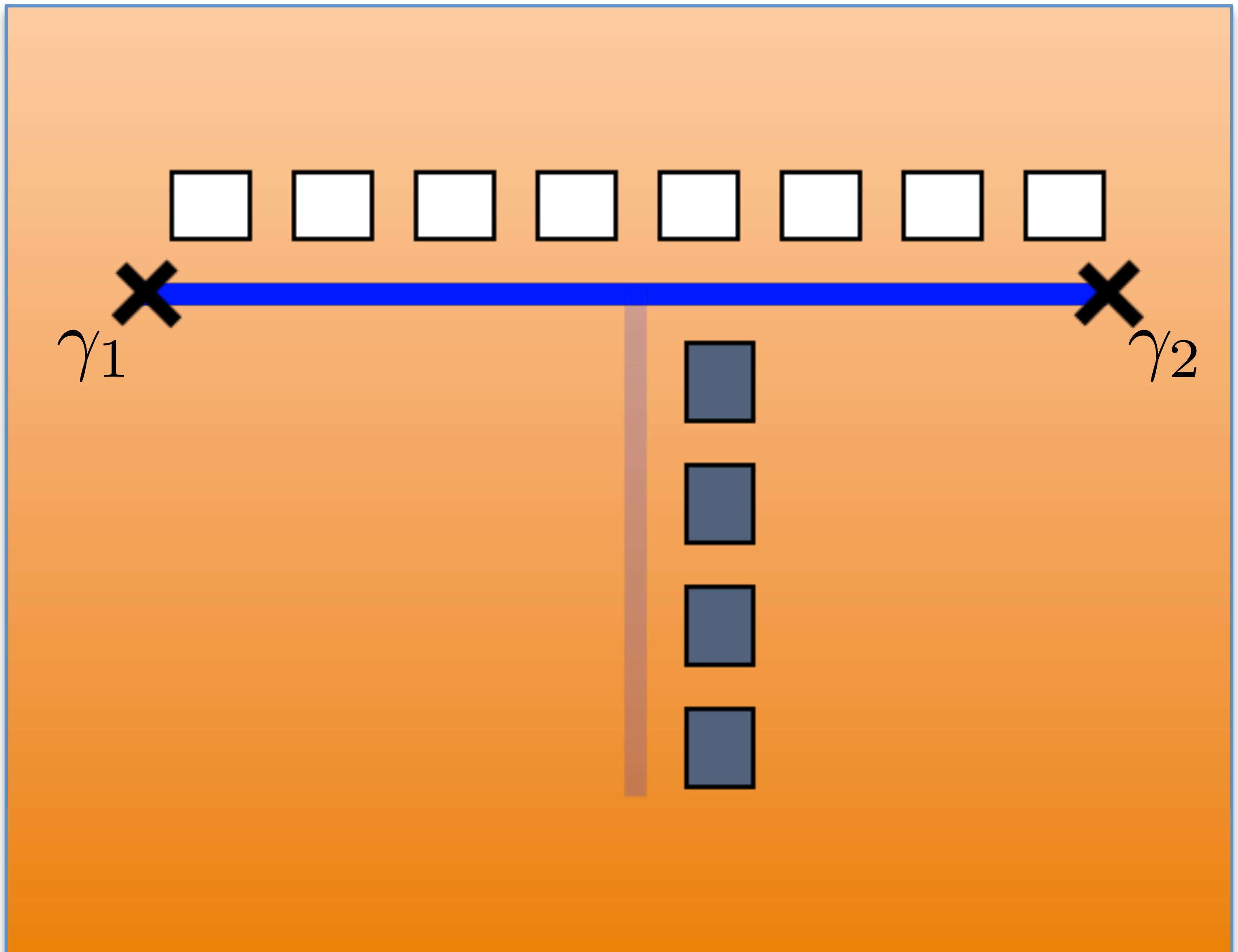


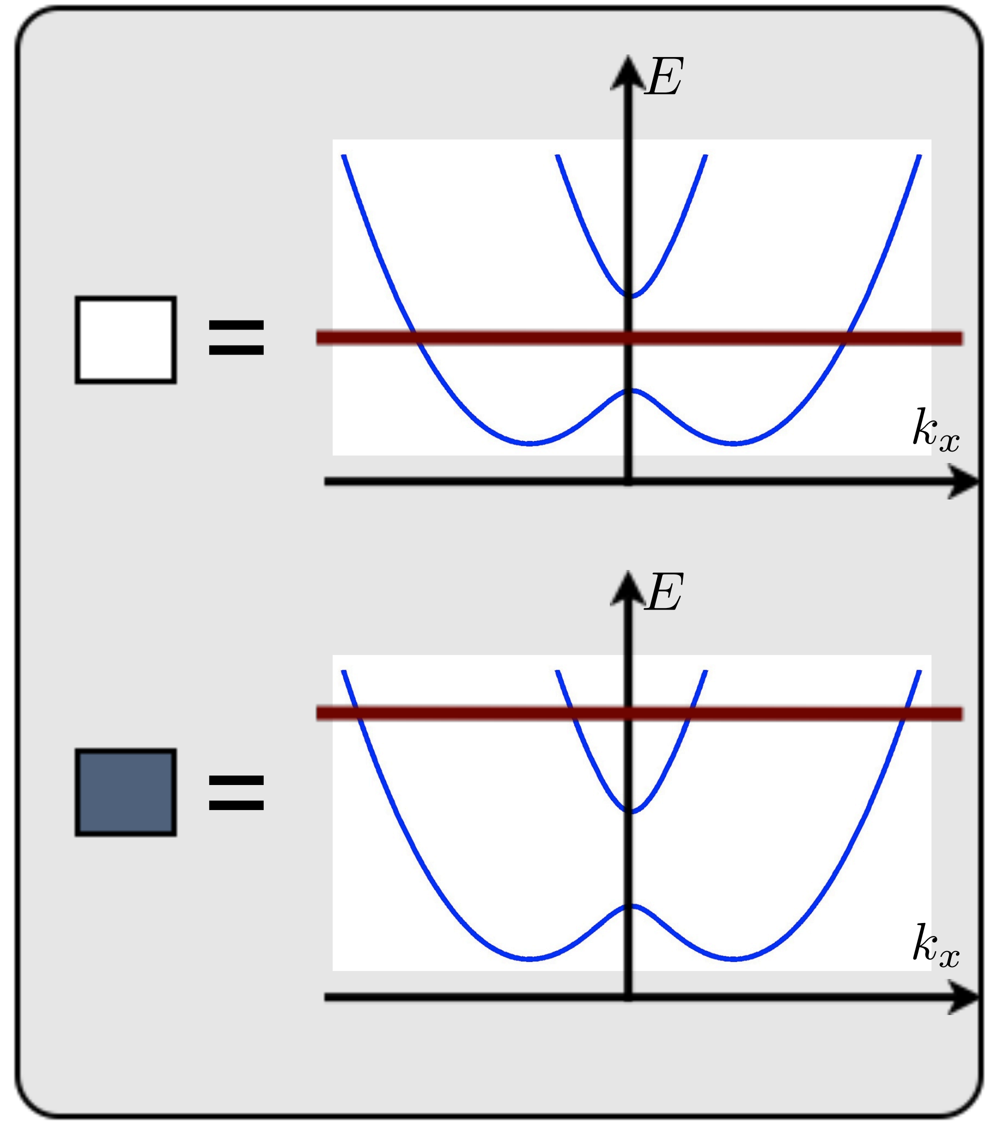
But Majorana modes also occur in 1D topological superconductors, where exchange statistics is ill-defined.

Question: Are Majoranas in 1D as interesting/useful as in 2D?

Answer: YES!!

Harnessing non-Abelian statistics





JA, Oreg, Refael, von Oppen, Fisher, Nature Phys. 2010
Clarke, Sau, Tewari, PRB 2010
Halperin, Oreg, Stern, Refael, JA, von Oppen, PRB 2011

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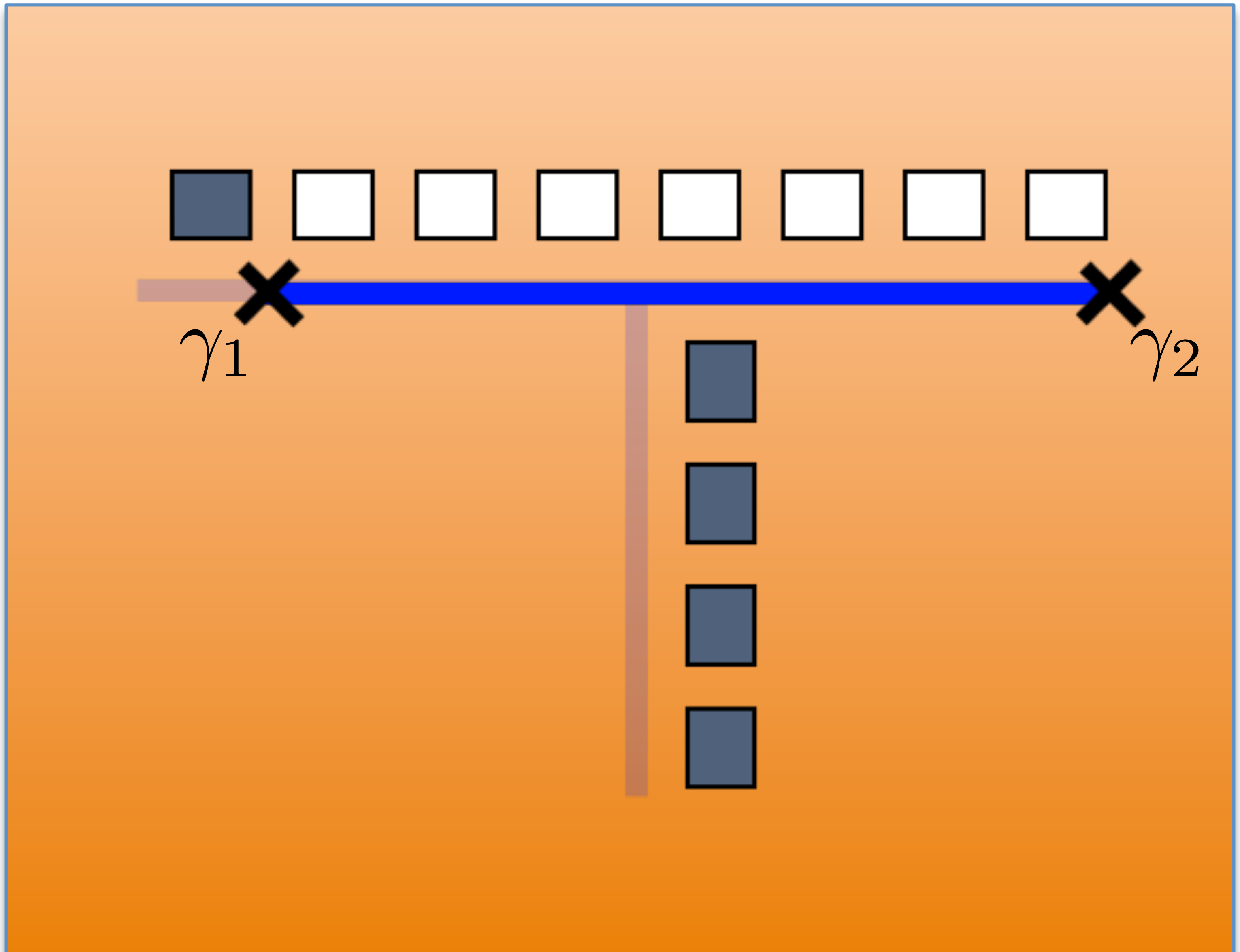

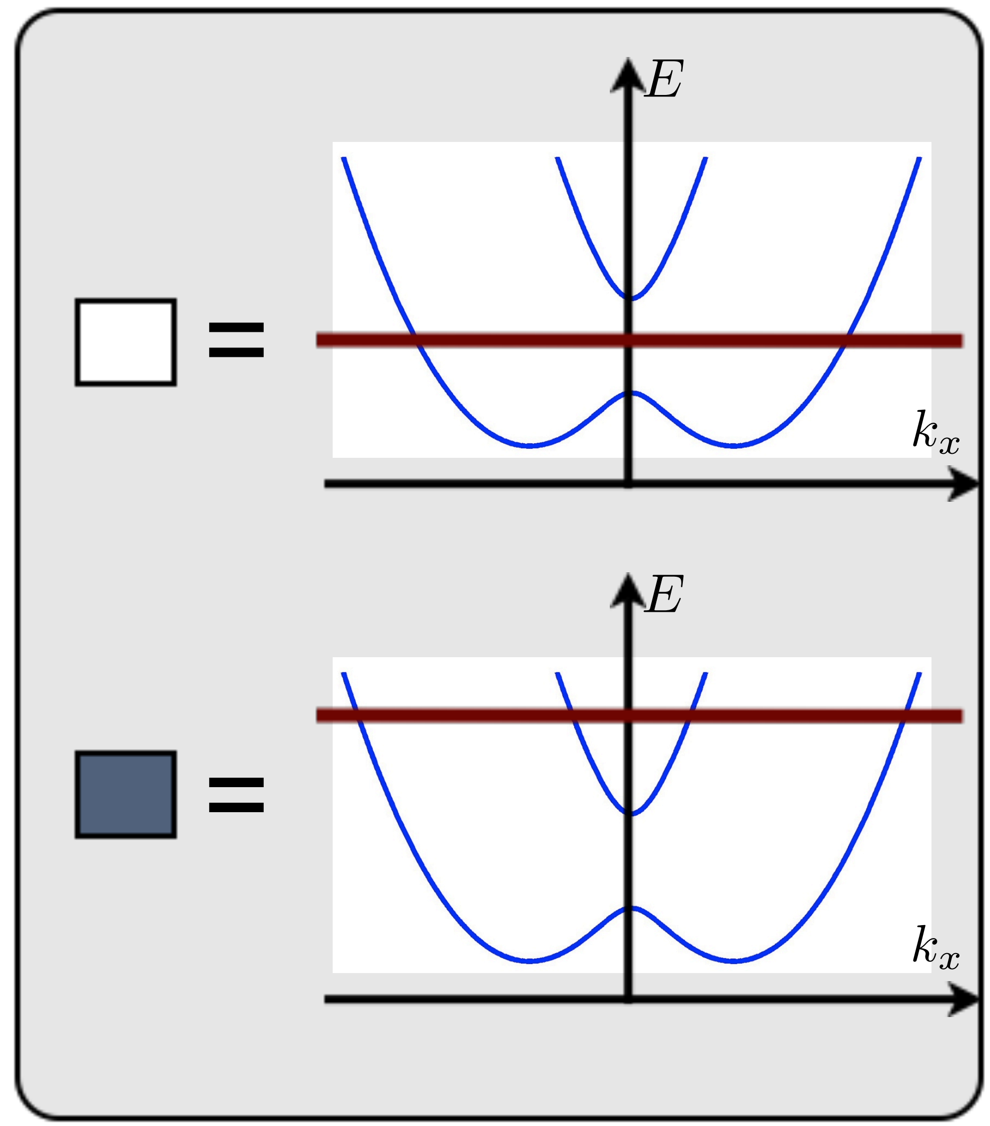

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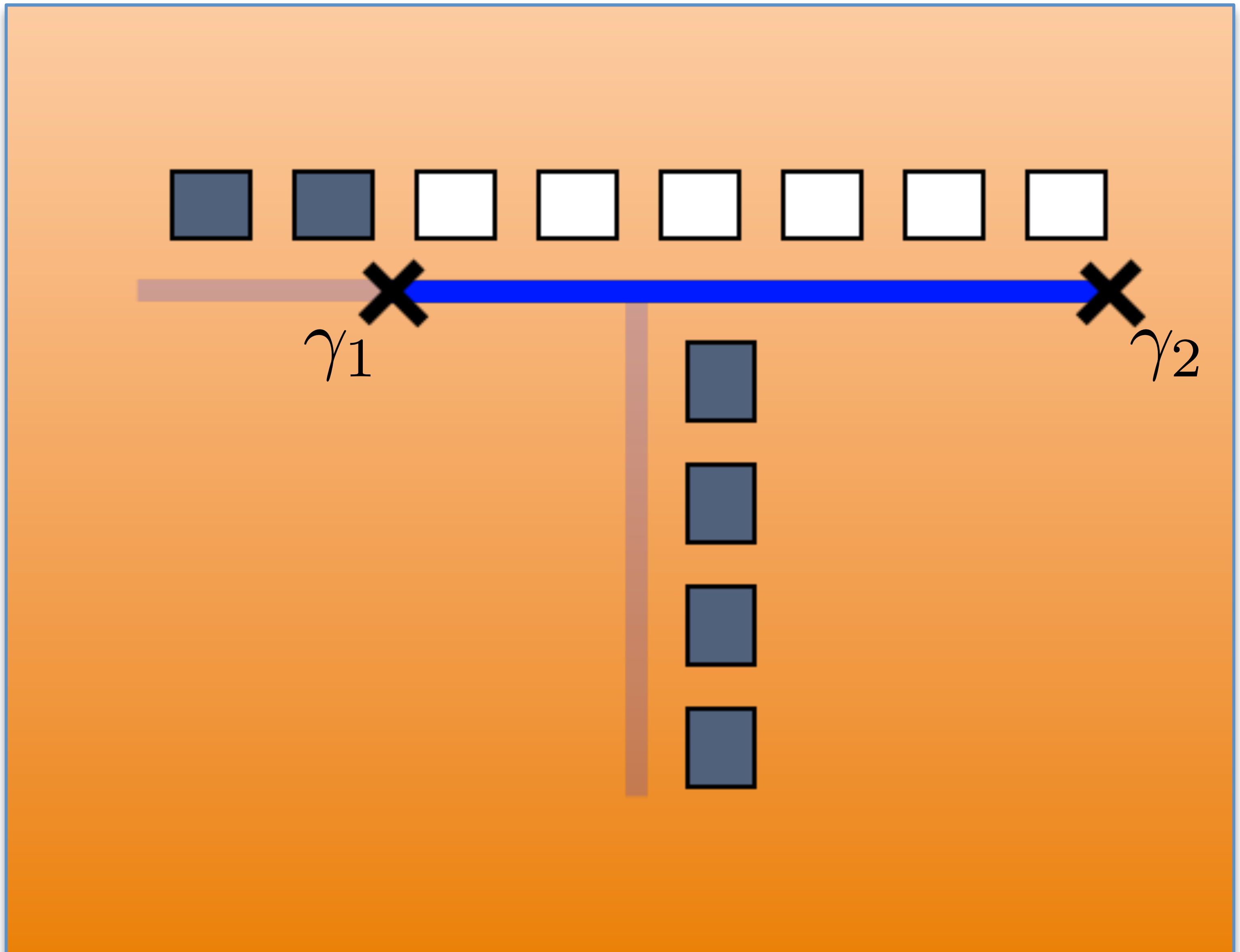

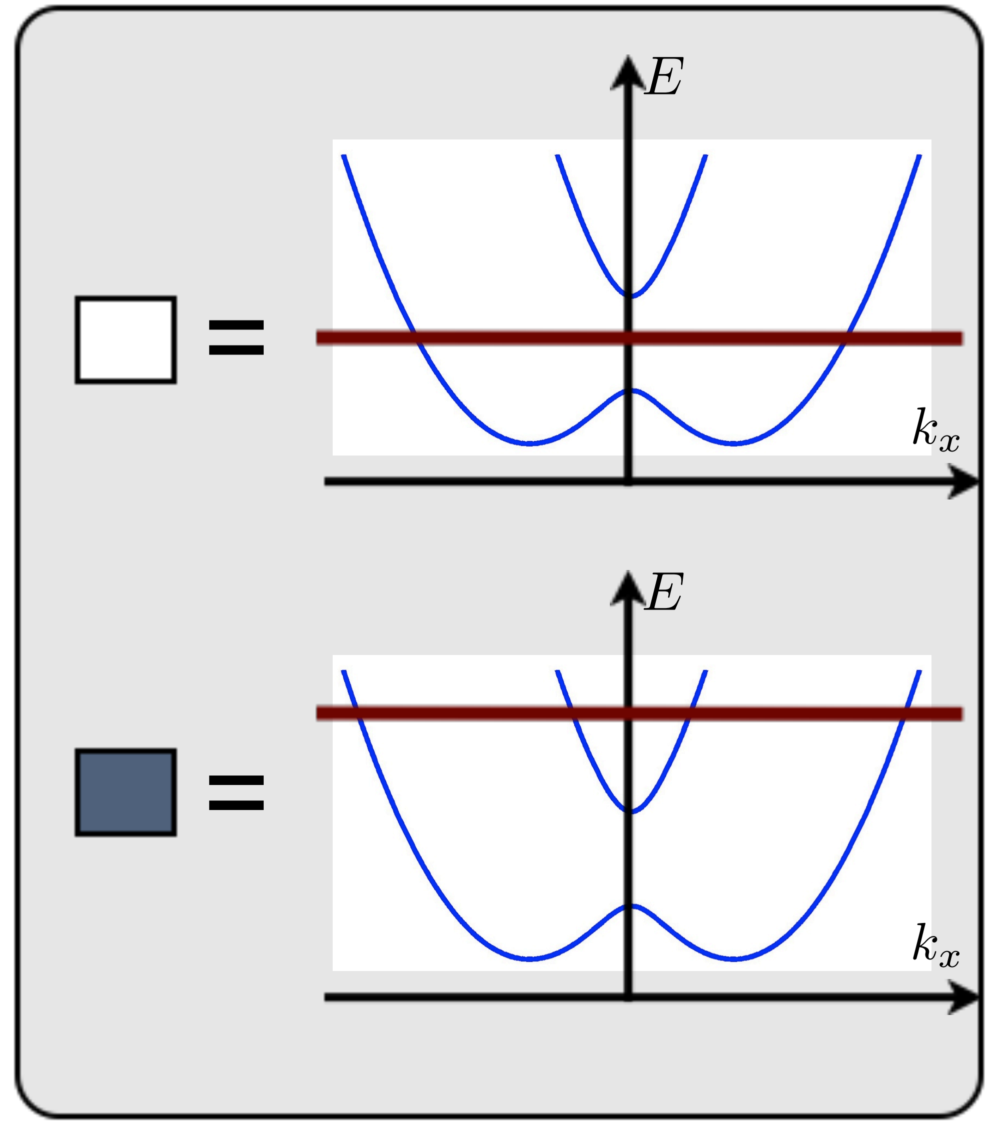

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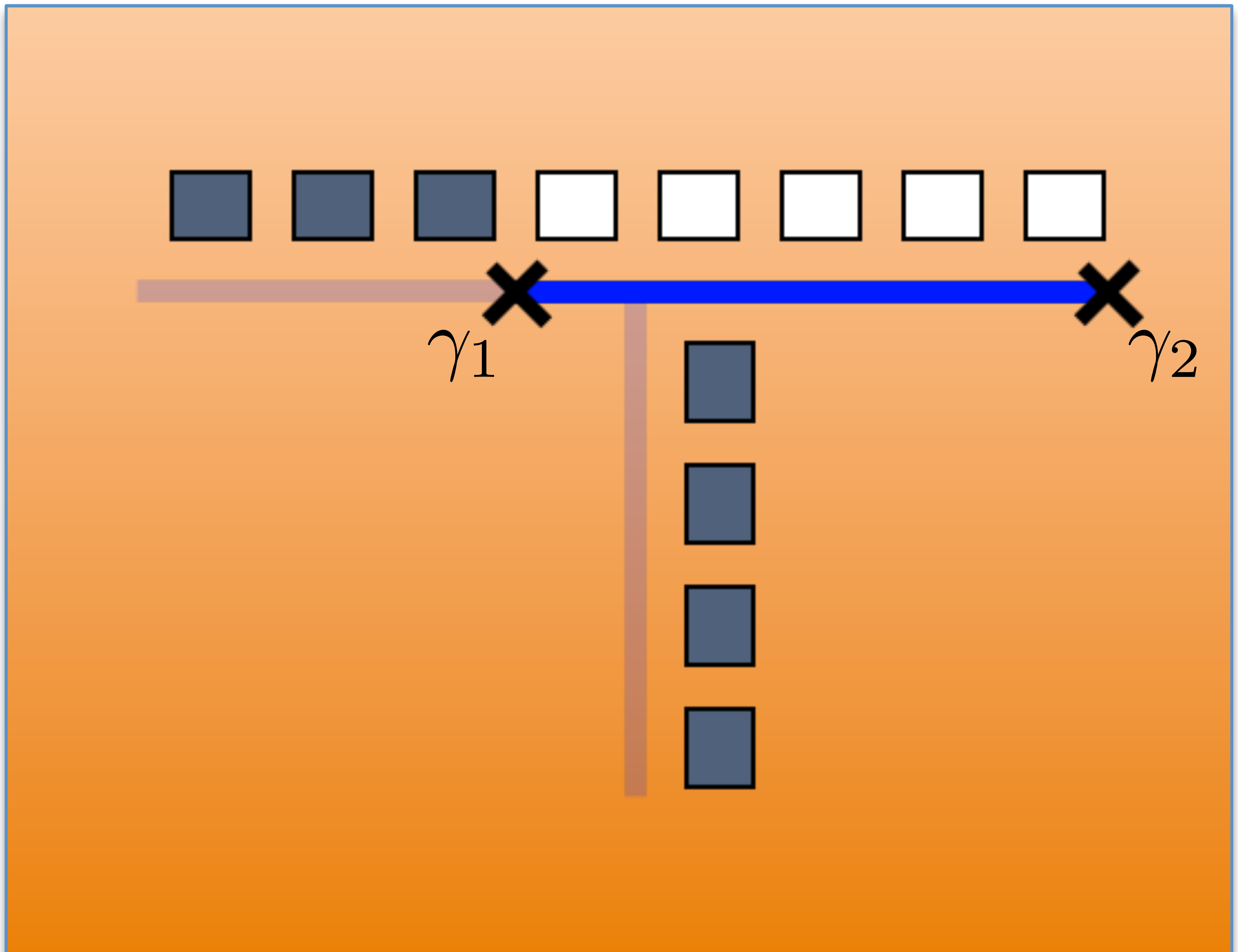

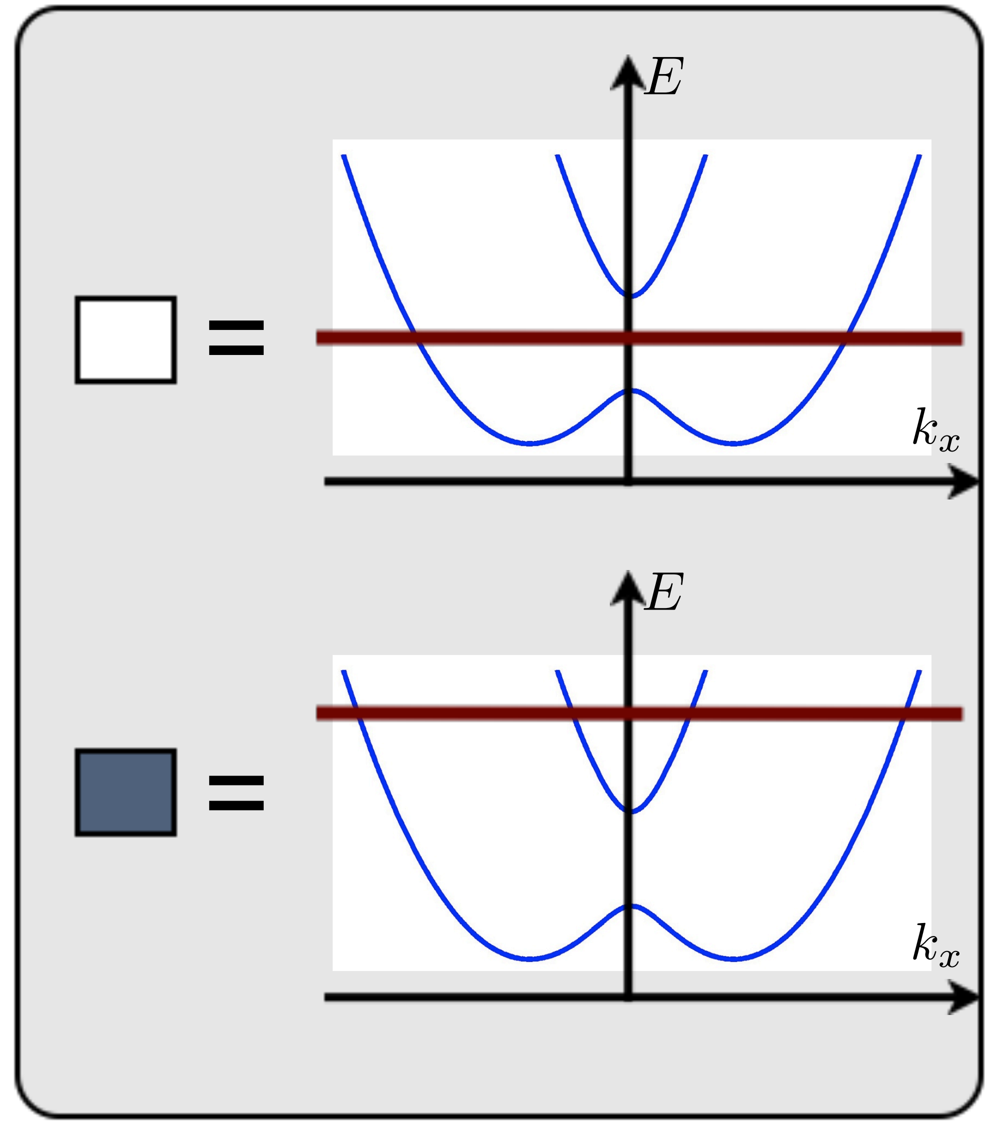

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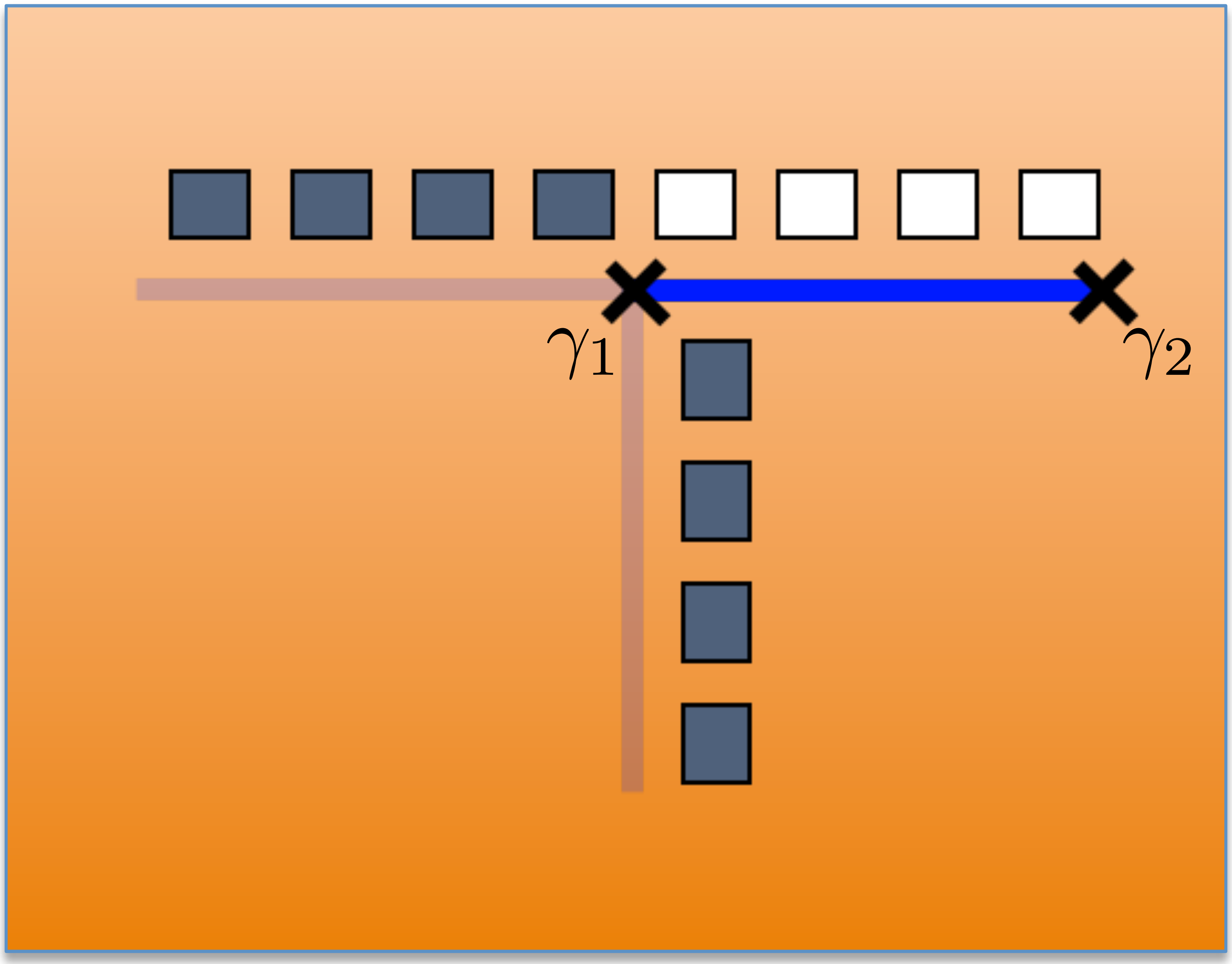

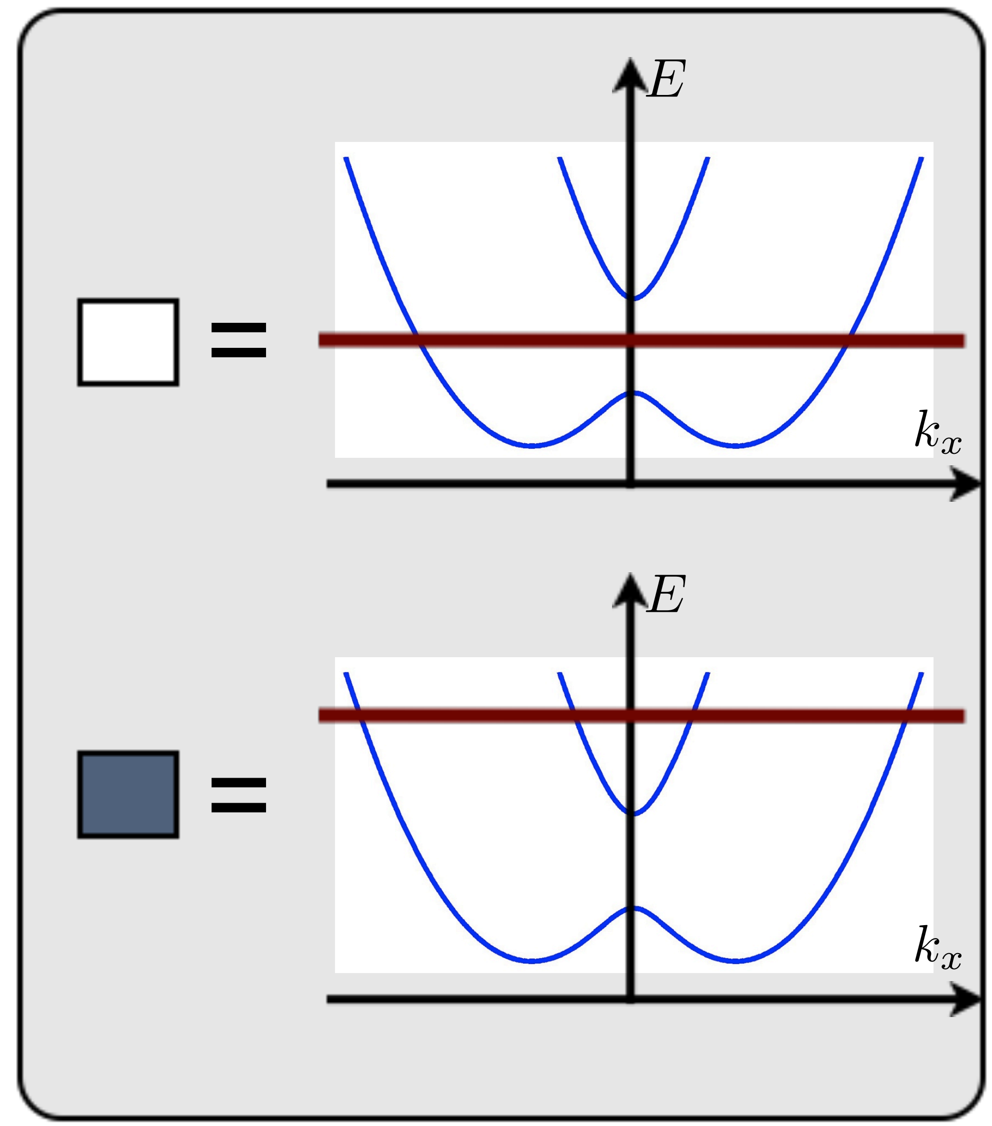

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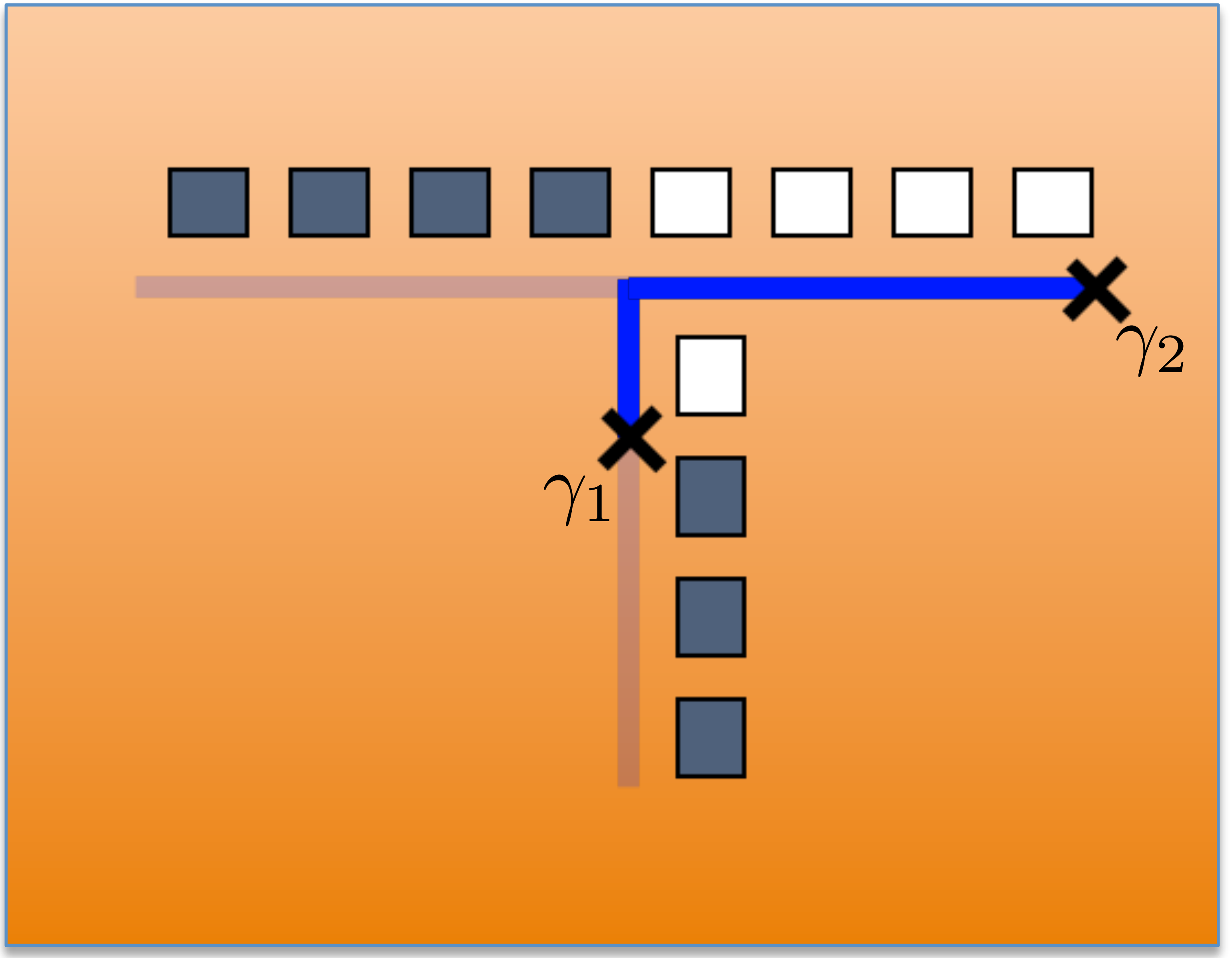

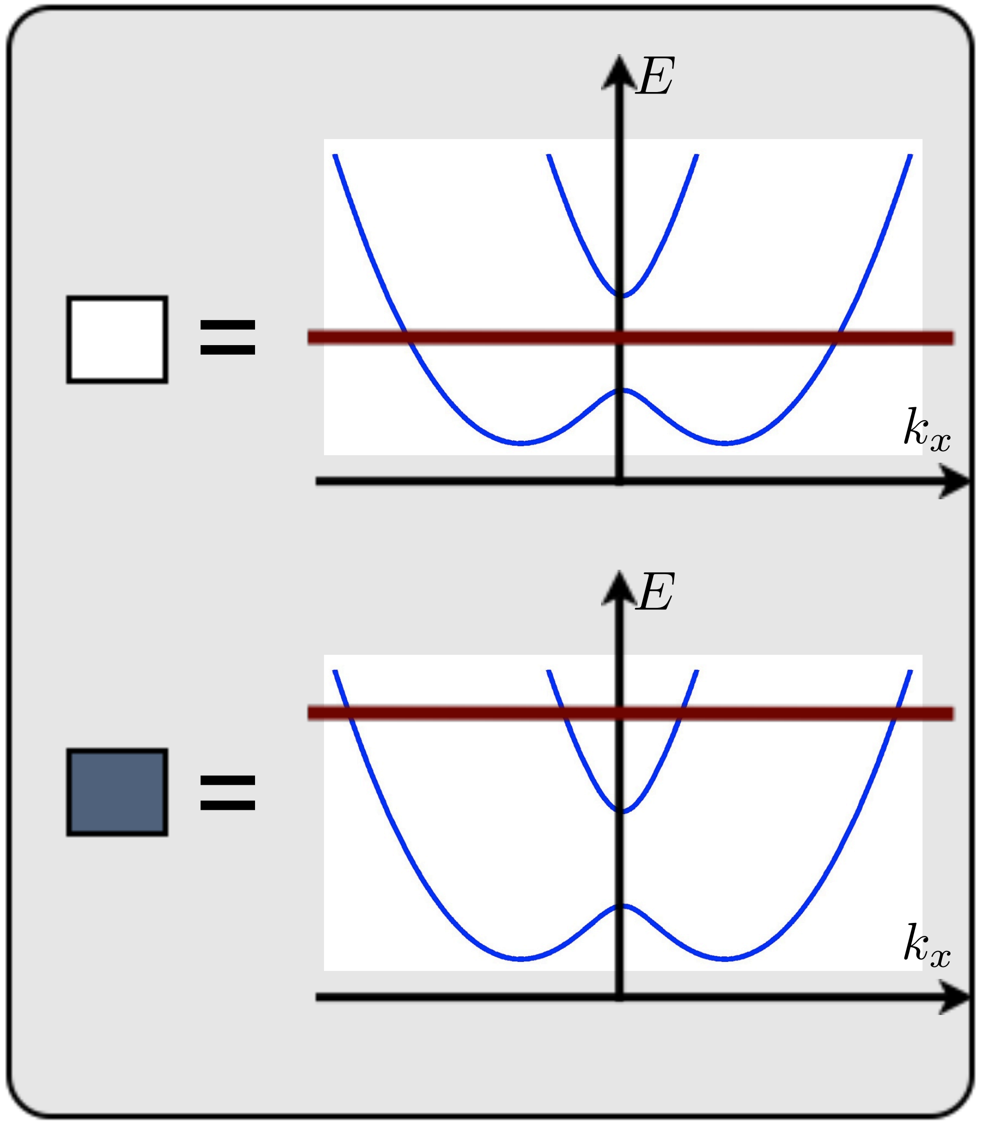

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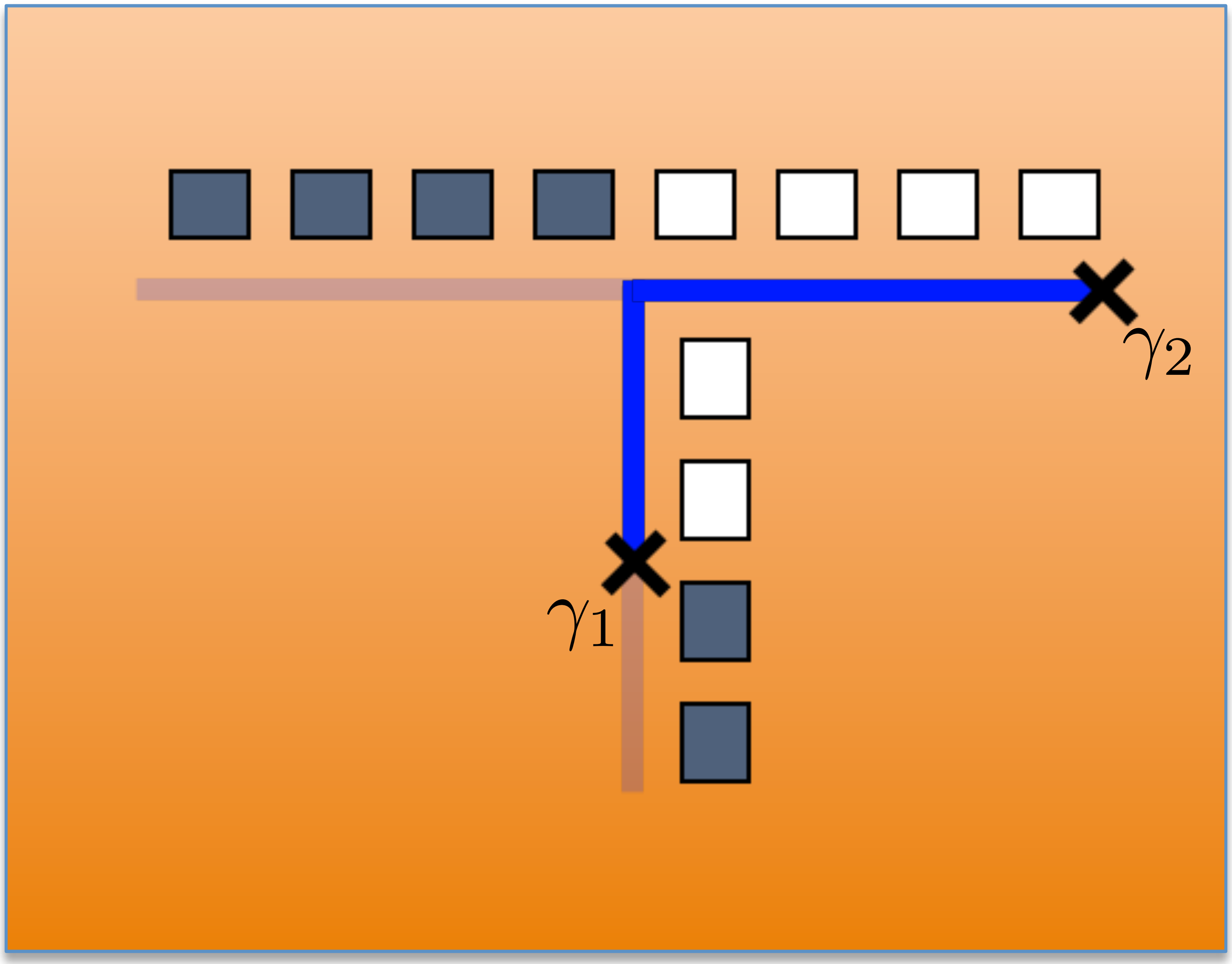

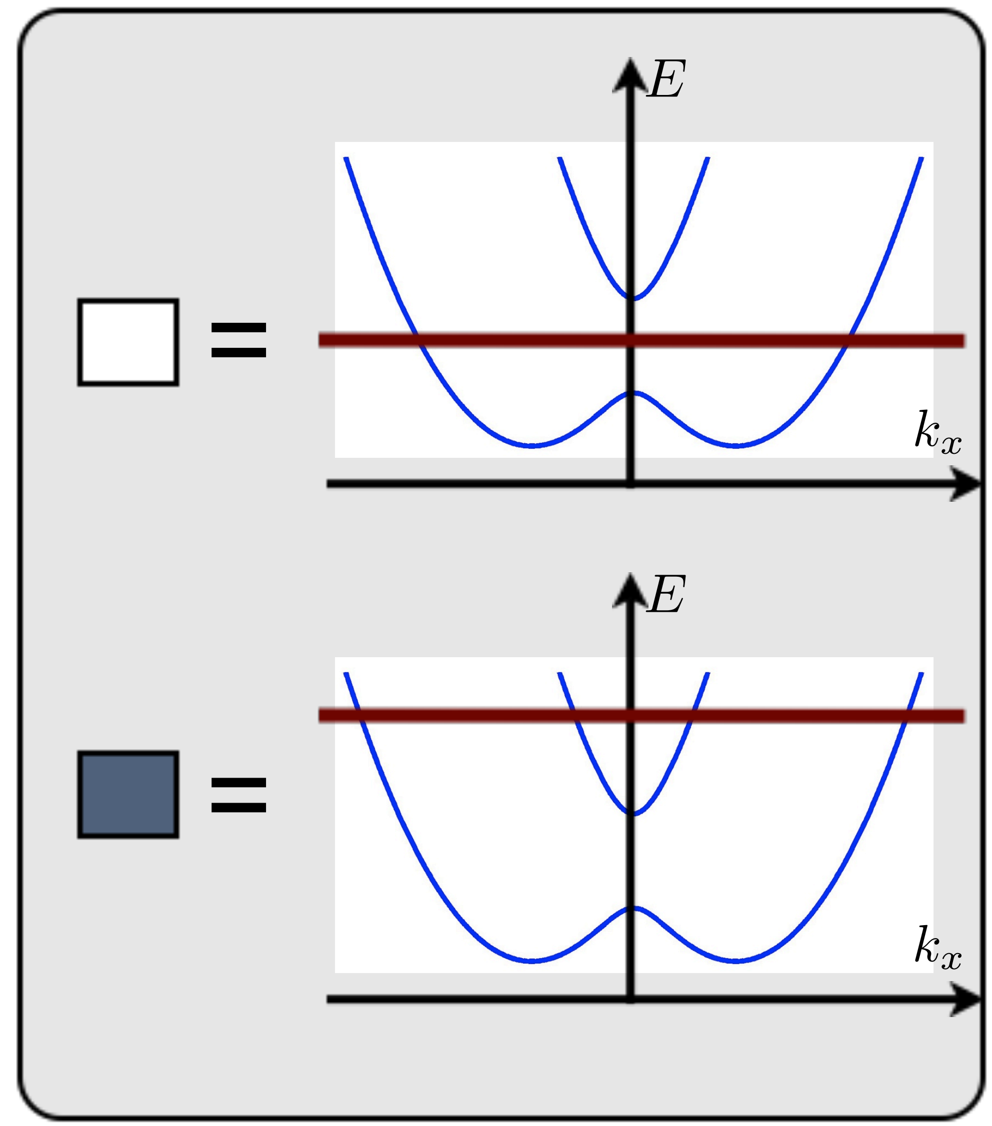

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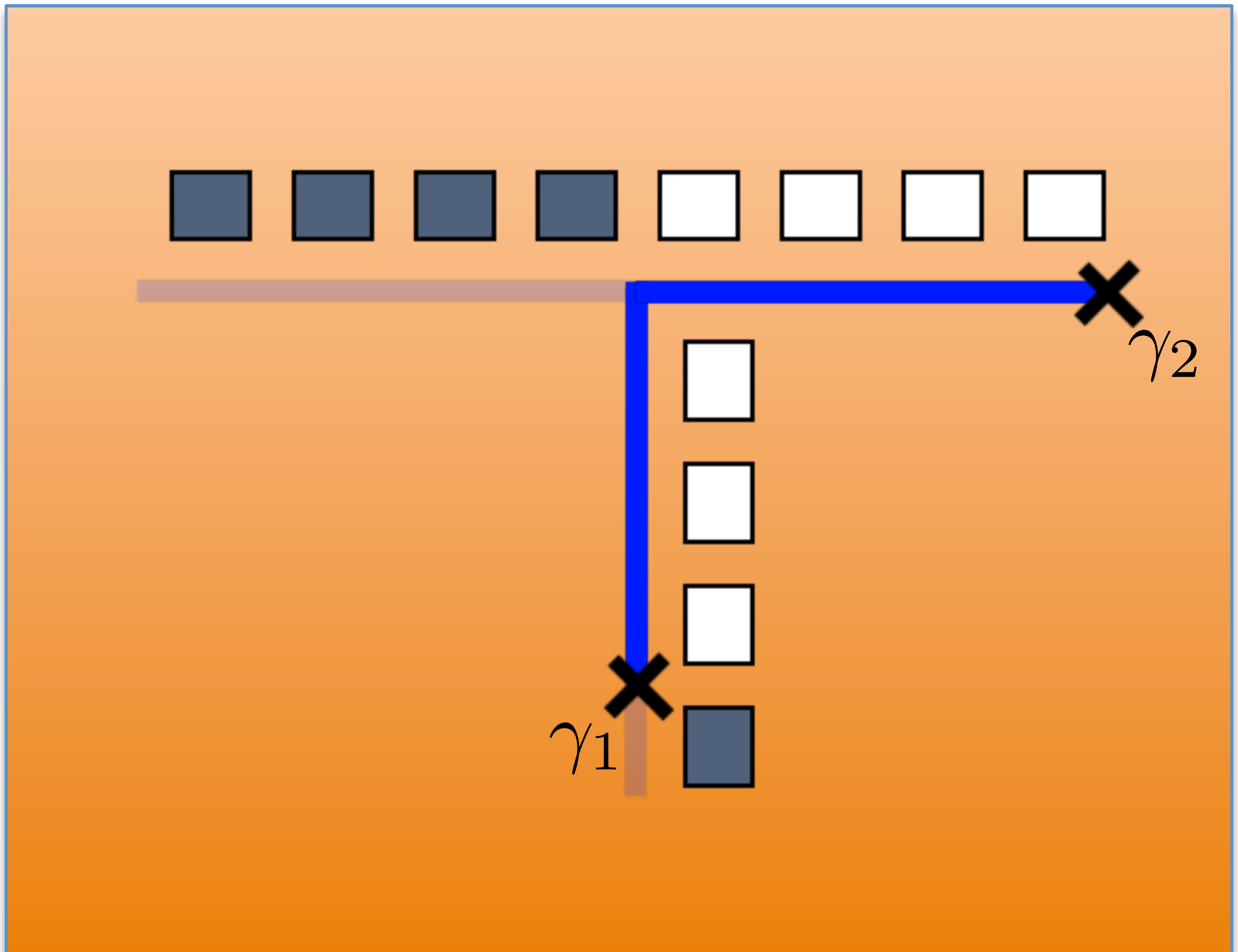

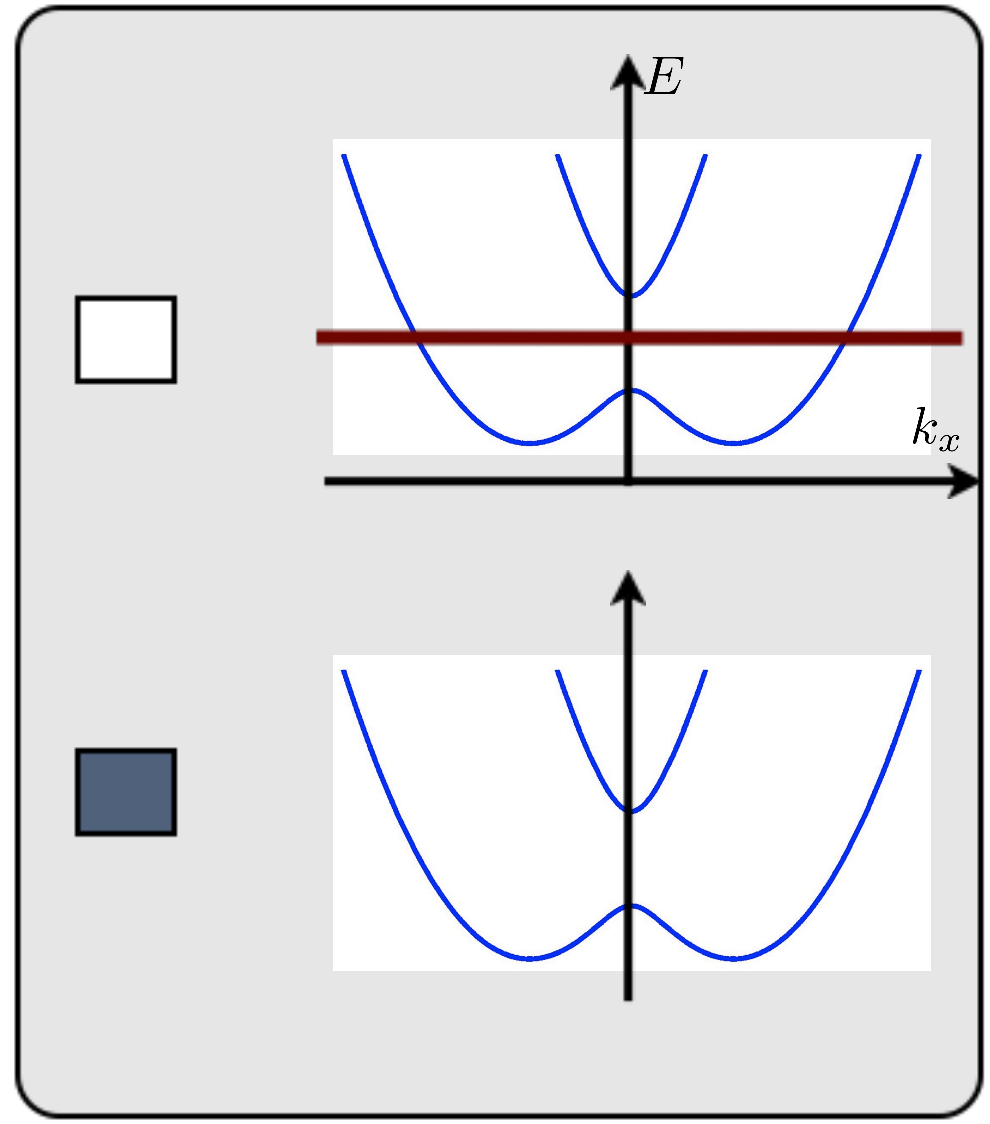

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