Field theories for non-Fermi liquids

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Outline

- Introduction
- Non-Fermi liquids
  - Hot Fermi surface (Ising-nematic QCP)
  - Hot Spots (SDW QCP)
- Field theories for NFL
  - Perturbative approaches
  - Non-perturbative approach

Quantum matter (partial list)

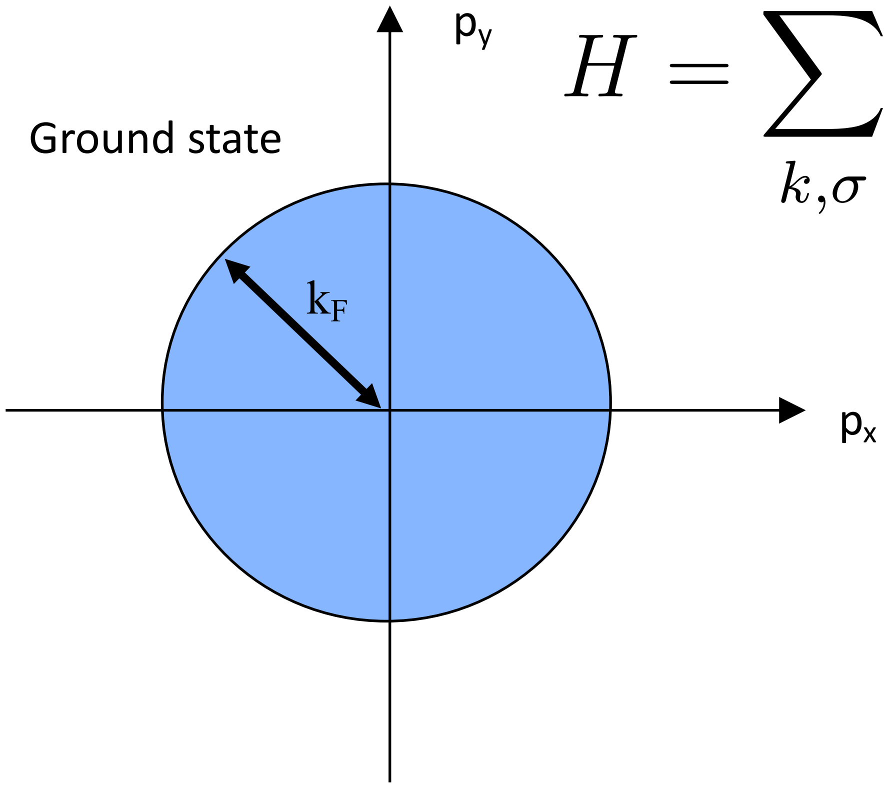
	Entropy	Low-energy Effective Theory
Trivial Insulator	Exp(-Δ/T)	0
Topological States	$(TL)^a$ $a < d$	TQFT / BCFT
Critical states with Lorentz invariance	$(TL)^d$	Relativistic QFT
Fermi surface	$k_F^{d-1} T L^d$ $= (TL)(k_F L)^{d-1}$	QFT with infinitely many degrees of freedom

(T : temperature, L : linear system size, d : space dimension, k_F : Fermi momentum)

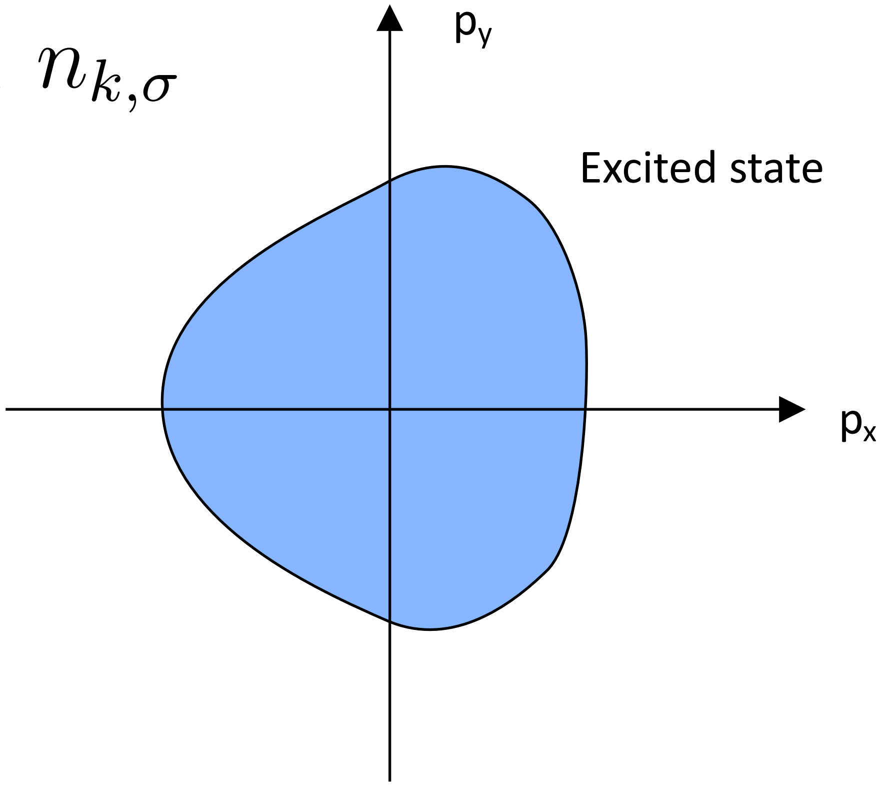
This lecture is about

low-energy effective theories of
strongly correlated metals (non-Fermi liquids)
that arise near QCP

Fermi Gas

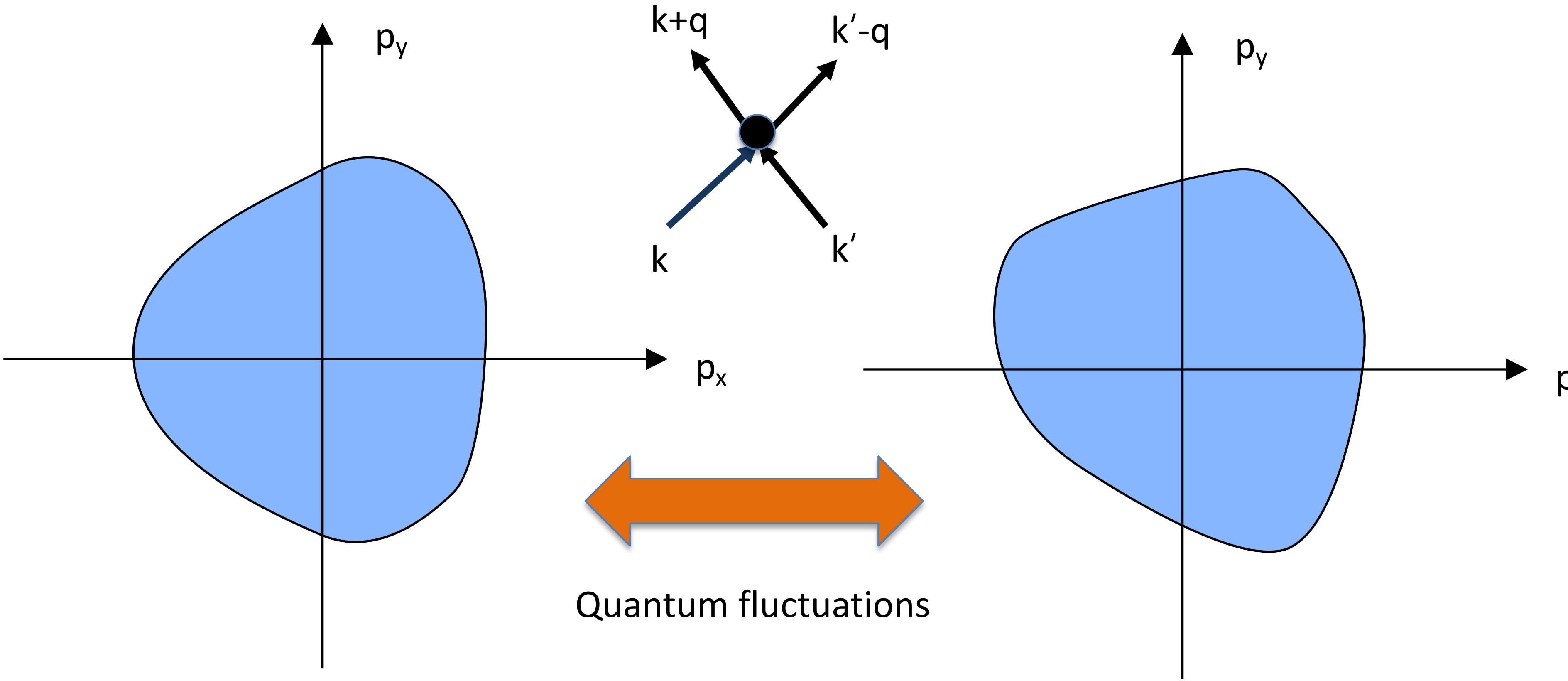


$$H = \sum_{k,\sigma} \epsilon_k^0 \; n_{k,\sigma}$$



Many-body eigenstates are labeled by the occupation numbers of single-particle states $|n_{k_1,\sigma_1}, n_{k_2,\sigma_2}, \dots >$

Interacting Fermions



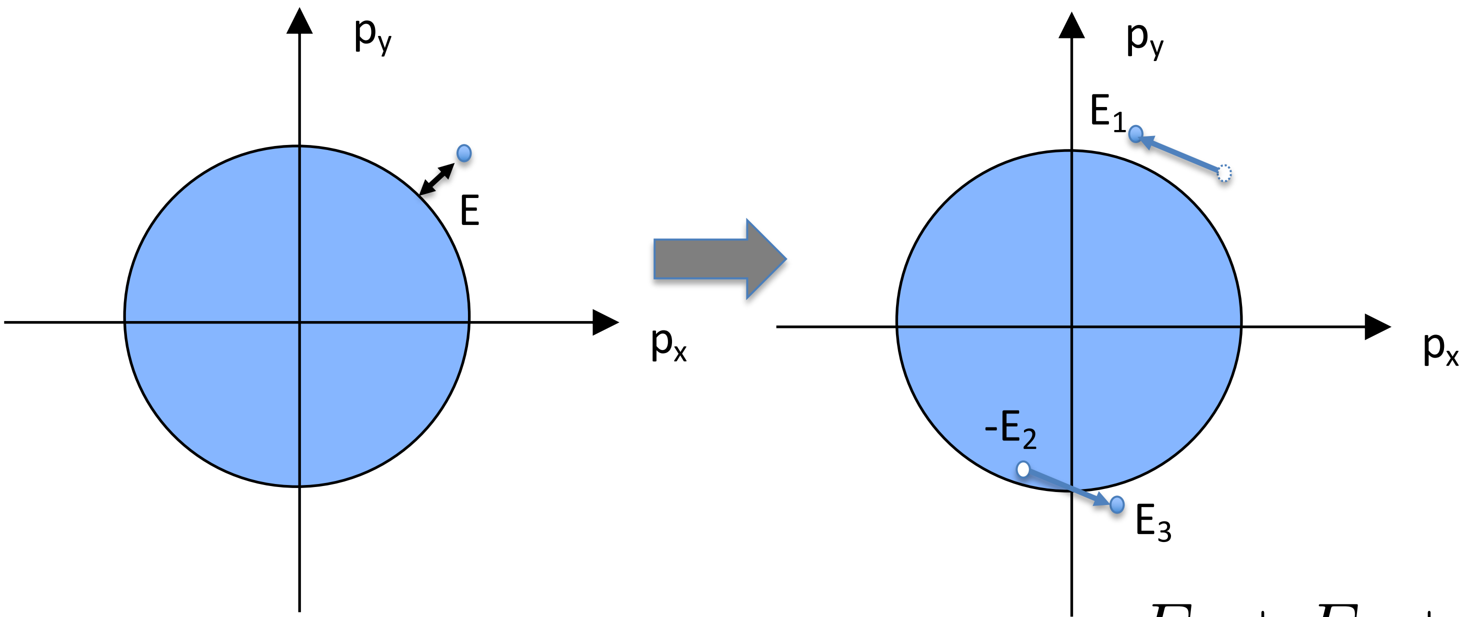
$$H' = \sum_{k,k',q} V_{k,k',q} \; c^{\dagger}_{k'+q} c^{\dagger}_{k-q} c_k c_{k'}$$

Shape of Fermi surface is subject to quantum fluctuations

Fermi Liquids

[Landau]

[Shankar, Polchinski]



$$E_1 + E_2 + E_3 = E$$

$$E_1, E_2, E_3 \geq 0$$

Particles close to the Fermi surface have long life-time

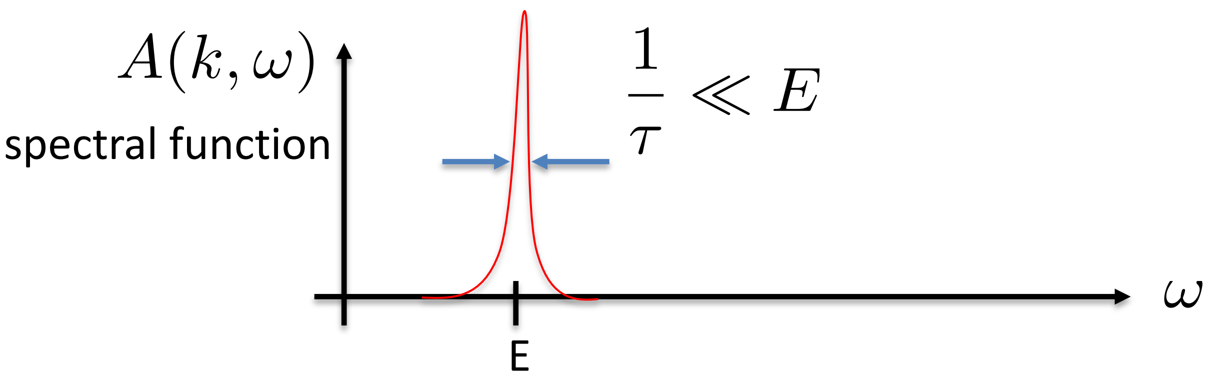
$$\frac{1}{\tau} = \alpha V^2 E^2$$

V : microscopic interaction
α : kinematic constants
(FS shape, size, velocity)

Fermi Liquids

[Landau]

[Shankar, Polchinski]



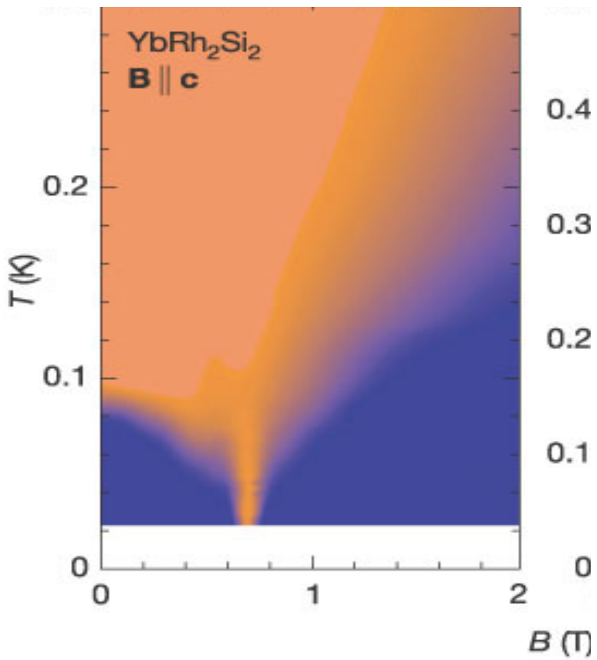
- Low-energy eigenstates of interacting electrons are labeled by the occupation numbers of single-particle states

$$|n_{k_1,\sigma_1}, n_{k_2,\sigma_2}, \dots >'$$

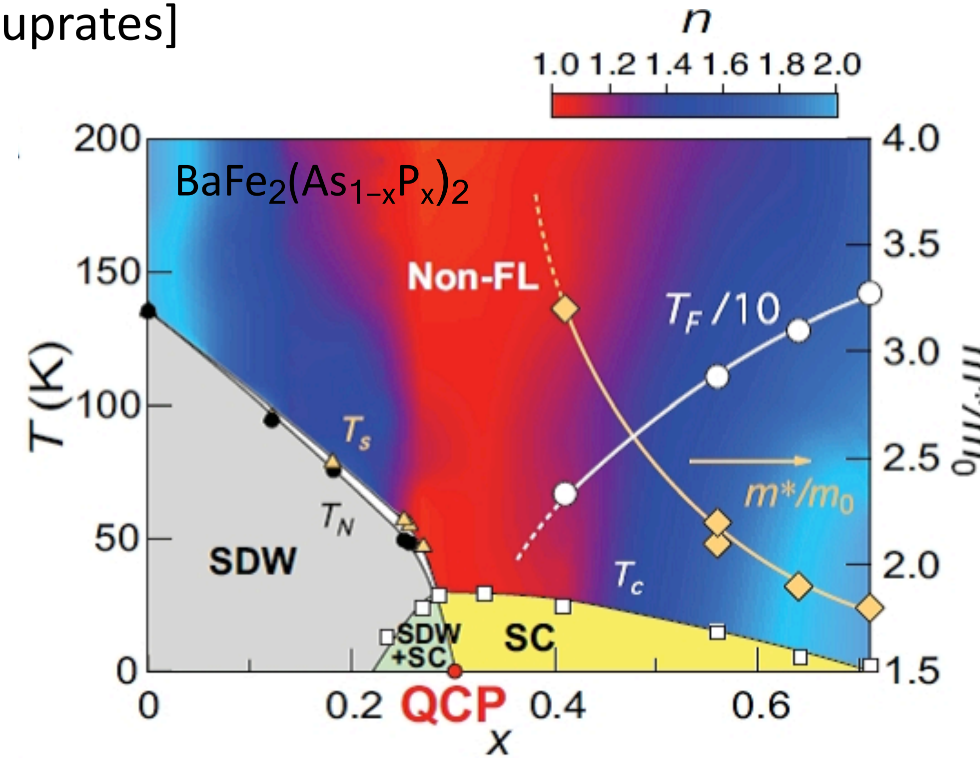
- The well-defined single-particle excitations are **quasiparticles** with renormalized mass
- In Fermi liquids, low temperature properties of interacting electrons are qualitatively similar to those of free fermions
  - Specific heat : C ~ T
  - Magnetic susceptibility : χ ~ const.

Breakdown of Fermi liquid near Quantum Critical Points

[heavy fermion; pnictides; cuprates]



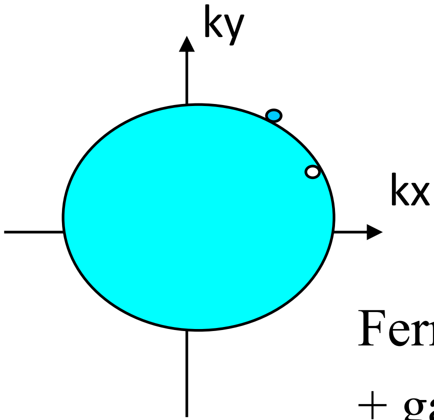
[Custers et al.(2003)]



[Hashimoto et al. Science 336, 1554 (2012)]

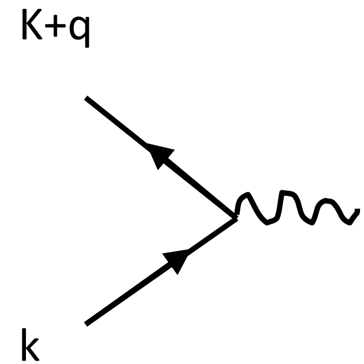
- Experimentally, NFL's are often characterized by anomalous thermodynamic / transport properties
- Spectroscopic evidences, while being more direct, are rarer

NFL @ QCP



Fermi surface

+ gapless collective mode



- At QCP, order parameter becomes gapless collective mode that mediates singular interactions between electrons
- Single-particle excitations created near FS no longer have long lifetime if the interaction is singular enough to generate strong non-forward scatterings

$$\frac{1}{\tau} = \alpha V(E)^2 E^2 > E$$

NFL's are described by interacting field theories that are not diagonalizable in single-particle basis

Two important factors that determines the nature of NFL near QCP

- space dimension
- wavevector of gapless collective mode

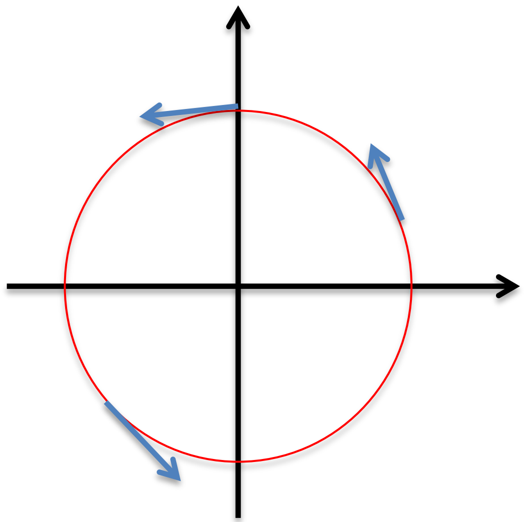
Space dimension

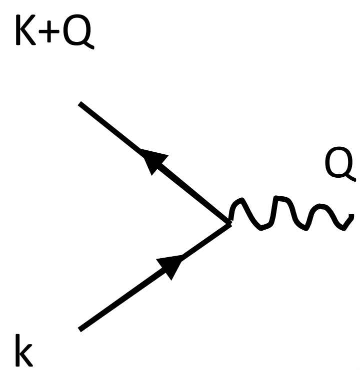
- 3d : quantum fluctuations are relatively weak
- 1d : no extended Fermi surface
- 2d : most challenging & interesting :
  - Extended Fermi surface
  - Strong quantum fluctuations at low energies

❊ We will focus on NFLs in d=2.

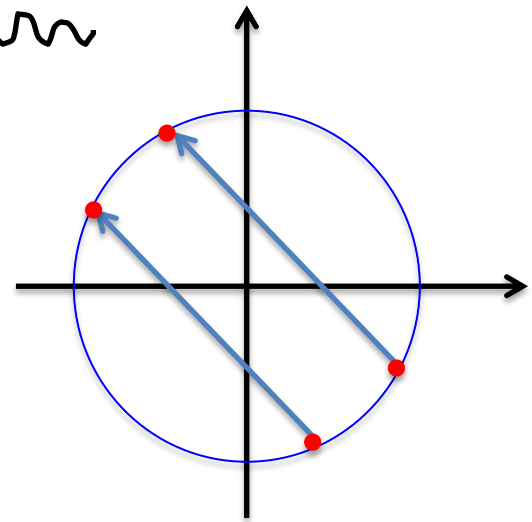
Wavevector of gapless collective mode (Q)

Hot Fermi surface





Hot spot



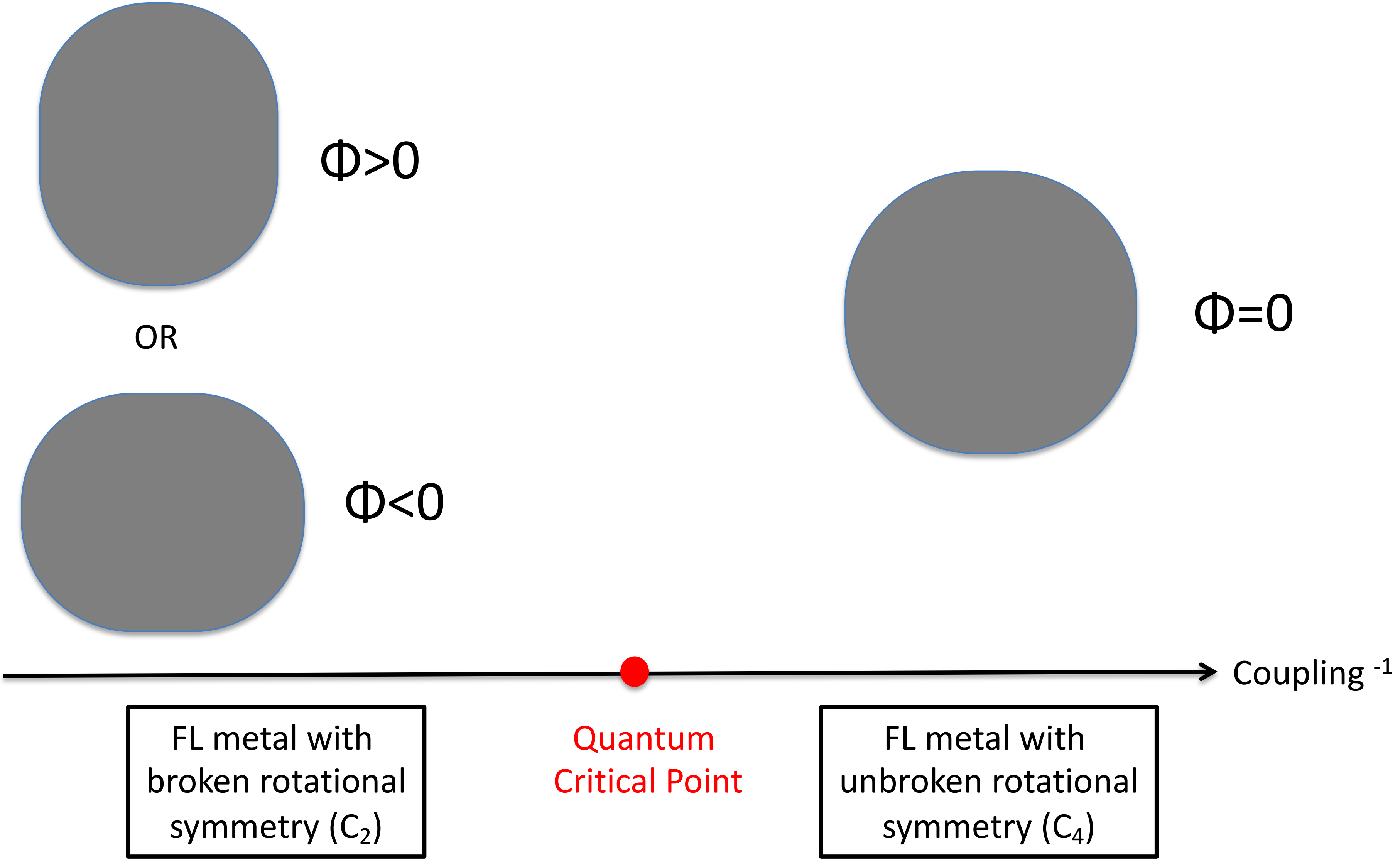
Q=0
Nematic, ferromagnetic QCP,
Spin liquids with emergent gauge boson,..

Q≠0
SDW, CDW,.. QCP

Hot Fermi Surface

Ising-nematic quantum critical metal

Ising-Nematic QCP



Low-energy theory at QCP

$$S = \sum_{j=\uparrow,\downarrow} \int d^3K \ c_j^\dagger(K) \left[iK_0 + \epsilon_{\vec{K}} \right] c_j(K)$$

Kinetic energy of electron

$$+ \ \frac{1}{2} \int d^3q \ \ [q_0^2 + c^2 |\vec{q}|^2] \ |\phi(q)|^2$$

Kinetic energy of the critical Ising order parameter

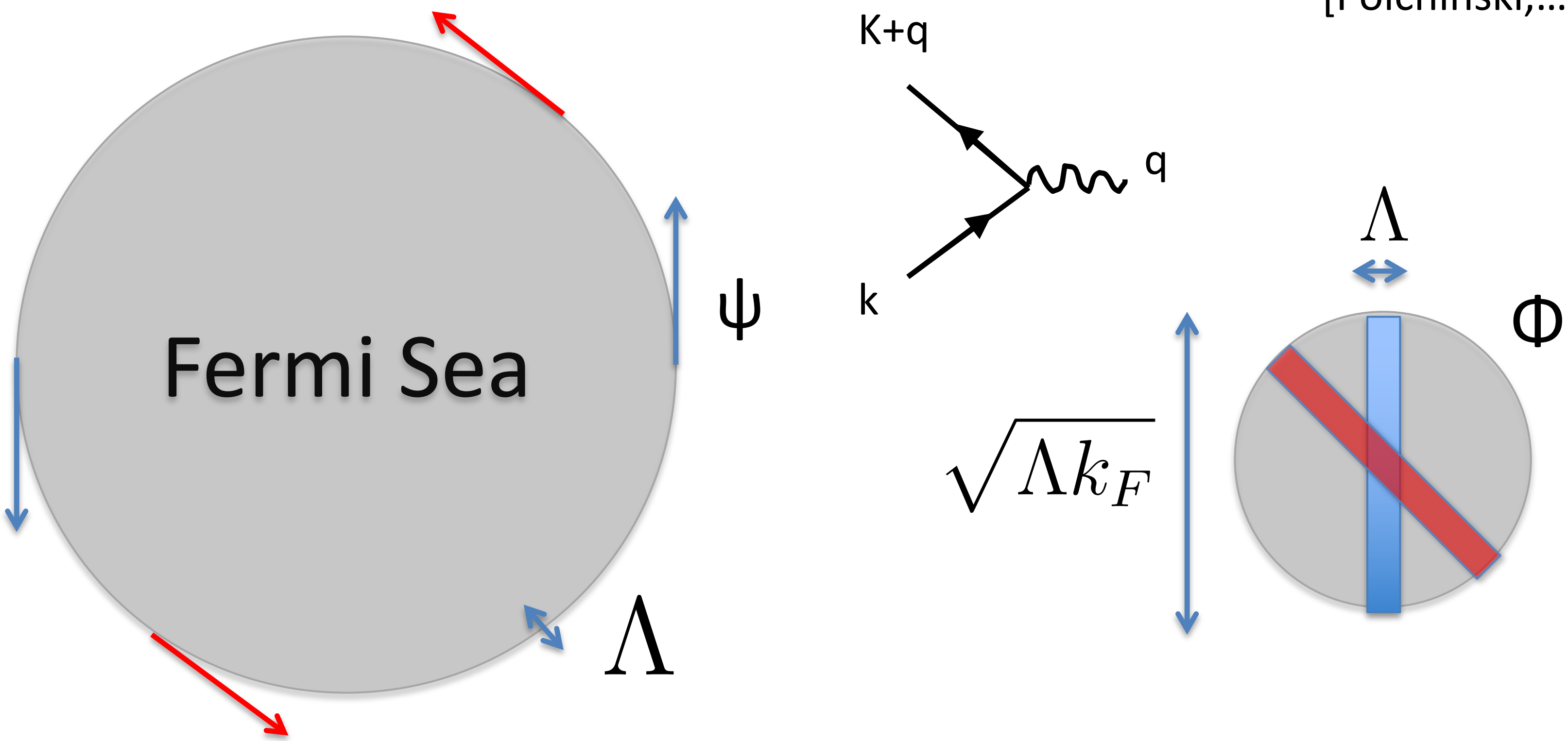
$$+ \ e \int d^3K d^3q \ \ (\cos K_x - \cos K_y) \phi(q) c_j^\dagger(K+q) c_j(K)$$

Coupling between electrons and the order parameter

The order parameter couples to fermions as a momentum-dependent chemical potential, which deforms the Fermi surface in the l=2 channel.

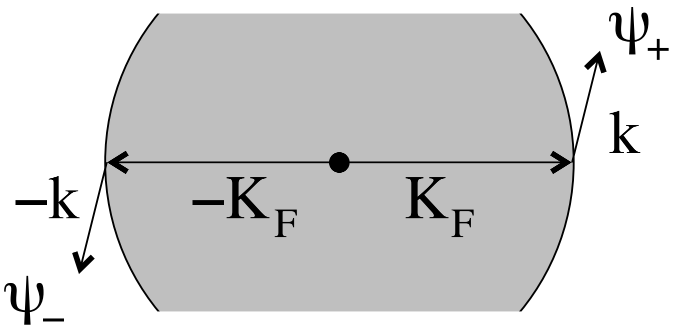
Emergent locality in momentum space

[Polchinski,...]



- At low energies, fermions are primarily scattered along the directions tangential to FS
- Fermions with non-parallel tangential vectors are decoupled from each other in the low-energy limit (modulo pairing interaction in the presence of superconducting instability)

Minimal theory : two-patch theory



$$S = \sum_{s=\pm}\sum_{j=1}^{N}\int d^3k\psi^\dagger_{s,j}(k)\left[ik_0 + sk_1 + k_2^2\right]\psi_{s,j}(k)$$
$$+ \frac{1}{2}\int d^3q\,\left[q_0^2 + c^2|\vec{q}|^2\right]\phi(-q)\phi(q)$$
$$+ \frac{e}{\sqrt{N}}\sum_{s=\pm}\sum_{j=1}^{N}\int d^3kd^3q\;\phi(q)\;\psi^\dagger_{s,j}(k+q)\psi_{s,j}(k)$$

*spin flavor generalized to N

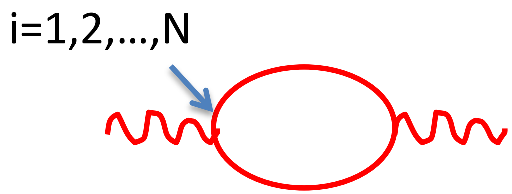
- $\psi_{s,j}(k)$ represents the electron field defined near two opposite points of FS (patches s=±), where k is momentum measured from the center of each patch
- The fermion-boson coupling is **relevant**, and the theory becomes strongly coupled at low energies : a small parameter is needed for controlled expansion

Perturbative approaches

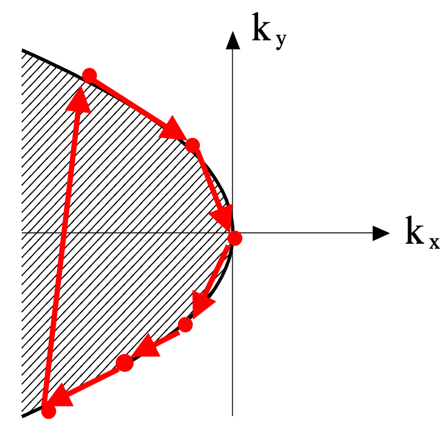
1/N - expansion

[Altshuler, Ioffe, Millis; Kim, Furusaki, Wen, Lee, Polchinski,…]

- Collective modes are heavily damped with fermions

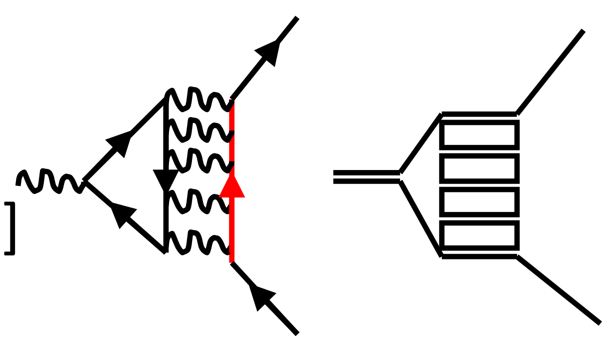


- In relativistic QFT, quantum fluctuations are tamed in the large N limit



- However, this is not the case in the presence of FS : fluctuations are amplified at low energies as fermions are scattered along the Fermi surface

- All planar graphs (in the one-patch theory) and beyond (in the two-patch theory) remain important [similar to a matrix model]

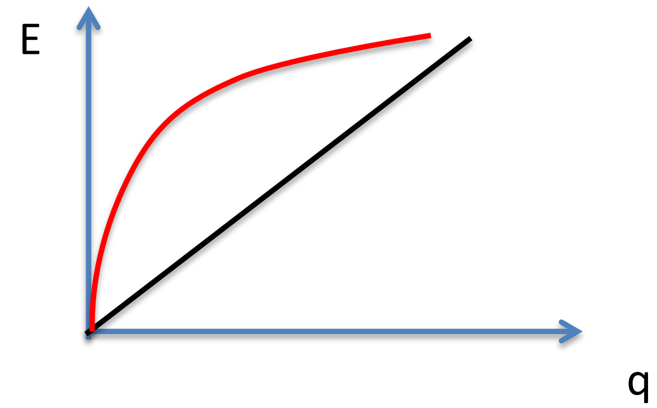


[Lee(09); Metlitski, Sachdev (10)]

Dynamical tuning

[Nayak, Wilczek(94); Mross, McGreevy, Liu, Senthil(10)]

$$|q|^2\phi^2 \rightarrow |q|^{1+\epsilon}\phi^2$$

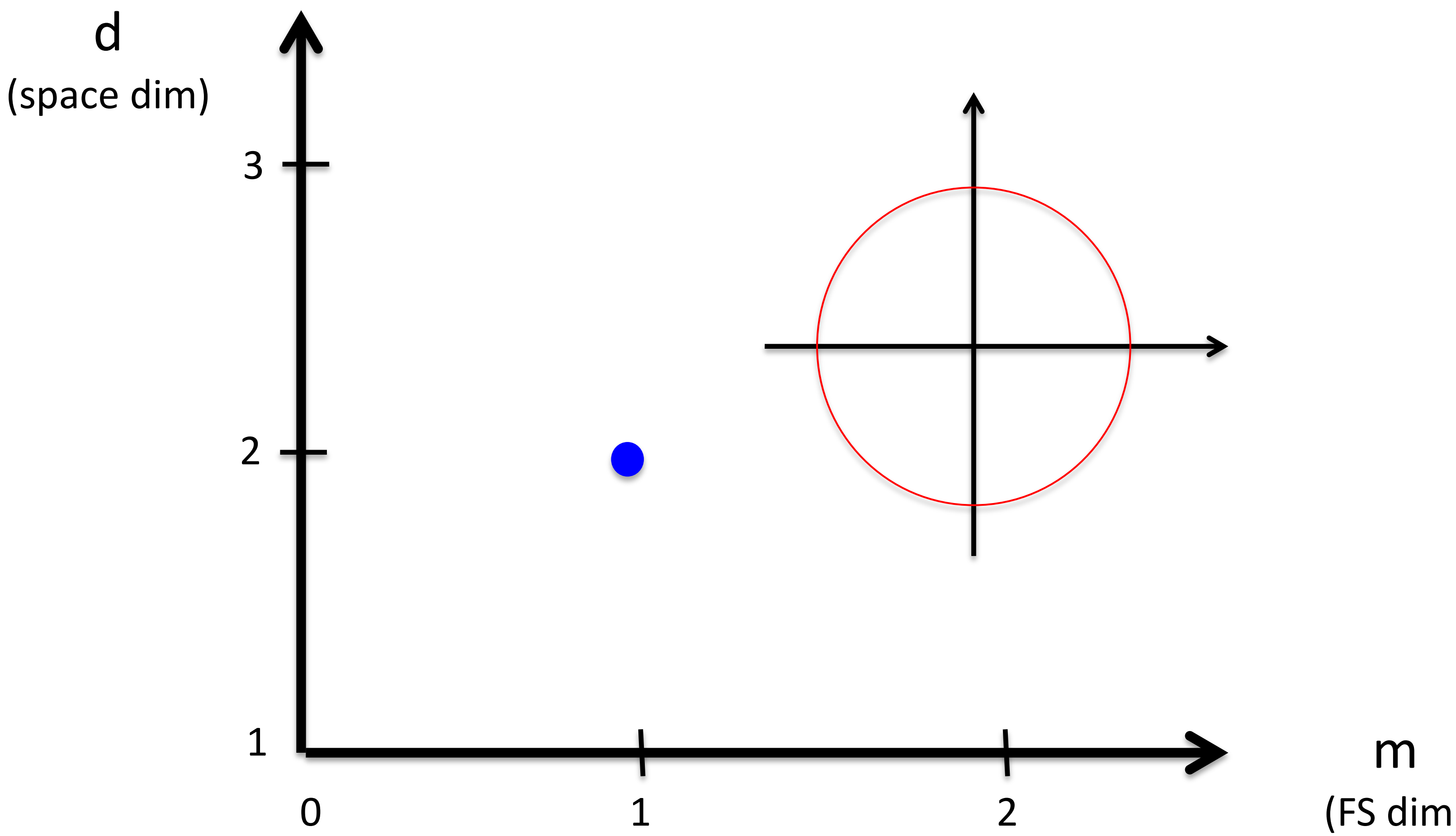


- Tame quantum fluctuations by suppressing DOS of critical boson
- All symmetries kept
- Breaks locality of the theory : the bosonic mode can not have an anomalous dimension perturbatively

$$\frac{|q|^{1+\epsilon+\eta}}{\Lambda^\eta} = q^{1+\epsilon}\,(1 + \eta \ln q/\Lambda + ..)$$

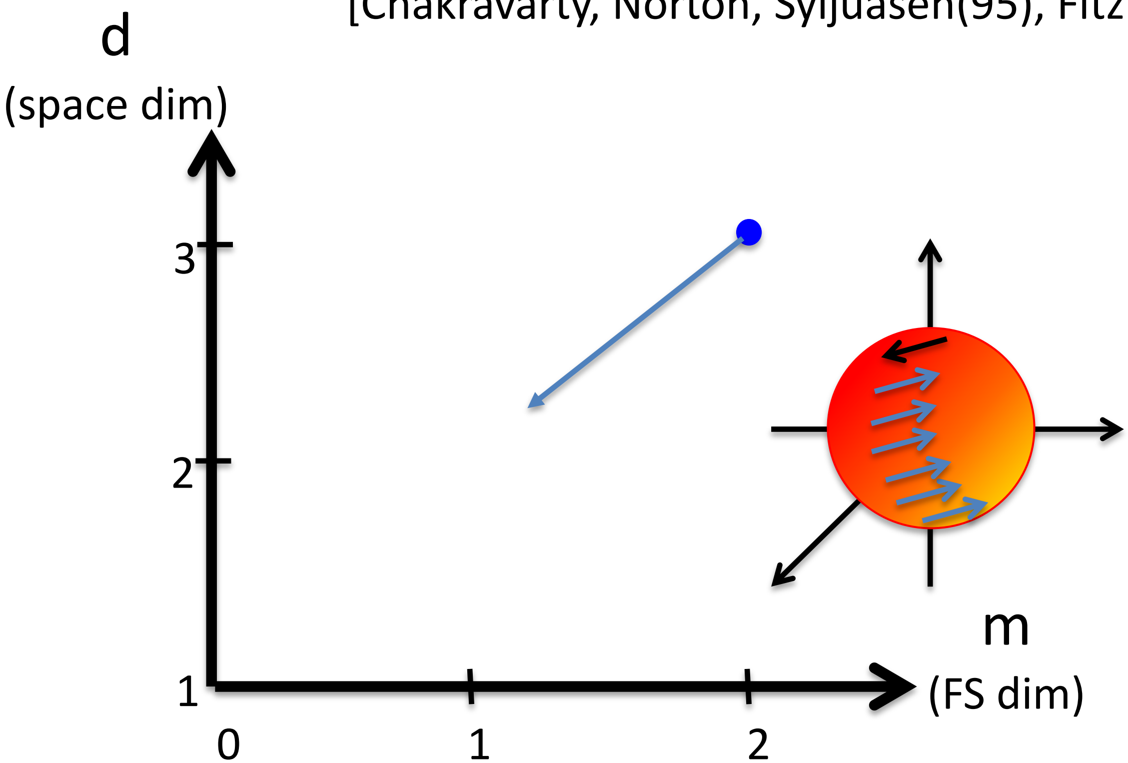
UV divergent non-local terms such as $q^{1+\epsilon}\ln\Lambda$ can not arise as perturbative quantum correction

Dimensional Regularization scheme : no unique way to extend dimension



Tuning dim of space along with the dim of FS

[Chakravarty, Norton, Syljuasen(95), Fitzpatrick, Kachru, Kaplan, Raghu (13),..]



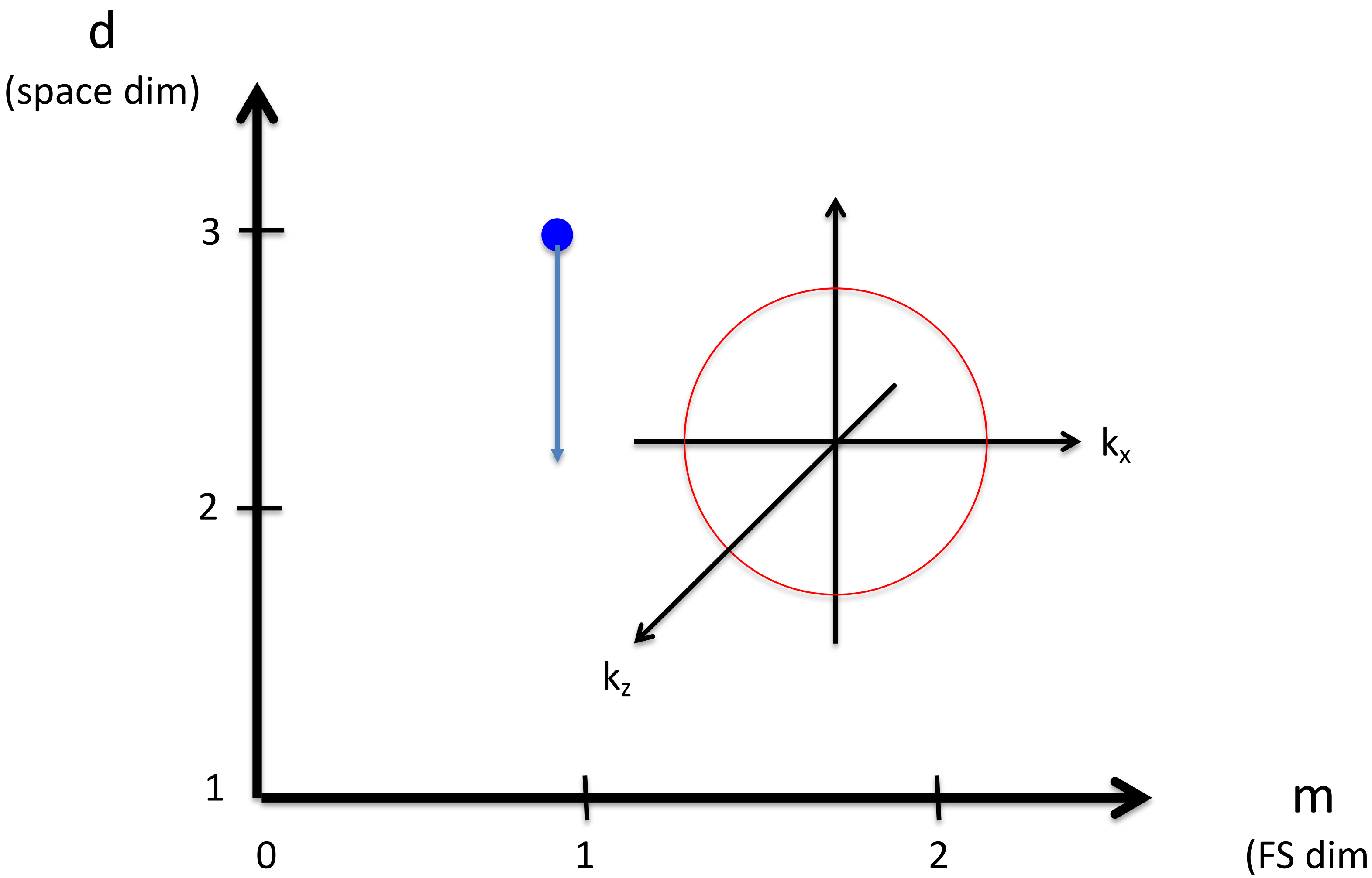
- Size of FS enters as a scale (UV/IR mixing)

[Mandal, SL (15)]

$$A(\omega) = \omega^{-\alpha} f\left(\left[\frac{\omega}{k_F}\right]^{m-1}\right)$$

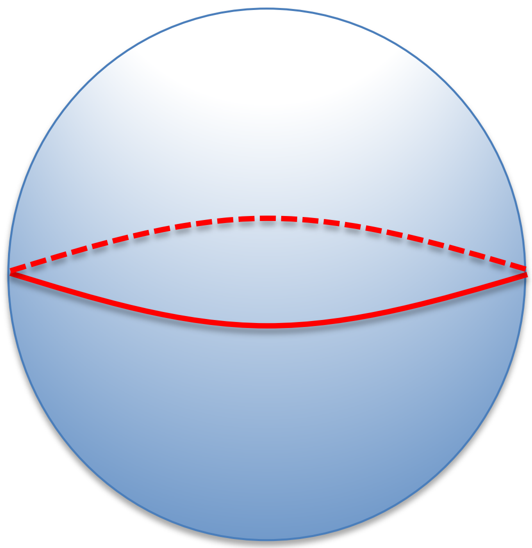
- Crossover function f(x) is singular in the small x limit, and m→1 limit and ω→0 limit do not commute
- You want to probe the region with f(1), but end up probing the f(0) limit in this scheme

Tuning co-dimension of FS



- A non-local version [Senthil, Shankar (09)]
- We will use local version [Dalidovich, Lee (13)]

The theory at d = 3 describes a spin triplet p-wave SC

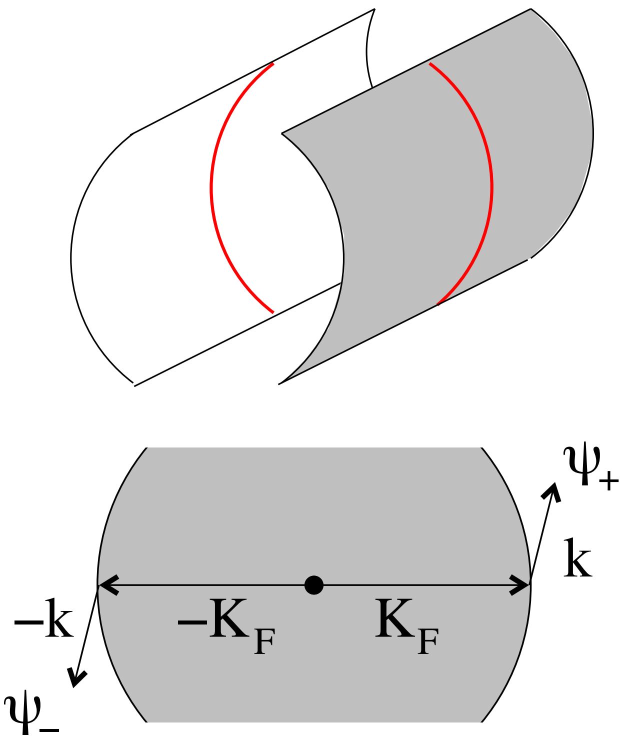


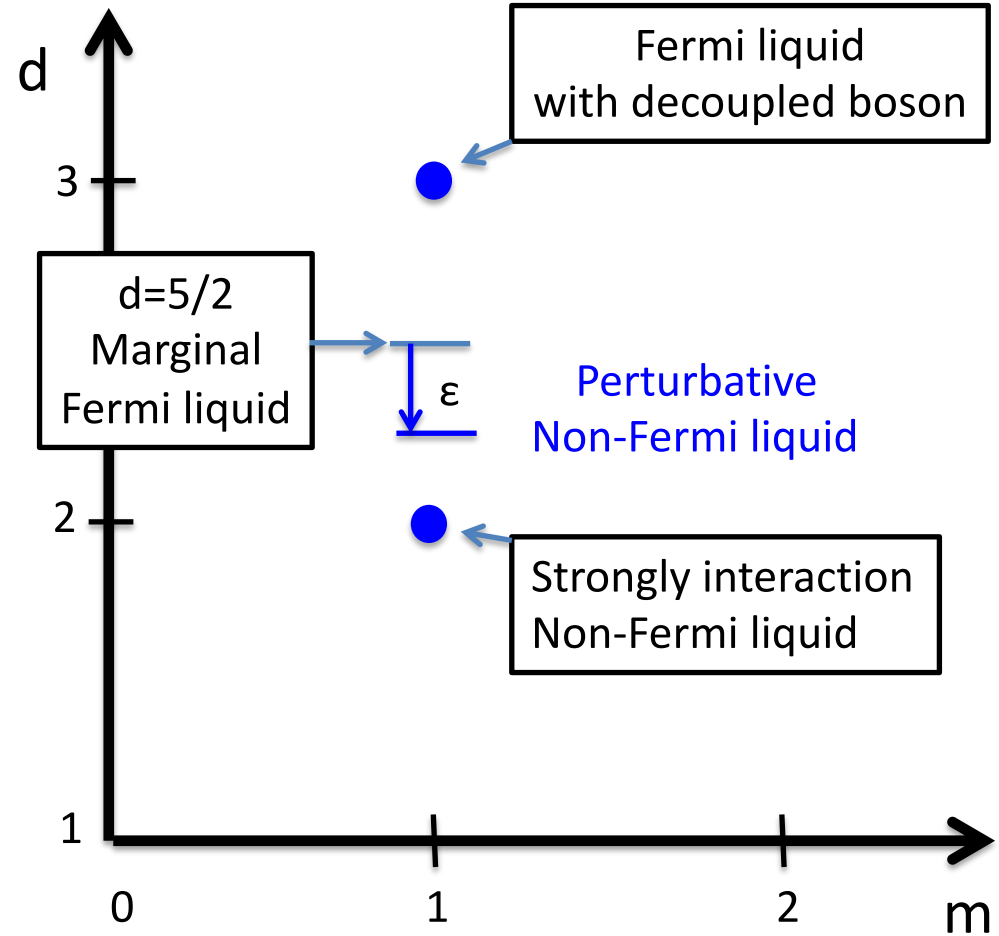
$$S = \int \frac{d^4k}{(2\pi)^3} \left\{ \sum_{s=\pm} \sum_{j=\uparrow,\downarrow} \psi^\dagger_{s,j}(k) \left(ik_0 + sk_2 + k_3^2 \right) \psi_{s,j}(k) \right.$$

$$\left. -k_1 \left(\psi^\dagger_{+,\uparrow}(k) \psi^\dagger_{-,\uparrow}(-k) + \psi^\dagger_{+,\downarrow}(k) \psi^\dagger_{-,\downarrow}(-k) + h.c. \right) \right\}$$

- breaks the global U(1)

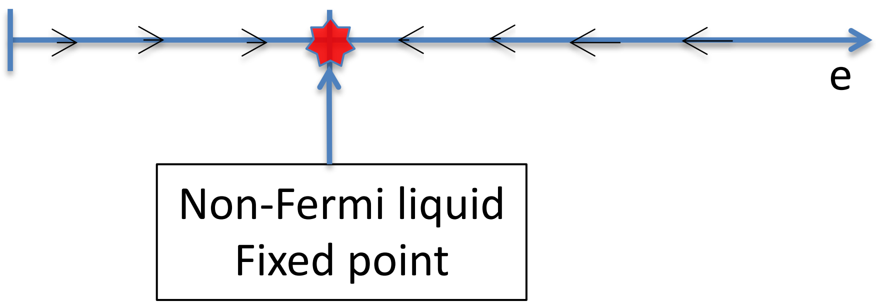
Perturbative NFL near d=5/2





Two-loop results for the Ising-nematic critical metal

$$\frac{de}{dl} = \frac{\epsilon}{2}e - 0.02920\left(\frac{3}{2}-\epsilon\right)\frac{e^{7/3}}{N} + 0.01073\left(\frac{3}{2}-\epsilon\right)\frac{e^{11/3}}{N^2}$$



$$\frac{e^{*4/3}}{N} = 11.417\epsilon + 55.498\epsilon^2$$

$$z = \frac{3}{3-2\epsilon}$$

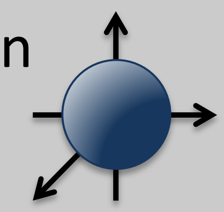
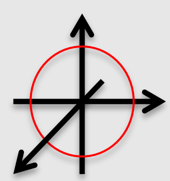
[Dalidovich, SL (13)]

Physical properties of the Ising-nematic critical metal

up $\boldsymbol{\epsilon}^2$ order

- Fermion propagator : $G(k) = \frac{1}{|\delta_k|^{1-0.1508\epsilon^2}} \; g\left(\frac{|\vec{K}|^{1/z}}{\delta_k}\right)$
- Boson propagator : $D(k) = \frac{1}{k_d^2} \; f\left(\frac{|\vec{K}|^{1/z}}{k_d^2}\right)$
- Specific heat : $c \sim T^{(d-2)+\frac{1}{z}}$ $\qquad z = \frac{3}{3-2\epsilon}$
  - The collective mode does not have an anomalous dimension up to three-loop, but it is likely that a non-trivial anomalous dimension arises at higher orders [Holder, Metzner(15)]
  - Near d=5/2, there is no SC instability
  - At d=2, low-energy scaling will be cut off by SC instability

Summary of proposed control schemes

	Deformation schemes	Pro	Con
1/N	Increase the # of flavors	Symmetry, locality	Not controlled
Dynamical tuning	Modify the dispersion $$\|q\|^2\phi^2 \to \|q\|^{1+\epsilon}\phi^2$$	symmetry	Locality lost
Dim. reg.	Tune the dimension of FS 	Symmetry, locality	spurious UV/IR mixing
Co-Dim. reg.	Tune the co-dimensions of FS 	Locality, No UV/IR mixing	Some symmetry broken

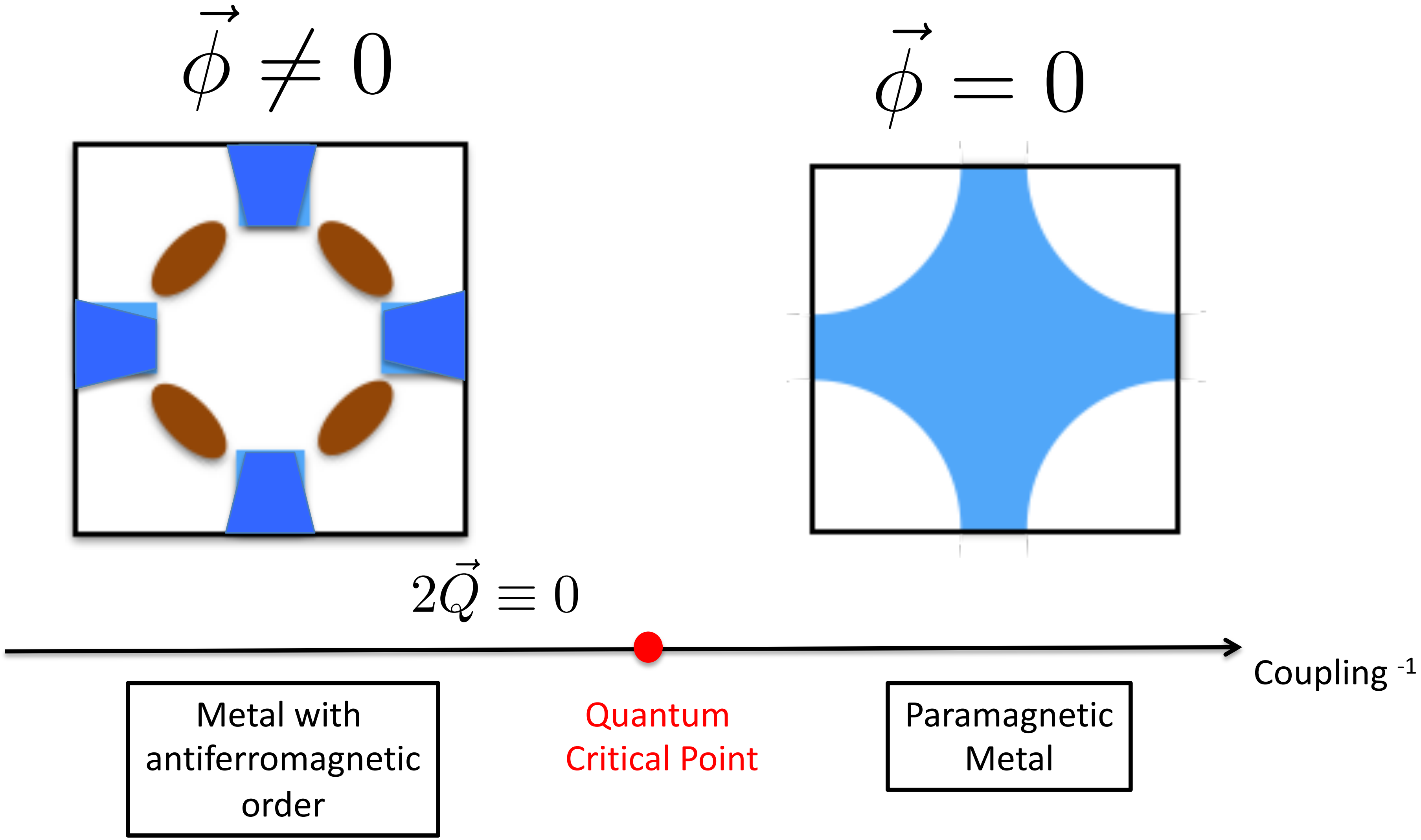
The main purpose of a perturbative expansion is to reveal new organizing principles, which may lead to non-perturbative understanding of physics in the deep quantum regime

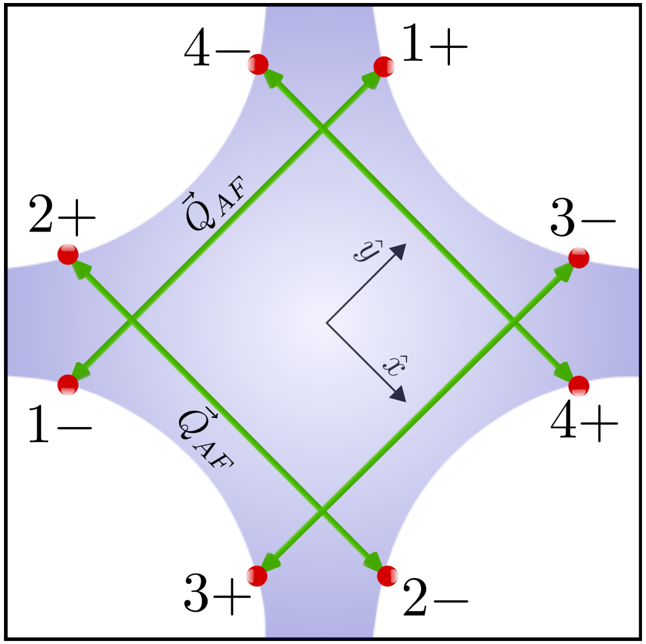
- For Q=0 NFL, non-perturbative solution is available only for the chiral NFL (without T, P symmetry) [Sur, SL (14)]
- Understanding non-chiral NFL with Q=0 in d=2 remain an open problem
- More progress made for NFL with Q≠0

Hot spots

Antiferromagnetic quantum critical metal

Antiferromagnetic phase transition in metal





Minimal Theory

[Abanov, Chubukov, Schmalian; Metlitski, Sachdev; ..]

$$e_1^{\pm}(\vec{k}) = -e_3^{\pm}(\vec{k}) = v k_x \pm k_y$$

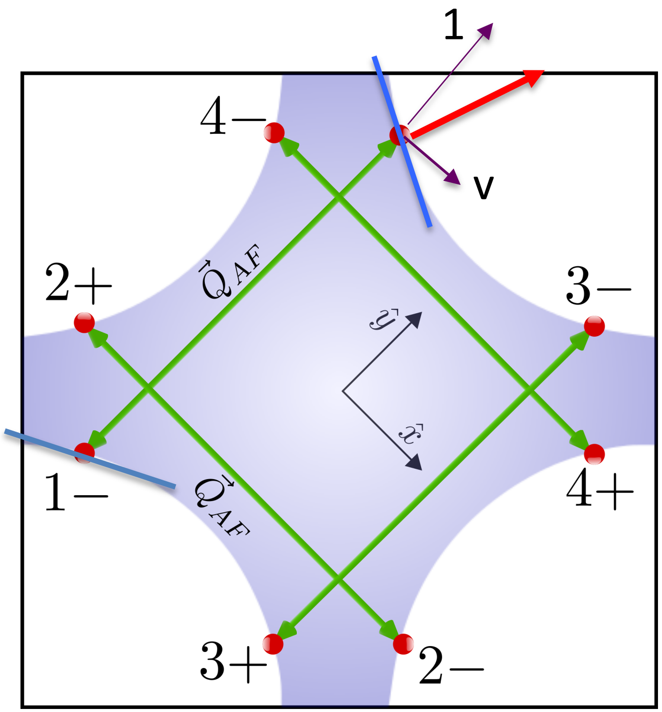
$$e_2^{\pm}(\vec{k}) = -e_4^{\pm}(\vec{k}) = \mp k_x + v k_y$$

$$\mathcal{S} = \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \, \psi_{l,\sigma}^{(m)*}(k) \left[i k_0 \; + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k)$$

$$+ \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q)$$

$$+ g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right]$$

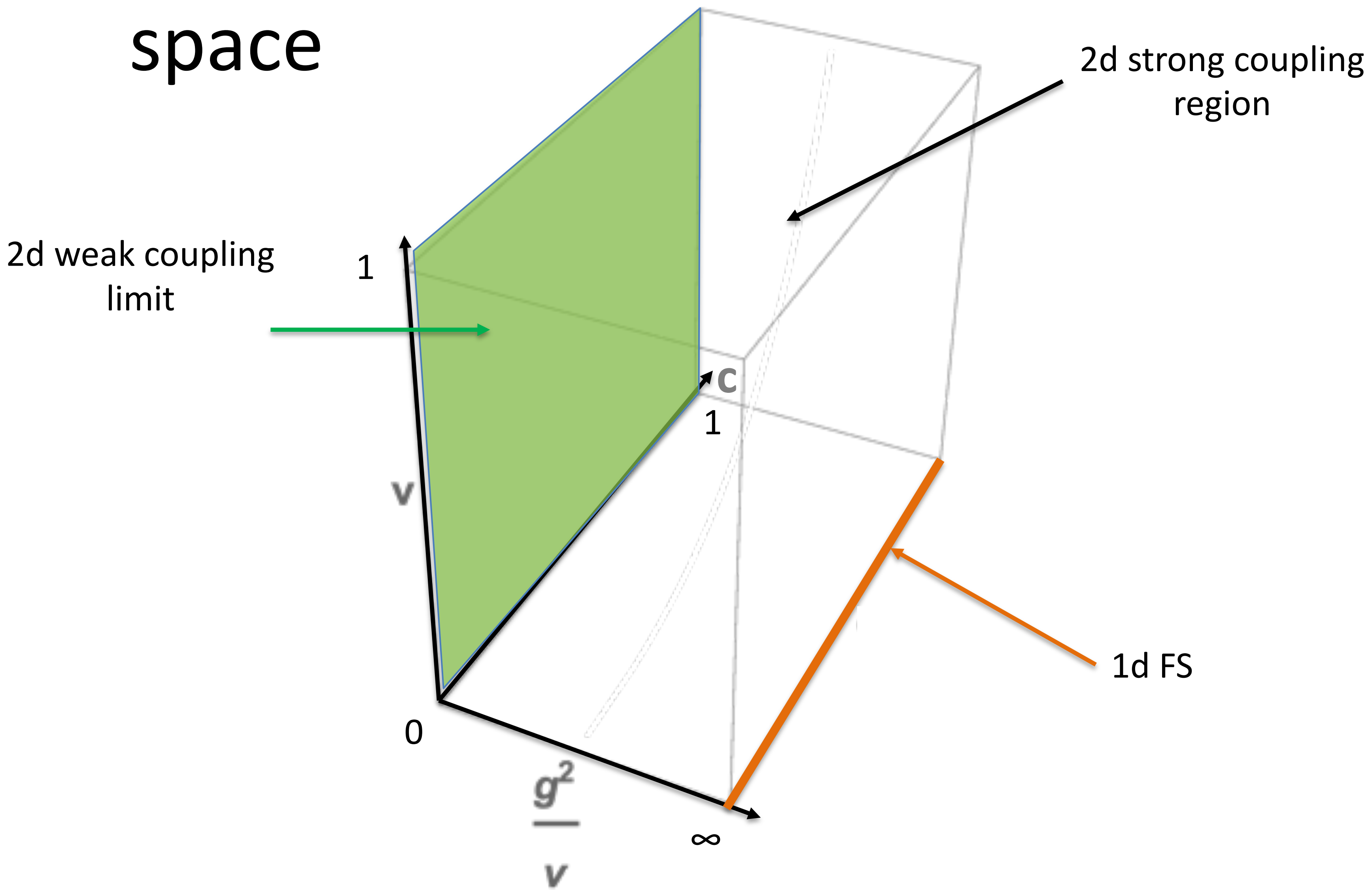
Parameters of the theory



- v : Fermi velocity perpendicular to Q_{AF}
- c : boson velocity
- g : coupling bet'n fermion and boson

- If v=0, hot spots connected by Q_{AF} are nested

parameter space



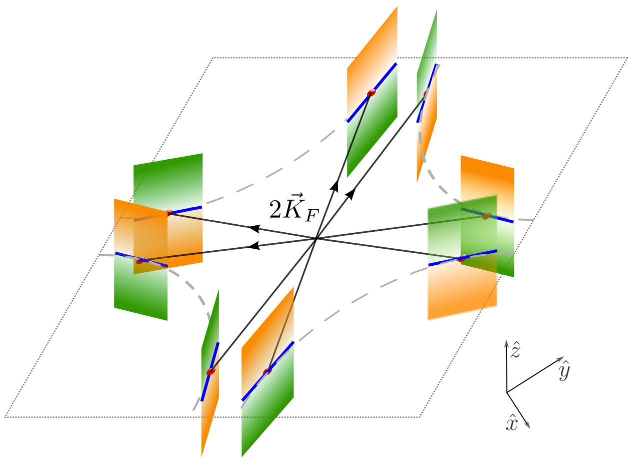
- g^2 /v is the coupling that controls perturbation

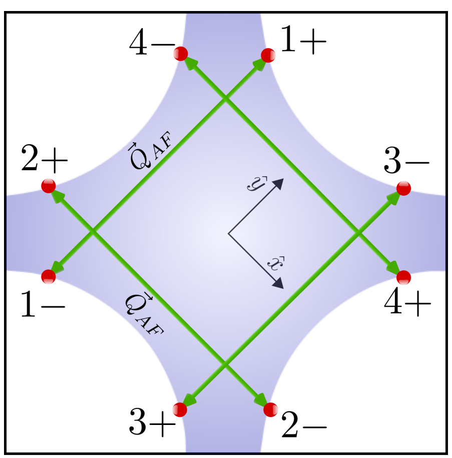
Gaussian scaling analysis

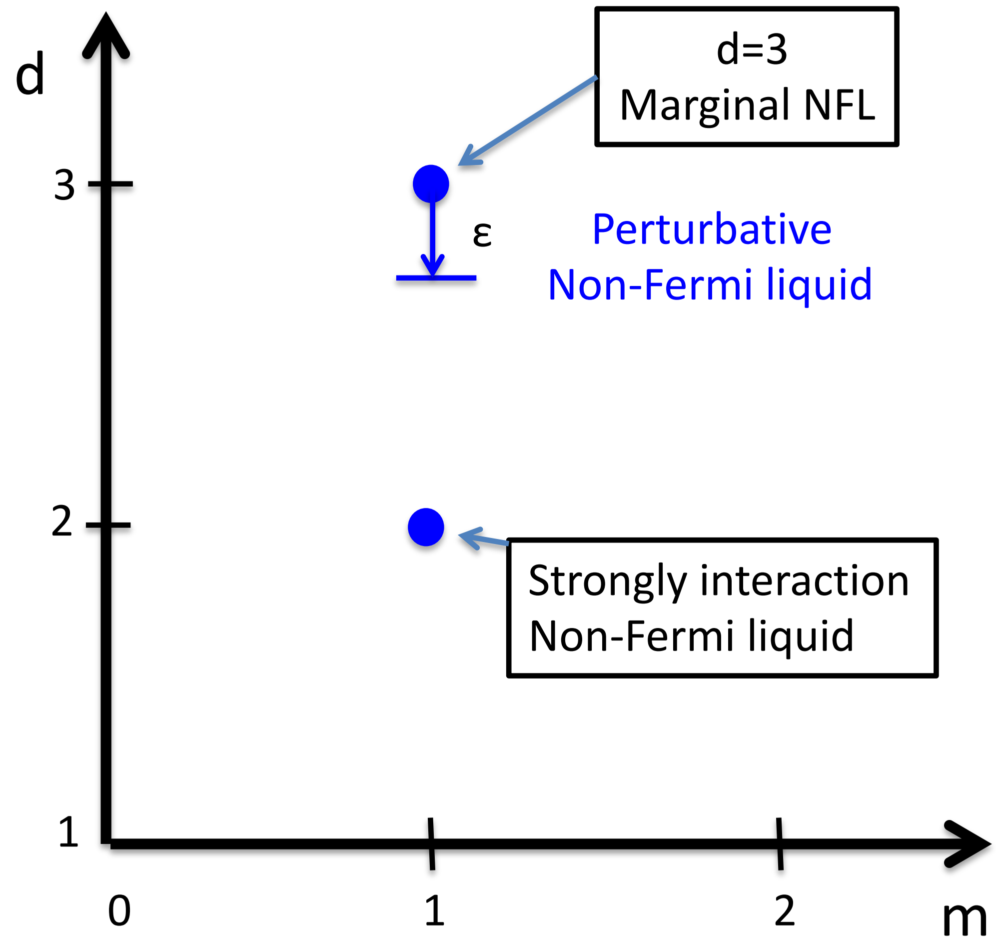
$$\mathcal{S} = \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \, \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 \ + e_l^m(\vec{k})\right] \psi_{l,\sigma}^{(m)}(k)$$
$$+\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2|\vec{q}|^2\right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q)$$
$$+\boxed{g_0} \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c.\right]$$

- Interaction is relevant
- The same perturbative approaches discussed for Q=0 case can be employed in this case

Perturbative window with tuning co-dim.







global U(1) can be kept

Lesson from the ε-expansion

[Sur, Lee (14); Lunts, Andres, Lee(17)]

- One-loop is not enough even to the leading order in ε: quantum fluctuations are not organized by number of loops

- Emergent quasi-locality with a hierarchy in velocities

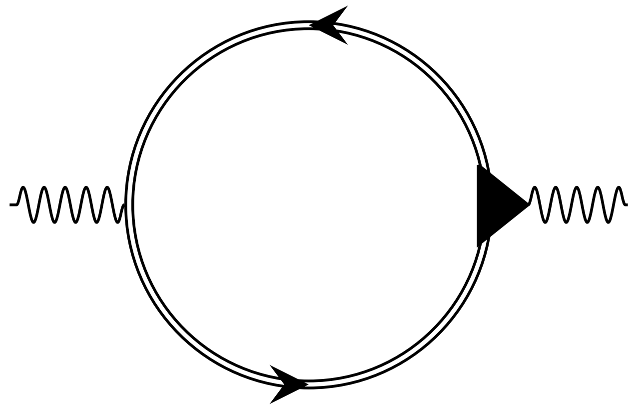
$$v, c \to 0 \ \left(\frac{v}{c} \to 0\right), \quad g \to 0 \left(\frac{g^2}{v} \to O(\epsilon)\right)$$

- Collective mode is damped by particle-hole excitation and acquires an O(ε) anomalous dimension
- Fermions remain largely coherent

Emergent hierarchy of velocities v << c << 1

- One may use the ratios of the velocities as small parameters to organize dynamics at d=2

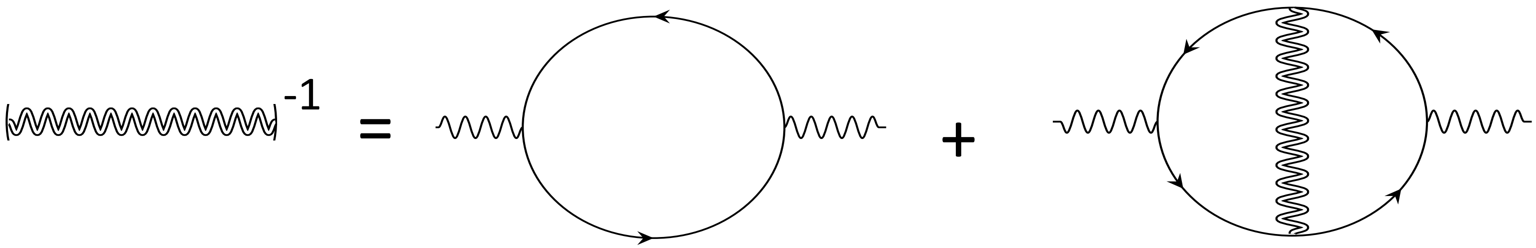
Non-perturbative solution in d=2



$$D(q)^{-1} = m_{CT} - \pi v \sum_n \int dk \; \mathrm{Tr}\left[\gamma_1 G_{\bar{n}}(k+q)\Gamma(k,q)G_n(k)\right]$$

- In general, it is hard to solve the self-consistent equation because G(k), Γ(k,q) depend on D(q)
- However, the Dyson equation can be first solved in the $v \ll c \ll 1$ limit, and the solution can be used to show that v, c and v/c flows to zero in the low-energy limit

Reduced Dyson equation in the v << 1 limit



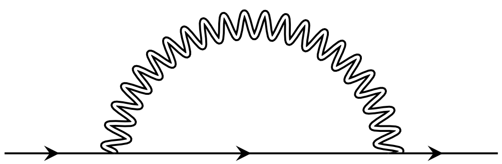
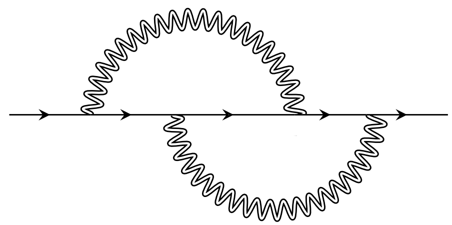
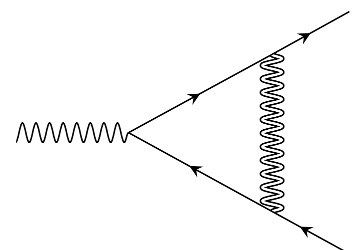
$$D(q)^{-1} = |q_0| + c(v)\Big[|q_x| + |q_y|\Big],$$

$$c(v) = \frac{1}{4}\sqrt{v\log(1/v)}$$

- Boson propagator is entirely generated from particle-hole fluctuations
- v << c(v) if v <<1

Flow of v

- In the small v limit, v indeed flows to zero in the low energy limit, which completes the cycle of self-consistency

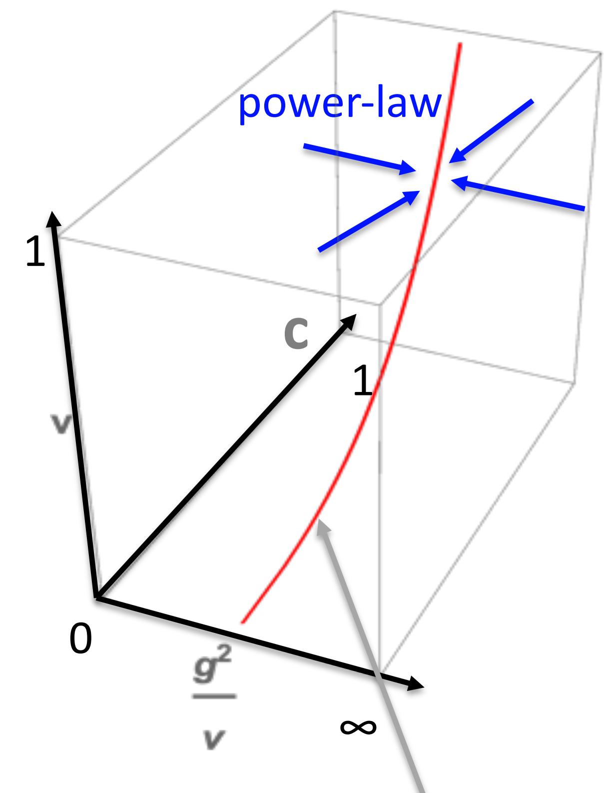
$$\frac{dv}{d\ln\mu} = \frac{6}{\pi^2} v^2 \log\left(\frac{1}{c(v)}\right) \qquad v = \frac{2\pi^2}{3}\left(\log\frac{1}{\mu}\ \log\log\frac{1}{\mu}\right)^{-1}$$

Exact exponents : $z = 1, \quad \eta_\phi = 1, \quad \eta_\psi = 0$

[Schlief, Lunts, SL (16)]

Crossovers

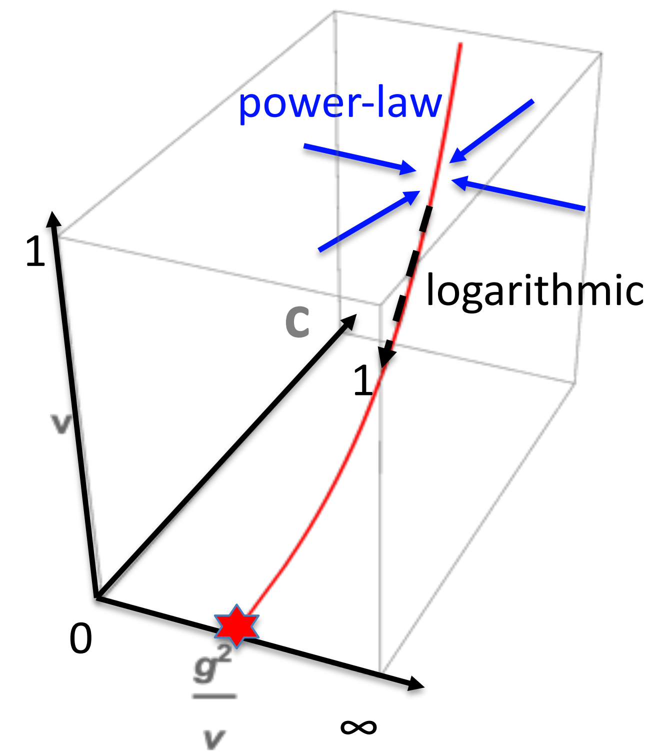
- Two-stage RG flow
- Stage I
  - Non-perturbative effects generate flow from the perturbative regime ($g^2/v \ll 1$) to strongly coupled regime ($g^2/v \sim 1$)
  - RG flow is quickly attracted to a universal manifold parameterized by v
  - The place where the initial RG flow lands on the attractor depends on the bare value of v



$$g = \sqrt{\frac{\pi v}{2}}$$

$$c(v) = \frac{1}{4}\sqrt{v \log(1/v)}$$

Crossovers



- Stage II
  - In the low-energy limit, v and v/c(v) flows to zero logarithmically
  - The expansion in v/c(v) becomes asymptotically exact
  - Eventually the system flows to the fixed point with z=1, η=1 (exact)
- Due to the slow flow of v, the attractor approximately acts as a line of fixed points
  - `Transient dynamical critical exponent' is determined by v

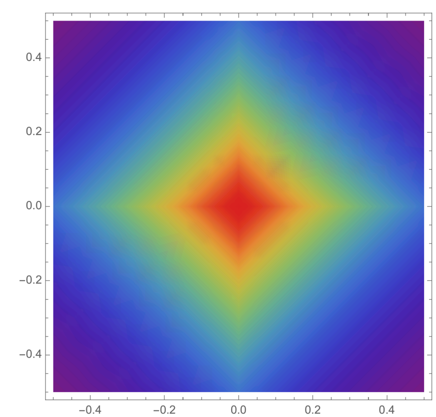
$$z = 1 + \frac{3}{4\pi}\frac{v}{c(v)}$$

Dynamical Spin Susceptibility $\chi''(\omega, Q_{AF})$

$\omega \gg \omega^*$	$\omega \ll \omega^*$
$\dfrac{1}{\omega^{1-\frac{1}{2\pi}\frac{v}{c(v)}\log\frac{c(v)}{v}}}$	$\dfrac{1}{\omega e^{\frac{2}{\sqrt{3}}\left(\log\frac{1}{\omega}\right)^{1/2}}}$

$$\omega^* \sim \Lambda e^{-1/v_0}$$

q_y



q_x

- Only C_4 symmetric; no emergent O(2)

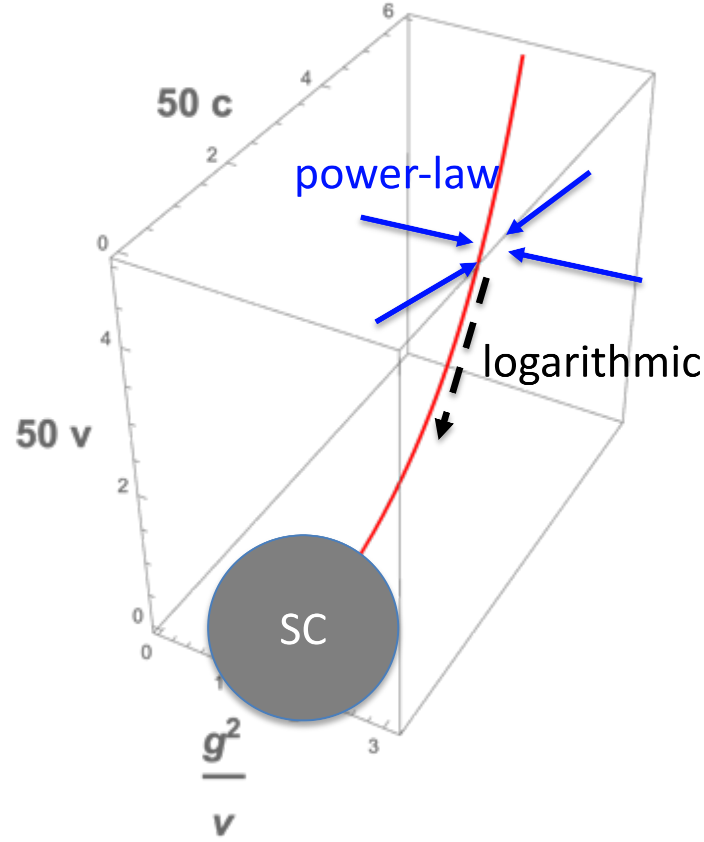
Electron spectral function at the hot spots

$\omega \gg \omega^*$	$\omega \ll \omega^*$
$\dfrac{1}{\omega^{1-\frac{3}{4\pi}\frac{v}{c(v)}}}$	$\dfrac{1}{\omega e^{2\sqrt{3}\frac{\left(\log\frac{1}{\omega}\right)^{1/2}}{\log\log\frac{1}{\omega}}}}$

- No quasiparticle at the hot spots

Superconductivity

- In d=2, SC kicks in, and RG flow is cut off
- T_c is not a universal quantity but depends on microscopic theory (bare parameters)



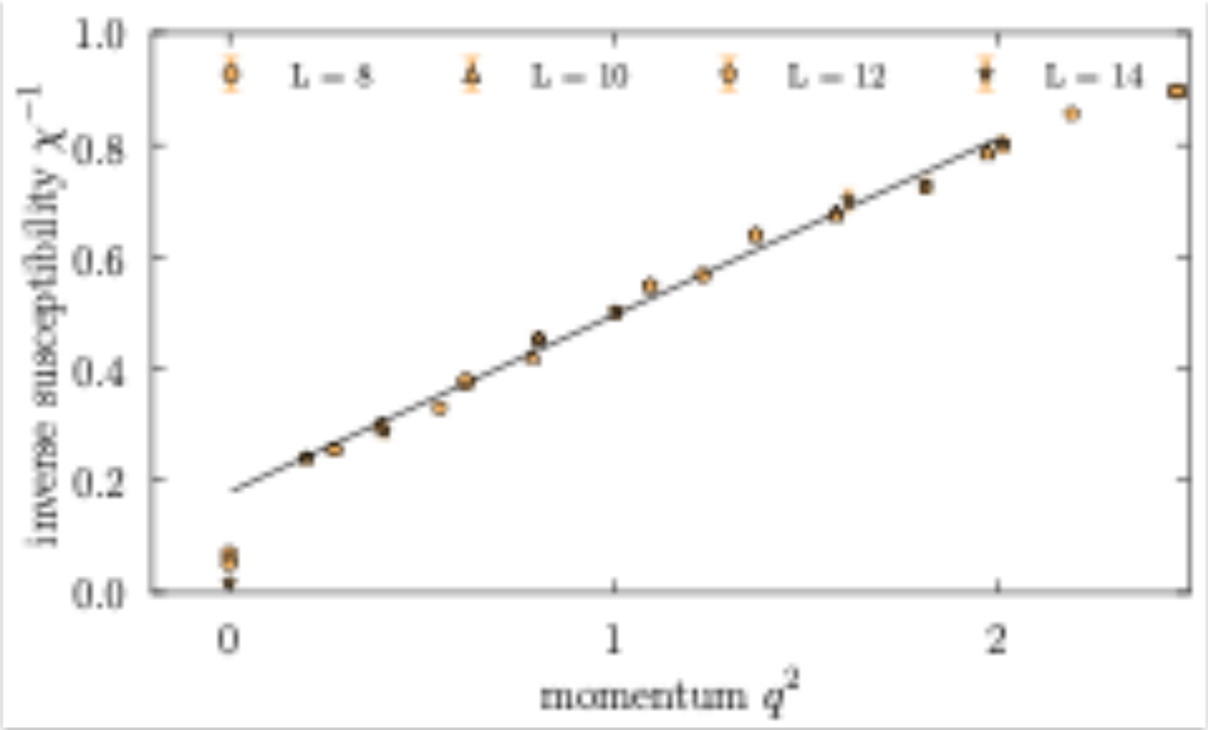
- Most likely, one needs small v with $g^2/v\sim1$ to enter z=1 scaling regime before SC sets in (a target for numerics)

Latest numerics

Hierarchy of energy scales in an O(3) symmetric antiferromagnetic quantum critical metal: a Monte Carlo study

Carsten Bauer,[1] Yoni Schattner,[2] Simon Trebst,[1] and Erez Berg[3]

arXiv : 2001.00586



Summary

- Theories of NFL @ QCP can be divided into two classes
  - Hot Fermi surface
  - Hot Spots
- Various control schemes have been developed
- Perturbative solutions obtained from tuning the co-dim of FS eventually led to the non-perturbative solution for the SU(2) symmetric AF quantum critical metal in 2+1D

Open problems

- Beyond patch theory
  - Capturing momentum dependent universal data
- Superconductivity
- Disorder
- Local moments
- Full scope of the non-perturbative method that uses hierarchy between velocities (Migdal-like)