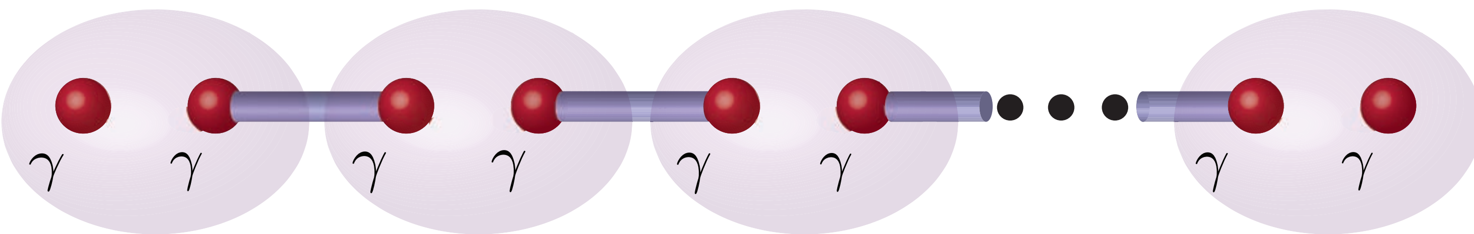
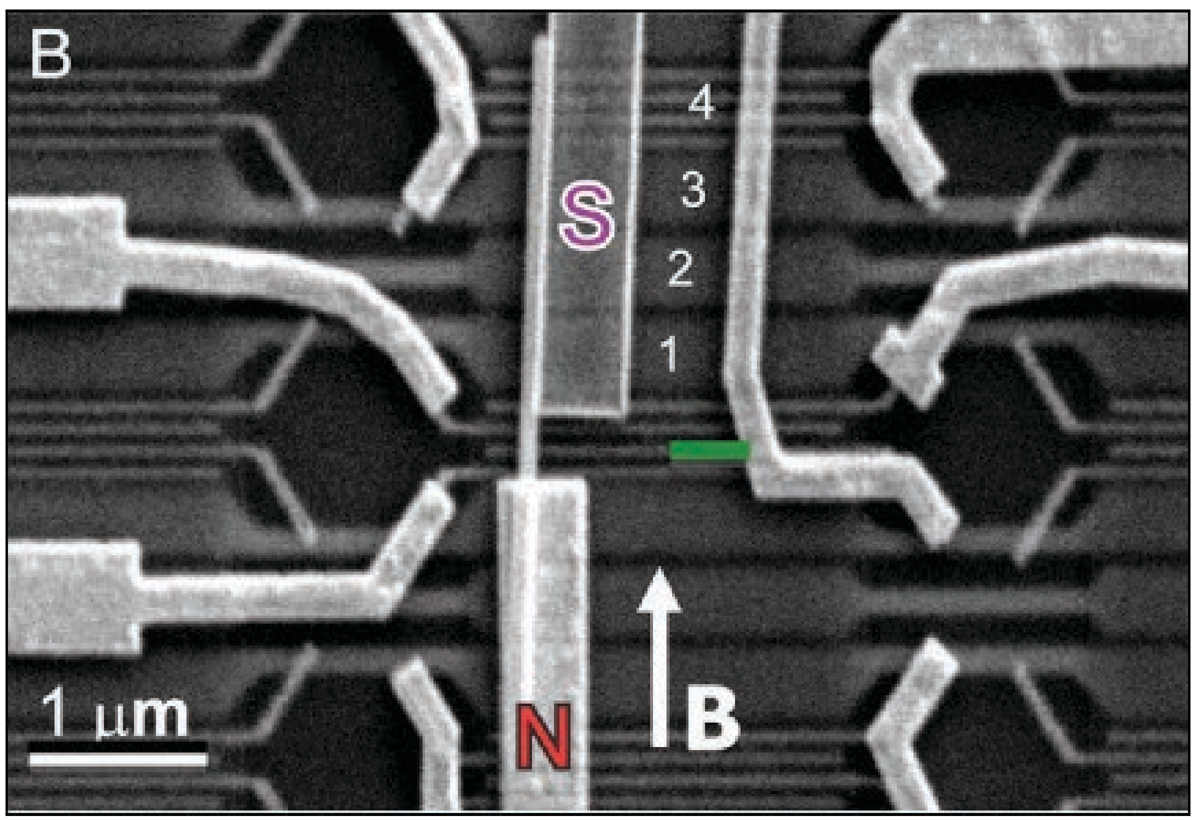
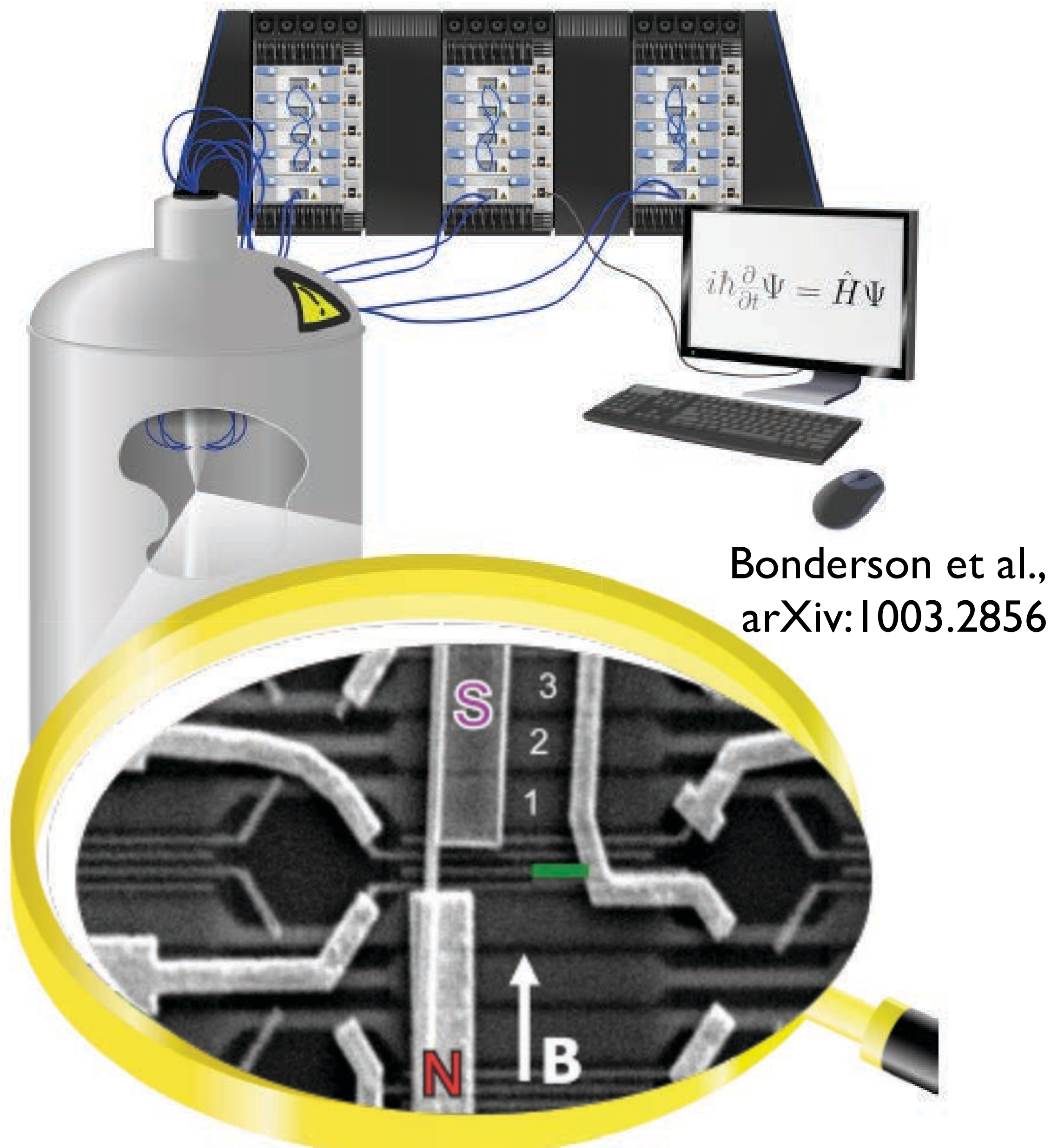
# The quest for Majorana I




Mourik et al., Science 2012




Bonderson et al., arXiv:1003.2856

Jason Alicea (Caltech)

# Acknowledgments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Amir Yacoby (Harvard)

INSTITUTE FOR QUANTUM INFORMATION AND MATTER

# Outline

- Some general remarks on Majorana fermions in condensed matter

- Toy models capturing Majoranas (continued)
  - Quick reminder of “Kitaev chain”
  - 2D topological superconductivity

- Select plausible experimental realizations

# Majorana fermions: high-energy vs. cond. matter

Ettore Majorana
(1906-1938?)

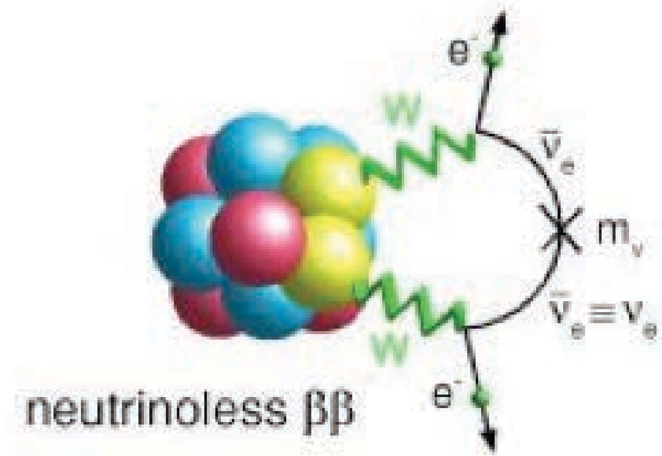
## Originally "invented" Majorana fermions

(fermionic particles that are their own antiparticle)

Neutrinos?

Dark Matter?

Wilczek, Nature Physics **5**, 614 (2009)



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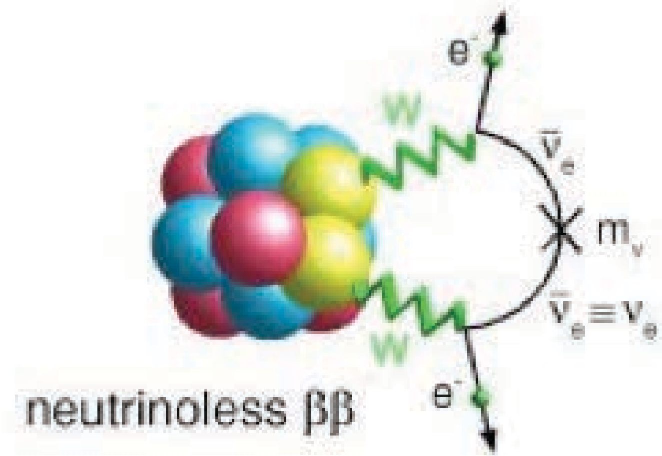
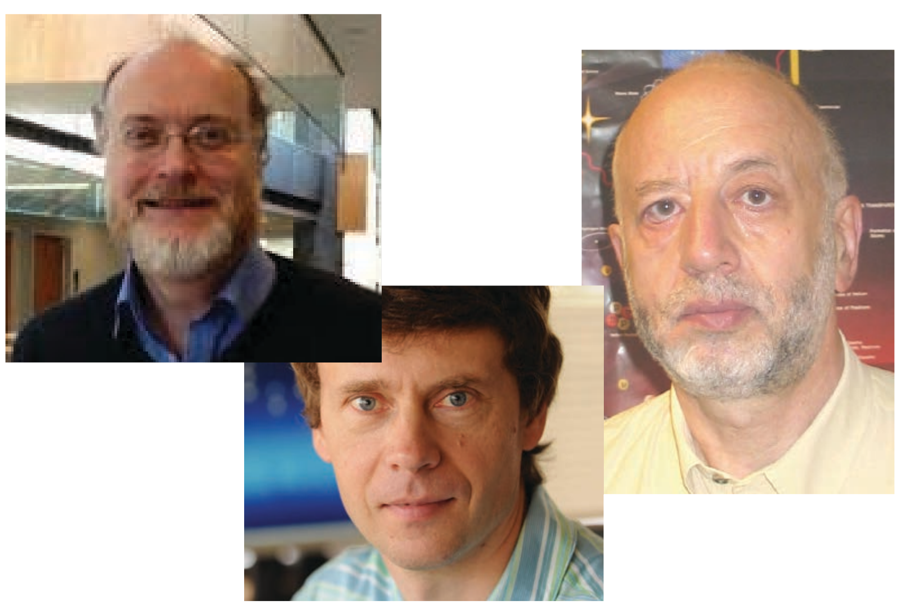
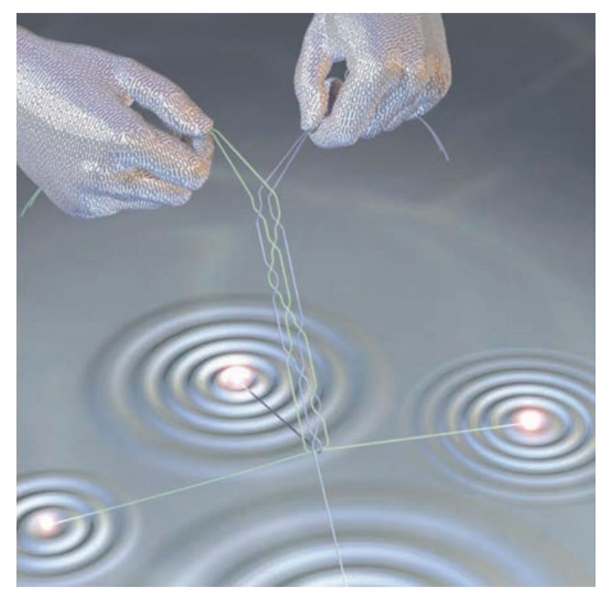



Wilczek, Nature Physics **5**, 614 (2009)

Condensed matter physicists mainly seek **Majorana fermion zero-modes** (which are not really particles!)



$$\gamma = \gamma$$



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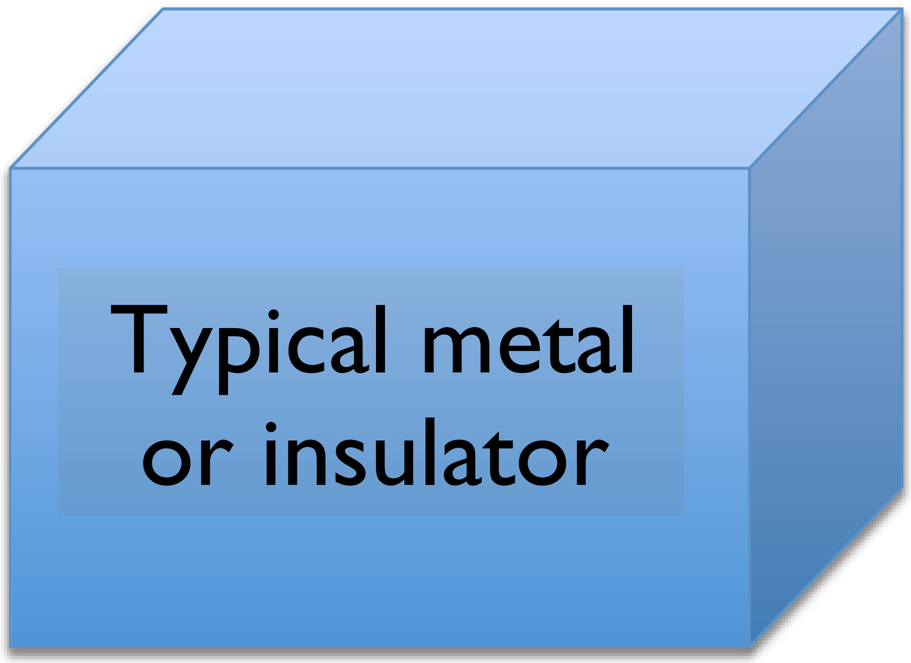

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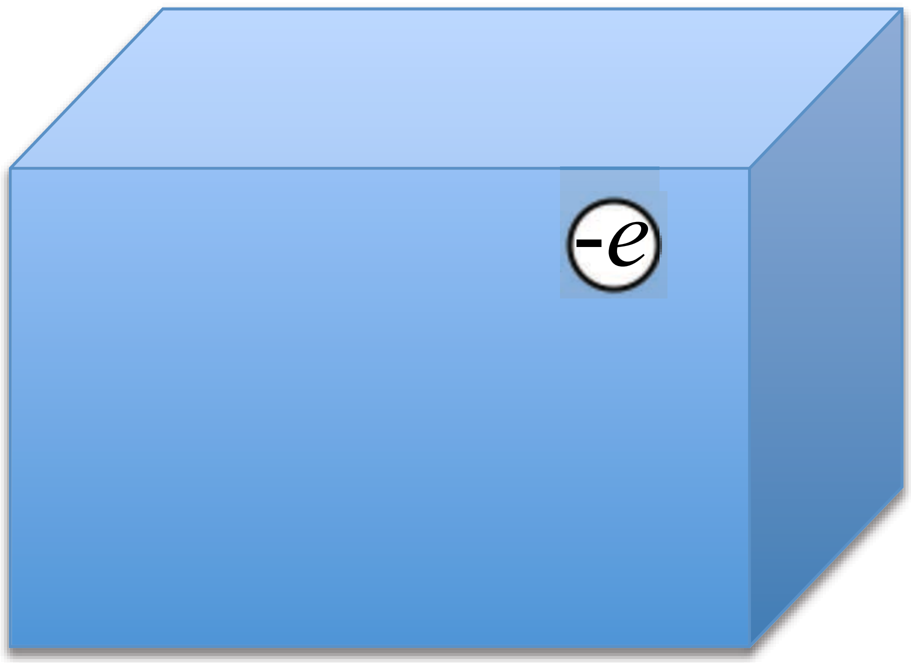


$c^\dagger|\psi\rangle$ (Adds an electron)

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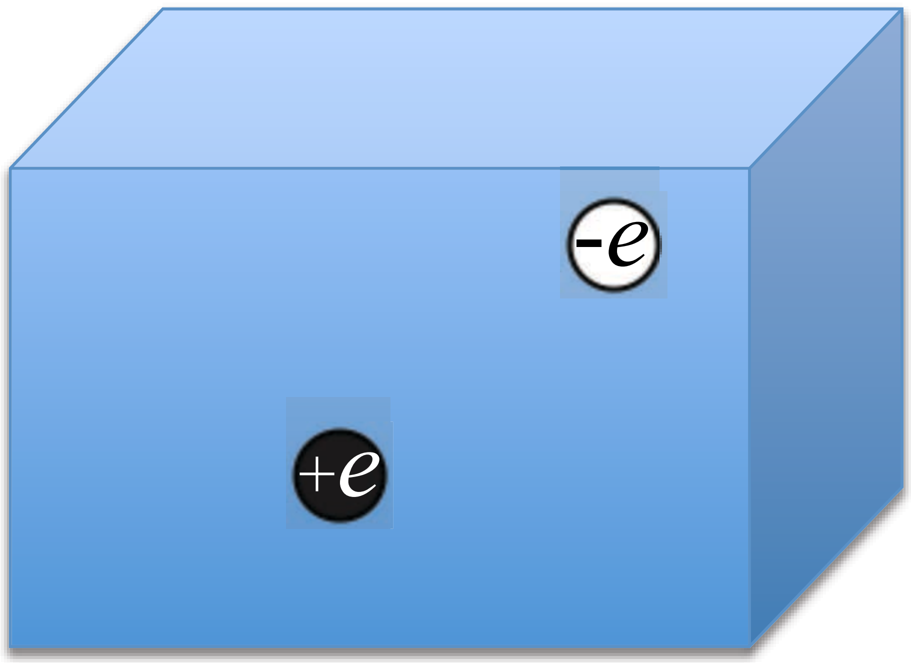


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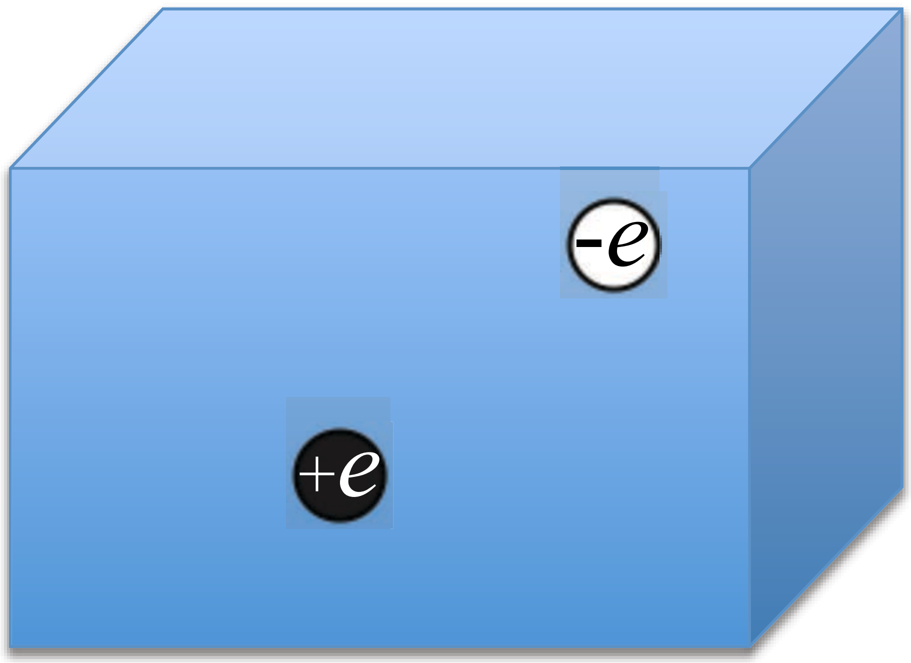


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Majorana appears only through ***emergent*** excitations

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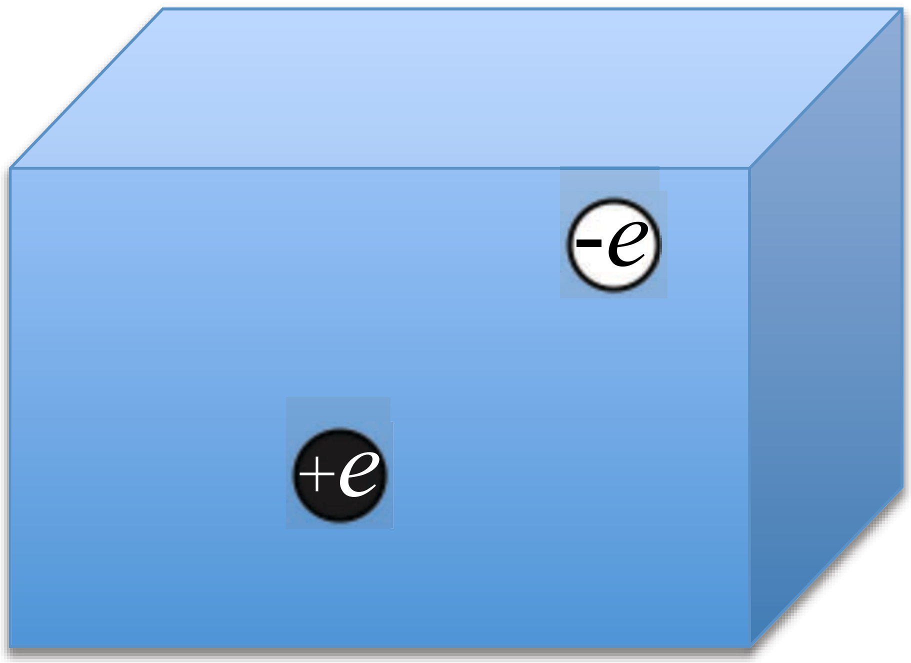


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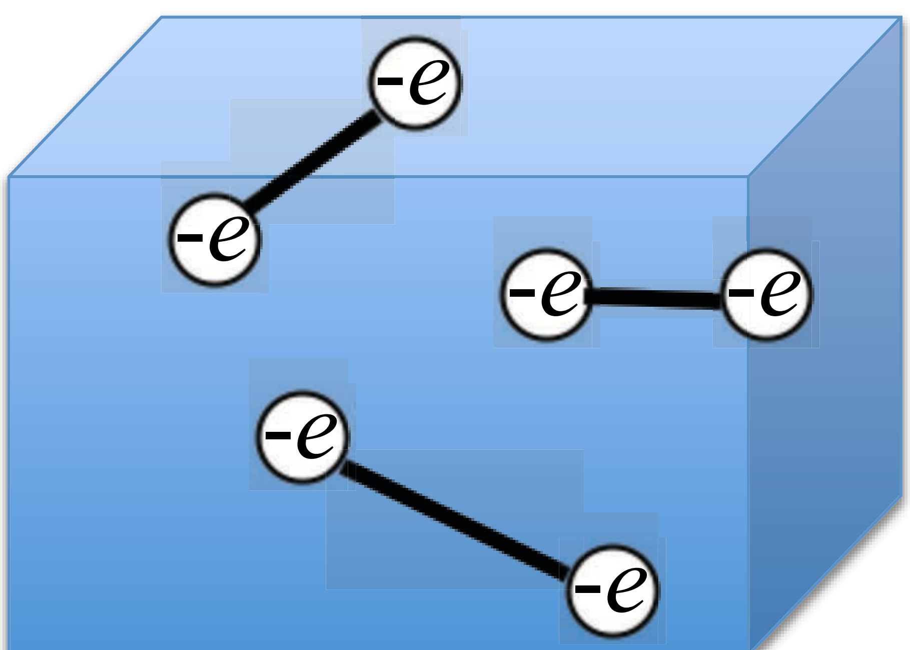


**Superconductors**
are natural platforms

$$f^\dagger \sim uc^\dagger + vc$$

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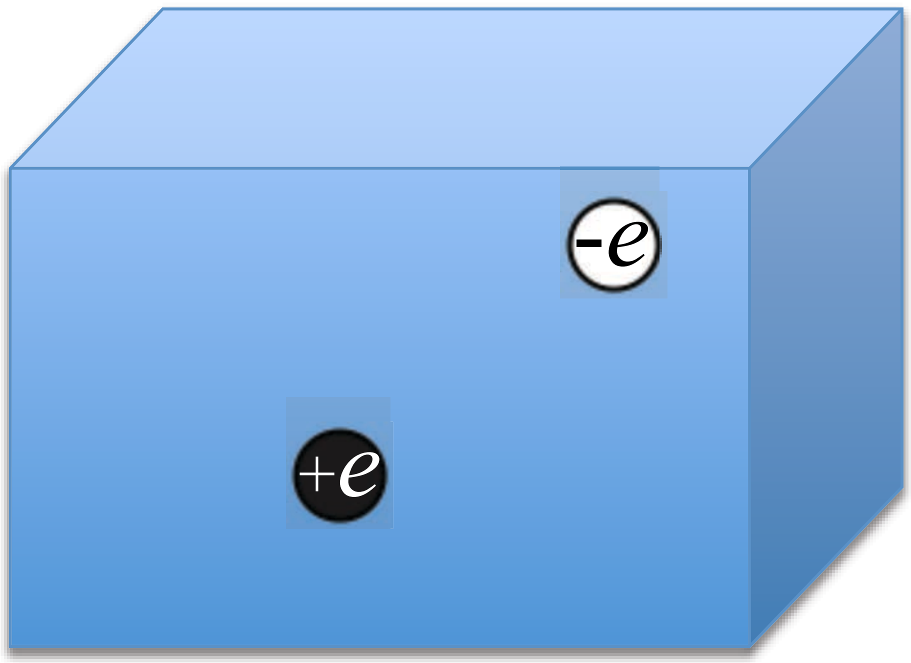


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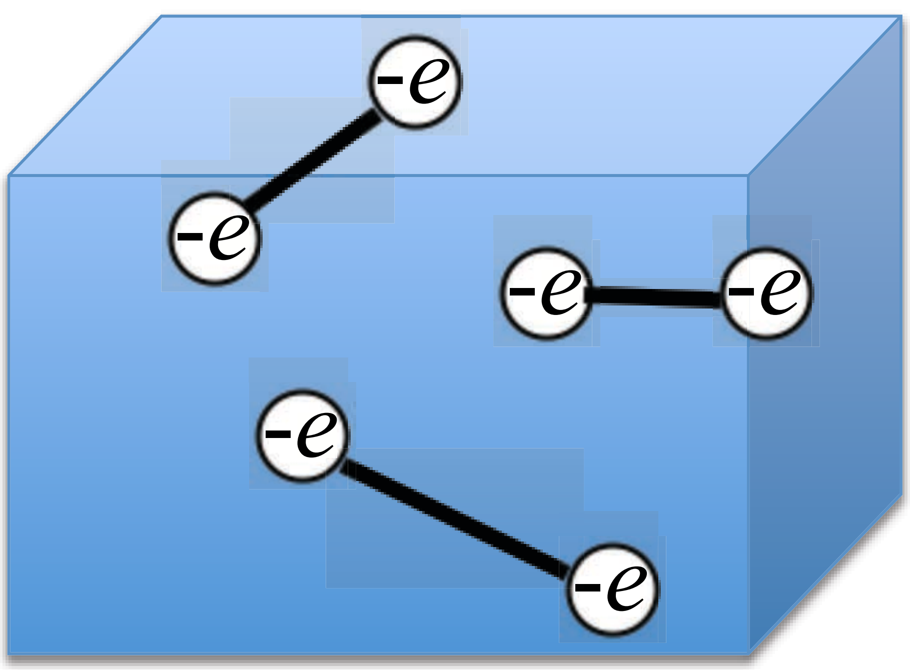


**Superconductors** are natural platforms ← but must be ***topological***

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$$\sum \qquad - \bar{\sum} \qquad \Delta \quad \phi$$

$\pi = 0$

$t = \Delta \qquad - \quad - \frac{\phi}{} \quad \gamma \qquad \gamma$

(Kitaev 2001)

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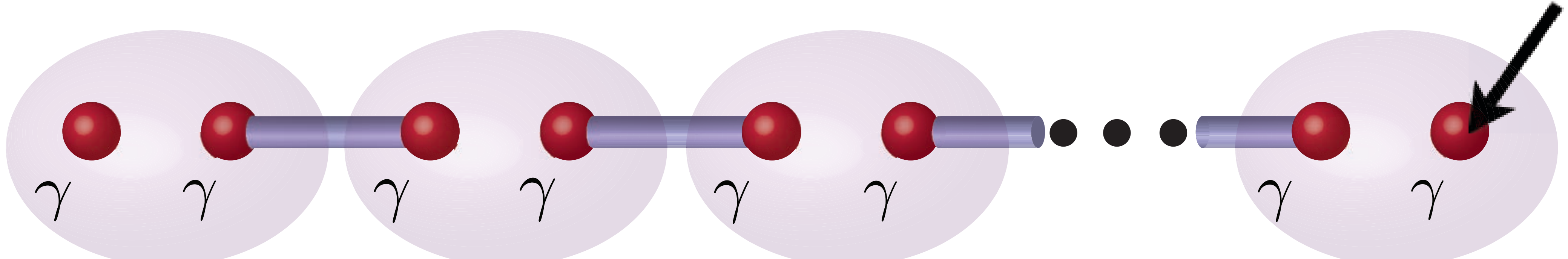
$$\Rightarrow \quad - \sum \gamma \quad \gamma$$

Unpaired Majorana fermion zero-modes!




(Kitaev 2001)

# D spinless p-wave superconductc

$$\gamma_i^\dagger = \gamma_i \qquad \gamma_i^2 = 1 \qquad \gamma_1\gamma_2 = -\gamma_2\gamma_1$$

$$f = \frac{1}{2}(\gamma_1 + i\gamma_2) \qquad f|0\rangle = 0 \qquad f^\dagger|0\rangle = |1\rangle$$

Majoranas encode a **two-fold ground-state degeneracy**, consisting of states w/ even and odd fermion #

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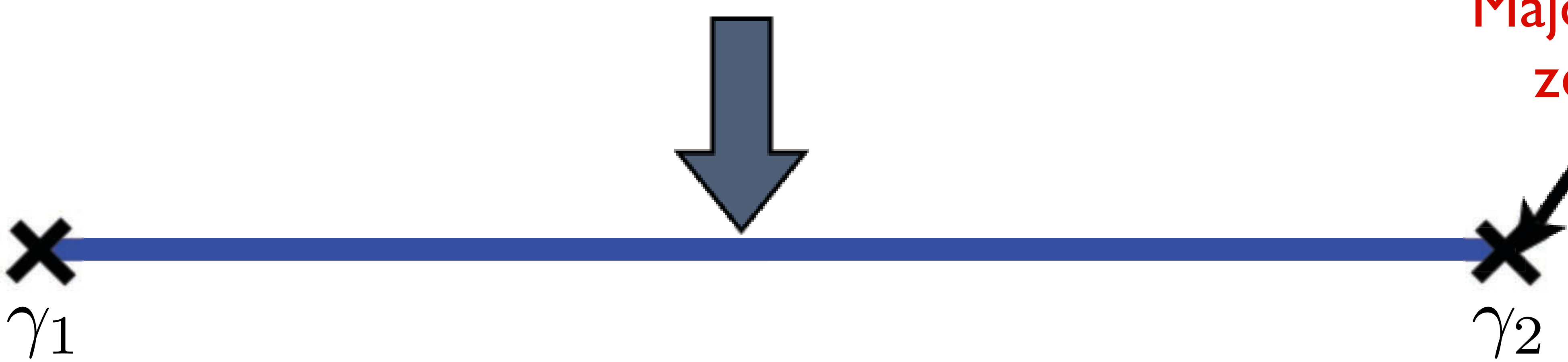


(Kitaev 2001)

# 2D spinless p+ip superconductor

$$H = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\left[(\epsilon_{\mathbf{k}} - \pi)\psi_{\mathbf{k}}\psi_{\mathbf{k}} + (\Delta_{\mathbf{k}}\psi_{\mathbf{k}}\psi_{-\mathbf{k}} + h\ c\ )\right]$$

$$\Delta_{\mathbf{k}} \propto k_x + ik_y$$

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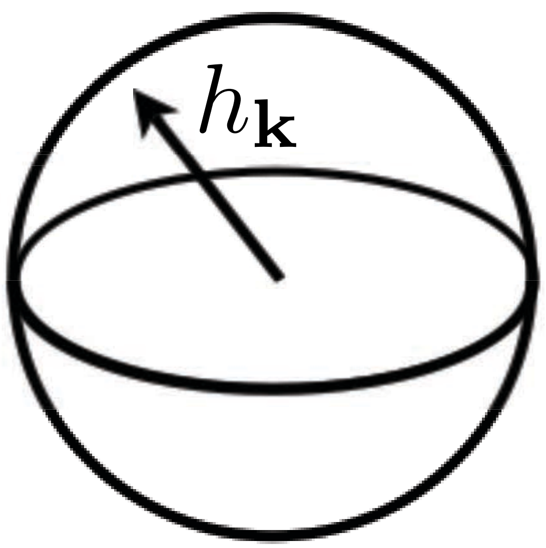


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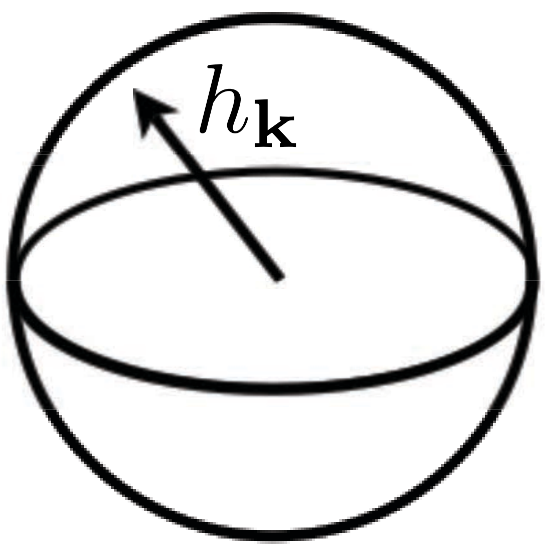


Upon sweeping all of k-space, how many times does $\vec{h}_\mathbf{k}$ cover the unit sphere? Defines **"Chern number"**.

$$C = \int \frac{d^2\mathbf{k}}{4\pi}\left[\hat{\mathbf{h}} \cdot (\partial_{k_x}\hat{\mathbf{h}} \times \partial_{k_y}\hat{\mathbf{h}})\right]$$

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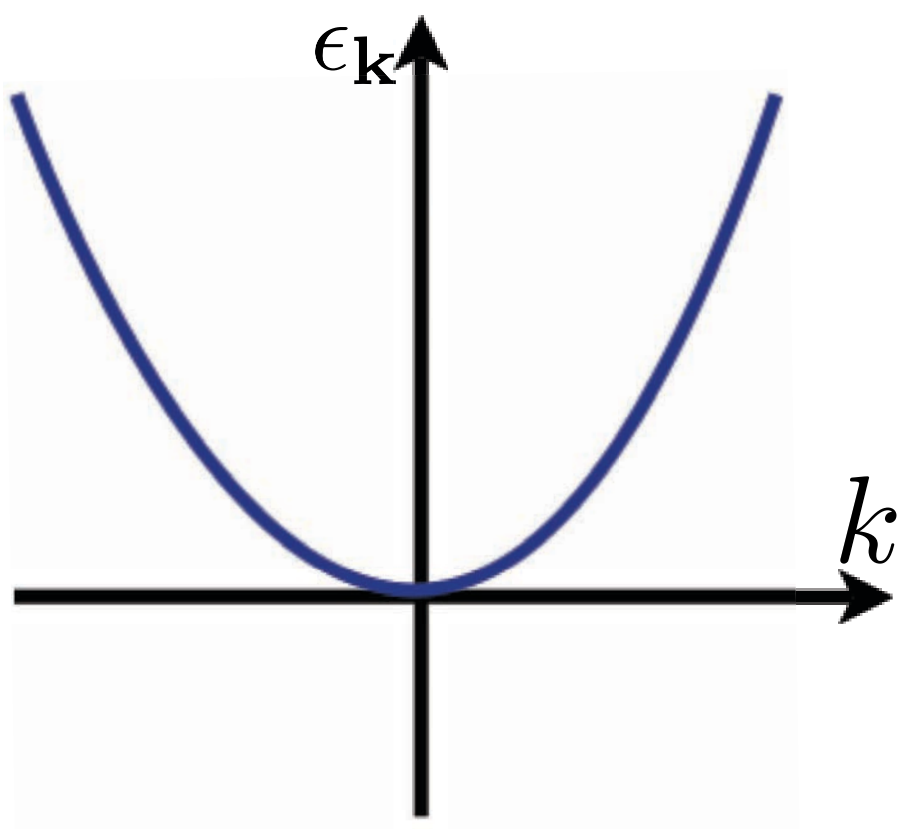


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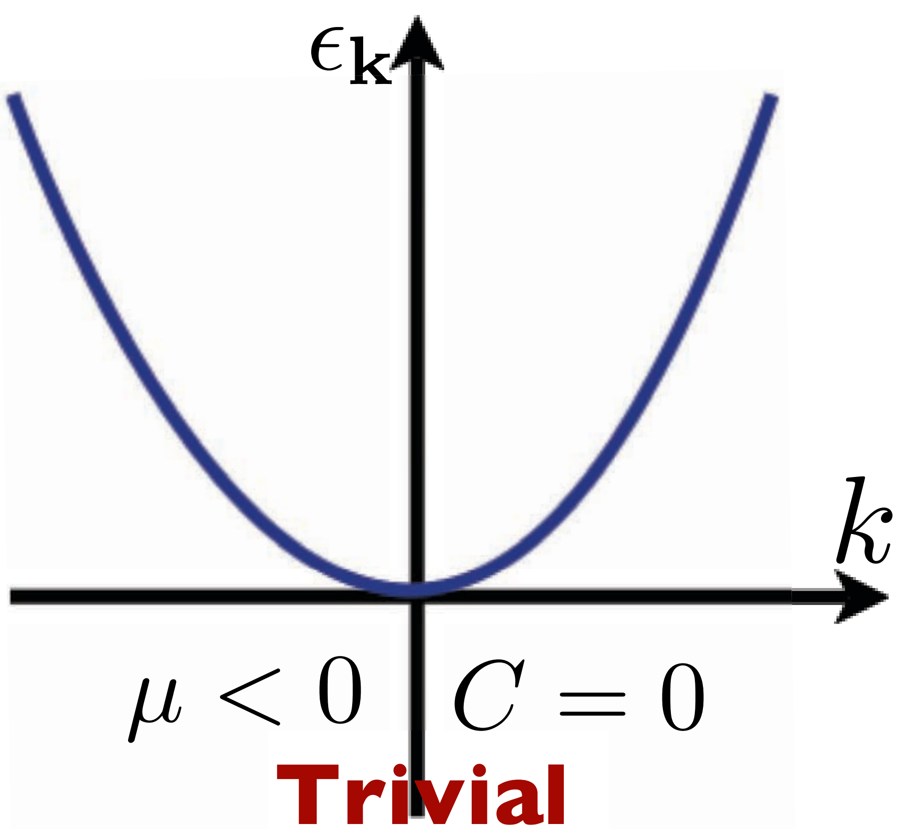


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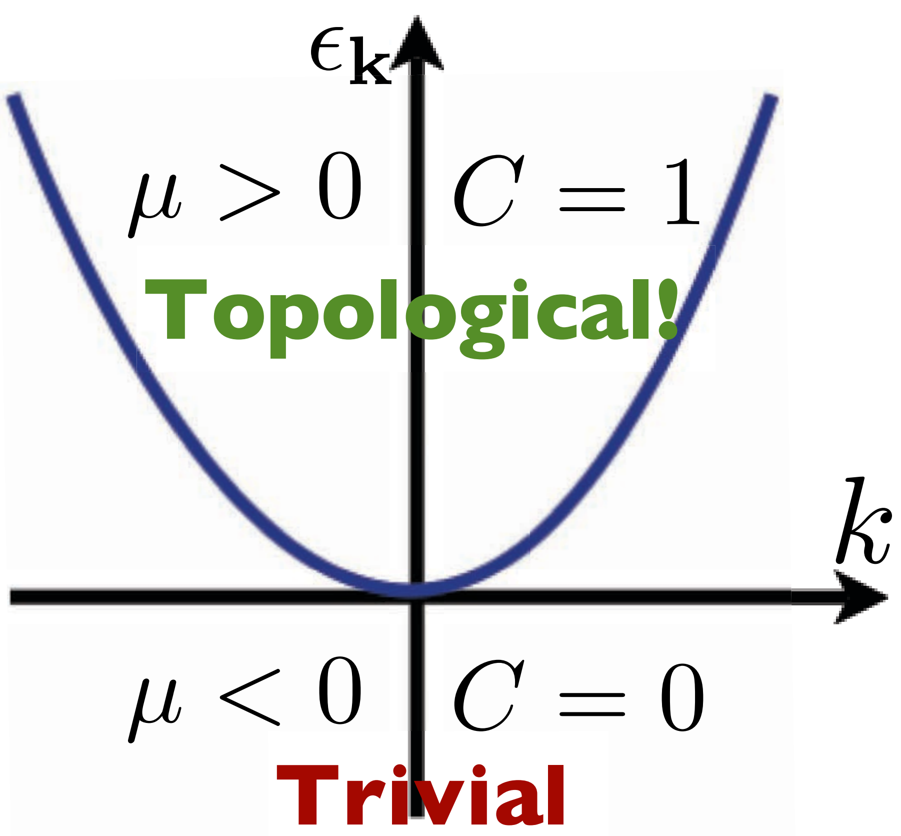


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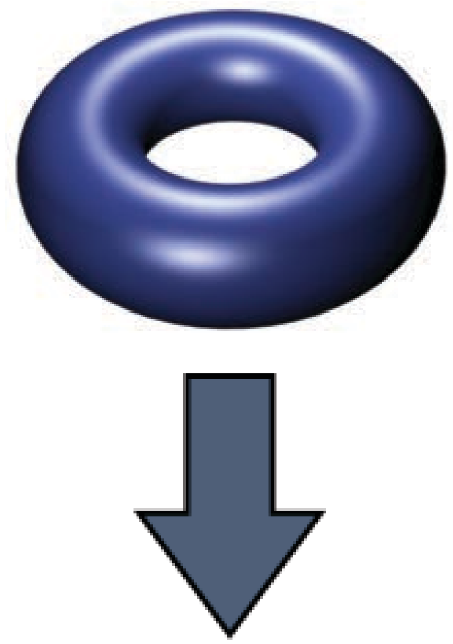
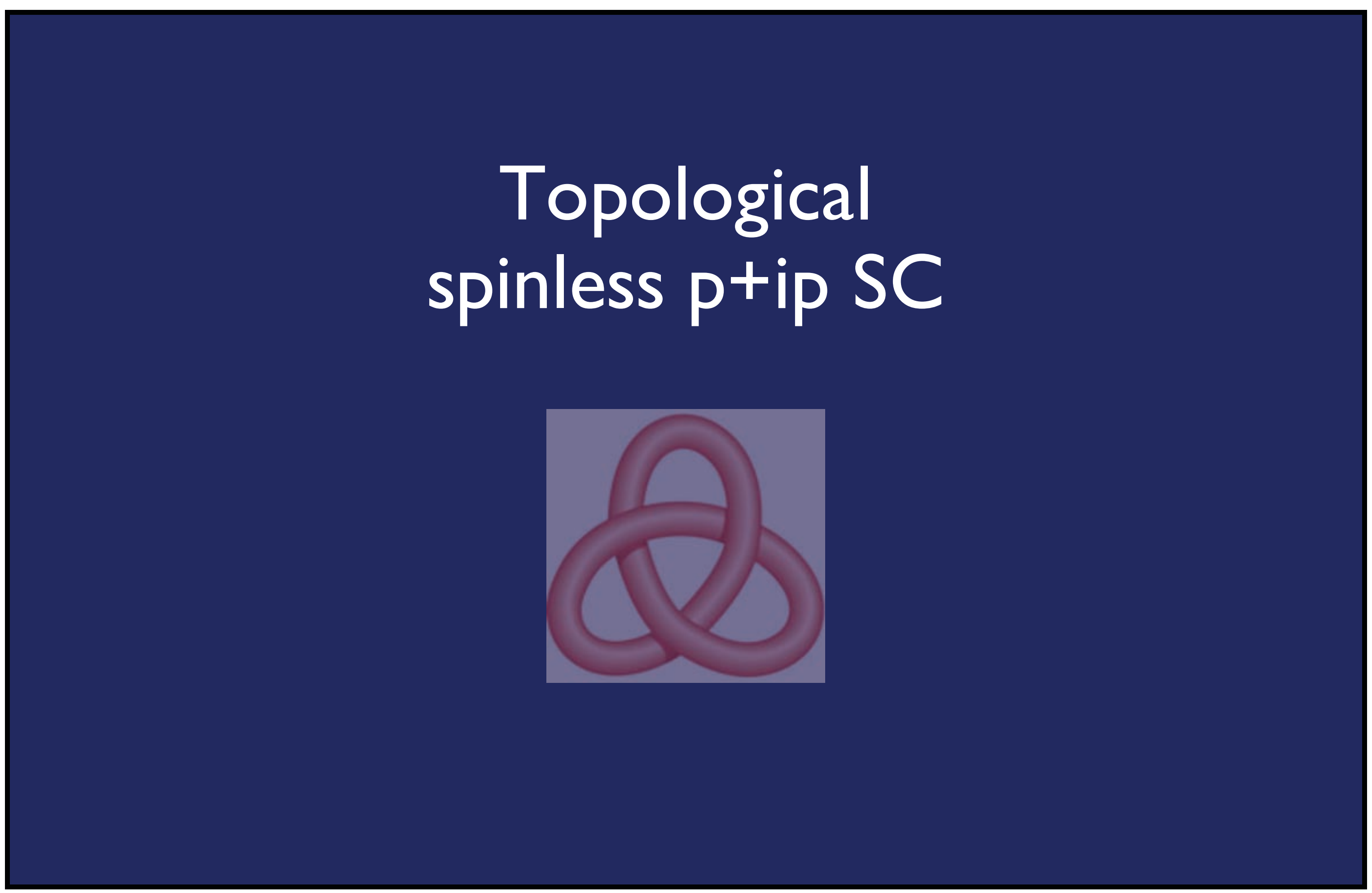
# Boundary physics & vortices

Cut torus into
planar geometry:






Vacuum

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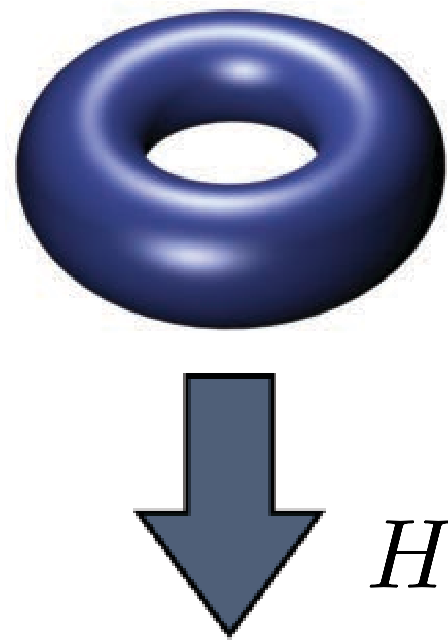
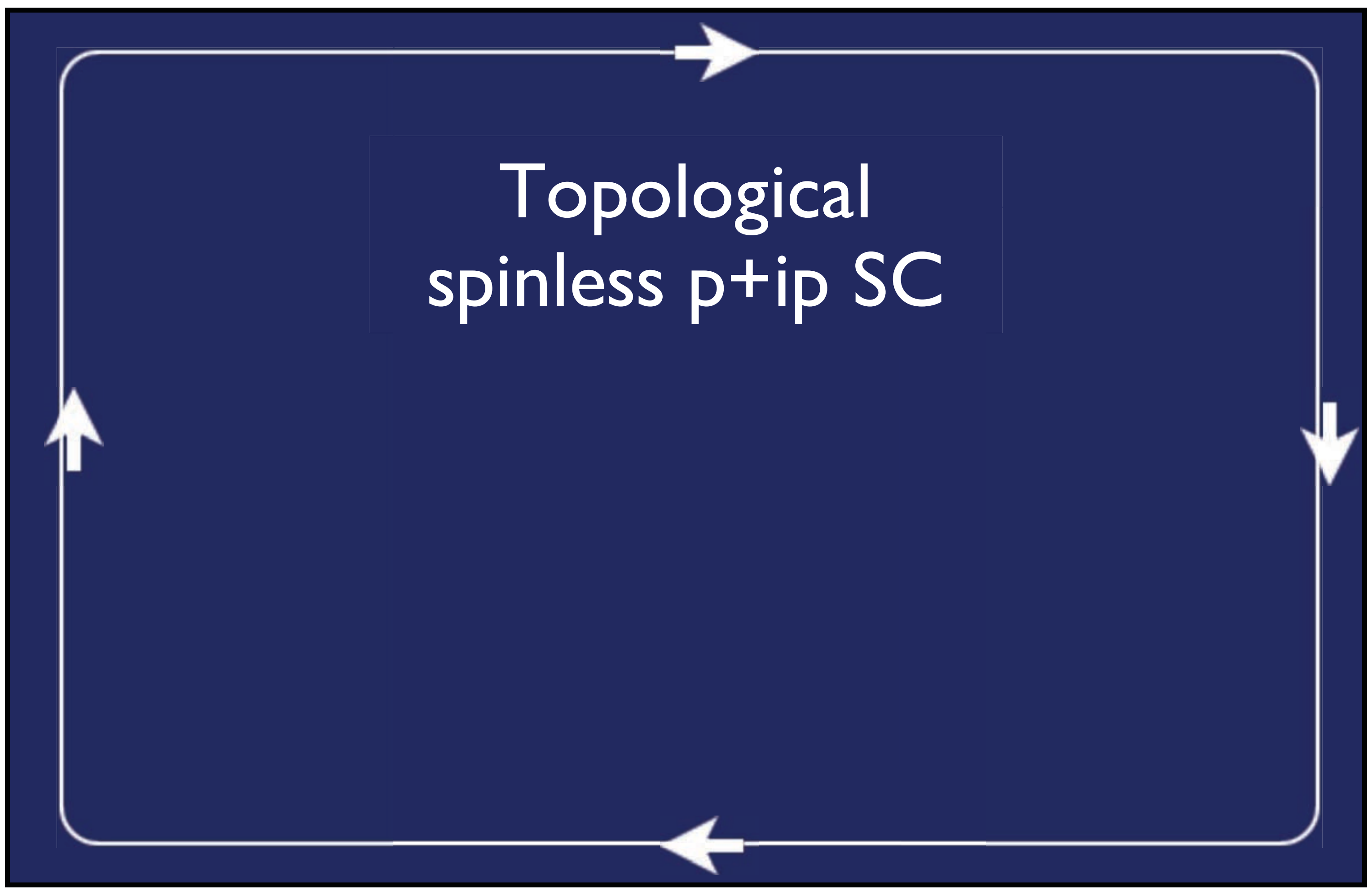
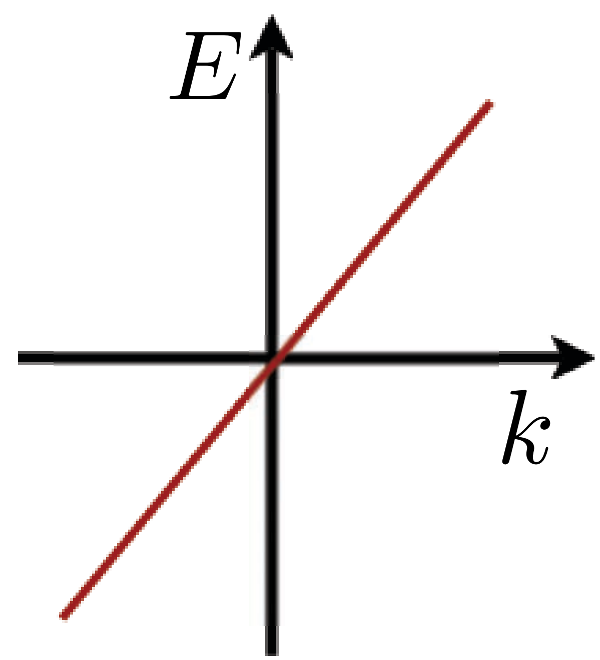


Gapless chiral Majorana edge modes = half of an integer quantum Hall edge state

$$H_{\text{edge}} = \int du \gamma_{\text{edge}}(-iv\partial_u)\gamma_{\text{edge}}$$

Vacuum

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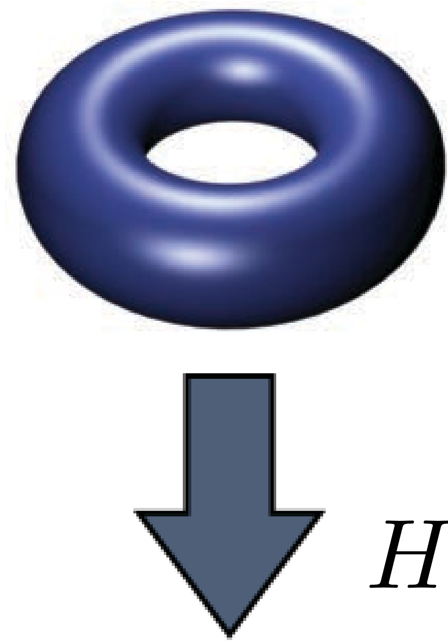

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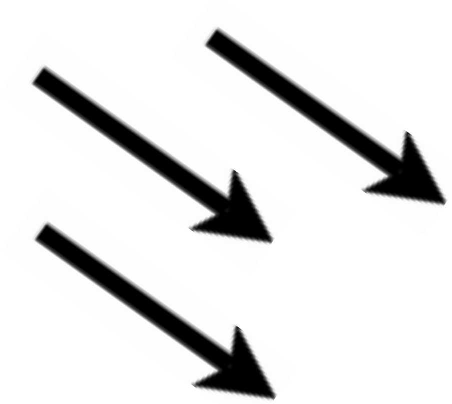

$\mathbf{B}$

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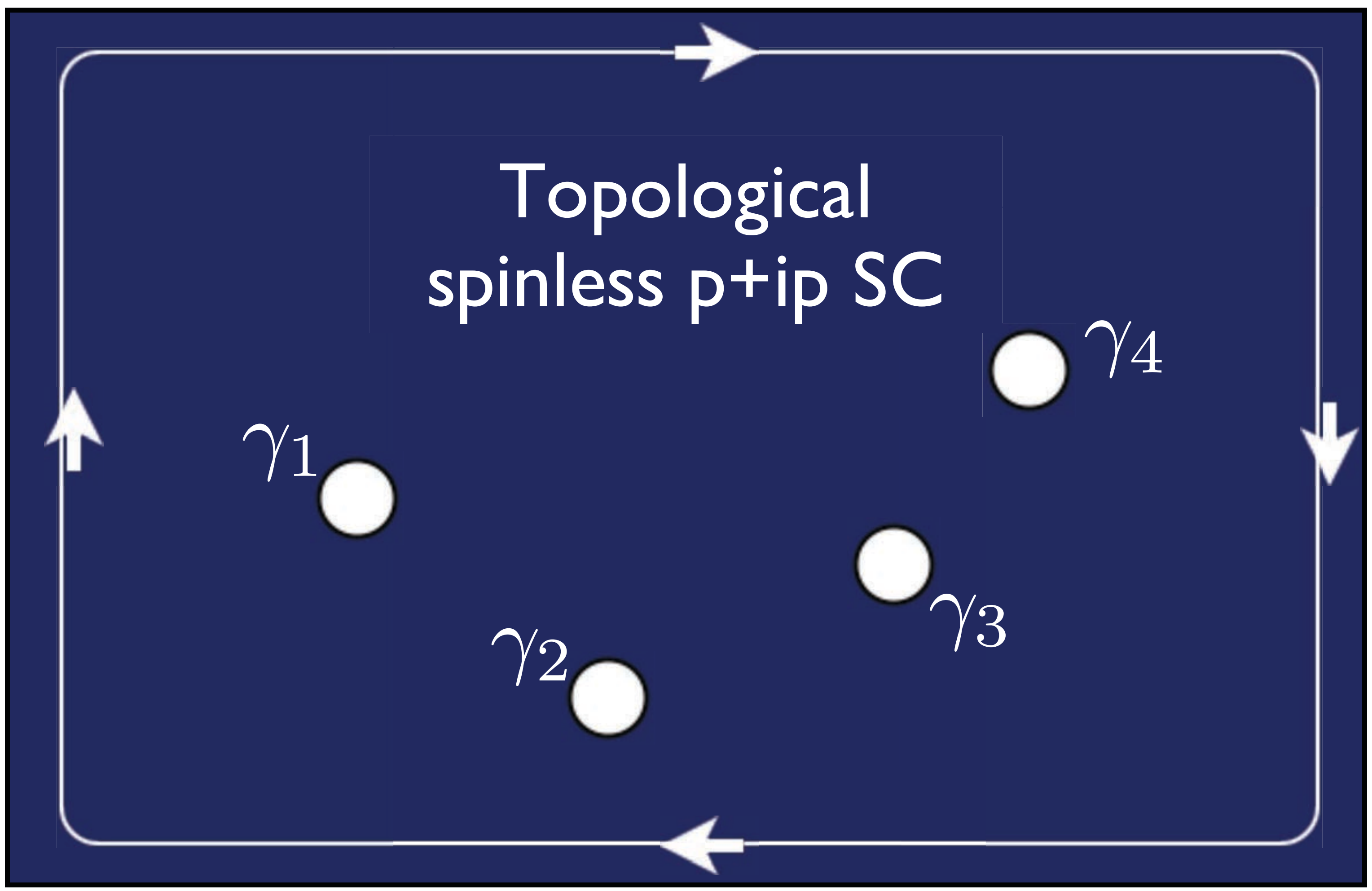


Vortices bind Majorana zero-modes...

$$f_A = \gamma_1 + i\gamma_2$$

$$f_B = \gamma_3 + i\gamma_4$$

(Read & Green 2000)

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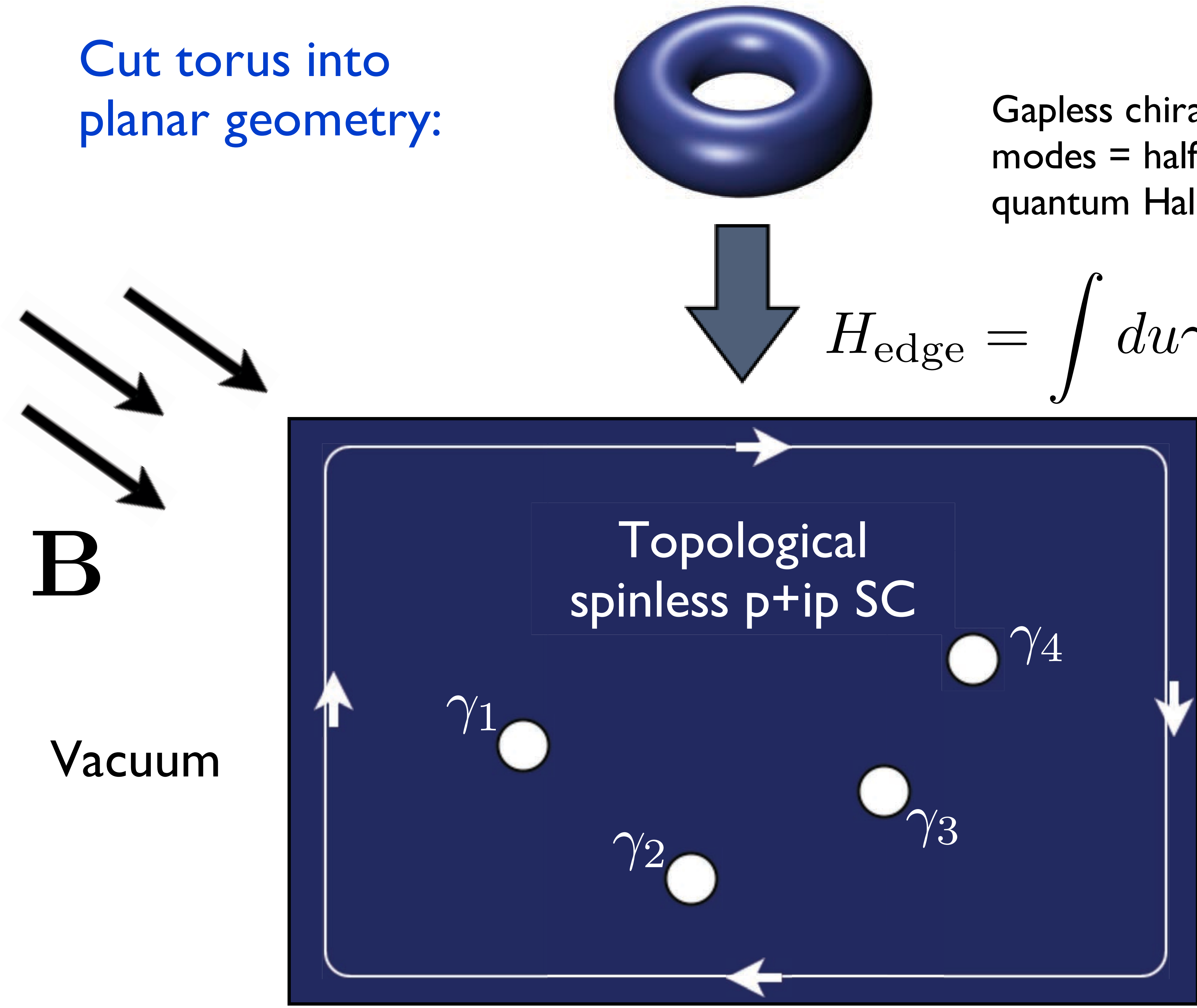


Vortices bind Majorana zero-modes...

$$f_A = \gamma_1 + i\gamma_2$$

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...and exhibit non-Abelian statistics!

(Read & Green 2000; Ivanov 2001)

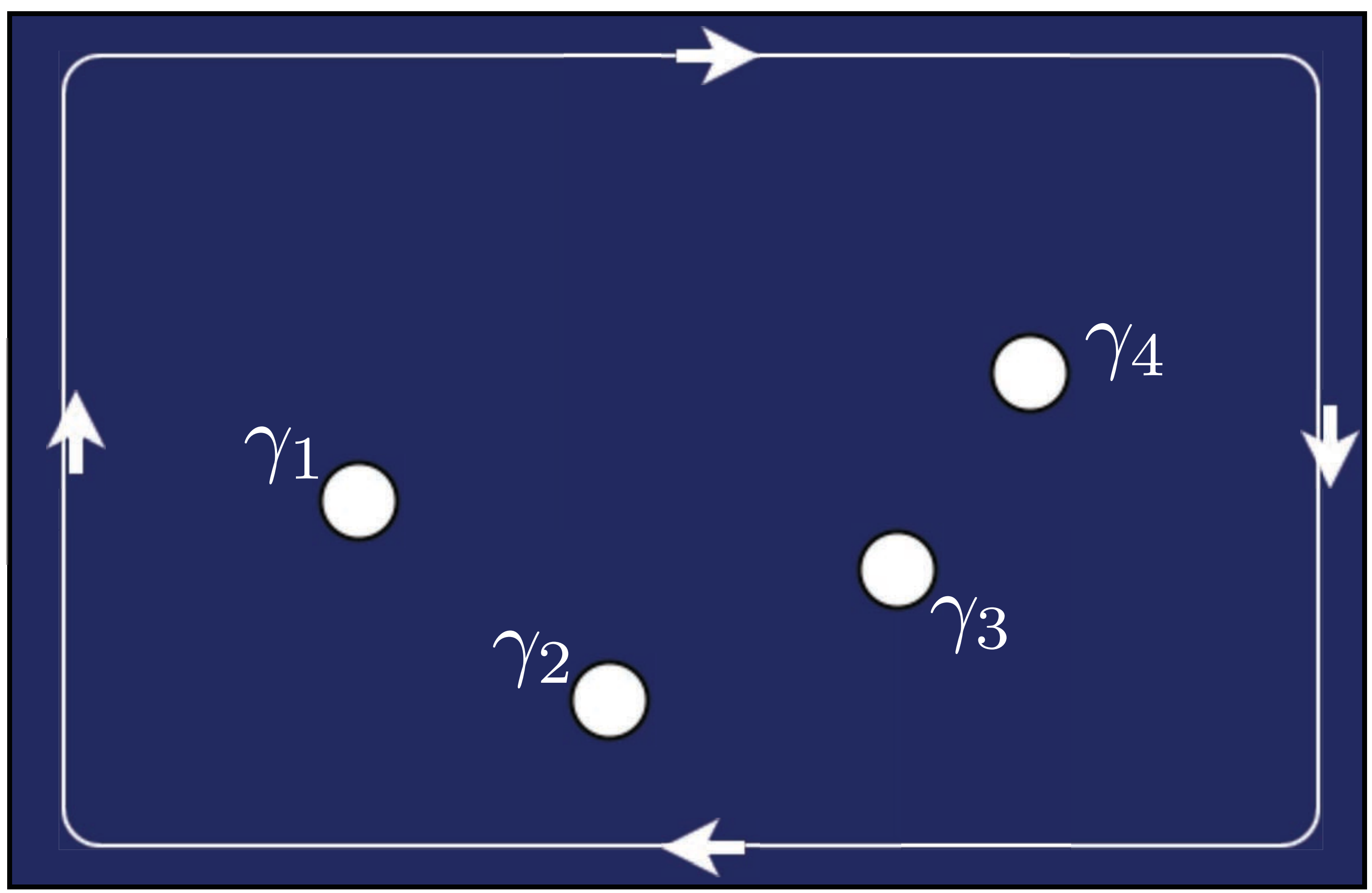
# Summary of toy models

End Majorana fermion zero-modes

1D spinless p-wave SC in topological phase


2D spinless p+ip SC in topological phase

Majorana fermion zero-modes bind to vortices...




...and edge supports gapless chiral Majorana mode

# Homework Set 1

1. For the Kitaev chain, find an expression for the topological invariant distinguishing the trivial and non-trivial phases of the model. Does it take on integer values, or is it a $Z_2$ index?

2. Find the Hamiltonian describing low-energy physics at the phase transition between topological and trivial phases of the Kitaev chain. Compare to the edge Hamiltonian for a p+ip superconductor.

3. For a 2D spinless p+ip superconductor, derive the chiral Majorana edge Hamiltonian. Does the Majorana operator satisfy periodic or antiperiodic boundary conditions? How does your answer change if the bulk has vortices?

$$H_{\text{edge}} = \int du \gamma_{\text{edge}} (-iv\partial_u) \gamma_{\text{edge}}$$

4. Using your answer to 3, argue that h/2e vortices bind Majorana zero-modes as claimed. (Hint: think about the vortex core as a small puncture in the system.) Are there other finite-energy modes bound to the vortex? If so what are their energies?

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- Some general remarks on Majorana fermions in condensed matter

- Toy models capturing Majoranas (continued)

  - Quick reminder of "Kitaev chain"

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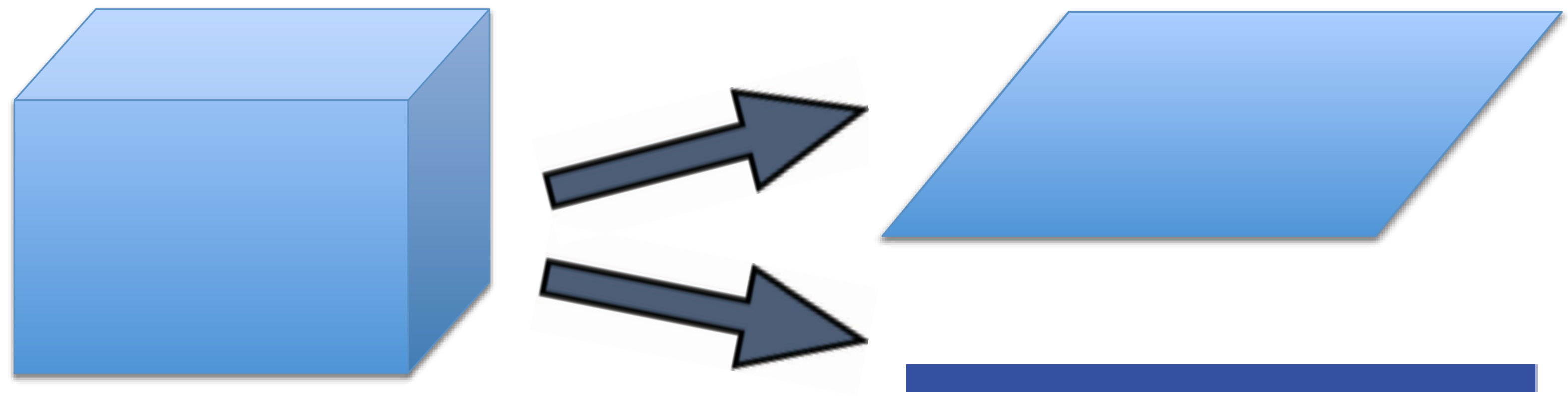
# The basic challenges

"Spinless" 1D, 2D p-wave superconductivity is hard to find.

1. We live in 3D



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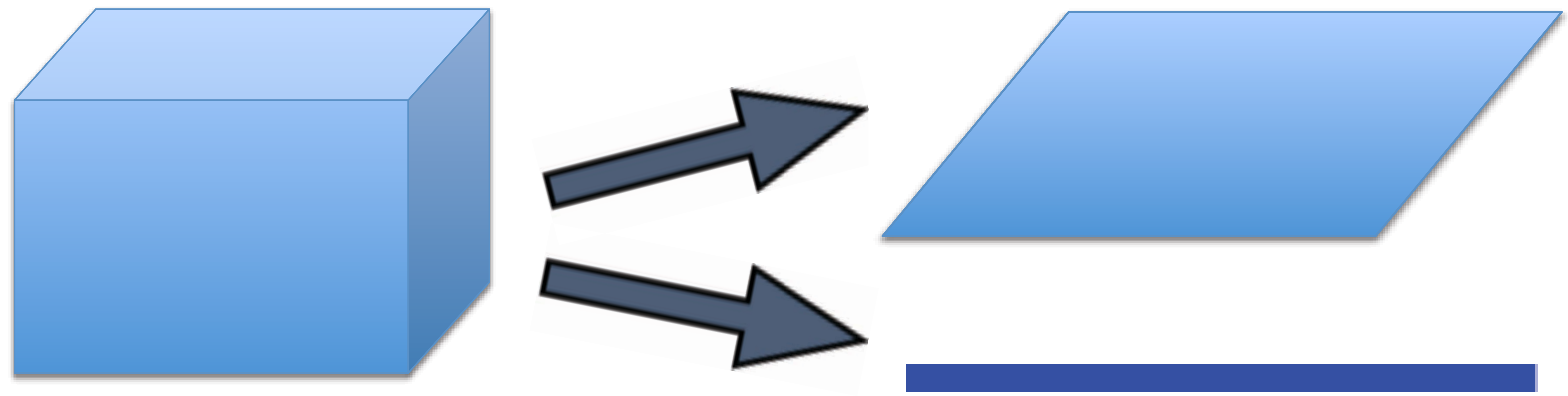

("Intrinsic" 1D and 2D superconductors also do not exhibit LRO at finite T, unlike toy models where this is assumed.)

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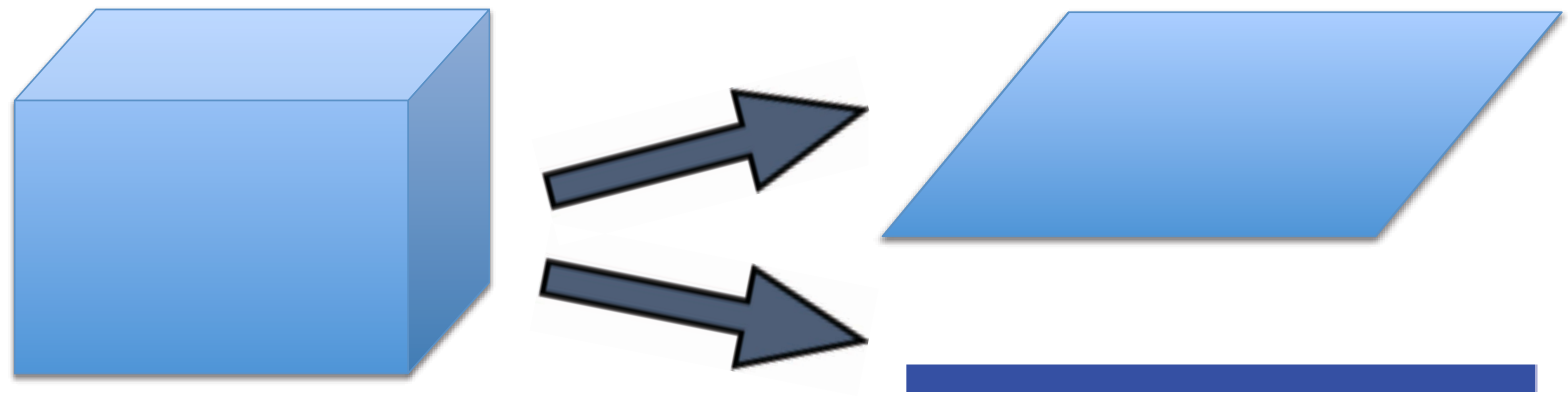
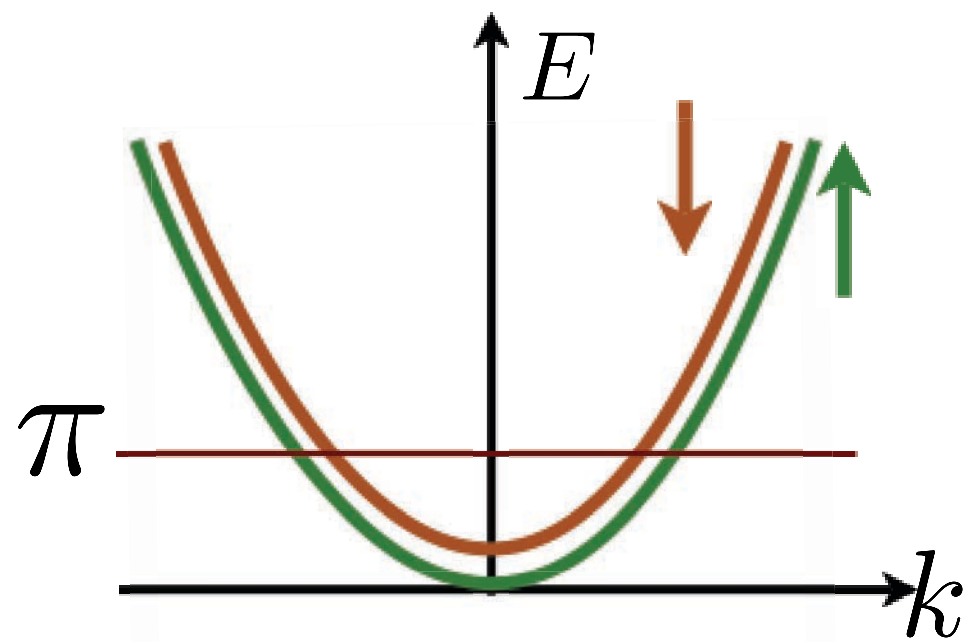


2. Electrons carry spin



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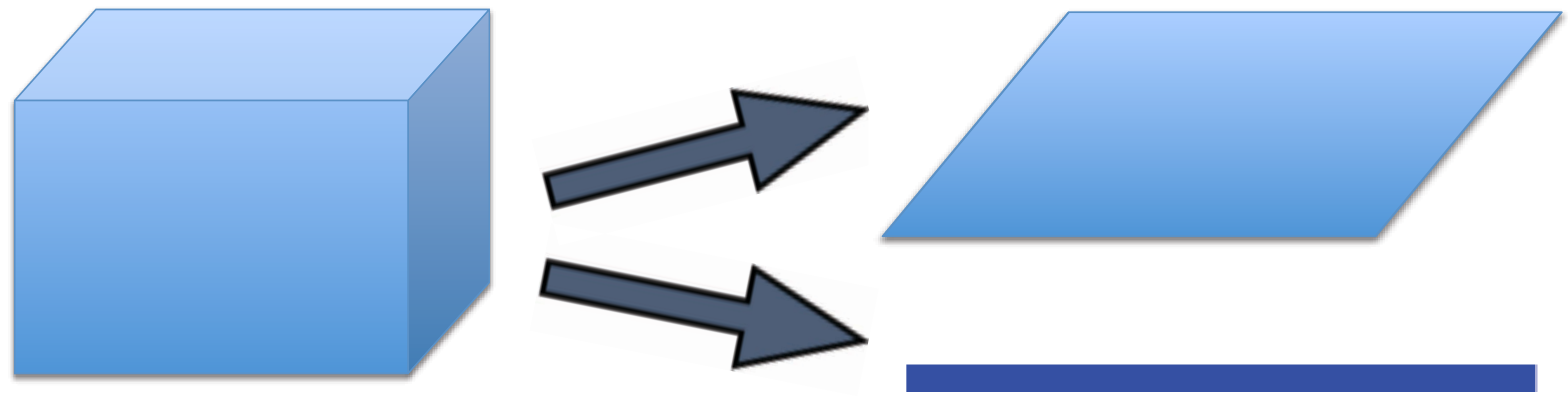

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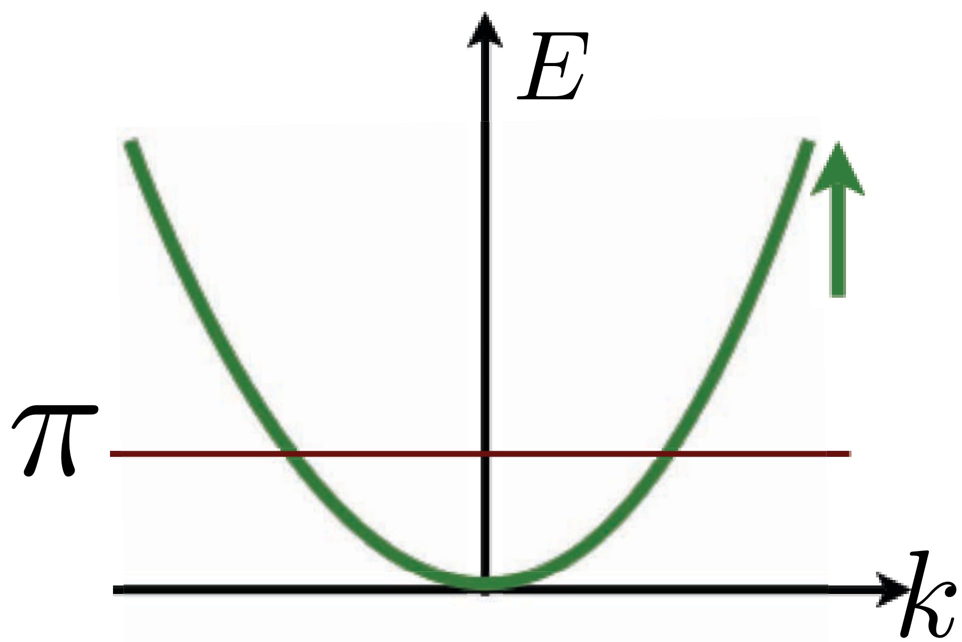
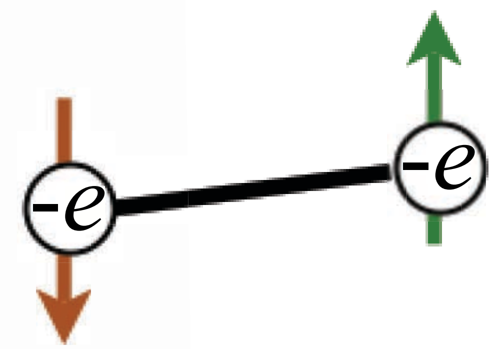



3. Vast majority of superconductors form **spin-singlet** Cooper pairs



# Two ways forward

1. Search for new compounds w/exotic superconductivity

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Matthias's 6th rule: Stay away from theorists!

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1. Search for new compounds w/exotic superconductivity

Matthias's 6th rule: Stay away from theorists!

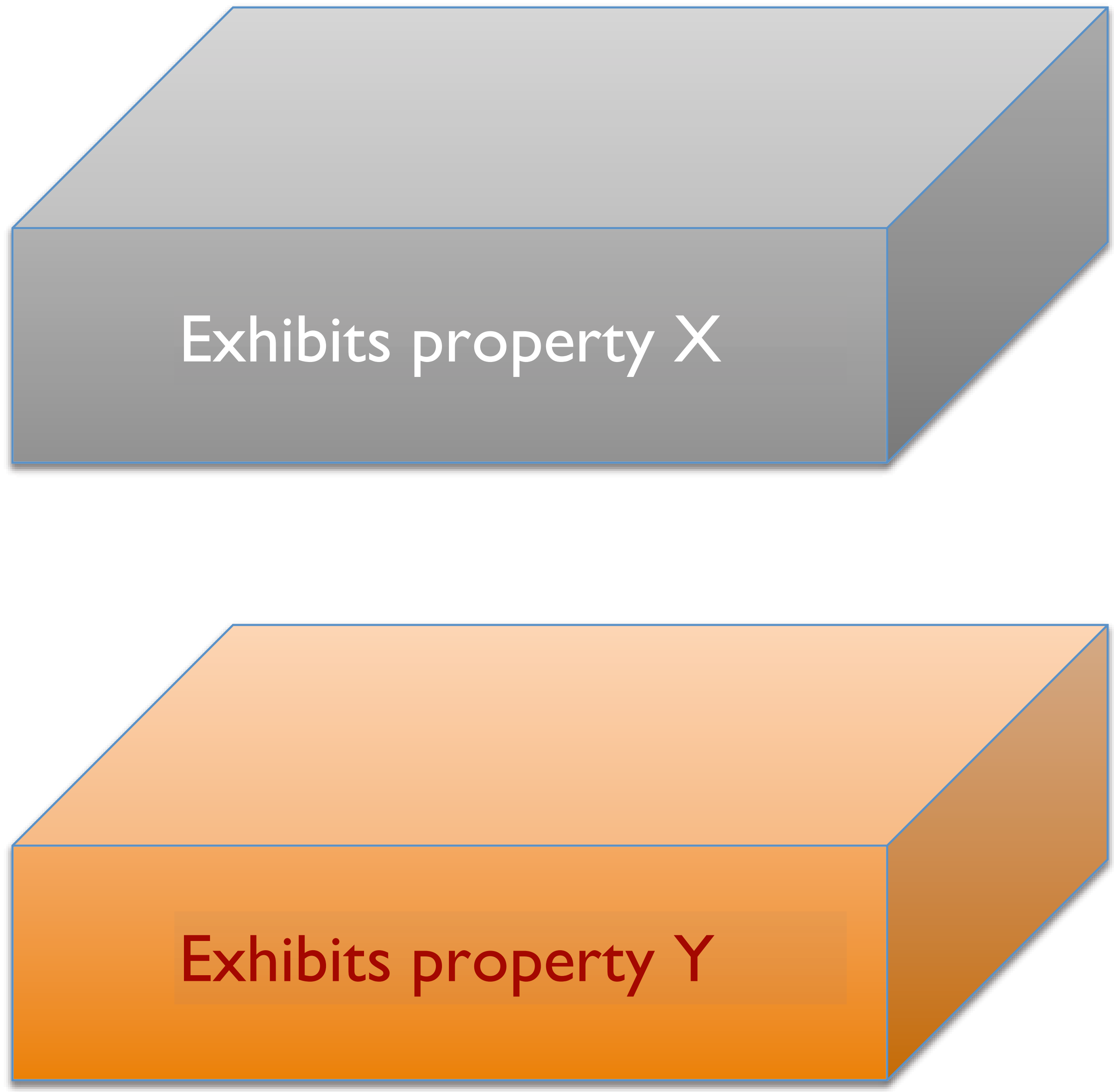
2. "Engineer" topological superconductivity from available materials

Theorists **can** be useful, particularly if methods involve **weakly interacting electrons**

(Approach originally pioneered by Fu & Kane.)

# General strategy for "engineering" topological superconducors (and other exotic phases)

Suppose you want a system with properties X and Y which seem incompatible...



# General strategy for "engineering" topological superconducors (and other exotic phases)

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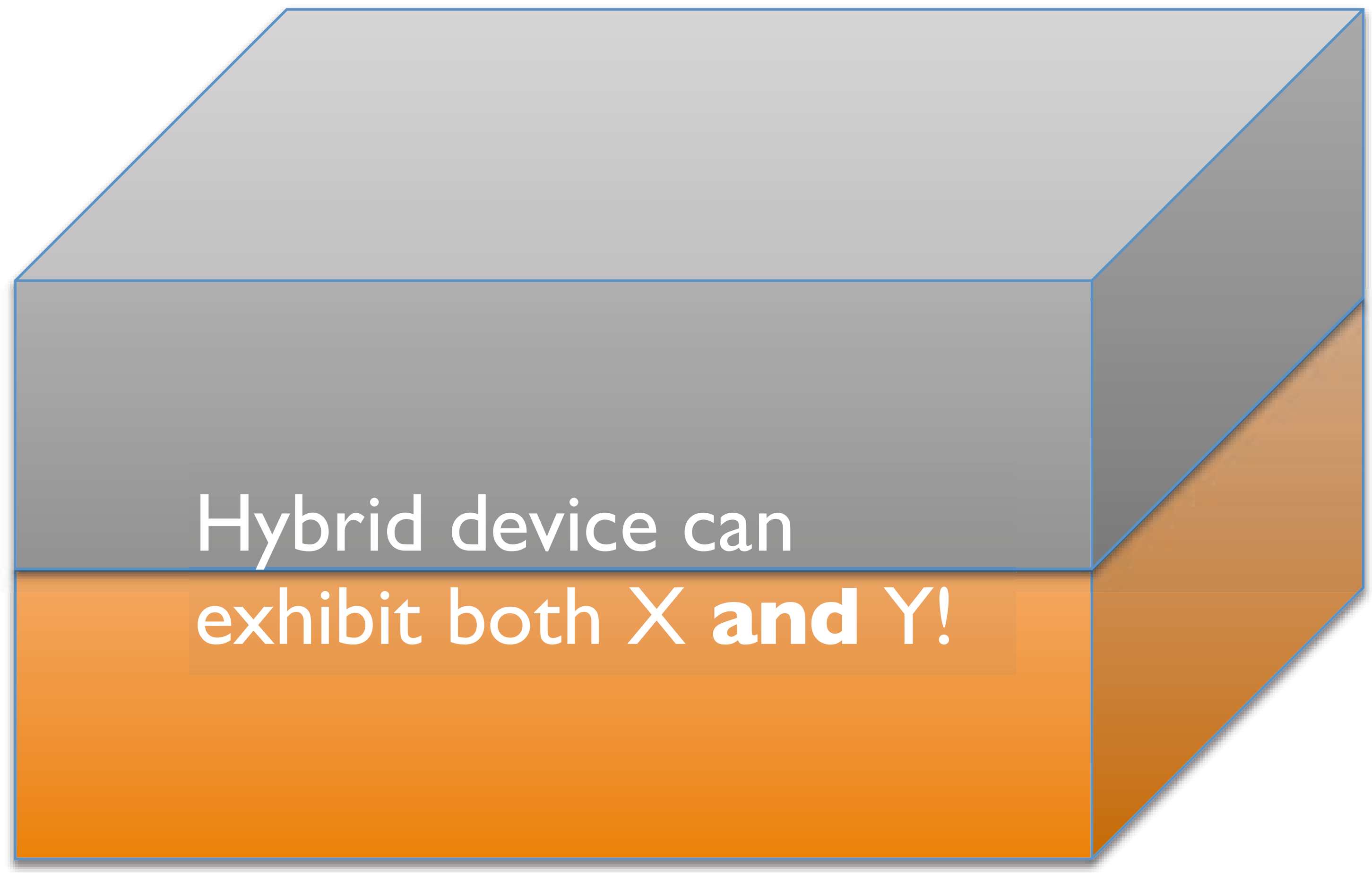
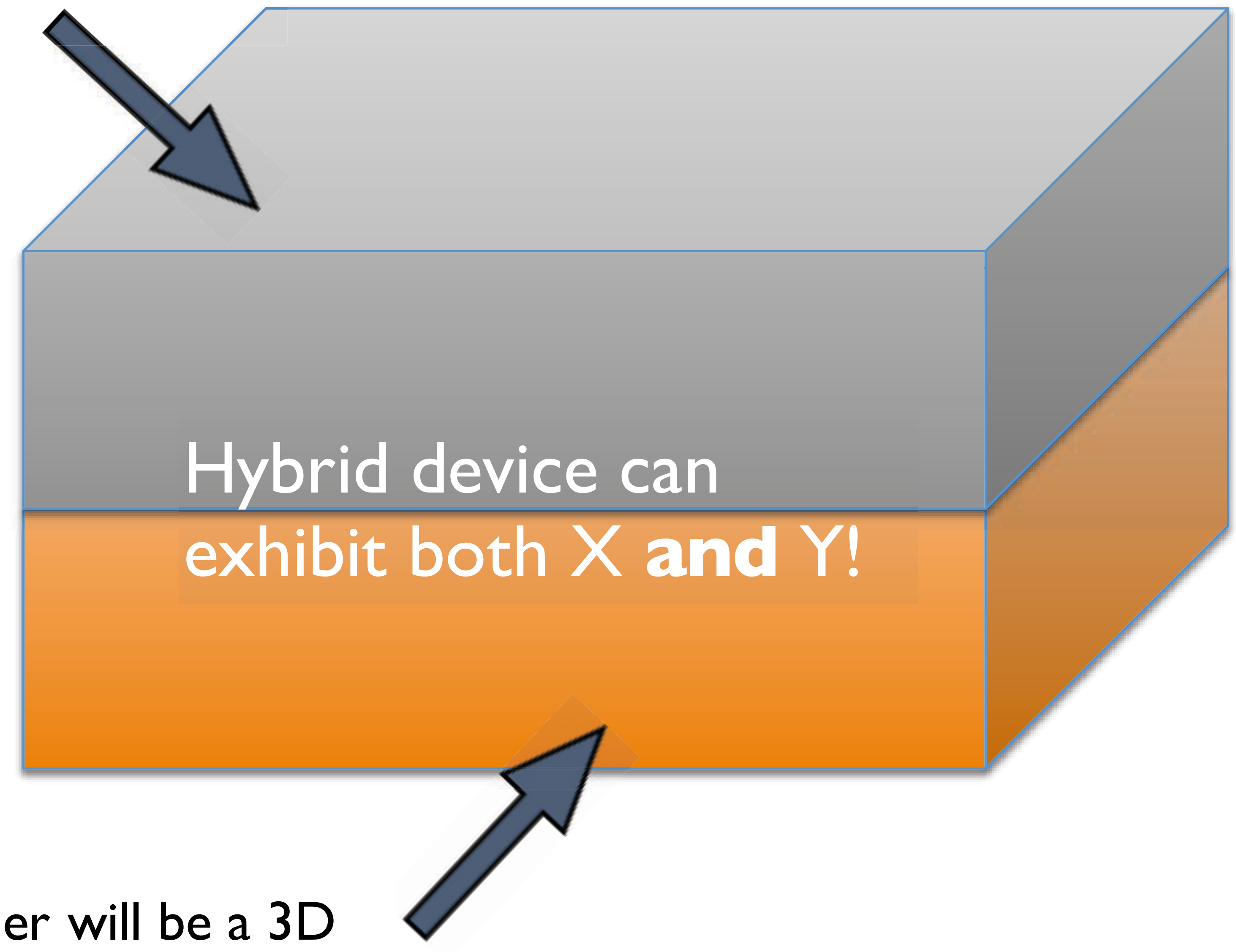



**Very useful concept, likely with lots of untapped potential!**

# General strategy for "engineering" topological superconducors (and other exotic phases)

Here, one subsystem will support 1D or 2D modes with **strong spin-orbit coupling**...

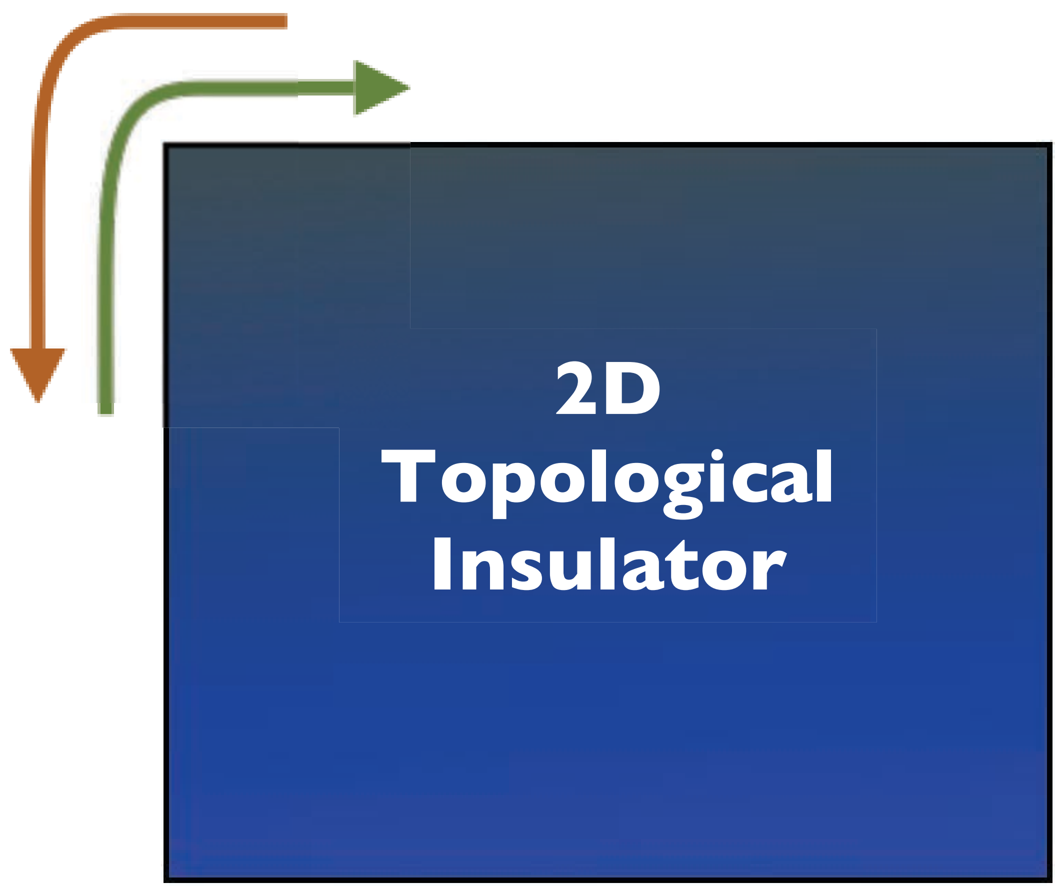
Suppose you want a system with properties X and Y which seem incompatible...

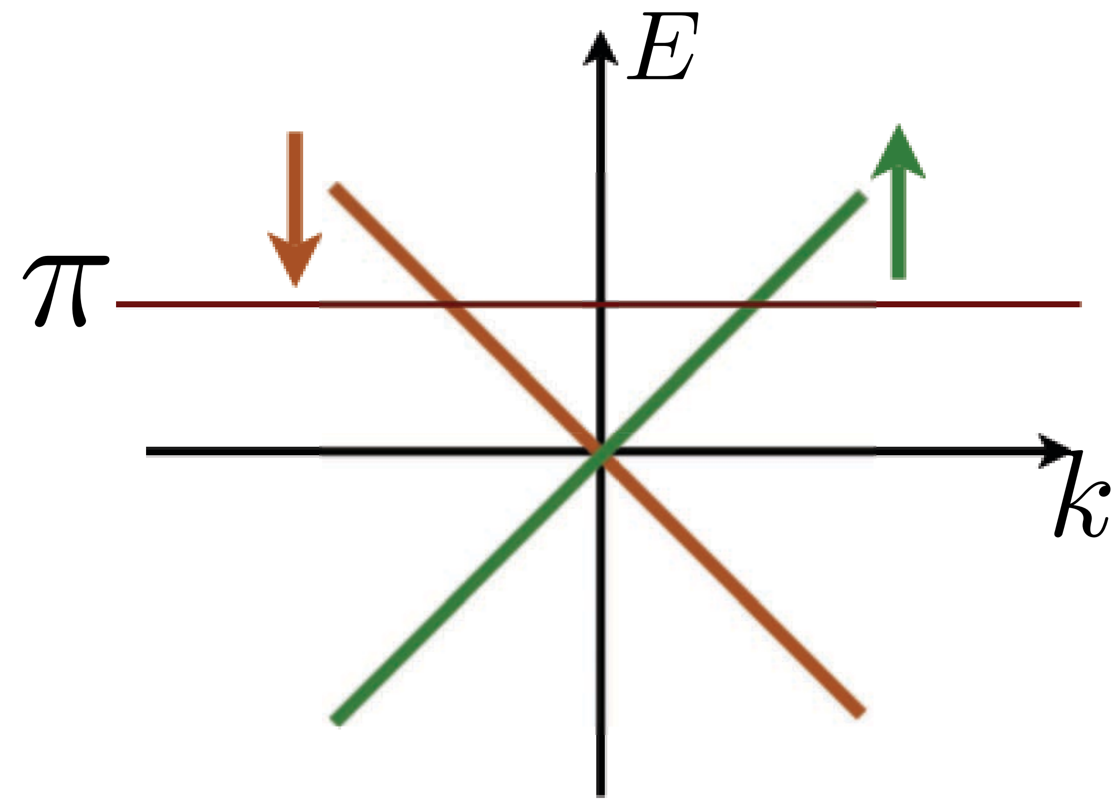



...the other will be a 3D **s-wave** superconductor (with LRO).

# Experimental Routes to 1D topological superconductivity

# 1D topological superconductivity via edge states



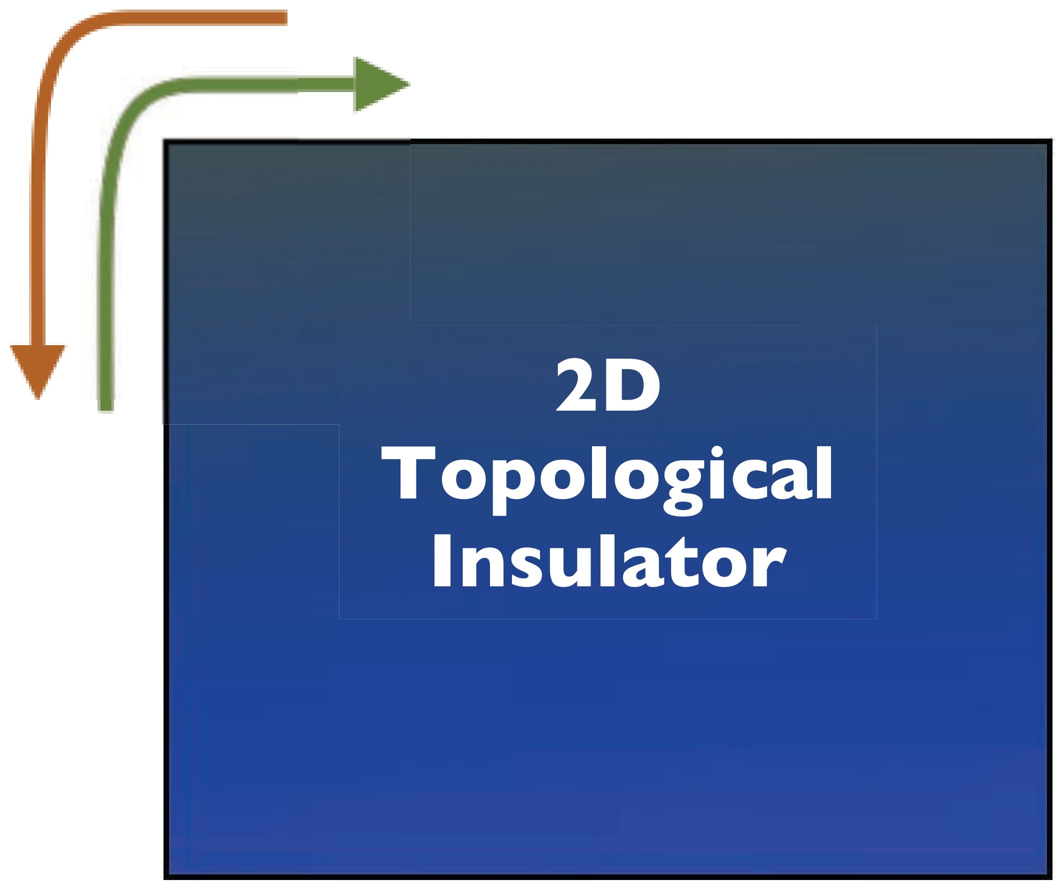

I. Gapless as long as time-reversal, U(1) particle conservation are present

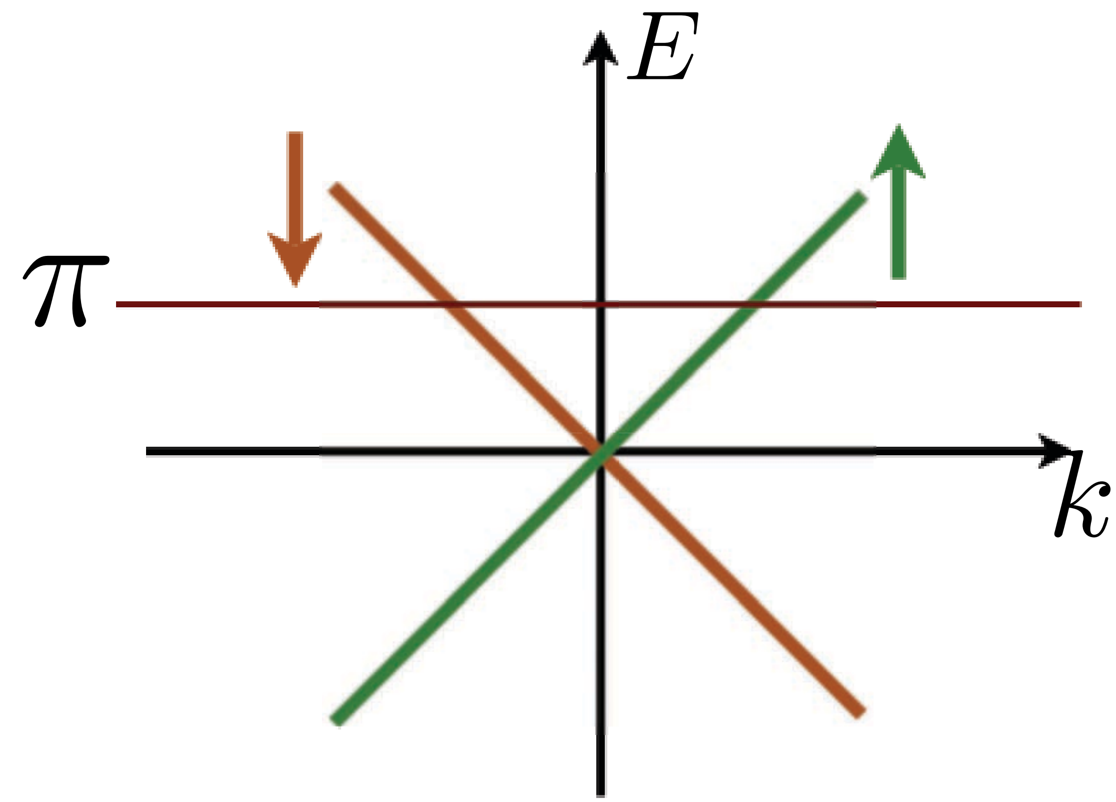
II. By construction 1D & "spinless"

III. Easy to make superconducting

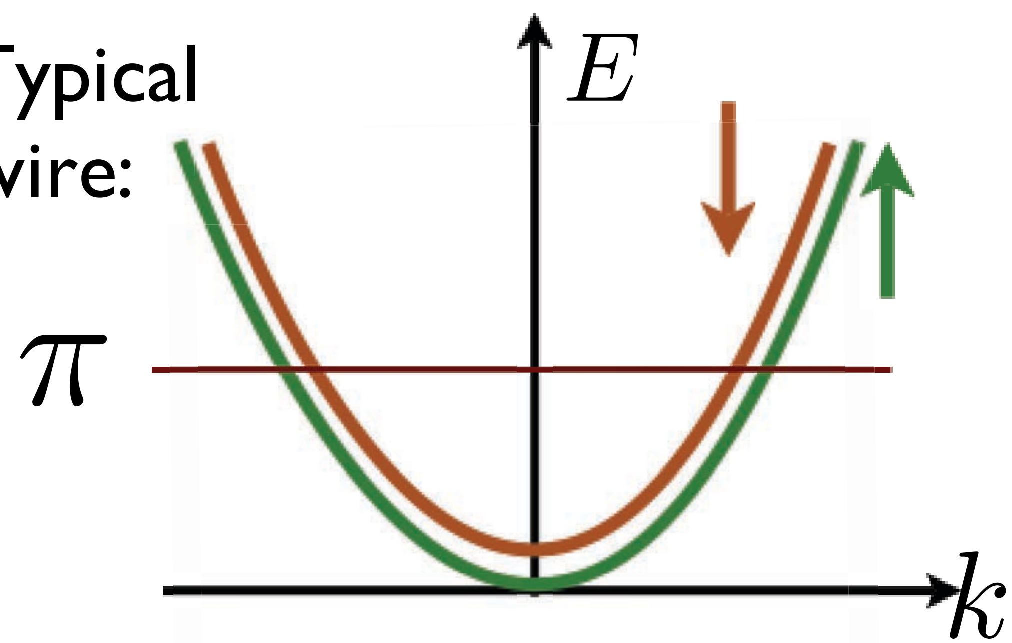
Fu & Kane 2009

# 1D topological superconductivity via edge states




Typical wire:

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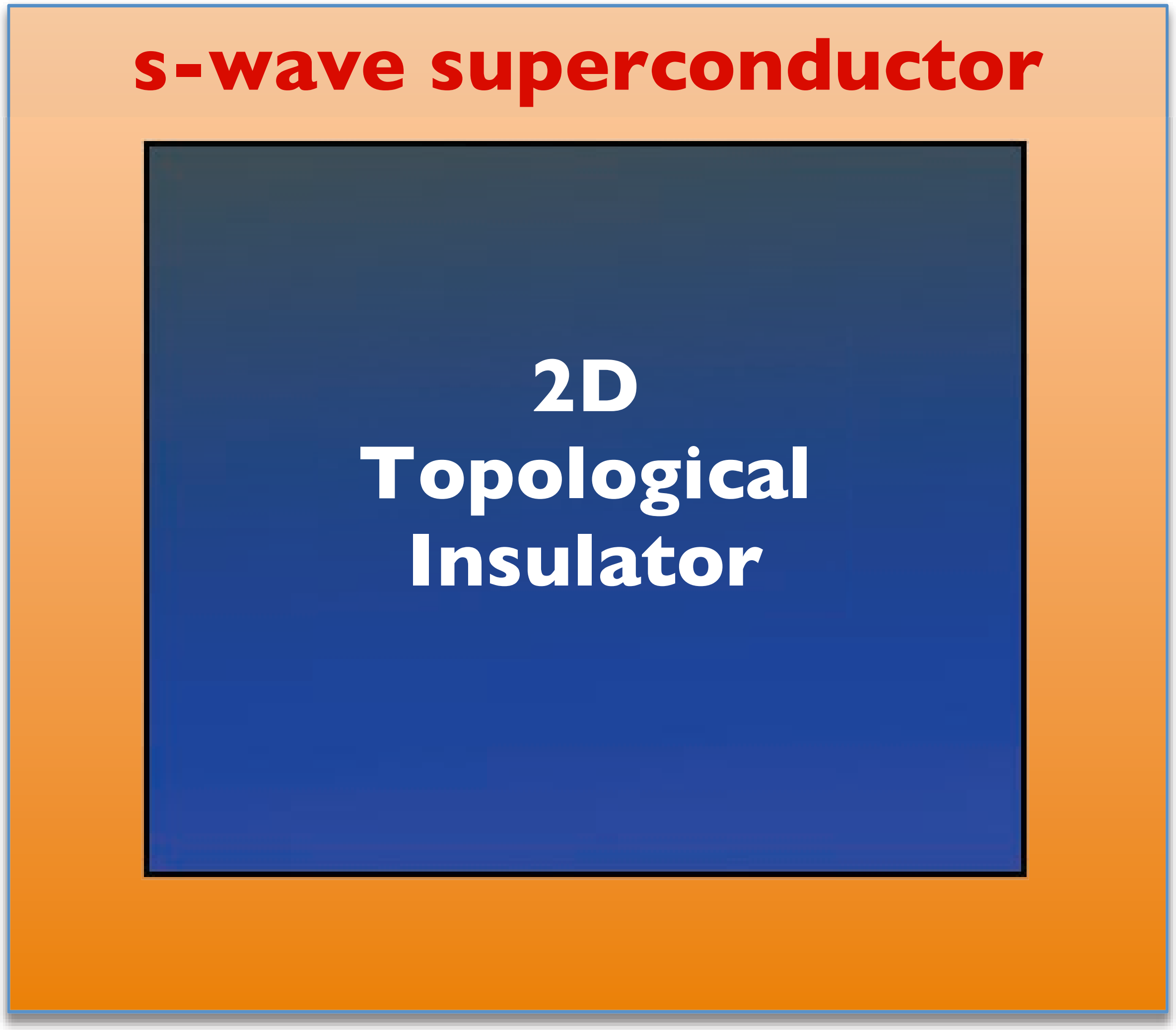
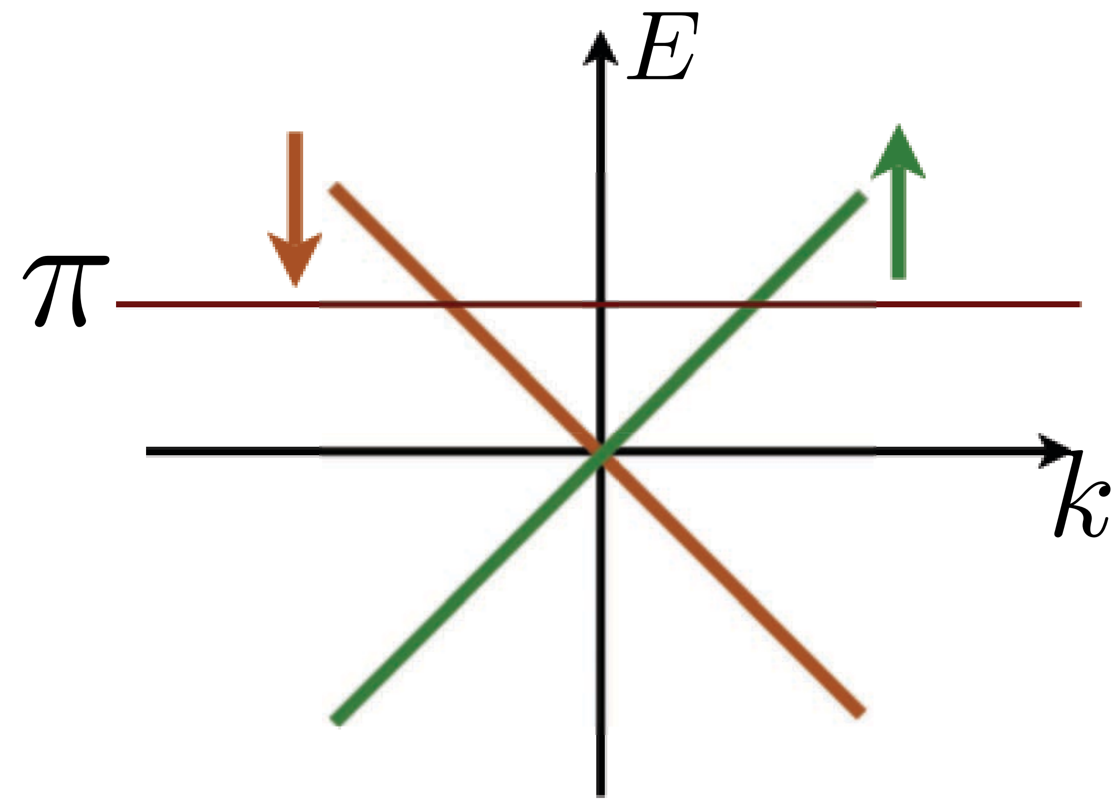
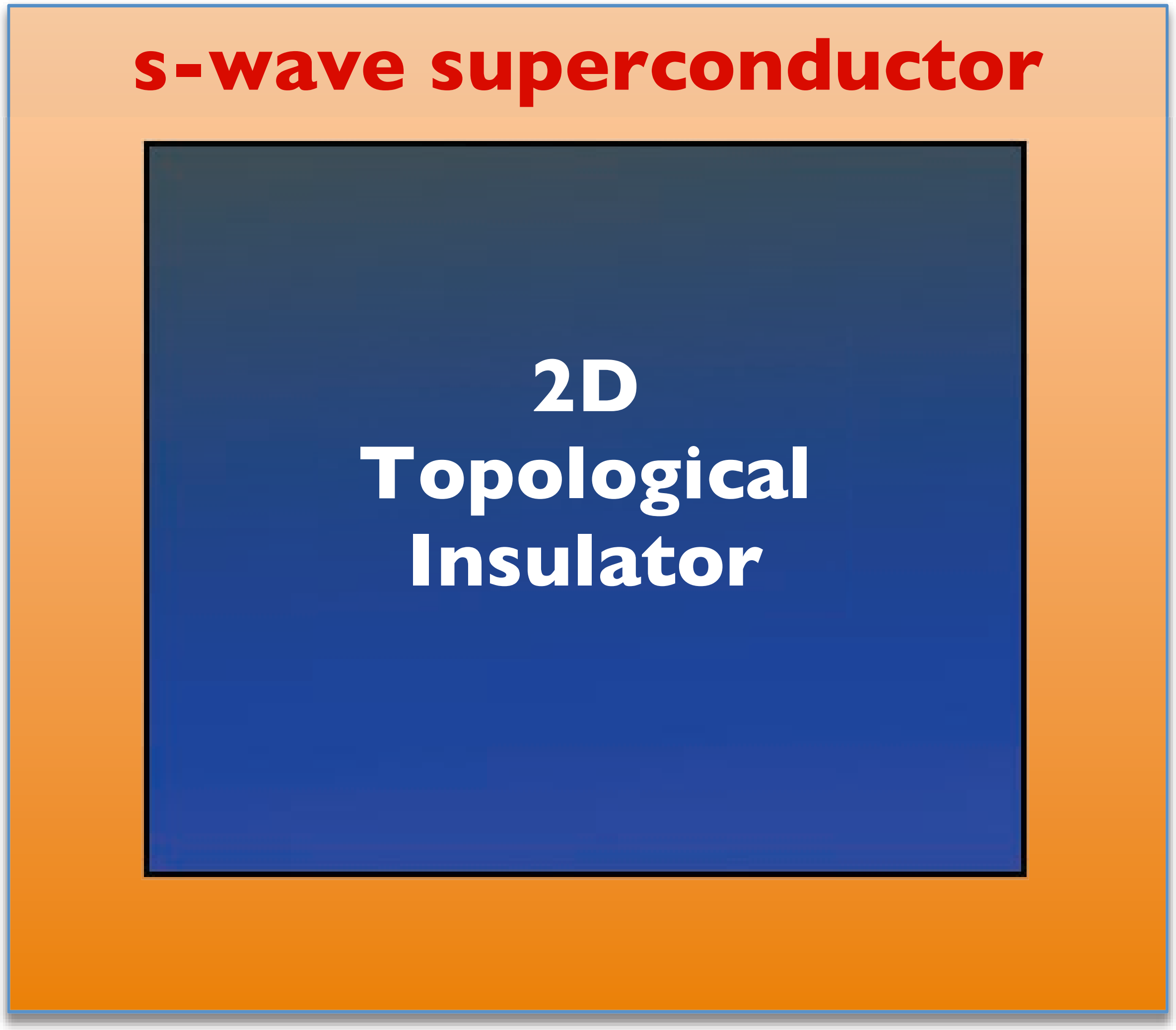
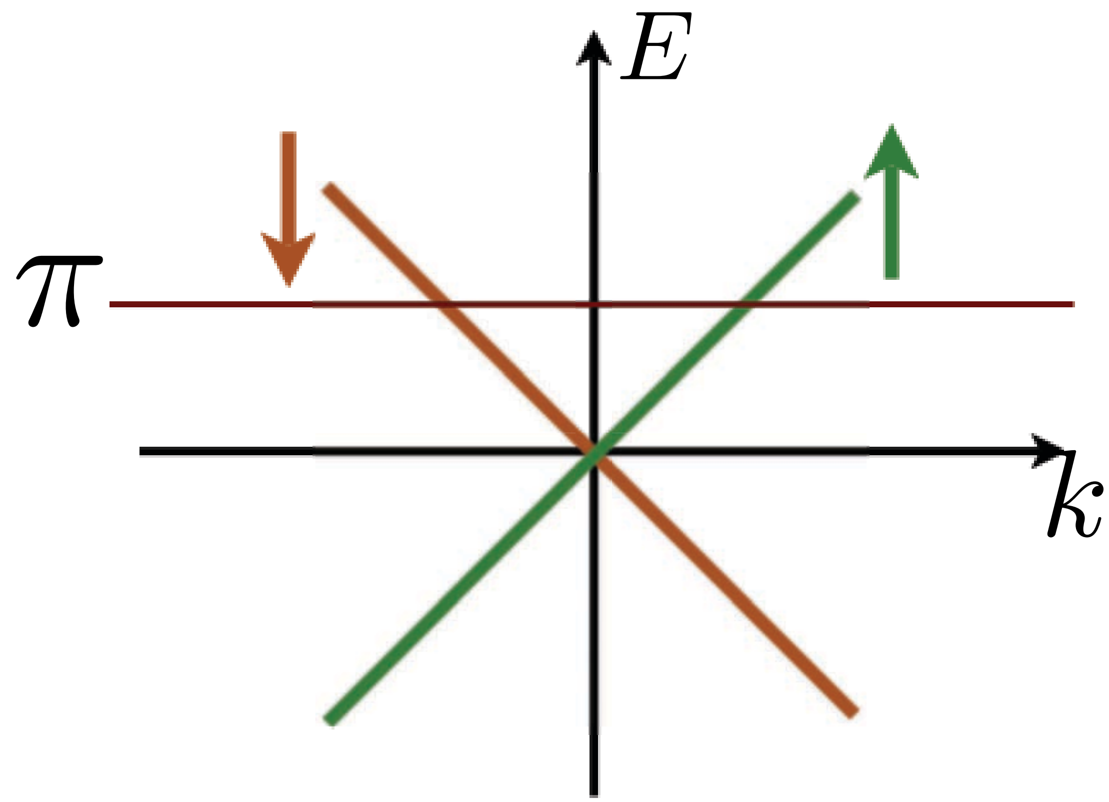



Fu & Kane 2009

# 1D topological superconductivity via edge states



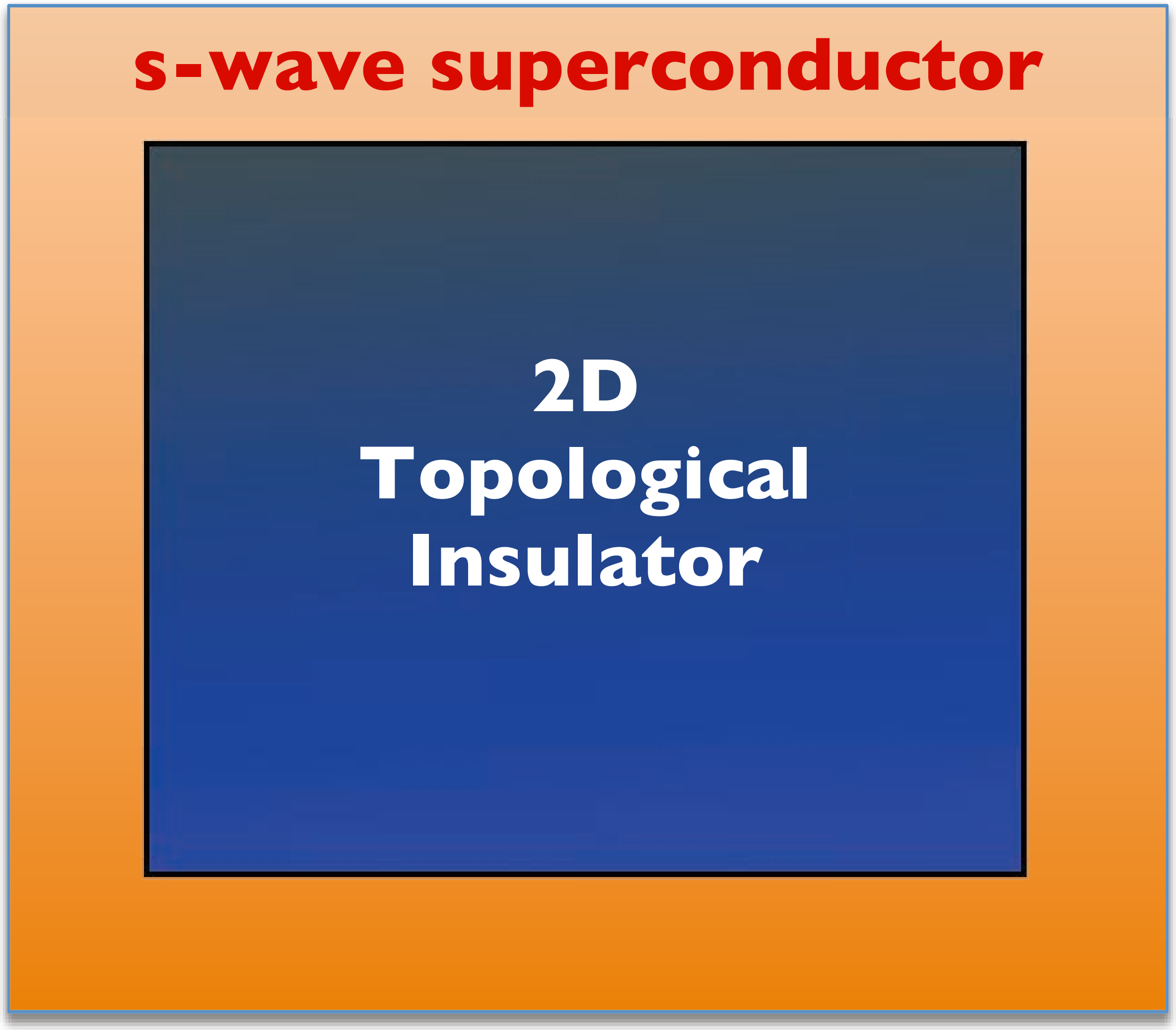

$$H_{\text{edge}} = \int dx[-\pi(\psi_R\psi_R + \psi_L\psi_L) - ihv(\psi_R\partial_x\psi_R - \psi_L\partial_x\psi_L)]$$

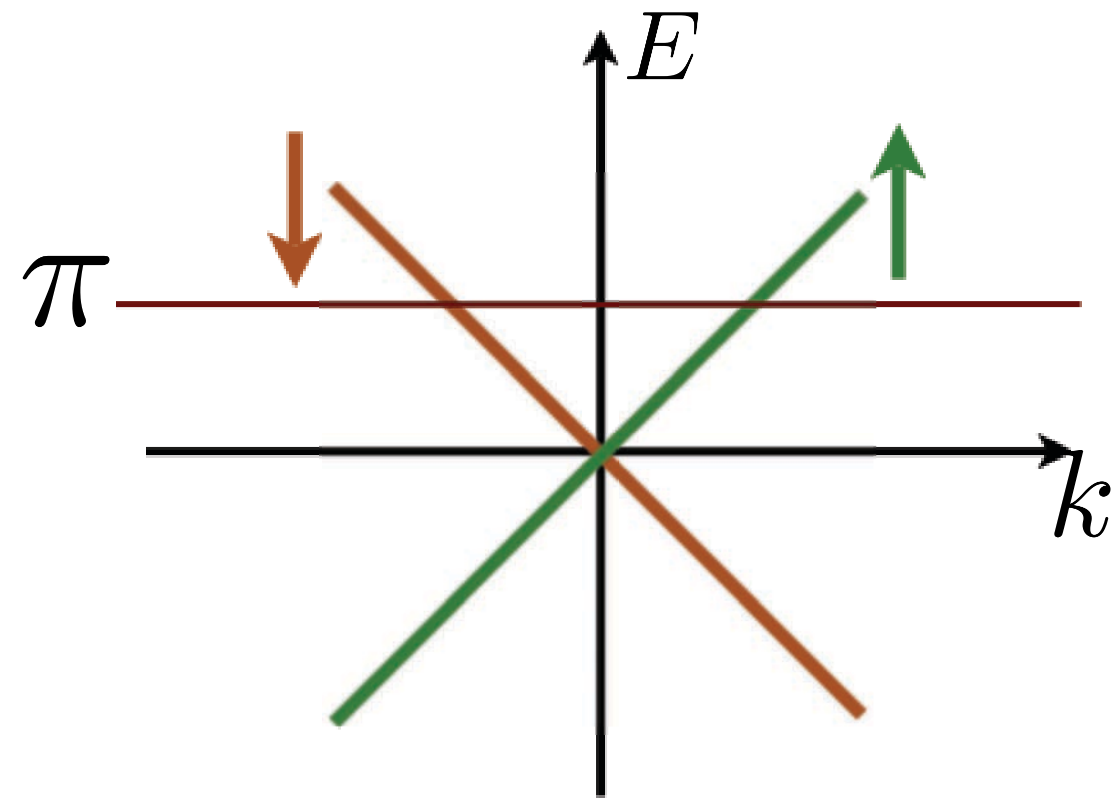
$$+H_{\text{hybridization}}$$

Can then "integrate out" gapped superconductor degrees of freedom

Fu & Kane 2009

# 1D topological superconductivity via edge states



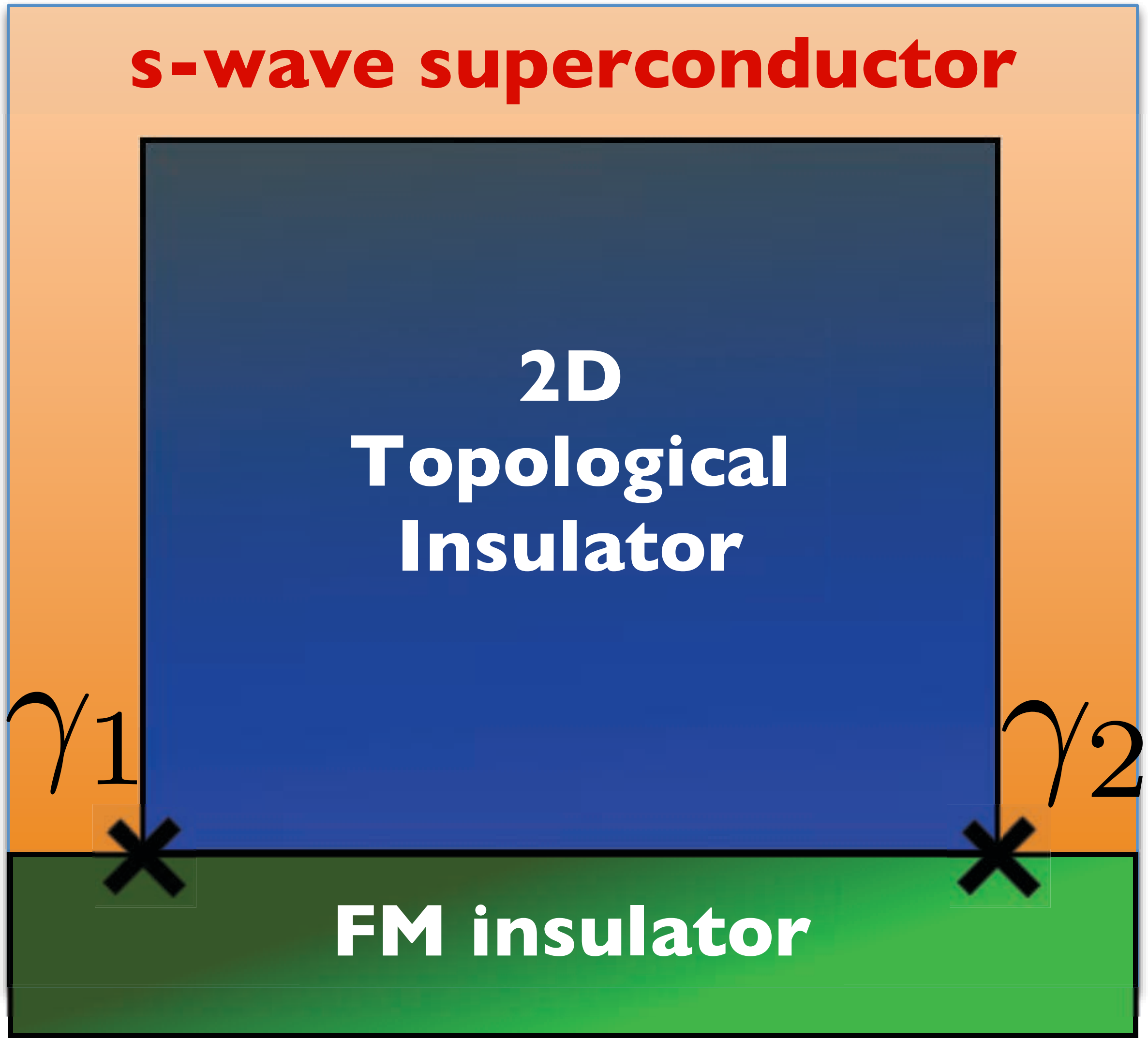




$$H_{\rm edge} = \int dx[-\pi(\psi_R\psi_R + \psi_L\psi_L) - ihv(\psi_R\partial_x\psi_R - \psi_L\partial_x\psi_L)]$$
$$+ \int dx\Delta(\psi_R\psi_L + H.c.)$$

Describes a 1D topological superconductor (on a ring)!

Fu & Kane 2009

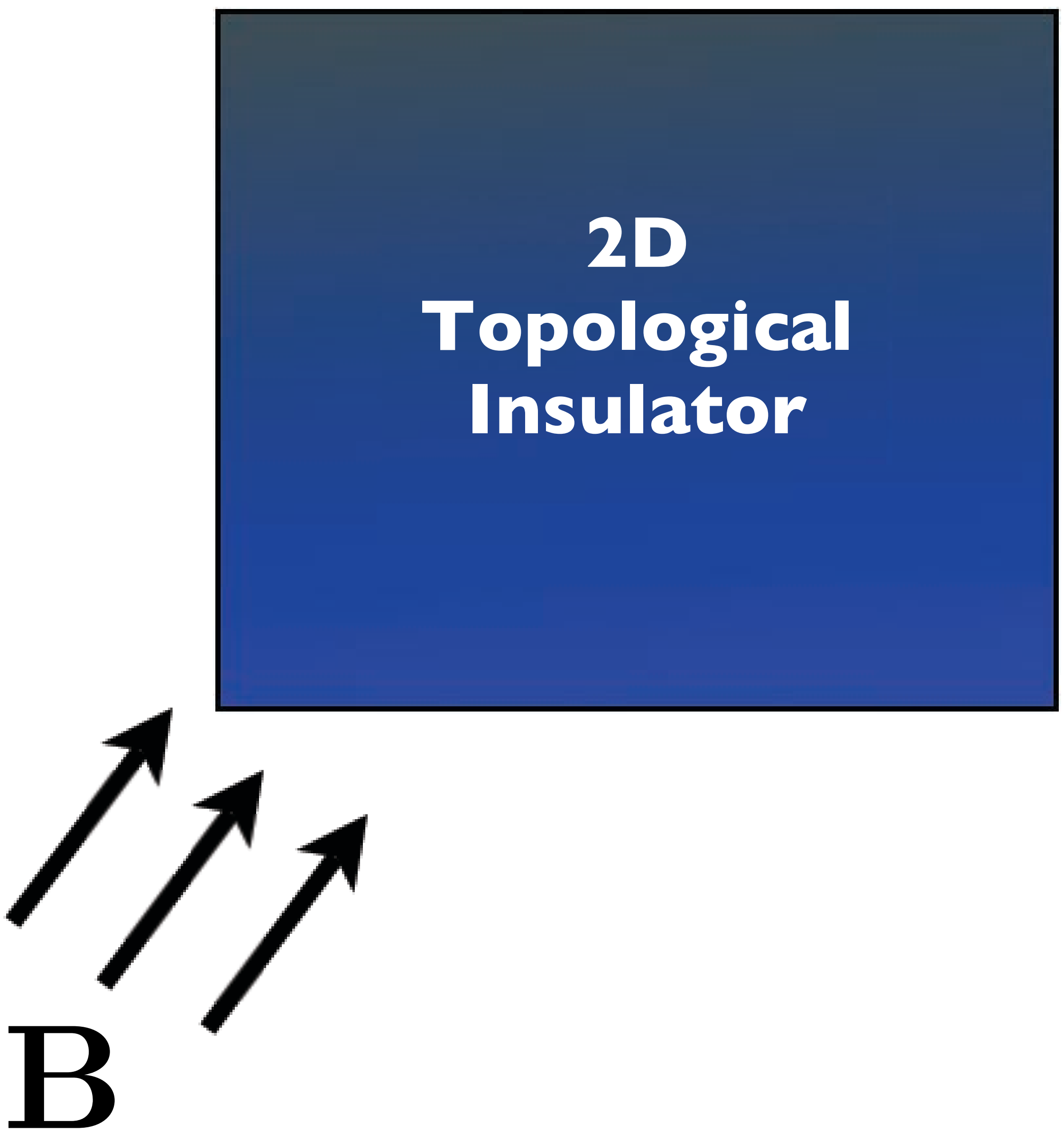
# Generating Majoranas at the edge I

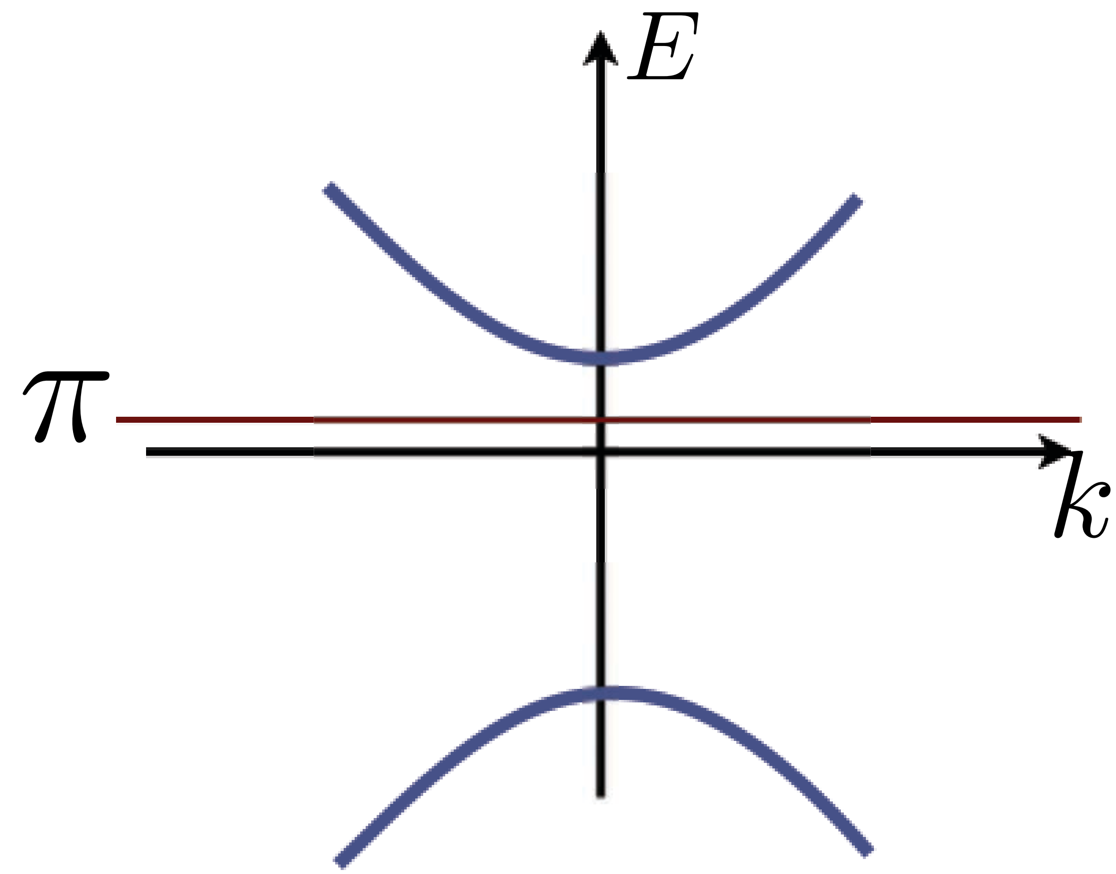



Majoranas arise, but are immobile. Look for a modified setup that allows more control.

Fu & Kane 2009

# Generating Majoranas at the edge II

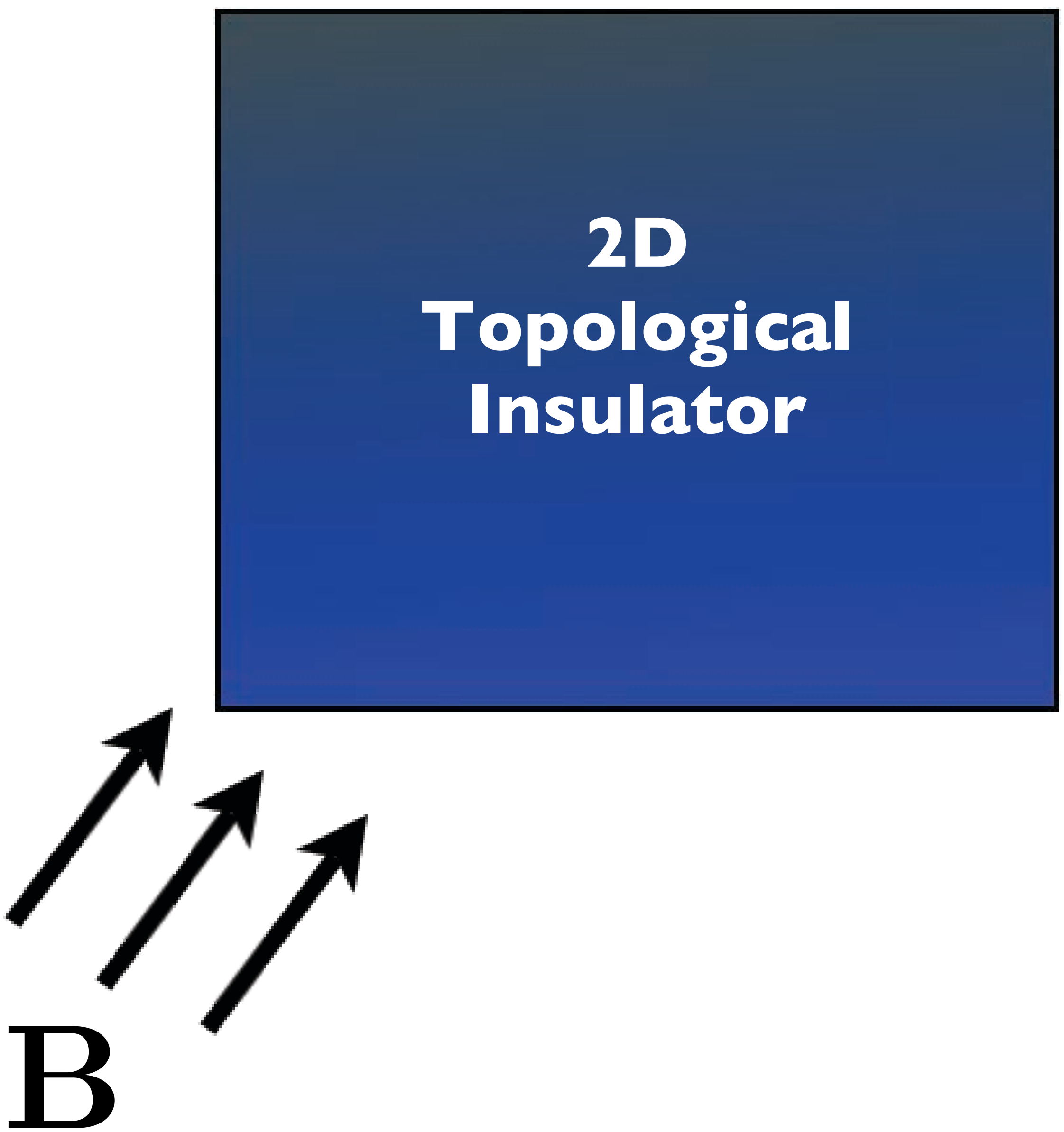


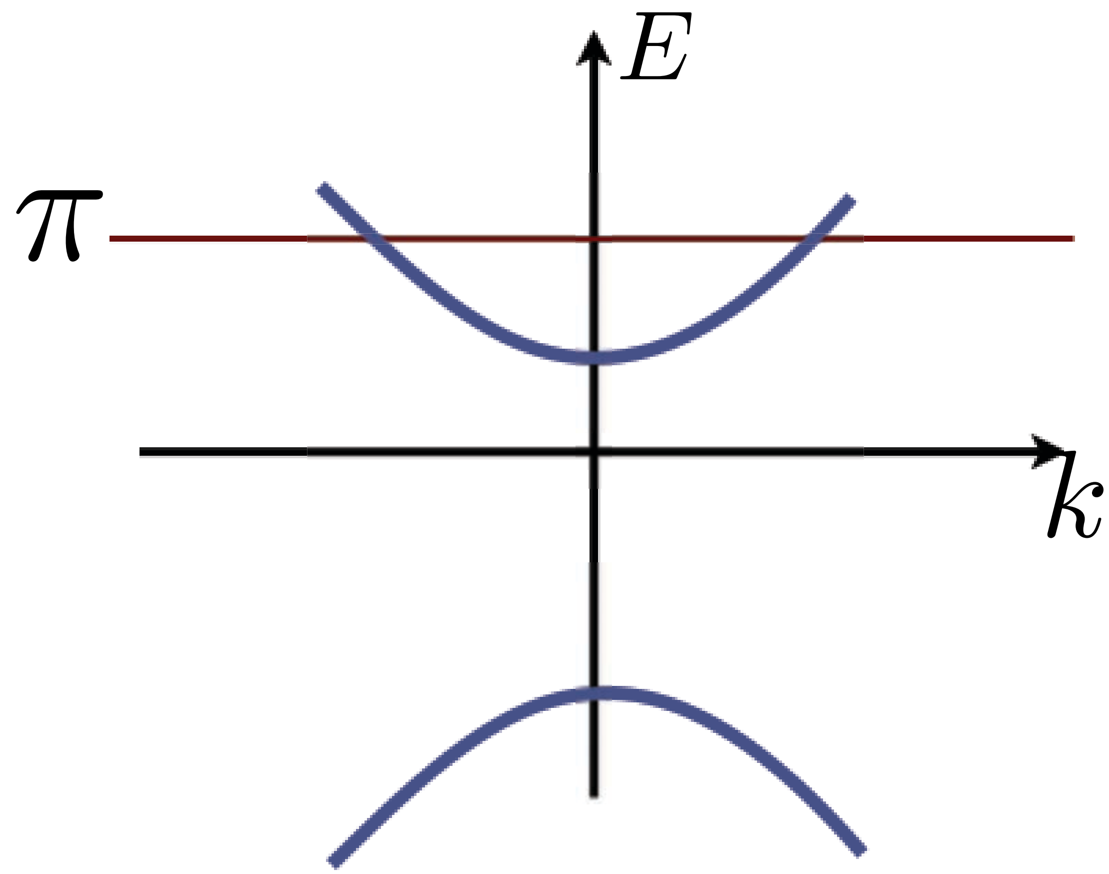




Gapped due to B-field

Alicea & Lindner

# Generating Majoranas at the edge II

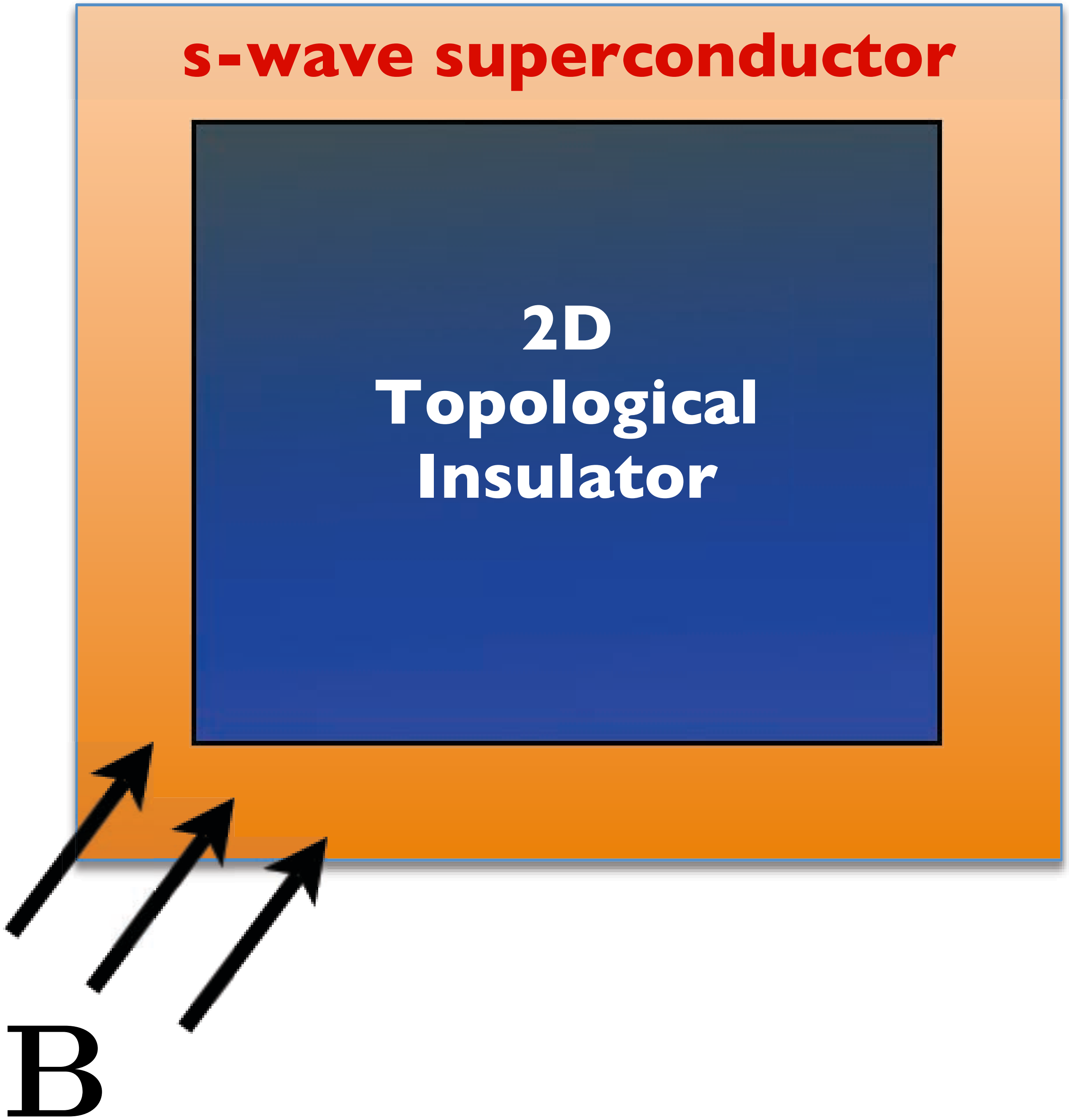


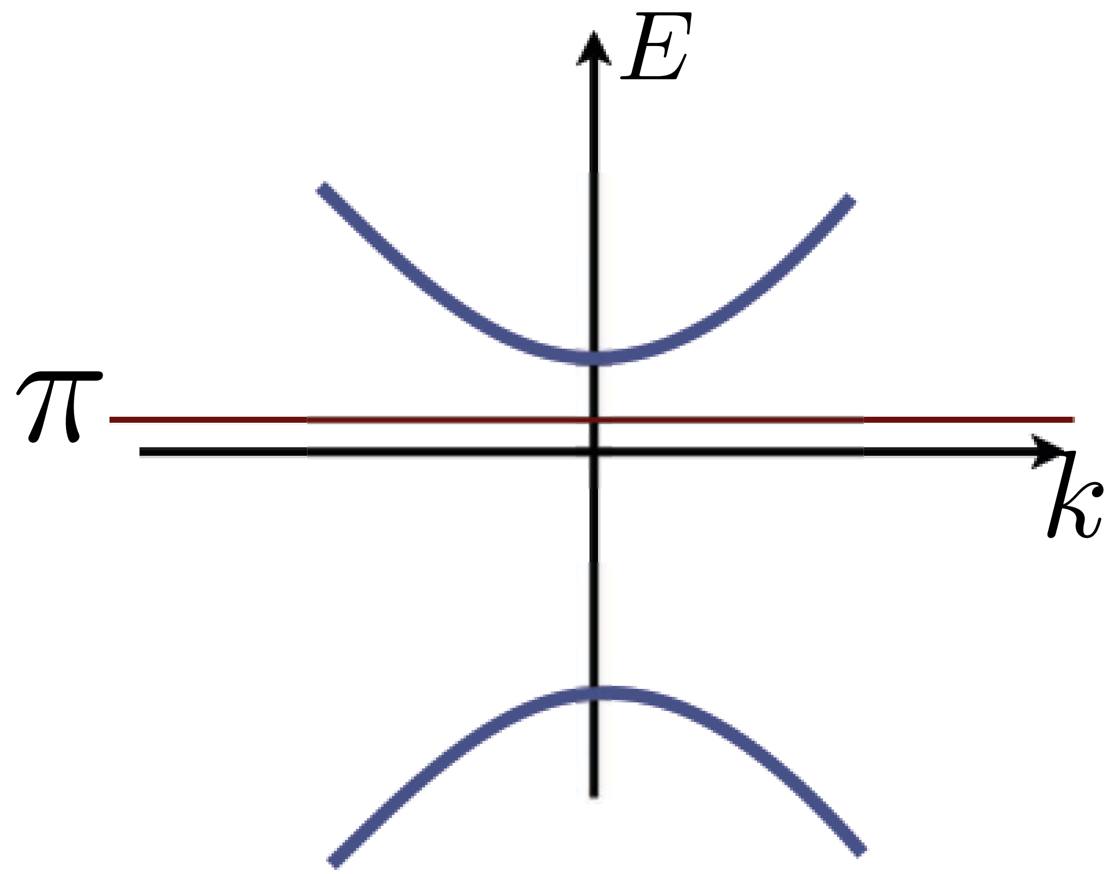




Gapless despite the B-field

Alicea & Lindner

# Generating Majoranas at the edge II




Gapped due to B-field

Alicea & Lindner

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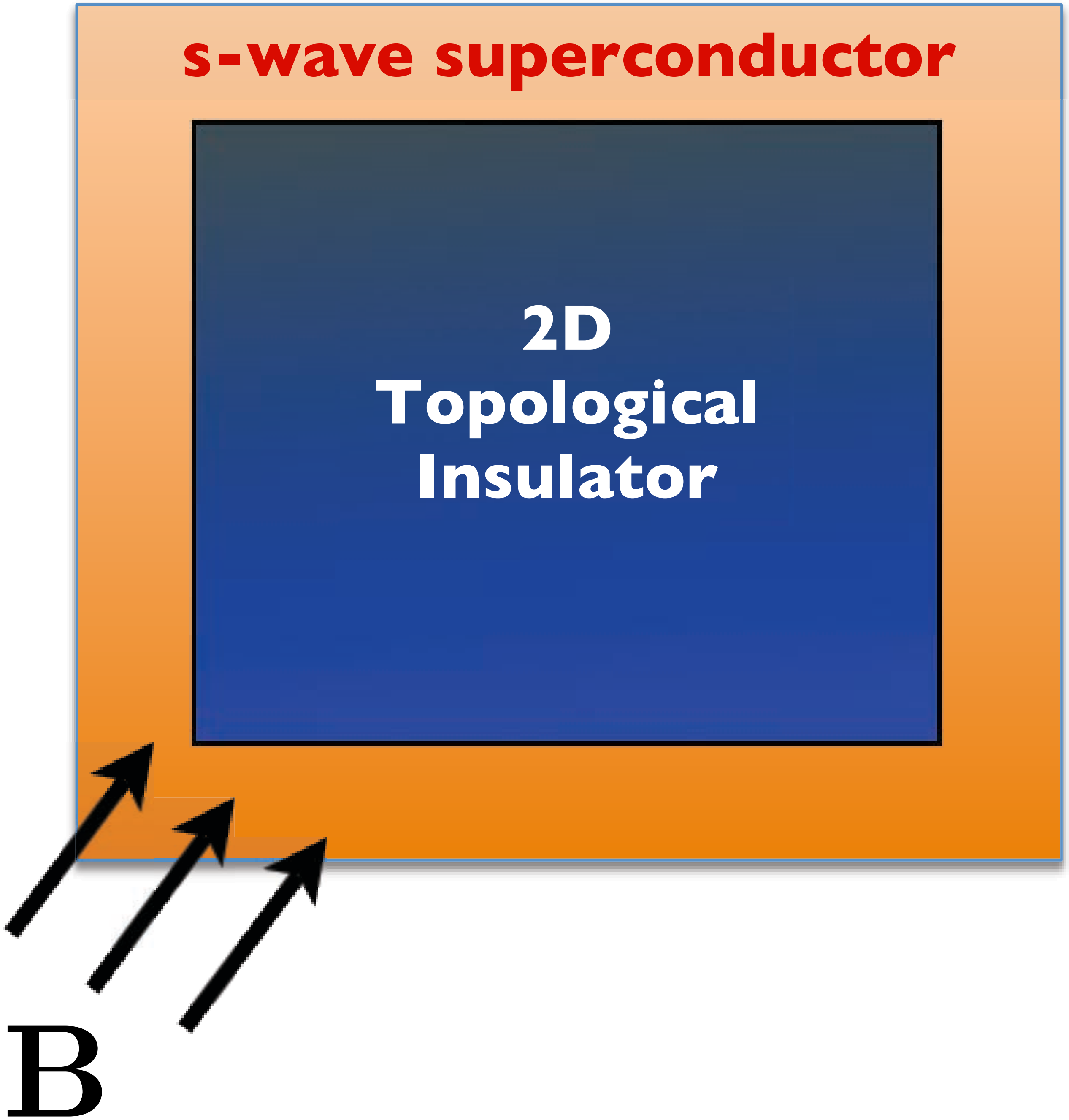
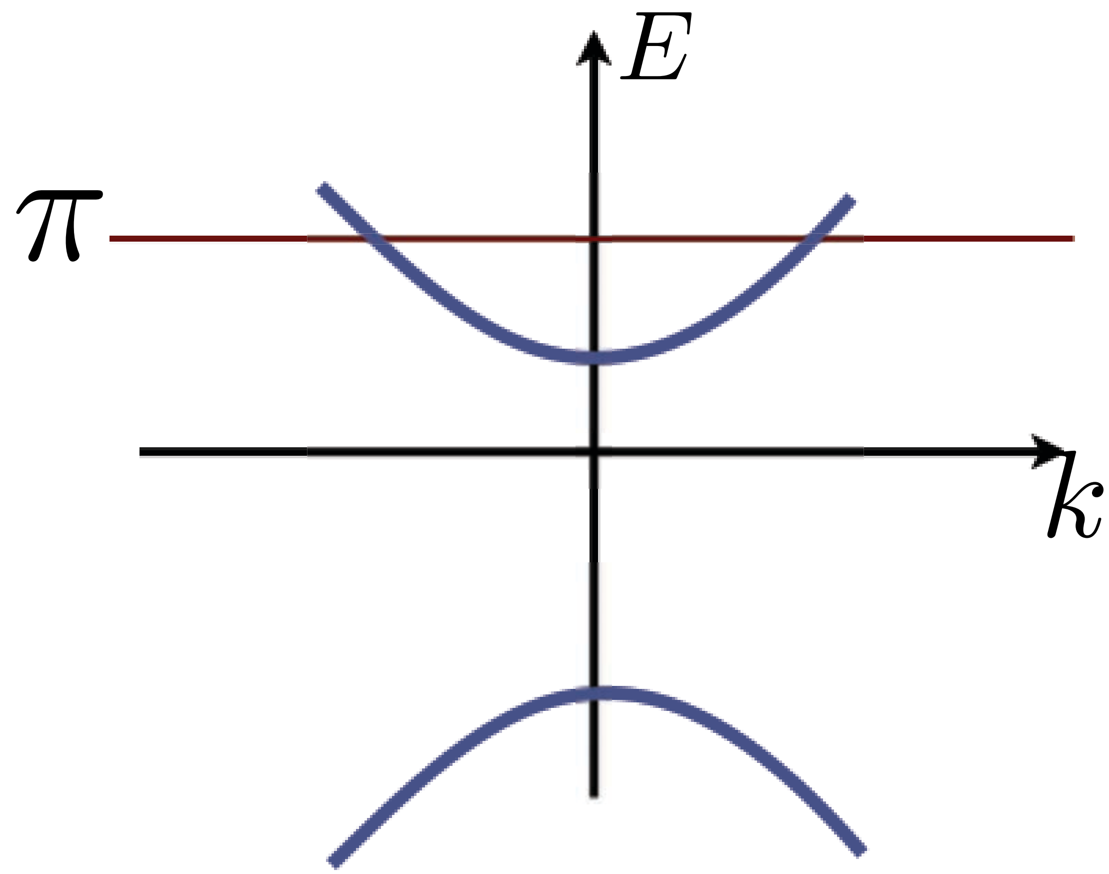
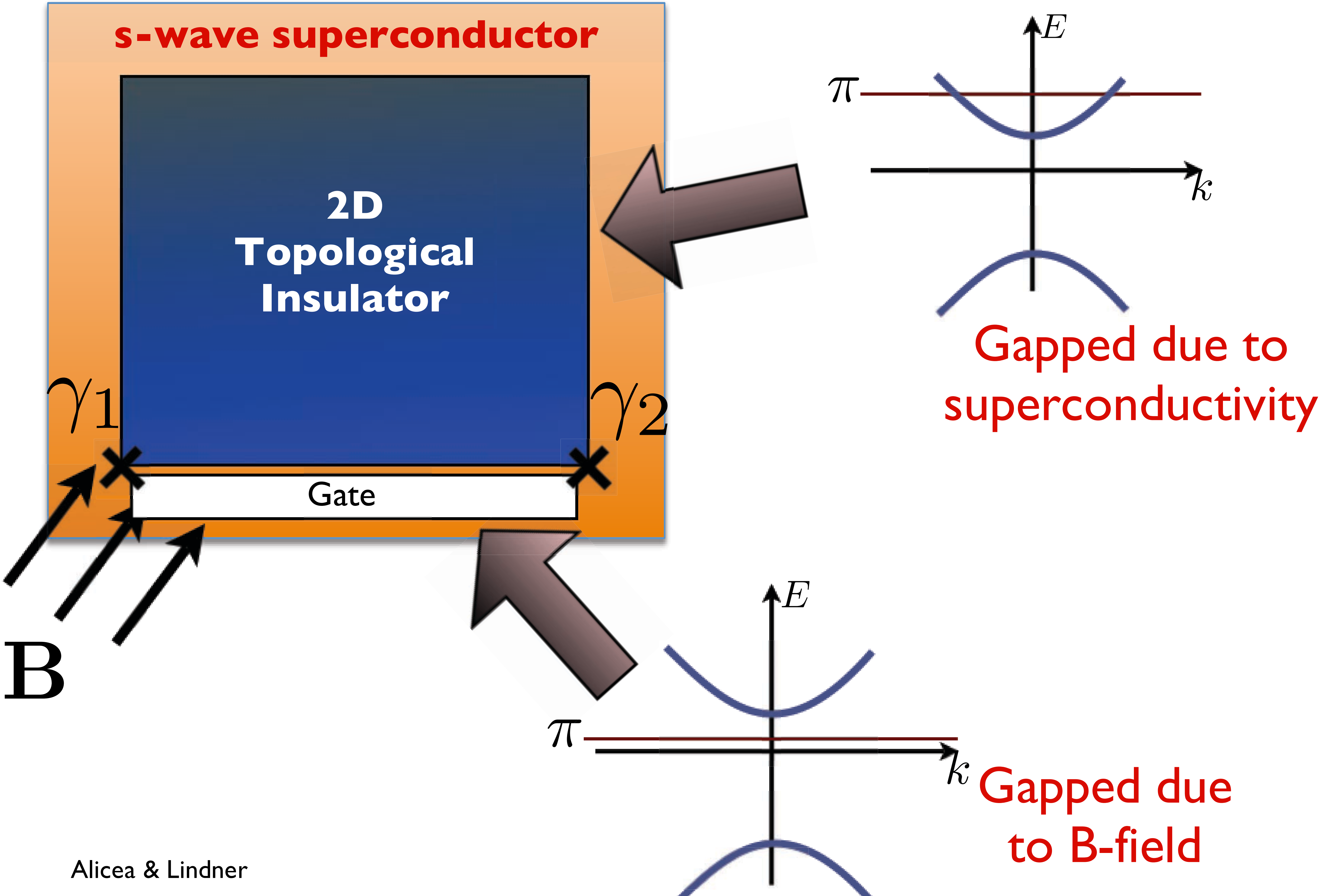



Gapped due to superconductivity

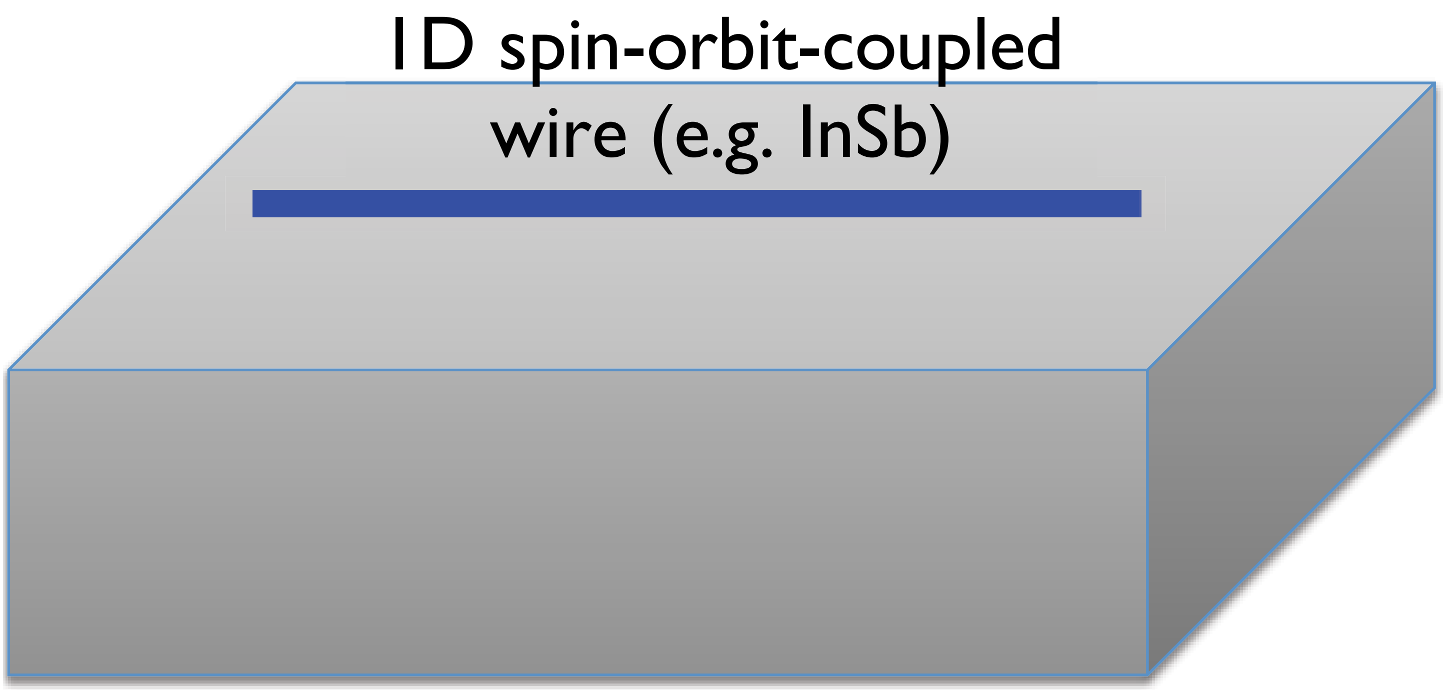
Alicea & Lindner

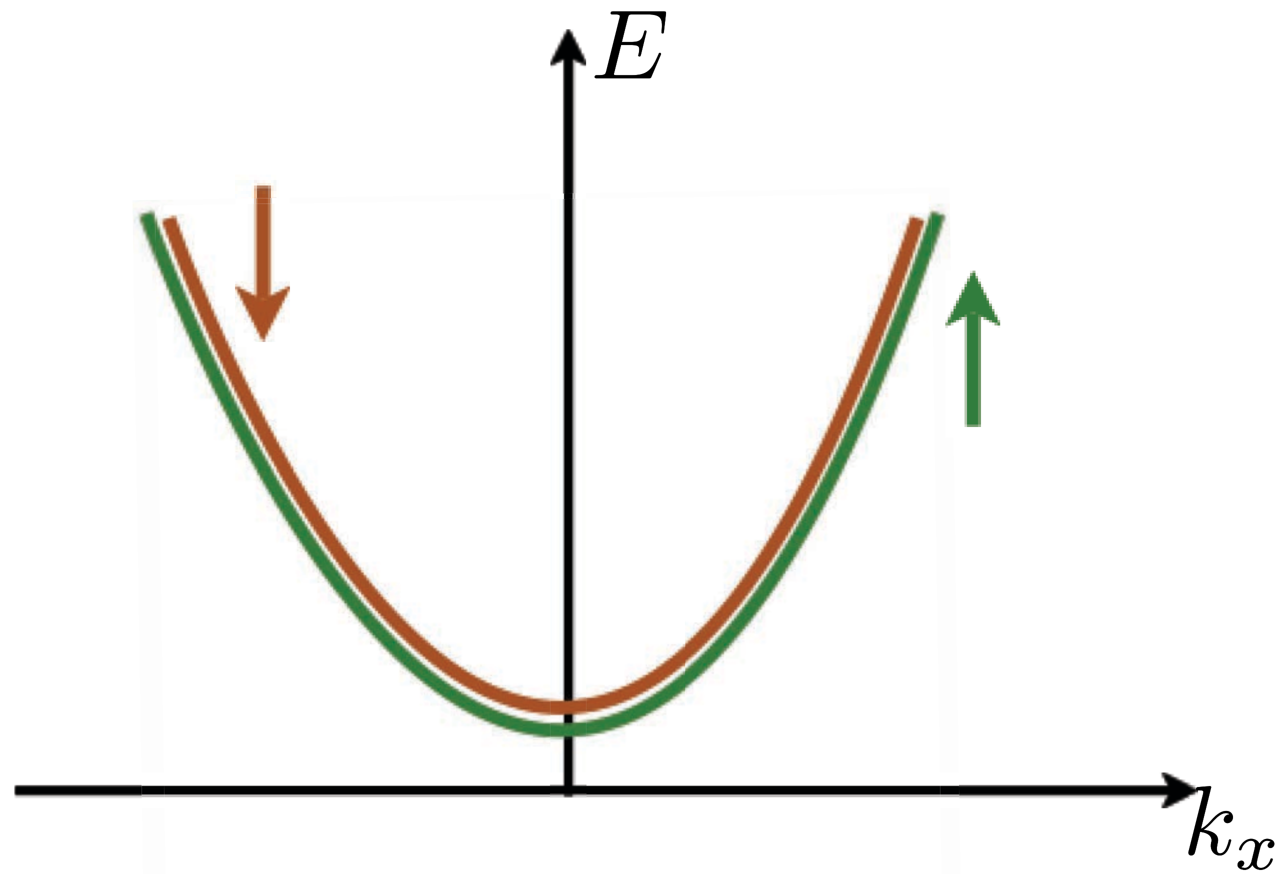
# Generating Majoranas at the edge II




Alicea & Lindner

# Majorana fermions in 1D wires

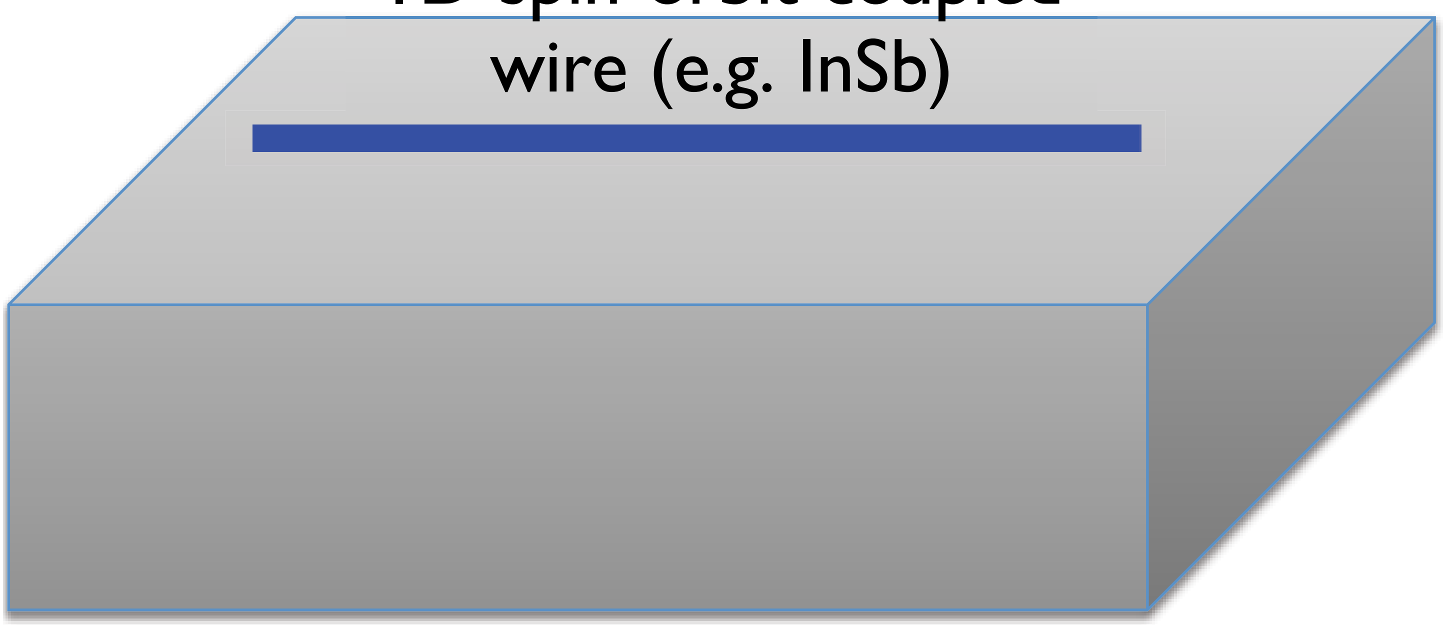


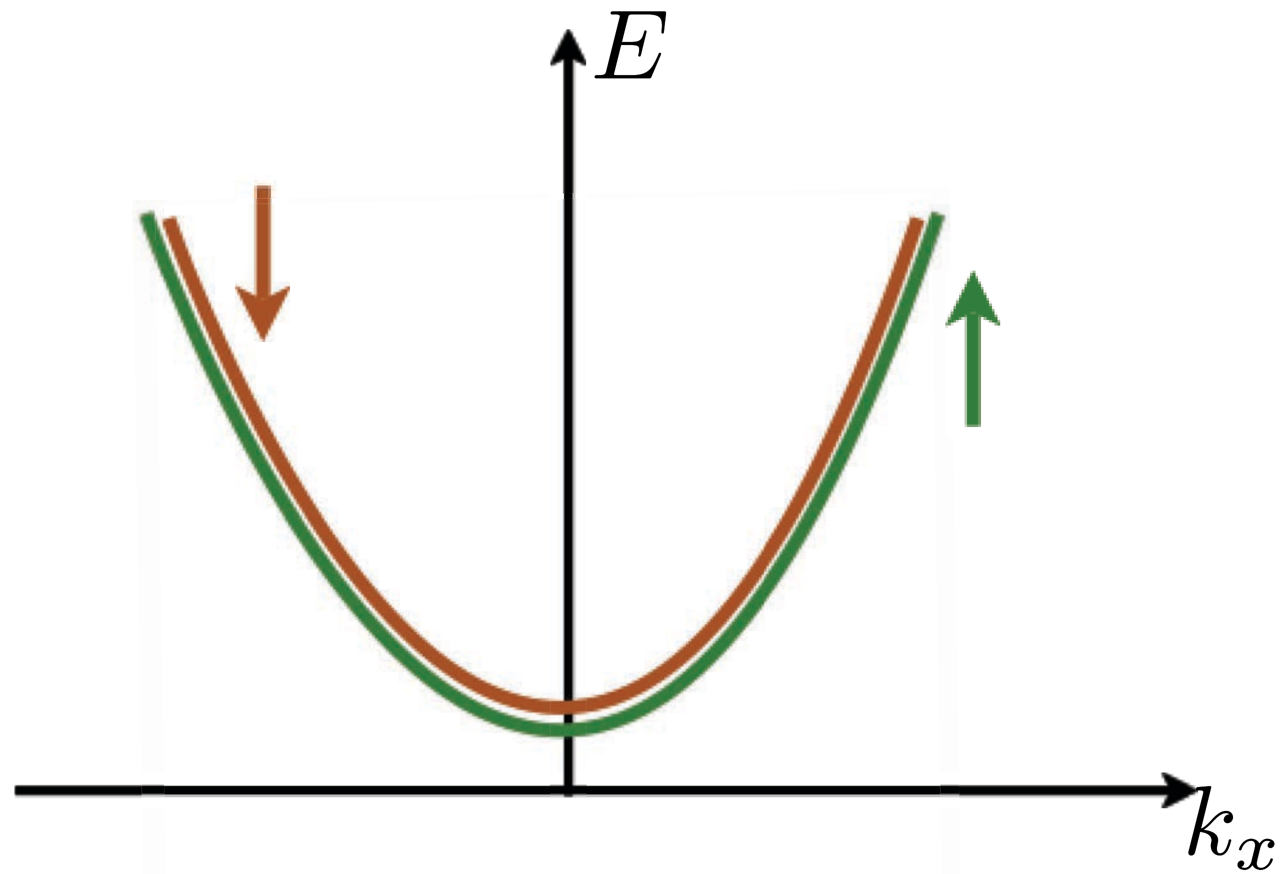




$$H = \int dx \psi \left[ -\frac{\partial_x^2}{2m} - \pi \right] \psi$$

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)

# Majorana fermions in 1D wires



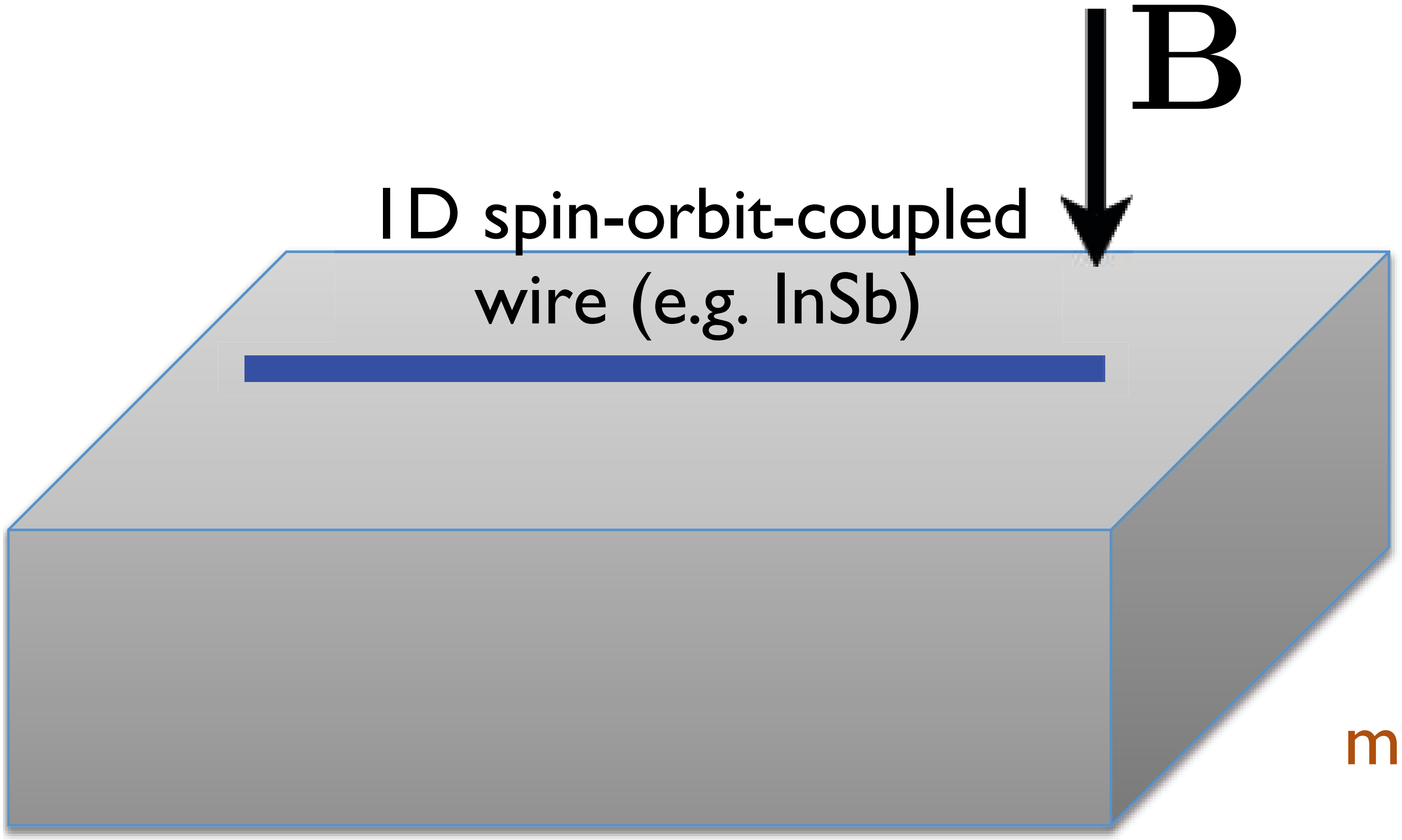



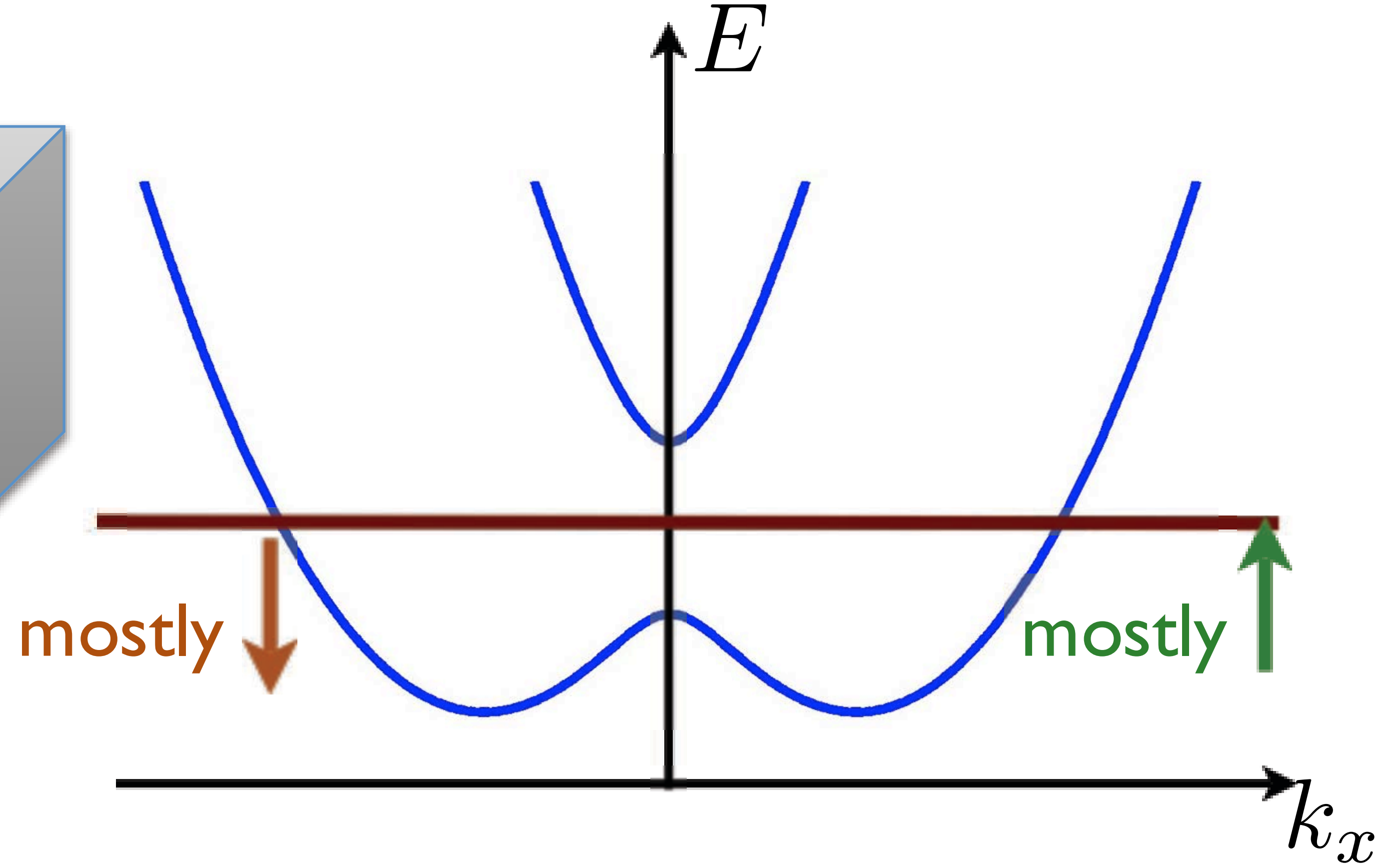

$$H = \int dx \psi \left[ -\frac{\partial_x^2}{2m} - \pi - i h v \partial_x \sigma^y \right] \psi$$

"Rashba spin-orbit coupling" $(\mathbf{E} \times \mathbf{P}) \cdot \sigma$

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)

# Majorana fermions in 1D wires

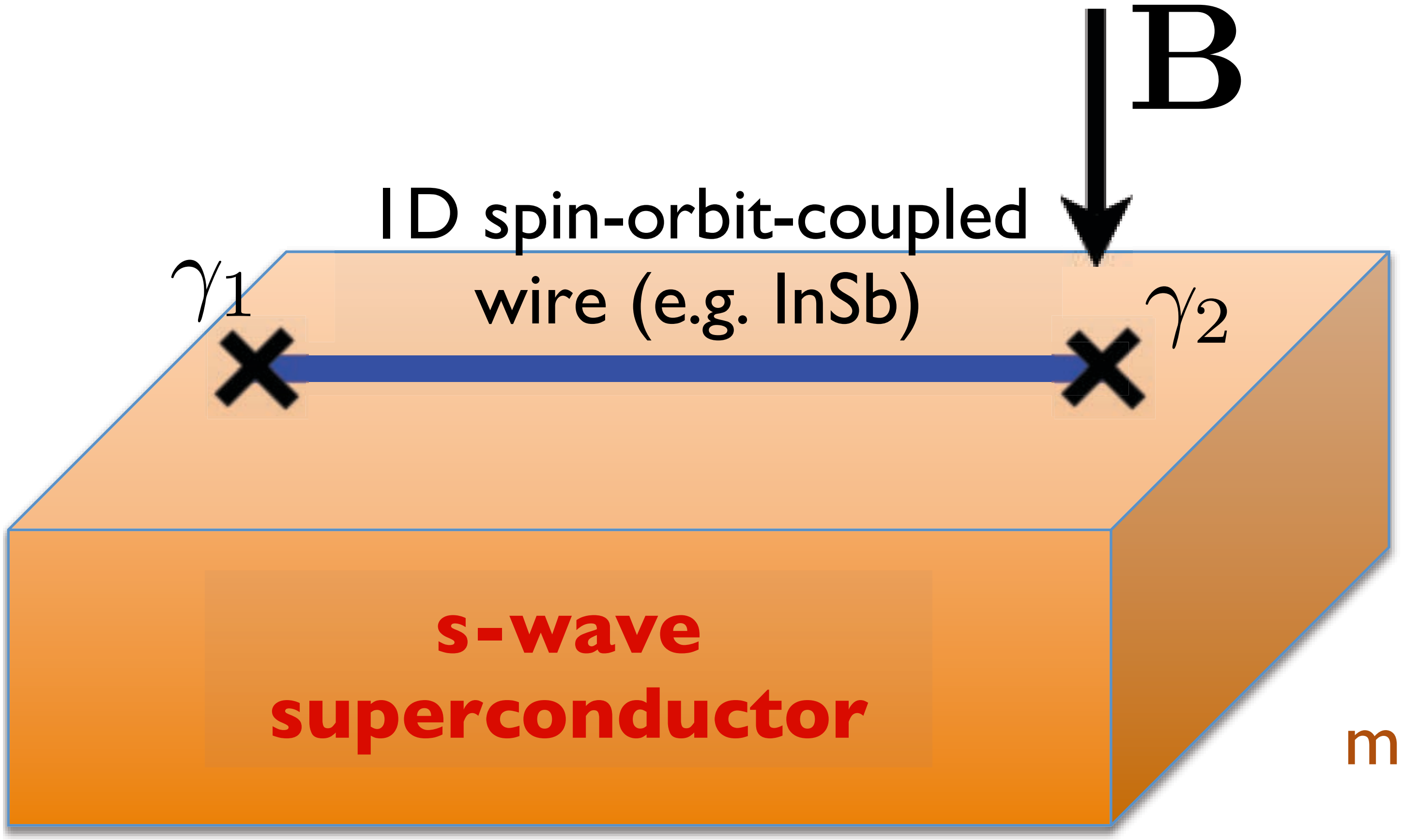


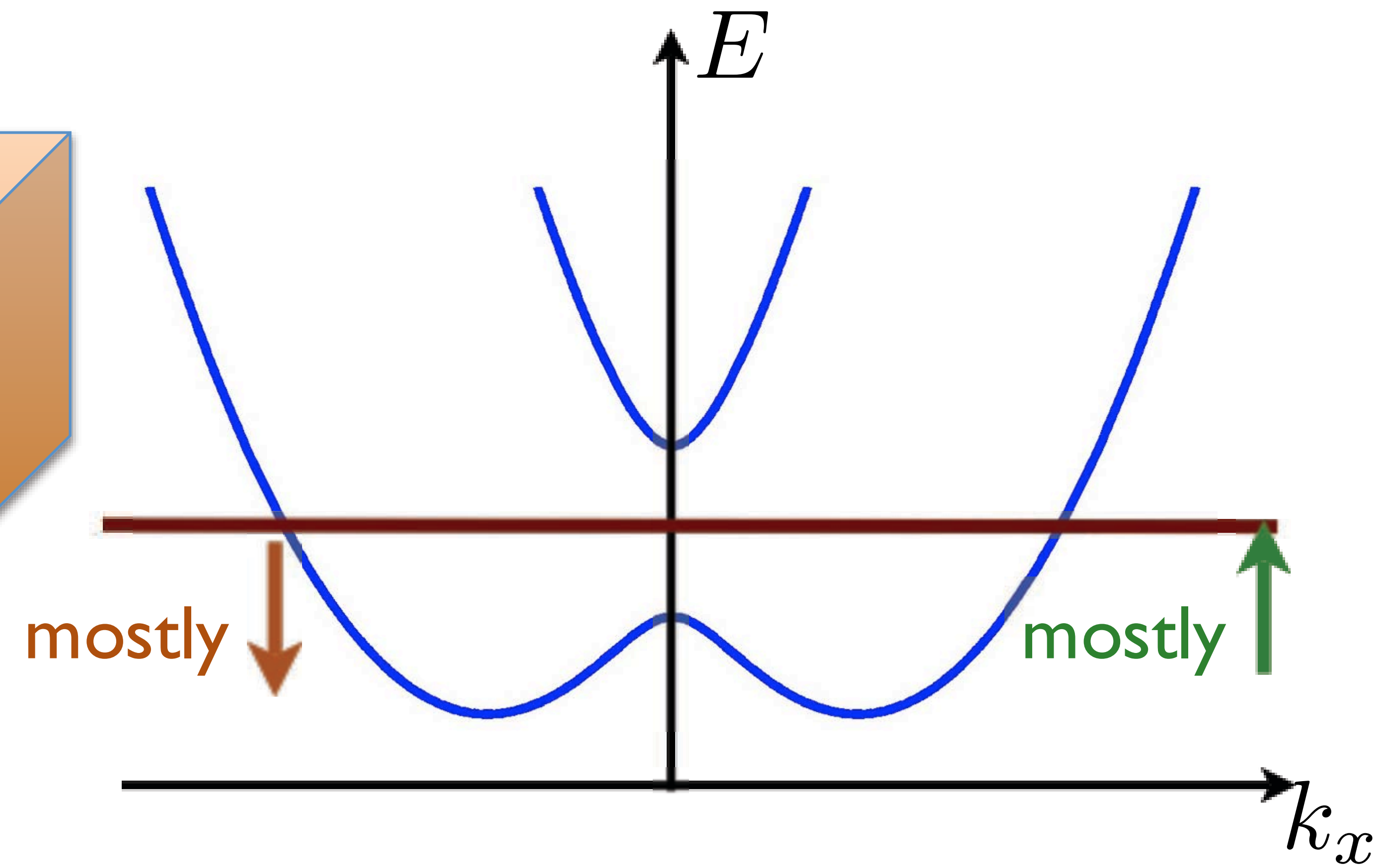




$$H = \int dx \psi \left[ -\frac{\partial_x^2}{2m} - \pi - i h v \partial_x \sigma^y - \frac{g\pi_B B}{2} \sigma^z \right] \psi$$

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)

# Majorana fermions in 1D wires




$$H = \int dx\psi \left[-\frac{\partial_x^2}{2m} - \pi - ihv\partial_x\sigma^y - \frac{g\pi_B B}{2}\sigma^z\right]\psi$$

$$+ (\Delta\psi_\uparrow\psi_\downarrow + h\ c\ )$$

Generates a **1D 'spinless' p-wave superconducting state with Majorana zero-modes!**

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)

# Homework Set 2

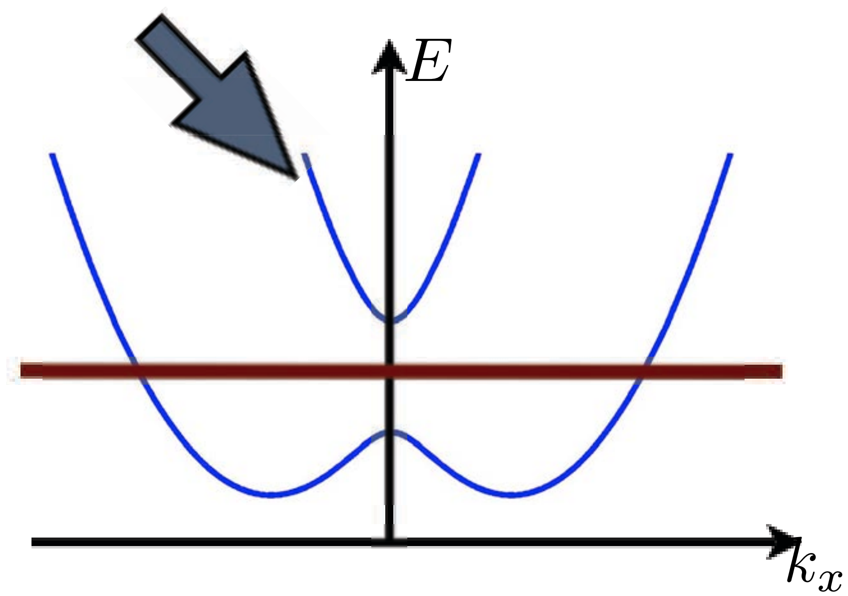
1. Show that integrating out the parent superconductor's degree of freedom indeed generates pairing terms for the edge modes/wire. Do other parameters in the Hamiltonian also get renormalized due to the hybridization?

2. Rewrite the wire Hamiltonian in terms of operators that add excitations to the upper/lower bands. Project out the upper band and compare the resulting effective Hamiltonian with the toy model for a spinless p+ip superconductor.

$$H = \int dx \psi \left[ -\frac{\partial_x^2}{2m} - \pi - ihv\partial_x \sigma^y - \frac{g\pi_B B}{2}\sigma^z \right] \psi$$

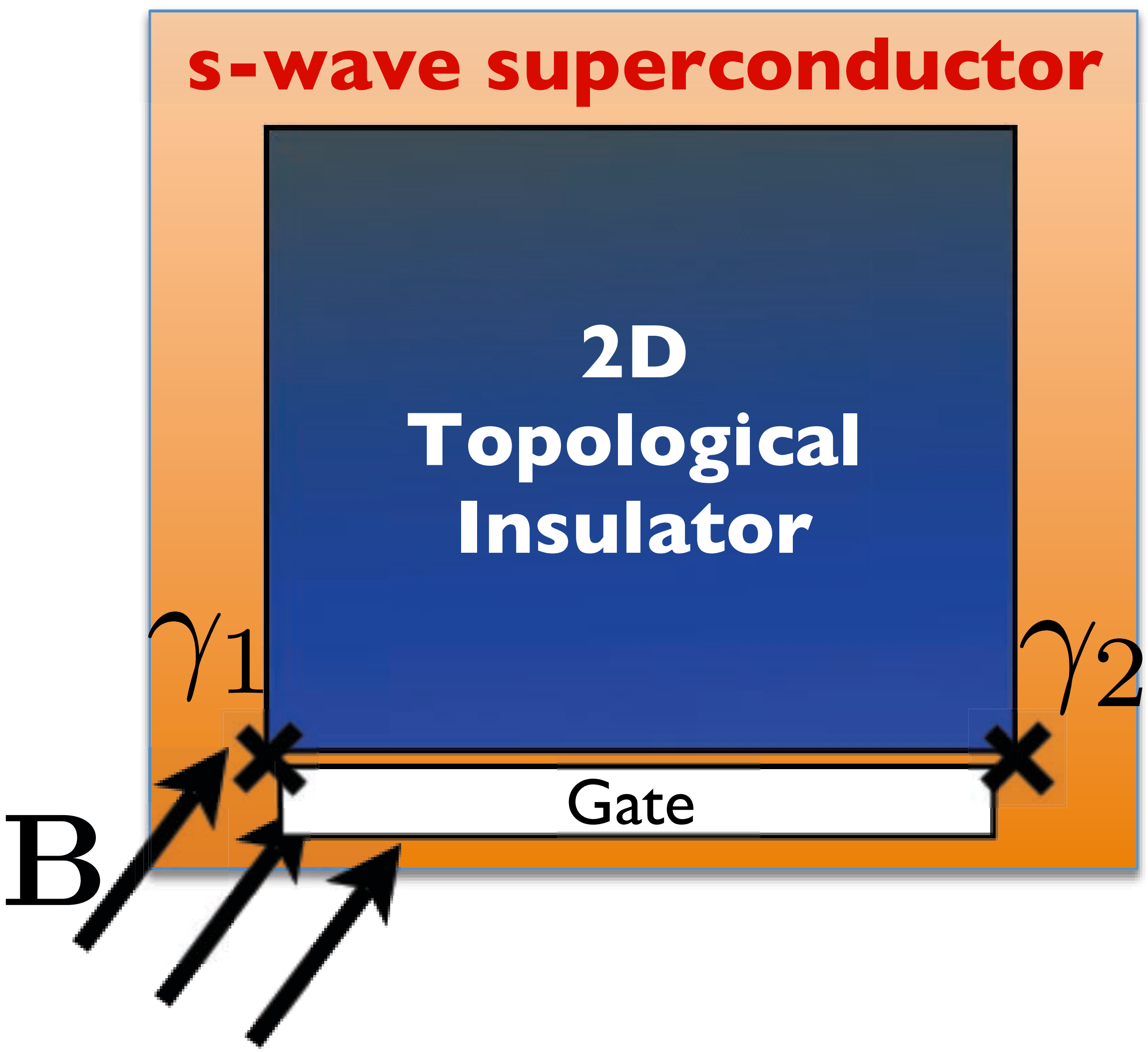
$$+ (\Delta \psi_\uparrow \psi_\downarrow + h\ c\ )$$

Project out this band




3. Find the parameter range (i.e., magnetic field, pairing energy, and chemical potential) over which the topological phase occurs in 1D wires.

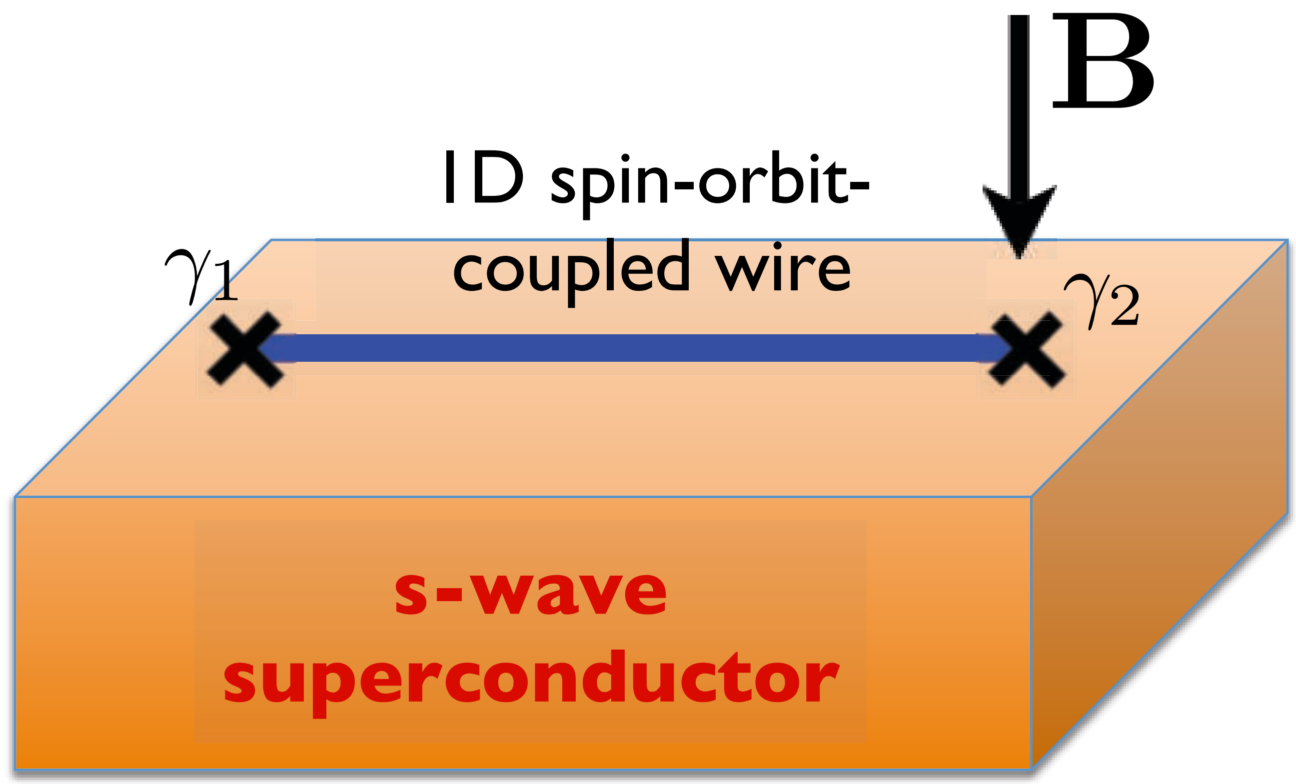
# 1D wire vs. 2D topological insulator setups




-Not much tuning required

-Built-in resilience against disorder (Anderson's theorem)

-Few materials (but situation is improving)




-Semiconductor technology well advanced

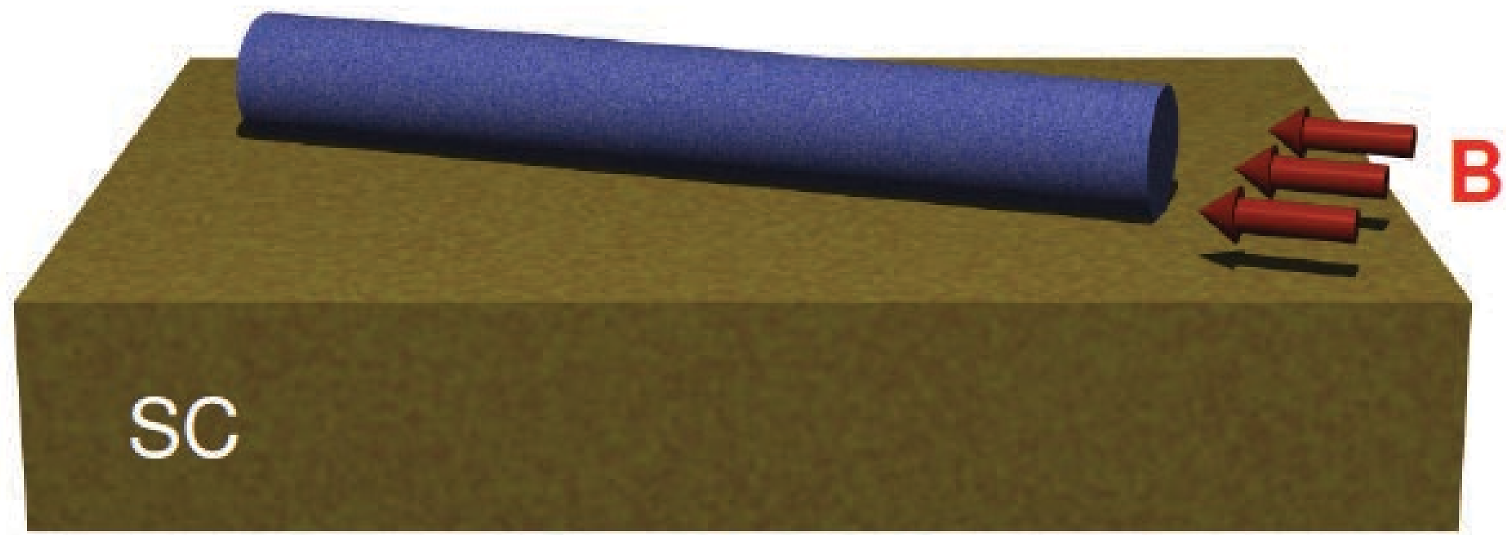
-Required ingredients demonstrated long ago

-Need to fine-tune chemical potential within (small) Zeeman gap

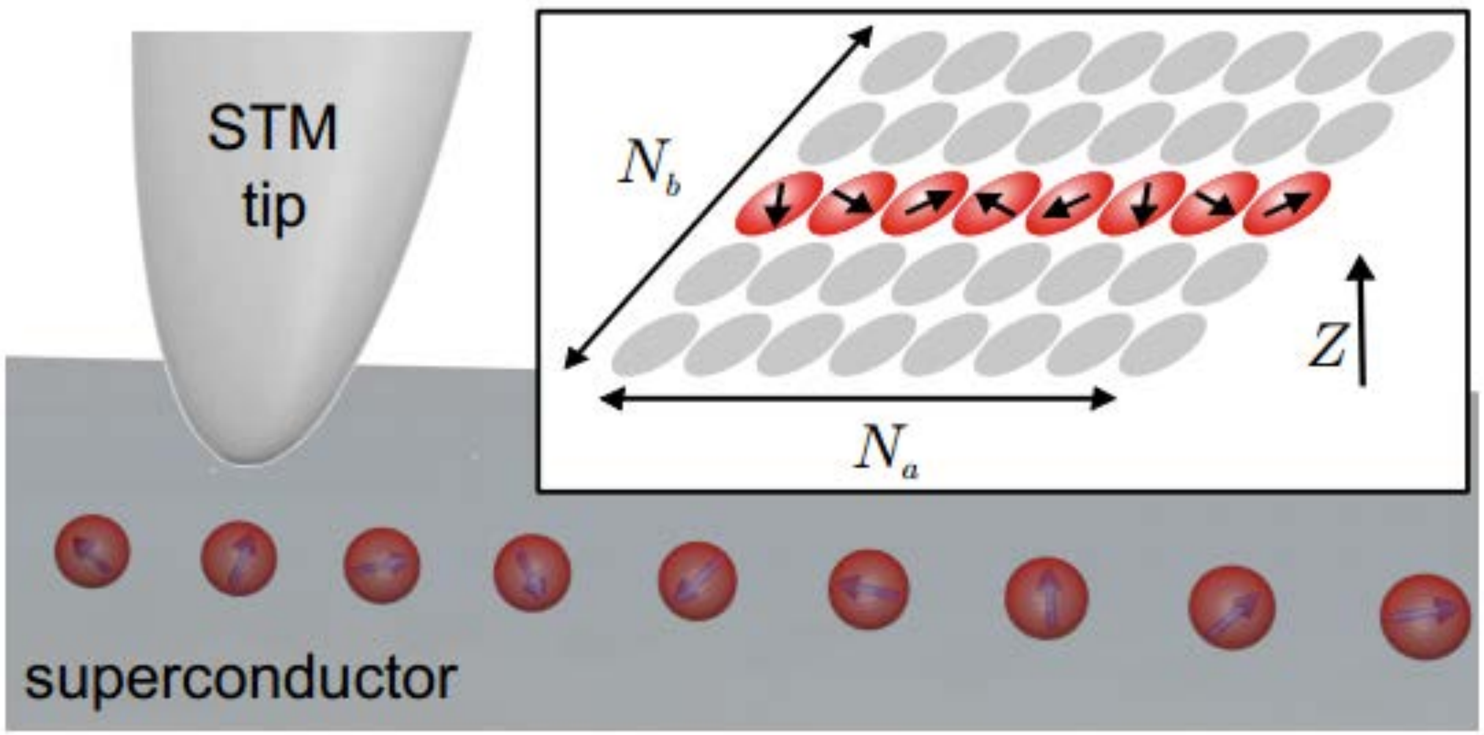
-Disorder poses more serious issue

# Other promising realizations

3D topological insulator nanowires




Cook and Franz, Phys. Rev. B **84**, 201105(R) (2011)

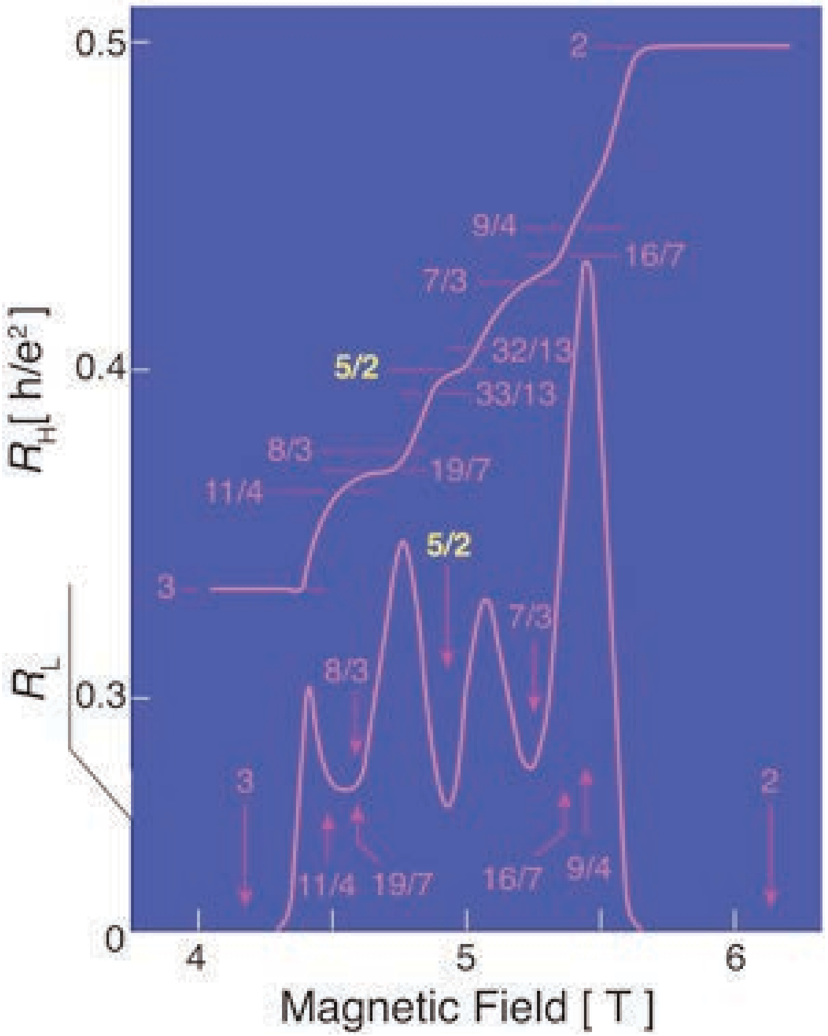



Magnetic-atom chains on a superconductor

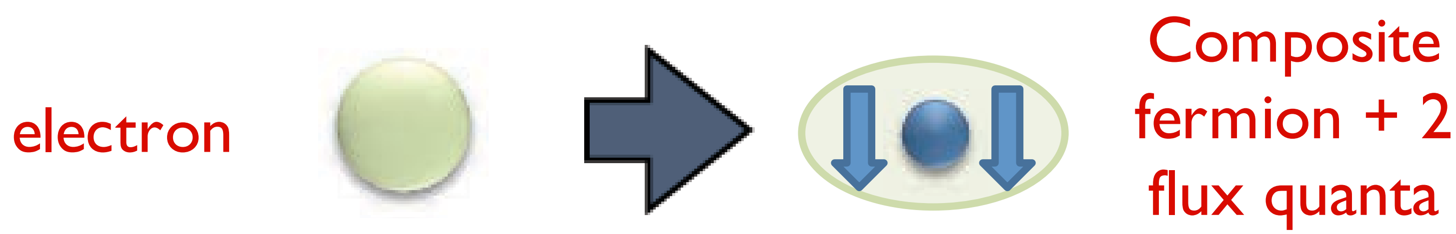
Beenakker et al., Phys. Rev. B **84**, 195442 (2011); Yazdani et al., Phys. Rev. B **88**, 020407(R) (2013)

# Experimental Routes to 2D topological superconductivity

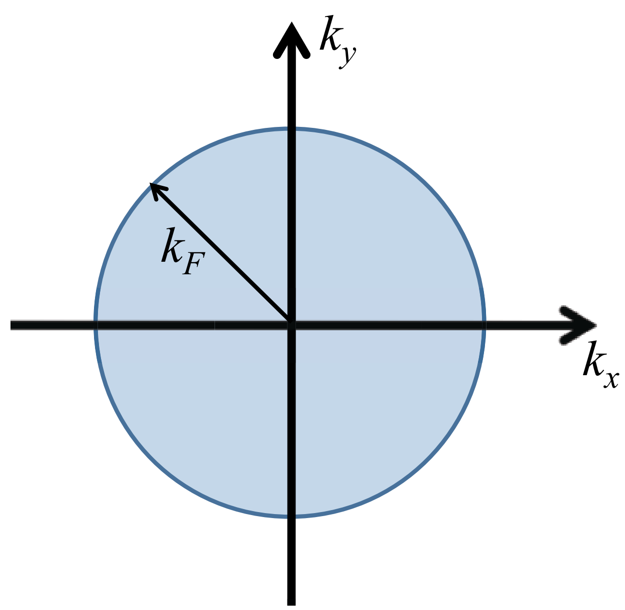
# An "intrinsic" realization




Willet, Eisenstein, et al. (1987)
Moore & Read (1991)
Bonderson, Kitaev, Shtengel (2006)
Stern & Halperin (2006)
W. Kang et al. (2011)




## ν = 1/2




Composite fermions form a **"spinless"** metal

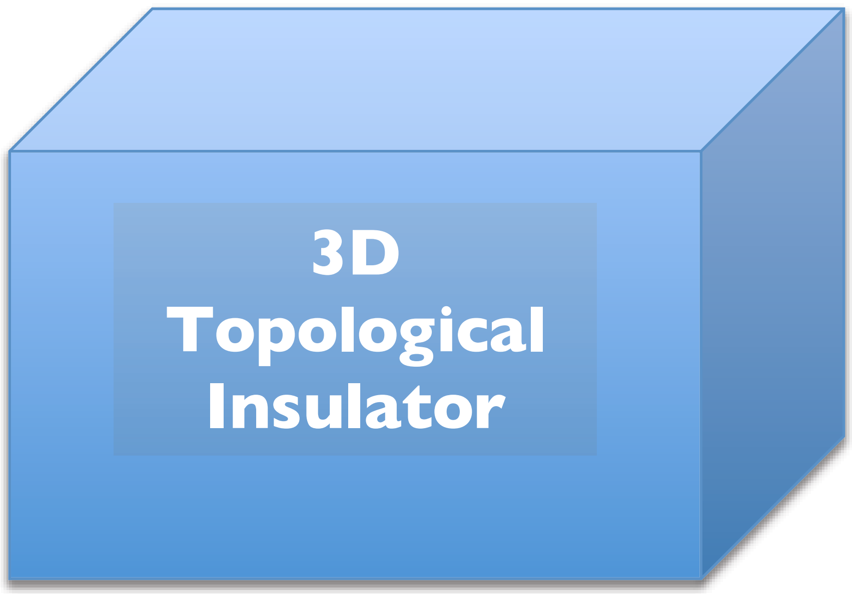
## ν = 5/2

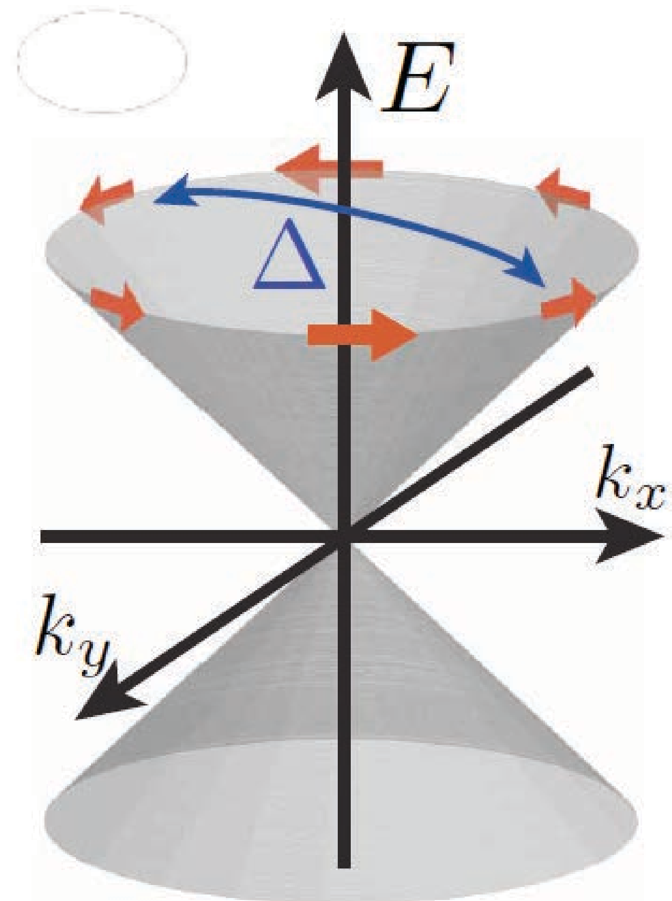
Composite Fermi sea is unstable towards ***p+ip* pairing!**

$$\Psi_{\rm Pf} = {\rm Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^2$$

# Majorana fermions in a 3D topological insulator

3D topological insulators: inert bulk but **odd # of Dirac cones on the surface**



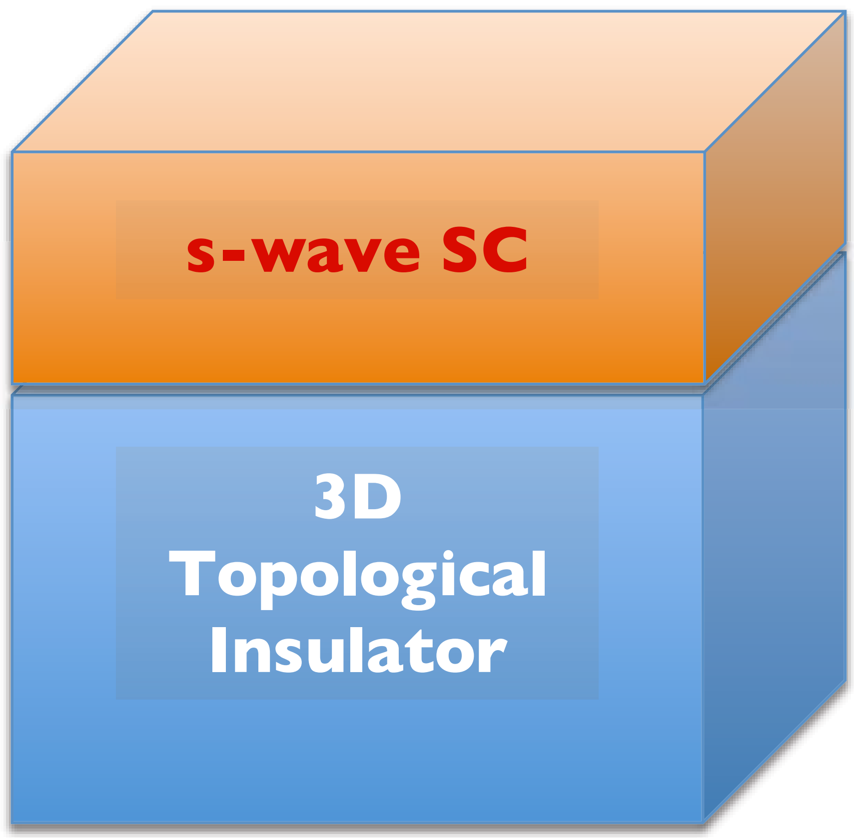



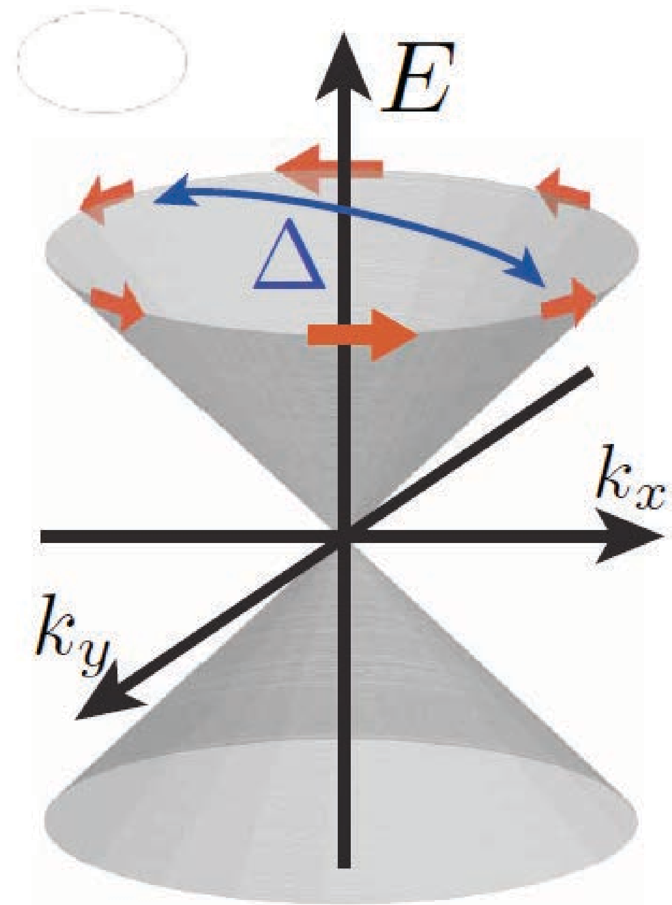

$$H = \int d^2\mathbf{r}\psi\ (-iv\vec{\sigma}\ \ \nabla - \pi)\psi$$

Surface looks **"spinless"!** (i.e., only one Fermi surface rather than two)

(Fu, Kane, & Mele 2006; Moore & Balents 2006; Roy 2006; Fu & Kane 2008)

# Majorana fermions in a 3D topological insulator



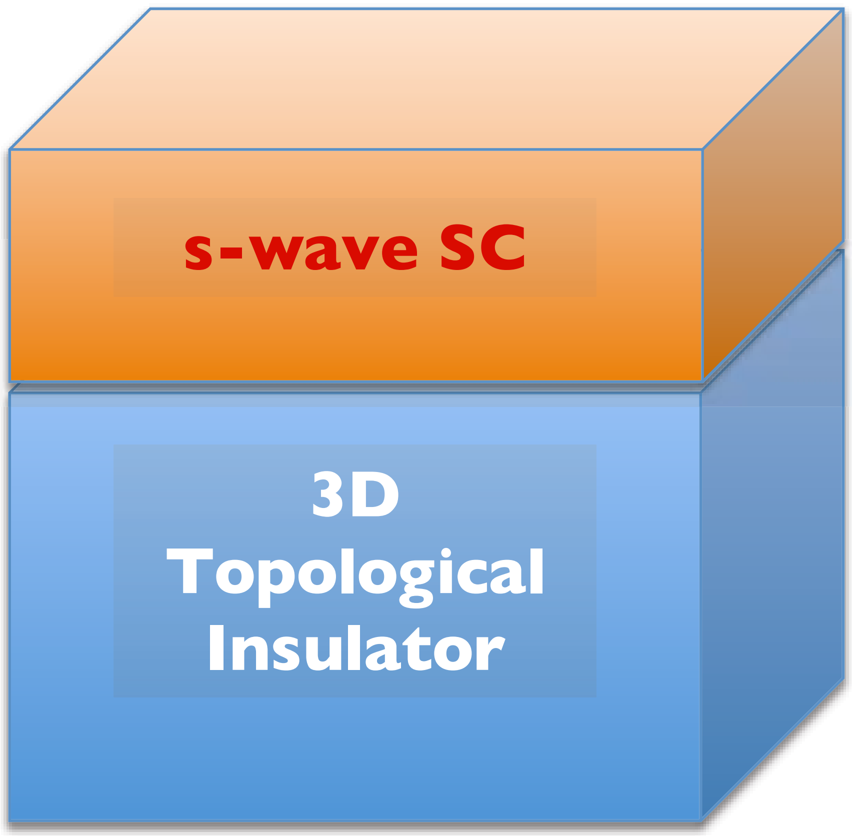



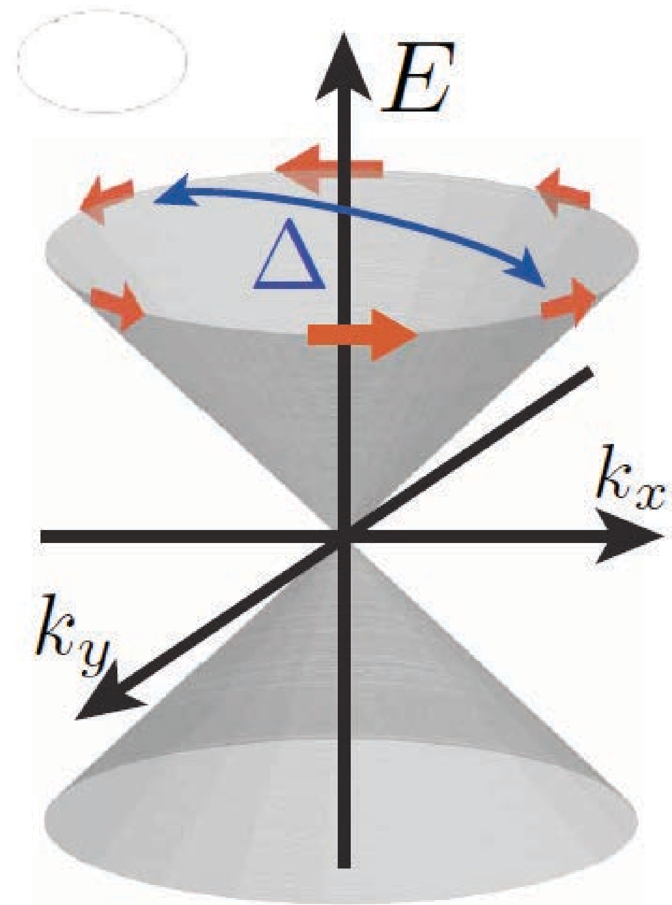

$$H = \int d^2\mathbf{r}[\psi\ \ (-iv\vec{\sigma}\ \ \ \nabla - \pi)\psi + (\Delta\psi_\uparrow\psi_\downarrow + h\ \ c\ \ )]$$

Surface inherits spin-singlet pairing...

(Fu, Kane, & Mele 2006; Moore & Balents 2006; Roy 2006; Fu & Kane 2008)

# Majorana fermions in a 3D topological insulator




$$H = \int d^2\mathbf{r}[\psi \ \ (-iv\vec{\sigma} \ \ \nabla - \pi)\psi + (\Delta\psi_\uparrow\psi_\downarrow + h \ c \ )]$$

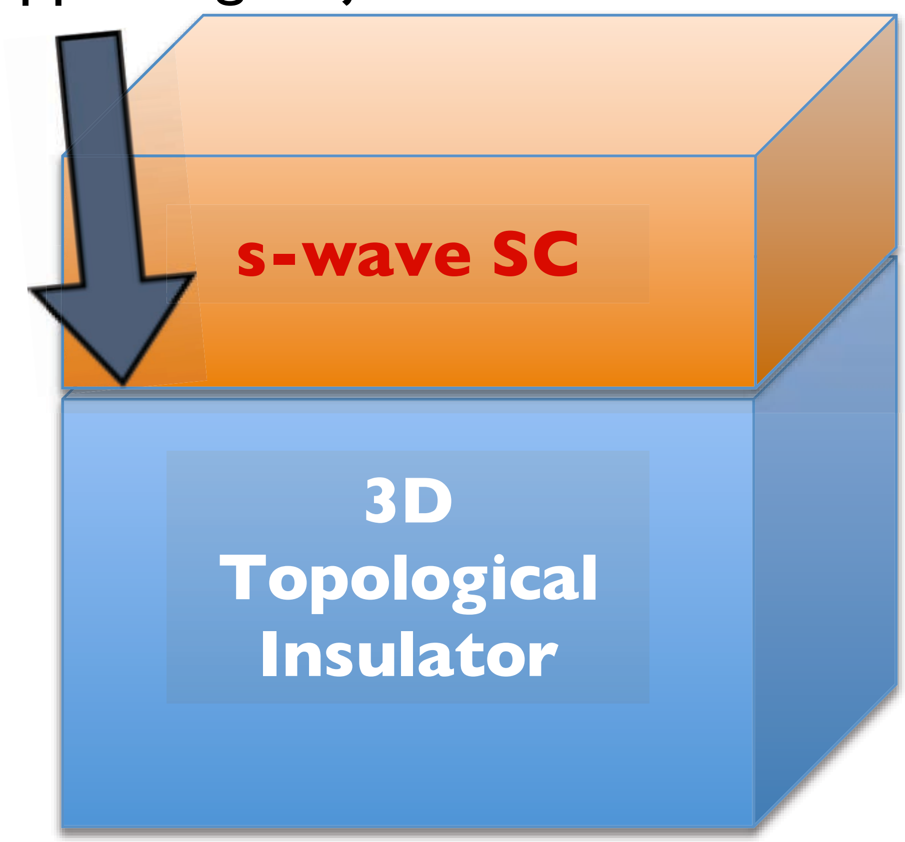
...but spin is not conserved, so this is NOT a simple s-wave superconductor

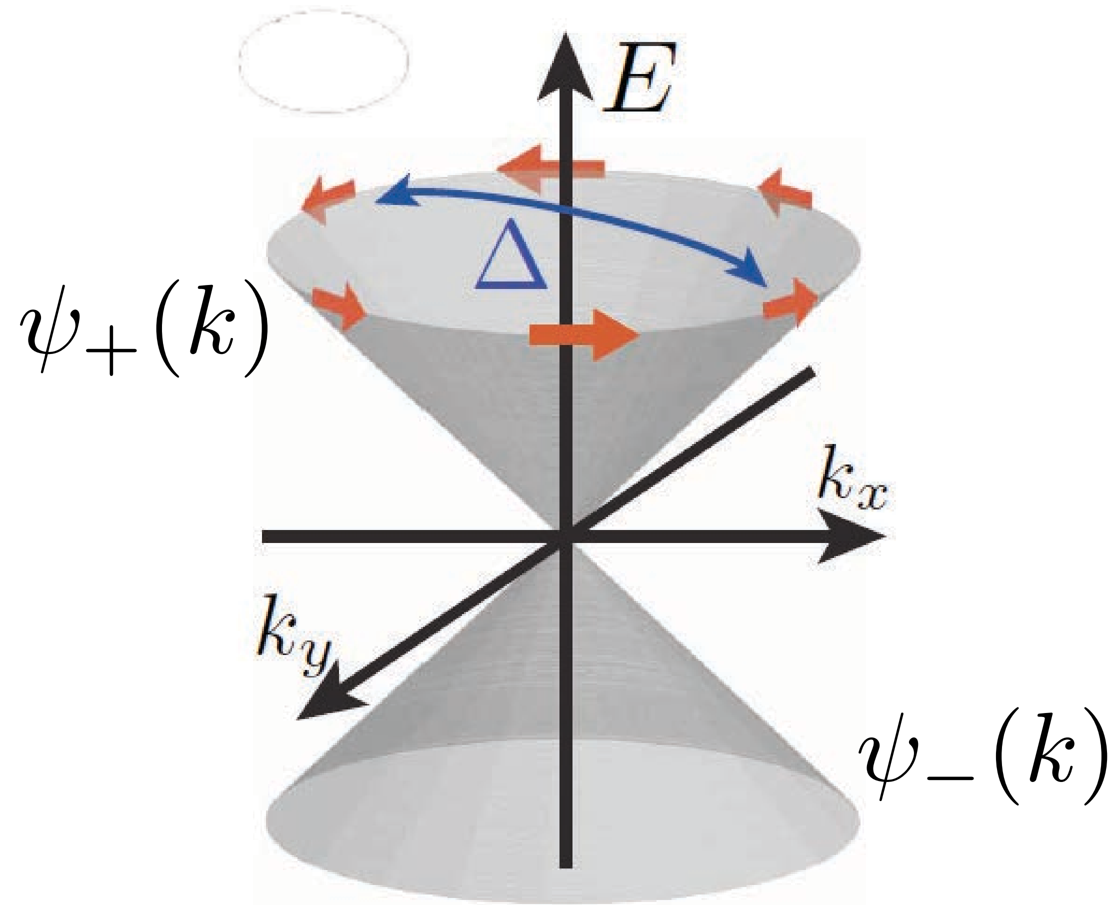
Surface inherits spin-singlet pairing...

(Fu, Kane, & Mele 2006; Moore & Balents 2006; Roy 2006; Fu & Kane 2008)

# Majorana fermions in a 3D topological insulator

Interface realizes 2D topological SC
supporting Majorana zero-modes at vortices!



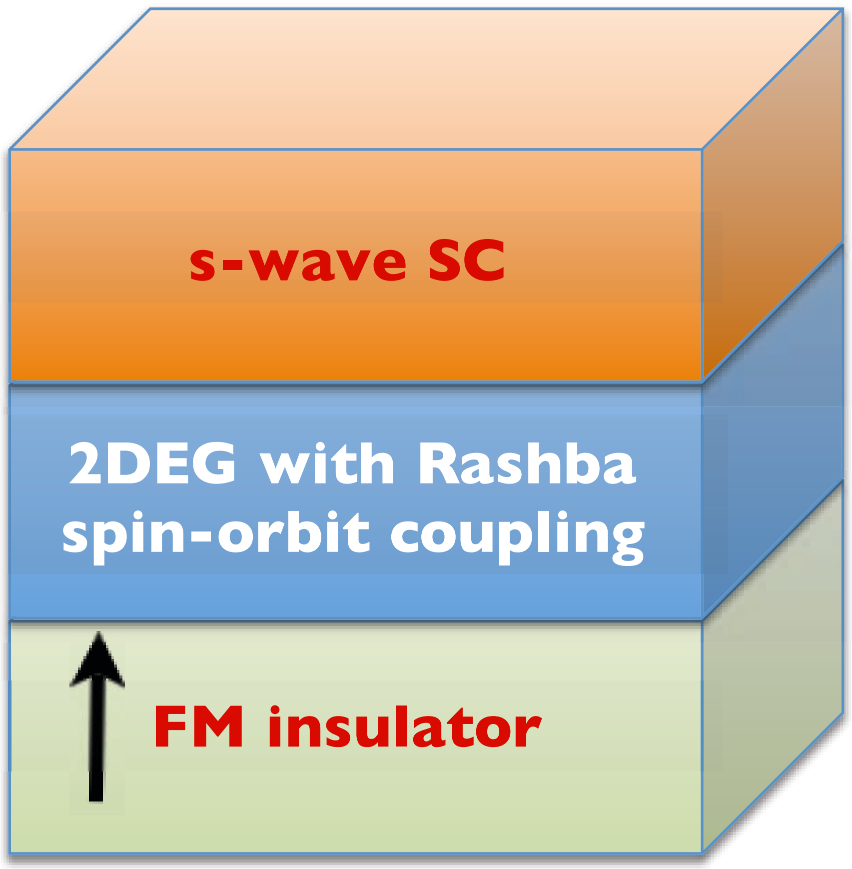

$$H = \int d^2\mathbf{k} \Big\{ [\epsilon_+(k)\psi_+\psi_+ + \epsilon_-(k)\psi_-\psi_-]$$

$$+\Delta\left[\left(\frac{k_x + ik_y}{2k}\right)[\psi_-(k)\psi_-(-k) - \psi_+(k)\psi_+(-k)] + h\,c\right]\Big\}$$

Pairing is *p+ip* in this basis!

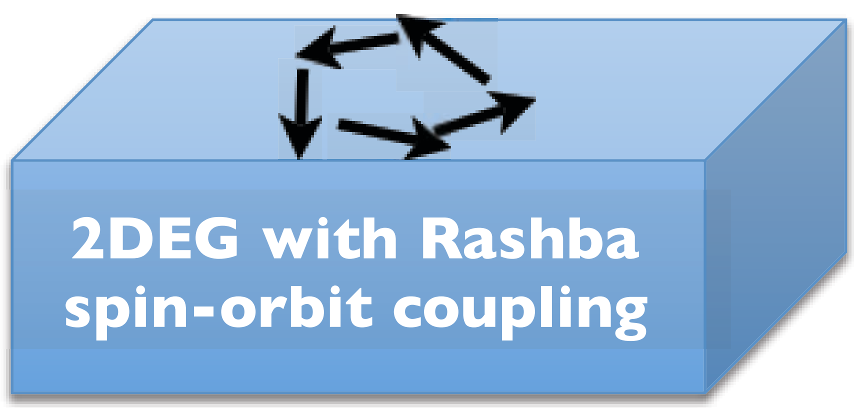
(Fu, Kane, & Mele 2006; Moore & Balents 2006; Roy 2006; Fu & Kane 2008)

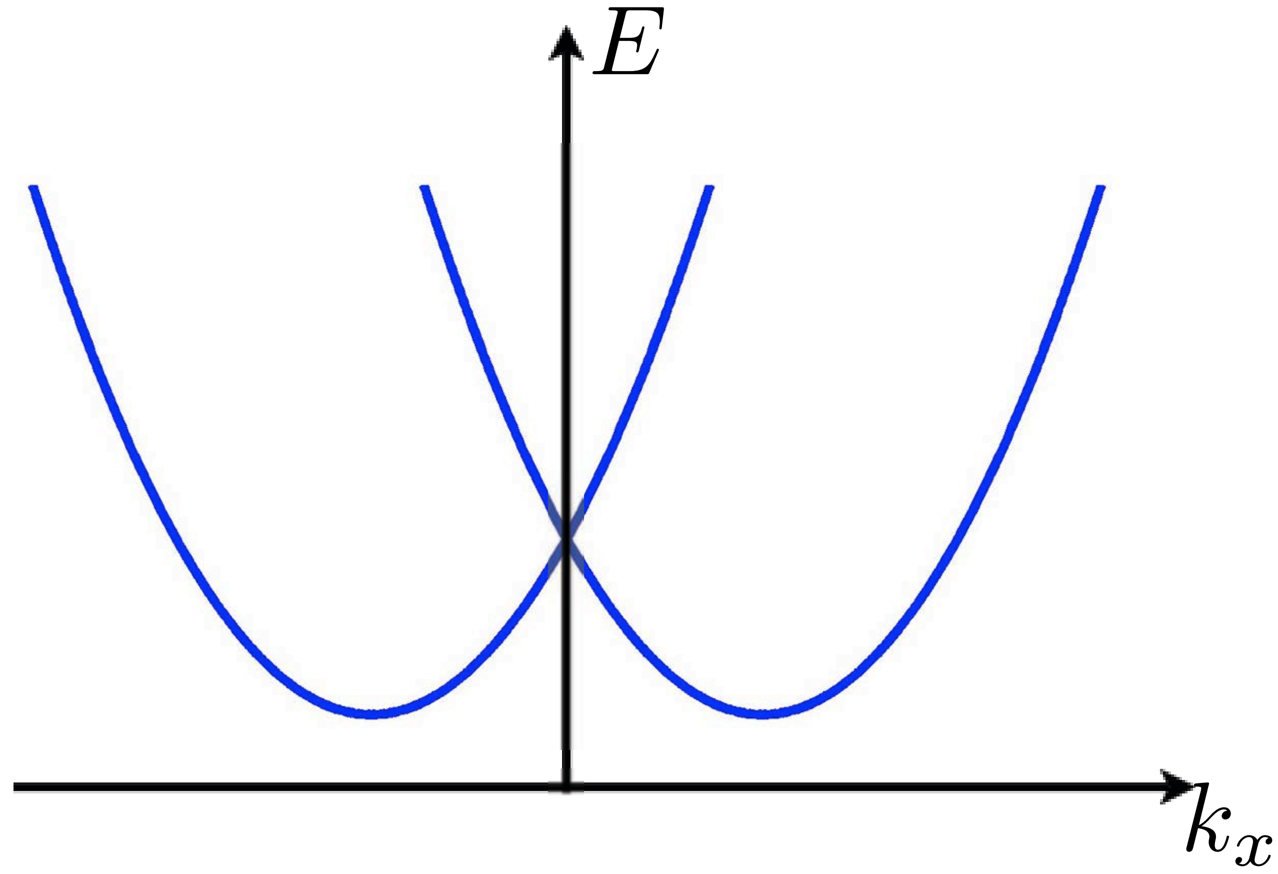
# Majorana fermions in semiconductor devices




(Sau, Lutchyn, Tewari, & Das Sarma 2009)

# Majorana fermions in semiconductor devices



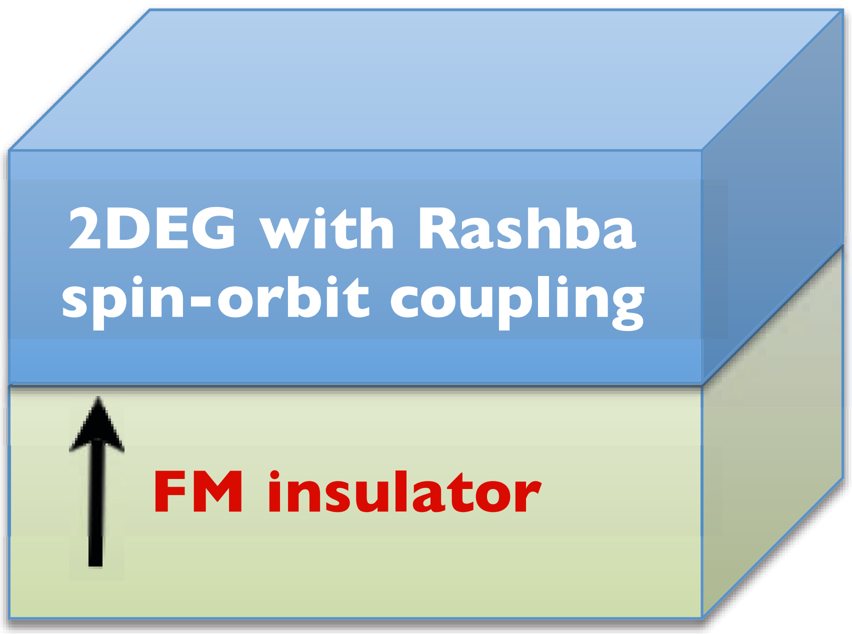



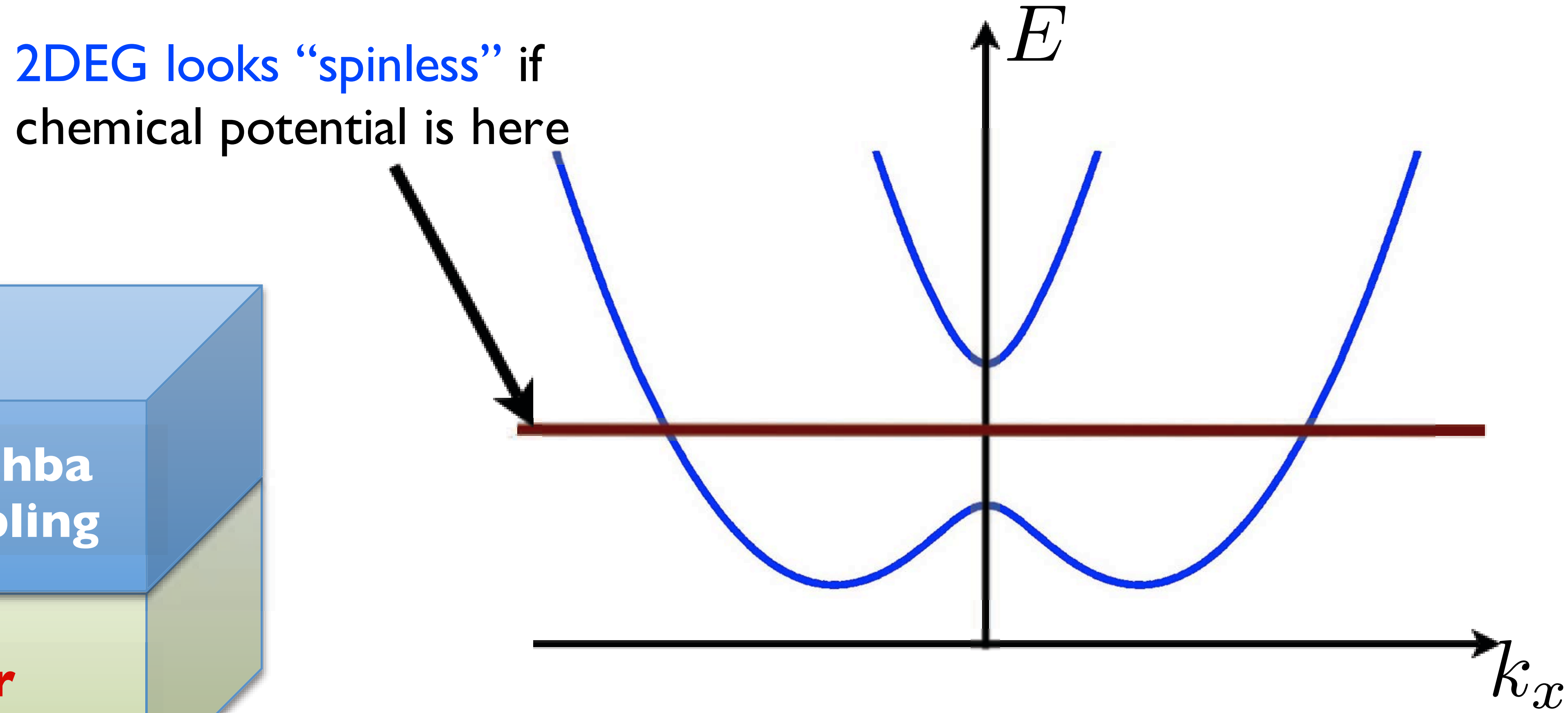

$$H = \int d^2\mathbf{r}\psi \left[ -\frac{\nabla^2}{2m} - \pi - i\alpha(\sigma^x \partial_y - \sigma^y \partial_x) \right] \psi$$

Rashba spin-orbit coupling in 2D

(Sau, Lutchyn, Tewari, & Das Sarma 2009)

# Majorana fermions in semiconductor devices




$$H = \int d^2\mathbf{r}\psi \left[ -\frac{\nabla^2}{2m} - \pi - i\alpha(\sigma^x \partial_y - \sigma^y \partial_x) + V_z \sigma^z \right] \psi$$

(Sau, Lutchyn, Tewari, & Das Sarma 2009)

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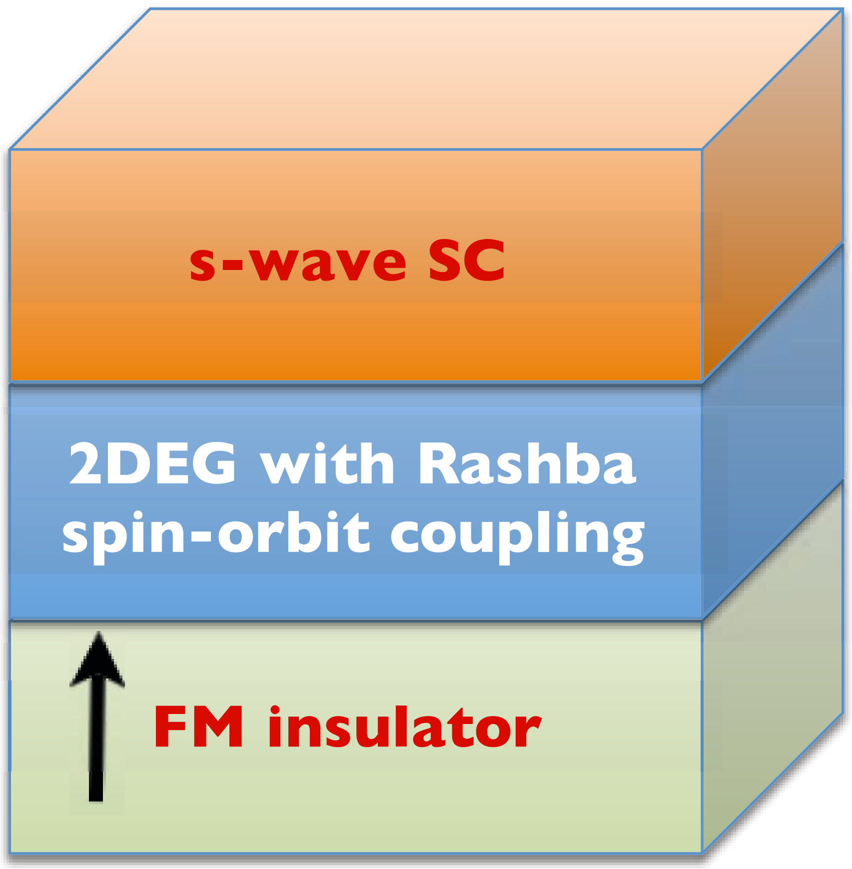


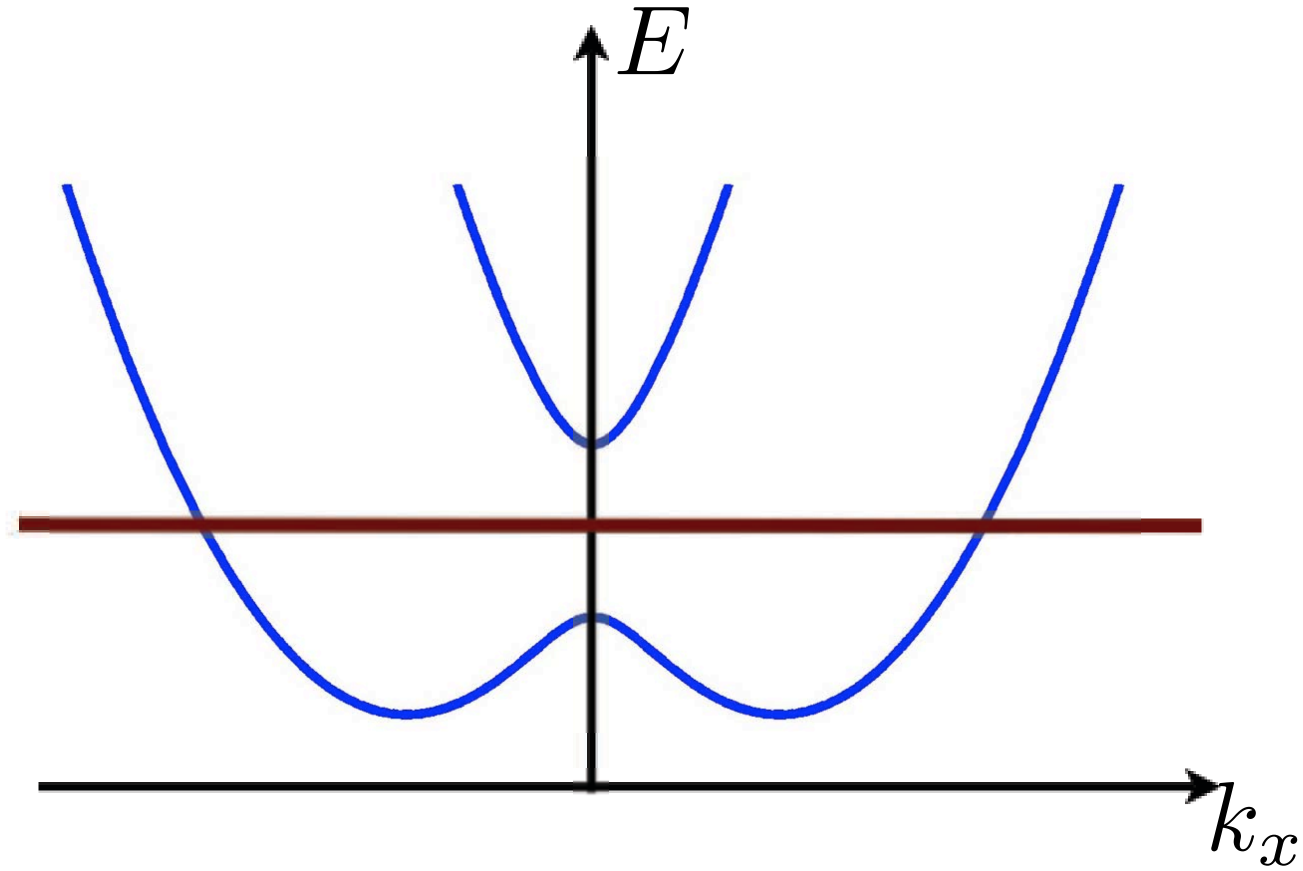


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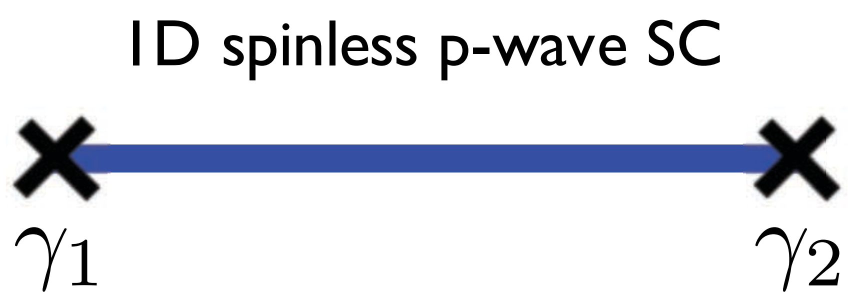
$$+ \int d^2\mathbf{r}(\Delta \psi_\uparrow \psi_\downarrow + h\ c\ )$$

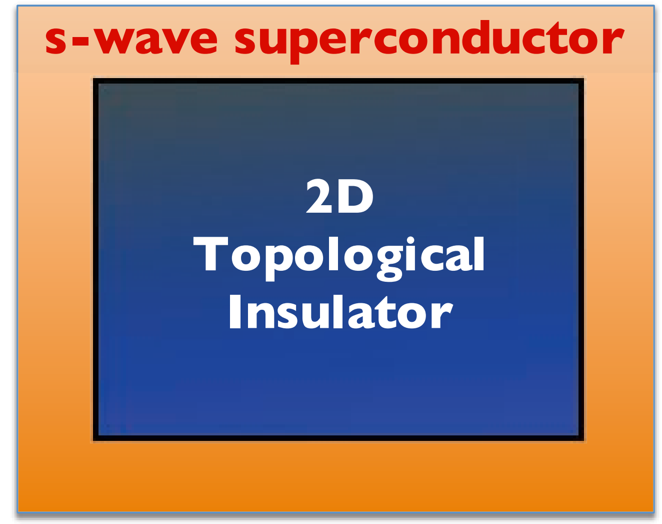
Realizes 2D topological SC supporting Majorana zero-modes (when chemical potential is tuned)

(Sau, Lutchyn, Tewari, & Das Sarma 2009)

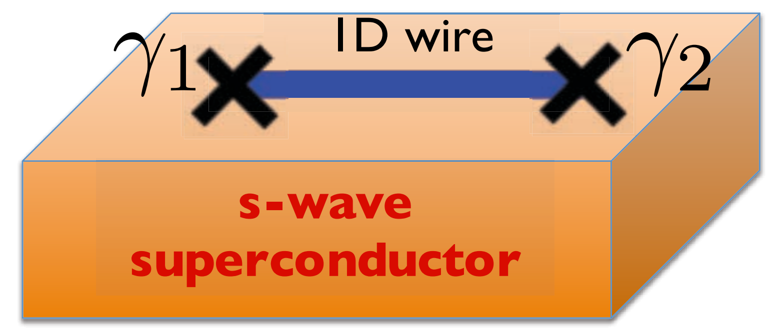
# Summary so far



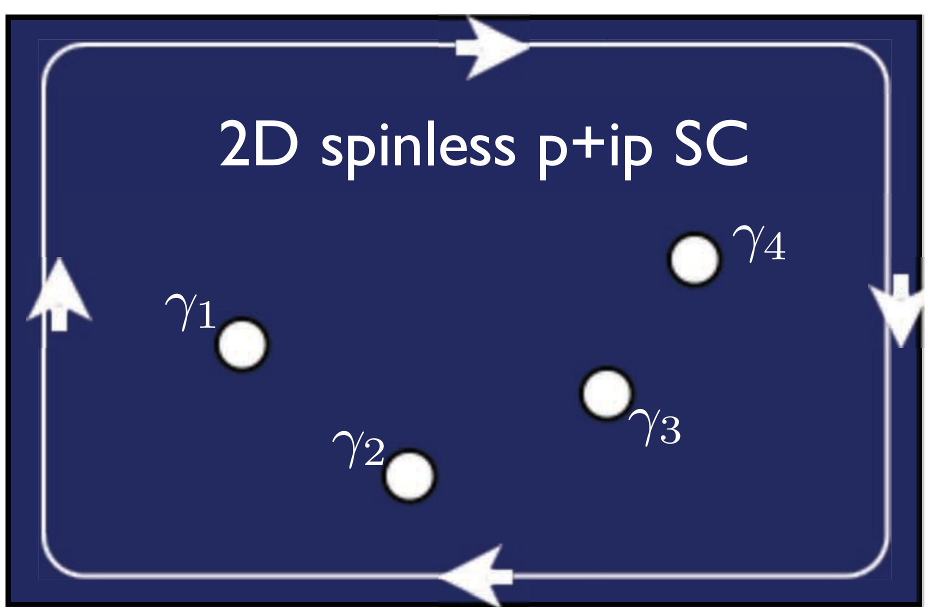


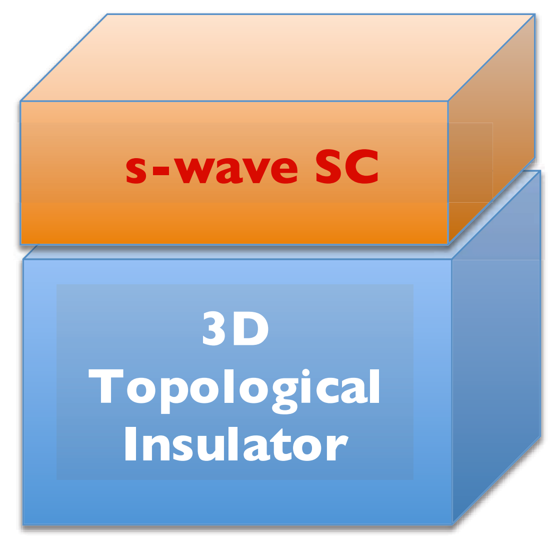


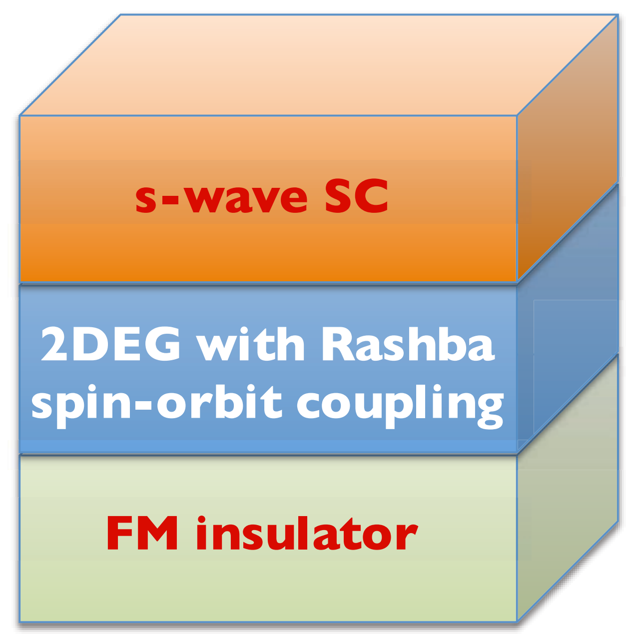

among other promising realizations!




Common theme:
spin-orbit coupling
+ proximity effects



Next time: Majorana detection schemes, experimental status, and applications