Spin-fluctuation theories of unconventional superconductivity

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Lecture I, summary

Weak coupling theory of unconventional (non-phonon) SC

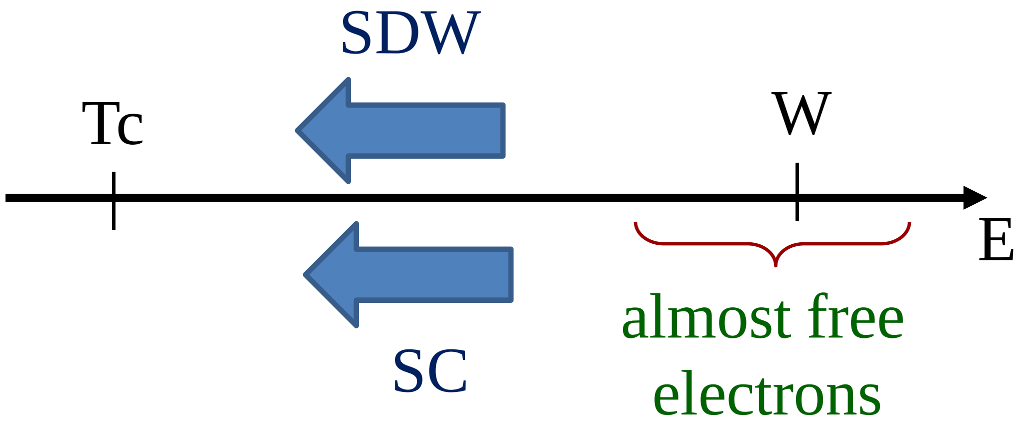
Kohn-Luttinger mechanism for U(q) =U:

p-wave pairing for isotropic dispersion
d-wave ($d_{x^2-y^2}$) pairing in the cuprates
d+id ($d_{x^2-y^2} + d_{xy}$) in doped graphene
s+- in Fe-pnictides

If first-order (bare) interaction U(q) in these channels is repulsive, SC is still possible when fluctuations in the density-wave channel are comparable to SC fluctuations (SC vertex is pushed up due to interaction with SDW)

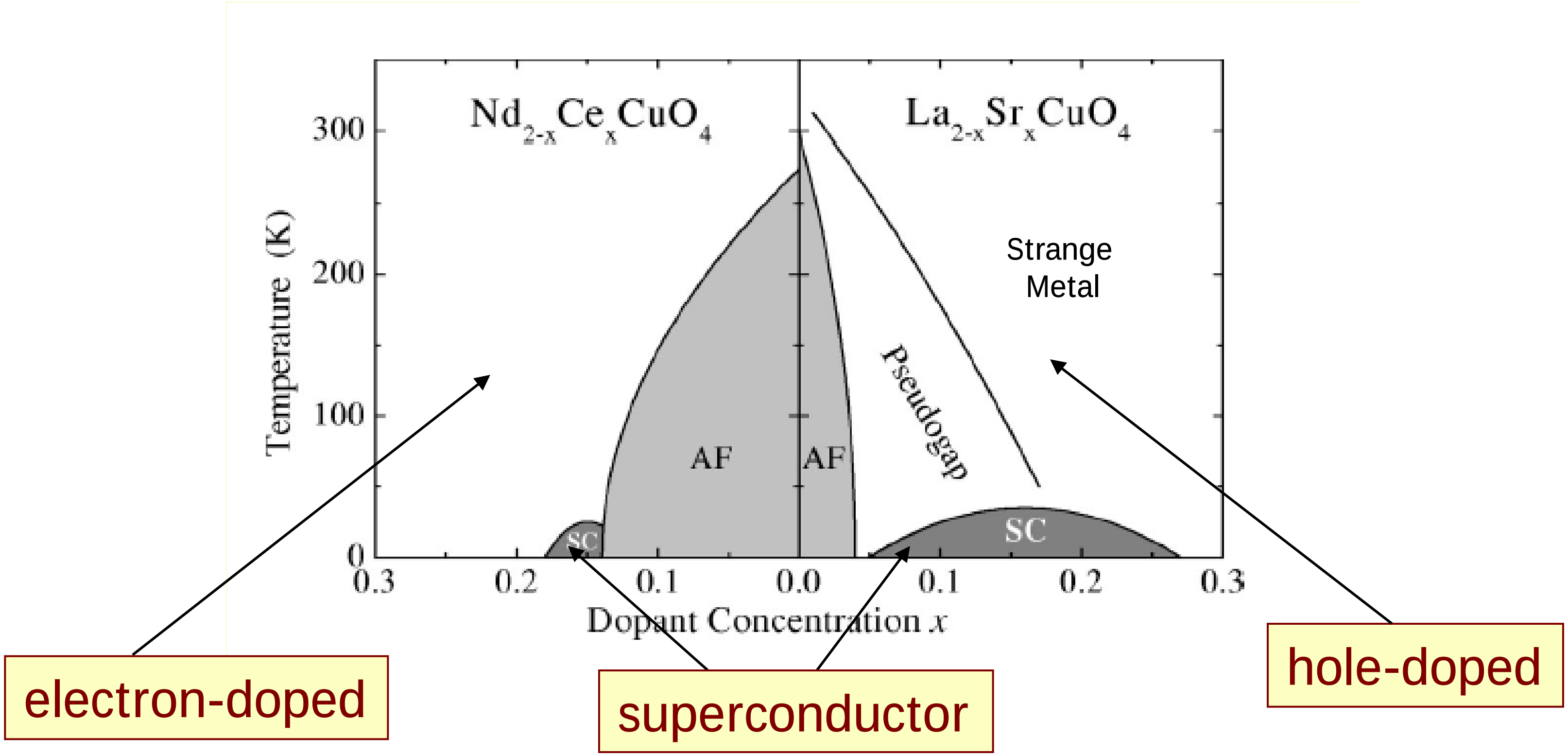
This is truly weak coupling theory: U/W <<1

W = bandwidth



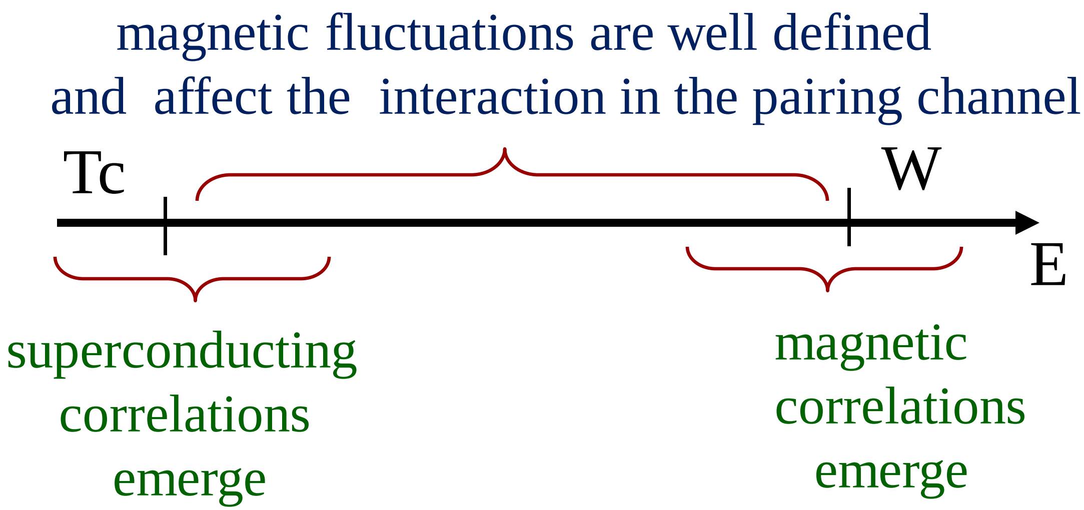
SDW and SC fluctuations develop simultaneously at smaller energies, comparable to Tc (when U log W/Tc ~ 1)

Cuprates: magnetism emerges at a larger scale than superconductivity



Magnetic J ~100 meV (2 magnon Raman peak at 300 meV),
superconducting Δ ~5-30 meV

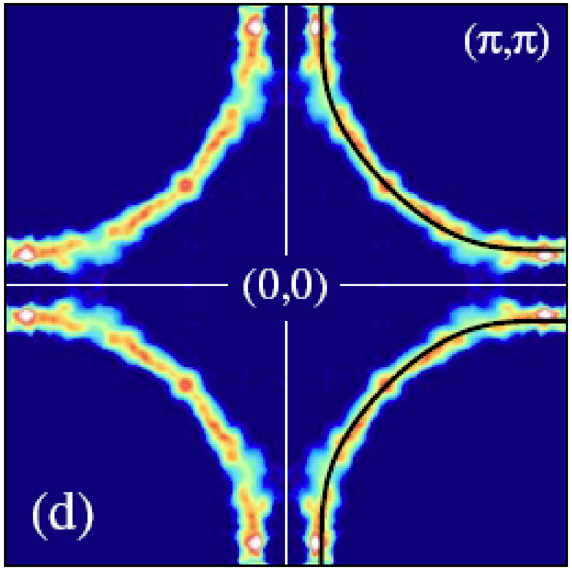
Another scenario: assume that magnetism emerges already at scales comparable to the bandwidth, W



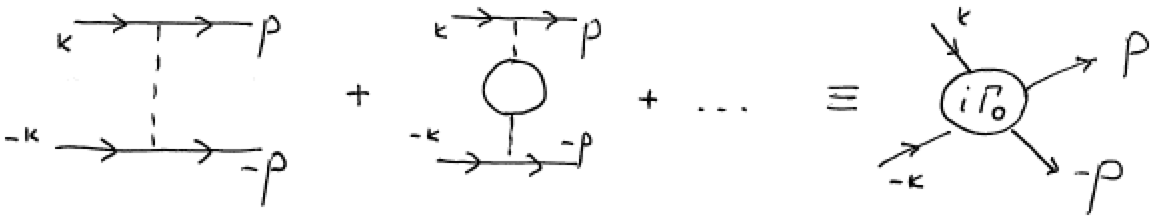
In this situation, one can introduce and explore the concept of spin-fluctuation-mediated pairing:
effective interaction between fermions is mediated by already well formed spin fluctuations

This is not a controlled theory: U/W ~1
(intermediate coupling)

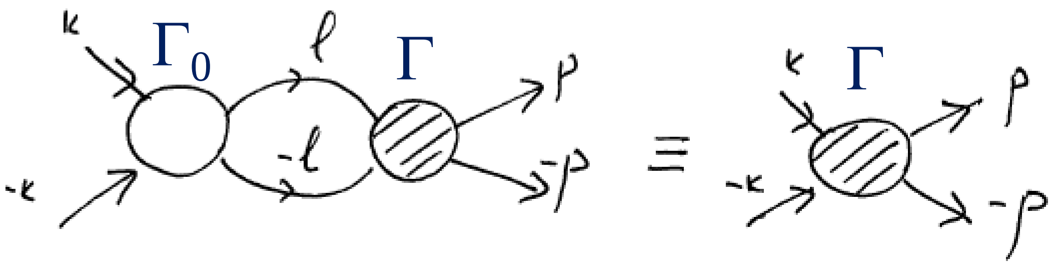
The key assumption is that at U/W ~1 Mott physics does not yet develop, and the system remains a metal with a large Fermi surface



Problem I: how to re-write pairing interaction as the exchange of spin collective degrees of freedom?



everything but the Cooper channel

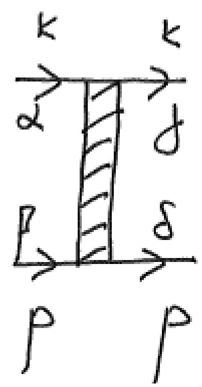


the Cooper channel

No well-defined perturbation theory for Γ_0 when U ~ W

Use the same strategy as for the derivation of Landau function for a Fermi liquid for a special case when Π(q) is peaked at q=0

$$\Gamma^{\Omega}_{\alpha\beta,\gamma\delta}(p,k) =$$



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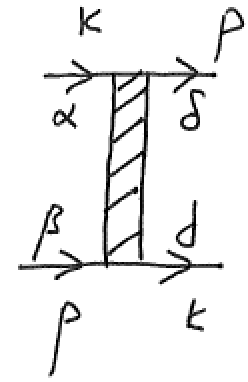
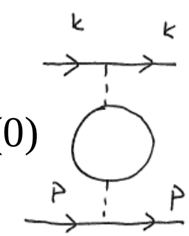


Suppose that p and k are close, and Π(k-p) is peaked at k=p

Π(0) is the largest, but for Γ^{Ω}

Π(0)



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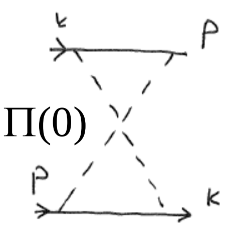
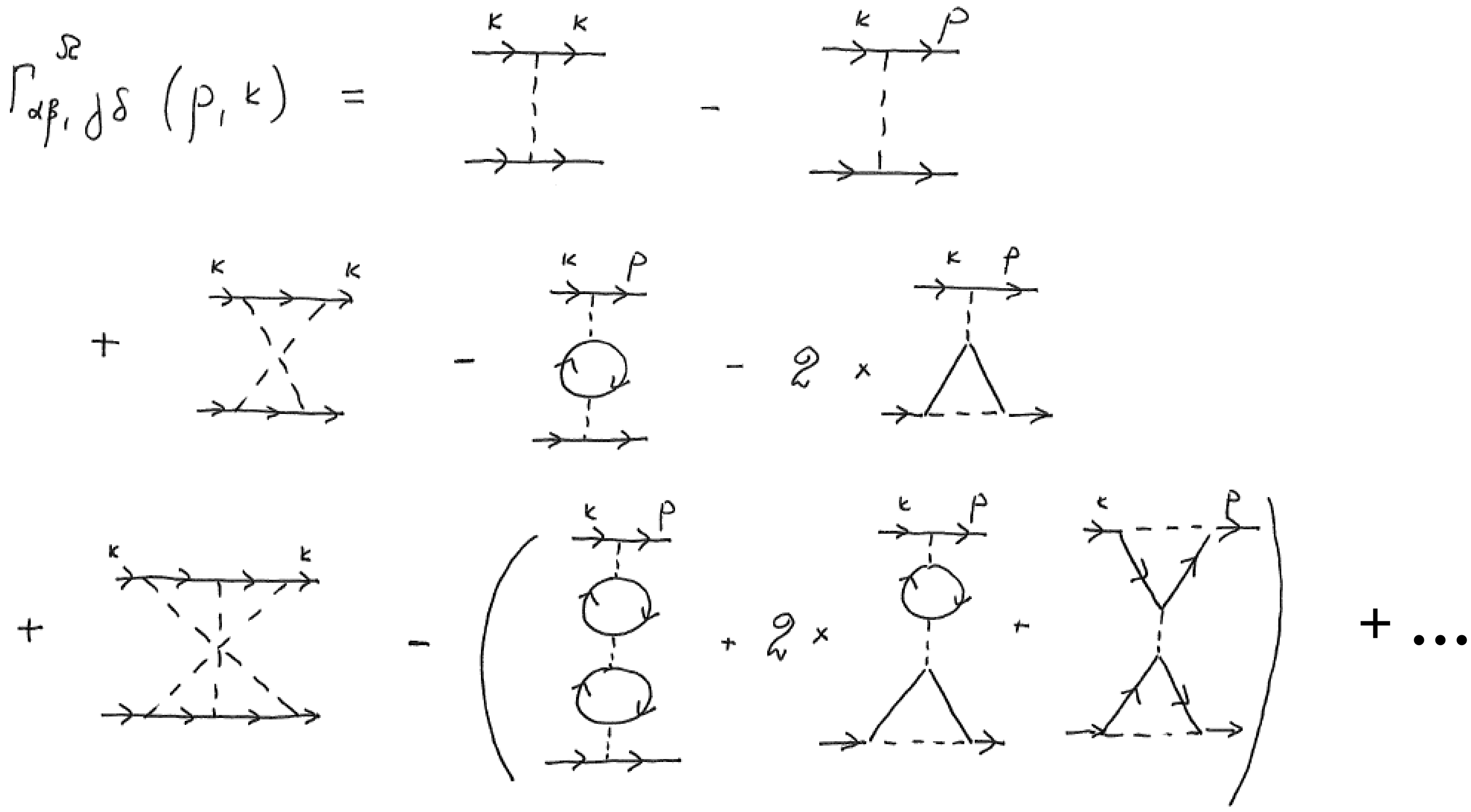
Π(0)



$\equiv 0$

$$\Pi(q=0,\Omega) = \int G_{\vec{l},\omega}\, G_{\vec{l},\omega+\Omega} = 0$$

Collect subleading terms with Π(k-p)



care has to be taken about summation of spin indices

The series can be summed up, and the result is

$$\Gamma^{\Omega}_{\alpha\beta,\gamma\delta} = -\delta_{\alpha\gamma}\delta_{\beta\delta}\frac{U}{1-U\Pi(k-p)} + \delta_{\alpha\delta}\delta_{\beta\gamma}\frac{U}{1-U^2\Pi^2(k-p)}$$

$$= \frac{U}{2}\frac{\delta_{\alpha\delta}\delta_{\beta\gamma}}{1+U\Pi(k-p)} - \frac{U}{2}\frac{\vec{\sigma}_{\alpha\delta}\vec{\sigma}_{\beta\gamma}}{1-U\Pi(k-p)}$$

For repulsive interaction U, the dominant term is in the spin channel

$$\left(\Gamma^{\Omega}_{\alpha\beta,\gamma\delta}\right)_{spin} = -\frac{U}{2}\frac{\vec{\sigma}_{\alpha\delta}\vec{\sigma}_{\beta\gamma}}{1-U\Pi(k-p)}$$

$$\left(\Gamma^{R}_{\alpha\beta,\gamma\delta}\right)_{spin} = -\frac{U}{2}\,\frac{\vec{\sigma}_{\alpha\delta}\,\vec{\sigma}_{\beta\gamma}}{1-U\,\Pi(k-p)}$$

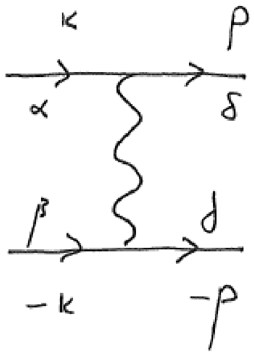
$$1-U\,\Pi(k-p) = \xi^{-2} + (k-p)^2$$ (in units of interatomic spacing)

$$\left(\Gamma^{R}_{\alpha\beta,\gamma\delta}\right)_{spin} = -\frac{U}{2}\,\frac{\vec{\sigma}_{\alpha\delta}\,\vec{\sigma}_{\beta\gamma}}{\xi^{-2} + (k-p)^2}$$

$$= -U/2\;\chi(k-p)\;\vec{\sigma}_{\alpha\delta}\,\vec{\sigma}_{\beta\gamma}$$

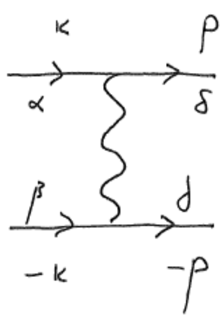
The same logics is applied, without proof, to the derivation of effective pairing interaction

Near a ferromagnetic instability



$$\Gamma_0 = -g \frac{\vec{\sigma}_{\alpha\delta}\,\vec{\sigma}_{\beta\gamma}}{\xi^{-2} + |k-p|^2} \qquad g \sim U$$

Near an antiferromagnetic instability



$$\Gamma_0 = -g \frac{\vec{\sigma}_{\alpha\delta}\,\vec{\sigma}_{\beta\gamma}}{\xi^{-2} + |k-p-Q|^2} \qquad g \sim U$$

The outcome of this analysis is the effective Hamiltonian for instantaneous fermion-fermion interaction in the spin channel

Near a ferromagnetic instability

$$H^{eff} = -g\,(c^{+}_{\alpha}\,\vec{\sigma}_{\alpha\delta}\,c_{\delta})\,(c^{+}_{\gamma}\,\vec{\sigma}_{\gamma\beta}\,c_{\beta})\,\chi(q)$$

$$\chi(q) = \frac{1}{q^2 + \xi^{-2}}$$

Near an antiferromagnetic instability

$$H^{eff} = -g\,(c^{+}_{\alpha}\,\vec{\sigma}_{\alpha\delta}\,c_{\delta})\,(c^{+}_{\gamma}\,\vec{\sigma}_{\gamma\beta}\,c_{\beta})\,\chi(q+Q)$$

$$\chi(q+Q) = \frac{1}{q^2 + \xi^{-2}}$$

THIS IS THE SPIN-FERMION MODEL

It can also be introduced phenomenologically, as a minimalistic low-energy model for the interaction between fermions and collective modes of fermions in the spin channel

Check consistency with Kohn-Luttinger physics

Antiferromagnetism for definitness

$$H^{eff} = -g\,(c^{+}_{\alpha}\,\vec{\sigma}_{\alpha\delta}\,c_{\delta})\,(c^{+}_{\gamma}\,\vec{\sigma}_{\gamma\beta}\,c_{\beta})\,\chi(q+Q)$$

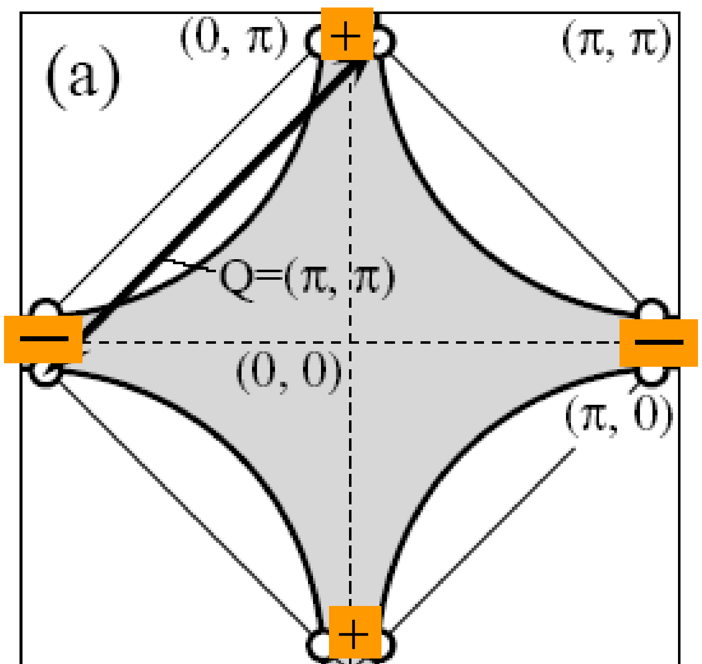
$$\chi(q+Q) = \frac{1}{q^2 + \xi^{-2}}$$

Effective interaction is repulsive, but is peaked at large momentum transfer

Check consistency with Kohn-Luttinger physics

Eqn. for a sc gap

$$\Delta(k) = -\int dq \frac{\Delta(q)}{\sqrt{\Delta^2(q) + E^2(q)}} \chi(k-q)$$



$d_{x^2-y^2}$ superconductivity

KL analysis assumes weak coupling
(static interaction, almost free fermions)

To properly solve for the pairing we need to know how fermions behave in the normal state

Energy scales:

- coupling g
- $v_F\ \xi^{-1}$
- bandwith W

Let's just assume for the next 30 min that g << W. Then high-energy and low-energy physics are decoupled, and we obtain a model with one energy scale g and one dimensional ratio $g/v_F\ \xi^{-1}$

$\lambda = \frac{g}{v_F \xi^{-1}}$ is the relevant parameter of the problem

$\lambda << 1$ truly weak coupling, KL pairing in a Fermi gas

$\lambda >> 1$, the system is still a metal, but with strong correlations

Problem II: how to construct normal state theory for $\lambda >> 1$

• fermions get dressed by the interaction with spin fluctuations

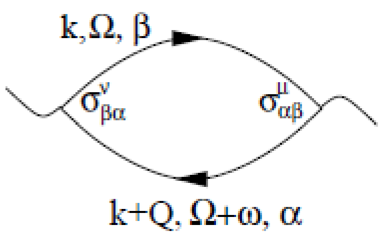
• spin fluctuations get dressed by the interaction with low-energy fermions

Bosonic and fermionic self-energies have to be computed self-consistently (c.f. Subir's talk)

Fermionic self-energy: mass renormalization & lifetime
Bosonic self-energy: Landau damping

At one loop level:

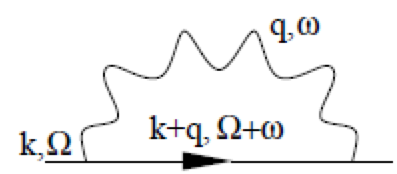
bosons (spin fluctuations) become Landau overdamped



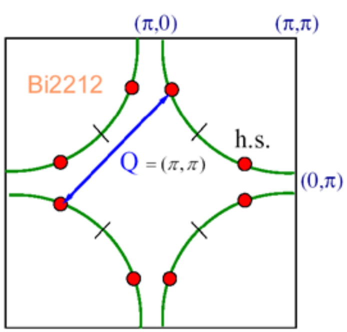
$$\chi(q,\omega) = \frac{1}{q^2 + \xi^{-2} - i\,\omega/\omega_{sf}}$$

$$\omega_{sf} \propto g/\lambda^2 \left(= \frac{9}{64\pi}\frac{g}{\lambda^2} \right)$$

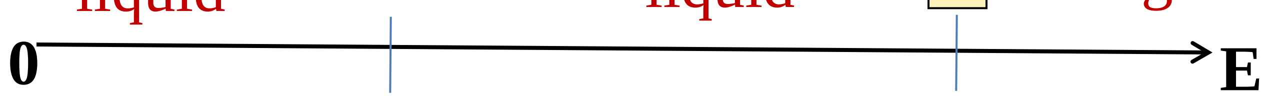
fermions acquire frequency dependent self-energy Σ(ω)



Hot spots



Fermi liquid | $\omega_{sf} \sim g/\lambda^2$ | Non-Fermi liquid | g | Fermi gas

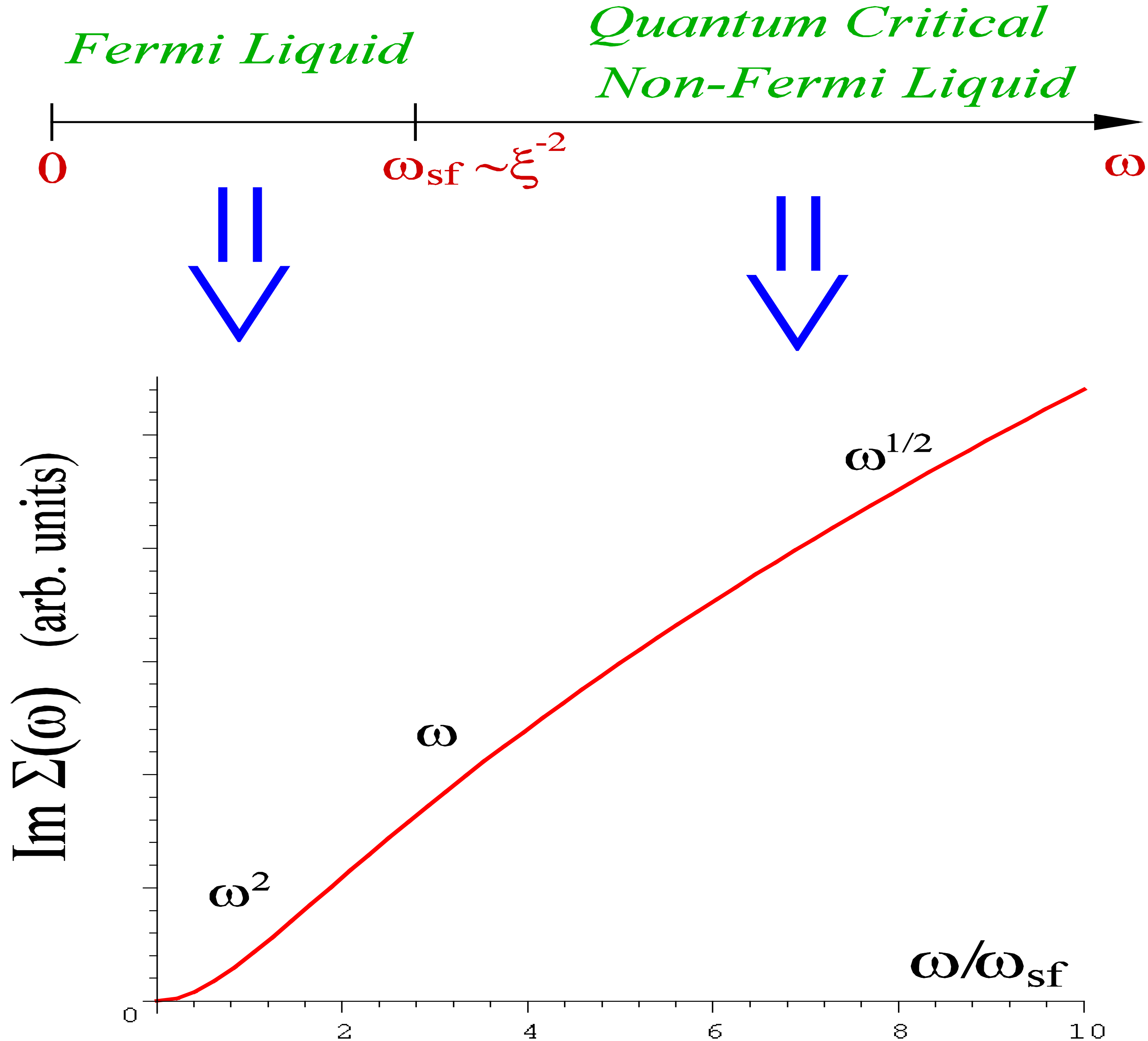


$$\Sigma'(\omega) = \lambda\omega, \quad \Sigma''(\omega) \propto \omega^2$$

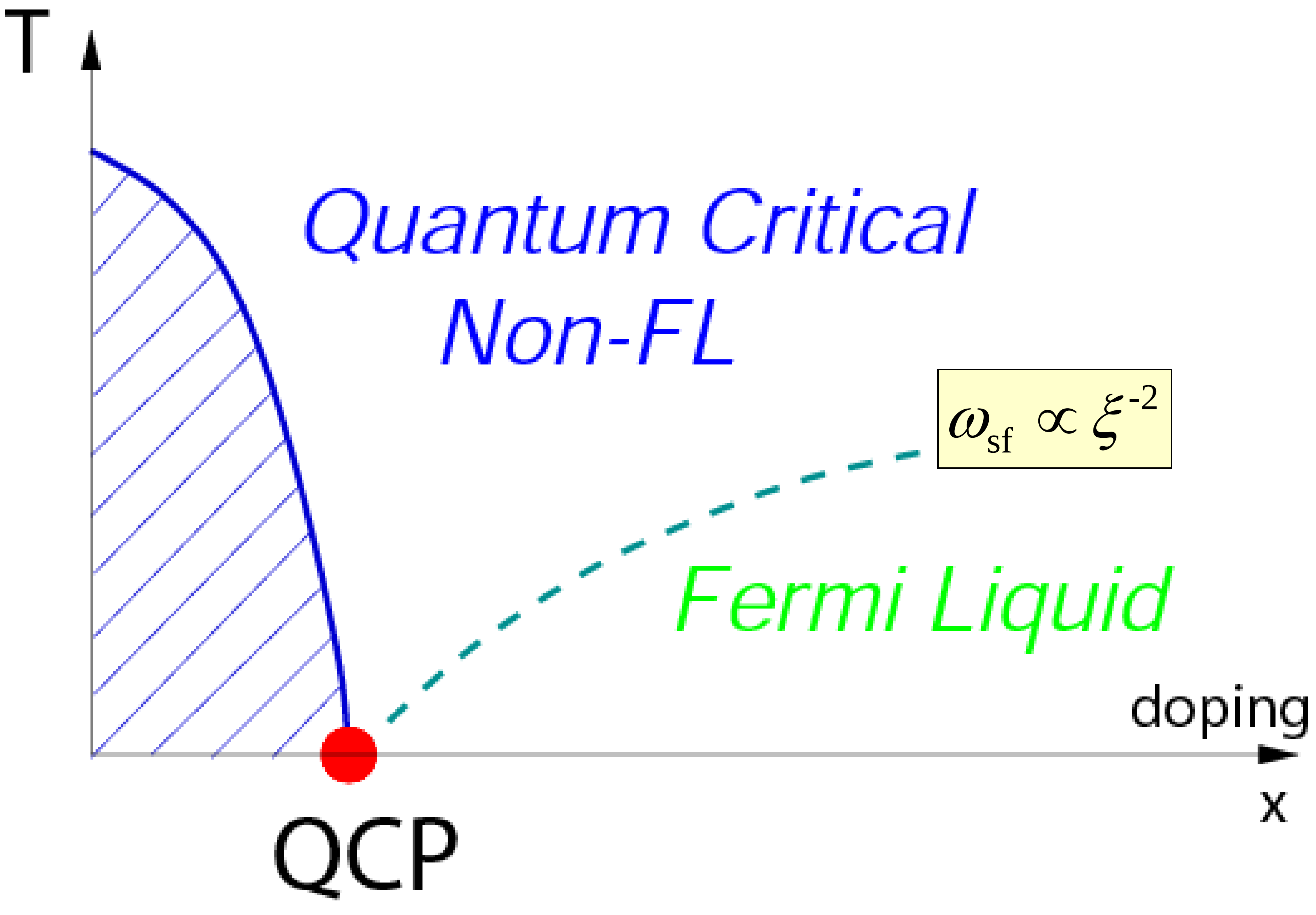
$$\Sigma'(\omega) \propto (\omega g)^{1/2} > \omega, \quad \Sigma''(\omega) \propto (\omega g)^{1/2} > \omega$$

$$\Sigma(\omega) < \omega$$

At $\xi^{-1} = 0$, Fermi liquid region disappears at a hot spot

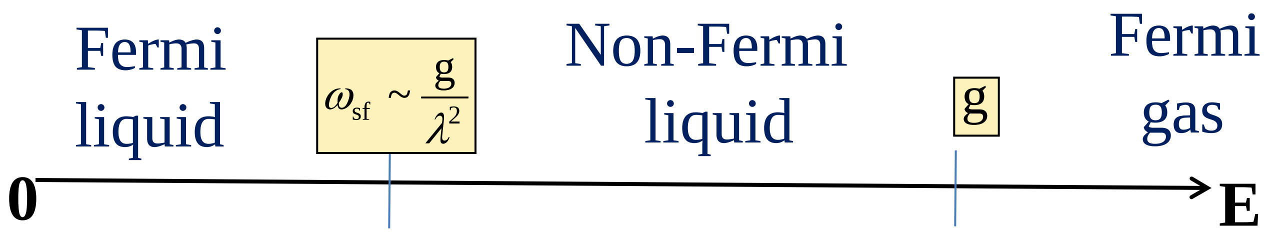


Fermi Liquid
Quantum Critical
Non-Fermi Liquid
0
ωsf ~ξ-2
ω
Im Σ(ω) (arb. units)
ω2
ω
ω1/2
ω/ωsf
0
2
4
6
8
10



T
Quantum Critical
Non-FL
$\omega_{sf} \propto \xi^{-2}$
Fermi Liquid
doping
x
QCP

Problem III: pairing at λ >>1



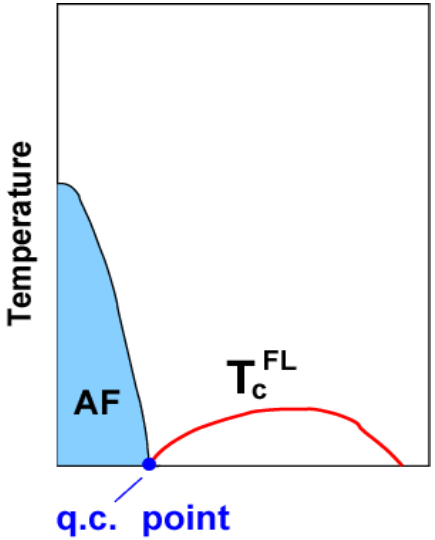
Pairing in the Fermi liquid regime is KL physics

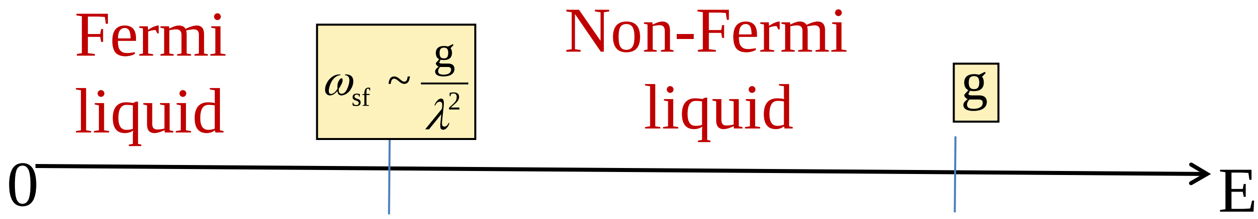
McMillan formula for phonons by analogy

$$T_c \sim \omega_D \exp[-(1+\lambda)/\lambda]$$

$$Tc \propto \xi^{-2} \exp(-\frac{\xi+\xi_0}{\xi})$$

If only Fermi liquid region would contribute to d-wave pairing, Tc would be zero at a QCP





Pairing in non-Fermi liquid regime is a new phenomenon

Pairing vertex Φ becomes frequency dependent $\Phi(\Omega)$

Gap equation has non-BCS form

$$\Phi(\Omega) = \frac{\pi}{2} T \sum_{\omega > \omega_{sf}} \frac{\Phi(\omega)}{|\omega|^{1/2} |\Omega - \omega|^{1/2}} \frac{1}{1 + (|\omega|/g)^{1/2}}$$

$\Sigma(\omega) \propto \omega^{1/2}$ — $\int dq \chi(q, \omega) \propto \omega^{-1/2}$ — $1 + \omega / \Sigma(\omega)$, soft cutoff

For comparison, in a Fermi liquid

$$\Phi(\Omega) = \frac{\lambda}{1+\lambda} \pi T \sum_{\omega} \frac{\Phi(\omega)}{|\omega|} \frac{1}{(1 + |\omega| / \omega_{sf})^{1/2}}$$

Compare BCS and QC pairings

Quantum-critical pairing

$$\Phi(\Omega) = \frac{\pi}{2} T \sum_{\omega > \omega_{sf}} \frac{\Phi(\omega)}{|\omega|^{1/2} |\Omega - \omega|^{1/2}} \frac{1}{1 + (|\omega|/g)^{1/2}}$$

BSC pairing

$$\Phi(\Omega) = \frac{\lambda}{1+\lambda} \pi T \sum_{\omega} \frac{\Phi(\omega)}{|\omega|} \frac{1}{(1 + |\omega|/\omega_{sf})^{1/2}}$$

- pairing problem in the QC case is universal (no overall coupling)

- pairing kernel is $|\omega|^{-1}$, like in BCS theory, only a half of $|\omega|$ comes from self-energy, another from interaction

Is the quantum-critical problem like BCS?

Let's check: Pairing kernel $|\omega|^{-1}$ → logarithms!

BCS:

$$\Phi(\Omega) = \bar{\lambda} T \sum^{\omega_{sf}} \frac{\Phi(\omega)}{|\omega|} + \Phi_0, \qquad \bar{\lambda} = \frac{\lambda}{1+\lambda}$$

sum up logarithms

$$\Phi = \Phi_0 \, (1 + \bar{\lambda} \log \frac{\omega_{sf}}{T} + \bar{\lambda}^2 \log^2 + \ldots) = \frac{\Phi_0}{\bar{\lambda} \log \frac{T}{T_c}}$$

pairing instability at any coupling

QC case:

$$\Phi(\Omega) = \frac{\pi T}{2} \sum^{g} \frac{\Phi(\omega)}{|\omega|^{1/2} |\Omega - \omega|^{1/2}} + \Phi_0$$

sum up logarithms

$$\Phi(\Omega = 0, T) = \Phi_0 \, (1 + \frac{1}{2} \log \frac{g}{T} + \frac{1}{2} \left(\frac{1}{2} \log \frac{g}{T} \right)^2 + \ldots) = \Phi_0 \left(\frac{g}{T} \right)^{1/2}$$

no divergence at a finite T

Let's look a bit more carefully

$$\Phi(\Omega) = \varepsilon \frac{\pi}{2} T \sum \frac{\Phi(\omega)}{|\omega|^{1/2} |\Omega - \omega|^{1/2}} \frac{1}{1 + (|\omega|/g)^{1/2}} + \Phi_0$$

At small ε perturbation theory should be valid

$$\Phi(\Omega = 0, T) = \Phi_0 \left(1 + \frac{\varepsilon}{2} \log\frac{g}{T} + \frac{\varepsilon^2}{2}\left(\frac{1}{2}\log\frac{g}{T}\right)^2 + \ldots\right) = \Phi_0 \left(\frac{g}{T}\right)^{\varepsilon/2}$$

Focus on the regime T< Ω <g, from which we get logarithms

$$\Phi(\Omega) = \frac{\varepsilon}{4} \int \frac{\Phi(\omega)}{|\omega|^{1/2}} \left(\frac{1}{|\Omega - \omega|^{1/2}} + \frac{1}{|\Omega + \omega|^{1/2}} \right)$$

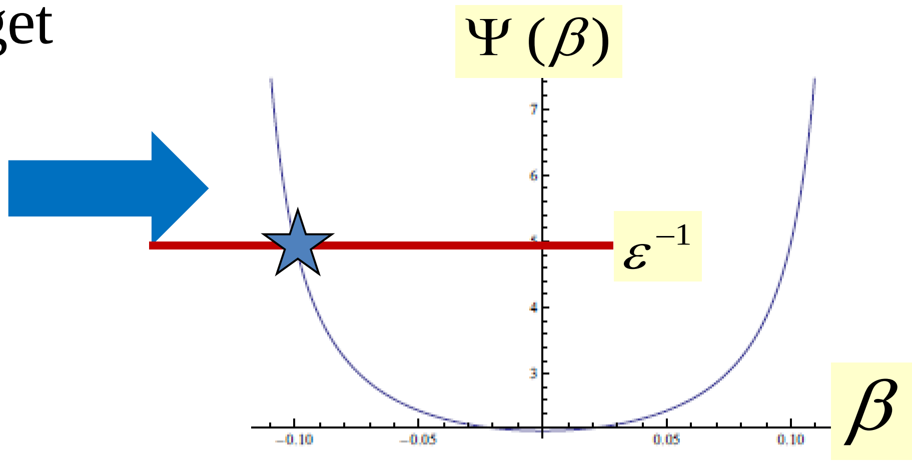
Solve in this regime, and then see whether we can satisfy boundary conditions at Ω =T and at Ω =g

$$\Phi(\Omega) = \frac{\varepsilon}{4}\int \frac{\Phi(\omega)}{|\omega|^{1/2}}\left(\frac{1}{|\Omega - \omega|^{1/2}} + \frac{1}{|\Omega + \omega|^{1/2}}\right)$$

Solution is a power-law $\quad \Phi(\Omega) = \Omega^{-(1/4-2\beta)}$

Substitute, we get

$$\varepsilon^{-1} = \Psi(\beta)$$

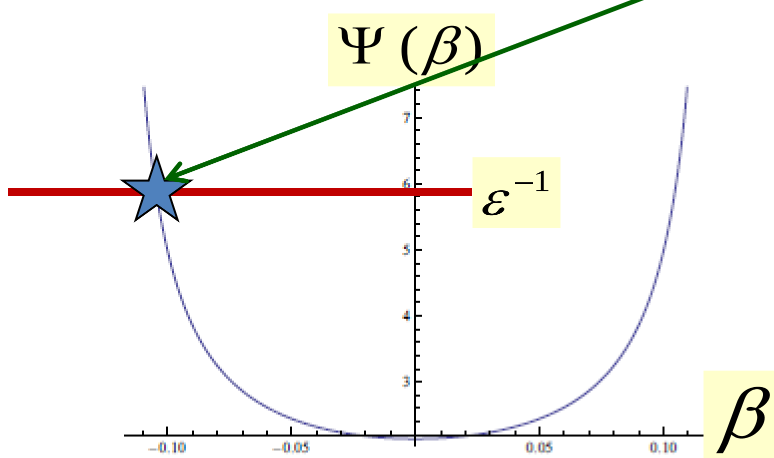


At small ε we do indeed reproduce perturbation theory

$$\Phi(\Omega, T=0) = \Phi_0 \left(1 + \frac{\varepsilon}{2}\log\frac{g}{\Omega} + \frac{\varepsilon^2}{2}\left(\frac{1}{2}\log\frac{g}{\Omega}\right)^2 + \ldots\right) = \Phi_0 \left(\frac{g}{\Omega}\right)^{\varepsilon/2}$$

$$\Phi(\Omega) = \Omega^{-(1/4-2\beta)}$$

$$1/4 - 2\beta = \varepsilon/2$$



Now recall that we need to satisfy boundary conditions:
an upper one at g and a lower one at T

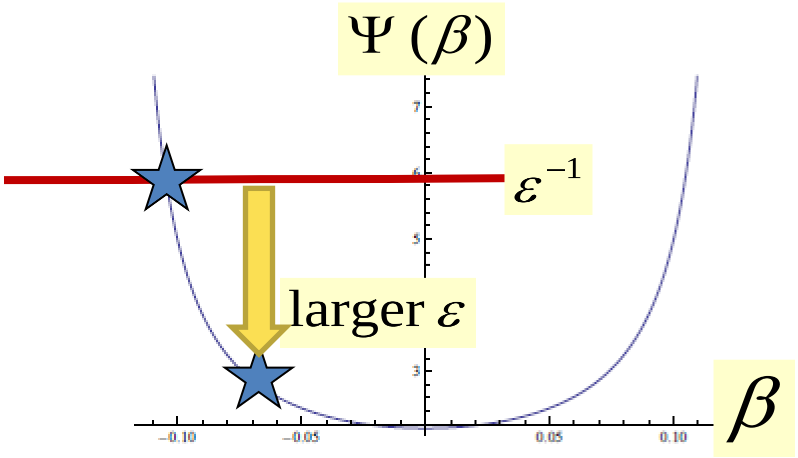
$$\Phi(\Omega) = \frac{\varepsilon}{4}\int_T^g \frac{\Phi(\omega)}{|\omega|^{1/2}}\left(\frac{1}{|\Omega - \omega|^{1/2}} + \frac{1}{|\Omega + \omega|^{1/2}}\right)$$

With $\Phi(\Omega) = \Omega^{-(1/4-2\beta)}$ one can satisfy one boundary condition by varying T, but not both

No QC superconductivity?

No QC superconductivity?

Not so fast….



$$\Psi(0) = 2.13 \Rightarrow \varepsilon_{\max} = 0.47$$

Perturbation theory does not work for $\varepsilon > \varepsilon_{max}$,
and, in particular, it does not work for $\varepsilon = 1$

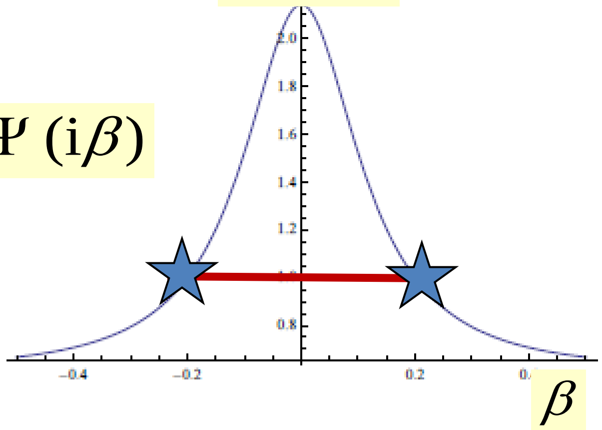
Set ε=1 $$\Phi(\Omega) = \frac{1}{4}\int \frac{\Phi(\omega)}{|\omega|^{1/2}} \left(\frac{1}{|\Omega - \omega|^{1/2}} + \frac{1}{|\Omega + \omega|^{1/2}} \right)$$

Still search for a a power-law solution $\Phi(\Omega) = \Omega^{-(1/4-2\beta)}$

But now take β to be imaginary, i β $1 = \Psi(i\beta)$

$\Psi(i\beta)$

$1 = \Psi(i\beta)$



Two solutions!

$$\Phi(\Omega) = C\left(\frac{1}{\Omega}\right)^{1/4} \cos(2\beta \log(\Omega) + \phi_0)$$

A free parameter: phase!

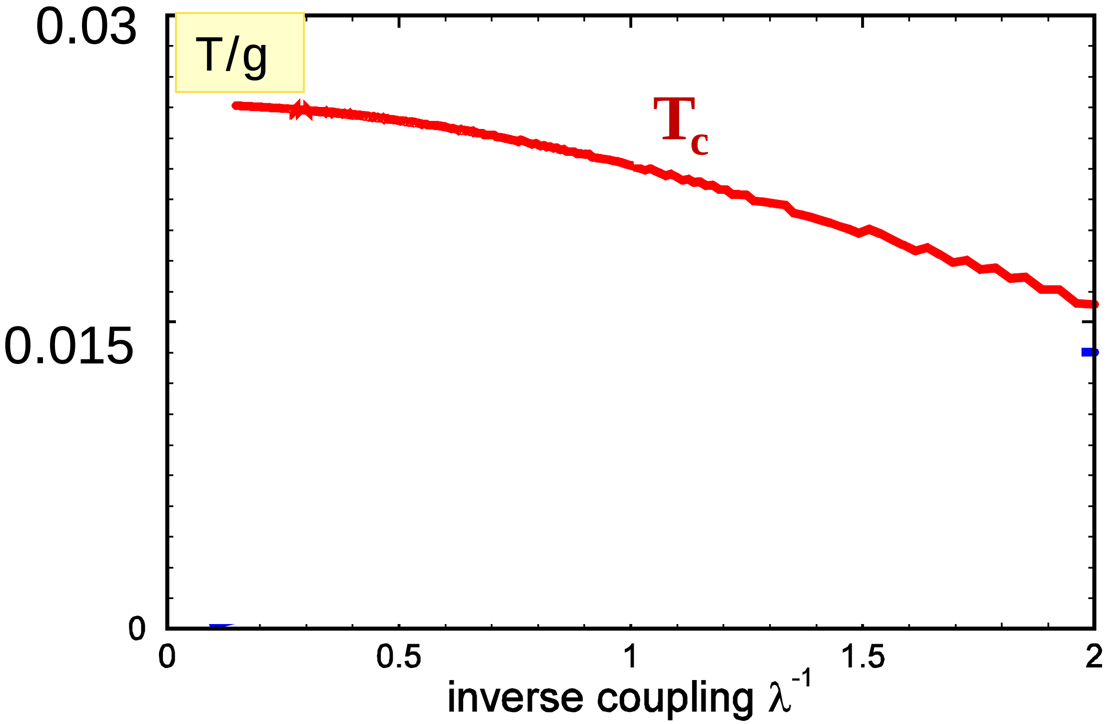
Now back to boundary conditions

$$\Phi(\Omega) = \frac{1}{4}\int_T^g \frac{\Phi(\omega)}{|\omega|^{1/2}} \left(\frac{1}{|\Omega - \omega|^{1/2}} + \frac{1}{|\Omega + \omega|^{1/2}} \right)$$

One boundary: fix the phase
another: set T=Tc

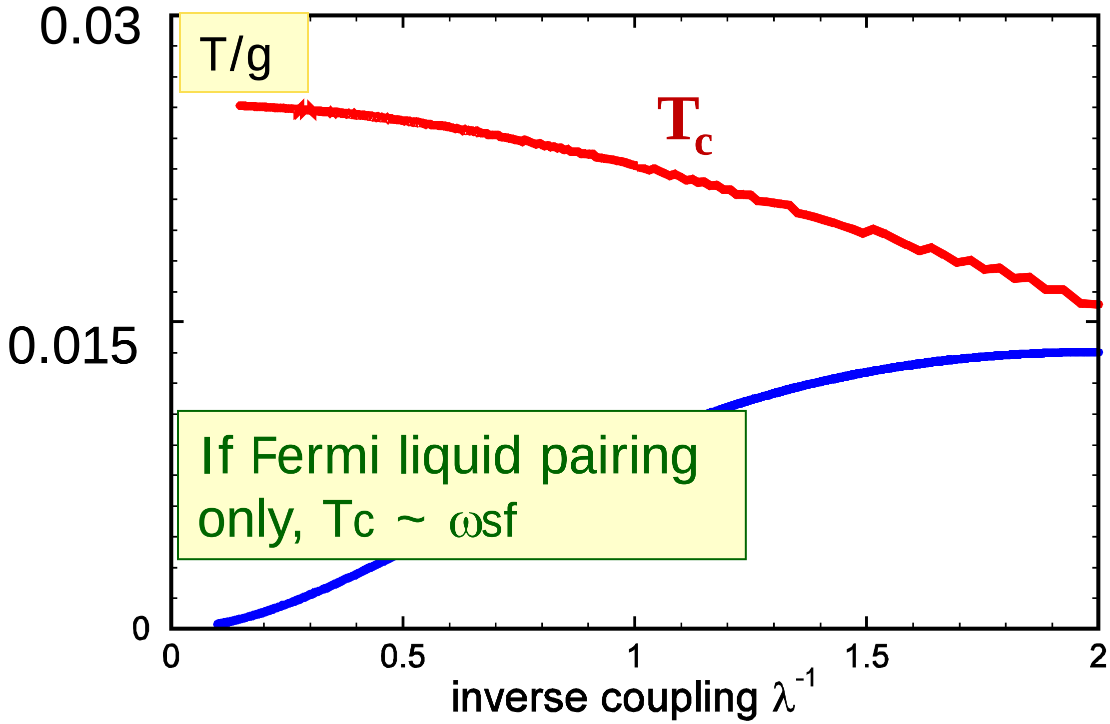
Now we get Tc at which the linearized gap equation has a solution!

The result: a finite Tc right at the quantum-critical point



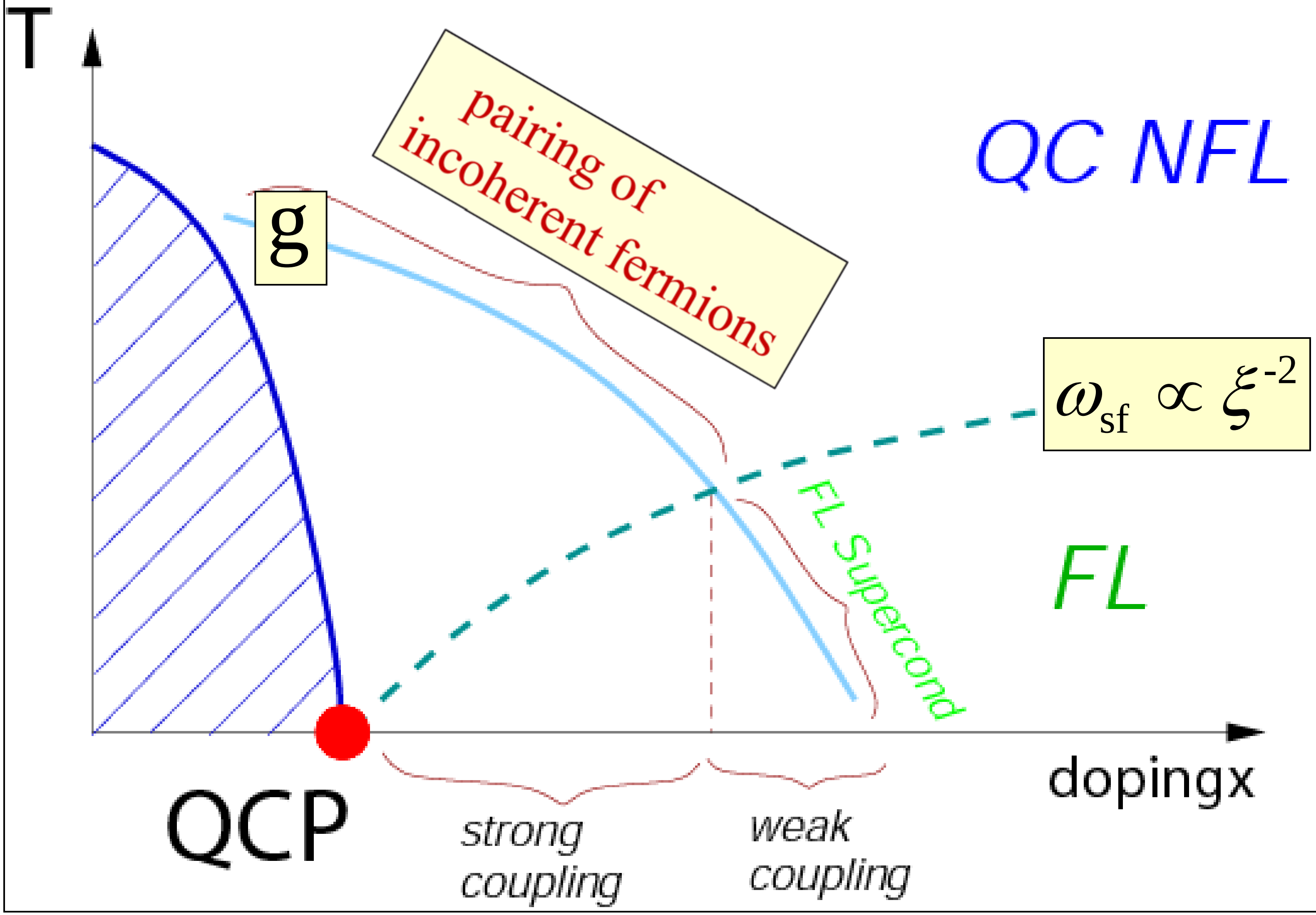
$$T_c = 0.025\, g \text{ at QCP}$$

The result: a finite Tc right at the quantum-critical point



$$T_c = 0.025\, g \text{ at QCP}$$

Dome of a pairing instability above QCP



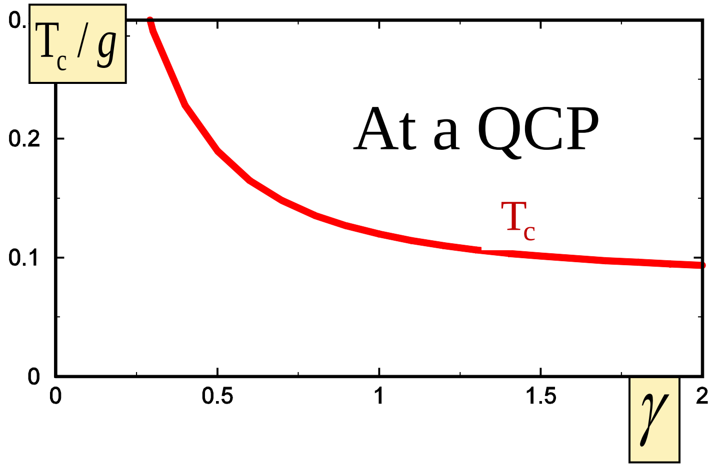
This problem is quite generic and goes beyond the cuprates

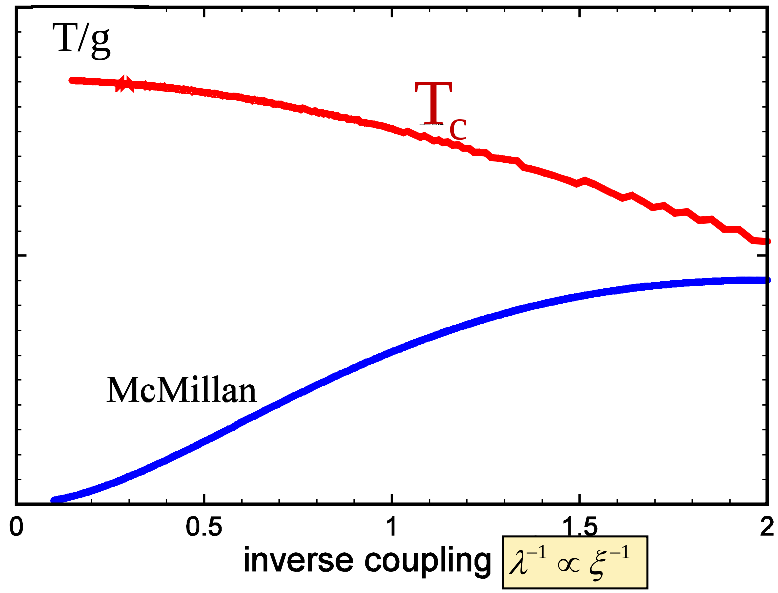
$$\Phi(\Omega) = \frac{1-\gamma}{2} \int_0^g d\omega \frac{\Phi(\omega)}{|\omega|^{1-\gamma}} \left(\frac{1}{|\Omega-\omega|^{\gamma}} + \frac{1}{|\Omega+\omega|^{\gamma}} \right)$$

Abanov et al, Moon, She, Zaanen

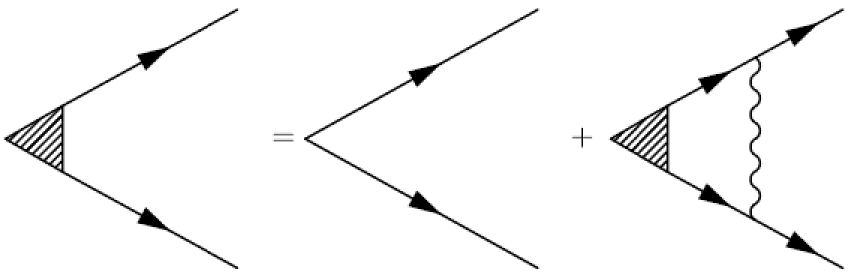
$\gamma = 1/2$	Antiferromagnetic QCP	Abanov et al, Metlitski, Sachdev
$\gamma = 1/3$	FM QCP, nematic, composite fermions, $\Omega^{2/3}$ problem	Bonesteel, McDonald, Nayak, Haslinger et al, Millis et al, Bedel et al…
$\gamma = +0 \ (\log \omega)$	3D QCP, Color superconductivity	Son, Schmalian, A.C, Metlitski, Sachdev
$\gamma = 1$	Z=1 pairing problem	Schmalian, A.C….
$\gamma = +0 \rightarrow \gamma = 1$	pairing in the presence of SDW	Moon, Sachdev
$\gamma \approx 0.7$	fermions with Dirac cone dispersion	Metzner et al
$\gamma = 2$	Pairing by near-gapless phonons $T_c^{ad} = 0.1827\, g$	Allen, Dynes, Carbotte, Marsiglio, Scalapino, Combescot, Maksimov, Bulaevskii, Dolgov, …..

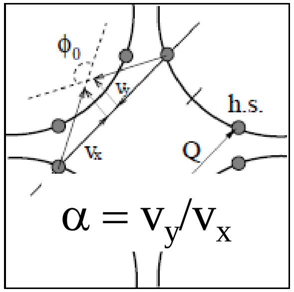
It turns out that for all γ, the coupling (1 - γ)/2 is larger than the threshold





The actual problem near antiferromagnetic QCP in a metal is more complex, because fermions away from hot spots have Fermi liquid self-energy at the lowest frequencies





$$\Phi(\omega \sim T, 0) = \Phi_0 \left(1 + \frac{\lambda}{2\pi} \log^2 \Lambda/T\right), \quad \lambda = \frac{2\alpha}{(1+\alpha^2)}$$

$T_c = 0.006\, g$ at QCP
(instead of $0.025g$)

Metlitski & Sachdev

Y. Wang, A.C.

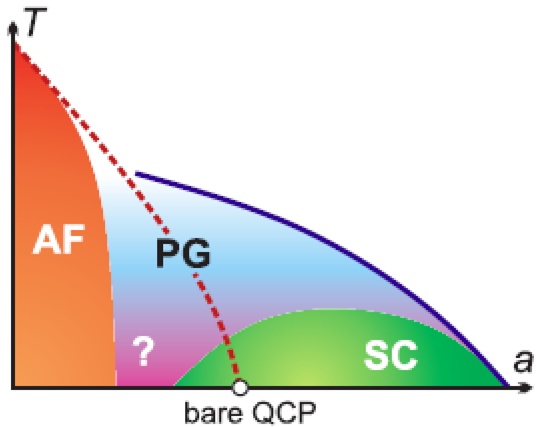
$g = 1.7$ eV, $T_c \sim 120$K

Re: Subir's talk

Superconducting and bond-density-wave order are almost degenerate at T ~ Tc

Metlitski, Sachdev
Efetov, Meier, Pepin

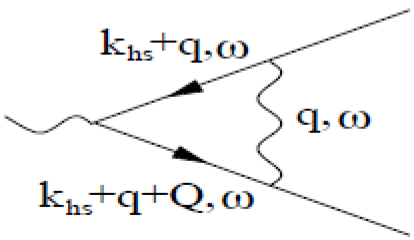
Tc ~ 0.006g is the temperature at which the "modulus" of the combined SC+ CDW order parameter develops.



Moon Sachdev
Metlitski, Sachdev
Efetov, Meier, Pepin

Accuracy: corrections are O(1), the leading ones can be accounted for in the 1/N expansion

Leading vertex corrections are log divergent



$$\frac{\Delta g}{g} = \frac{Q(v)}{N} \log \lambda$$

$$Q(v) = \frac{4}{\pi} \tan^{-1} \frac{v_x}{v_y}$$

To order O(1/N):

$$\chi(\mathrm{q}, \omega) \propto \frac{1}{(\mathrm{q}^2 + |\omega|)^\eta}$$

$$\eta = 1 - \frac{1}{2\mathrm{N}}$$

The only change is

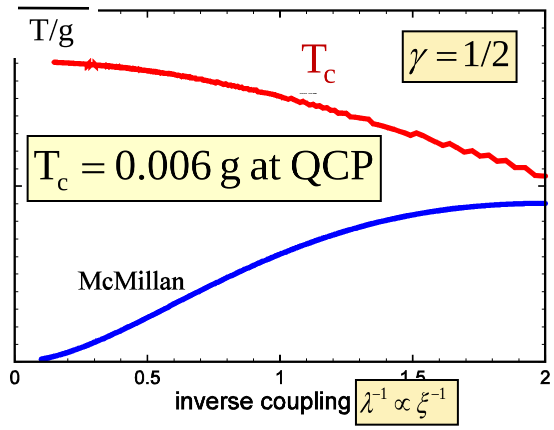
$$\gamma \rightarrow \frac{1}{2} - \frac{1}{2\mathrm{N}}$$

Now, we assumed before that g is smaller than $W \sim v_F/a$

What if the coupling is comparable to bandwidth

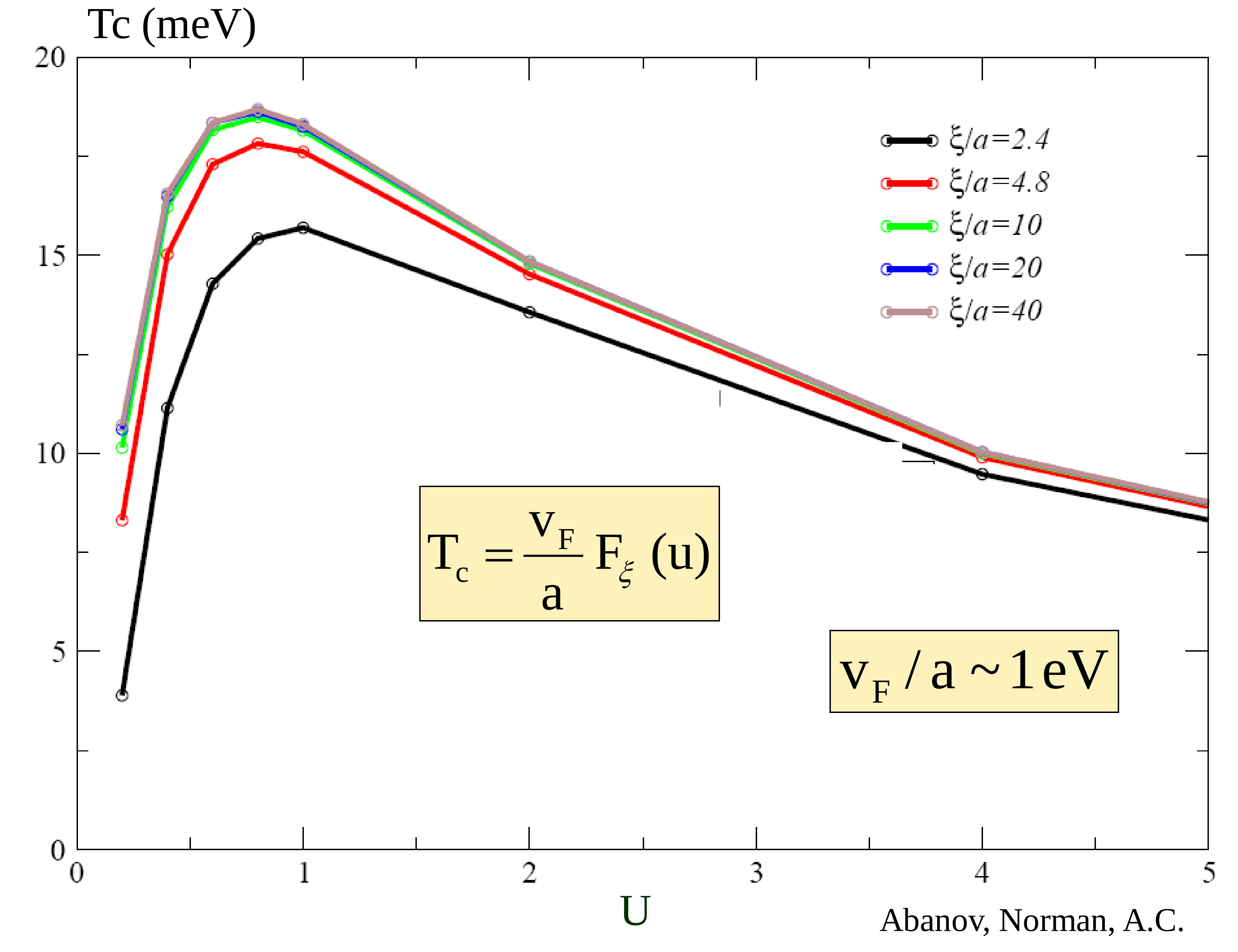
$$T_c = g\,\Psi_\xi$$

The larger is g, the larger is T_c



In general, we have two parameters, $u = \frac{g}{W} = \frac{g\,a}{v_F}$ and $\lambda = u\,\xi$

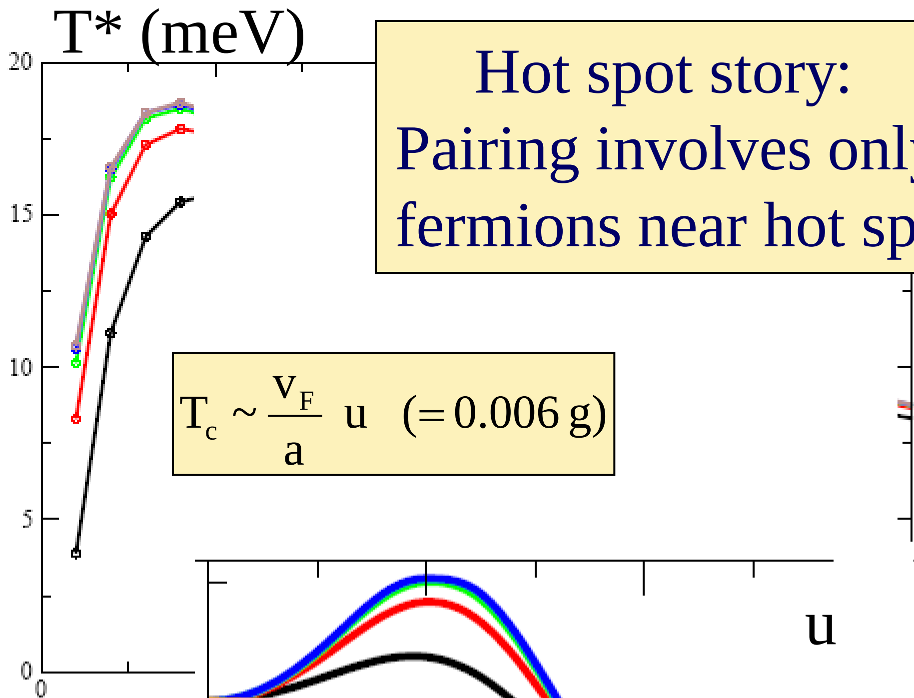
$$T_c = \frac{v_F}{a} F_\xi(u) \quad \left(\Rightarrow g\,\Psi_\xi \text{ when u is small}\right)$$



Abanov, Norman, A.C.

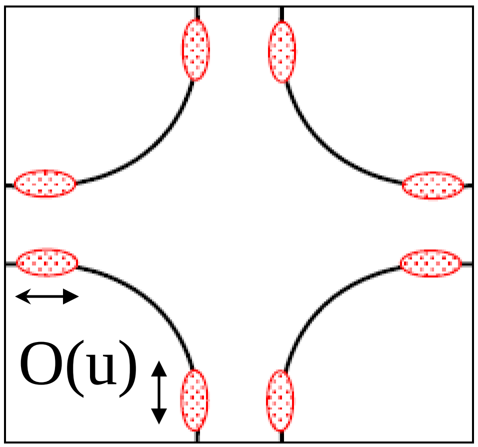
$u < 1,\ \lambda = u\,\xi > 1$

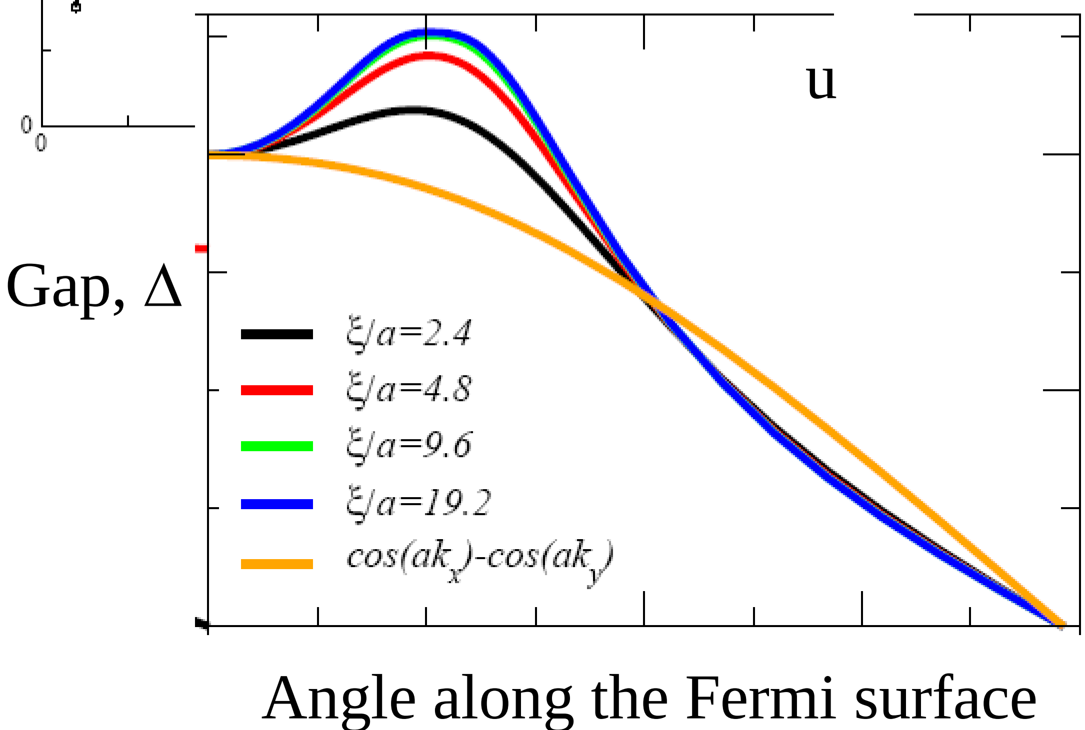
$u \sim g\,a/v_F$



Hot spot story:
Pairing involves only fermions near hot spots

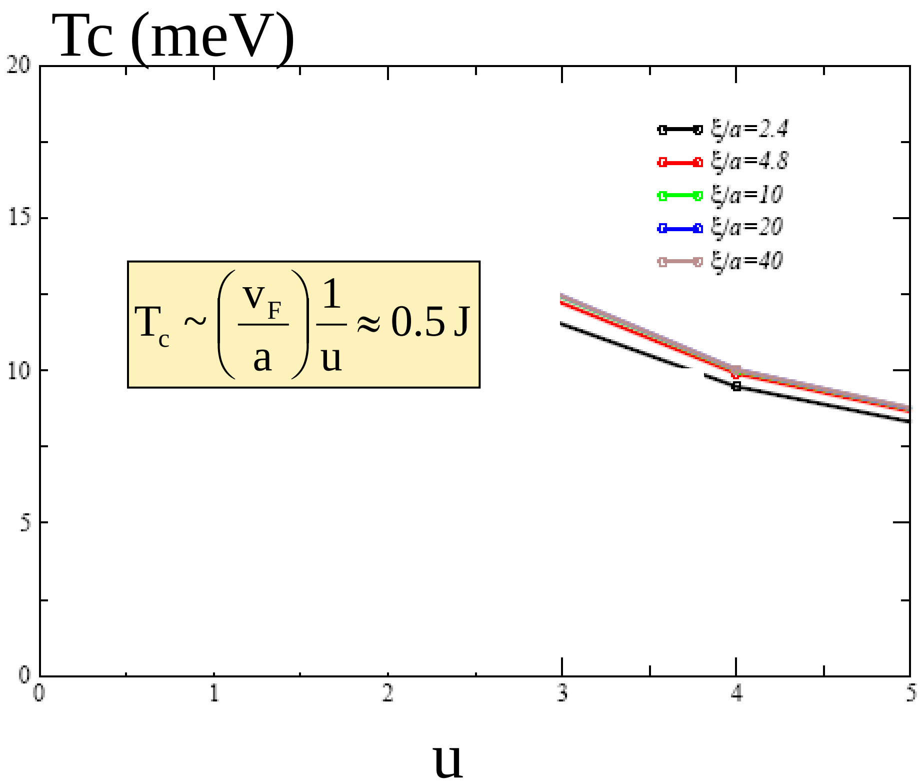
$$T_c \sim \frac{v_F}{a}\,u \quad (= 0.006\,g)$$





The gap is anisortopic, not simply $\cos k_x - \cos k_y$

Strong coupling, u >1



$$T_c \sim \left(\frac{v_F}{a}\right)\frac{1}{u} \approx 0.5\,J$$

$$\chi_q(\Omega) = \frac{1}{(q-\pi)^2 + \xi^{-2} + \frac{16}{3}u\frac{|\Omega|}{v_F\, a}}$$

d-wave attraction

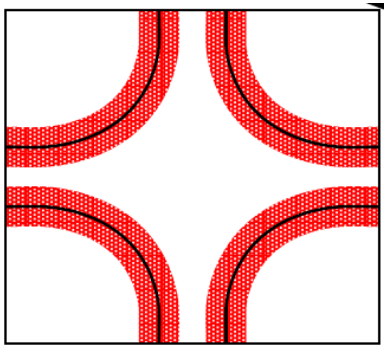
A tendency towards $\chi(\Omega) \sim 1/\Omega$

Balance when $T\,u \sim v_F/a$

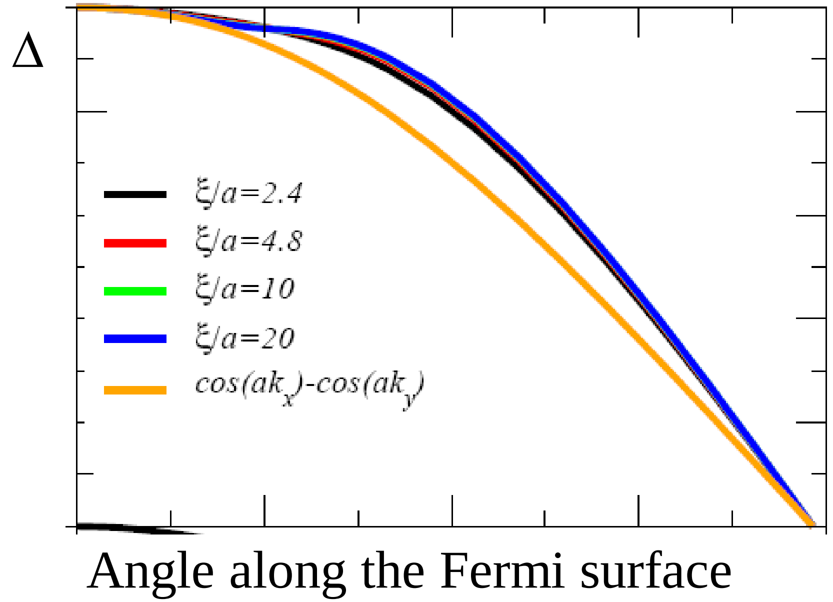
At strong coupling, Tc scales with the magnetic exchange J

Strong coupling, u >1

The whole Fermi surface is involved in the pairing

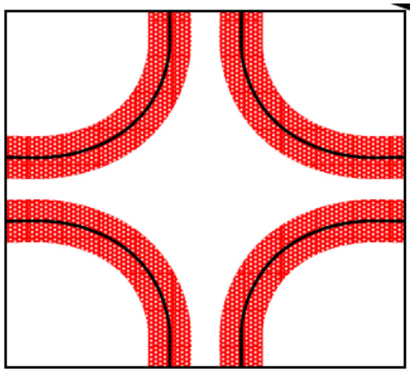


Gap variation along the Fermi surface



Almost $\cos k_x - \cos k_y$ d-wave gap
(as if the pairing is between nearest neighbors)

Strong coupling, u >1



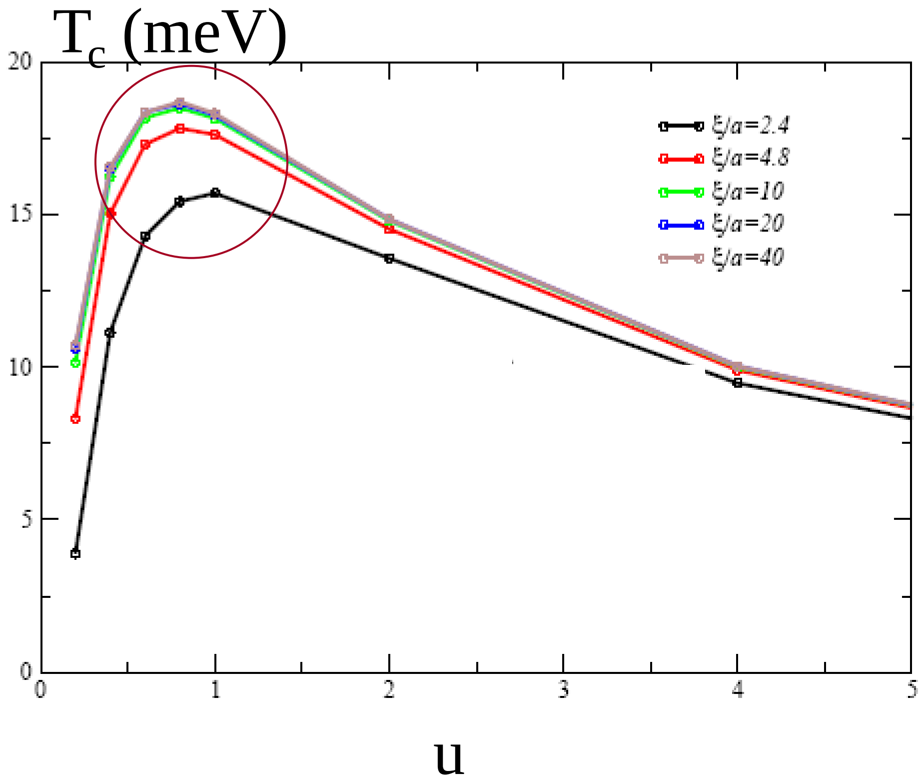
On one hand, the whole Fermi surface is involved in the pairing

On the other, the fact that Tc does not grow with u, restricts relevant fermionic states: $\varepsilon_k \approx v_F (k - k_F) \sim J << v_F/a$

$$|k - k_F| \sim \frac{(3-4)\,J}{v_F} \sim 0.1\frac{\pi}{a} << \frac{\pi}{a}$$

d-wave pairing at strong coupling still involves fermions in the near vicinity of the Fermi surface

Intermediate u = O(1)



$$T_{c,max} \sim 0.02 \frac{v_F}{a}$$

$$T_{c,max} \sim 200 - 250 \text{ K}$$

Universal pairing scale

Robustness of $T_{c,max}$

FLEX (similar, but not identical to our calculations)

Monthoux, Scalapino
Monthoux, Pines,
Eremin, Manske,
Bennemann, Schmalian,
Dahm, Tewordt ….

$$T_c \sim (0.01 - 0.015)\ \frac{v_F}{a} \sim 100 - 150\ \mathrm{K}\ \ \text{for } u \sim 0.25$$

CDA, cluster DMFT

Majer, Jarrell, …
Haule, Kotliar, Capone …
Tremblay, Senechal, ….

$$T_c \sim 0.01 \frac{v_F}{a}\ \ \text{for } u \sim 0.25,\ \ T^* \sim 0.015\ \frac{v_F}{a}\ \ \text{for } u = 0.75$$

FLEX with experimental inputs

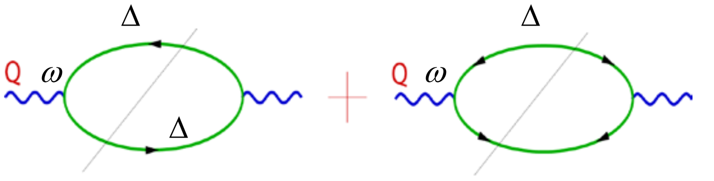
Scalapino, Dahn, Hinkov,
Hanke, Keimer, Fink, Borisenko,
Kordyuk, Zabolotny, Buechner

$$T_c \sim 170\ \mathrm{K}$$

The superconducting phase

Spin dynamics changes because of d-wave pairing -- the resonance peak appears

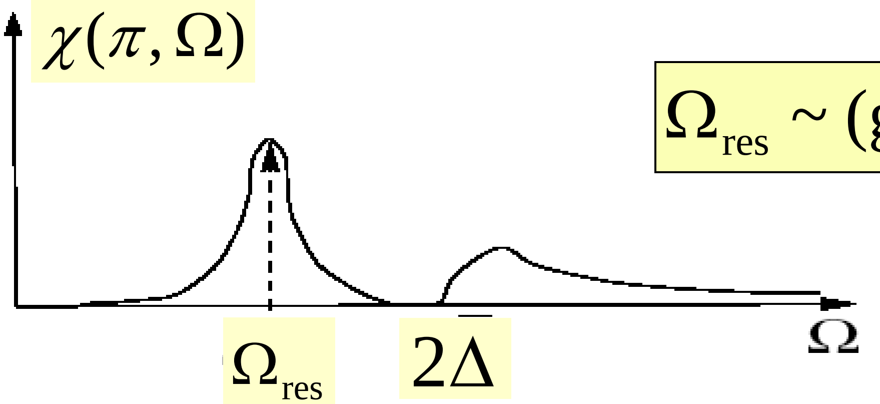
- no low-energy decay below 2Δ due to fermionic gap



- residual interaction is "attractive" for d-wave pairing

Collective spin fluctuation mode at the energy well below 2Δ

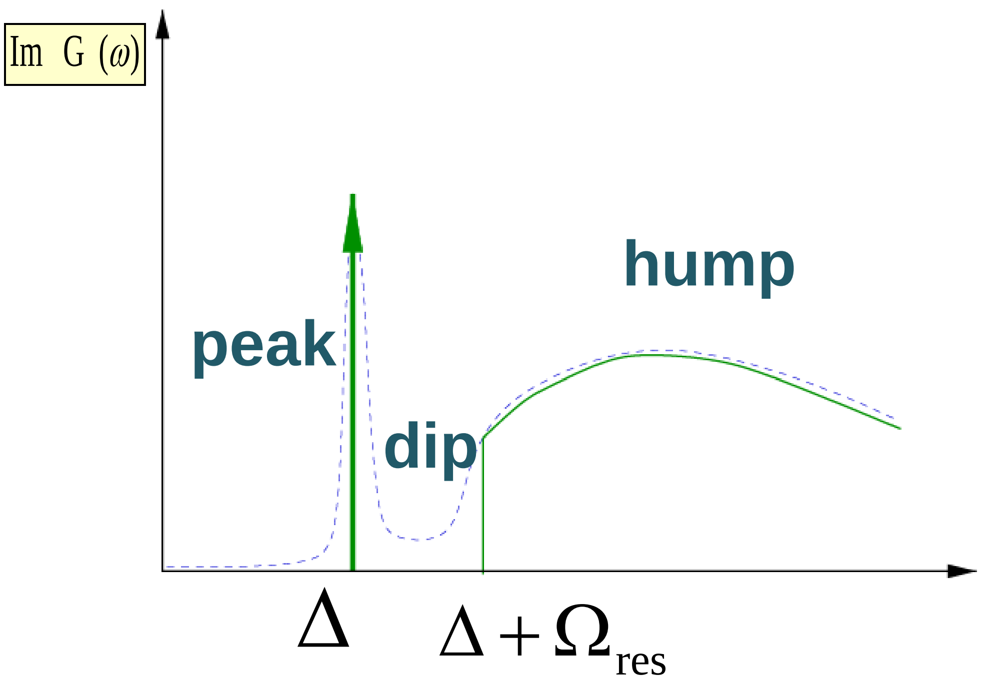
$$\chi(\Omega) \sim \frac{1}{\Omega^2 - \Omega^2_{res}}$$

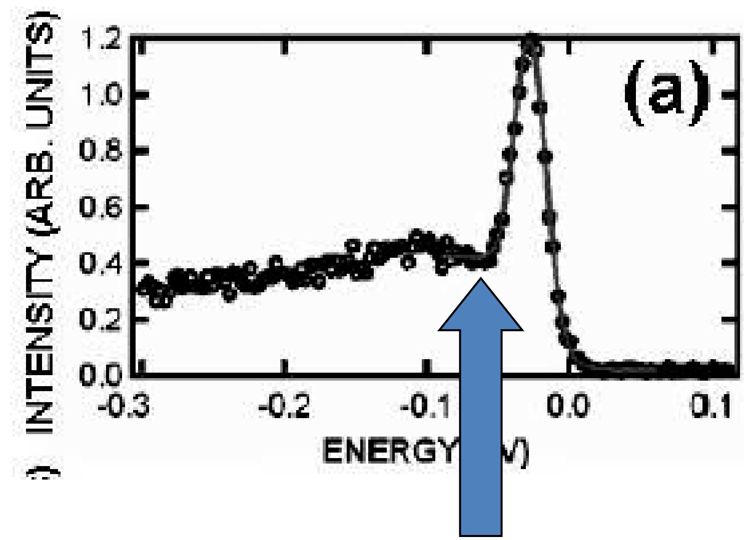


$$\Omega_{res} \sim (g\,\omega_{sf})^{1/2} \sim \xi^{-1}$$

By itself, the resonance is NOT a fingerprint of spin-mediated pairing, nor it is a glue to a superconductivity

A fingerprint is the observation how the resonance peak affects the electronic behavior, if the spin-fermion interaction is the dominant one



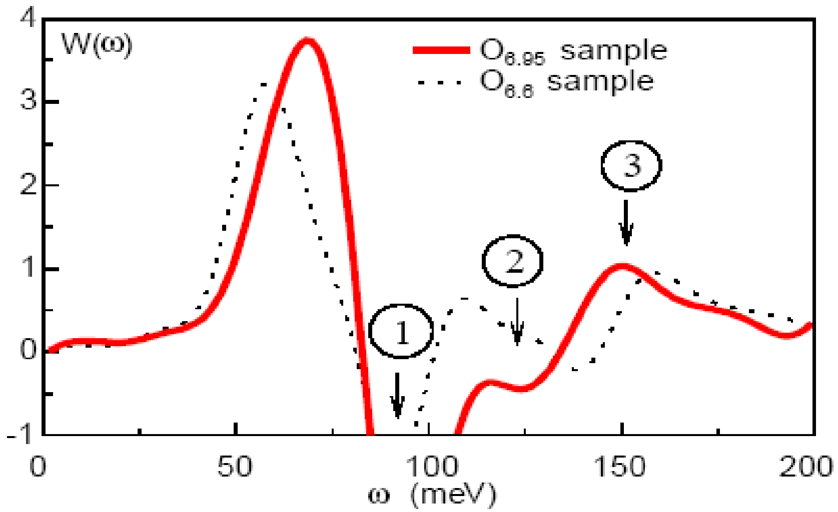


$$\Omega_{mode} \sim 38 - 40 \ \text{meV}$$

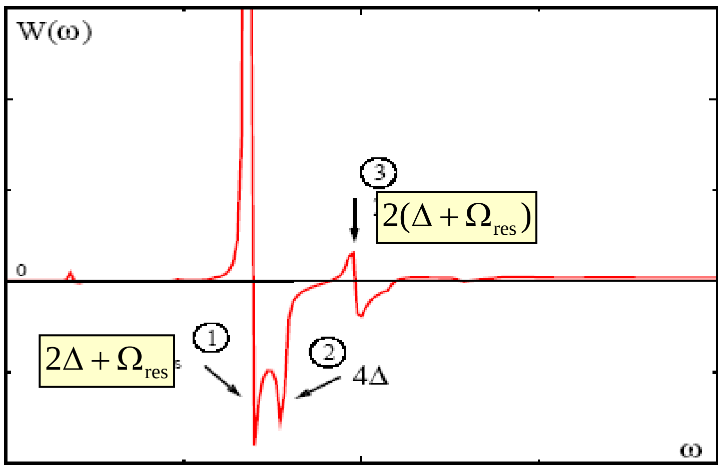
The resonance mode also affects optical conductivity

$$W(\omega) = \frac{d^2}{d\omega^2}\left[\omega \,\mathrm{Re}\,\frac{1}{\sigma(\omega)}\right]$$

YBCO6.95



Theory



$$\Delta \approx 30\text{ meV},\quad \Omega_{res} \approx 40\text{ meV}$$

Basov et al,
Timusk et al,
J. Tu et al…..

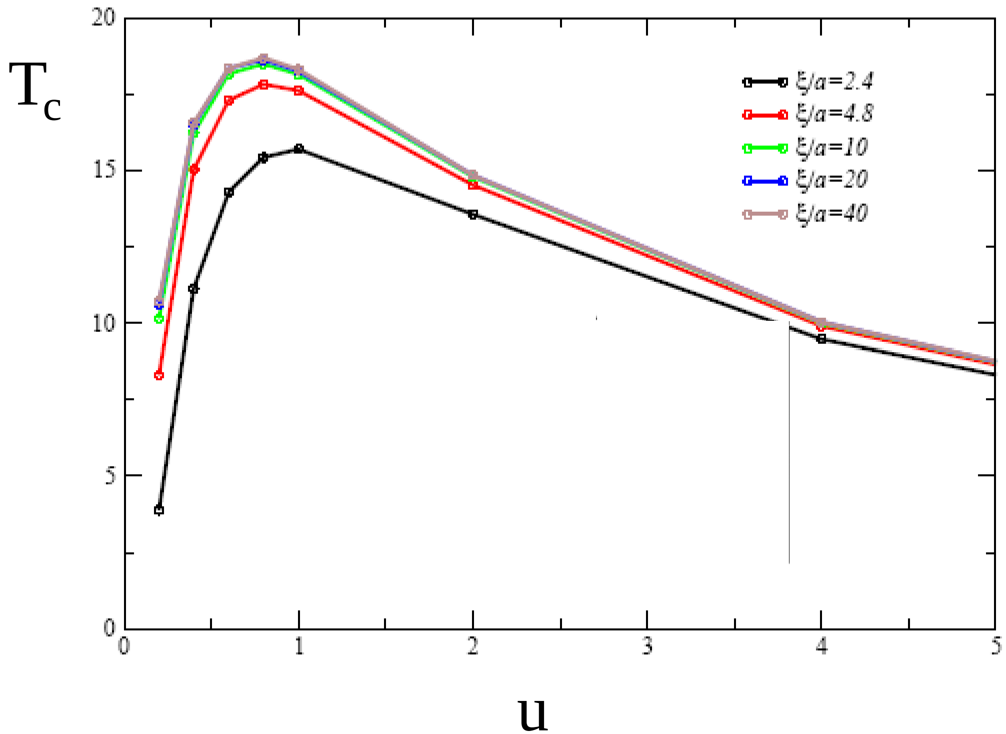
Abanov et al
Carbotte et al

Conclusions

Spin-fermion model: the minimal model which describes the interaction fermions, mediated by spin collective degrees of freedom

Some phenomenology is unavoidable (or RPA)

Once we selected the model, how to get $\Sigma(\omega)$ and the pairing are legitimate theoretical issues (and not only for the cuprates).



Universal pairing scale

$$T_{c,\,max} \sim 0.02\,\frac{v_F}{a}$$

The gap

$$\Delta(k) \approx \Delta\,(\cos k_x - \cos k_y)$$

Low-energy collective mode

$$\Omega_{res} \sim \Delta/\lambda < \Delta$$

THANK YOU