JAN 2026

Bilayer Excitons and Quantum Condensates

Maglab winter school

JIA LI
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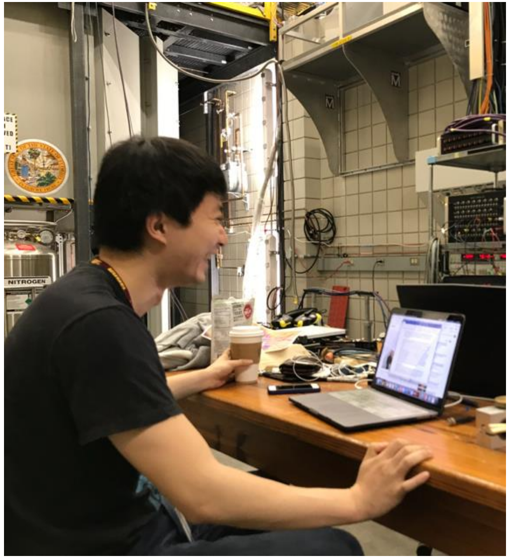
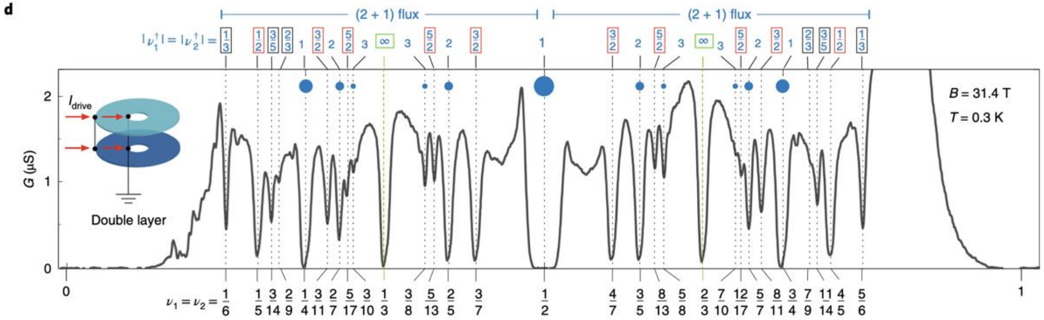
WHAT STARTS HERE CHANGES THE WORLD





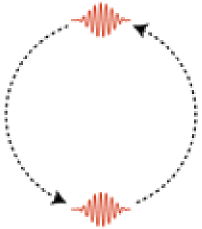
Outline

Introduction:
Exciton superfluid

2-component FQHE
and fractional excitons

Exciton solid

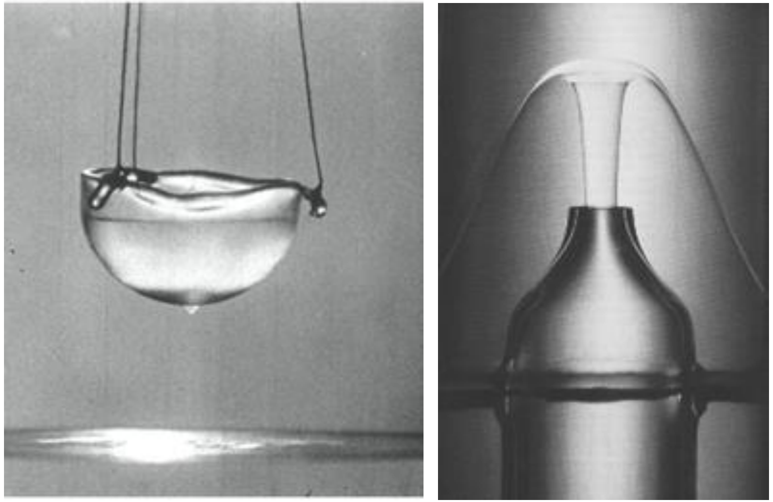
Bosonic condensate



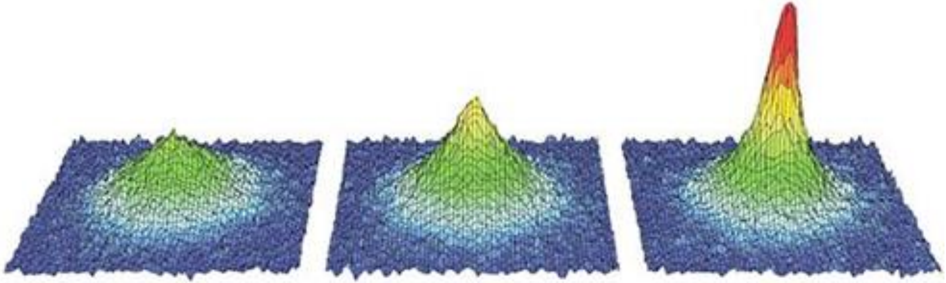
Bosons: $\phi = 2\pi$

→ Bose-Einstein statistics

→ Bose-Einstein condensate

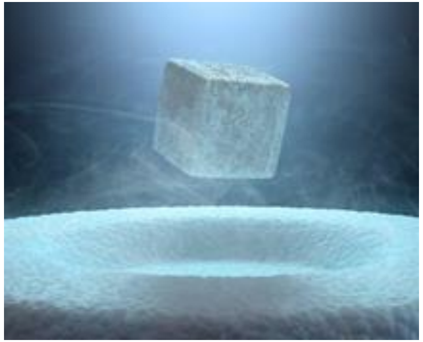


Superfluid in Helium-4 (1937)



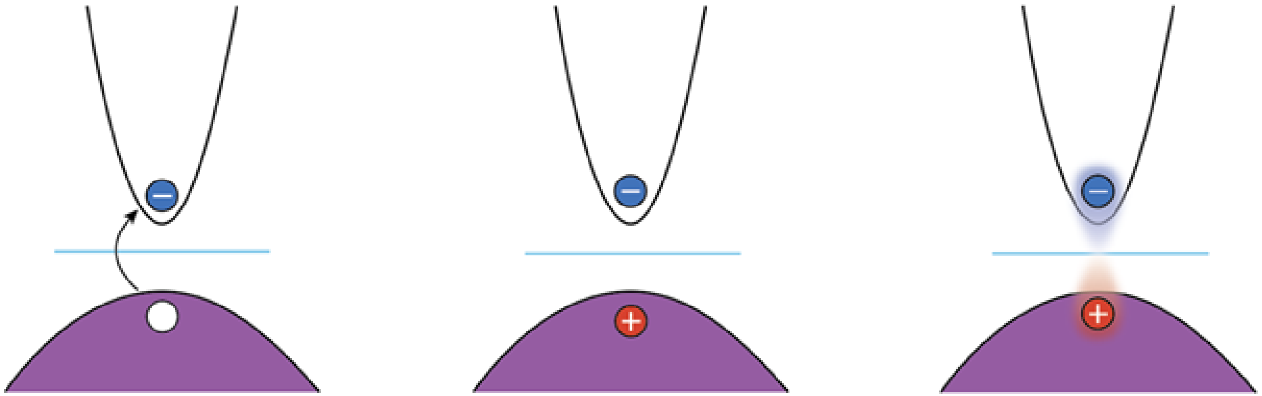
BEC condensate in cold atomic gas (1995)

Engineered Bosonic condensate



Superconductivity

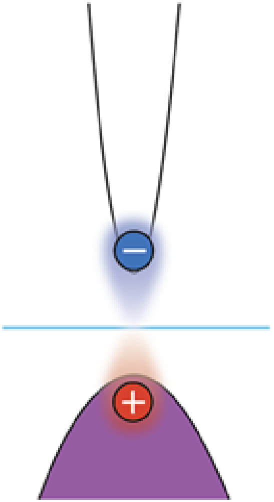
Feasibility of superfluidity of paired spatially separated electrons and holes; a new superconductivity mechanism,
Lozovik, Y.E. and Yudson, V.I., *JETP Lett* (1975)

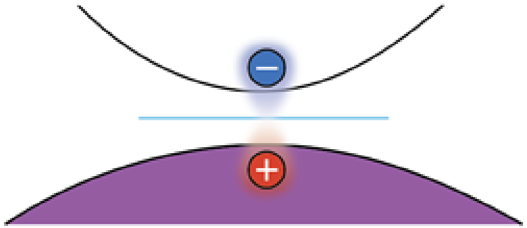


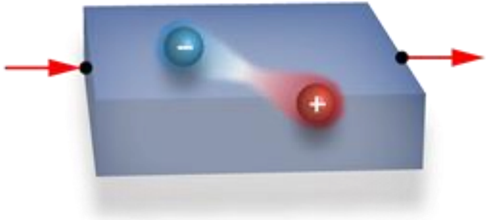
Excitons

BEC

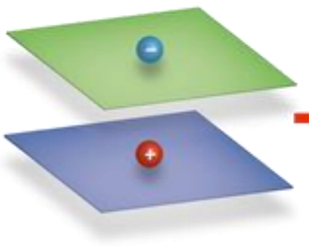
$$T_c = \left(\frac{n}{\zeta(3/2)}\right)^{2/3} \frac{2\pi\hbar^2}{m k_B} \approx 3.3125 \frac{\hbar^2 n^{2/3}}{m k_B}$$

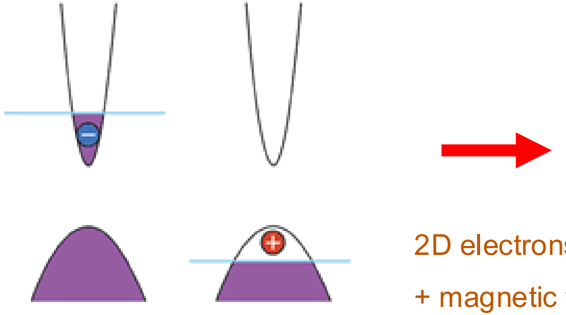






Quantum Hall bilayer

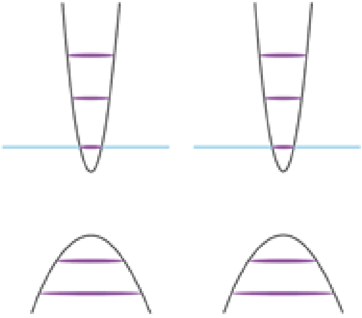




2D electrons

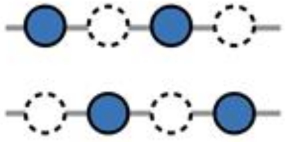
+ magnetic field

 Landau level



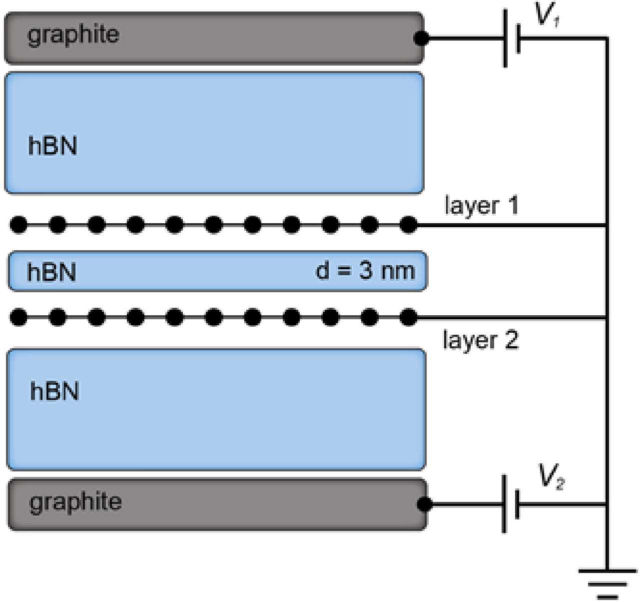
$\nu_1 = 1/2$

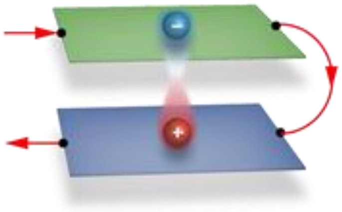
$\nu_2 = 1/2$



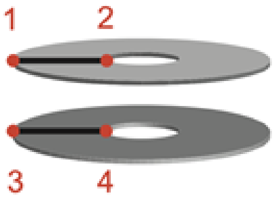
$$\Psi_{(lmn)} = \prod (z_i - z_j)^l \prod (w_i - w_j)^m \prod (z_i - w_j)^n \exp\left[-\tfrac{1}{4} \left(\sum |z_i|^2 + \sum |w_i|^2 \right) \right]$$

Quantum Hall graphene bilayer





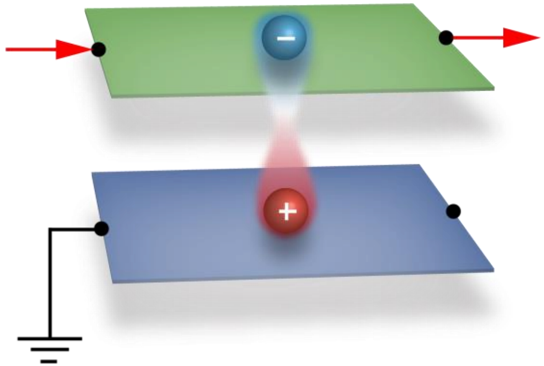
Double hall bar

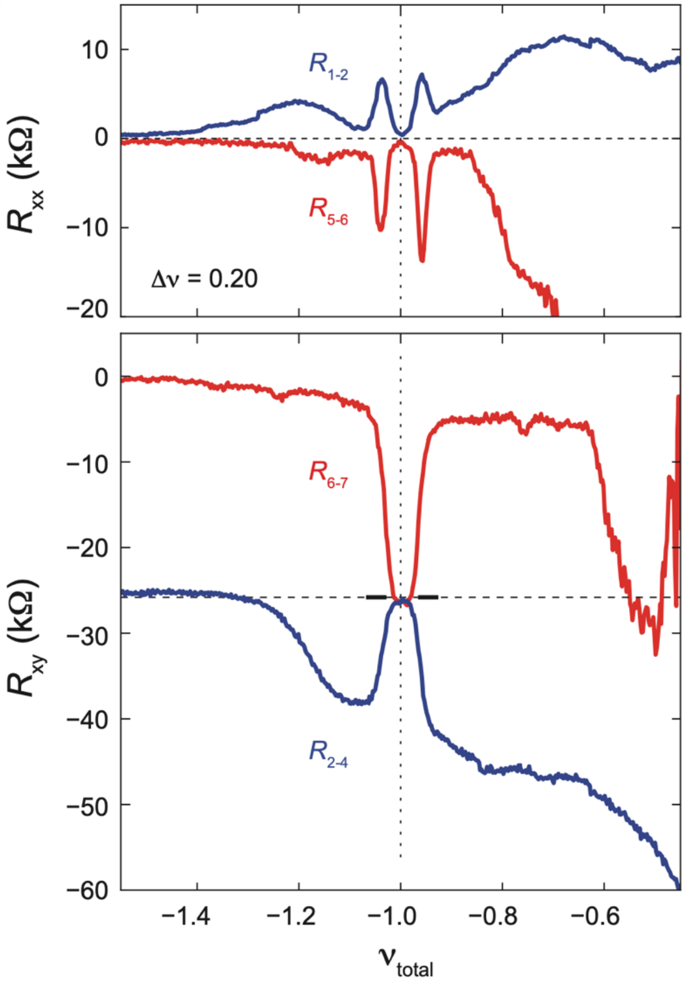


Double Corbino

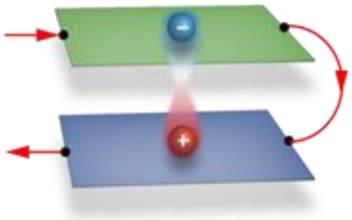
X. Liu et.al. *Science* **375** (6577), 205 (2022)

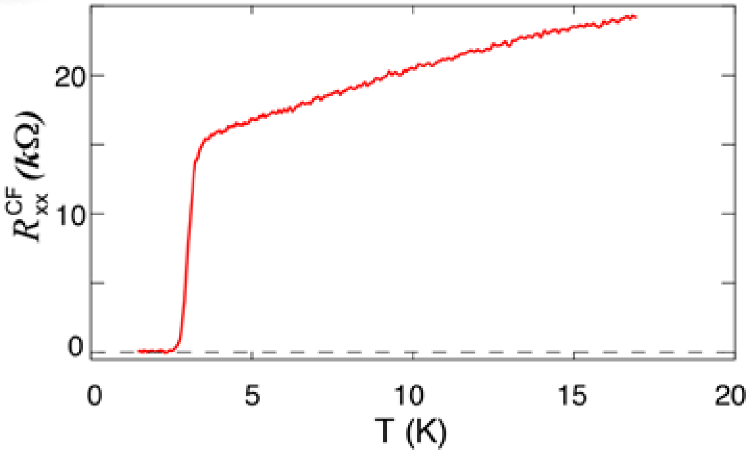
Transport signatures of exciton condensate: counterflow drag (hall bar)

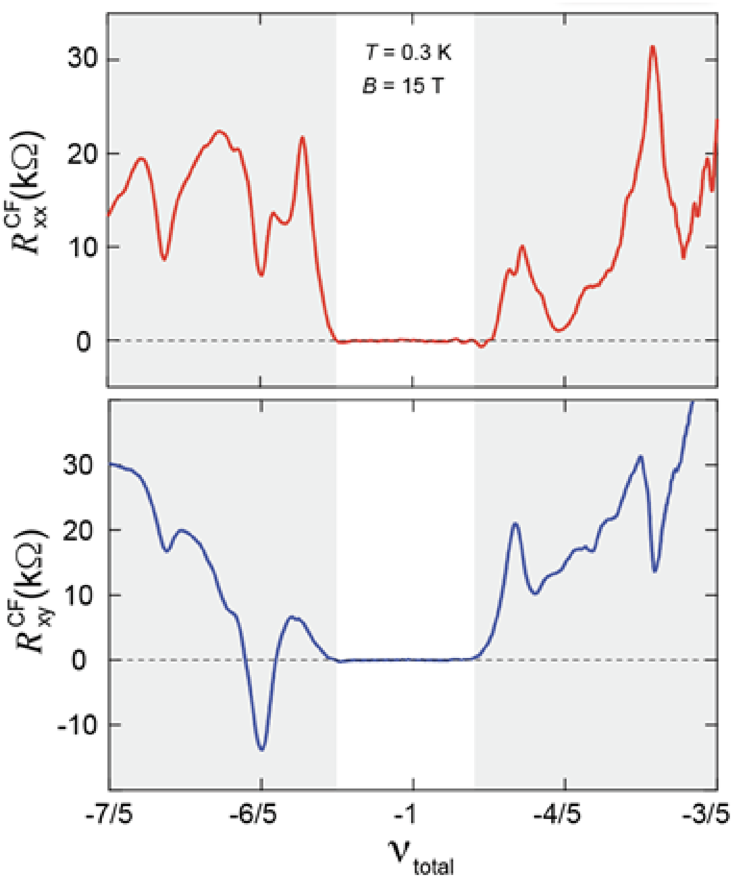




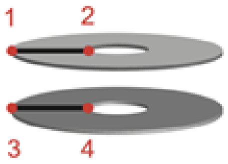
Transport signatures of exciton condensate: counterflow (Hall bar)







Transport signatures of exciton condensate: Corbino



LETTER

doi:10.1038/nature11302

Exciton condensation and perfect Coulomb drag

D. Nandi[1], A. D. K. Finck[1], J. P. Eisenstein[1], L. N. Pfeiffer[2] & K. W. West[2]

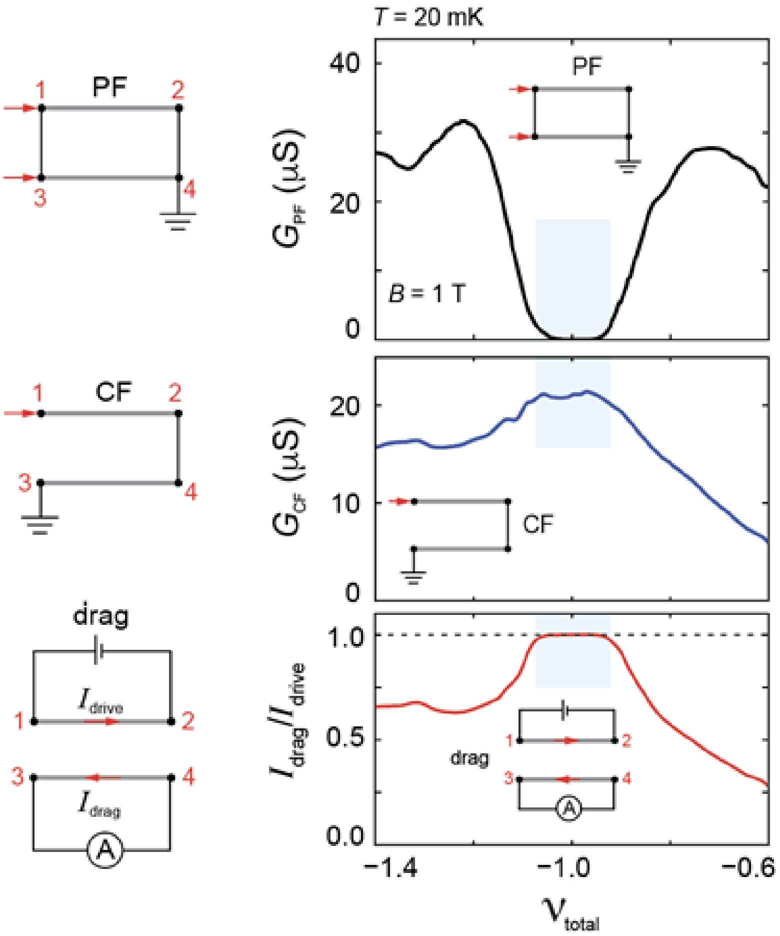
Article

Excitons in the fractional quantum Hall effect

https://doi.org/10.1038/s41586-024-08274-3

Received: 5 August 2024

Naiyuan J. Zhang[1,6], Ron Q. Nguyen[1,6], Navketan Batra[1,2,6], Xiaoxue Liu[1,5], Kenji Watanabe[3], Takashi Taniguchi[4], D. E. Feldman[1,2] & J. I. A. Li[1,2,3]



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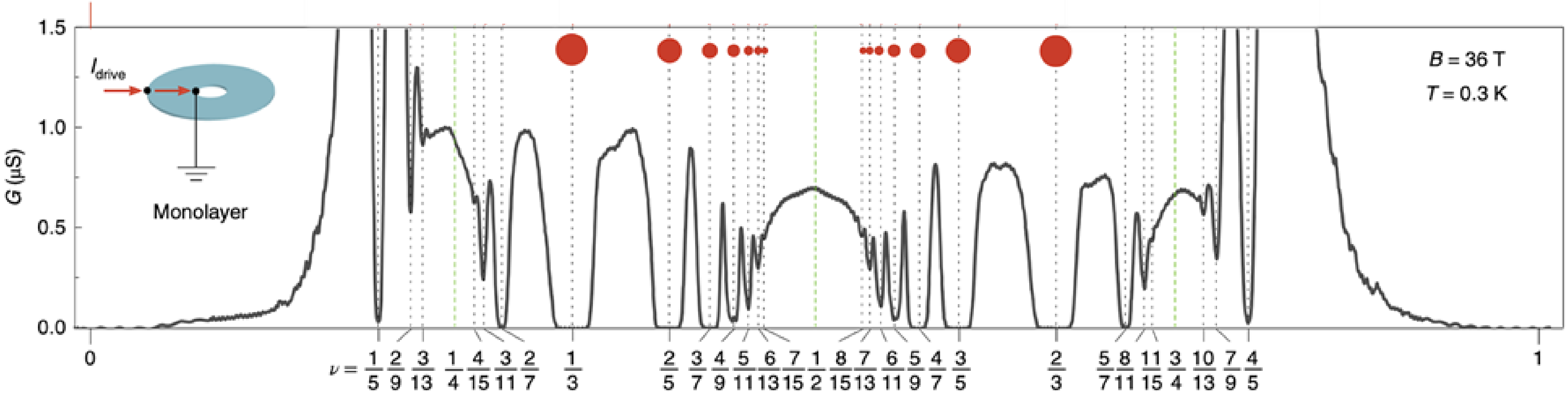
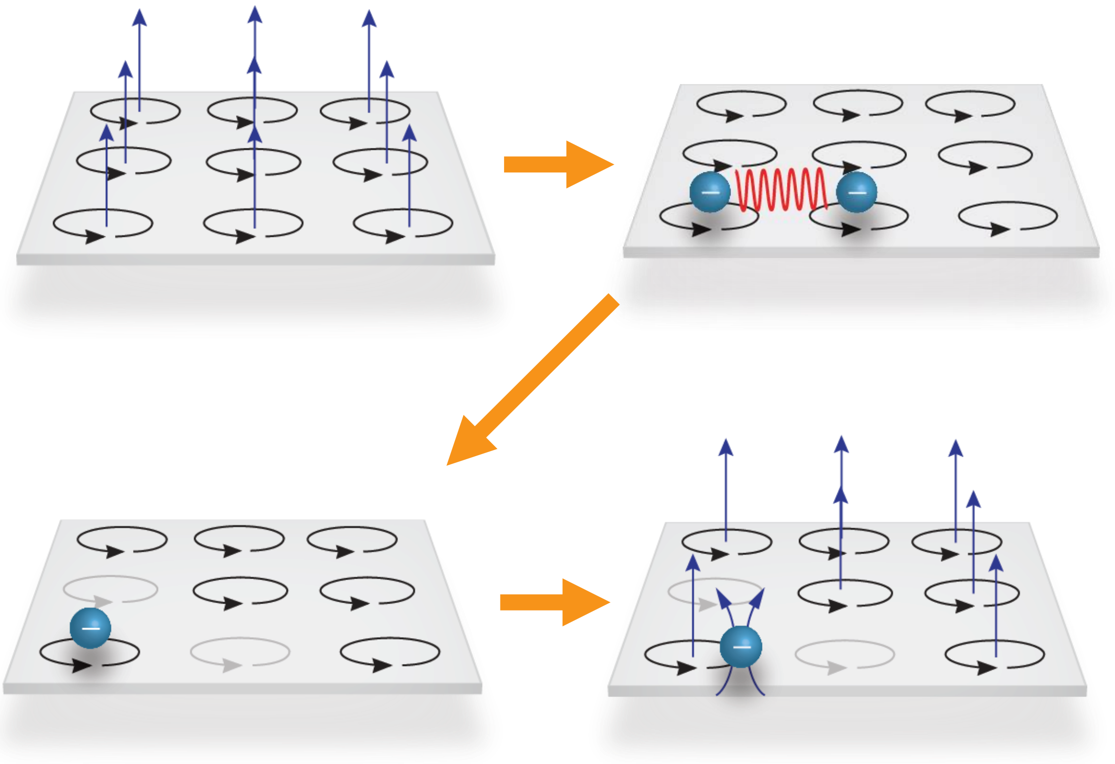
FQHE in monolayer graphene



Y. Zeng*, J.I.A. Li* et.al. PRL (2019)



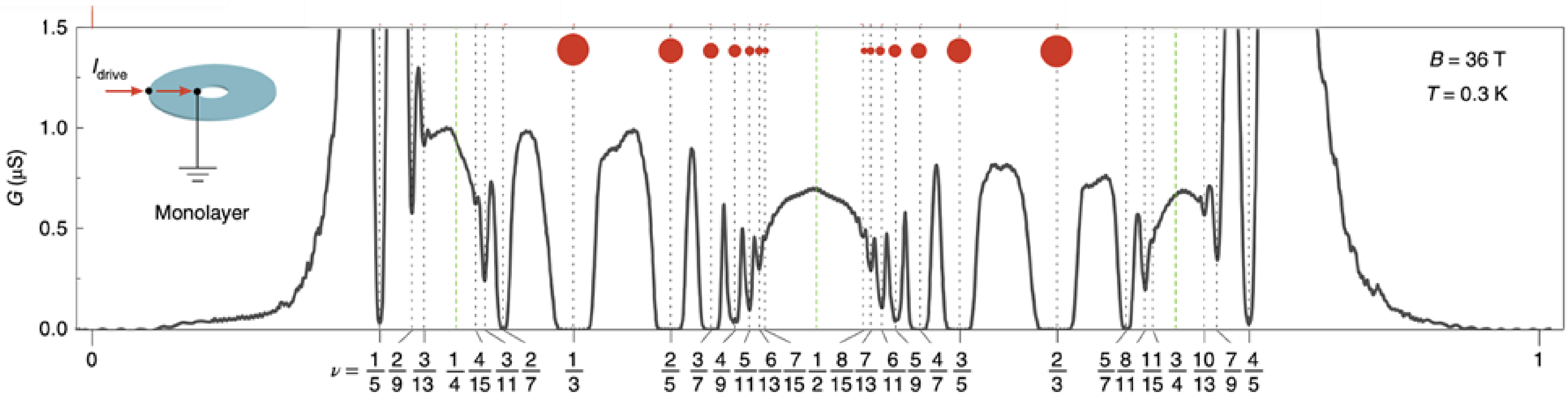
$$\nu^* = \frac{\nu}{1-\phi\nu}$$

ϕ number of attached flux

FQHE in monolayer graphene

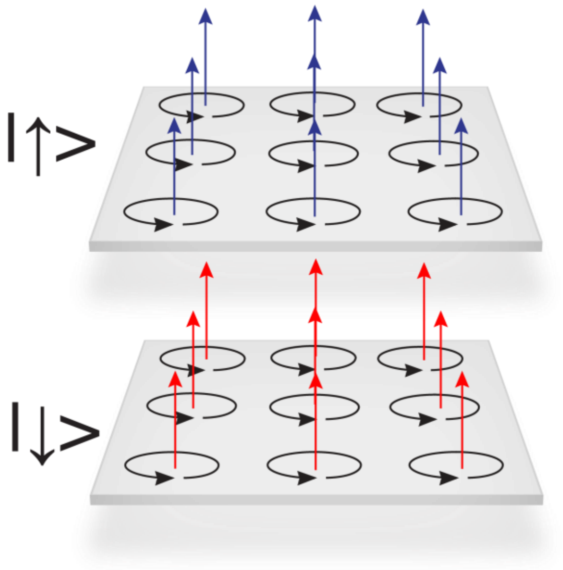
$$\nu^* = \frac{\nu}{1-\phi\nu}$$

ϕ number of attached flux



FQHE can be viewed as effective IQHE for CFs!

Layer degrees of freedom

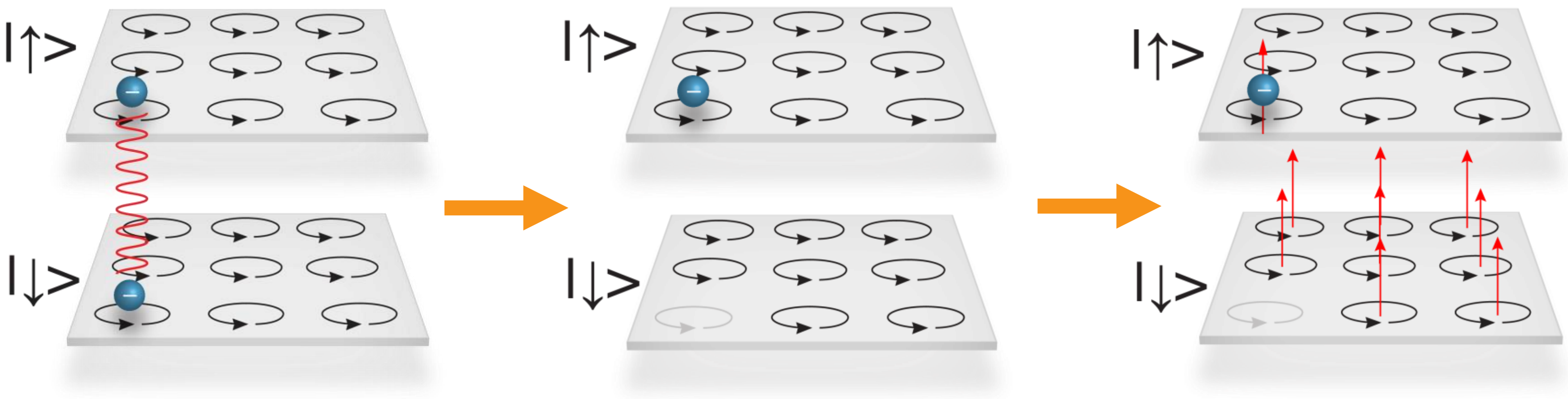


$\nu_\uparrow$: filling fraction of layer ↑
$\nu_\downarrow$: filling fraction of layer ↓

$S_z \sim \nu_\uparrow - \nu_\downarrow = \Delta\nu$

Coupling between two spin species could lead to a new type of FQHE state

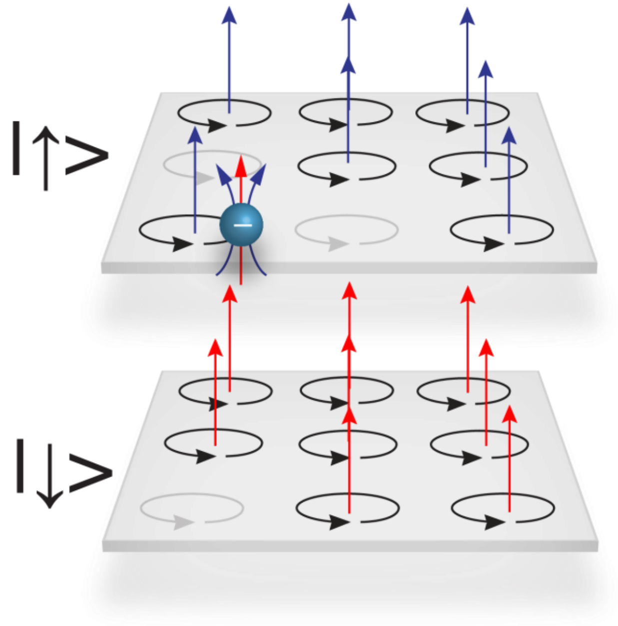
CF model with two layer species



Correlation between different layer species could be modeled by flux attachment as well.

Inter-species flux attachment affects effective B-field of the opposite spin species

B. I. Halperin, Helv. Phys. Acta 56, 75–102 (1983)
J. K. Jain, Composite fermions (Cambridge University Press, 2003)



$$\nu_{\uparrow}^{eff} = \frac{\nu_{\uparrow}}{1-(a\nu_{\uparrow}+b\nu_{\downarrow})}$$

a: number of intra-species flux
b: number of inter-species flux

For a=2, b=1

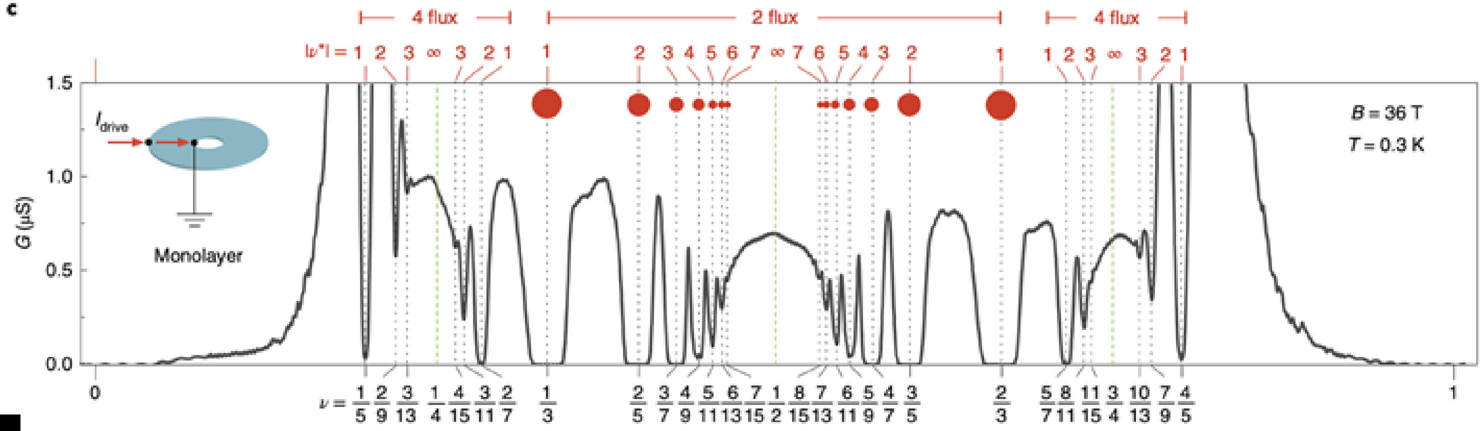
$$\nu_{\uparrow}^{eff} = \frac{\nu_{\uparrow}}{1-(2\nu_{\uparrow}+\nu_{\downarrow})}$$

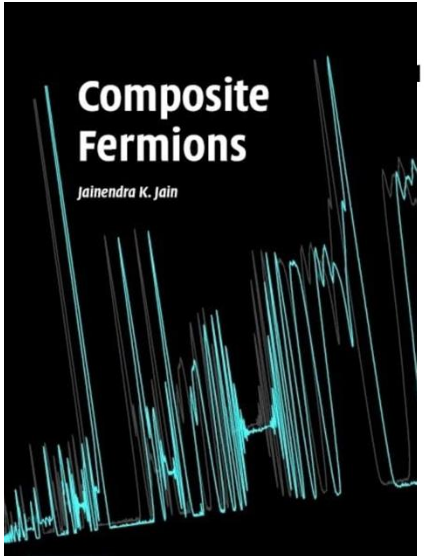
When $\nu_{\uparrow} = \nu_{\downarrow} = 1/4$, $\nu_{total} = 1/2$

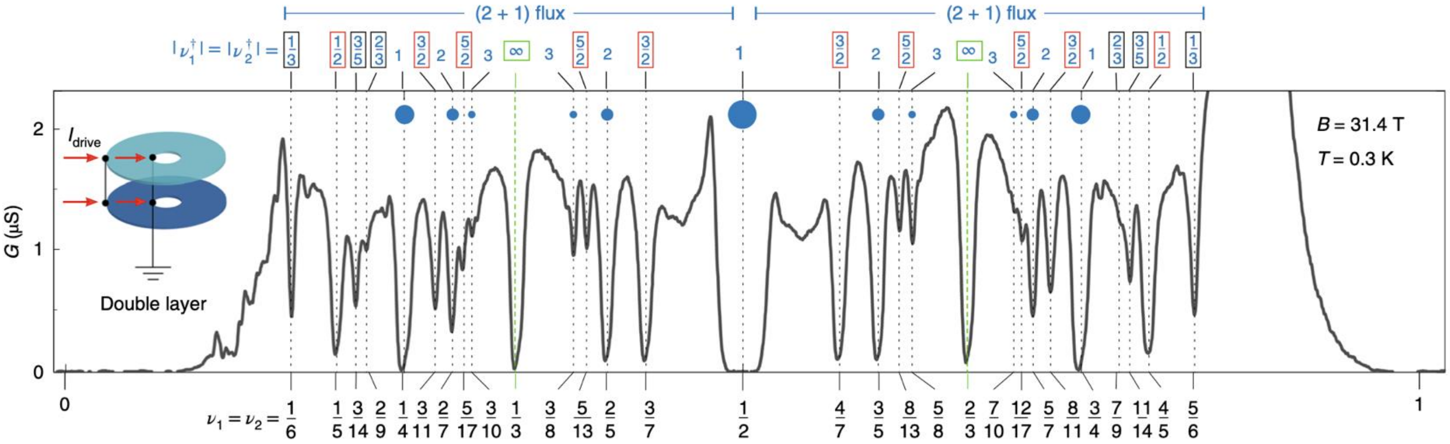
$\nu^{eff}_1 = \nu^{eff}_2 = 1$

B. I. Halperin, Helv. Phys. Acta 56, 75–102 (1983)

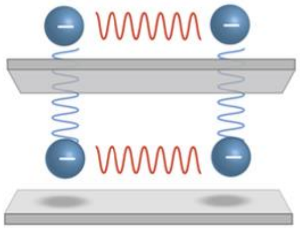
J. K. Jain, Composite fermions (Cambridge University Press, 2003)

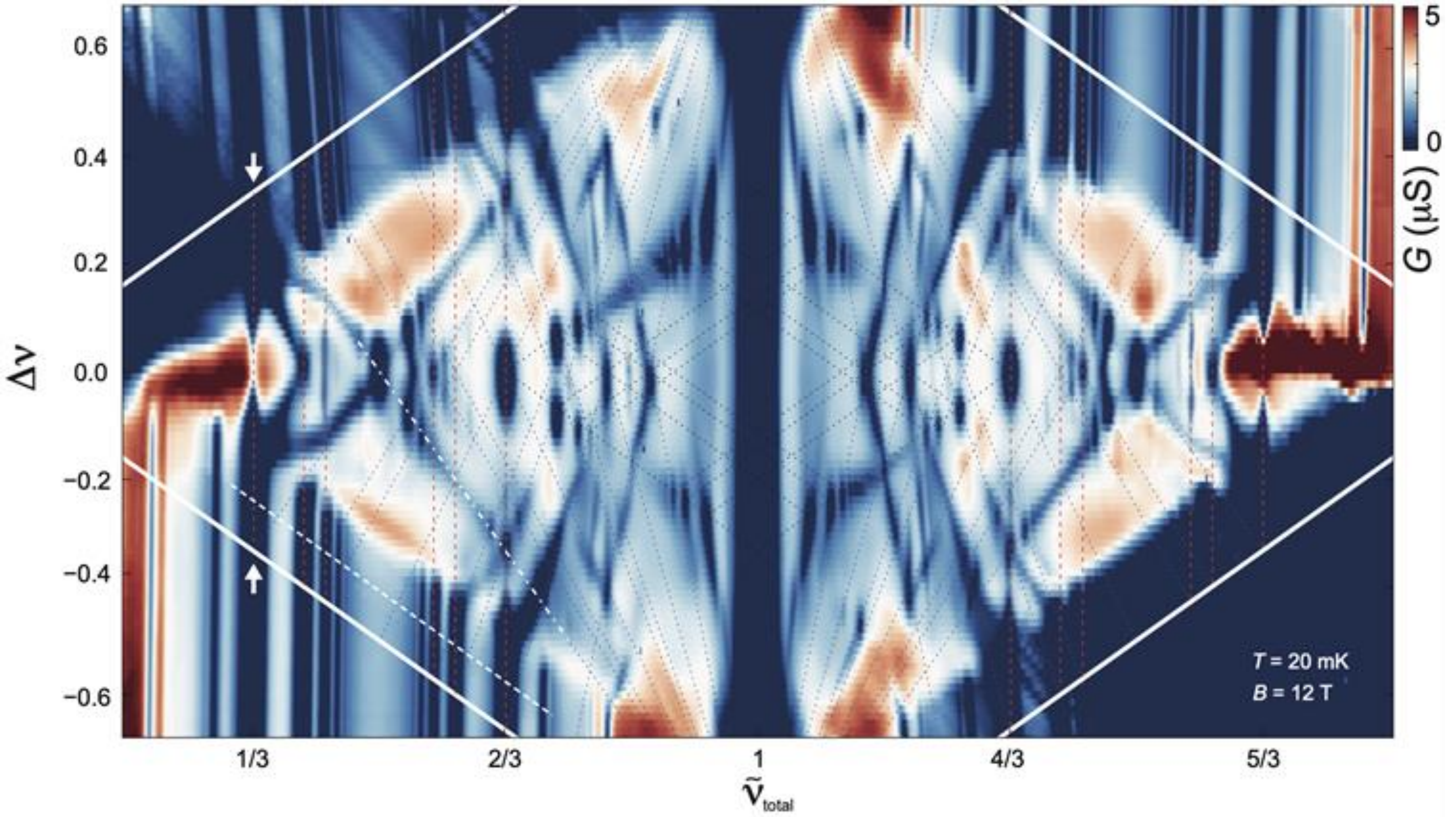

c
4 flux
2 flux
4 flux
B = 36 T
T = 0.3 K
G (μS)
1.5
1.0
0.5
0.0
0
1
I_{drive}
Monolayer


Composite Fermions
Jainendra K. Jain


(2 + 1) flux
(2 + 1) flux
B = 31.4 T
T = 0.3 K
G (μS)
2
1
0
0
1
I_{drive}
Double layer

FQHE in quantum Hall bilayer

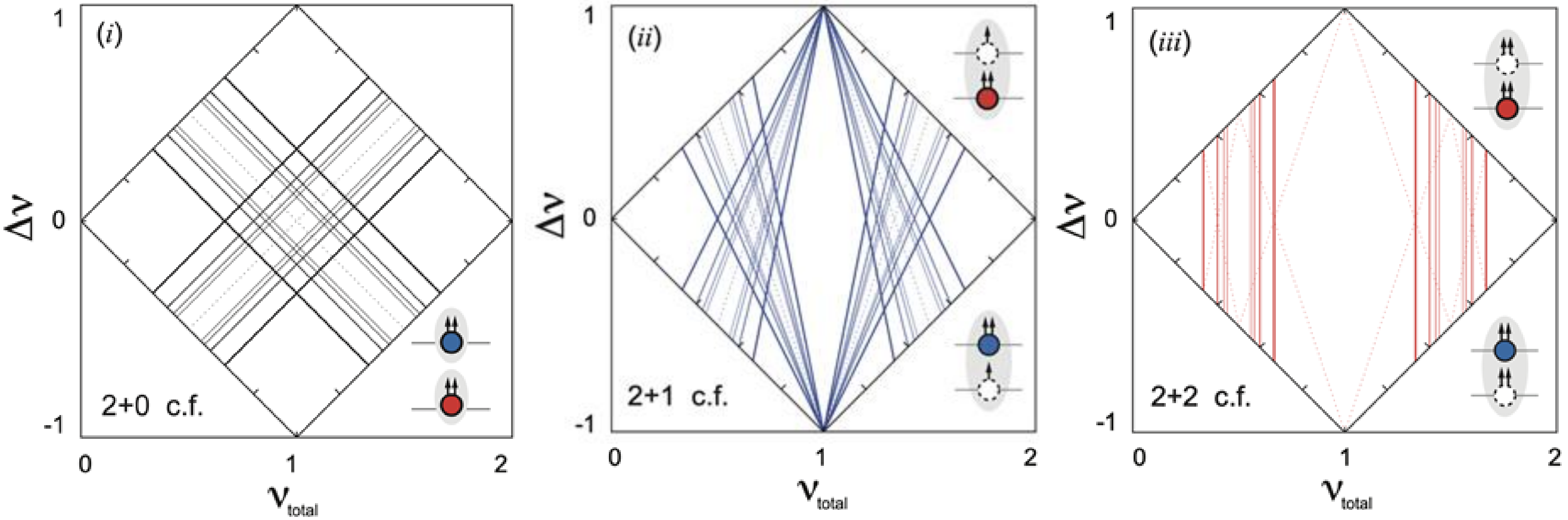




2-component FQHE

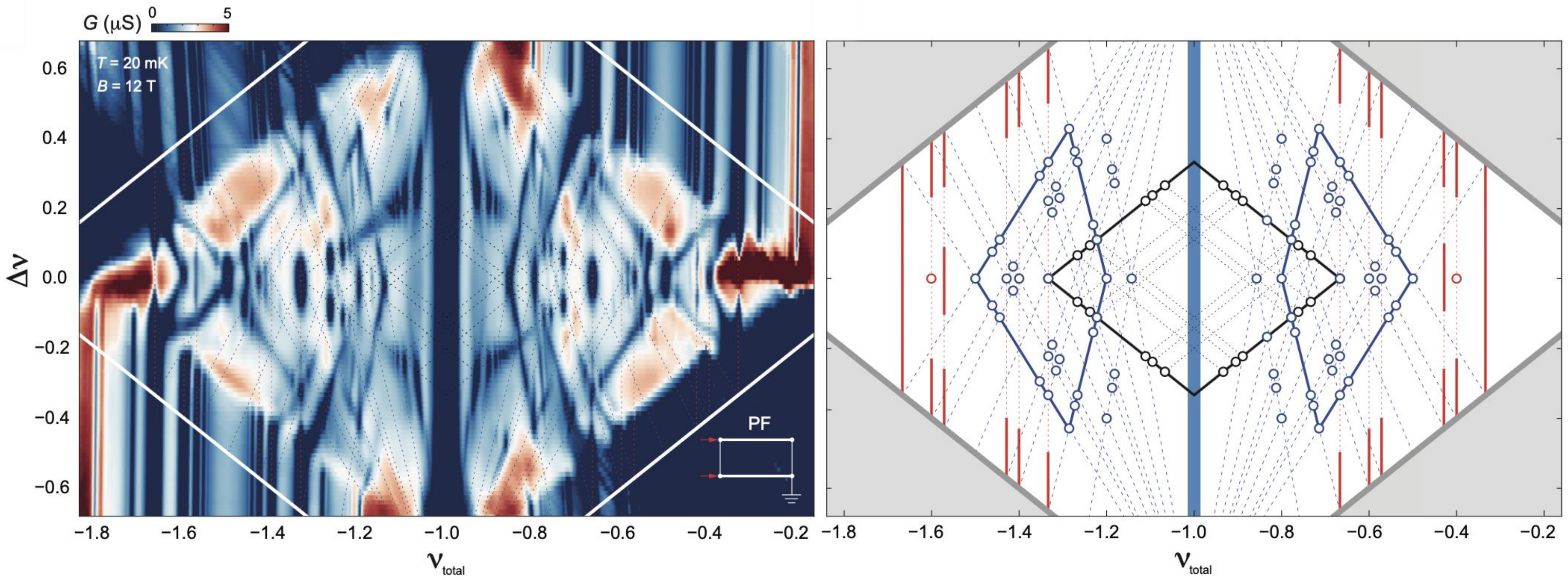
$$ {}^{2}_{1}\nu_1^* = \frac{\nu_1}{1 - 2\nu_1 - m\nu_2} $$

$$ {}^{2}_{1}\nu_2^* = \frac{\nu_2}{1 - 2\nu_2 - m\nu_1} $$

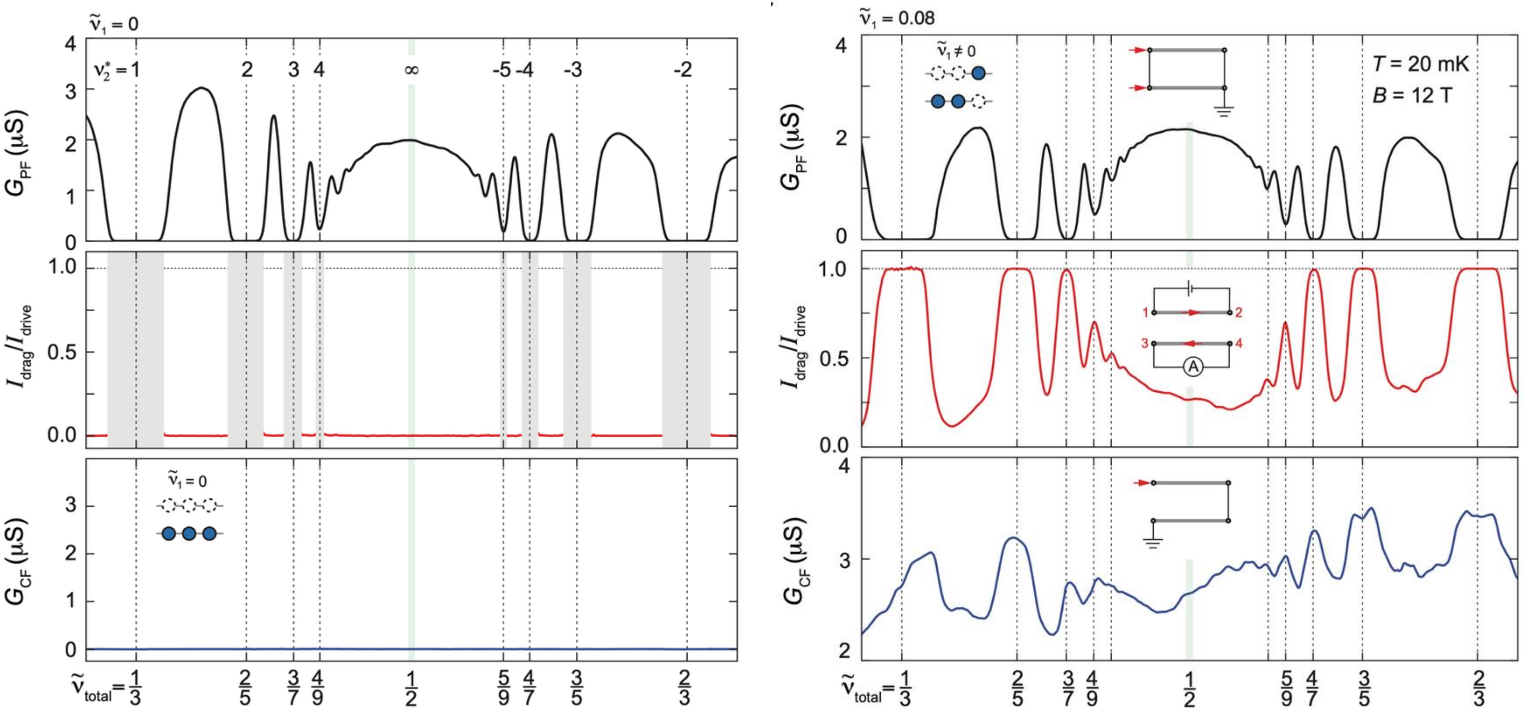


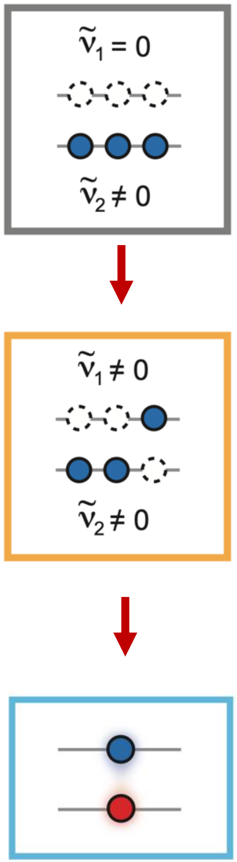
Increasing interlayer correlation

FQHE in quantum Hall bilayer

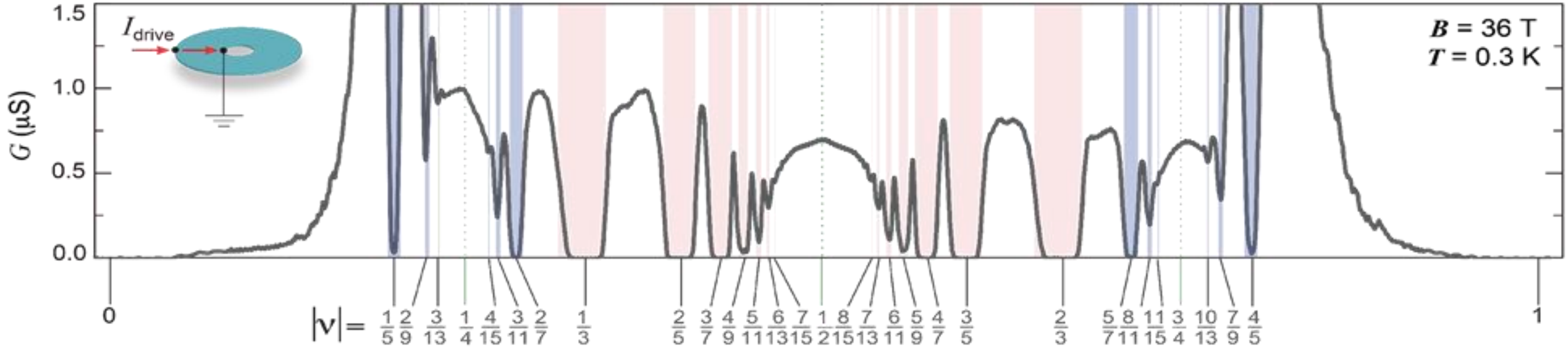


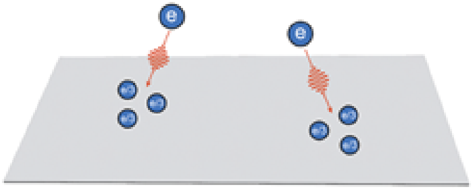
$$\Psi_{(lmn)} = \prod (z_i - z_j)^l \prod (w_i - w_j)^m \prod (z_i - w_j)^n \exp\left[-\frac{1}{4} \left(\sum |z_i|^2 + \sum |w_i|^2 \right) \right]$$

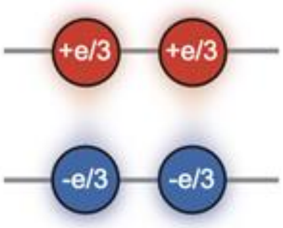




Exciton pairing between Laughlin states







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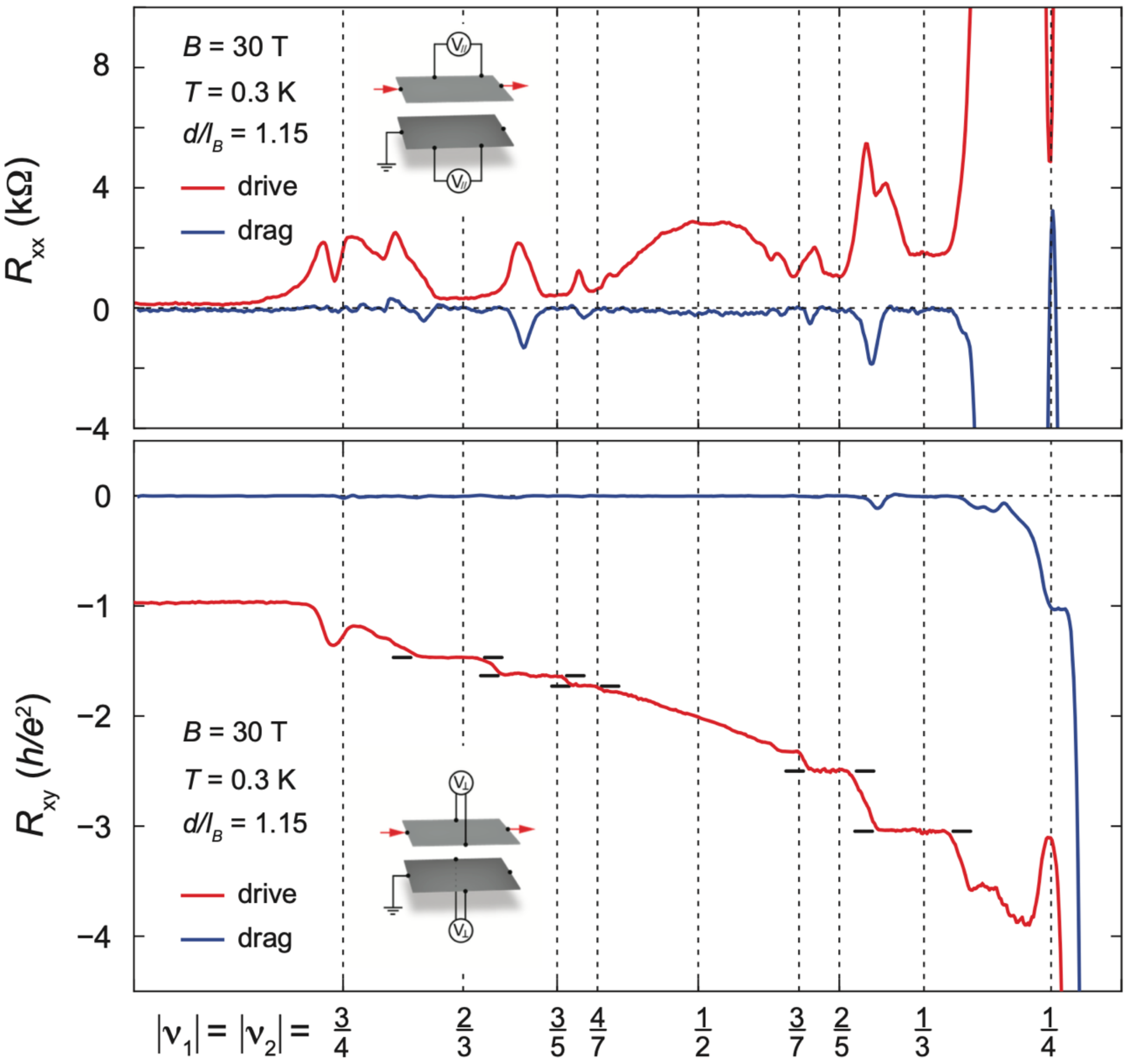
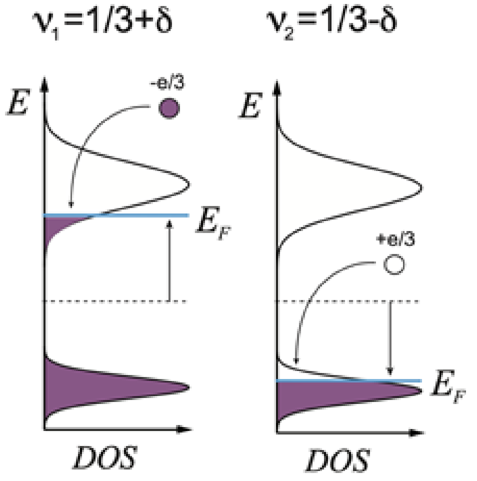
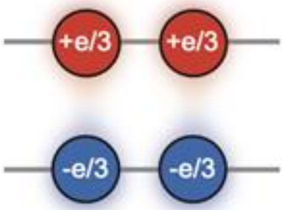
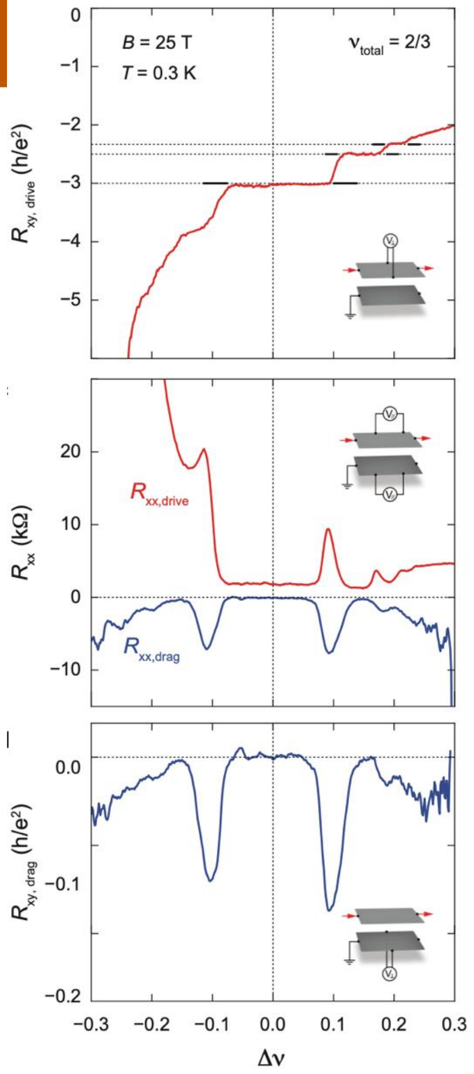


Doping excitons at 1/3+δ/1/3-δ







Outline

Introduction:
Exciton superfluid

2-component FQHE
and fractional excitons

Exciton solid

PHYSICAL REVIEW B **84**, 075130 (2011)

Crystallization of an exciton superfluid

J. Böning, A. Filinov,* and M. Bonitz†
Institut für Theoretische Physik und Astrophysik, Christian-Albrechts-Universität, Leibnizstrasse 15, D-24098 Kiel, Germany
(Received 15 April 2011; revised manuscript received 20 July 2011; published 9 August 2011)

PHYSICAL REVIEW LETTERS **130,** 057001 (2023)

Chester Supersolid of Spatially Indirect Excitons in Double-Layer Semiconductor Heterostructures

Sara Conti,[1,*] Andrea Perali,[2] Alexander R. Hamilton,[3] Milorad V. Milošević,[1,4] François M. Peeters,[1,5] and David Neilson[1,3]
[1]*Department of Physics, University of Antwerp, 2020 Antwerp, Belgium*
[2]*Supernano Laboratory, School of Pharmacy, University of Camerino, 62032 Camerino (MC), Italy*
[3]*ARC Centre of Excellence for Future Low Energy Electronics Technologies, School of Physics, University of New South Wales, Sydney 2052, Australia*
[4]*NANOlab Center of Excellence, University of Antwerp, 2020 Antwerp, Belgium*
[5]*Universitade Federal do Ceará, Departamento de Física, 60455-760 Fortaleza, Brazil*

PHYSICAL REVIEW B **108**, 235158 (2023)

Excitonic phases in a spatially separated electron-hole ladder model

DinhDuy Vu and Sankar Das Sarma

PHYSICAL REVIEW B **110**, 195307 (2024)

Exciton crystal melting and destruction by disorder in a bilayer quantum Hall system with a total filling factor of one

Zhengfei Hu and Kun Yang

Bosonic crystal

Physics | *Physics* **4**, 109 (2011)

How Solid is Supersolid?

Anatoly B. Kuklov
Department of Engineering Science and Physics, CSI, CUNY, Staten Island, NY 10314, USA

Nikolay V. Prokof'ev and Boris V. Svistunov
Department of Physics, University of Massachusetts, Amherst, MA 01003, USA and Russian Research Center "Kurchatov Institute," 123182 Moscow, Russia

Published December 19, 2011

Macroscopic quantum properties of helium-4, one of the simplest and oldest elements in the universe, continue to puzzle and amaze scientists. Supertransport in solid helium-4 is the most elusive and controversial conundrum of all.

J Low Temp Phys (2013) 172:317–363
DOI 10.1007/s10909-013-0882-x

Overview on Solid ^{4}He and the Issue of Supersolidity

M.H.W. Chan · R.B. Hallock · L. Reatto

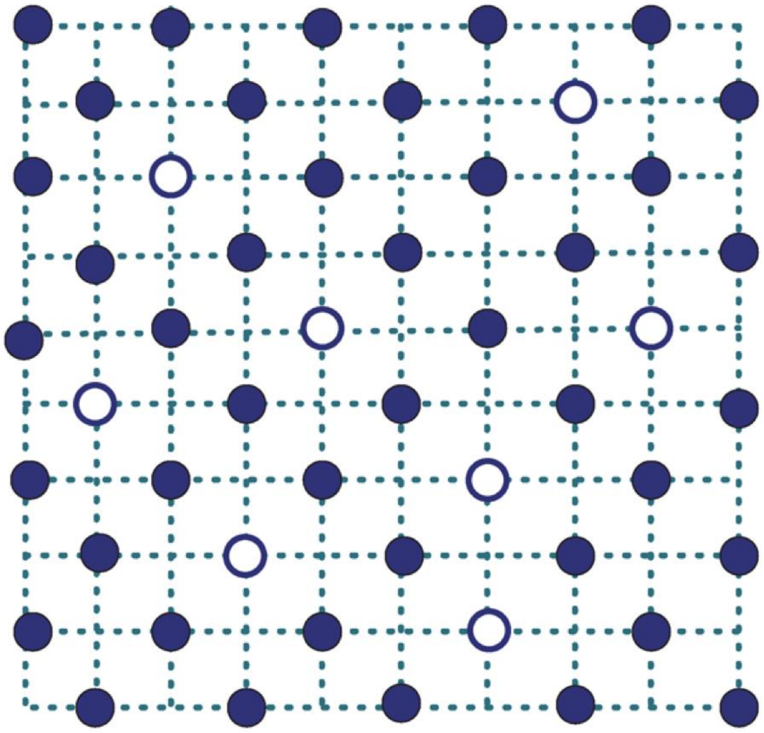
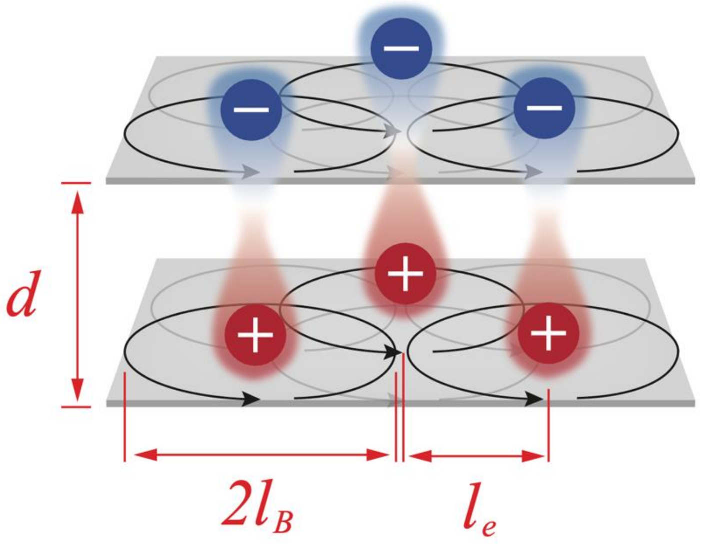


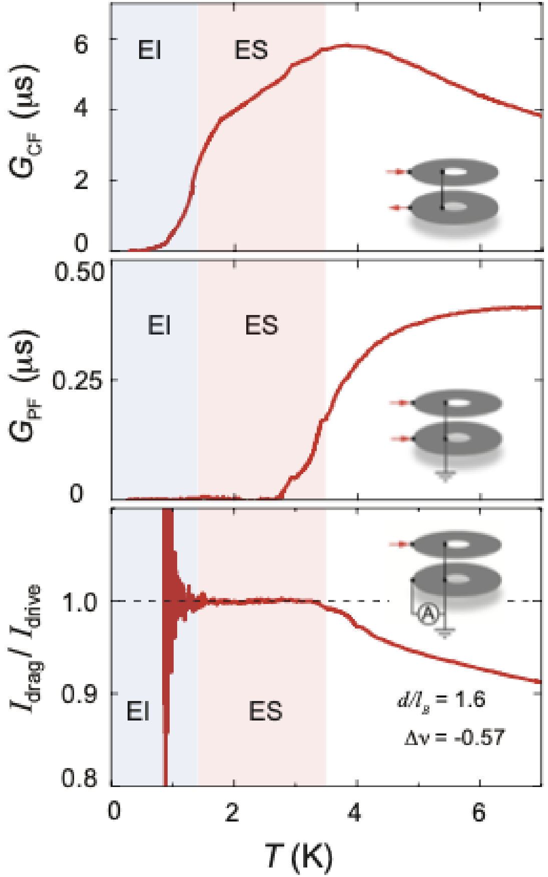
FIG. 1: An example of the lattice supersolid: the checkerboard order formed by particles (filled circles) in the underlying square lattice coexists with the superfluidity of mobile vacancies (open circles). (APS/A. B. Kuklov *et al.*)

Exciton solid in the dilute limit



$$\nu_1 = \nu_2 = 1/2 \quad \Delta\nu = 0$$

Exciton solid



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WHAT STARTS HERE CHANGES THE WORLD

nature physics

LETTERS

PUBLISHED ONLINE: 22 MAY 2017 | DOI: 10.1038/NPHYS4140

Excitonic superfluid phase in double bilayer graphene

J. I. A. Li[1*], T. Taniguchi[2], K. Watanabe[2], J. Hone[3] and C. R. Dean[1*]

LETTERS

https://doi.org/10.1038/s41567-019-0547-z

nature physics

Pairing states of composite fermions in double-layer graphene

J. I. A. Li[1,2,5], Q. Shi[2,5], Y. Zeng[2], K. Watanabe[3], T. Taniguchi[3], J. Hone[4] and C. R. Dean[2*]

RESEARCH

2D MATERIALS

Crossover between strongly coupled and weakly coupled exciton superfluids

Xiaomeng Liu[1]†, J. I. A. Li[2]†, Kenji Watanabe[3], Takashi Taniguchi[4], James Hone[5], Bertrand I. Halperin[1], Philip Kim[1*], Cory R. Dean[6*]

Article

Excitons in the fractional quantum Hall effect

https://doi.org/10.1038/s41586-024-08274-3

Received: 5 August 2024

Naiyuan J. Zhang[1,6], Ron Q. Nguyen[1,6], Navketan Batra[1,2,6], Xiaoxue Liu[1,5], Kenji Watanabe[3], Takashi Taniguchi[4], D. E. Feldman[1,2] & J. I. A. Li[1]✉

Article

Observation of a superfluid-to-insulator transition of bilayer excitons

https://doi.org/10.1038/s41586-025-09986-w

Received: 29 June 2023

Yihang Zeng[1,2,8], Dihao Sun[1,8], Naiyuan J. Zhang[3], Ron Q. Nguyen[3], Qianhui Shi[1], A. Okounkova[1], K. Watanabe[4], T. Taniguchi[5], J. Hone[6], C. R. Dean[1]✉ & J. I. A. Li[7]✉

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Brown University/UT Austin

Naiyuan James Zhang*, Ron Nguyen*,

Navketan Batra*, Xiaoxue Liu, Harshadeep Maddali, Ishika Tulsian, Dima Feldman

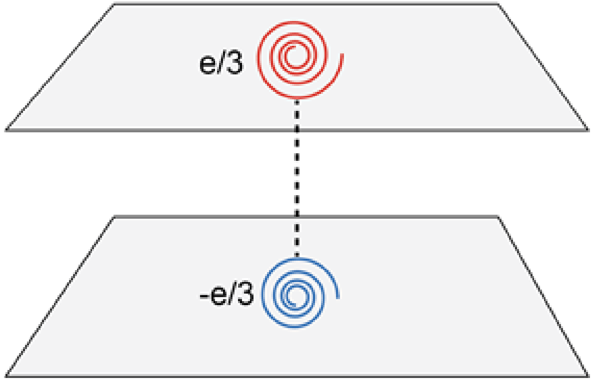
NIMS

Kenji Watanabe, Takashi Taniguchi

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
UNITED STATES AIR FORCE

NSF

NATIONAL MAGLAB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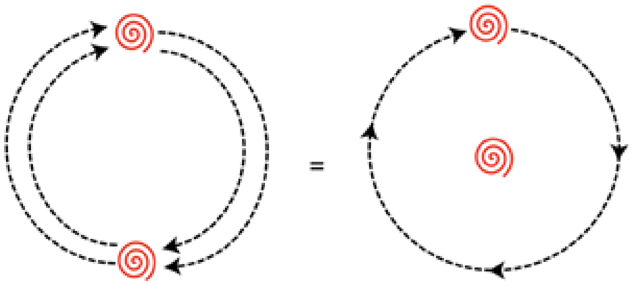
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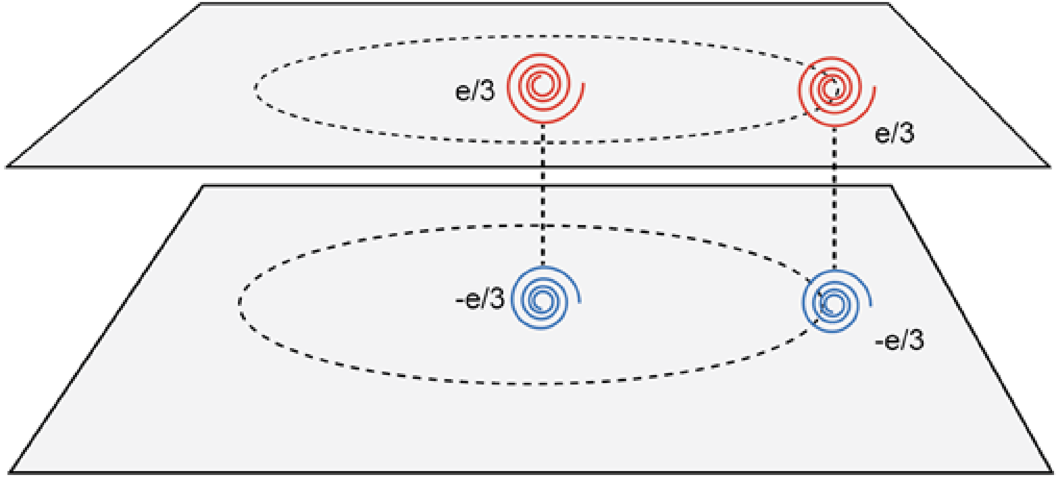
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