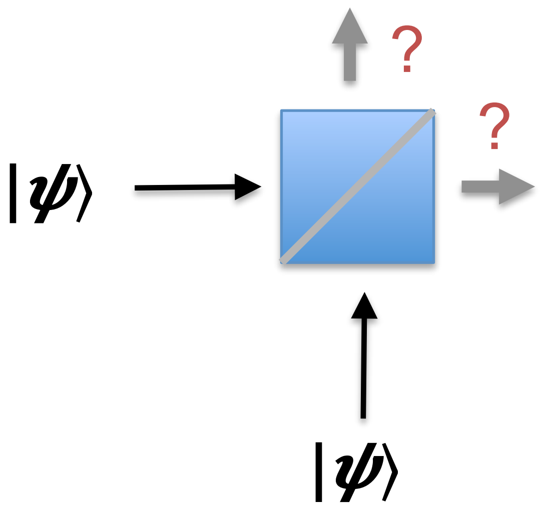
Measuring entanglement in synthetic quantum systems



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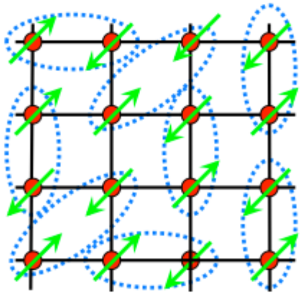
research.iqc.uwaterloo.ca/qiti/

University of Waterloo | IQC Institute for Quantum Computing

Theory Winter School, Tallahassee, Florida
Jan 8-12, 2018

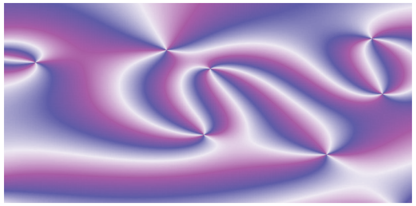
Entanglement in Many-body Systems



- **Resource for quantum information processing**

- **Novel states of matter:**

Order beyond simple broken symmetry



Example - Topological order, spin liquid, fractional quantum Hall - characterized by quantum entanglement !

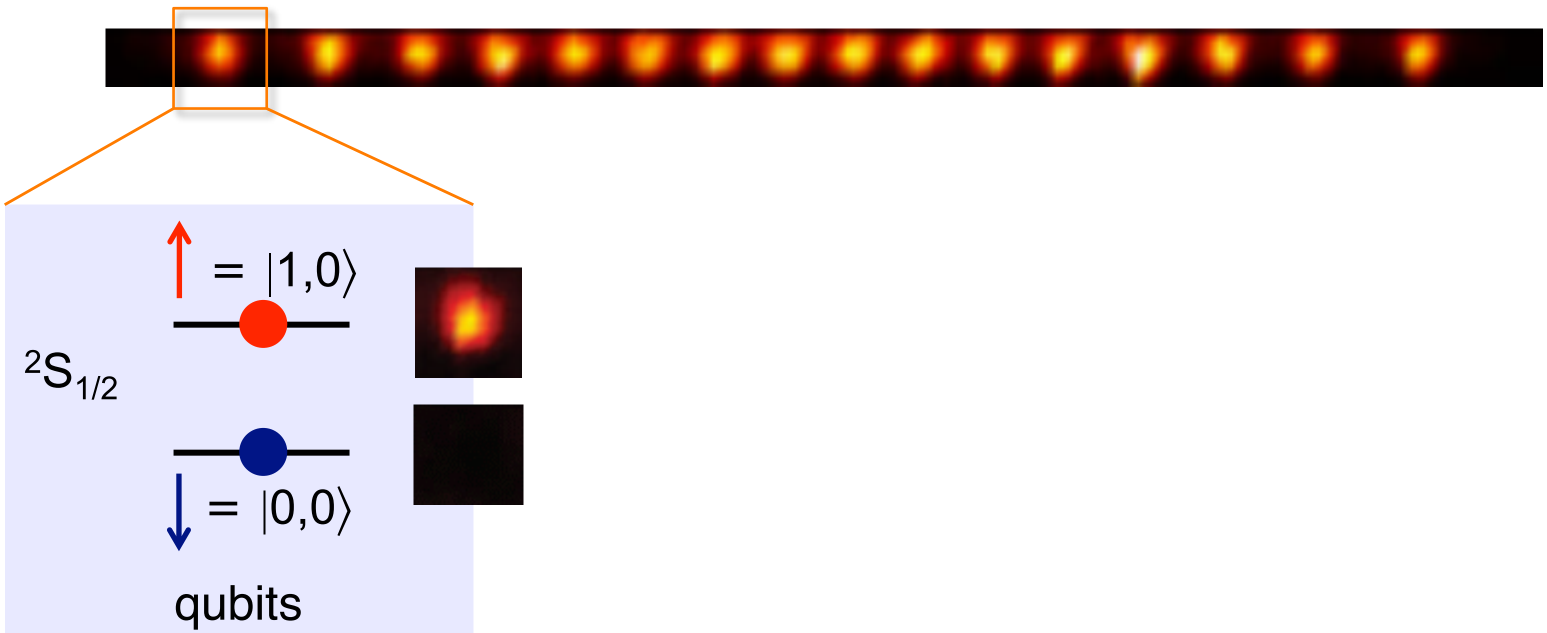
- **Quantum criticality**
- **Quantum dynamics …**

- **Challenge: Entanglement not detected in traditional CM experiments**

Outline

- Recap – preparing entangled states with ion qubits/spins
- State tomography
- Witness operators
- Replica method – measuring second Renyi entropy

Preparing entangled states with ions

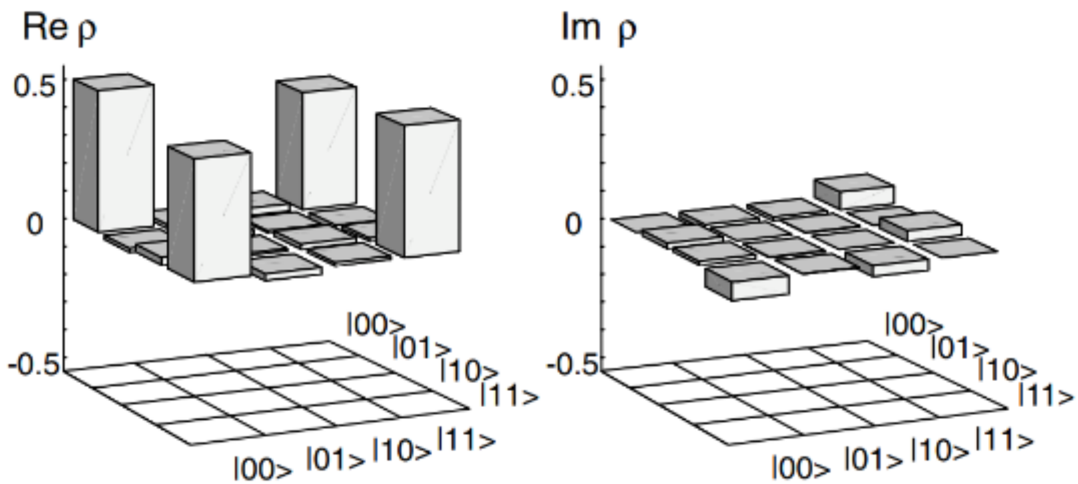


1. Initialize the qubits to a (product) state
2. Evolve the state under single qubit unitary rotations and laser-induced phonon mediated spin-spin interactions [digital ‘circuit’ of logic gates or analog Hamiltonian evolution]
3. **Measurement** – unitary rotation of measurement basis + Spin dependent fluorescence

State Tomography

Reconstruct the entire density matrix

$$|\phi^+\rangle = 1/\sqrt{2}\ (|00\rangle + |11\rangle)$$



Individual qubit addressing required to measure ZZ,XX,YY,ZX, …

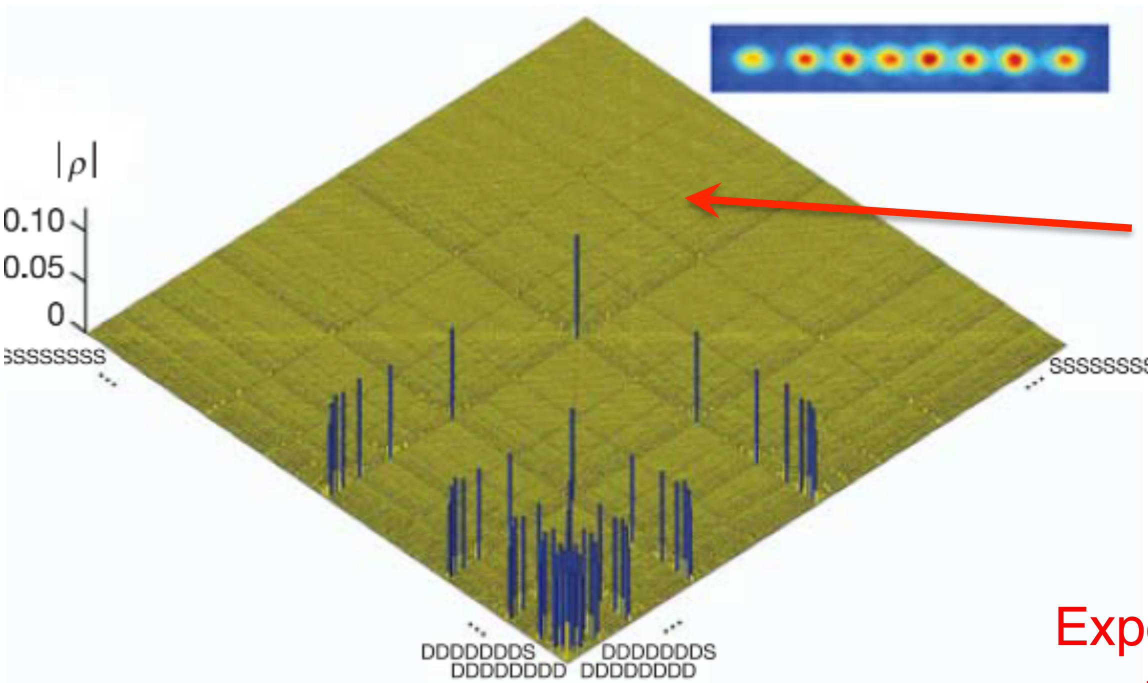
No. of qubits, $N = 2$, # measurements = 1800 (~ 3^N) , total time taken = 40 sec

Roos, C. F. et al, *PRL* **92**, 220404 (2004)

State Tomography

Reconstruct the entire density matrix

$$|W_N\rangle = (|D\cdots DDS\rangle + |D\cdots DSD\rangle + |D\cdots DSDD\rangle + \cdots + |SD\cdots D\rangle)/\sqrt{N}$$



Mostly empty!

Exponential number of measurements!

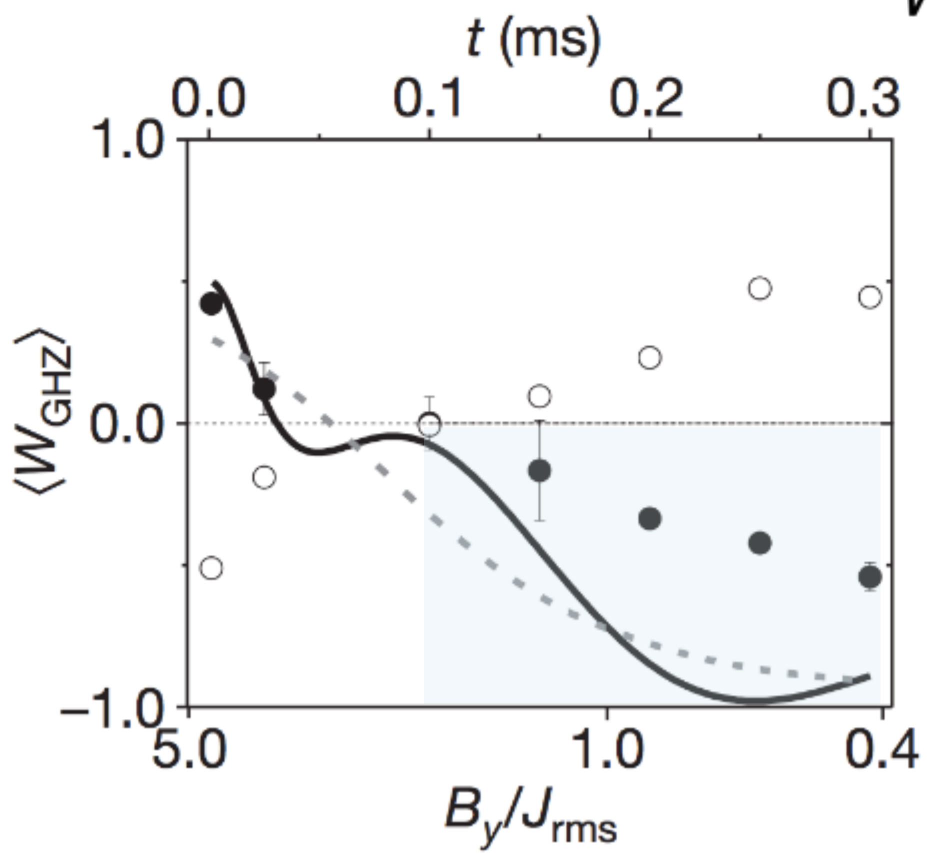
No. of qubits, $N = 8$, # measurements > 656,100 (~ 3^N) , total time taken = 10 hr

Haffner, H. et al, *Nature* **438**, 643 (2005)

Witness operators

Make your most educated guess!

$\langle W \rangle < 0$ ➡ has entanglement of the particular kind!



$$W_{\mathrm{GHZ}} = \frac{1}{2} - |\mathrm{GHZ}\rangle\langle\mathrm{GHZ}|$$

$$= 9/4 - \hat{\mathcal{J}}_x^2 - \sigma_\phi^{(1)}\sigma_\phi^{(2)}\sigma_\phi^{(3)}$$

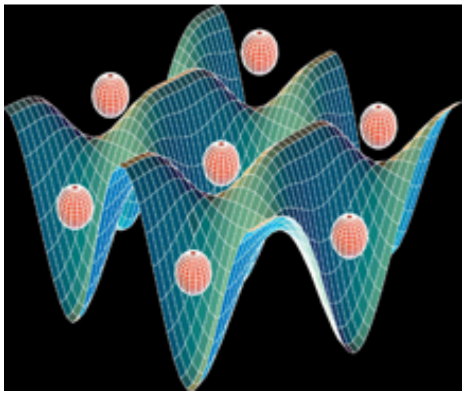
$$|\mathrm{GHZ}\rangle = |\downarrow\downarrow\downarrow\rangle - |\uparrow\uparrow\uparrow\rangle$$

Kim, K et al

Nature 465, 590 (2010)

Quantum Simulation : Platforms

Trapped ions
Nature Physics 8, 277–284 (2012)



Neutral atoms
in optical lattices
Nature Physics 8,
267–276 (2012)

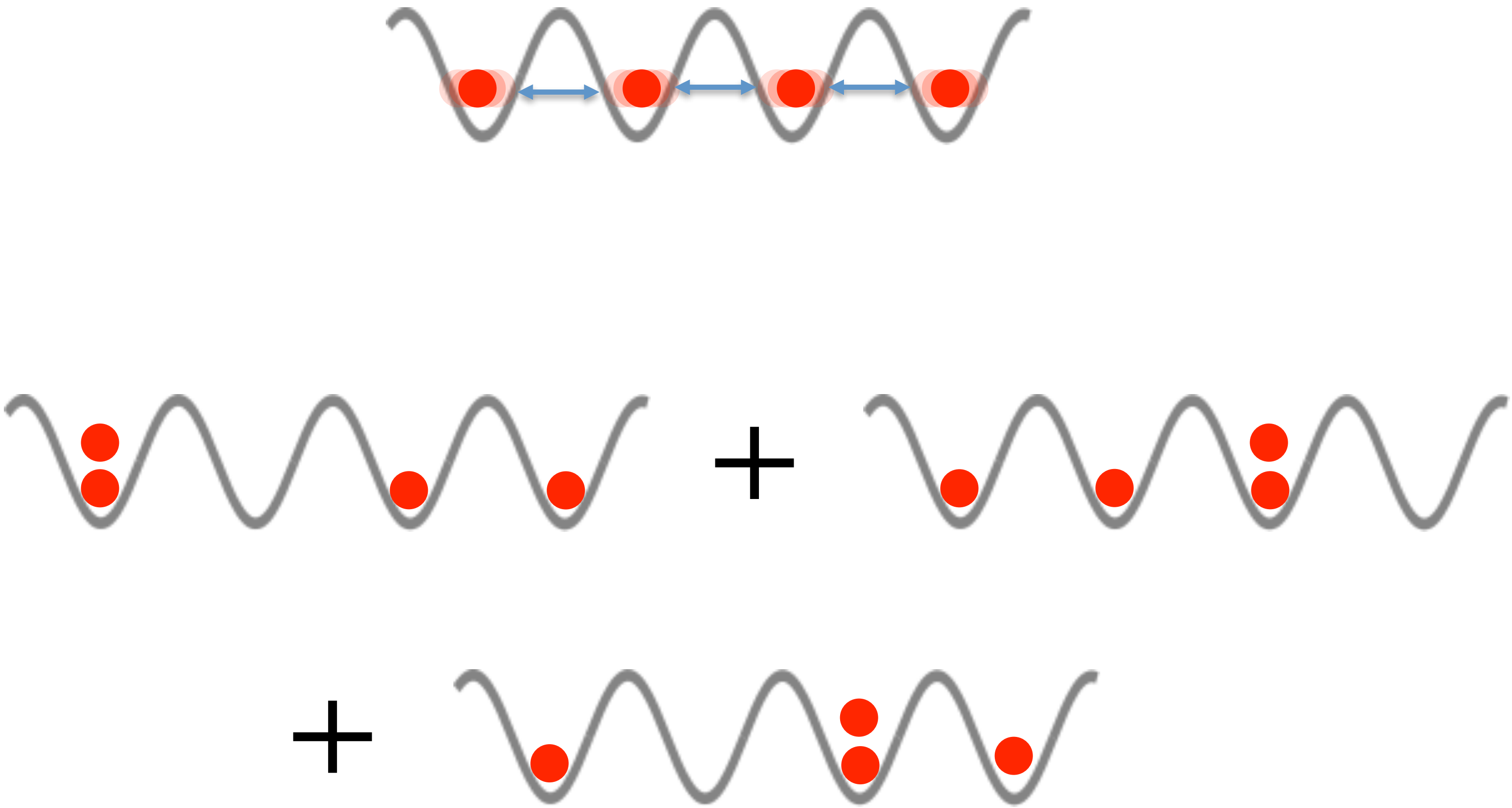
Superconducting circuits
Nature Physics 8,
292–299 (2012)

Photonic
networks
Nature Physics
8, 285–291 (2012)

NV defects in diamonds
Physics Today 67(10), 38(2014)

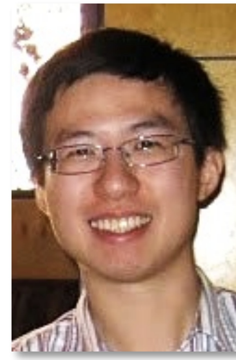
Entanglement

Itinerant many-body systems



Markus Greiner

Alex Ruichao Ma
→ Simon lab, Chicago



Philipp Preiss
→Jochim Lab, Heidelberg

Eric Tai

Matthew Rispoli

Theory:

Andrew Daley
Hannes Pichler
Peter Zoller
Dieter Jaksch …

Alex Lukin

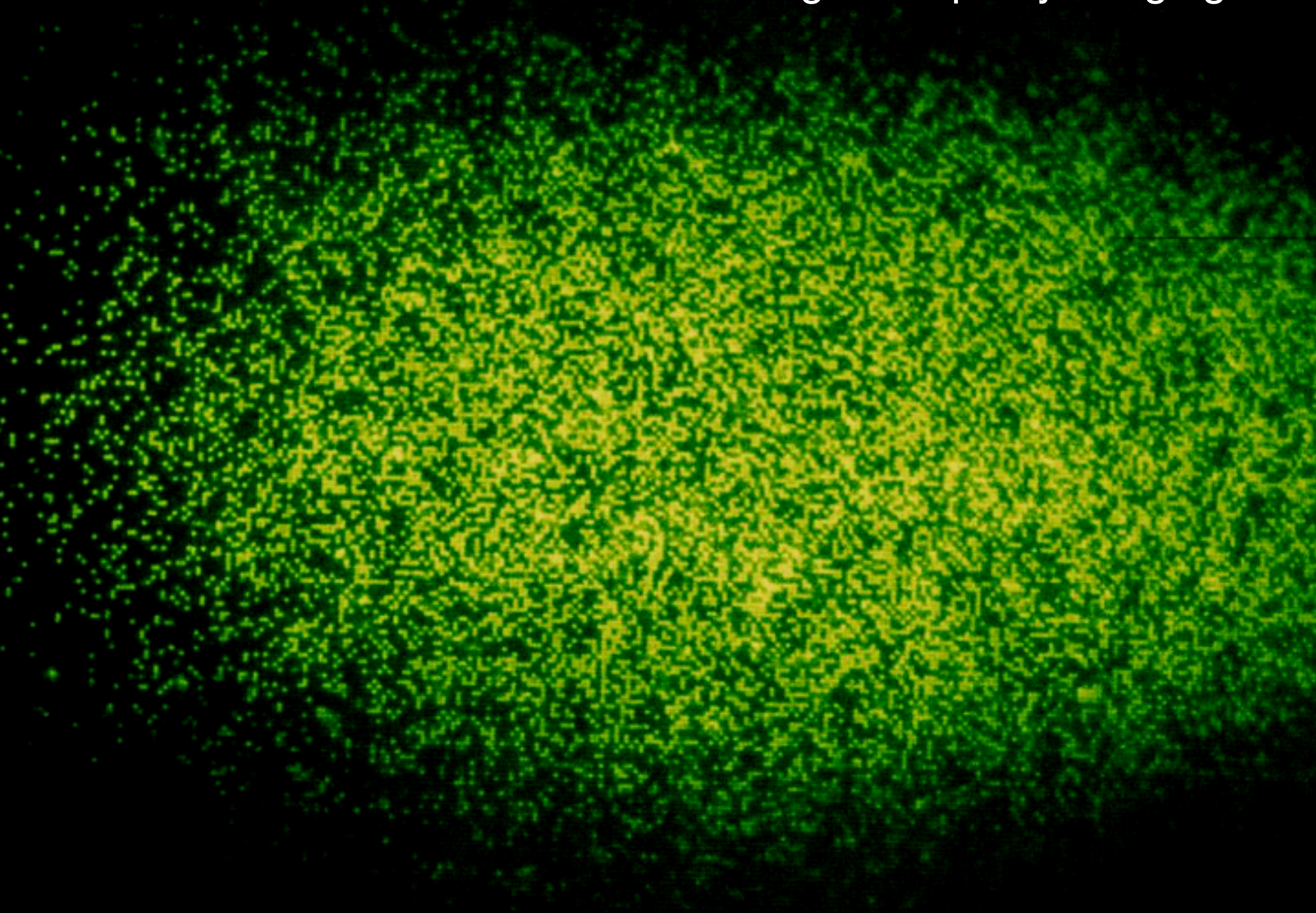
Quantum gas microscope

Bakr *et al*., Nature 462, 74 (2009), Bakr *et al*., Science.1192368 (June 2010)

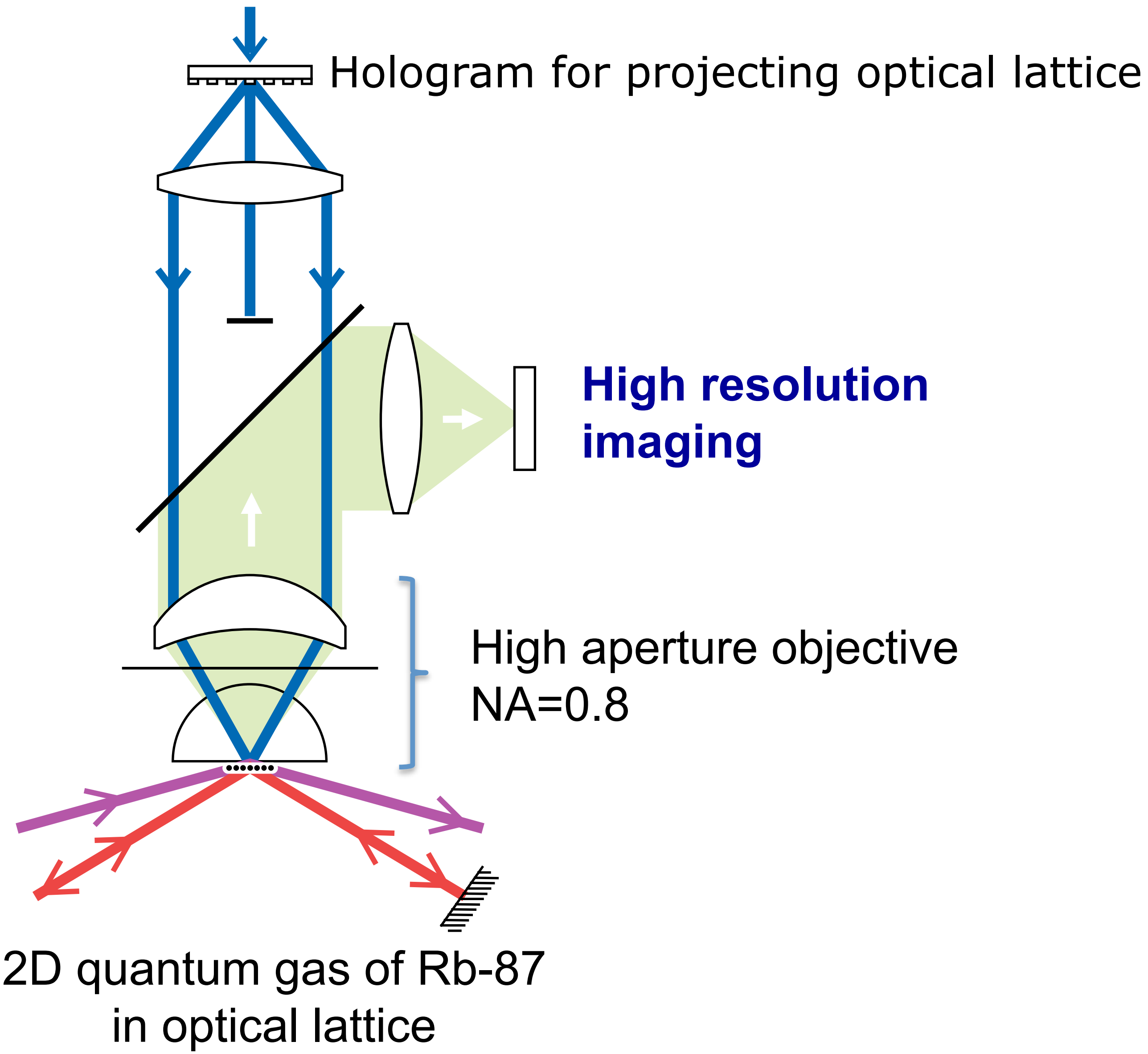
Previous work on single site addressability in lattices:

Detecting single atoms in large spacing lattices (**D. Weiss**) and 1D standing waves (D. Meschede), Electron Microscope (H. Ott), Absorption imaging (J. Steinhauer), single trap (P. Grangier, Weinfurter/Weber), **few site resolution (C. Chin),** See also: Sherson *et al*., Nature

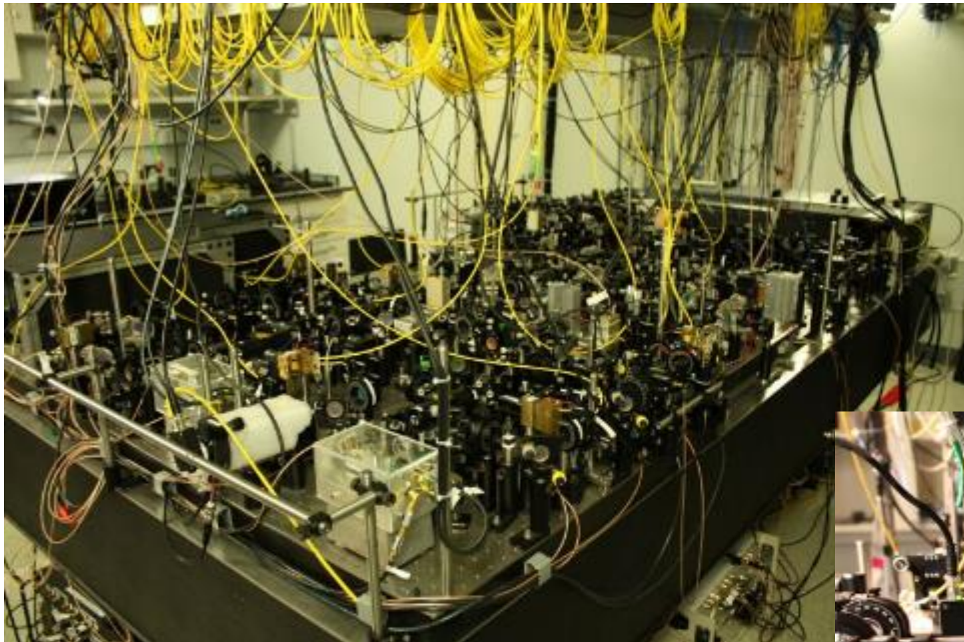
Single site parity Imaging

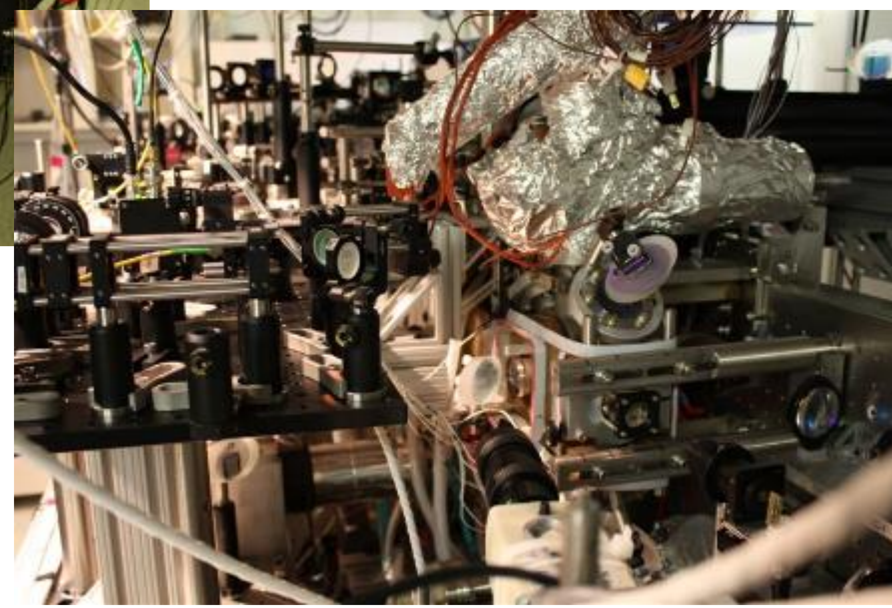


Quantum gas microscope

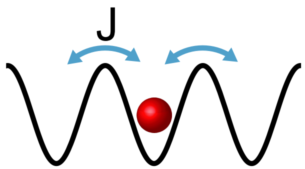


… and the whole apparatus

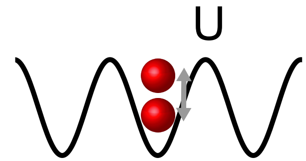




tunneling J

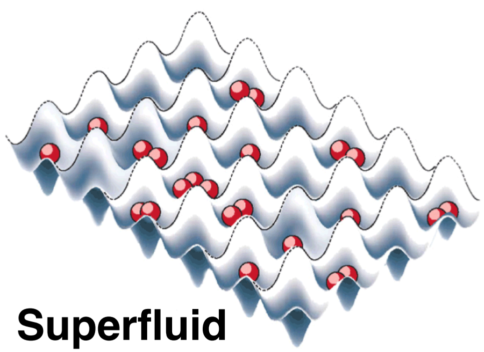


interaction U

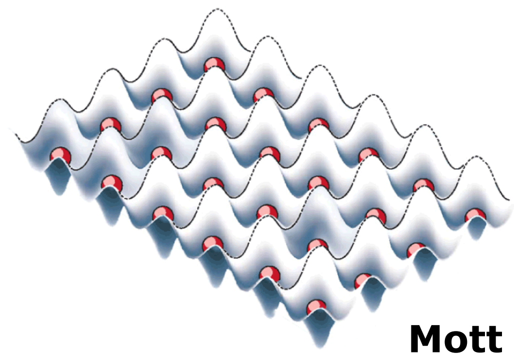


$$H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2} \sum_i n_i(n_i - 1)$$

Bose Hubbard Model

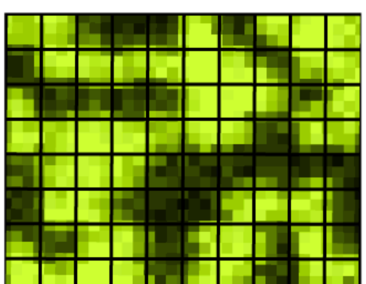


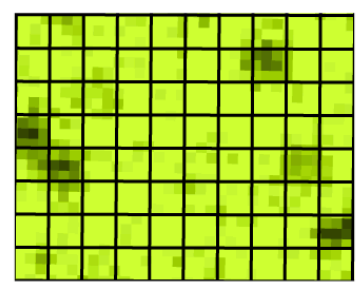
Superfluid



Mott insulator

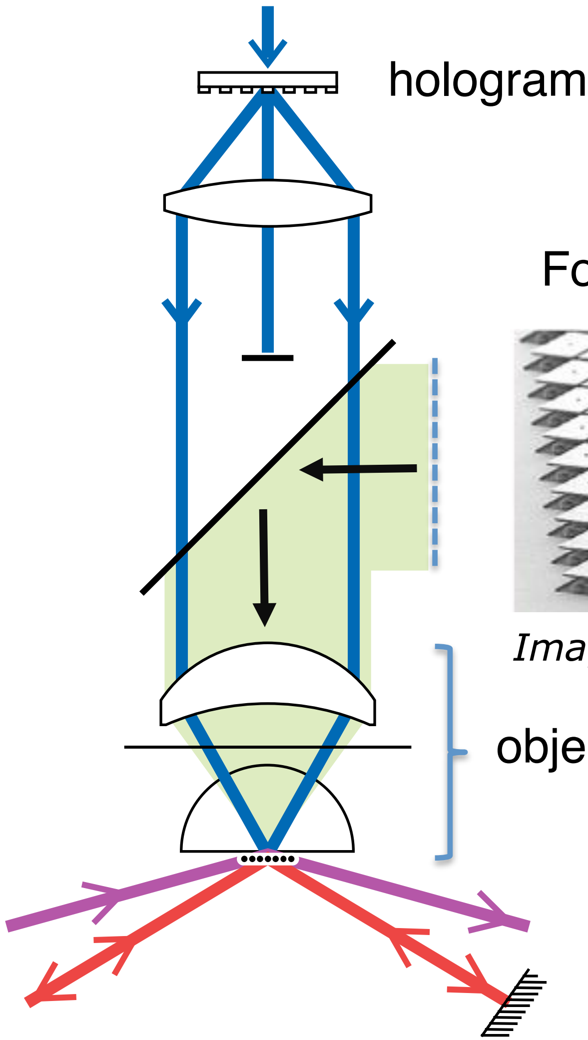
$U \ll J$ — U/J → $J \ll U$





Bakr *et al.*, Science. 329, 547 (2010)

Projecting arbitrary potential landscapes



2D quantum gas of Rb-87
in optical lattice

Fourier hologram

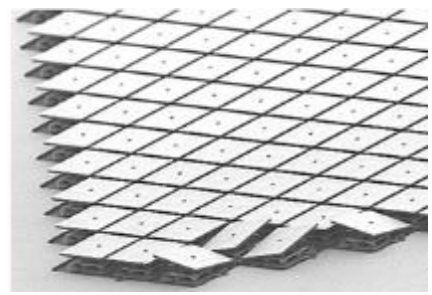
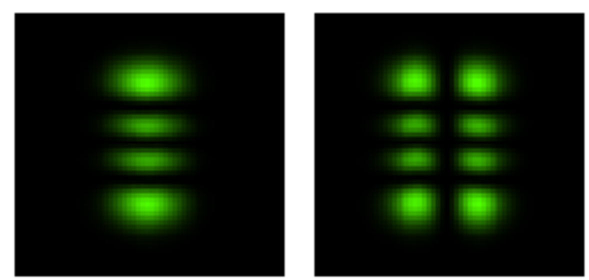


Image: EKB Technologies

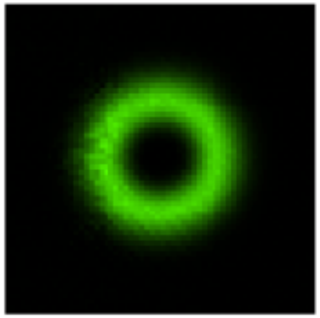


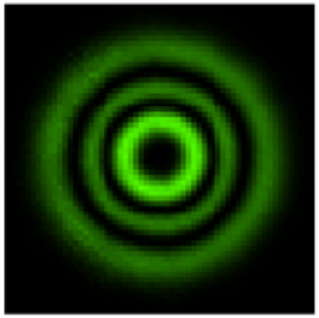
Thesis : P. Zupancic (LMU/Harvard, 2014)

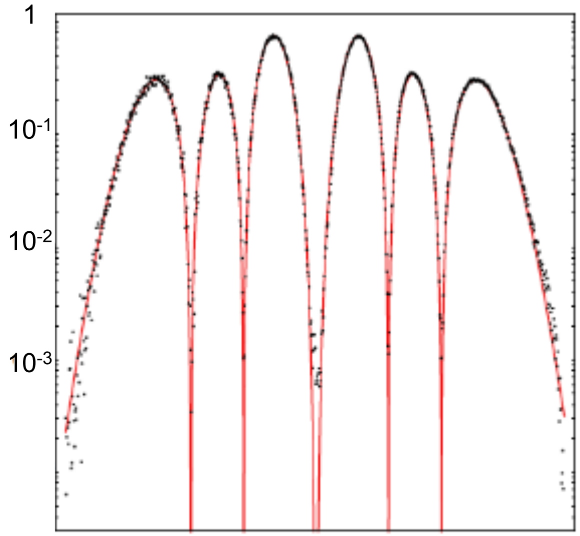
Arbitrary beam shaping

- Weitenberg et al., **Nature** 471, 319-324 (2011) Zupancic, P., Master's Thesis, LMU Munich/ Harvard 2013
- Cizmar, T *et al.,* **Nature Photonics** 4, 6 (2010)

High-order Laguerre Modes

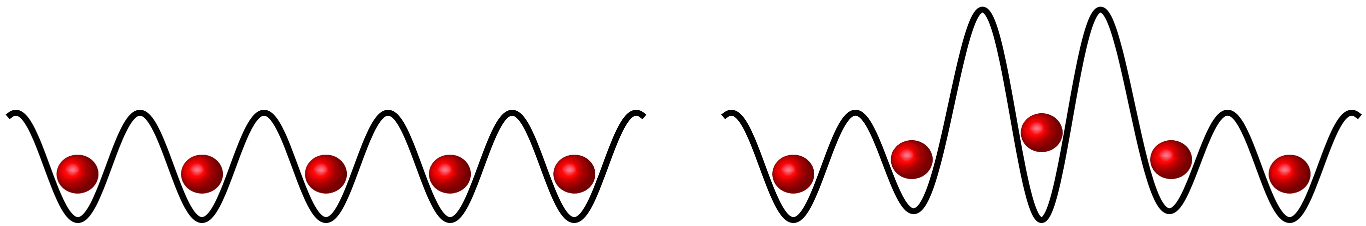






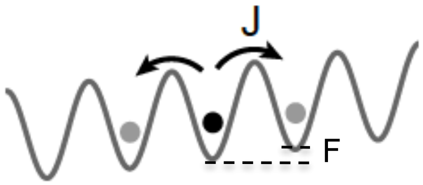
Laguerre-Gauss profile

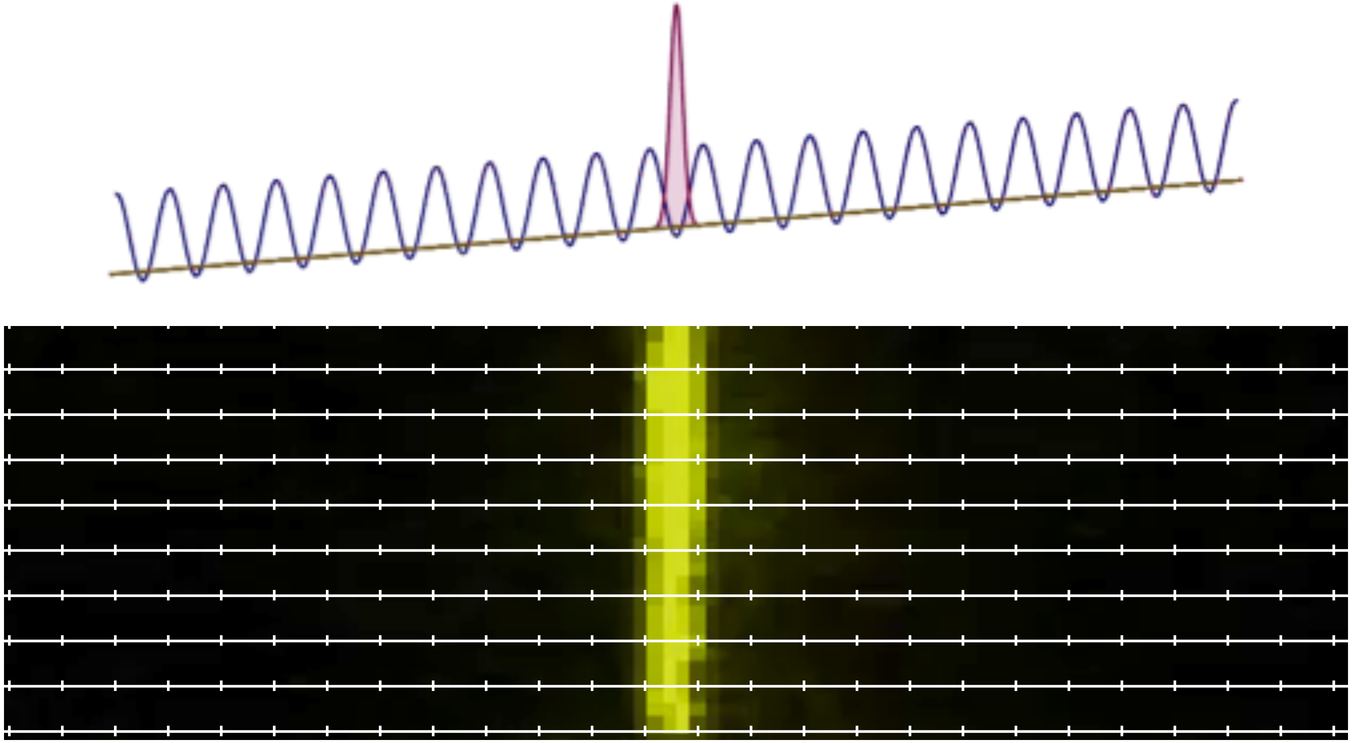
A bottom-up system for neutral atoms



(Single shot image)

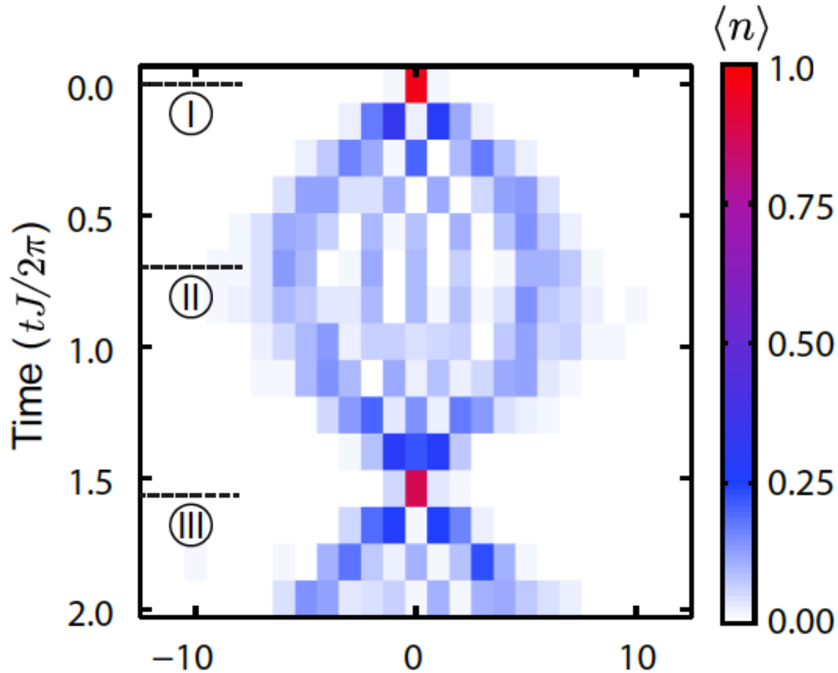
Single-Particle Bloch oscillations

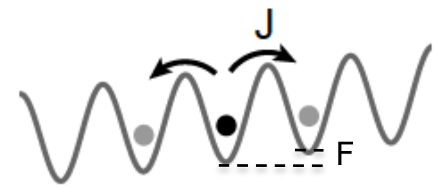




- P. M. Preiss, R. Ma, M. E. Tai, A. Lukin, M. Rispoli, P. Zupancic, Y. Lahini, *R. Islam,* M. Greiner **Science** 347, 1229 (2015)

Single-Particle Bloch oscillations



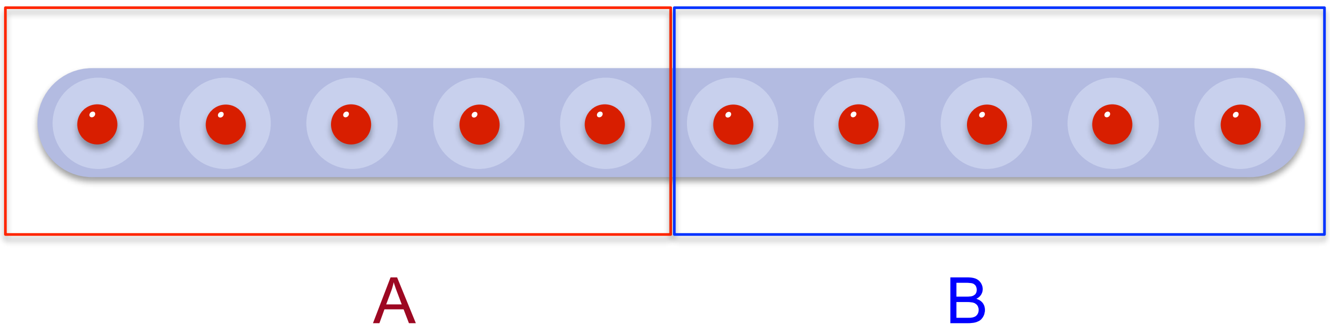


- Temporal period $T_B = \dfrac{2\pi}{F}$, spatial width $L_B = \dfrac{4J}{F}$
- Delocalized over ~14 sites = 10μm.
- Revival probability 96(3)%

- P. M. Preiss, R. Ma, M. E. Tai, A. Lukin, M. Rispoli, P. Zupancic, Y. Lahini, *R. Islam,* M. Greiner **Science** 347, 1229 (2015)

Entanglement in Many-body Systems

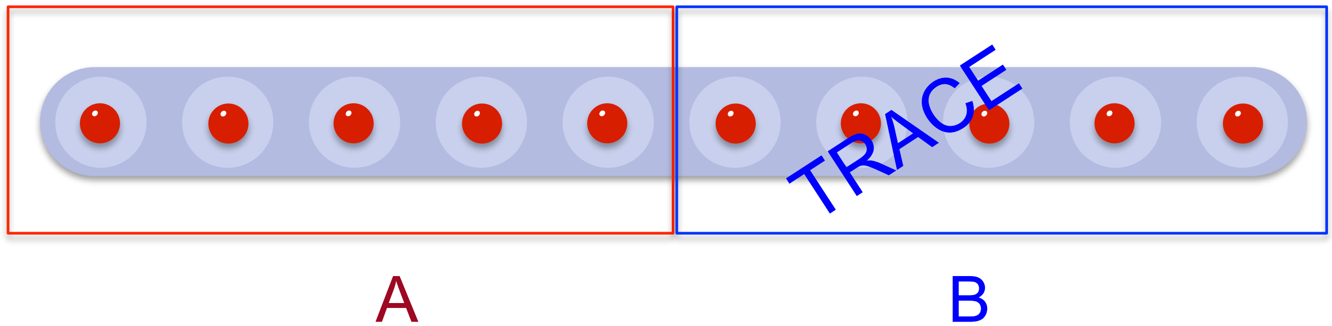
Many-body system: Bipartite entanglement



Product state: $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$ e.g. Mott insulator

Entangled state: $|\Psi\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$ e.g. Superfluid

Entanglement Entropy



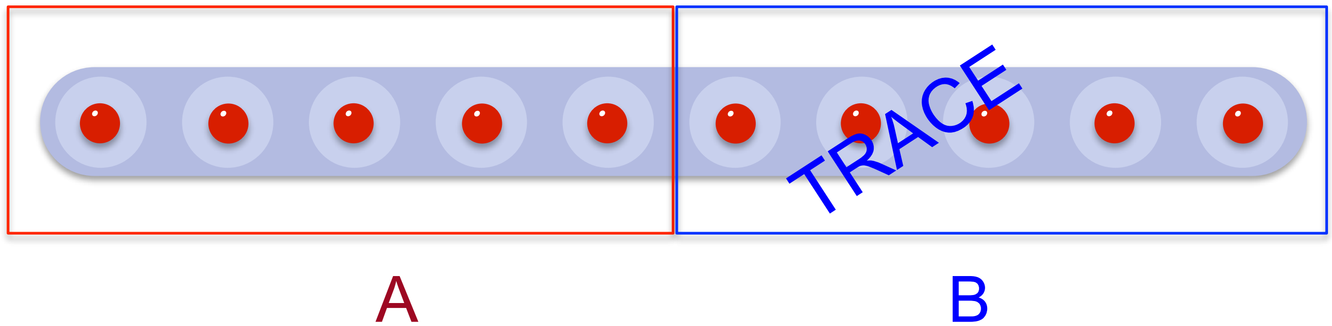
Reduced density matrix:

$$\rho_A = \mathrm{tr}_B\{\rho\} = |\Psi_A\rangle \otimes \langle\Psi_A|$$

	Product state → Pure state	**Entangled state → Mixed state**
Quantum purity = $\mathrm{Tr}(\rho_A^2)$	$=1$	<1
$S_2(\rho_A) = -\log \mathrm{Tr}(\rho_A^2)$	$=0$	>0

Renyi Entanglement Entropy $S_n(\rho_\alpha) = \frac{1}{1-n} \log \mathrm{Tr}\{\rho_\alpha^n\}$

Entanglement Entropy



Reduced density matrix:

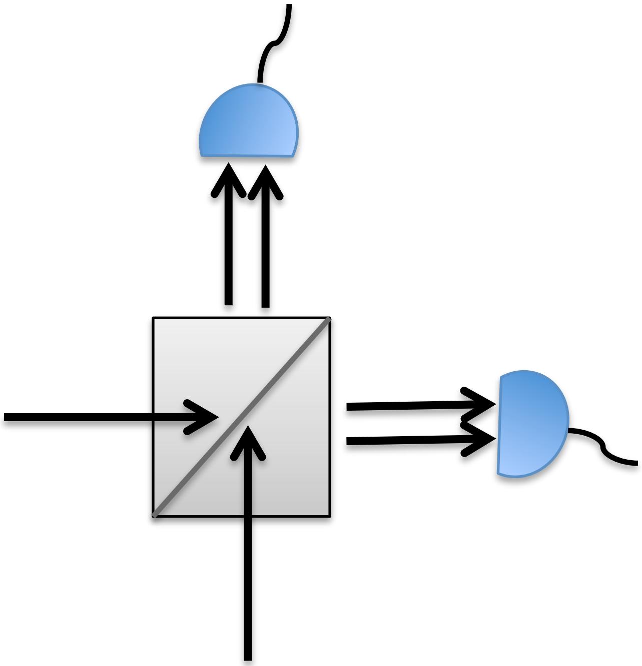
$$\rho_A = \mathrm{tr}_B\{\rho\} = |\Psi_A\rangle \otimes \langle\Psi_A|$$

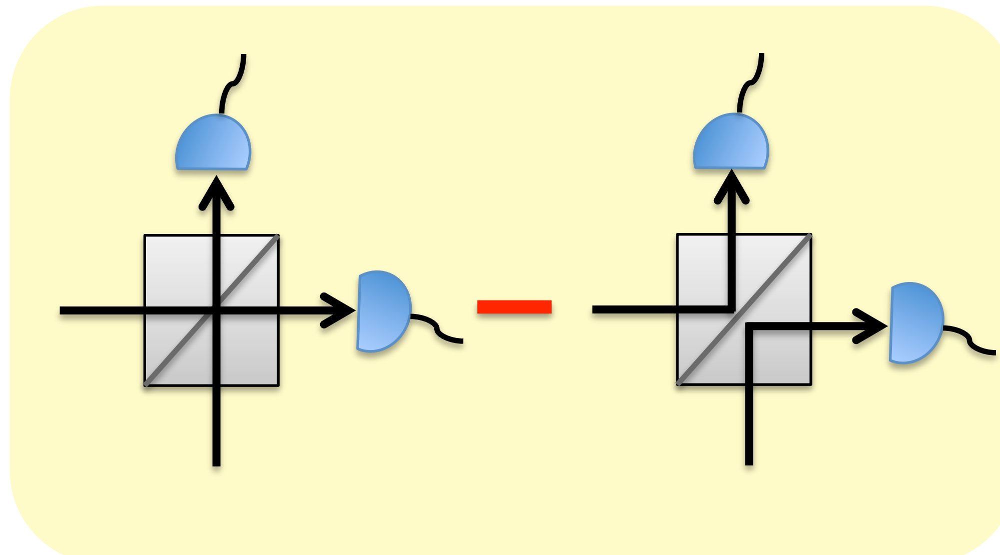
	Product state → Pure state	Entangled state → Mixed state
Quantum purity = $\mathrm{Tr}(\rho_A^2)$	$=1$	<1

Many-body Hong-Ou-Mandel interferometry

Alves and Jaksch, PRL 93, 110501 (2004)
Mintert et al., PRL 95, 260502 (2005)
Daley et al., PRL 109, 020505 (2012)

Hong-Ou-Mandel interference

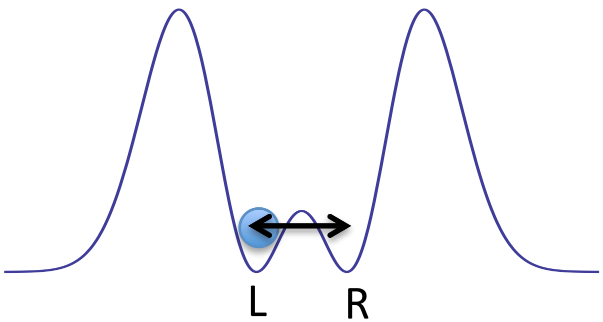


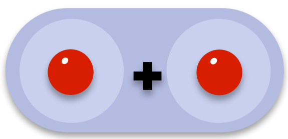


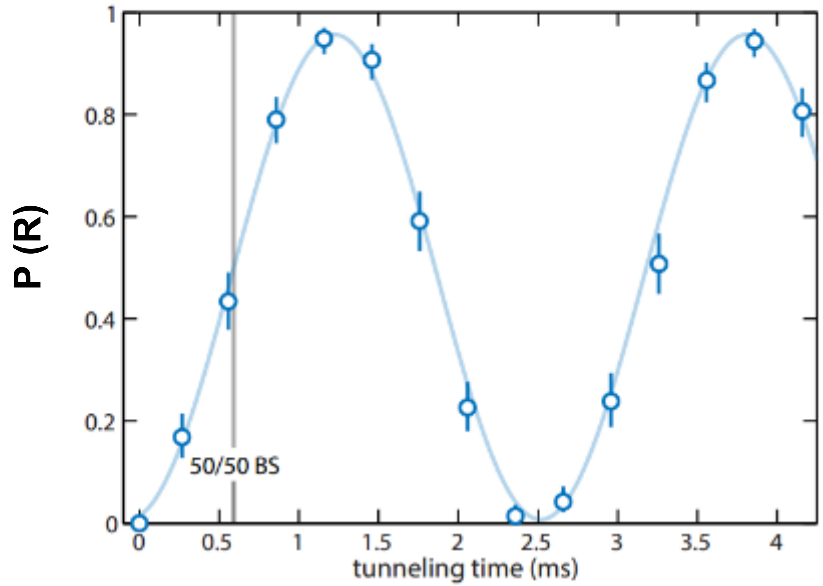
No coincidence detection
for **identical** photons

Hong C. K., Ou Z. Y., and Mandel L. Phys. Rev. Lett. 59 2044 (1987)

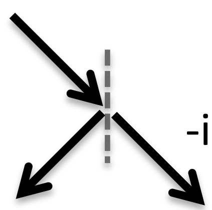
Beam splitter operation: Rabi flopping in a double well



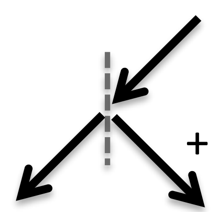




$$a_L^\dagger \to a_L^\dagger - i a_R^\dagger$$



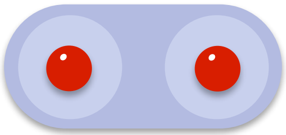
$$a_R^\dagger \to a_L^\dagger + i a_R^\dagger$$



Also see: Kaufman A M *et al.*,
Science 345, 306 (2014)
Without single atom detection:
Trotzky et al., PRL 105, 265303 (2010)
also Esslinger group

Two bosons on a beam splitter

Hong-Ou-Mandel interference

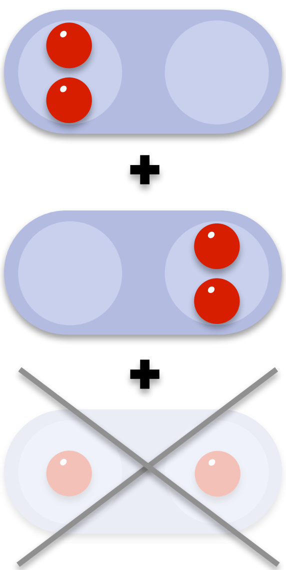


$$a_L^\dagger a_R^\dagger$$

↓ Beam splitter

$$a_L^\dagger \to a_L^\dagger - i a_R^\dagger$$

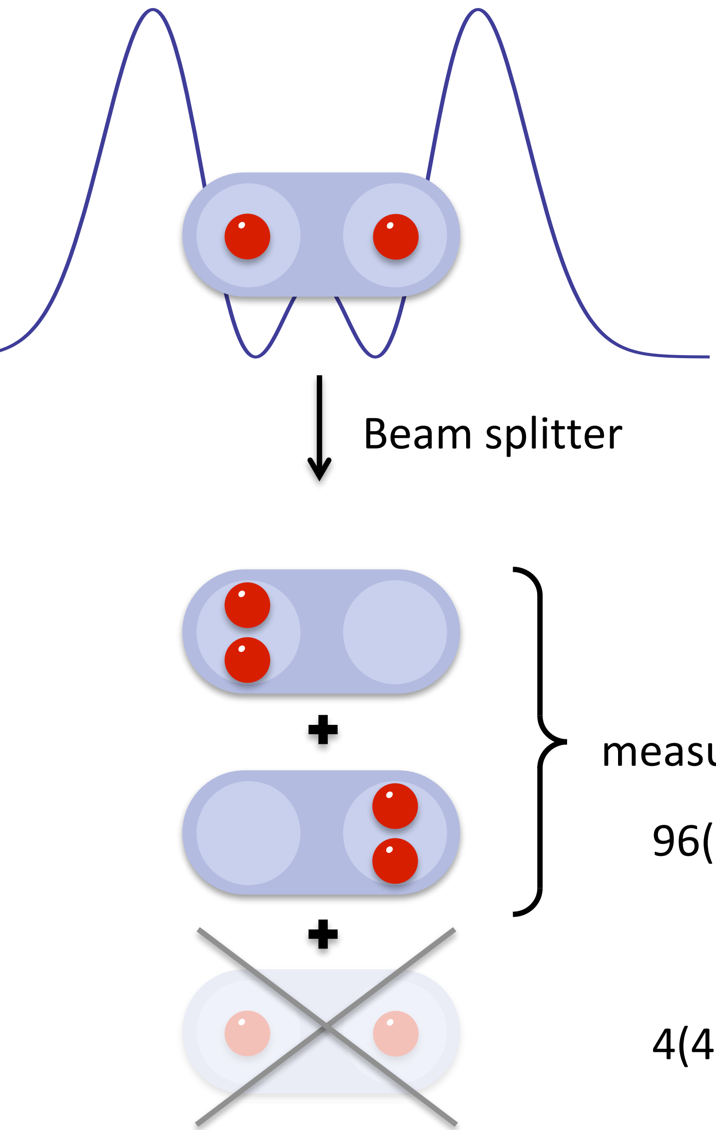
$$a_R^\dagger \to a_L^\dagger + i a_R^\dagger$$



$$a_L^\dagger a_L^\dagger$$

$$+$$

$$a_R^\dagger a_R^\dagger$$

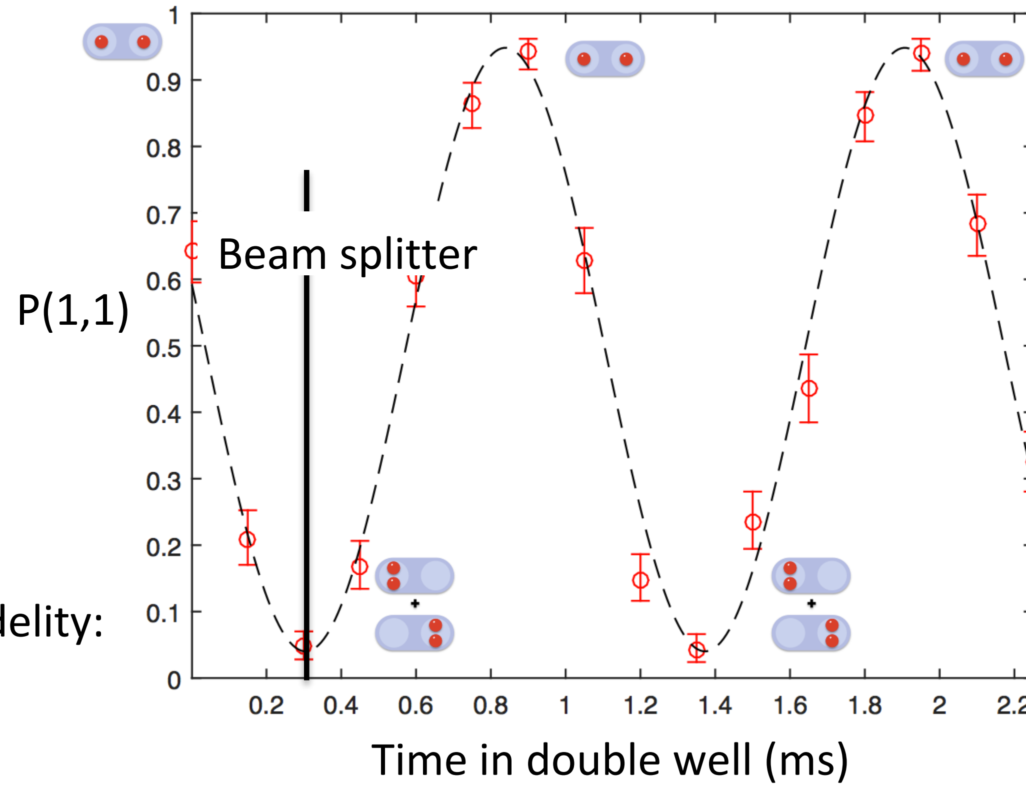


measured fidelity:

96(4)%

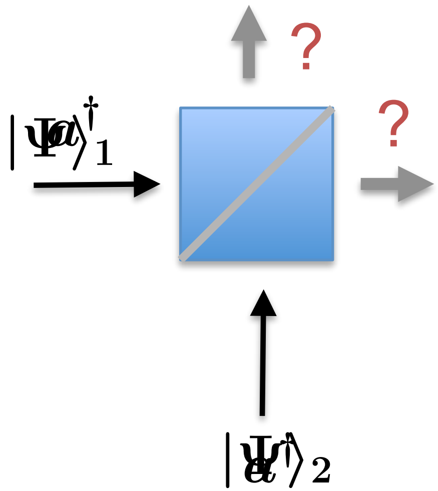
4(4)%

limited by interaction



Also see : Kaufman A M *et al.*,
Science 345, 306 (2014),
R. Lopes *et al*, Nature 520, 7545 (2015)

Quantum interference of bosonic many body systems



How "identical" are the **particles**?

vs.

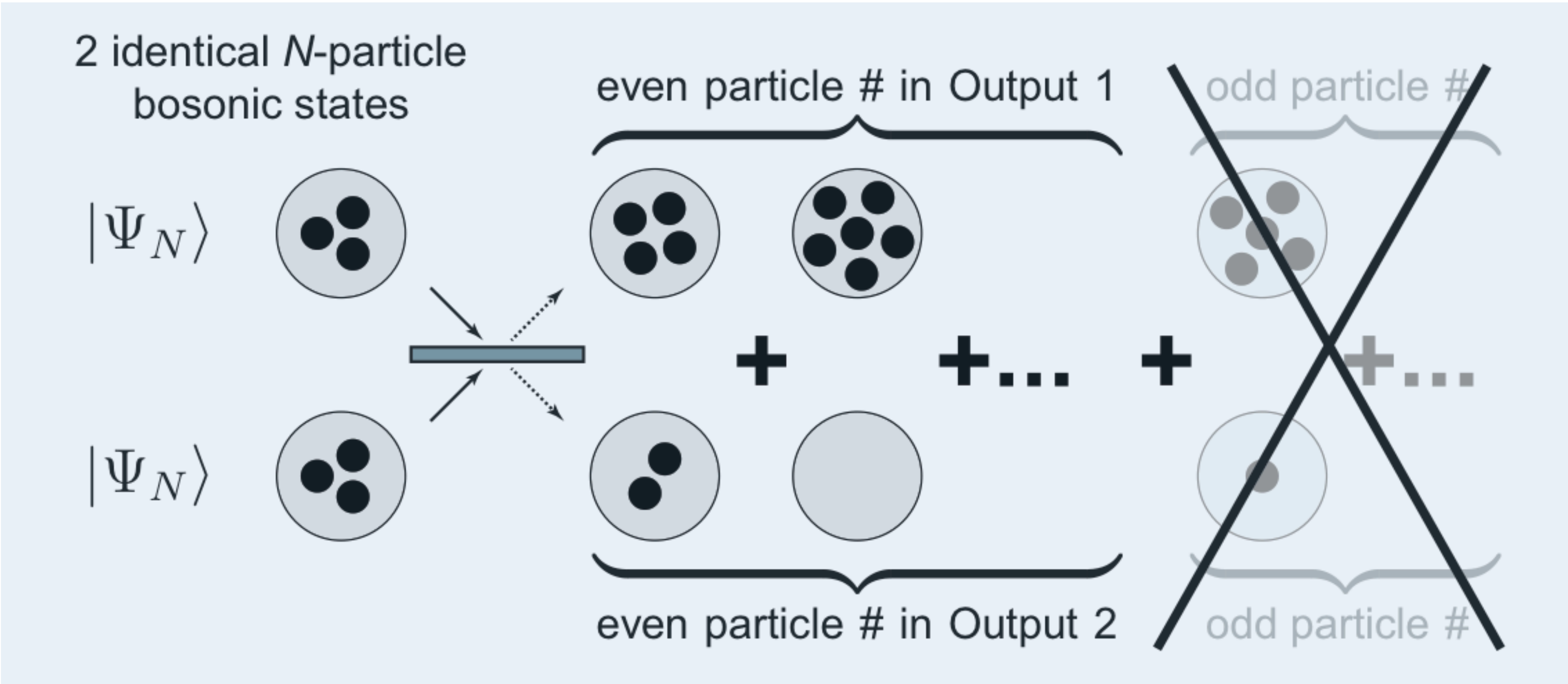
How "identical" are the **states**?

If $|\Psi\rangle_1 = |\Psi\rangle_2$, **deterministic number parity** after beam splitter

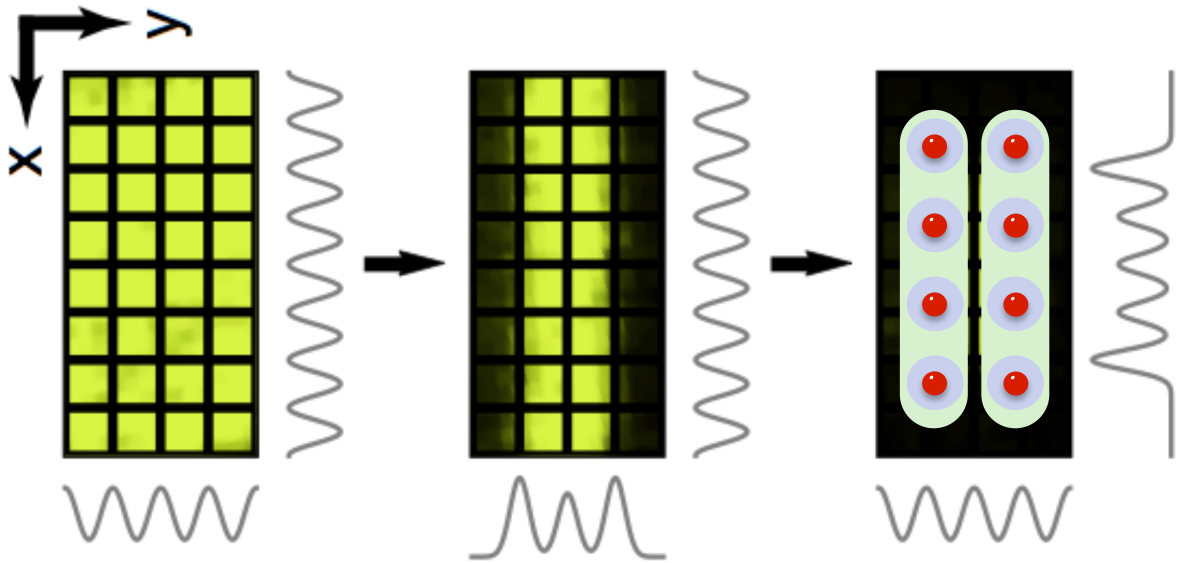
Alves and Jaksch, PRL **93** (2004)
Daley et al., PRL **109** (2012)

Also see Linke et al, arXiv1712.08581 for experiments on two copies of a trapped ion system simulating Fermi-Hubbard model.

Quantum interference of bosonic many body systems



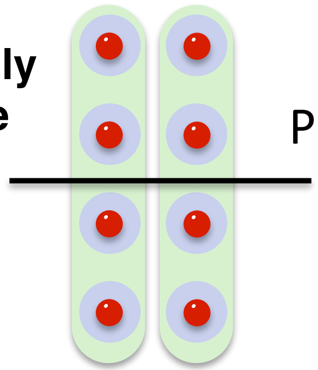
Making two copies of a many-body state



Measuring many-body entanglement

Mott Insulator

Locally pure

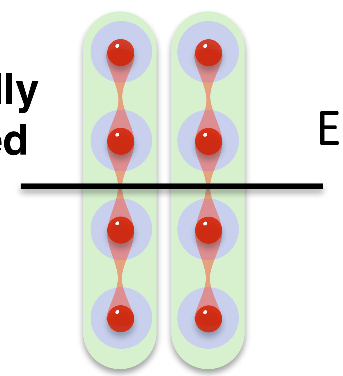


Product State

Globally pure

Superfluid

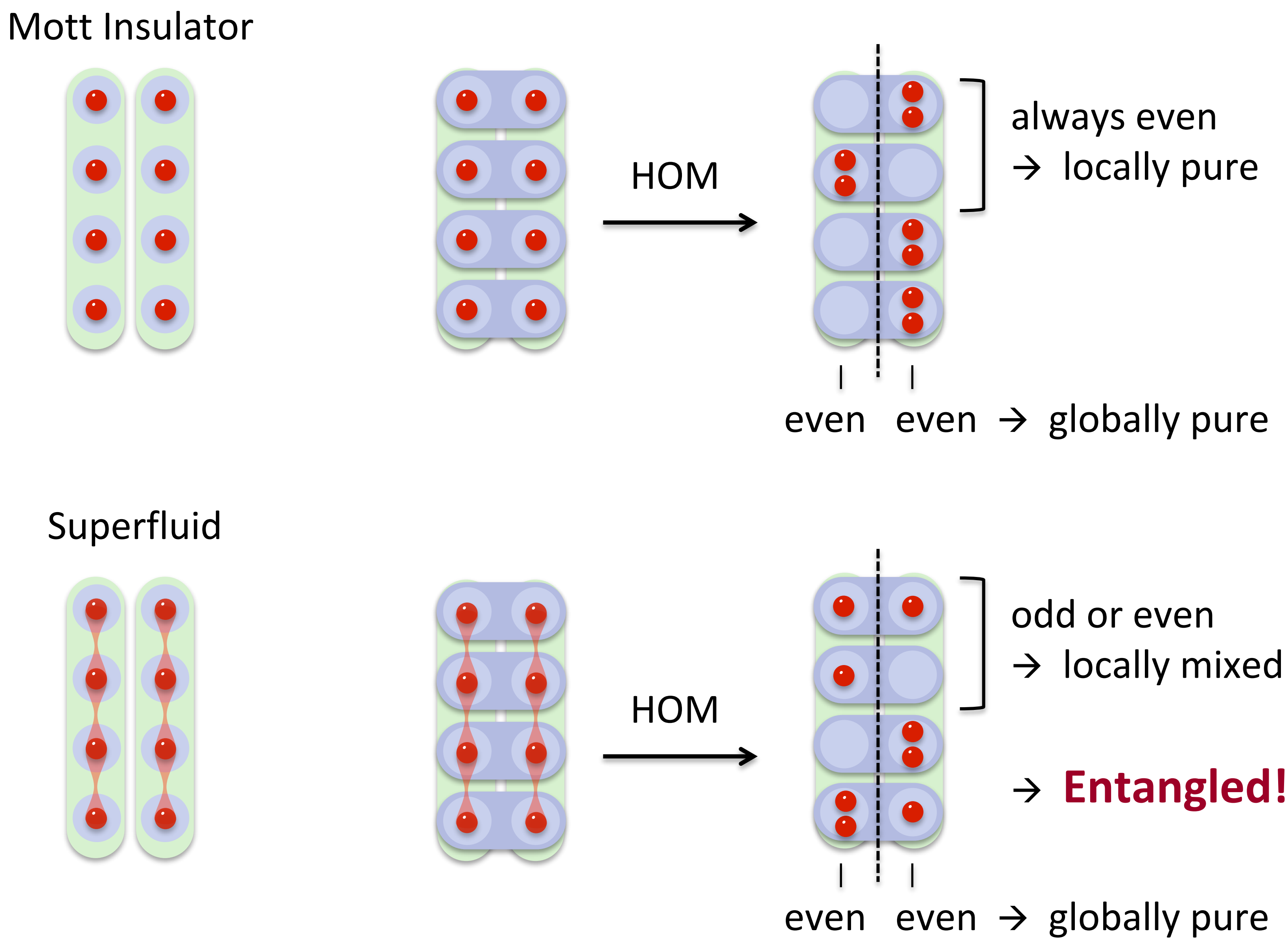
Locally mixed



Entangled

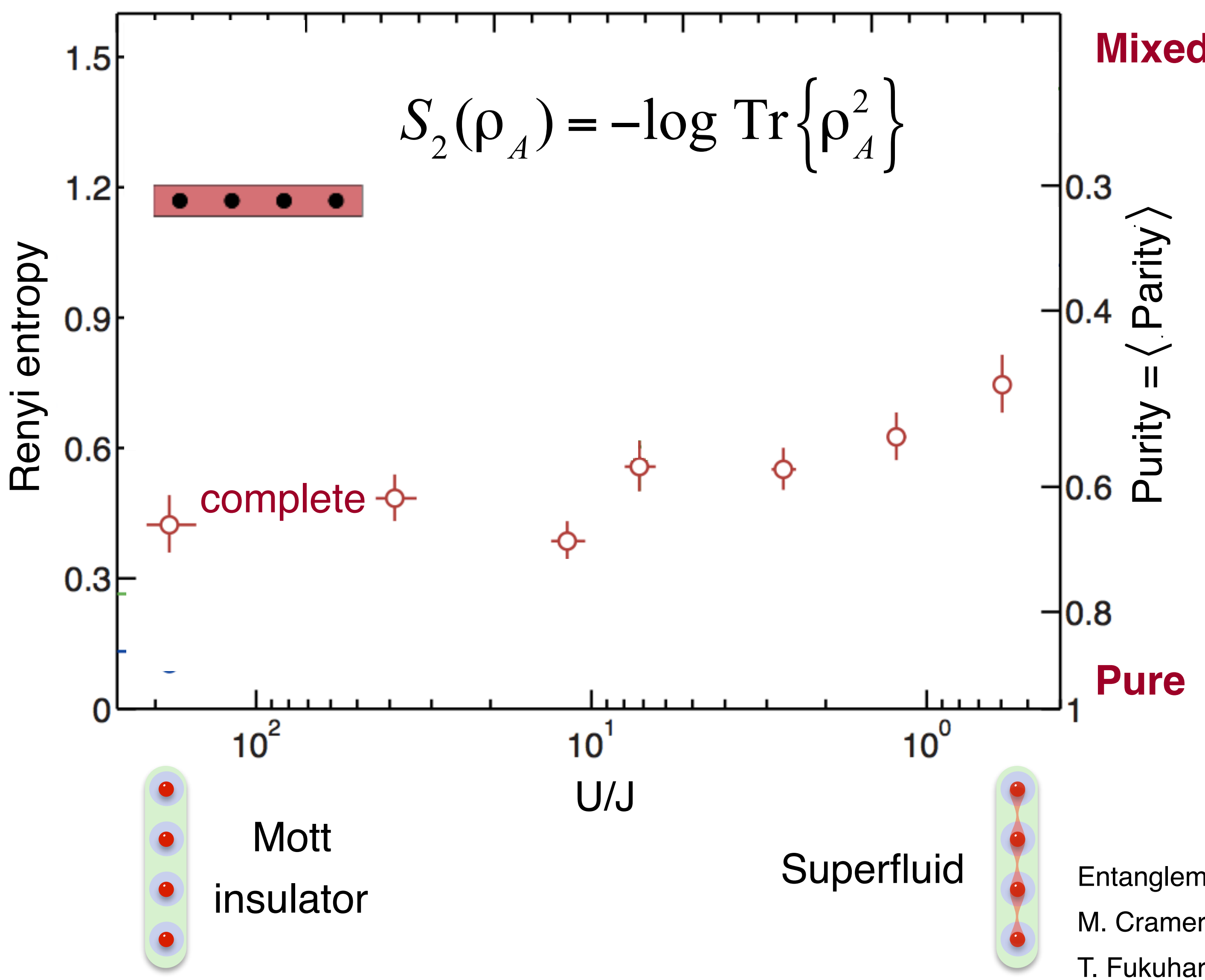
Globally pure

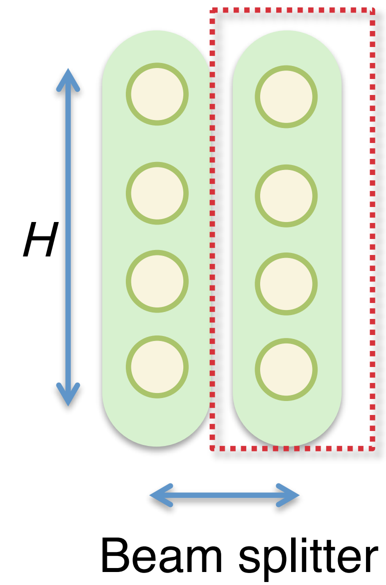
Measuring many-body entanglement



Ref: Alves C M, Jaksch D, PRL 93, 110501 (2004), Daley A J et al, PRL 109, 020505 (2012)

Entanglement in the ground state of a Bose-Hubbard system

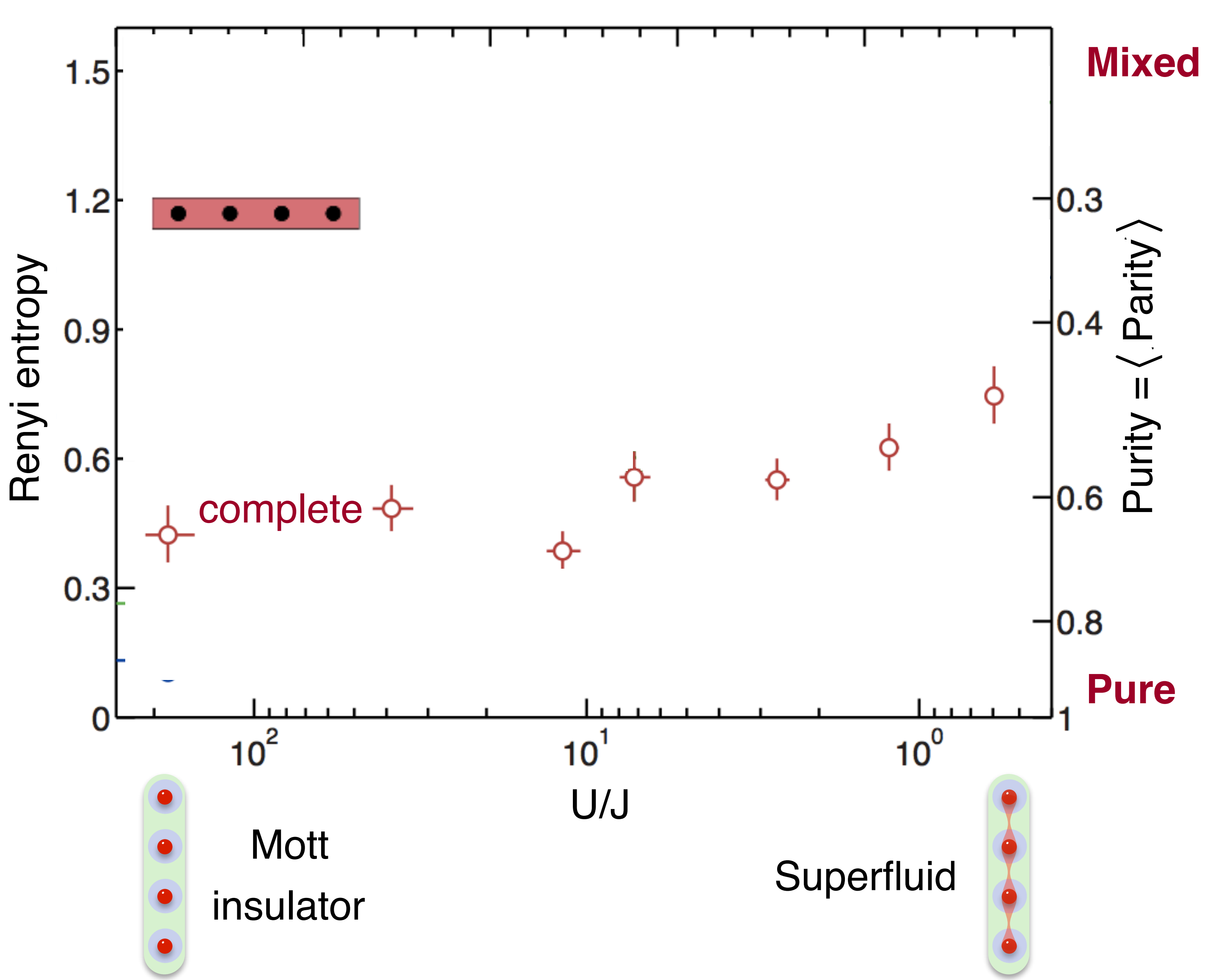


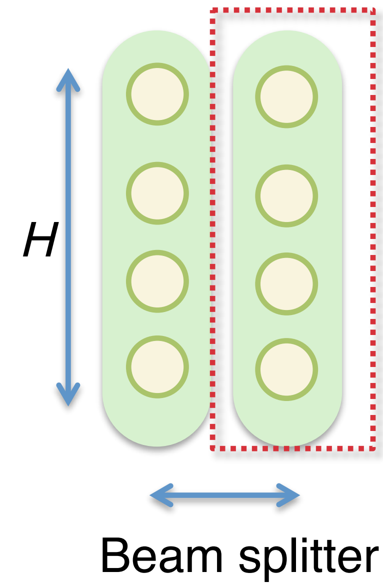


Rajibul Islam et al,
Nature 528, 77 (2015)

Entanglement in optical lattice systems:
M. Cramer *et al,* Nature Comm, 4 (2013),
T. Fukuhara *et al*, PRL 115, 035302 (2015)

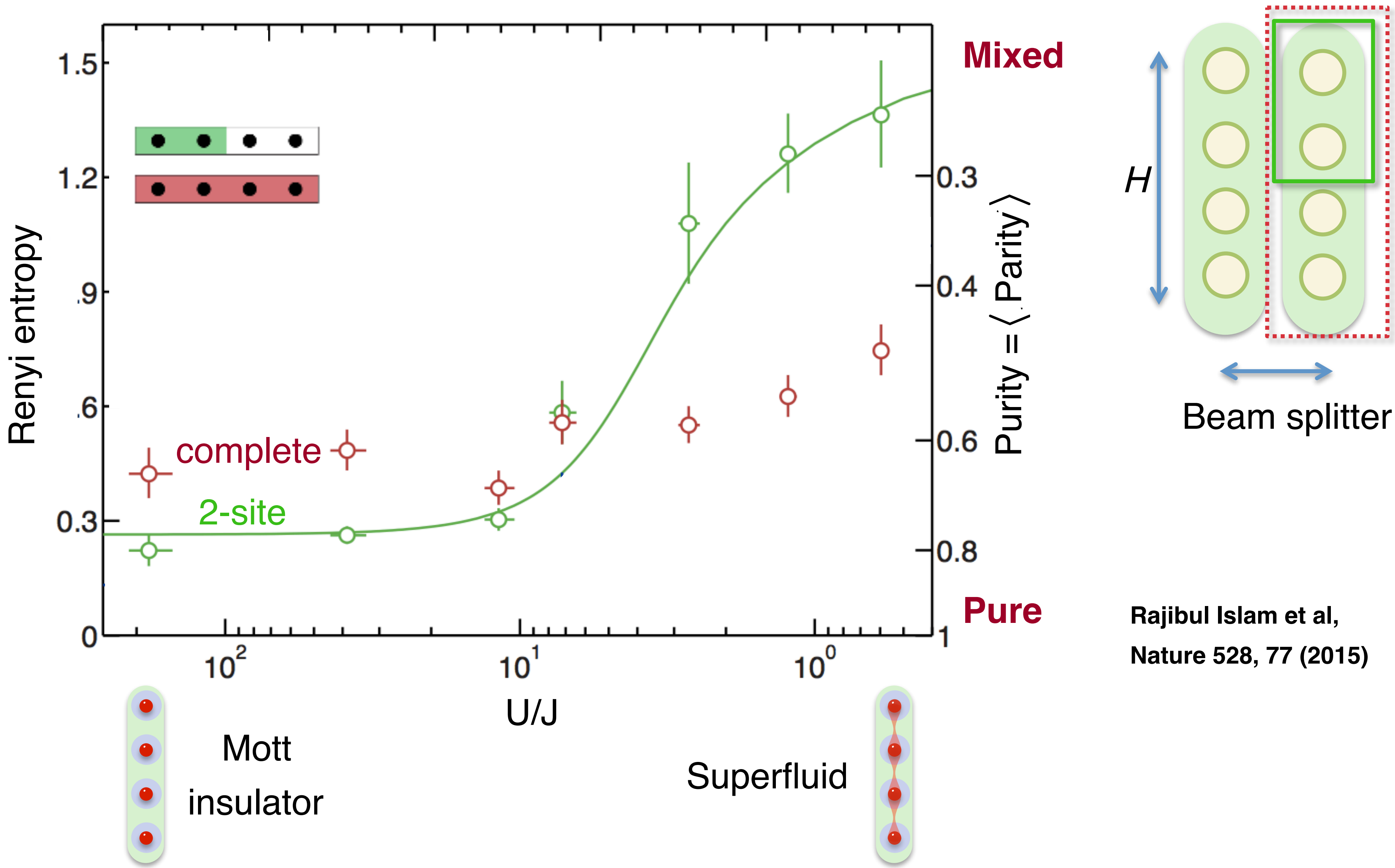
Entanglement in the ground state of a Bose-Hubbard system





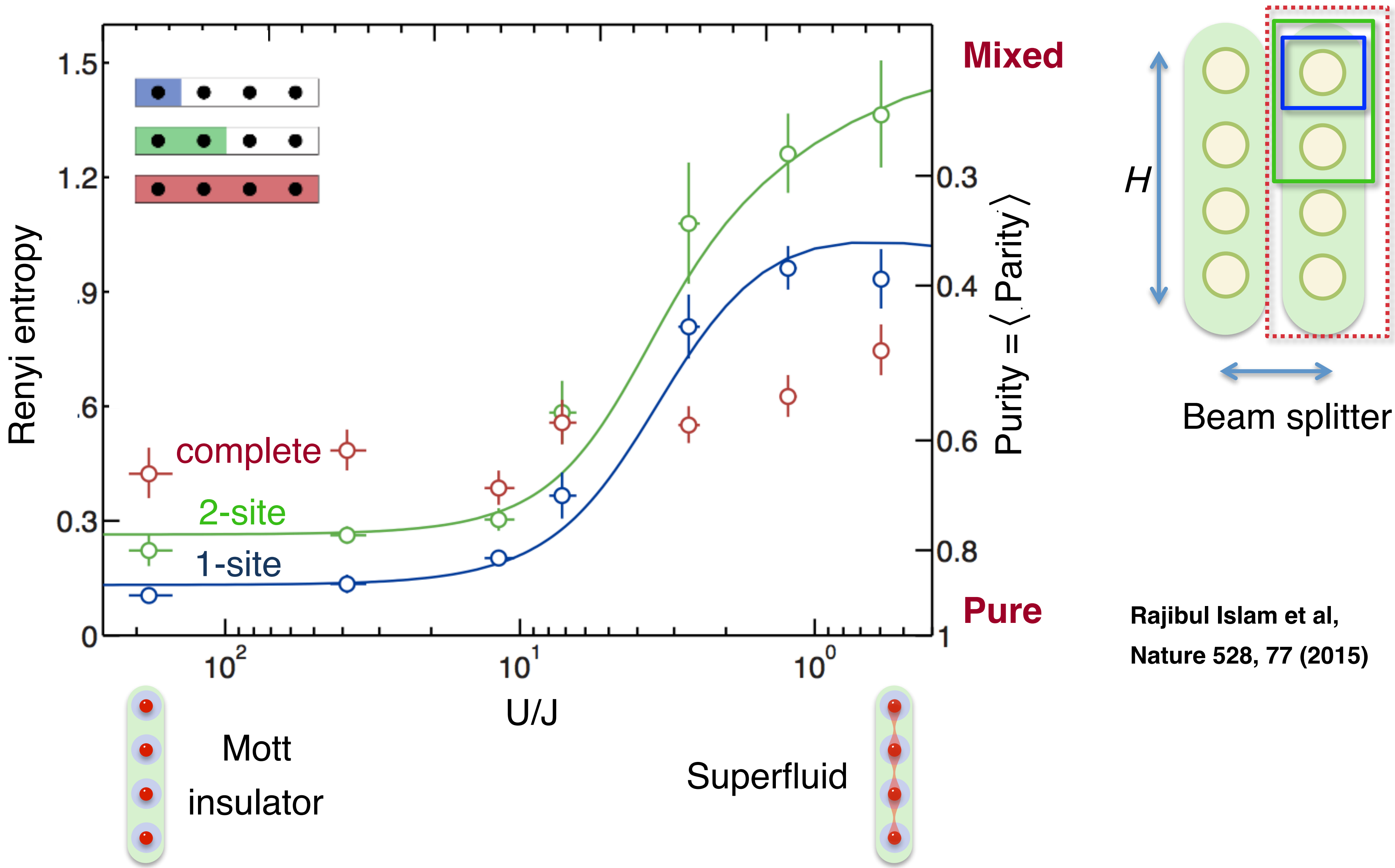
Rajibul Islam et al,
Nature 528, 77 (2015)

Entanglement in the ground state of a Bose-Hubbard system



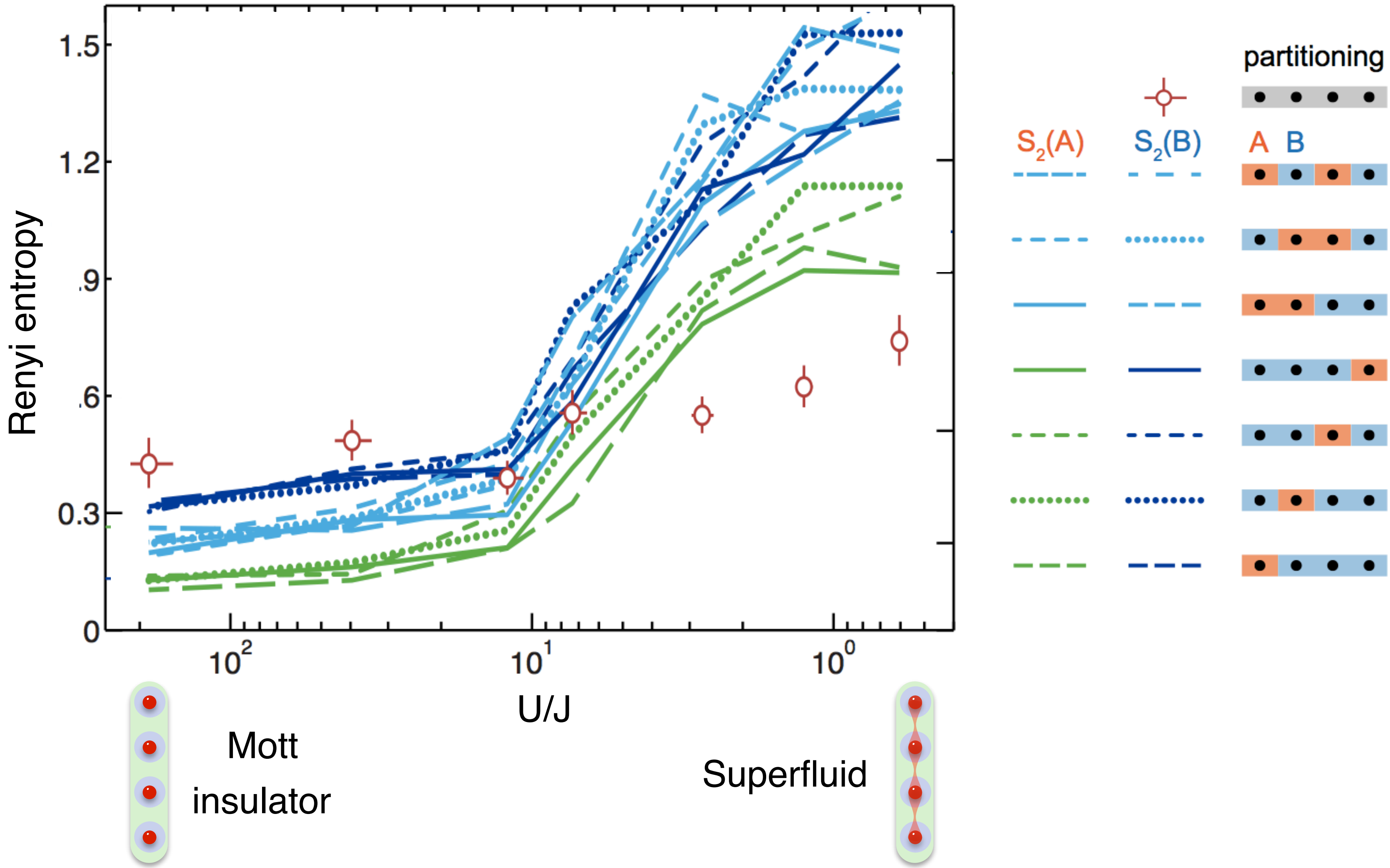
Rajibul Islam et al, Nature 528, 77 (2015)

Entanglement in the ground state of a Bose-Hubbard system

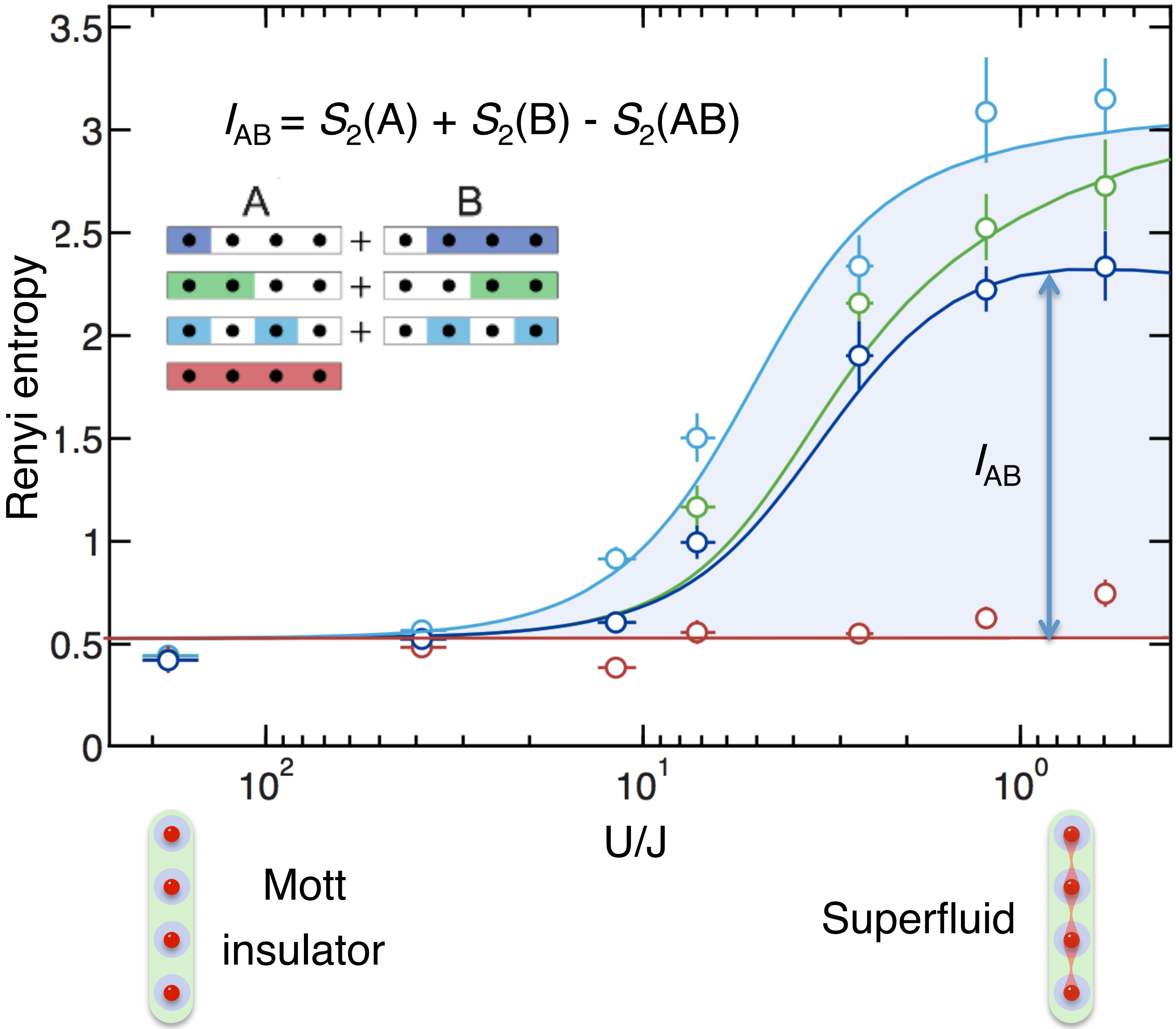


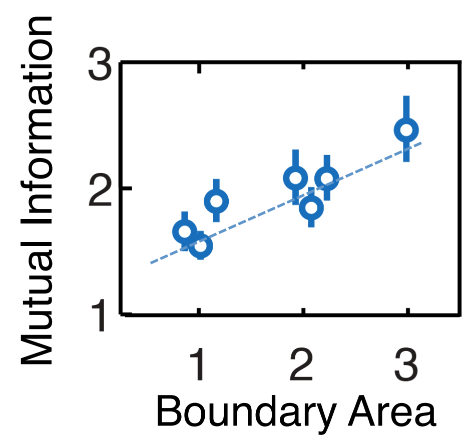
Rajibul Islam et al, Nature 528, 77 (2015)

Entanglement in the ground state of a Bose-Hubbard system

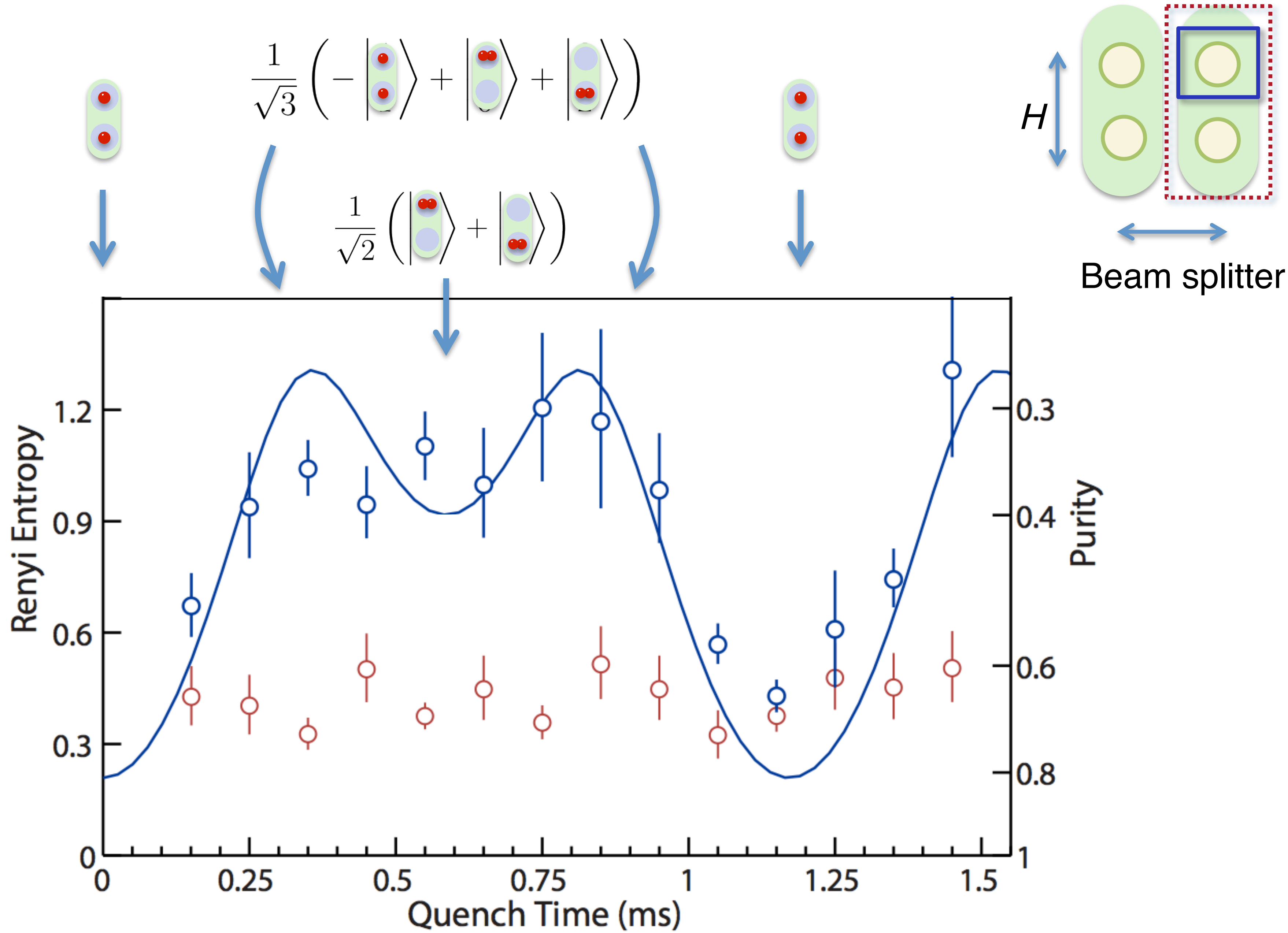


Mutual Information I_{AB}

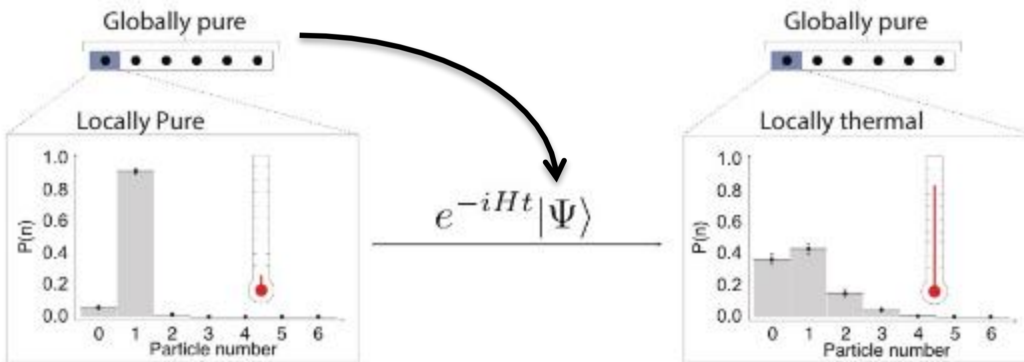




Non equilibrium: Quench dynamics

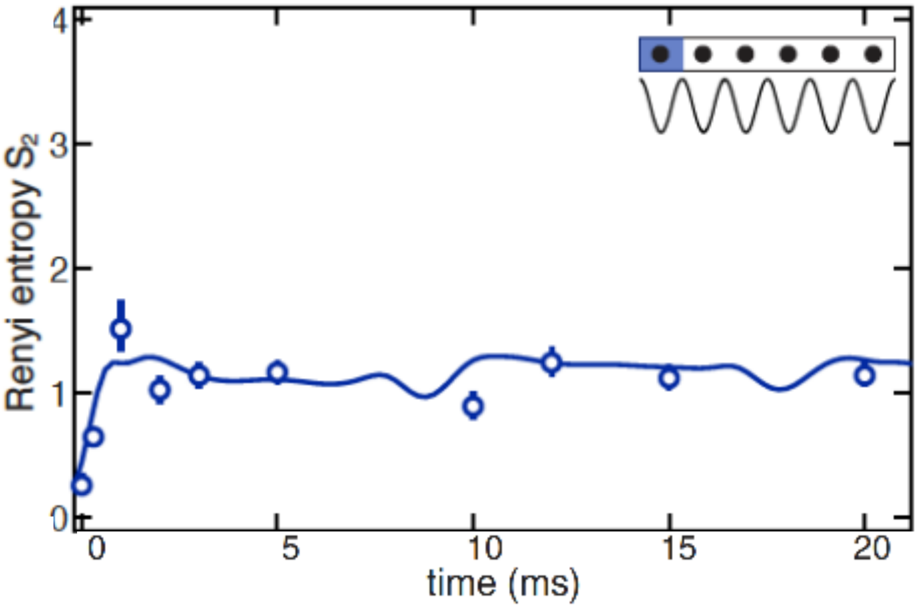


Probing thermalization of a pure state

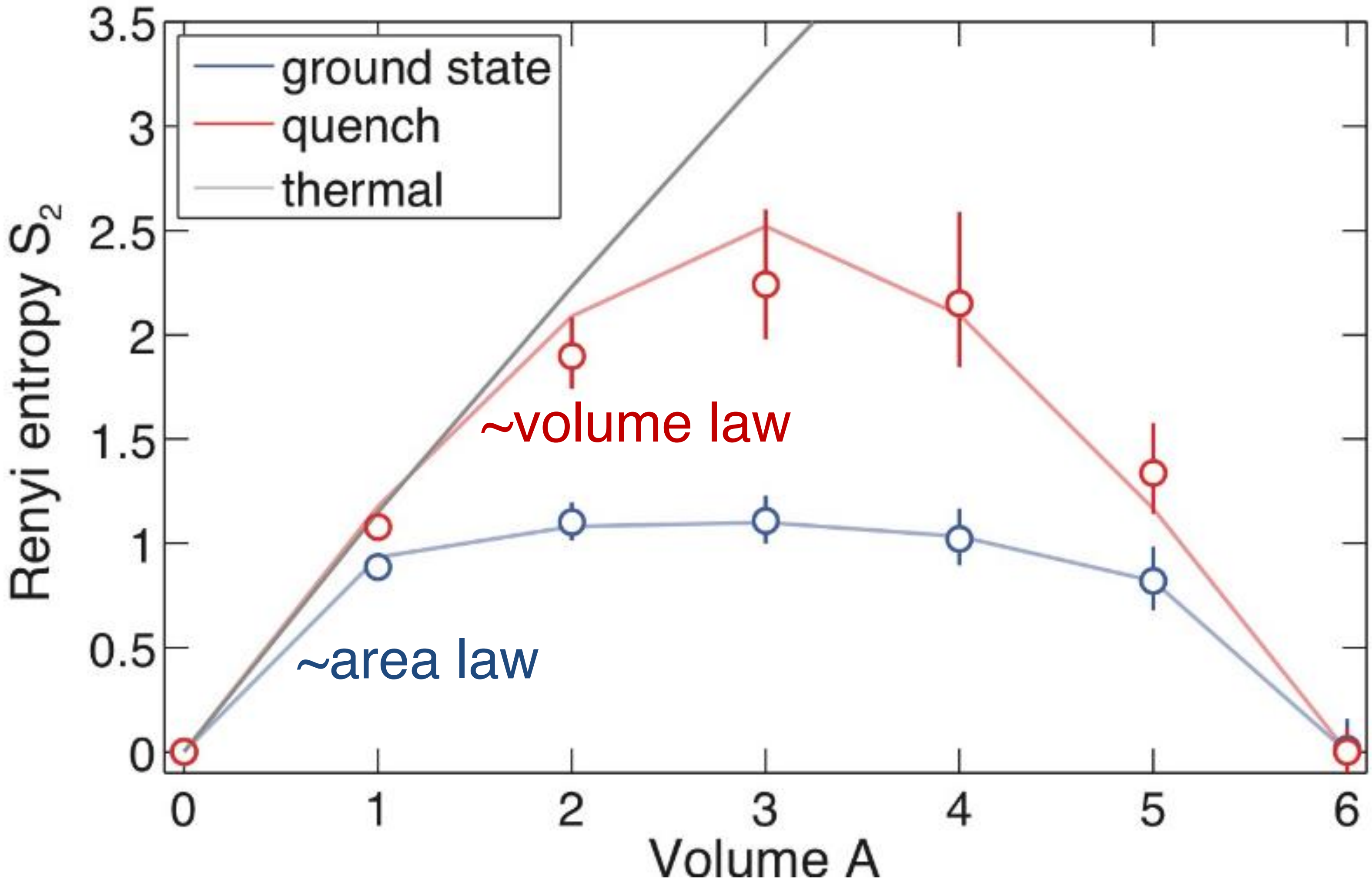


Kaufman, A. et al
Science ***353***, 794 (2016)

Non equilibrium: Quench dynamics



Approximate scaling laws on six sites



Local canonical (thermal) statistics ~ local statistics from entanglement

Calabrese…Cardy, J.Stat.Mech.0504:P04010; **Deutsch…Sharma, PRE 87, 042135 (2013)**; Santos...Rigol, PRE 86, 010102 (2013); Eisert…Plenio, 82, 277 (2010)

Thank You!