Universal logical gate sets with constant-depth circuits for topological and hyperbolic quantum codes

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A fundamental question in the theory of quantum computation is to understand the ultimate space-time resource costs for performing a universal set of logical quantum gates to arbitrary precision. To date, common approaches for implementing a universal logical gate set, such as schemes utilizing magic state distillation, require a substantial space-time overhead.

In this work, we show that braids and Dehn twists, which generate the mapping class group of a generic high genus surface and correspond to logical gates on encoded qubits in arbitrary topological codes, can be performed through a constant depth circuit acting on the physical qubits. In particular, the circuit depth is independent of code distance d and system size. The constant depth circuit is composed of a local quantum circuit, which implements a local geometry deformation, and a permutation of qubits. When applied to anyon braiding or Dehn twists in the Fibonacci Turaev-Viro code based on the Levin-Wen model, our results demonstrate that a universal logical gate set can be implemented on encoded qubits in O(1) time through a constant depth unitary quantum circuit, and without increasing the asymptotic scaling of the space overhead. Our results for Dehn twists can be extended to the context of hyperbolic Turaev-Viro codes as well, which have constant space overhead (constant rate encoding). This implies the possibility of achieving a space-time overhead of O(d/log d). From a conceptual perspective, our results reveal a deep connection between the geometry of quantum many-body states and the complexity of quantum circuits.

References: *arXiv:1806.06078*, *arXiv:1806.02358*, *arXiv:1901.11029.*