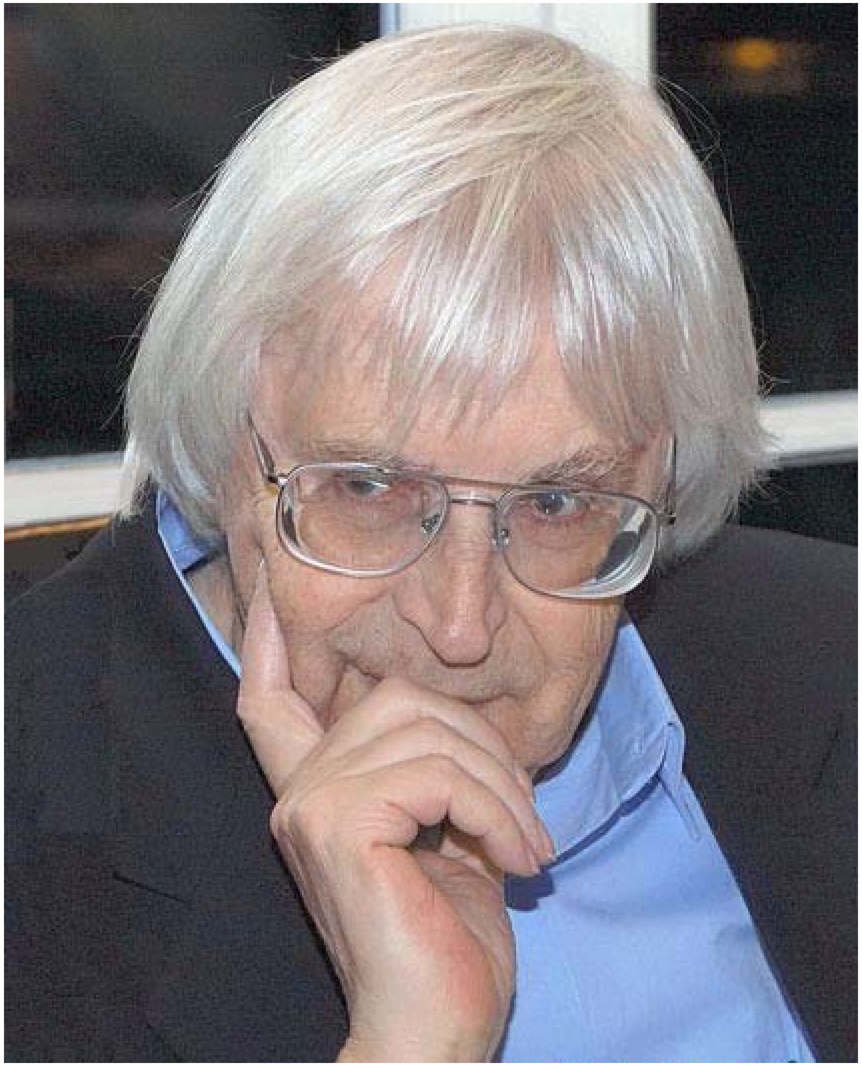
Lev Petrovich Gor‘kov

a Pioneer of the Modern Condensed Matter Theory



Thermal transport in disordered electron liquid

Georg Schwiete the University of Alabama

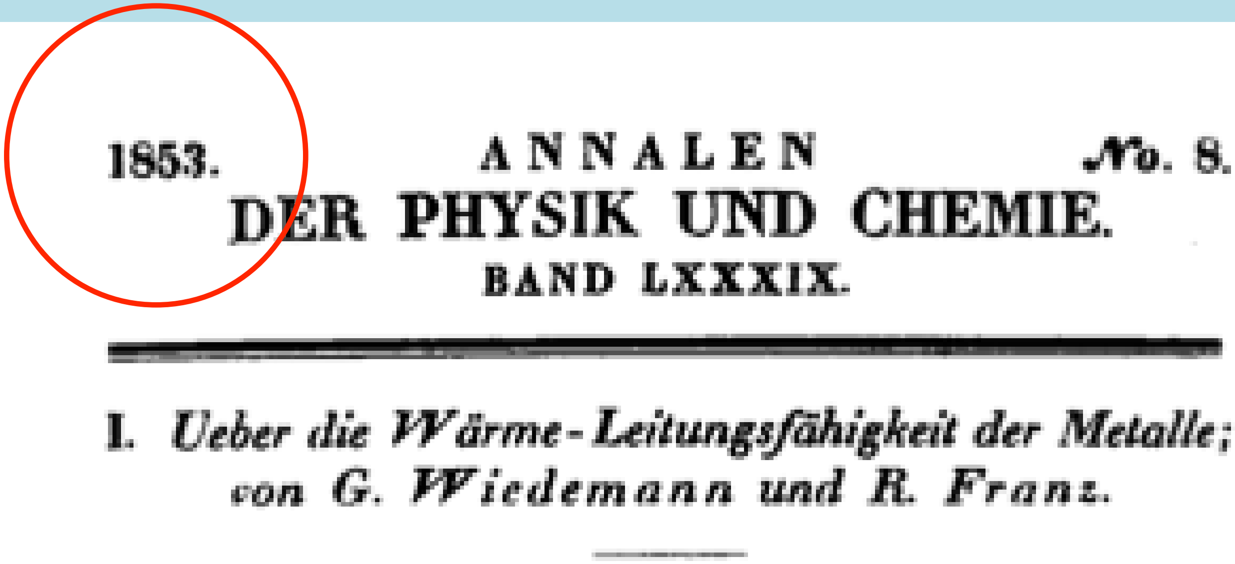
Alexander Finkel'stein

Texas A&M University and the Weizmann Institute of Science

Heat transport in a non-trivial interacting system

- How to generalize the RG approach to thermal transport?
- Wiedemann-Franz law in disordered interacting conductors

The Wiedemann-Franz law (WFL)



1853. ANNALEN №. 8.
DER PHYSIK UND CHEMIE.
BAND LXXXIX.

I. *Ueber die Wärme-Leitungsfähigkeit der Metalle; von G. Wiedemann und R. Franz.*

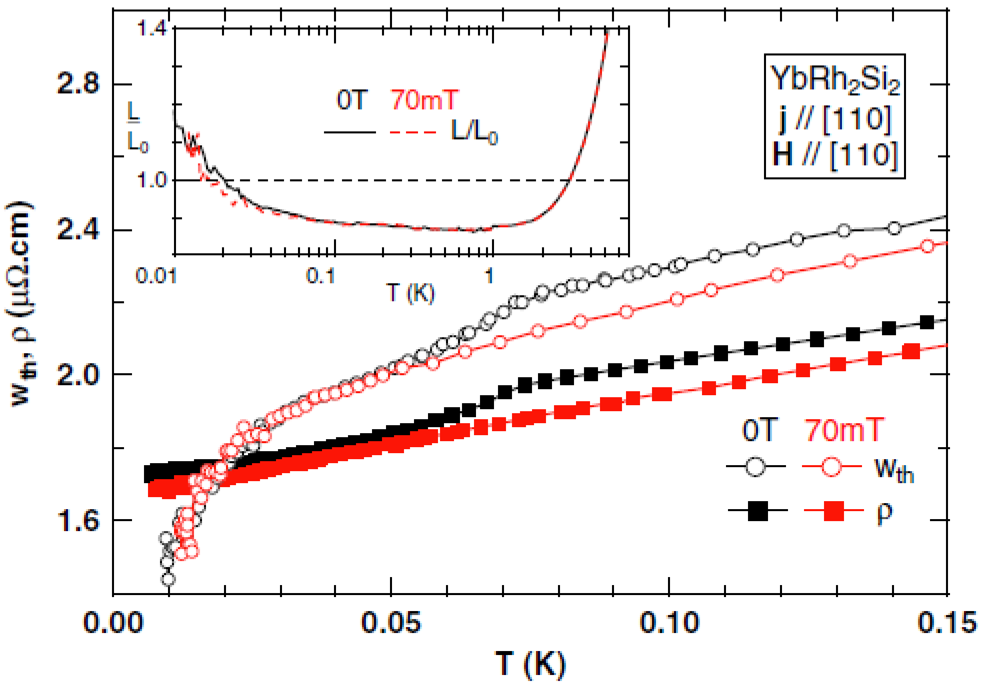
G. Wiedemann

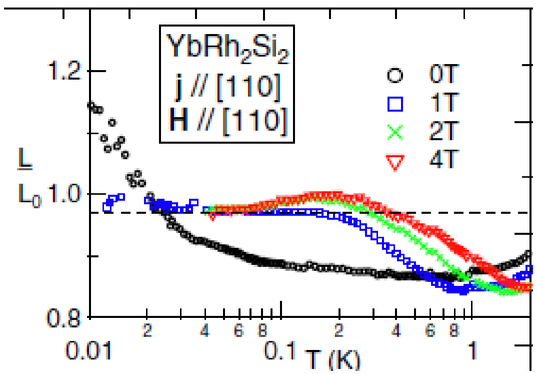
Nowadays: the WFL is used as a criterion for non-Fermi Liquid systems

The WFL as a criterion for non-Fermi Liquid systems:

Thermal Conductivity through the Quantum Critical Point in YbRh2Si2 at Very Low Temperature

M. Taupin…J. Flouquet PRL 115, 046402 (2015)





Electrons with or without magnons at very low temperature L/L_0=0.97 ??

The Wiedemann-Franz law (for heat transport, electron channel only)

Electric conductivity σ

α Seebeck coefficient

Peltier coefficient Π

$\tilde{\kappa}$ Thermal conductivity

$\mathbf{j}_e = 0$

$$\kappa = \tilde{\kappa} + \mathcal{O}\left(\left(T/\varepsilon_F\right)^2\right)$$

$\mathbf{E} = 0$

The Wiedemann-Franz "law"

$$\kappa = \mathcal{L}_0 \sigma T$$

Lorenz number

$$\mathcal{L}_0 = \frac{\pi^2}{3}\frac{k_B^2}{e^2} = \frac{c_{FL}}{2\nu e^2 T}$$

Unlike the Onsager relation, the Wiedemann Franz law is an approximate relation for itinerant electron systems. What is the range of validity?

Naïve arguments: heat and charge are transported together by the same carriers. Hence, the WFL

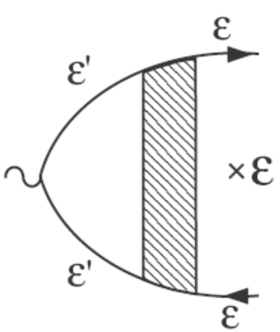
However, energy current is a not conserving quantity, while momentum is.

Inelasticity? **Nernst theorem?**

Technical complications:

how to get from the “entrance” to actual transporting with effective velocity multiplied by energy.

Langer (1962)-Fermi Liquid at T→0; Karen Michaeli and AF (2009)



Energy is carried by the renormalized velocity;

$$\mathbf{j}^{con}_{e,\,h} = -i\int \frac{d\varepsilon}{2\pi}\left[\hat{\mathbf{v}}(\varepsilon)\,\chi_{e,h}(\varepsilon)\,\hat{G}(\varepsilon)\right]^{<}$$

The renormalized velocity

For the Fermi Liquid the WF law automatically fulfills

$$\chi_h(\varepsilon) = \varepsilon$$

$$\chi_e(\varepsilon) = -e \quad ,$$

Heat transport in a non-trivial interacting system (2D electron liquid)

At low temperatures the Wiedemann-Franz Law is violated: there are additional corrections to the heat conductivity

Can one construct a comprehensive theory (including RG and additional log-corrections) ?

What is the ultimate fate of the WFL in a 2D metallic system at low temperatures?

i.e. in the presence of strong quantum corrections....

RG with a temperature gradient

How to approach the problem?

Kinetic equation approach? Including a temperature gradient is straightforward, but how to do RG?

NLσM? how to account for a temperature gradient?

$$\mathrm{Nl}\sigma\mathrm{M} \overset{source\ \varphi}{\longrightarrow} \langle nn \rangle \overset{Einstein}{\longrightarrow} \sigma$$

renormalize the NLσM with "gravitational potentials"

$$\mathrm{Nl}\sigma\mathrm{M} \overset{source\ ??}{\longrightarrow} \langle kk \rangle \overset{Einstein}{\longrightarrow} \kappa$$

Luttinger's „gravitational potential(s)" mimics/induces temperature variation;

Tolman - Ehrenfest effect (1930)

Tolman:
"On the weight of heat and thermal equilibrium in general relativity", PR 35 (1930)

Idea:
the gravitational potential couples not only to masses, but also to energy;
in the presence of gravity, temperature is not constant in equilibrium

In the Newtonian limit

$$\frac{\nabla T}{T} = \frac{\mathbf{g}}{c^2}$$

gravitational acceleration

Very large masses are needed for observable effects.
On the surface of the earth:

$$\frac{\nabla T}{T} = 10^{-16}\mathrm{cm}^{-1}$$

This idea inspired **Luttinger** to introduce an (artificial) gravitational potential in condensed matter physics to describe temperature variations.

The gravitation potentials (GPs): source fields for the heat density correlation function

hint:

$$
\begin{aligned}
& e^{-\frac{H}{T+\delta T}} \\
= \quad & e^{-\frac{H}{T(1+\delta T/T)}} \\
\approx \quad & e^{-\frac{H(1-\delta T/T)}{T}} \\
= \quad & e^{-\frac{H(1+\eta)}{T}}
\end{aligned}
$$

$$\chi_{kk} = \frac{i}{2} \frac{\delta^2 Z}{\delta \eta^2}$$

Variation of the temperature can be considered as a gravitation potential.

In equilibrium, the action of the gravitation potential can be compensated by the variation of temperature.

Einstein's relation:
response to the temperature variation

can be found from the response to the gravitation potential

Unlike electric field, the gravitational potentials act on everything

Action:

$$S[\psi^*,\psi]=\int_{\mathbf{r},t}\left(\psi^* i\partial_t\psi - k[\psi^*,\psi]\right)$$

$$\mathcal{Z}=\int D(\psi,\psi^*)\,\mathrm{e}^{iS} \qquad k=h_0+h_{int}-\mu n$$

$$S[\psi^*,\psi]=\int_{\mathbf{r},t}\left(\psi^* i\partial_t\psi - (1+\eta)k[\psi^*,\psi]\right)$$

$$\chi_{kk}=\frac{i}{2}\frac{\delta^2\mathcal{Z}}{\delta\eta_{\mathbf{r}_1 t_1}\delta\eta_{\mathbf{r}_2 t_2}}$$

Gravitational potential

Luttinger (1964)

Source fields for the heat-density correlation function

$$S[\psi^*,\psi] = \int_{\mathbf{r},t} (\psi^* i\partial_t \psi - (1+\eta) k[\psi^*,\psi])$$

Problem:

$$S_{dis} = -\int_{\mathbf{r},t} (1+\eta)\psi^* u_{dis} \psi$$

$$\psi \to \frac{1}{\sqrt{1+\eta}}\psi \qquad \psi^* \to \psi^* \frac{1}{\sqrt{1+\eta}}$$

After this transformation, the derivation of the NLσM is straightforward:

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q)^2 + 2iz\{\hat{\epsilon},\lambda\}Q\right] + Q\lambda(\Gamma_1+\Gamma_2)Q$$

$$\lambda = \frac{1}{1+\eta} \approx 1 - \eta + \eta^2 + \dots$$

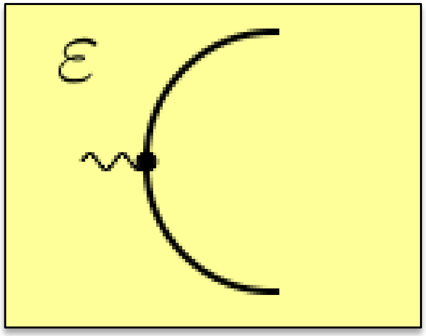
nonlinear in η !

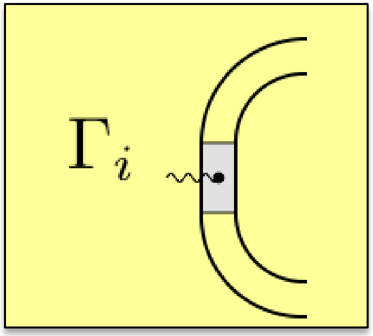
Keldysh NLσM extended by the "gravitation potentials"

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q)^2 + 2iz\{\hat{\epsilon}, \lambda\}Q\right] + Q\lambda(\Gamma_1 + \Gamma_2)Q$$

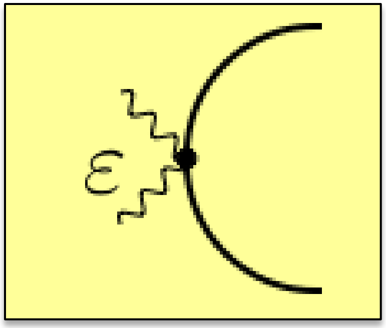
$$\lambda \approx 1 - \eta + \eta^2$$

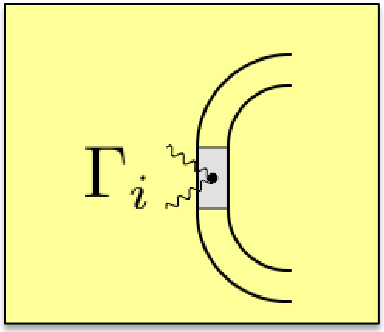
η





η^2

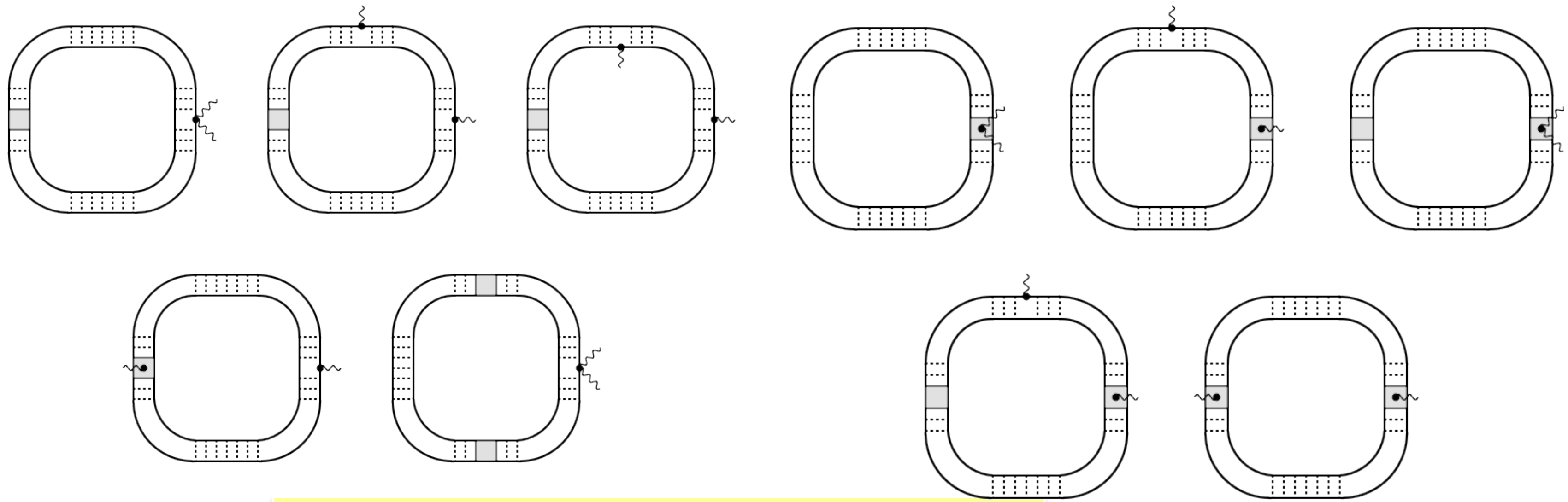




$$\chi_{kk} = \frac{i}{2} \frac{\delta^2 \mathcal{Z}}{\delta\eta_{\mathbf{r}_1 t_1} \delta\eta_{\mathbf{r}_2 t_2}}$$

The heat-density correlation function and the specific heats

Crosscheck: **The terms linear and quadratic in the gravitational potentials η are consistent with each other and also with thermodynamics**



$$\chi^{st}_{kk} = \frac{i}{2}\frac{\delta^2 Z}{\delta\eta^2} = -Tc$$

$$C = zC_{FL}$$

RG and sub-*T* region; the WFL and its violation

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}[D(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, \lambda\}Q] + \sum_i Q\lambda\Gamma_i Q$$

Energy scales

$$\lambda \approx 1 - \eta + \eta^2$$

ε_F

$\frac{1}{\tau}$

RG interval

Fixed point in the space of the gravitation potentials (GPs)

T

sub-T regime

beyond the RG interval

$\frac{T^2}{D\kappa_s}$

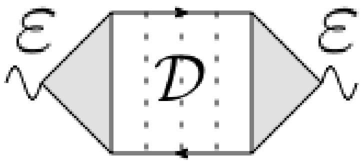
Two-stage calculation

1) The WFL holds despite strong renormalizations **(Rome-Boston 1988)**

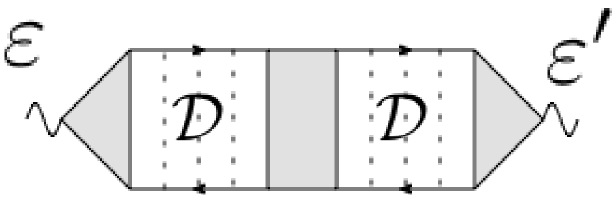
2) ~~WFL~~

Additional logarithmic corrections from the *imaginary part of* the *Coulomb* interaction

Dynamic part of the heat-density correlation function: illusion of simplicity (and the origin of the naïve WFL)



$$\varepsilon^2 \to T^2$$



$$\varepsilon \Gamma \varepsilon' \to 0$$

Rescattering is uneffective for constant amplitudes Γ !

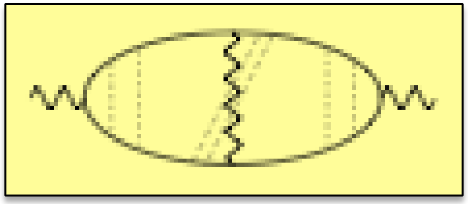
$$\int_\varepsilon \varepsilon \left[\mathcal{F}\left(\varepsilon+\frac{\omega}{2}\right) - \mathcal{F}\left(\varepsilon-\frac{\omega}{2}\right)\right] = 0$$

Finite result only for $\Gamma = \Gamma(\varepsilon, \varepsilon')$

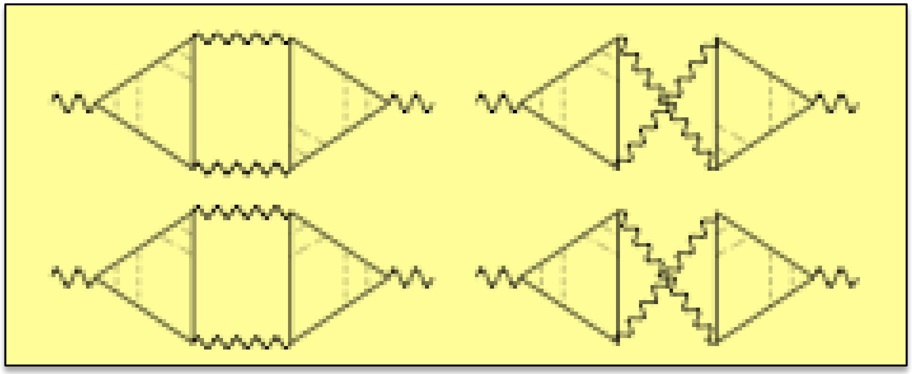
Such contributions are not considered in the Fermi Liquid and even in traditional RG scheme ! (off-shell versus on-shell)

Coulomb problem - Violation of the WFL

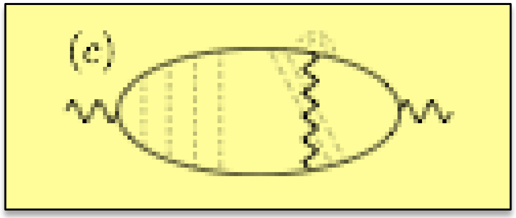
vertical diagrams



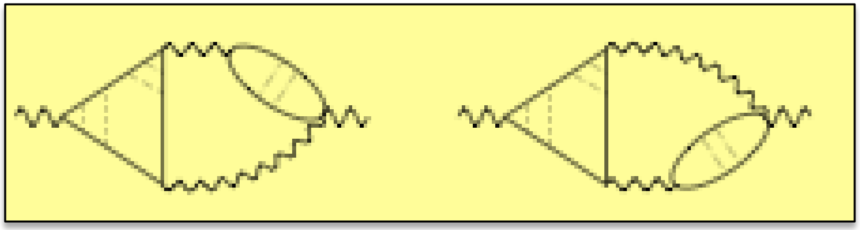
drag diagrams



regular vertex corrections



anomalous vertex corrections



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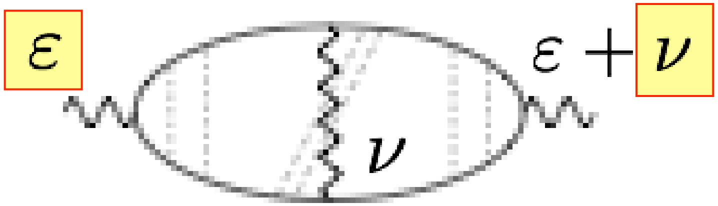
why vertical?

$\varepsilon - \varepsilon$ WFL

$\varepsilon - \nu$ ~~WFL~~



Coulomb problem - Violation of the WFL

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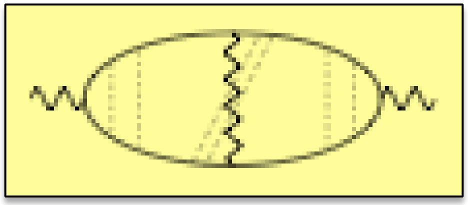

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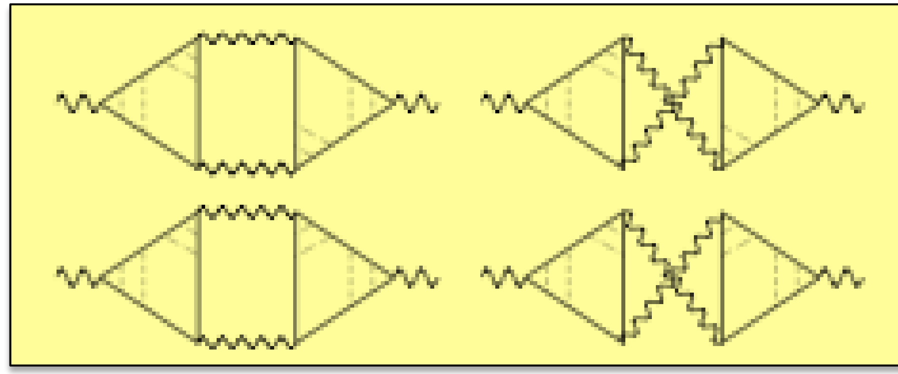
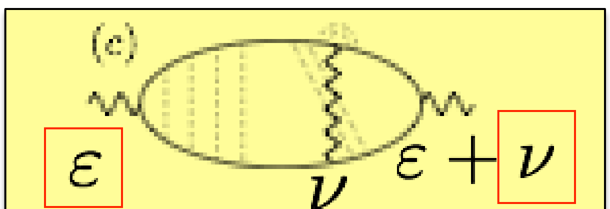


regular vertex corrections



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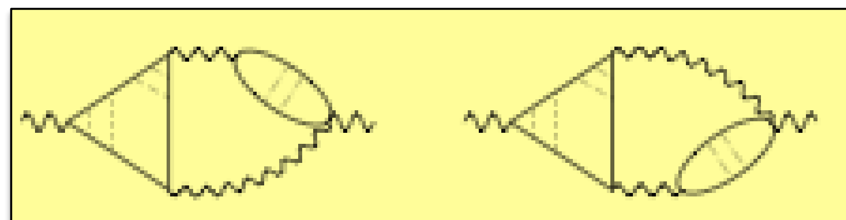


All aditional contributions

are proportional to Im(U^R): Decay into particle-hole pairs.

Example:

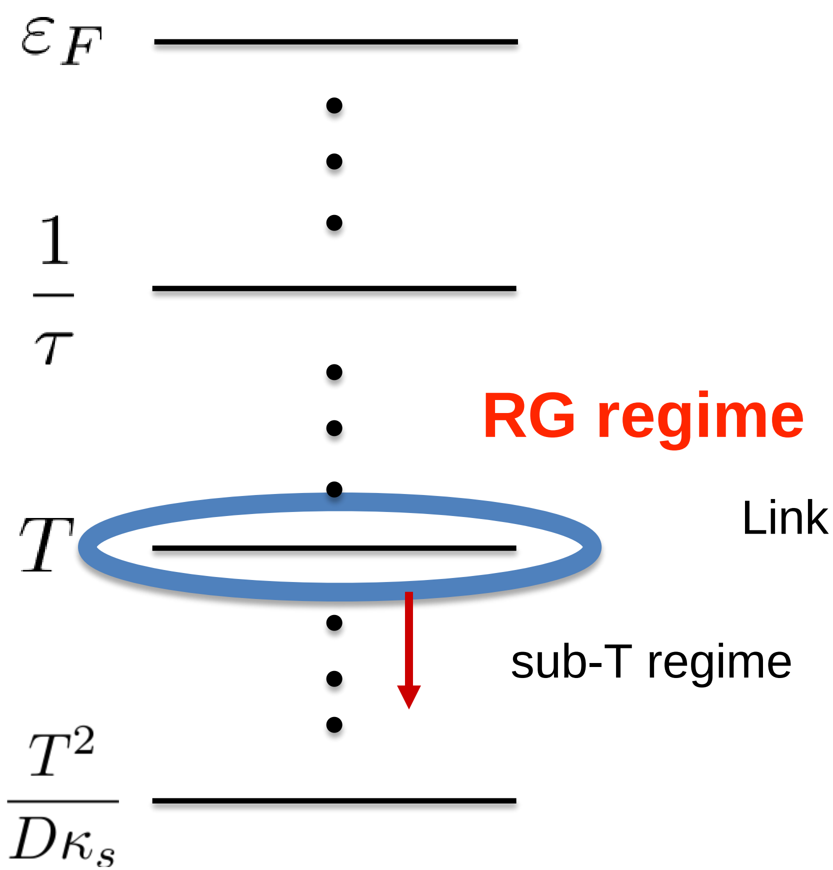
$$\delta\chi_{kk} \propto \int_{\mathbf{k},\varepsilon,\nu} \varepsilon\nu\partial_\varepsilon F_\varepsilon (F_{\varepsilon+\nu} + F_{\varepsilon-\nu}) \mathrm{Re}\mathcal{D}^2(\mathbf{k},\nu)\mathrm{Im}U^R(\mathbf{k},\nu)$$

Two stage RG procedure –
how does the renormalization affect δκ ?

the correction is positive:

$$\delta\kappa = \frac{T}{12}\log\frac{D\kappa_s^2}{T}$$

κ_s: screening radius



We checked that
all Fermi-liquid renormalizations,
the parameter z and all other
RG renormalizations
drop out when calculating δκ.

We thereby expect that the answer obtained for δκ is final.

RG-calculations

Result: ***Fixed point in the multi-parametric space of the gravitation potentials (a very special structure)***

$$\Delta\zeta_D = \Delta\zeta_z = \Delta\zeta_{\Gamma_1} = \Delta\zeta_{\Gamma_2} = 0$$

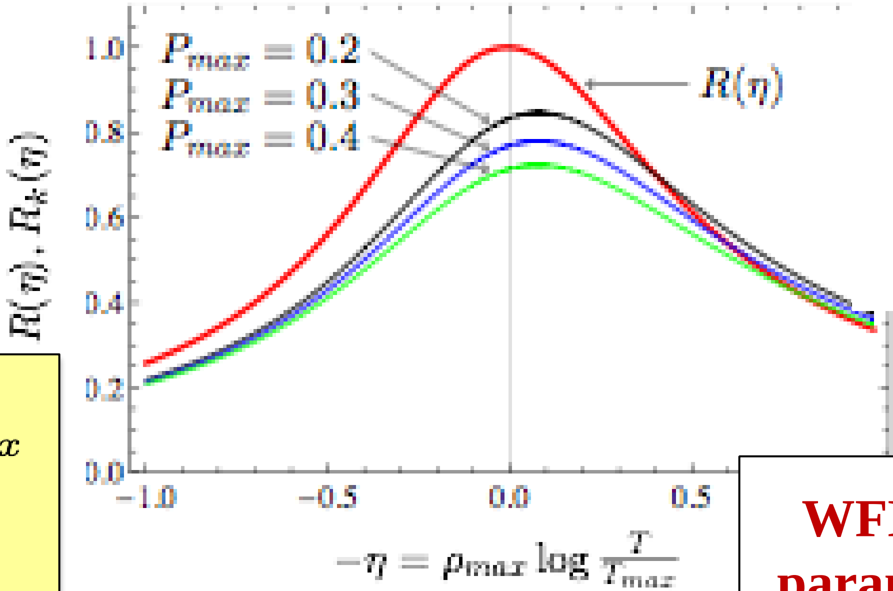
$$\zeta_D = 0 \qquad \zeta_z = \zeta_{\Gamma_1} = \zeta_{\Gamma_2} = -\eta$$

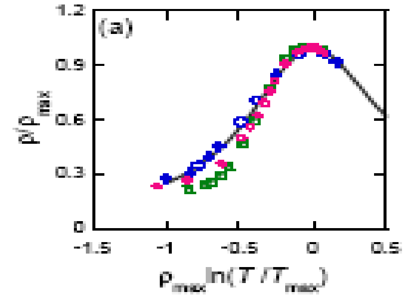
it holds only for the correct initial conditions

$$S = \int \mathrm{tr}[D(1+\zeta_D)(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, 1+\zeta_z\}Q] + \sum_{i=1,2} Q(1+\zeta_{\Gamma_i})\Gamma_i Q$$

Conservation of energy, which is "encrypted" in the NLsM

Application: Thermal transport on the metallic side of the metal-insulator transition in Si MOSFETs





$$R_k(\eta) = \rho_k(\eta)/\rho_{max}$$

$$\rho_k = \frac{e^2}{2\pi^2}\frac{\mathcal{L}_0 T}{\kappa}$$

WFL violation parametrized by:

$$P_{max} = \frac{\rho_{max} - \rho_k(0)}{\rho_k(0)}$$

R(η): normalized resistance

$R_k(\eta)$: normalized thermal resistance

Maximum in R_k at universal η (P_{max} - independent): η_{max}=-0.0785

Summary

- We developed a field theoretic model with „gravitational potentials“ suitable for the analysis of heat density correlation function in the disordered electron liquid;

- **For short range interactions the renormalization of κ and of σ are linked through the WF law.** **Fixed point for the gravitational potentials:**

$$\Delta\zeta_D = \Delta\zeta_z = \Delta\zeta_{\Gamma_1} = \Delta\zeta_{\Gamma_2} = 0$$

- For long-range (Coulomb) interaction there are additional logarithmic corrections originating from energies smaller than of the RG-interval. They lead to a violation of the WF law.

- Additional corrections are not renormalied neither by Fermi Liquid, nor the RG corrections. An element of universality has been found for the thermal resistance

Georg Schwiete & AF:
Phys. Rev. B **93** (2016) **Coulomb interaction**;
JETP (Keldysh issue) (2016) Theory of Thermal Conductivity in the Disordered Electron Liquid

Lev Petrovich Gor‘kov