Pedagogical Overview

Unconventional Superconductivity (The Phenomenon) and Unconventional Superconductors (The Materials)

- Conventional Superconductivity
- Unconventional Superconductivity and Superconductors
- Some Personal Favorites
Superconductivity arises when phases lock
\[ \varphi_1 = \varphi_2 = \varphi_3 = \ldots \varphi_i \ldots = \varphi \]

\[ \Psi_{p_i} = \phi_{\text{int}}(\rho_i) \psi_{\text{cm}}(r_i) = \phi_{\text{int}}(\rho_i) \psi_{\text{cm}}(r_i) e^{\varphi_i} \]

**Microscopic Specific Properties**
- \( q = 2 \)
- Pairing symmetry, pair size
- Single particle excitations
- Density of states
- Material parameters of GL theory
- The mechanism

**Macroscopic Emergent Properties**
- GL Order parameter/GL theory
- \( R = 0, B = 0, \Phi = \Phi_0 = h c / q \)
- Type II superconductivity
- Josephson effect
Some History

Adapted from a DoE Report

Conventional Era
- GL Theory, BCS, Gorkov and Eliashberg

Unconventional Era
- BCS Theory
- HgBaCaCuO @ 30 GPa: record T_c at 164 K
- TiBaCaCuO
- BiSrCaCuO
- HgBaCaCuO
- HgTIBaCaCuO
- YBaCuO
- LaBaCuO
- BKBO
- RbCsC_60
- Cs_3C_60 @ 1.4 GPa
- MgB_2
- YbPd_2B_2C
- K_3C_60
- Li @ 33 GPa
- PuCoGa_5
- PuRhGa_5
- CeCoul_9
- CaC_6
- CeCoul_6
- YbC_6
- diamond

Conventional Superconductivity
- Nighttime on the Moon
- Liquid nitrogen
- Surface of Pluto
- Liquid neon
- Liquid hydrogen
- Liquid helium
- CNT
Key Results of BCS Theory

- Superconductivity arises when electrons bind into pairs and these pairs lock their mutual phases to form a macroscopic quantum state. \((k \uparrow, -k \downarrow)\)

- The single-particle excitations of a superconductor are comprised of a phase-coherent linear combinations of electrons and holes. \(\psi_k^s = u_k \psi_k^e + v_k e^{i\phi} \psi_k^h\)

- \(T_c = \Omega o e^{-1/\lambda}\), which implies that if no other phases intervene, all normal metals will become superconductors as \(T \to 0\). Here \(\lambda\) is the attractive interaction parameter (not necessarily the electron-phonon interaction).

- Size of Cooper pair \(\xi \approx \hbar v_F / kT_c\)

In general, time reversed pairs (Anderson)
The Triumvirate Defining Conventional Superconductivity

A Complete Theory (BCS + Mechanism + Equations for the Material Parameters)

\[ F(r) = \int d^3r \left\{ \alpha |\psi|^2 + \frac{1}{2} \beta |\psi|^4 \frac{\hbar^2}{2m^*} \left( -i\nabla - \frac{e^*}{\hbar c} A \right) \psi \right\}^2 + \frac{B^2}{8\pi} \]

\[ T_c = \Theta_0 e^{\frac{-1}{\mu^*}} \]

\[ \mu^* = \frac{\mu}{1 + \mu \ln \left( \frac{\varepsilon_F}{\omega_D} \right)} \]
# Unconventional Superconductivity

<table>
<thead>
<tr>
<th>Conventional Superconductivity</th>
<th>Unconventional Superconductivity</th>
<th>Even More Unconventional Superconductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak coupling $\lambda \ll 1$ $T_c = \Theta_0 e^{\frac{-\lambda}{\lambda}}$</td>
<td>Strong coupling $\lambda &gt; 1$ Maximum in $T_c$</td>
<td>Very strong coupling $\lambda &gt;&gt; 1$ Phase fluctuations – $T_c \to 0$</td>
</tr>
<tr>
<td>Mean field theory</td>
<td>Quantum fluctuations when $\xi$ is small $\xi \approx \hbar v_F / kT_p$</td>
<td>Bose metal of pairs? $T_c = \min{T_\phi, T_p}$</td>
</tr>
<tr>
<td>S-wave singlet Time reversed pairs</td>
<td>Higher angular momentum pairing $p$, $d$, ...... pairing $\Delta(k) = \int dk^3 V(k,k')\Delta(k')$</td>
<td>S-wave triplet odd-frequency pairing Broken TRS pairs – $s + i s$, $d + i s$, etc</td>
</tr>
<tr>
<td>Retarded el-ph interaction $T_c = e^{\frac{\lambda - \mu^<em>}{\lambda - \mu^</em>}}$</td>
<td>Any other mechanism: Electronic interactions -- spin or charge</td>
<td>Non-Fermi liquid in normal state Quantum critical points Competing ordered phases</td>
</tr>
<tr>
<td>Anderson’s theorem $T_c$ independent of disorder</td>
<td>Enhanced coulomb repulsion due to disorder $T_c = \Theta_0 e^{\frac{\lambda - \mu^* - \delta \mu^<em>}{\lambda - \mu^</em>}}$</td>
<td>Disorder driven Superconductor/Insulator Transition</td>
</tr>
<tr>
<td>Conventional Classic BCS and Extensions</td>
<td>Unconventional Superconductivity</td>
<td>More Unconventional Superconductivity</td>
</tr>
<tr>
<td>----------------------------------------</td>
<td>----------------------------------</td>
<td>---------------------------------------</td>
</tr>
<tr>
<td>Single band superconductivity</td>
<td>Weakly coupled bands/orbitals</td>
<td>Josephson effects in $k$ space</td>
</tr>
<tr>
<td>Q = 0 pairing</td>
<td>Finite Q pairing</td>
<td>Pair density waves</td>
</tr>
<tr>
<td>Quasi-particles</td>
<td>SN proximity effect</td>
<td>Majorana Fermions</td>
</tr>
<tr>
<td>Phase coherent superposition of $e$’s and $h$’s</td>
<td>In the presence of a Dirac point</td>
<td></td>
</tr>
<tr>
<td>Earthly</td>
<td></td>
<td>Other worldly</td>
</tr>
</tbody>
</table>
Unconventional Superconductivity Along the El-Ph Line

Adapted from a DoE Report
Note similarity to the phase diagrams of the cuprate superconductors but where the ordered insulating state is in the charge sector.
Charge-Disproportionated (Negative U) Superconductors (e.g., BaBiO$_3$)

A Failure of LDA Theory → A new class of correlated insulator

2$\text{Bi}^{4+}(4s^1) \rightarrow \text{Bi}^{3+}(4s^2) + \text{Bi}^{5+}(4s^0)$

and

Oxygen atoms move to screen charge (Breathing mode)

Superconductivity arises upon doping ($\text{Ba}_{1-x}\text{K}_x\text{BO}_3$ and $\text{BaPb}_{1-x}\text{Bi}_x\text{O}_3$)
Disorder in the High $T_c$ Bismuthates

Conventional theory for $T_c$

- **Clean limit (BCS/Eliashberg):**
  \[ T_{c0} = \Theta_0 e^{\frac{-1}{\lambda - \mu^*}} \]
  
  $\lambda = el$-ph interaction parameter
  $\mu^* = retarded$ coulomb repulsion

- **Dirty limit (BCS/Eliashburg/Fukuyama):**
  \[ T_{cd} = \Theta_0 e^{\frac{-1}{\lambda - \mu^* - \delta\mu^*}} \]
  
  $\delta\mu^* = increase$ in $\mu^*$ due to disorder-induced localization

Measured $T_c = T_{cd}$ and independently estimated $\delta\mu^*$ permits estimate of $T_{c0}$

Disorder matters in region of highest $T_c$
Heavy Fermions and Magnetic Mechanisms

Adapted from a DoE Report
Cuprates and Strong Correlation – A Mixed Blessing

Adapted from a DoE Report
In Nature the Transition Temperature Can Be “Astronomically” High

FIG. 1. (Color online) A schematic outline for the phase diagram of matter at ultrahigh density and temperature. The CFL phase is a superfluid (like cold nuclear matter) and has broken chiral symmetry (like the hadronic phase).
ALABAMA: NASA’s Chandra X-ray Observatory has discovered the first direct evidence for a superfluid - a bizarre, friction-free state of matter - at the core of a neutron star.

Superfluids created in laboratories on Earth exhibit remarkable properties, such as the ability to climb upward and escape airtight containers. The finding has important implications for understanding nuclear interactions in matter at the highest known densities.

“The rapid cooling in Cas A’s neutron star, seen with Chandra, is the first direct evidence that the cores of these neutron stars are, in fact, made of superfluid and superconducting material,” said lead author Peter Shternin of the Ioffe Institute in St Petersburg, Russia, of a paper accepted in the Monthly Notices of the Royal Astronomical Society.

Unusual rapid decline in temperature

Neutron stars contain the densest known matter that is directly observable. One teaspoon of neutron star material weighs six billion tonnes. The pressure in the star’s core is so high that most of the charged particles, electrons and protons, merge resulting in a star composed mostly of uncharged particles called neutrons.
Any Room Temperature Superconductor Will be Extremely Unconventional

Local pairing, small unit cell and 3D (to minimize phase fluctuations), strong interactions, mean-field breakdown, not a single band.
If you want to find something unconventional, don’t look under the street light.
## Empirical Guidance on Specific Interactions

<table>
<thead>
<tr>
<th>Material Archetype</th>
<th>$T_c$</th>
<th>Interaction</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bismuthates</td>
<td>30K</td>
<td>Charge + Lattice</td>
<td><strong>Doped Negative U Insulator</strong></td>
</tr>
<tr>
<td>(i.e., doped BaBiO$_3$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cs$<em>3$C$</em>{60}$</td>
<td>40K</td>
<td>Lattice + Correlation (charge)</td>
<td><strong>El-Ph Covalent Bonds</strong></td>
</tr>
<tr>
<td>MgB$_2$</td>
<td>40K</td>
<td>Lattice</td>
<td><strong>El-Ph Covalent Bonds Prediction</strong></td>
</tr>
<tr>
<td>Fe-Based</td>
<td>50K</td>
<td>Spin</td>
<td><strong>Antiferromagnetism</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Multiple orbitals</strong></td>
</tr>
<tr>
<td>Cuprates</td>
<td>130K</td>
<td>Spin</td>
<td><strong>Doped Antiferromagnetic Positive U</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Mott Insulator</strong></td>
</tr>
<tr>
<td>Trace High $T_c$ Anomalies</td>
<td>&gt; Room Temperature</td>
<td>?</td>
<td><strong>Shouldn’t Ignore</strong></td>
</tr>
</tbody>
</table>

Electronic (charge and spin) interactions look good
To Determine $T_c$ There are Two Characteristic Temperatures to Consider

Superconductivity arises when electrons (or holes) form pairs and the quantum phases of these pairs order (lock) to form a coherent macroscopic quantum state with a single phase.

Each process has its own characteristic temperature.
Theoretical Guidance

An always controversial subject

- How to optimize the el-ph mechanism – Modern electronic structure calculations (It is clear that theory could have predicted the high $T_c$ superconductivity of Mg B$_2$)
- New mechanisms
- Electronic structure calculations as a tool in the search
- Strong interactions, but honor the “Goldilocks Principle”

High Temperature Superconductivity is a Crossover Phenomenon

Steven Kivelson, Stanford

E.g., from weak coupled BCS (delocalized) limit to strong coupled (local) limit. Typically also for doping.
…. And Maybe We Can Have Our Cake and Eat It Too

High $T_c$ and High Pair Density Superconductor Using a Normal Metal/Negative-U Insulator Proximity Effect:

$$H = -t \sum c_m^\dagger c_m - U \sum n_p^\uparrow n_p^\downarrow - t_\perp \sum c_m^\dagger c_p$$