Weak coupling theories of unconventional superconductivity (I)

(Superconductivity from repulsion)

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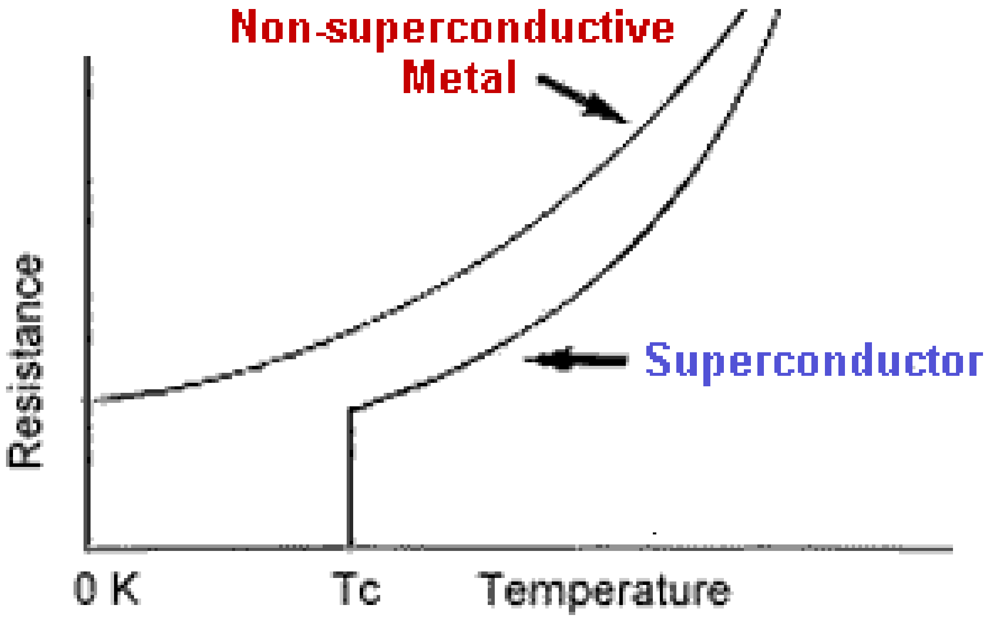
Superconductivity:

Zero-resistance state of interacting electrons



Let's start with some basics:

Consider a system of fermions with $k^2/2m$ dispersion and (screened) Coulomb interaction $U(r)$.

Single-particle (fermionic) excitations are described by the poles of the fermionic Green's function $G(k, \omega)$

For free fermions, $G(k, \omega) = 1/(\omega - v_F (k-k_F))$

In a Fermi liquid, $G(k, \omega) = Z/(\omega - v^*_F (k-k_F))$

Superconductivity is a two-particle instability of a system of interacting fermions

Collective two-particle (bosonic) excitations are described by the poles of the vertex function Γ (q,Ω) =fully renormalized interaction

Examples: sound (or zero sound) waves in a Fermi liquid

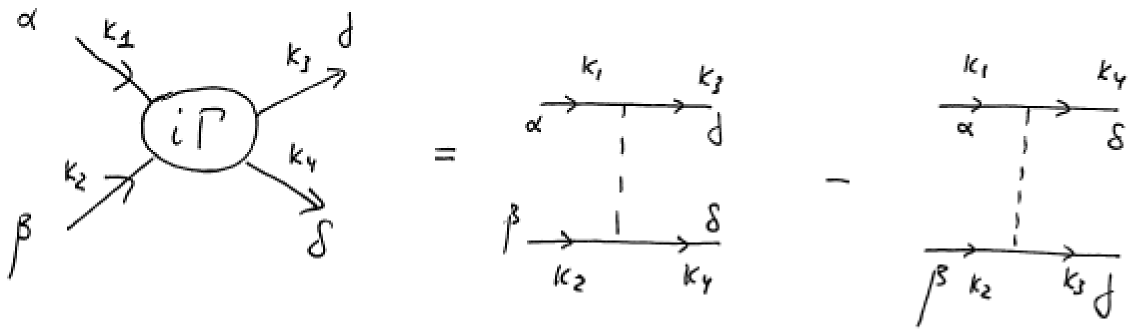
$$\Gamma \propto \frac{1}{\Omega - v_s q + i0} \quad \Rightarrow \quad \Omega = v_s q - \underline{i0}$$

Pole in the lower frequency half-plane

If Γ (q,Ω) had a pole in the upper half-plane, perturbations would increase with time and eventually destroy a Fermi liquid

Superconducting instability is of this kind

To first order in the interaction, Γ (q,Ω) is just an antisymmetrized interaction – no poles!



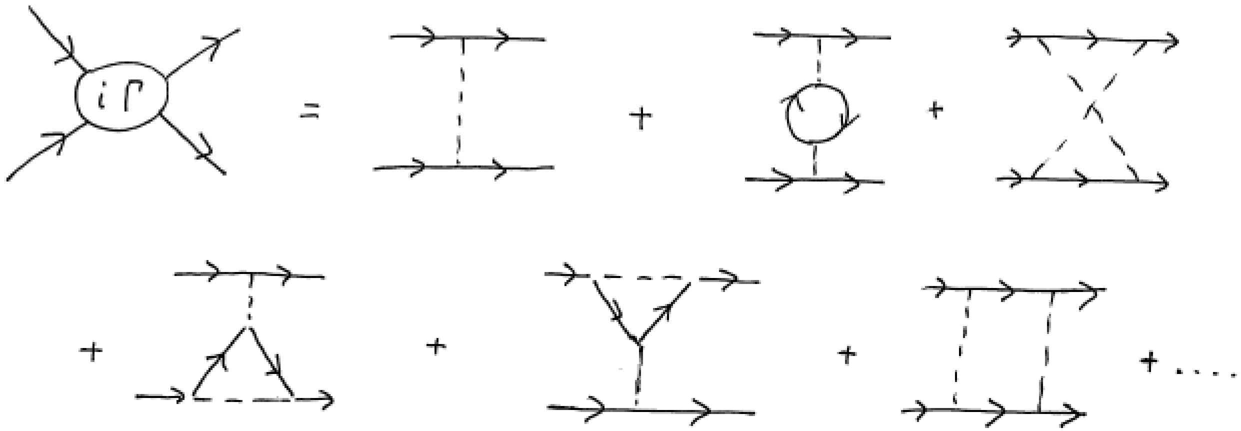
Roughly, Γ = -U

$$\Gamma = U(k_1-k_4)\,\delta_{\alpha\delta}\,\delta_{\beta\gamma} - U(k_1-k_3)\,\delta_{\alpha\gamma}\,\delta_{\beta\delta}$$

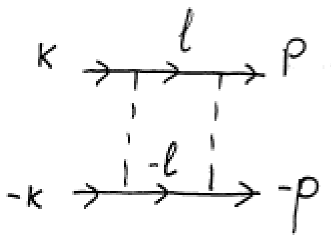
$$= \frac{1}{2}\left[U(k_1-k_4) + U(k_1-k_3)\right]\left(\delta_{\alpha\delta}\,\delta_{\beta\gamma} - \delta_{\alpha\gamma}\,\delta_{\beta\delta}\right)$$ **singlet**

$$+ \frac{1}{2}\left[U(k_1-k_4) - U(k_1-k_3)\right]\left(\delta_{\alpha\delta}\,\delta_{\beta\gamma} + \delta_{\alpha\gamma}\,\delta_{\beta\delta}\right)$$ **triplet**

Let’s now include higher-order terms:

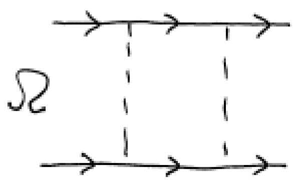


For generic momenta, a boring perturbation theory, but if total incoming momenta and frequency are zero, there is a singularity



$$\int \frac{d^d \ell \, d\omega_\ell}{(2\pi)^{d+1}} G_\ell G_{-\ell} = \frac{i N_F}{2} \int_{-\Lambda}^{\Lambda} \frac{d\varepsilon_\ell}{|\varepsilon_\ell|} = \log \text{ singular}$$

Let’s keep total frequency non-zero

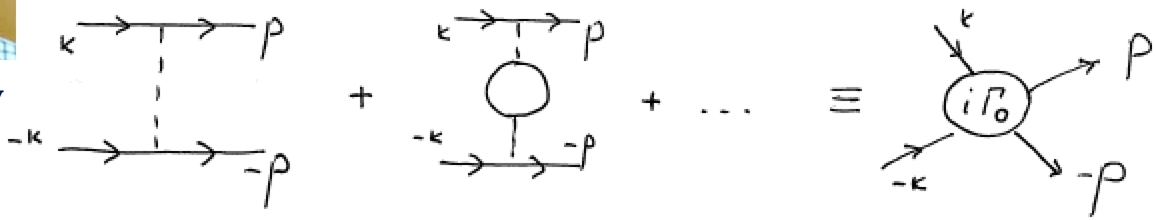


$$\frac{1}{2}\left(\log \text{ singular}\right) \Rightarrow \log \frac{\Lambda}{|\Omega|} + i\,\pi/2$$

L.P. Gorkov

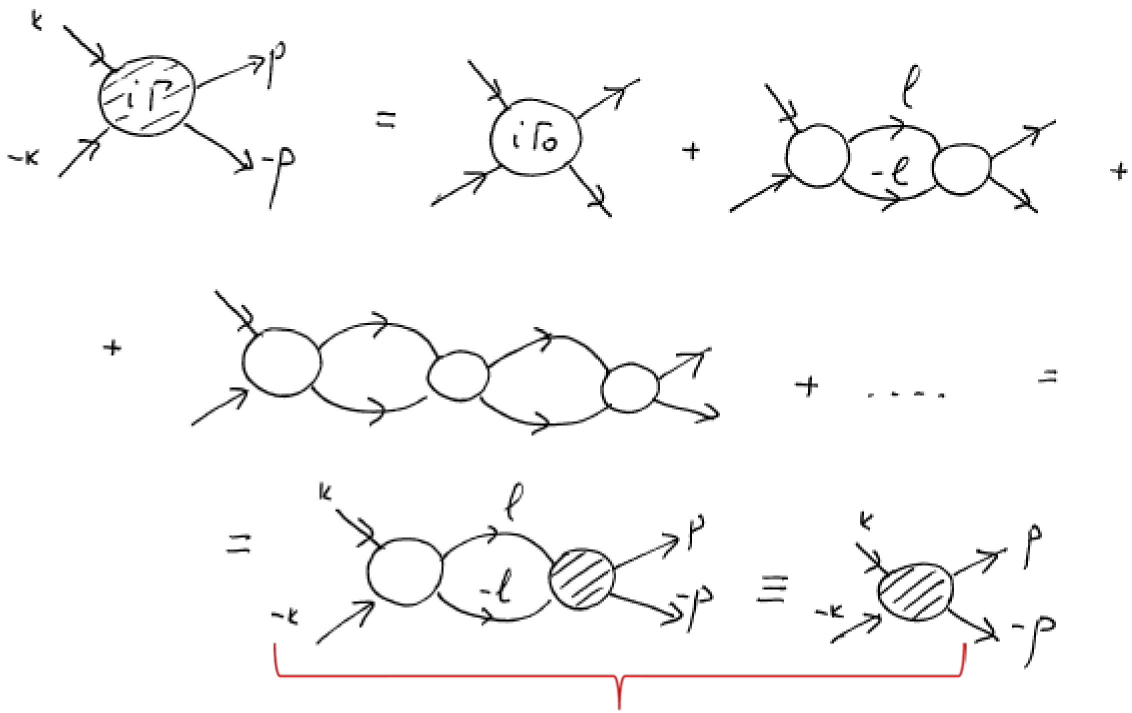
There is a recepe how to proceed

Collect all non-singular diagrams into a "bare" vertex



This is a regular pert. theory

And keep adding singular particle-particle renormalizations



Integral equation on the full vertex

Take Γ_0 = const (Γ_0 = - U in the Hubbard model)

$$\Gamma = \frac{\Gamma_0}{1 - \Gamma_0 N_F \left[\log \frac{\Lambda}{|\Omega|} + i\pi/2\right]}$$

For repulsive interaction , Γ_0 is negative, and

$$\Gamma \propto \frac{1}{N_F \log \frac{\Lambda}{|\Omega|}} \rightarrow 0$$

But if Γ_0 >0 (attraction)

$$\Gamma \propto \frac{1}{\Omega - i\Omega_0} \qquad \Omega_0 = \Lambda e^{-\frac{1}{\Gamma_0 N_F}}$$

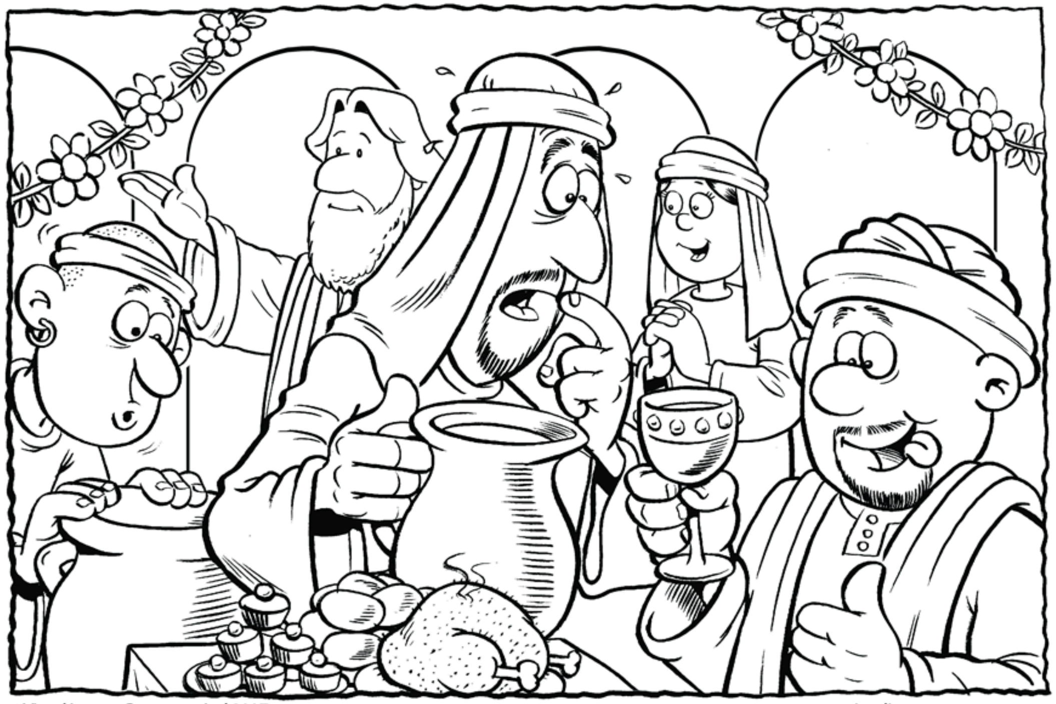
Pole in the upper frequency half-plane, i.e., perturbations grow with time and destroy the normal state

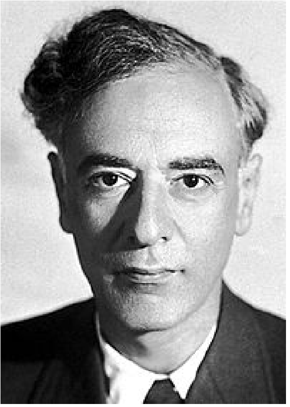
This is true only at small total momentum

$$\Gamma \propto \frac{1}{\Omega - i\Omega_0 \left(1 - \frac{V_F^2 q^2}{6\Omega_0^2}\right)}$$

Superconductivity from repulsive interaction

How one possibly get Γ_0>0 out of repulsion?





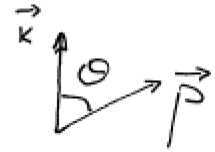
Lev Landau

What if Γ_0 is a function of momentum?

Lev Pitaevskii

$$\Gamma_0(\vec{k}, \vec{p})$$

$$|\vec{k}| = p_F \qquad |\vec{p}| = p_F$$



$$\Gamma_0(\theta) = \sum_{\ell=0}^{\infty} (2\ell+1)\, \Gamma_{\ell,0}\, P_\ell(\cos\theta)$$

$$\Gamma(\theta) = \sum_{\ell=0}^{\infty} (2\ell+1)\, \Gamma_\ell\, P_\ell(\cos\theta)$$

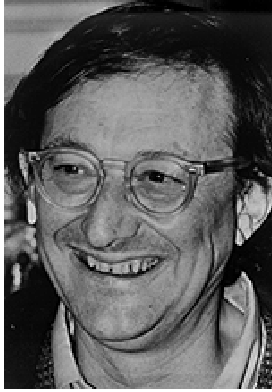
Different angular harmonics decouple

$$\Gamma_\ell = \frac{\Gamma_{\ell,0}}{1 - \Gamma_{\ell,0} N_F \left[\log\frac{\Lambda}{|\Omega|} + \frac{i\pi}{2}\right]}$$

It is sufficient to have $\Gamma_{l,0} > 0$ for just one value of l

Walter Kohn

Kohn-Luttinger mechanism 1965

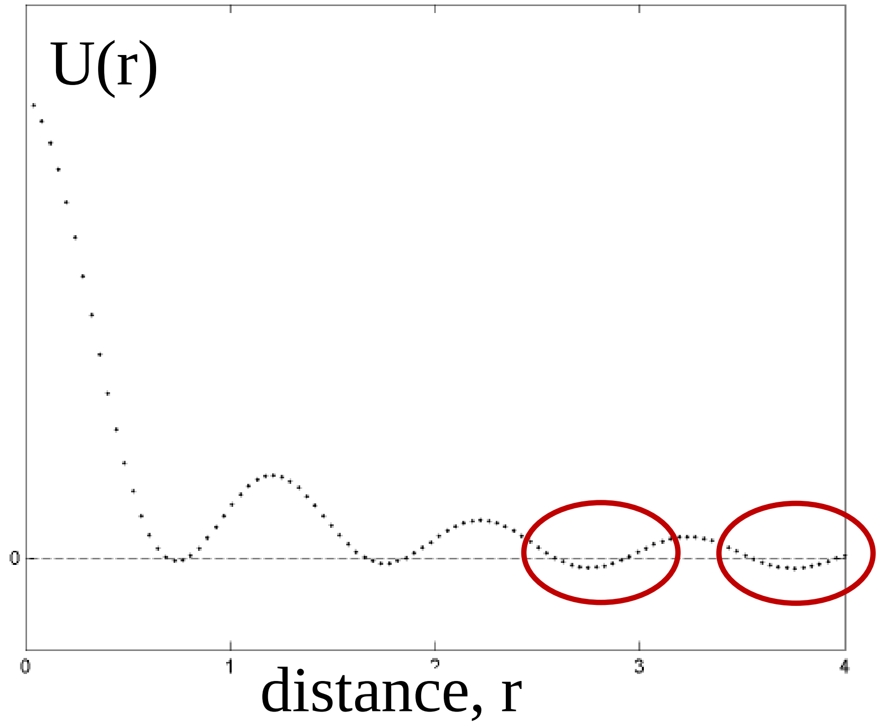


Joaquin Luttinger

Components of the interaction with large *l* come from large distances. At such distances, bare repulsive interaction occasionally gets over- screened and acquires oscillations

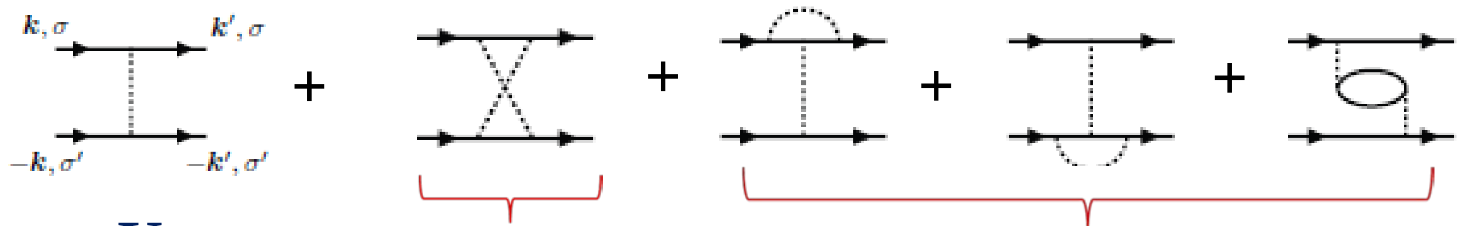
$$[U(r) = \cos(2k_F r)/r^3],$$

often called Friedel oscillations



How this actually works?

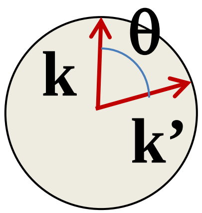
$\Gamma_0(\theta) =$



-U

this one gives θ dependence

these three cancel out



$$\Gamma_0(\theta) = -U^2\,\Pi(2p_F \cos\theta/2)$$

Particle-hole polarization bubble has a non-analyticity at $2p_F$ (i.e., at $\theta=0$)

$$\Pi(2p_F + x) - \Pi(2p_F) \sim x \log x$$

Because of non-analyticity, components $\Gamma_{l>>1}$ decay by a power-law, as $1/l^4$

$$\Gamma_{l>>1} = U^2 N_F/(2l^4) > 0$$

Components of the screened Coulomb interaction with large l are attractive, independent on the parity of l

Later developments:

Fay&Layzer, Kagan&A.C…

The attraction extends down to l=1, and Γ_1 is the largest:

$$\Gamma_1 = U^2\ (2 \log 2 - 1) > 0$$

p-wave instability

There is no interference with the bare U because bare U only contributes to s-wave channel

If U=U(q), situation is different, one needs to overcome bare $U_{l=1}$

However, $U_{l=1} \sim p^2_F$, while the second order term $\sim p_F$, and it definitely wins at low density

Kohn and Luttinger applied their result to 3He

At that time (1965) the general belief was that the pairing in 3He must be d-wave (l=2)

KL obtained Tc ~ E_F exp [-2.5 l^4], substituted l=2, and found Tc~ 10^{-17} K

A few years later it was found that l=1 for ^{3}He.

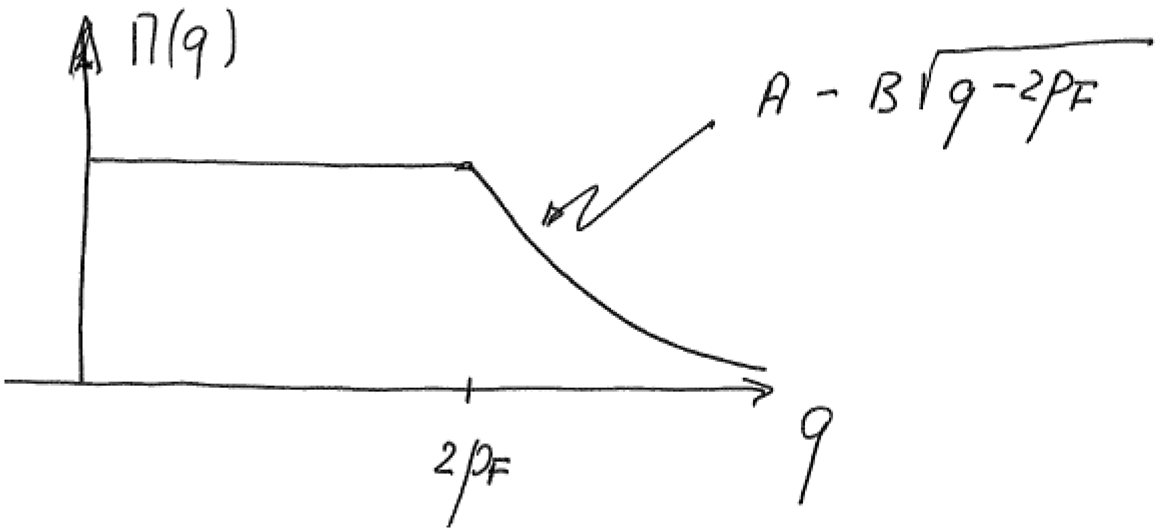
Tc (l=1) ~ 10^{-3} E_F ~ 10^{-3} K (Tc ~3 mK in ^{3}He)

I will focus on 2D systems for the rest of the lectures

Kohn-Luttinger effect in 2D

$$\Gamma_0(\theta) = -U^2\,\Pi(2p_F\cos\theta/2)$$

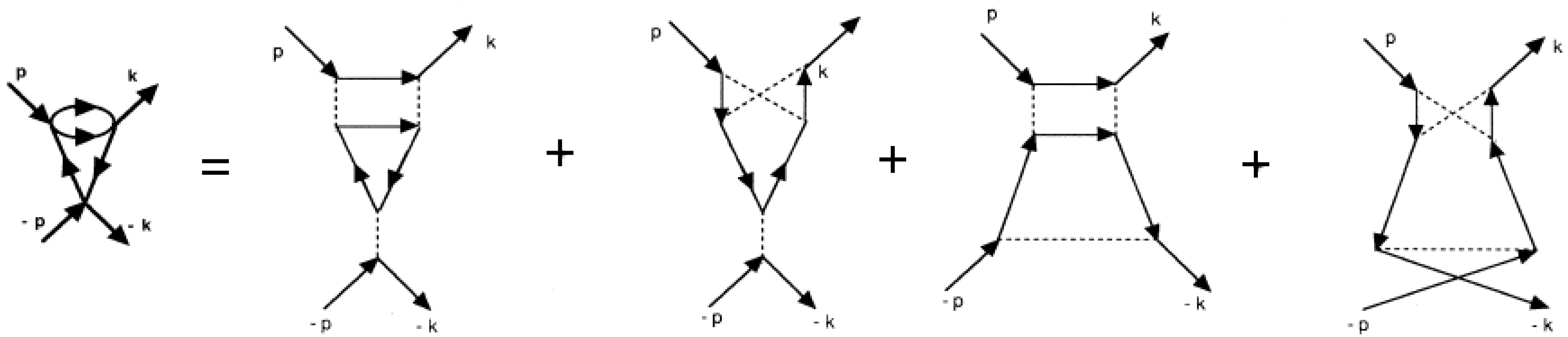
$$\Pi(q < 2p_F) = m/(2\pi) = \text{const}$$



No superconductivity at this stage

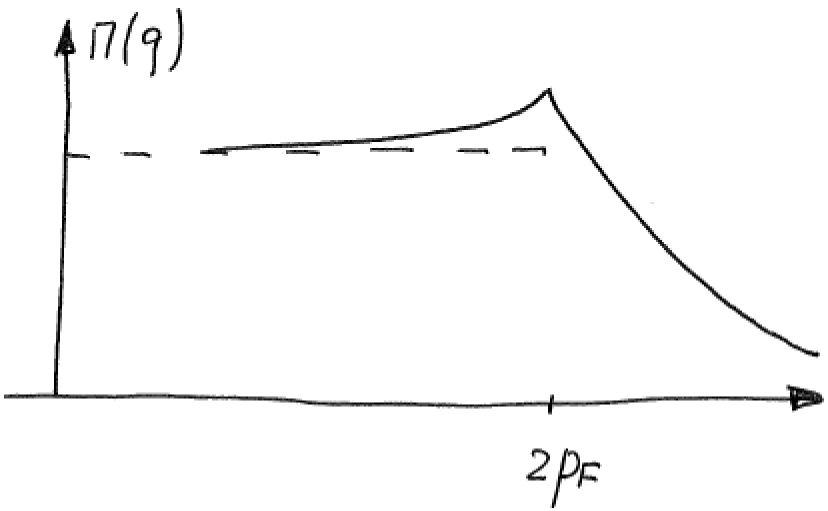
Two ways to extend the analysis:

I. Go to higher order in U (U^3)



Two ways to extend the analysis:

To order U^3



$$\Gamma_{l>>1} \sim U^3 \; N^2_F / l^2 \; > 0$$

Attraction again persists down to l=1, and $\Gamma_{l=1}$ is the largest

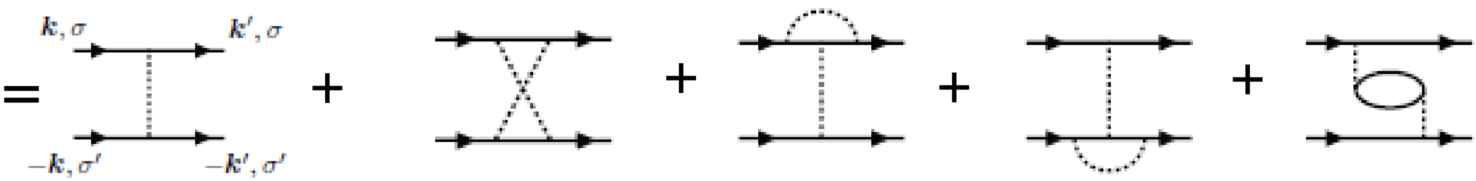
p-wave instability in a 2D isotropic Fermi liquid

Two ways to extend the analysis:

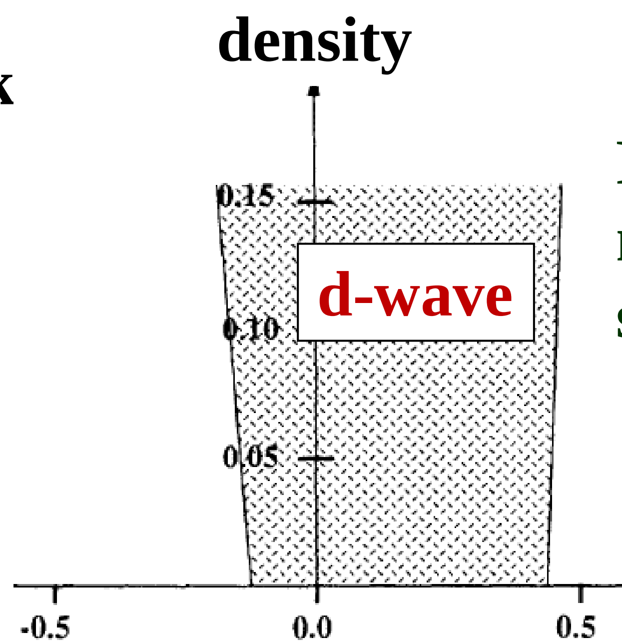
II. Do calculations on a lattice, with full E(k)

Still, second order

Γ_0 (k, k') =



$\Gamma_0 (k, k') = - U^2 \, \Pi \, (k+k$



For hoping between nearest (t) and second nearest (t') neighbors

Details matter, but most likely outcome is d-wave

(Raghu's lectures)

For the rest of the lectures I will explore KL idea that the effective pairing interaction is different from a bare repulsive U due to screening by other fermions, and it may have attractive components in some channels

- **cuprates**
- **doped graphene**
- **Fe-pnictides**

Each case will represent different lattice version of KL physics

Cuprates (1986…)

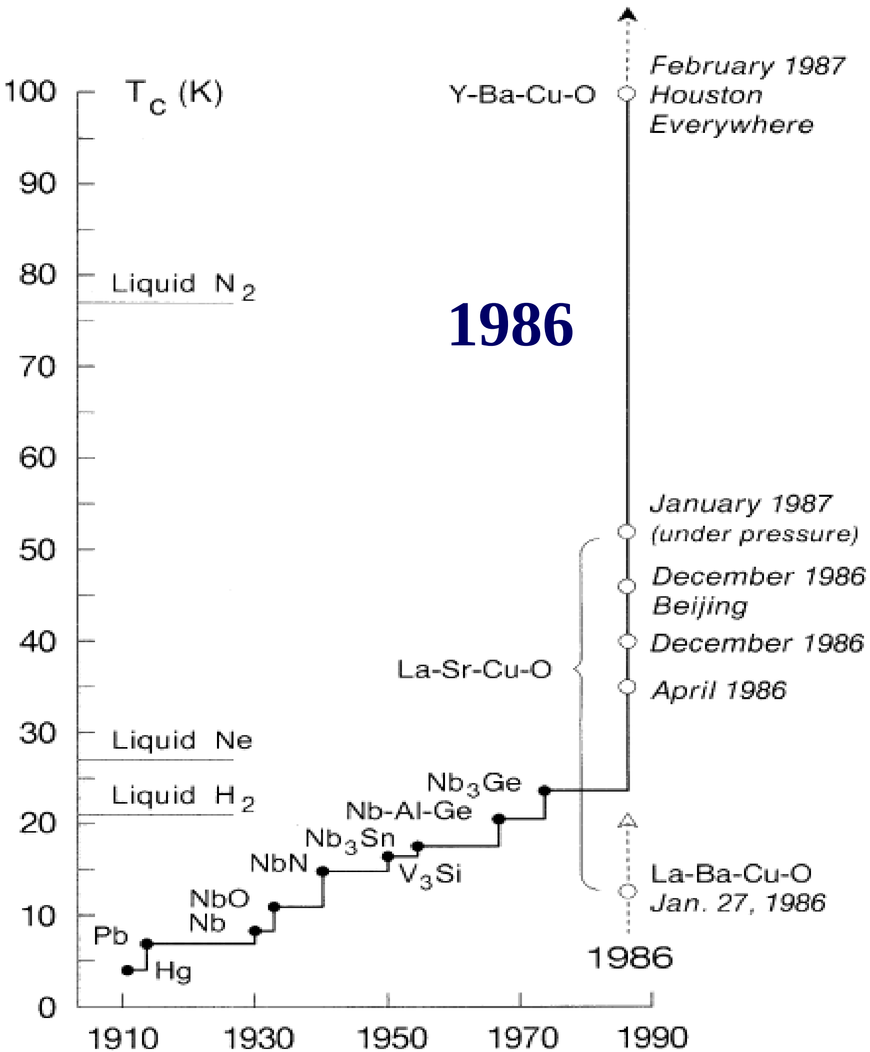
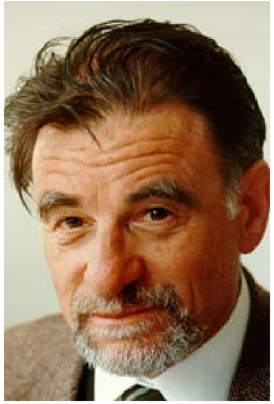


Fig. 1. Evolution of the superconductive transition temperature subsequent to the discovery of the phenomenon.

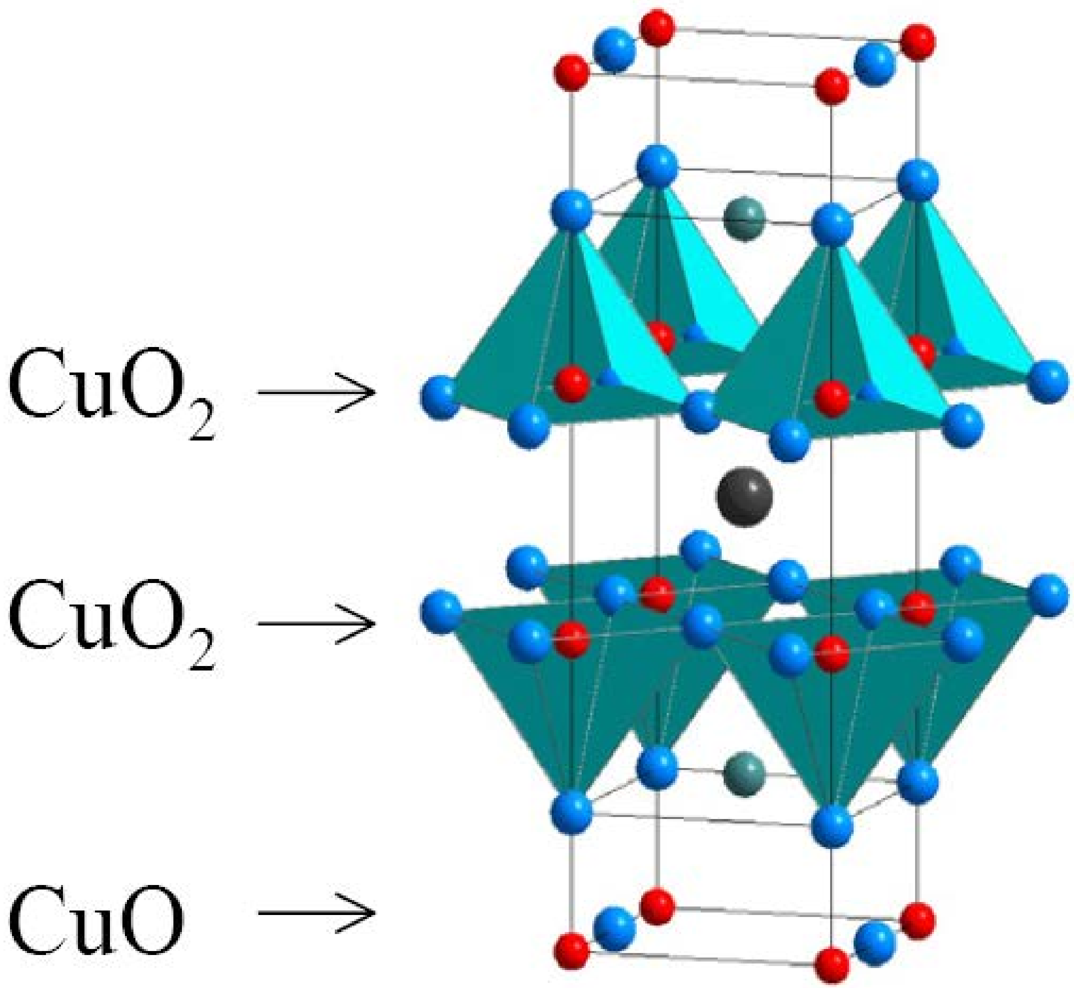


Alex Muller and Georg Bednortz

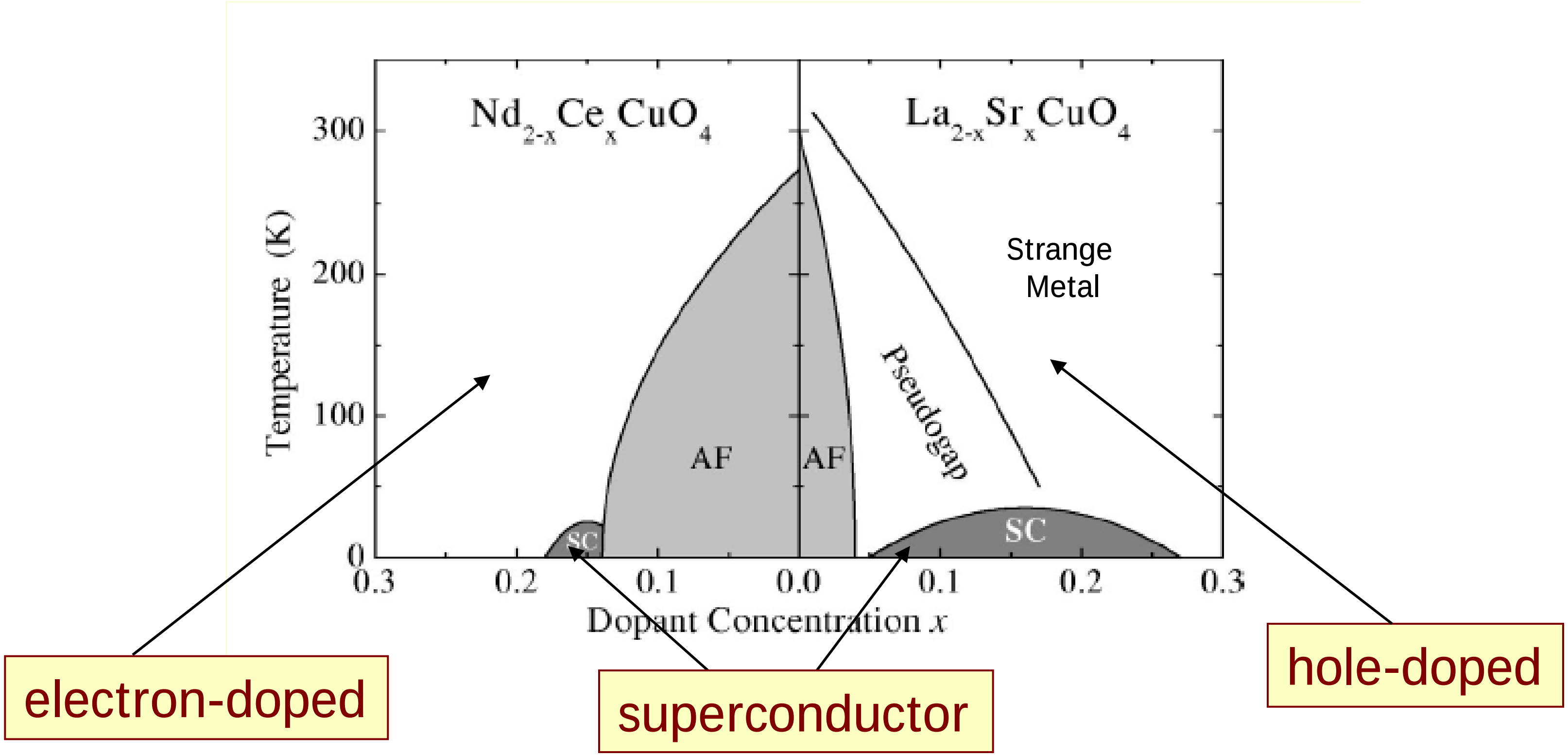
Nobel prize, 1987

Quasi-2D

$YBa_2Cu_3O_{6+x}$



Phase diagram

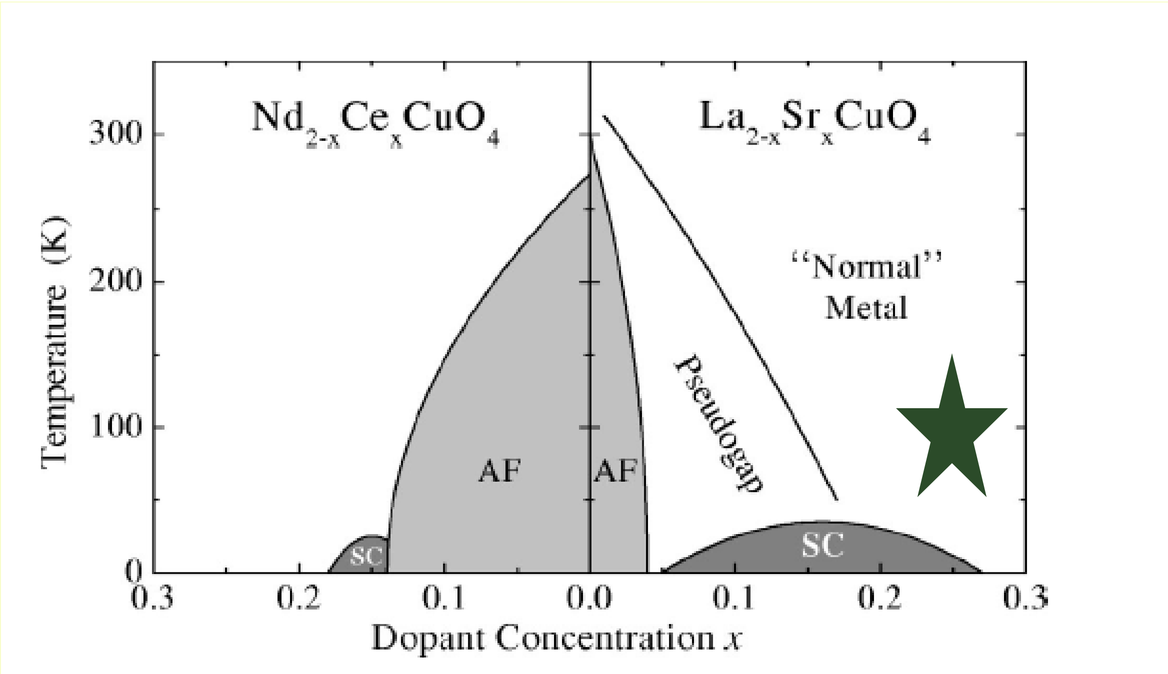


Parent compounds are antiferromagnetic insulators

Superconductivity emerges upon either hole or electron doping

Overdoped compounds are metals and Fermi liquids

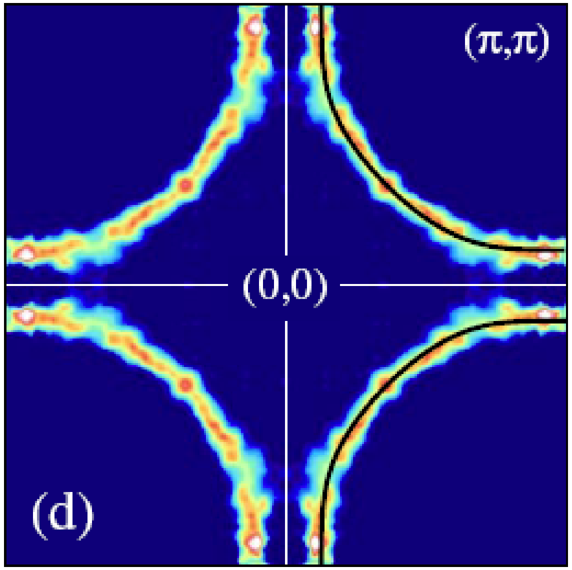
$Tl_2Ba_2CuO_{6+\delta}$



Overdoped compounds are metals and Fermi liquids

$Tl_2Ba_2CuO_{6+\delta}$

Photoemission



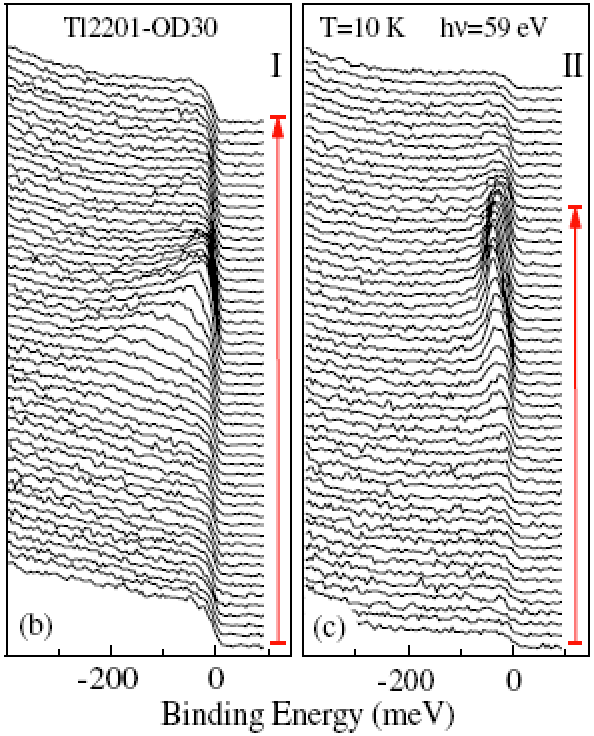
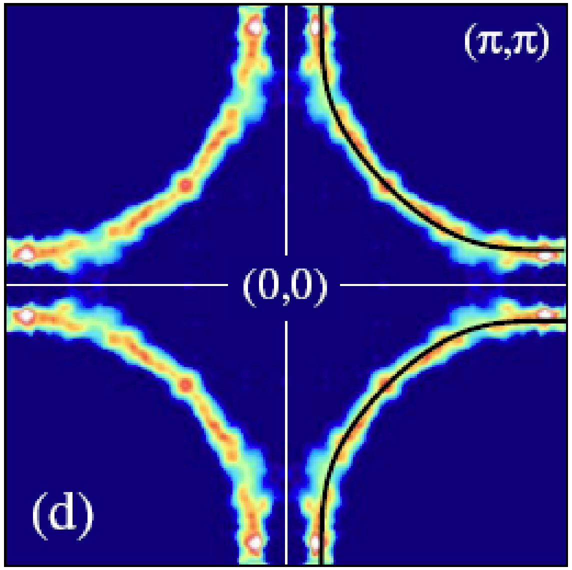


Plate et al

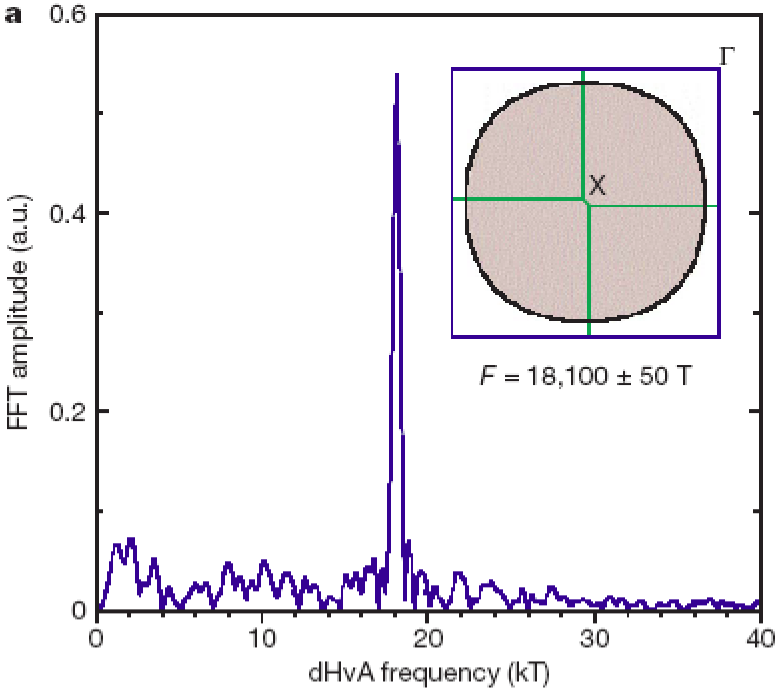
Areas are consistent with Luttinger count for electrons in a Fermi liquid

Overdoped compounds are metals and Fermi liquids

$Tl_2Ba_2CuO_{6+\delta}$



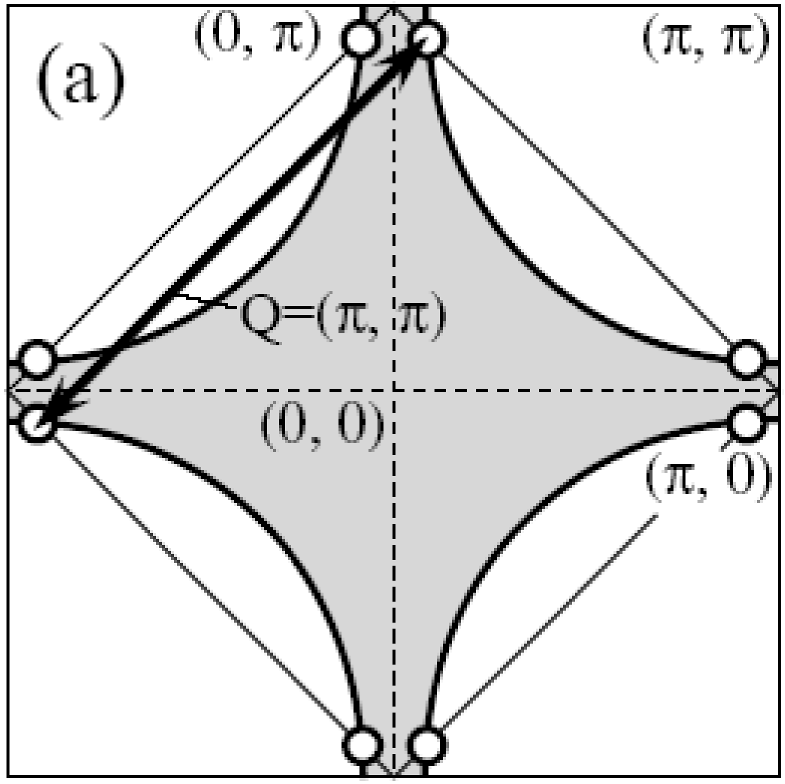
Oscillations in magnetoresistance

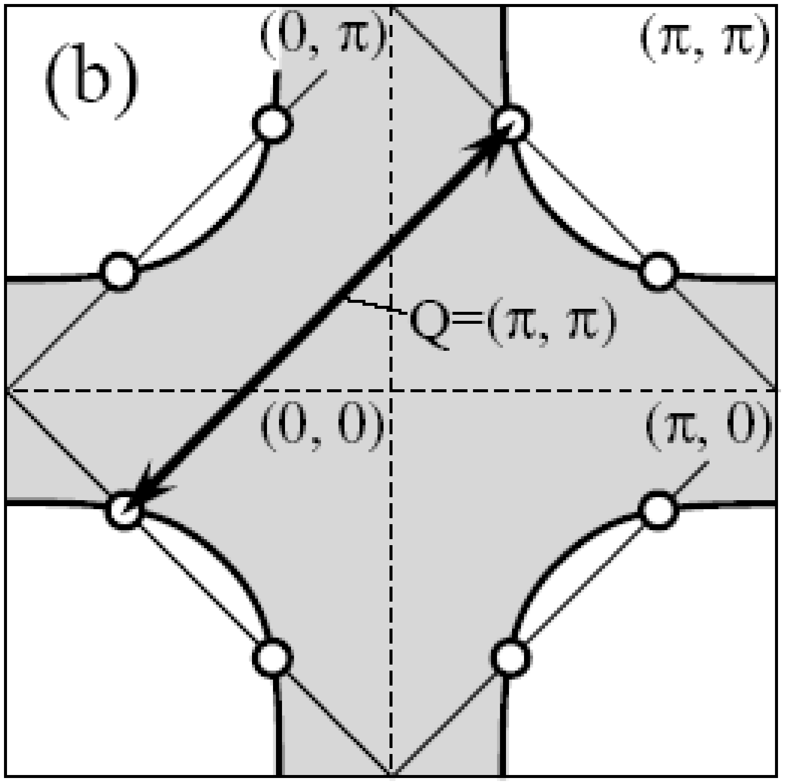


Vignolle et al

Areas are consistent with Luttinger count for electrons in a Fermi liquid

Fermi surface





Hole-doped **Electron-doped**

$$E(k) = -2t (\cos kx + \cos ky) + 4t' \cos kx \cos ky - \mu$$

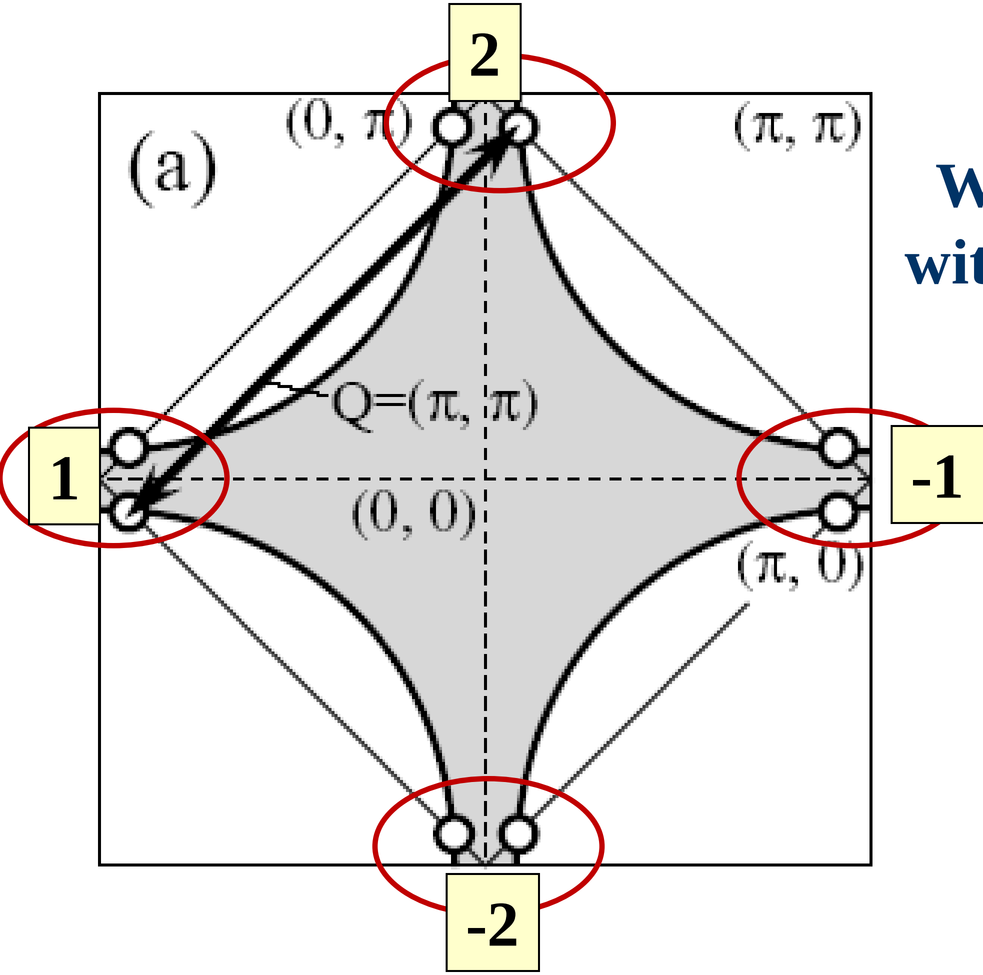
For a square lattice, the symmetry group is D_{4h}

$$X \leftrightarrow Y \quad \text{and} \quad X, Y \leftrightarrow -X, -Y$$

Four 1D representations:

		$X \leftrightarrow Y$	$X \leftrightarrow -X$
s-wave	**A1g : cos kx + cos ky,**	+	+
$d_{x^2-y^2}$	**B1g : cos kx - cos ky,**	-	+
d_{xy}	**B2g : sin kx * sin ky,**	+	-
g-wave	**A2g : (cos kx - cos ky)* sin kx * sin ky,**	-	-

Kohn-Luttinger-type consideration



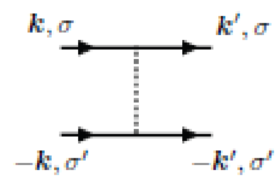
We have repulsive interactions within a patch and between patches

Consider Hubbard U

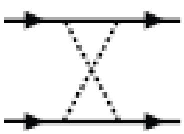
To first order, we only have a repulsive s-wave component.

To order U^2

$\Gamma_0 =$



+

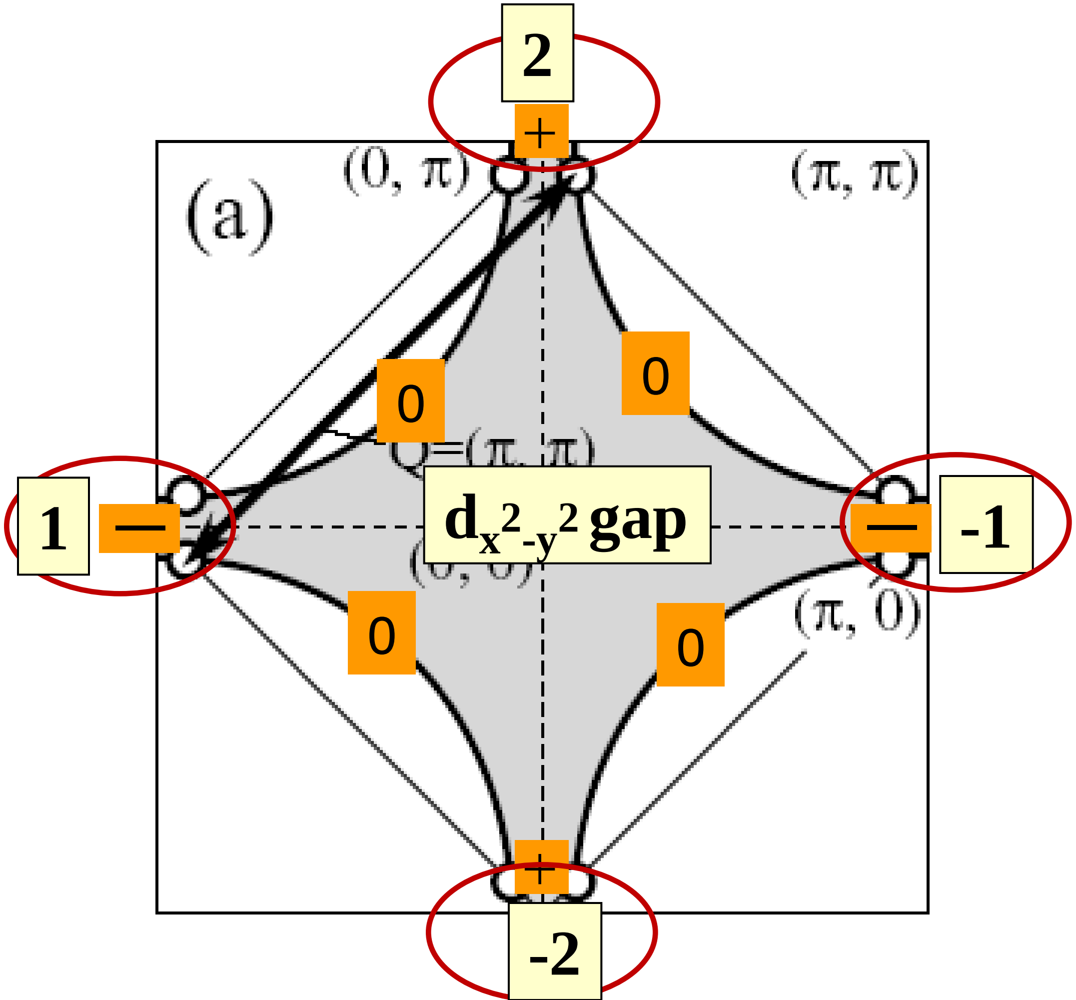


$$\Gamma_0(1,2) > \Gamma_0(1,1)$$

Let's momentarly consider only a larger $\Gamma_0(1,2)$

Eqn. for a sc gap

$$\Delta(1) = -\int dq_2 \frac{\Delta(2)}{\sqrt{\Delta^2(2) + E^2(q_2)}} \Gamma_0(1,2)$$

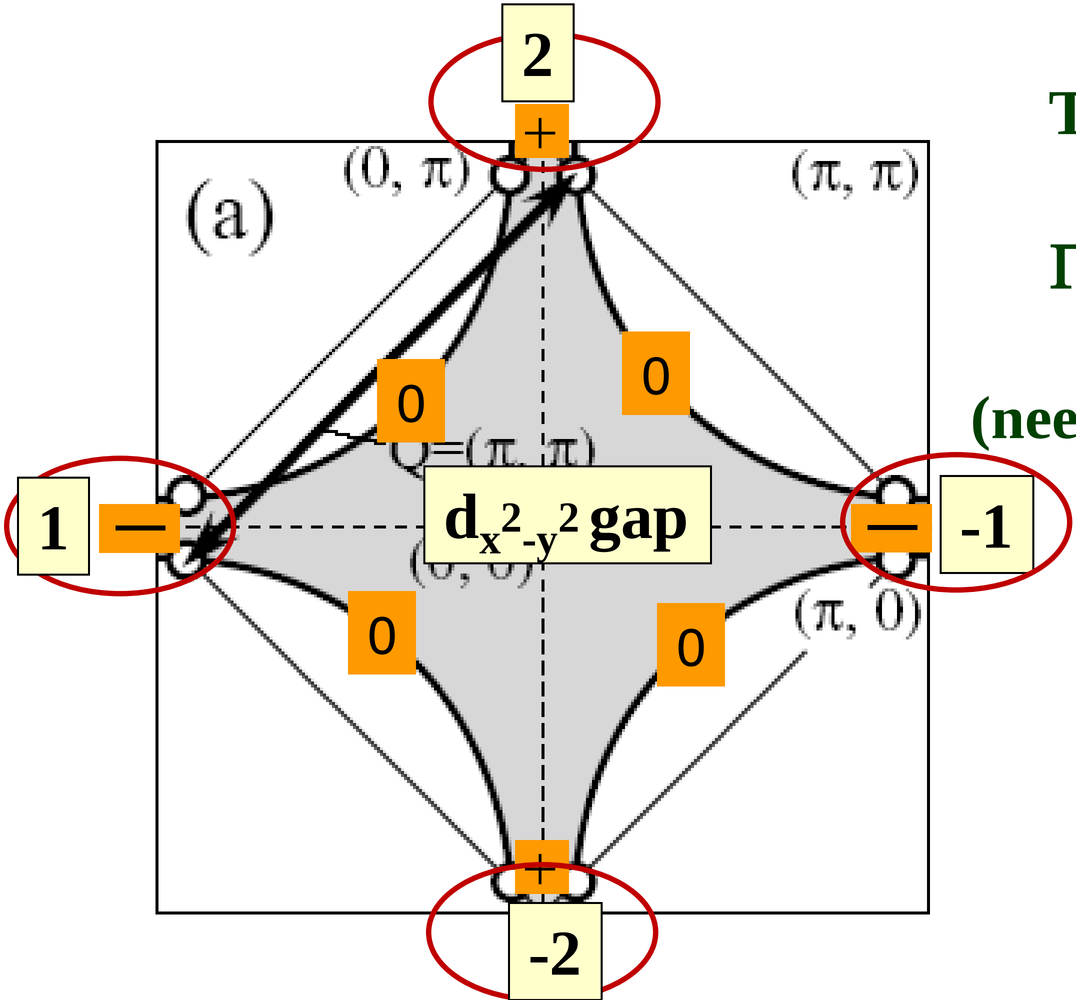


$$\Delta(1) = -\Delta(2)$$

Let's momentarly consider only a larger $\Gamma_0(1,2)$

Eqn. for a sc gap

$$\Delta(1) = -\int d\,q_2 \frac{\Delta(2)}{\sqrt{\Delta^2(2) + E^2(q_2)}}\,\Gamma_0(1,2)$$



The full solution is

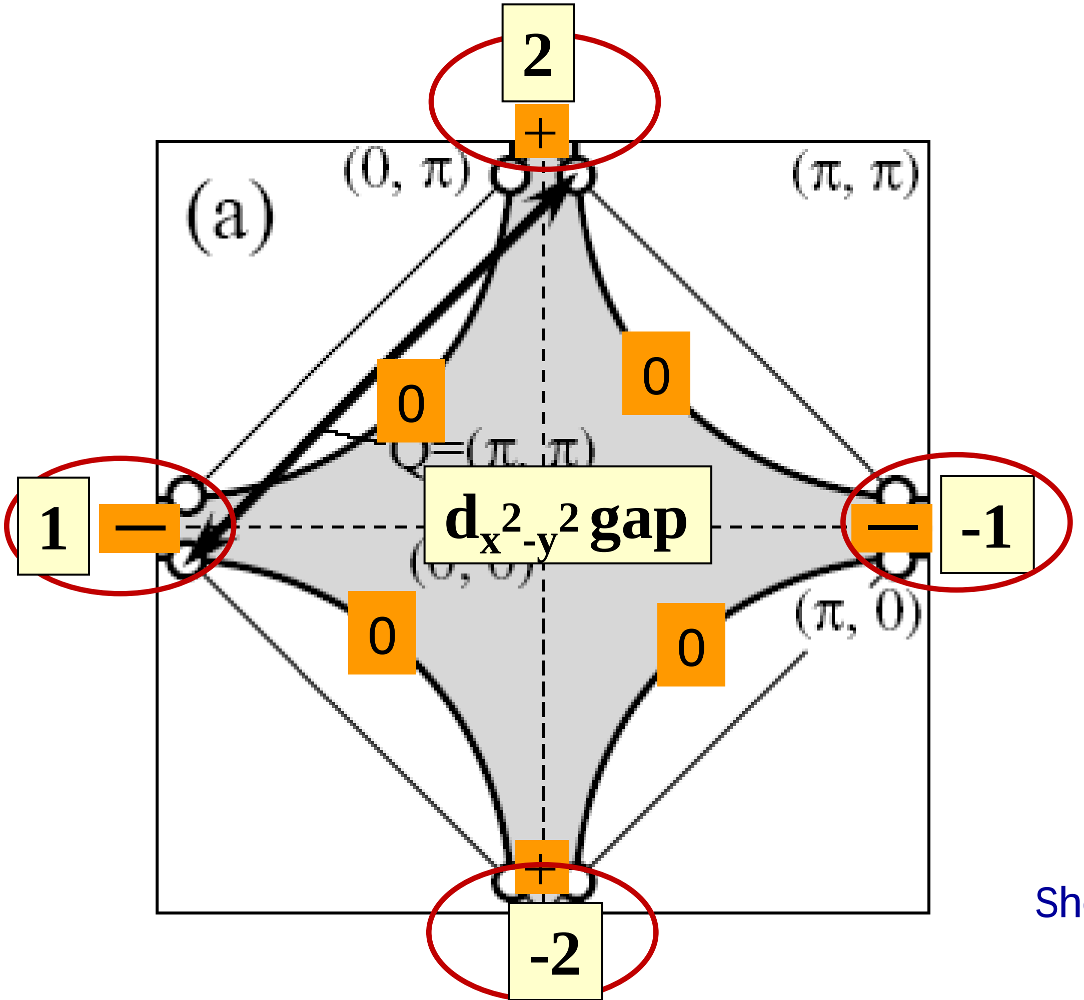
$$\Gamma_{d,0} = \Gamma_0(1,2) - \Gamma_0(1,1)$$

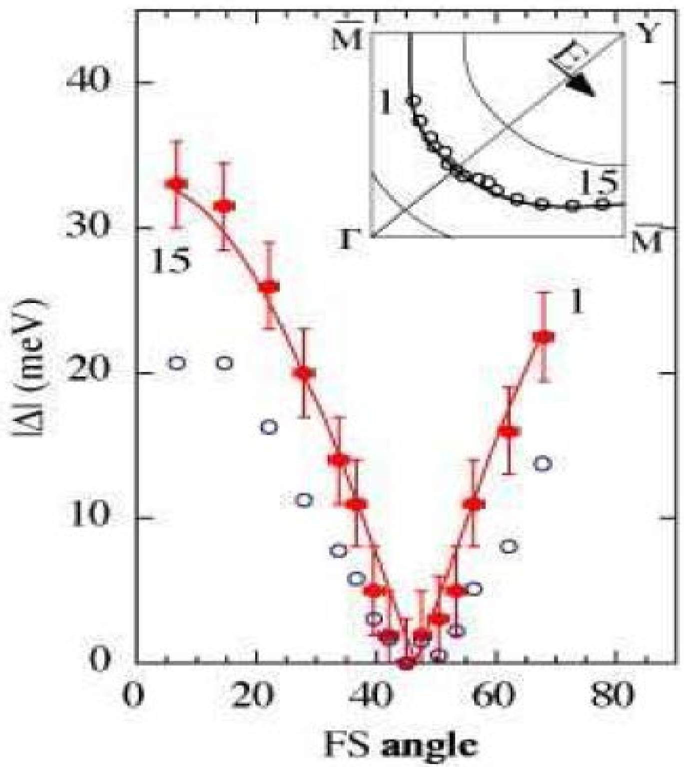
(need $\Gamma_{d,0} > 0$ for d-wave instability)

Let's momentarly consider only a larger $\Gamma_0(1,2)$

Eqn. for a sc gap

$$\Delta(1) = -\int d\,q_2 \frac{\Delta(2)}{\sqrt{\Delta^2(2)+E^2(q_2)}}\,\Gamma_0(1,2)$$

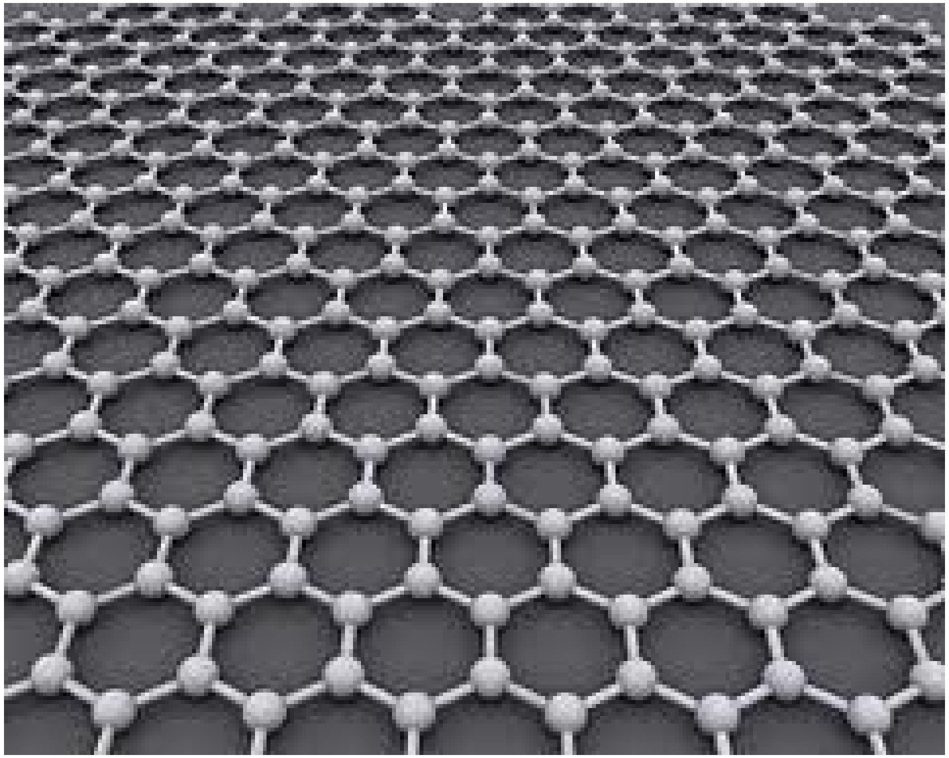




Shen, Dessau et al 93, Campuzano et al, 96

Doped graphene

Graphene -- an atomic-scale honeycomb lattice made of carbon atoms.

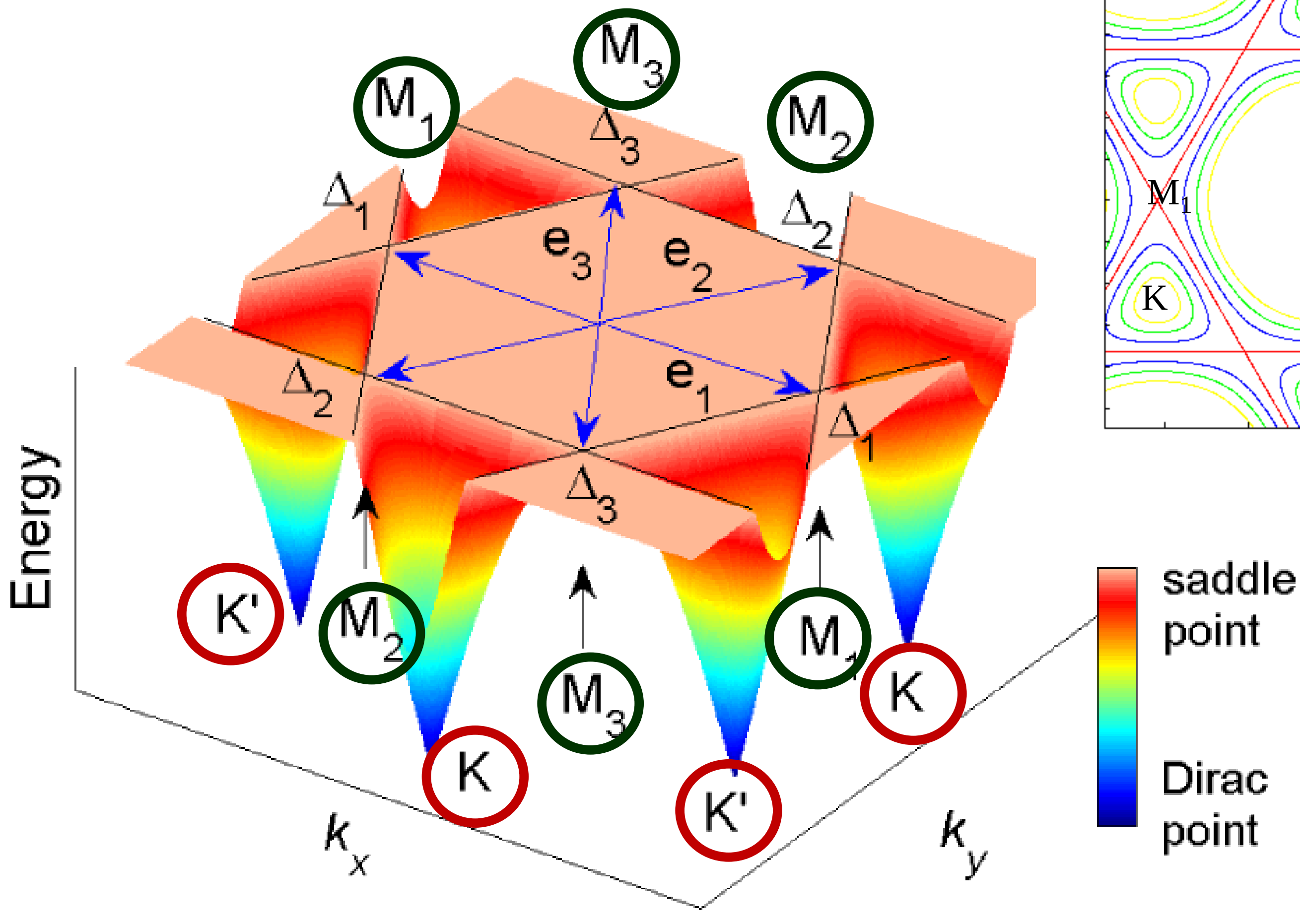


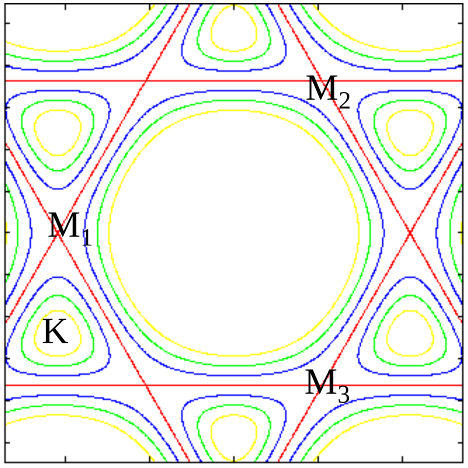
Nobel Prize 2010
Andre Geim, Konstantin Novoselov

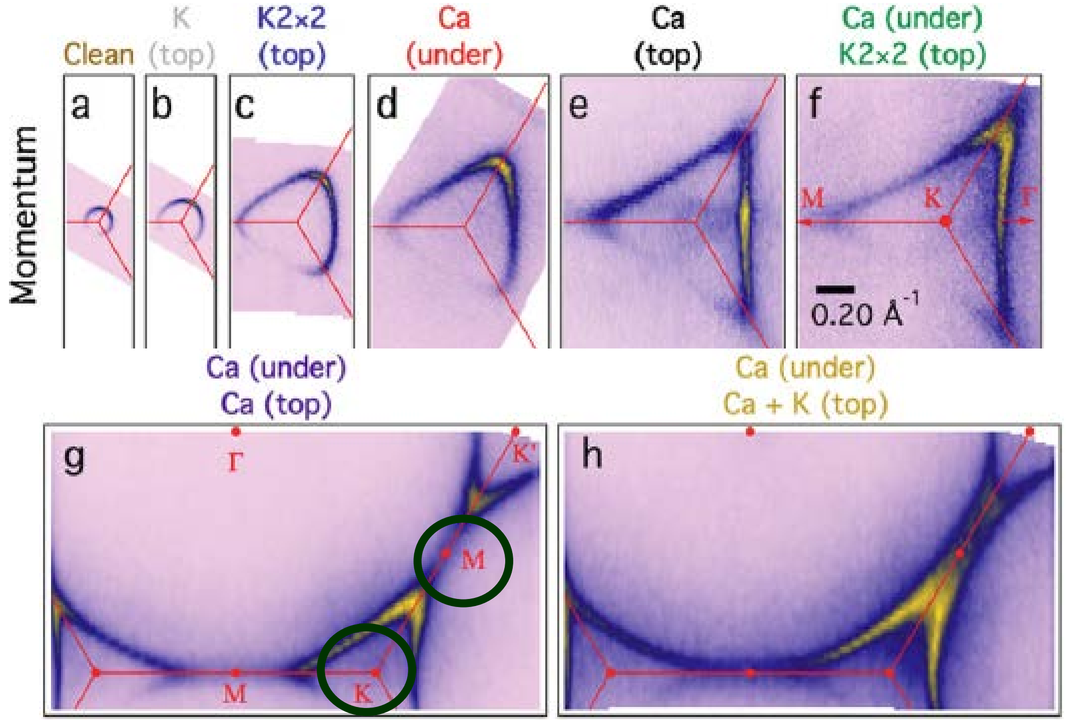
$$\varepsilon(k) = t_1\sqrt{1 + 4\cos\frac{k_y\sqrt{3}}{2}\cos\frac{3k_x}{2} + 4\cos^2\frac{k_y\sqrt{3}}{2}} - \mu$$

$\mu = 0$, Dirac points

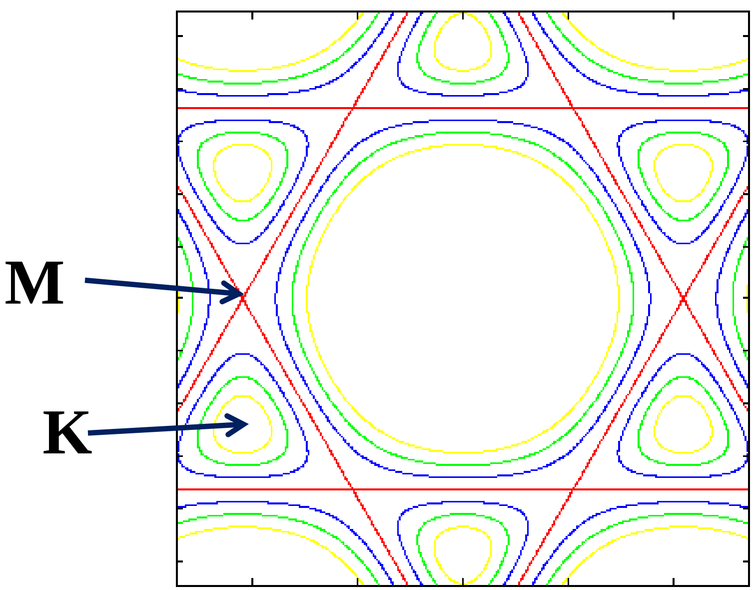
$\mu = t_1$, van Hove points





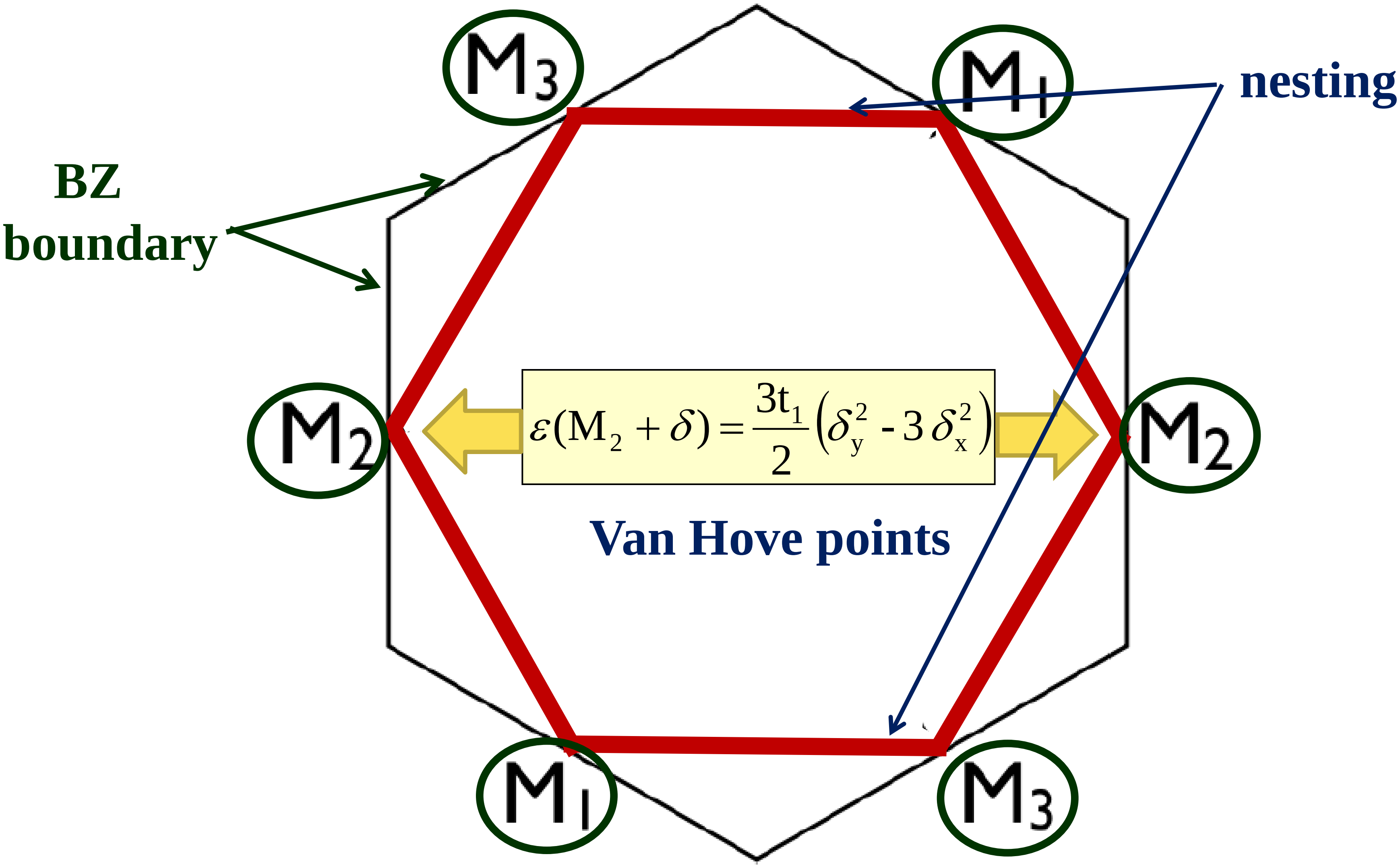


E. Rotenberg et al
PRL 104, 136803 (2010)

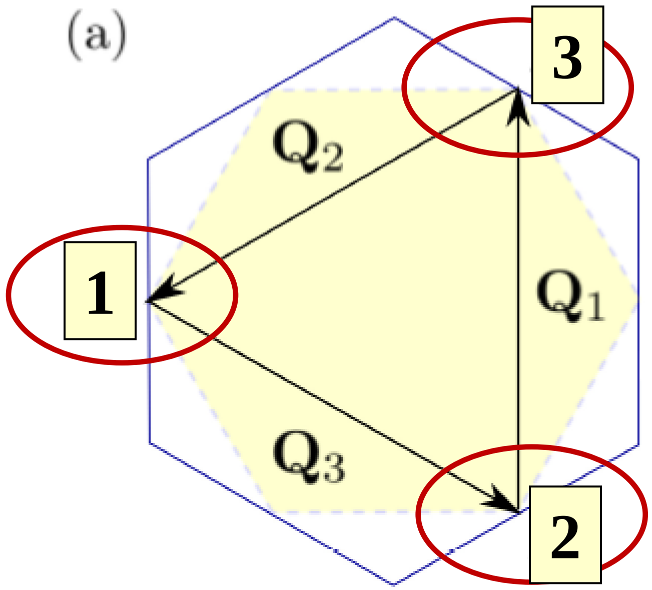


At van Hove doping

$$\varepsilon(\mathrm{k}) = t_1\sqrt{1 + 4\cos\frac{k_y\sqrt{3}}{2}\cos\frac{3k_x}{2} + 4\cos^2\frac{k_y\sqrt{3}}{2}} - t_1$$

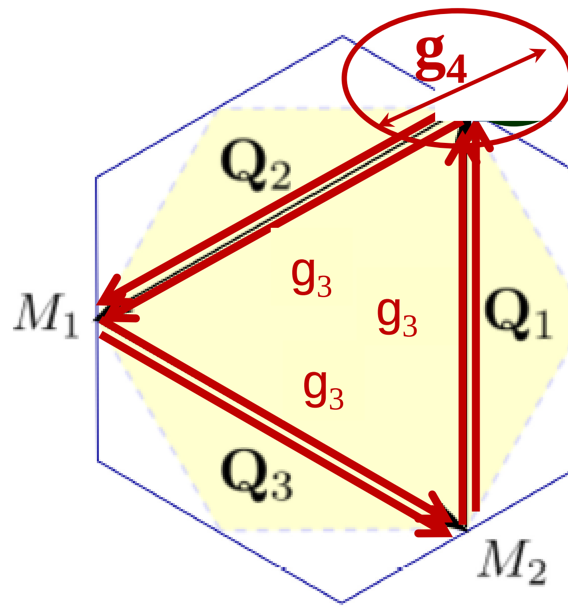


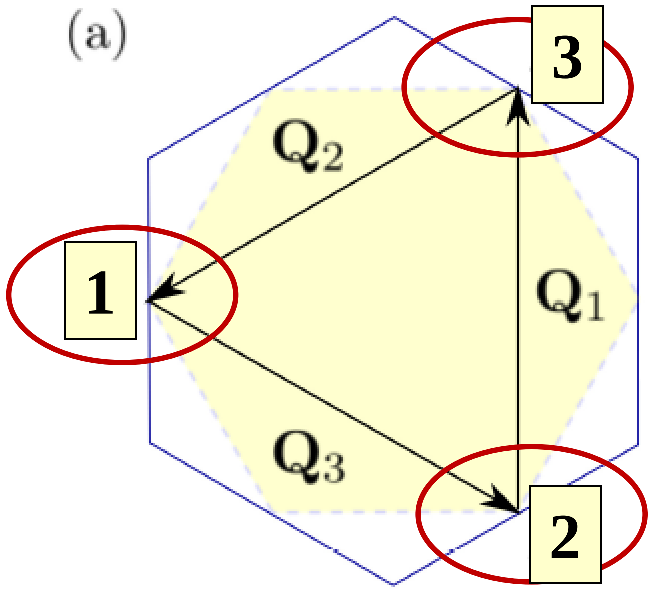
$$\varepsilon(M_2 + \delta) = \frac{3t_1}{2}\left(\delta_y^2 - 3\,\delta_x^2\right)$$



$$\Gamma_0(1,1) = \Gamma_0(2,2) = \Gamma_0(3,3) = \mathbf{g}_4$$

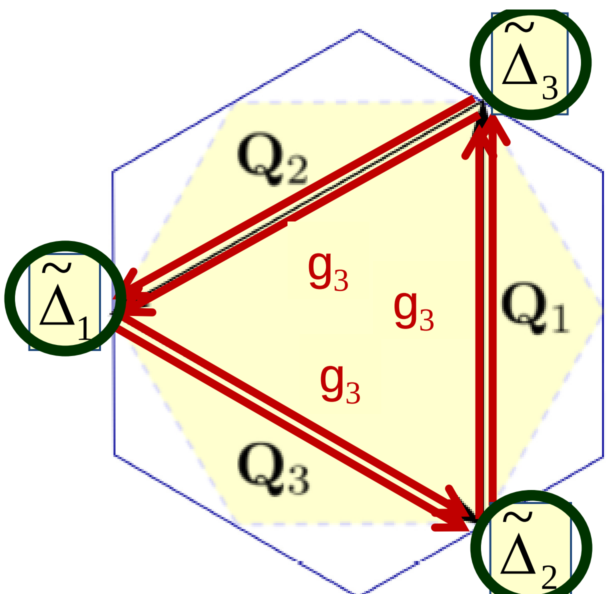
$$\Gamma_0(1,2) = \Gamma_0(2,3) = \Gamma_0(1,3) = \mathbf{g}_3$$





$$\Gamma_0(1,1) = \Gamma_0(2,2) = \Gamma_0(3,3) = g_4$$

$$\Gamma_0(1,2) = \Gamma_0(2,3) = \Gamma_0(1,3) = g_3$$



Eigenfunctions

$$\tilde{\Delta}_a = \frac{\tilde{\Delta}}{\sqrt{2}}(0,1,-1), \quad \tilde{\Delta}_b = \sqrt{\frac{2}{3}}\tilde{\Delta}\left(1,-\frac{1}{2},-\frac{1}{2}\right)$$

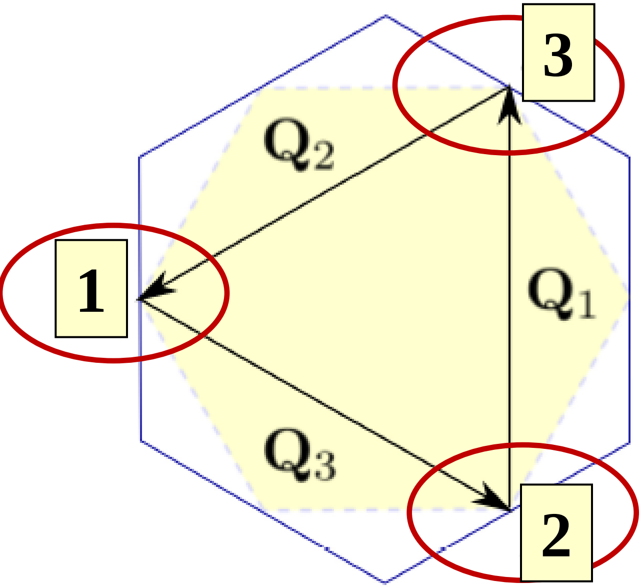
$$\tilde{\Delta}_c = \frac{\tilde{\Delta}}{\sqrt{3}}(1,1,1).$$

Eigenvalues

$$\Gamma_{c,0} = -g_4 - 2g_3$$

$$\Gamma_{a,b,0} = -g_4 + g_3$$

doubly degenerate solution



$$\Gamma_0(1,1) = \Gamma_0(2,2) = \Gamma_0(3,3) = g_4$$
$$\Gamma_0(1,2) = \Gamma_0(2,3) = \Gamma_0(1,3) = g_3$$

$\Gamma_{c;0} = - g_4 - 2g_3, \quad \Gamma_{a,b;0} = - g_4 + g_3,$

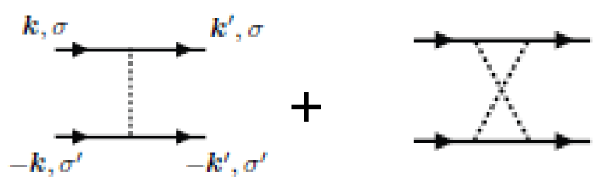
need $\Gamma > 0$ for pairing

Do Kohn-Luttinger analysis:

Consider Hubbard U

To first order in U, $g_4=g_3=U$, and we only have a repulsive s-wave component $\Gamma_{c,0} < 0$, $\Gamma_{a,b,0} = 0$

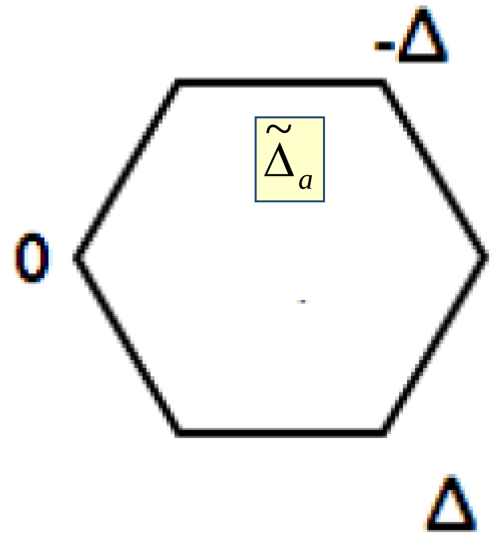
To order U^2

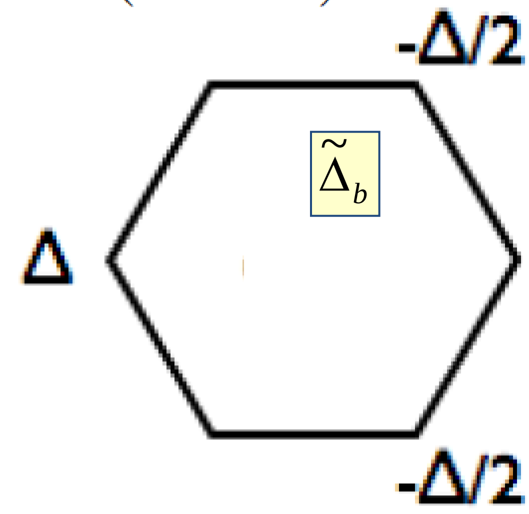
$\Gamma_0 =$ 

$\Gamma_0(1,2) > \Gamma_0(1,1)$, i.e., $g3 > g4$ and $\Gamma_{a,b;0} > 0$

$\Gamma_{a,b;0} > 0$
doubly degenerate
solution for SC

$$\tilde{\Delta}_a = \frac{\Delta}{\sqrt{2}}(0,1,-1) \qquad \tilde{\Delta}_b = \sqrt{\frac{2}{3}}\Delta\left(1,-\frac{1}{2},-\frac{1}{2}\right)$$





The two d-wave solutions are degenerate by symmetry

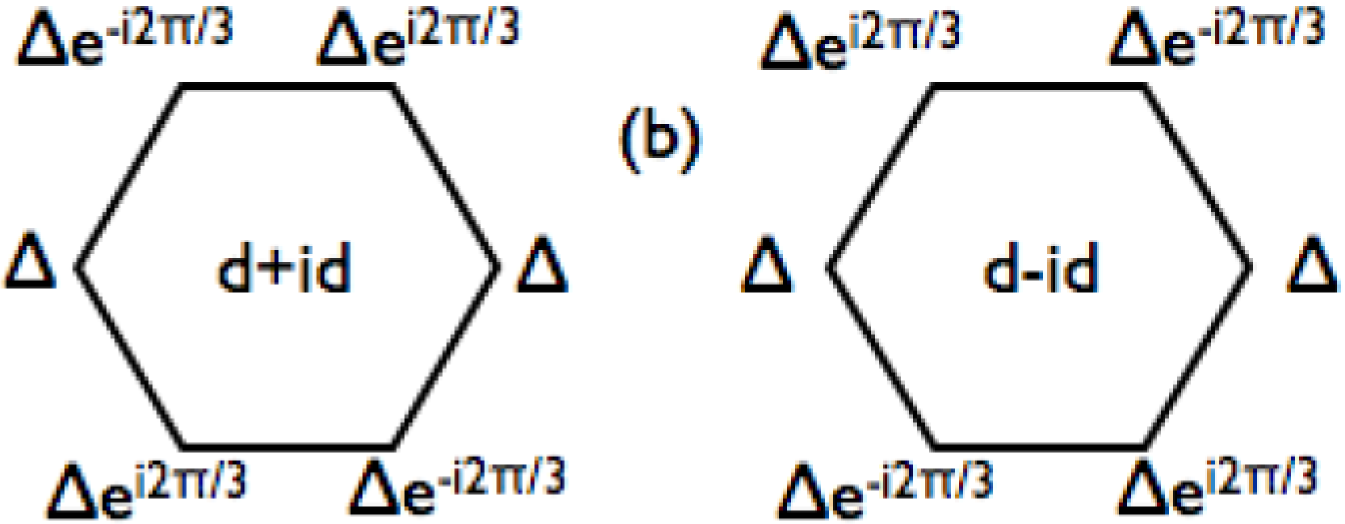
Gonzales

Landau-Ginzburg expansion

$$F = \alpha(T-T_c)(|\Delta_a|^2+|\Delta_b|^2) + K_1\left(|\Delta_a|^2+|\Delta_b|^2\right)^2 + K_2|\Delta_a^2+\Delta_b^2|^2 + O(\Delta^6)$$

d+id state wins

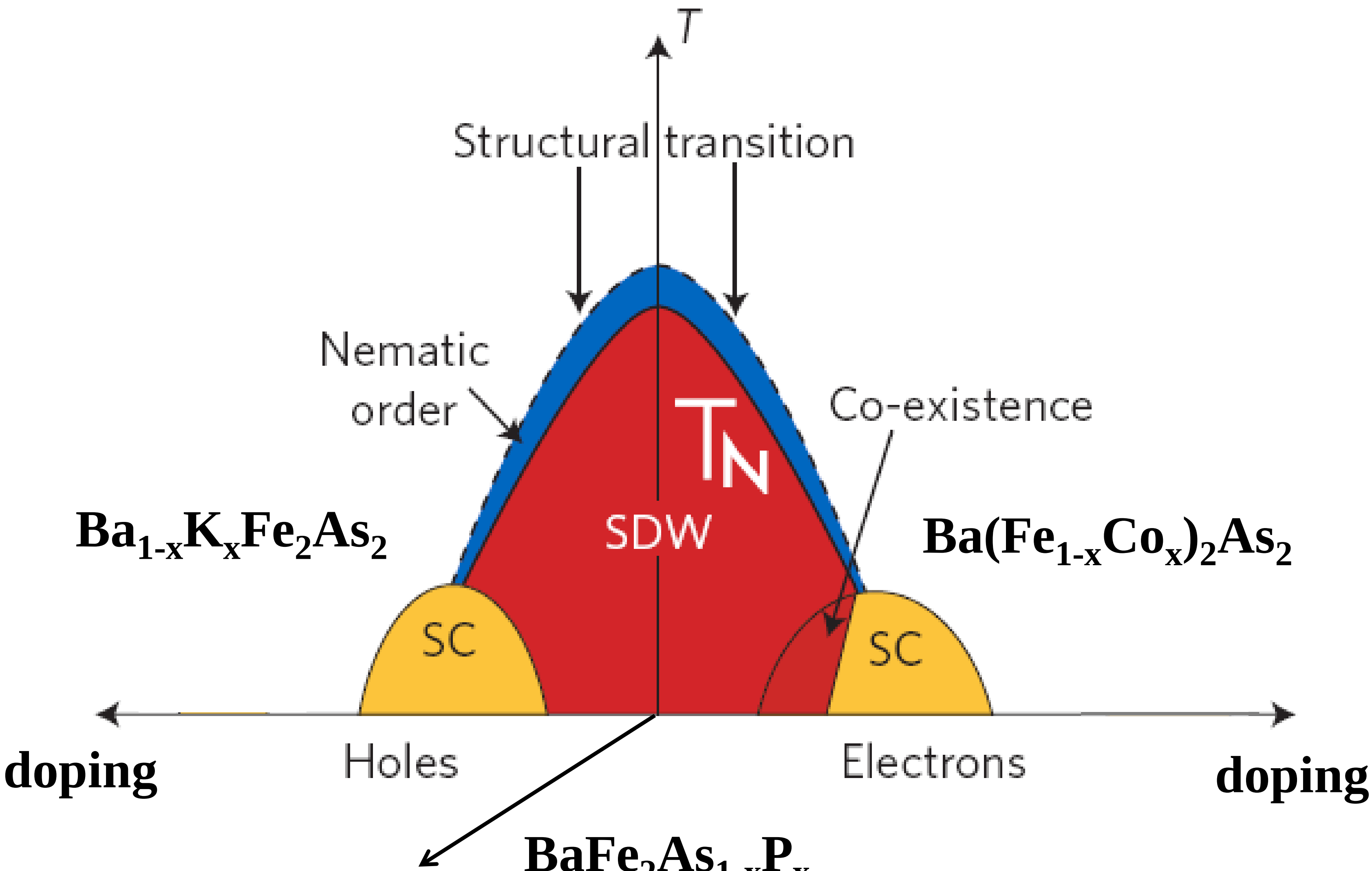
chiral superconductivity (phase winds up by 4 π)



d+id state

chiral superconductivity (phase winds up by 4 π)

Weakly/moderately doped systems:



New breakthrough in 2008: Fe-pnictides

$$LaFeAsO_{1-x}F_x, \; Tc = 26K$$

$$SmFeAsO_{1-x}F_x, \; Tc = 43K$$

Hideo Hosono

nature International weekly journal of science

Letter

Nature 453, 761-762 (5 June 2008) |

Superconductivity at

X. H. Chen[1 (#a1)], T. Wu[1 (#a1)]

1. Hefei National Laboratory for Phys

Correspondence to: X. H. Chen

Since the discovery of hig
been devoted to exploring
from Bardeen–Cooper–Sc
copper oxide supercondu
(ref. 2 (/nature/journal/v453
La–Nd, Sm and Gd) are n
$LaO_{1-x}F_xFeAs$ (ref. 3 (/natu
superconductivity in the r
measurements reveal a tr
high-temperature superc

nature International weekly journal of science

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nature International weekly journal of science

Letter

Nature 459, 64-67 (7 May 2009) | doi:10.1038/nature07981; Received 4 November 2008; Accepted 13 March 2009

A large iron isotope effect in $SmFeAsO_{1-x}F_x$ and $Ba_{1-x}K_xFe_2As_2$

R. H. Liu[1], T. Wu[1], G. Wu[1], H. Chen[1], X. F. Wang[1], Y. L. Xie[1], J. J. Ying[1], Y. J. Yan[1], Q. J. Li[1], B. C. Shi[1], W. S. Chu[2,3], Z. Y. Wu[2,3] & X. H. Chen[1]

1. Hefei National Laboratory for Physical Sciences at Microscale and Department of Physics, University of Science and Technology of China, Hefei, Anhui 230026, China
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Accepted 5 May 2008; Published online 4 June 2008

$F_{0.15}$

& C. L. Chien[1 (#a1)]

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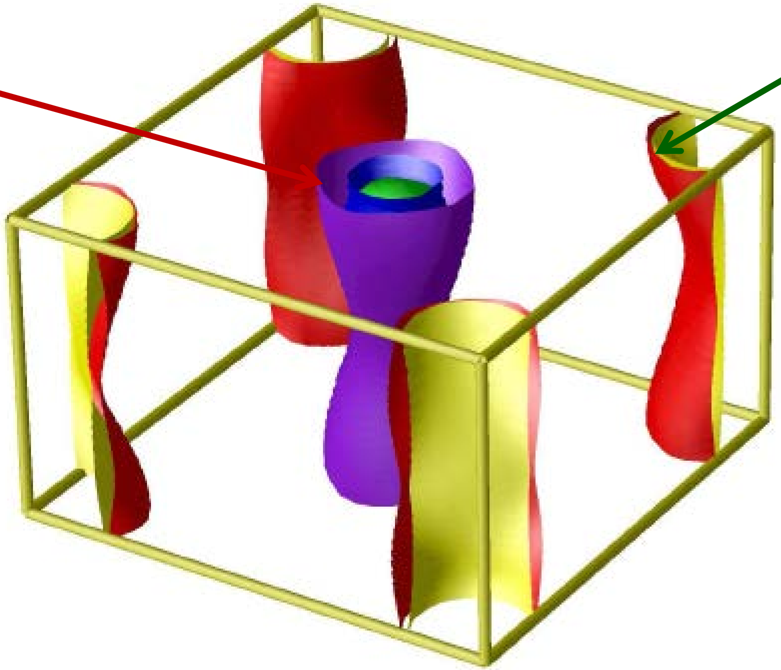
Band theory calculations for Fe-pnictides agree with experiments

Lebegue, Mazin et al, Singh & Du, Cvetkovic & Tesanovic…

Hole Fermi surface

Electron Fermi surface



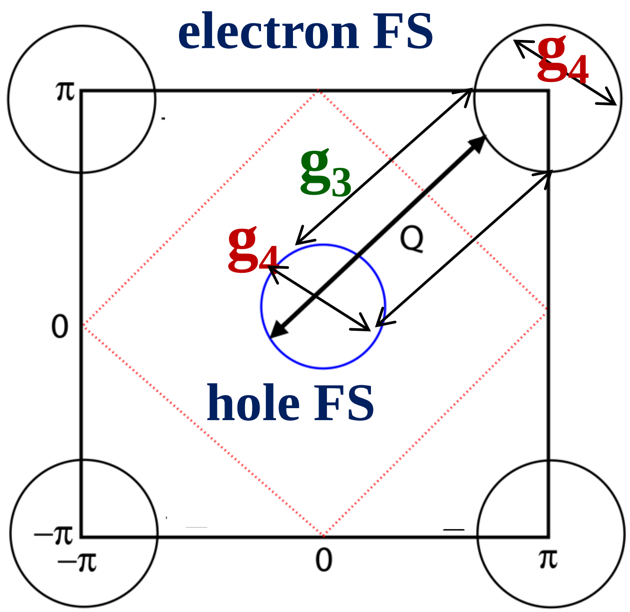
2-3 circular hole pockets around (0,0)

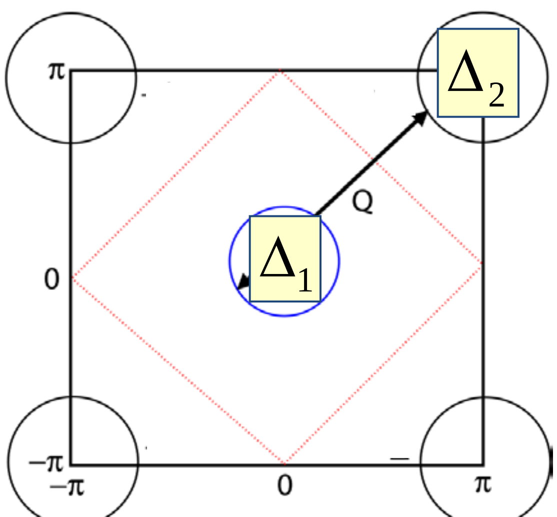
2 elliptical electron pockets around (π,π) (folded BZ), or $(0,\pi)$ and $(\pi,0)$ (unfolded BZ)

A toy model: one hole and one electron pocket

Inter-pocket repulsion g_4

Intra-pocket repulsion g_3





Eigenfunctions

$\Delta_a\,(1,1)$ $\Delta_b\,(1,-1)$

Eigenvalues

$$\Gamma_{a,0} = -g_3 - g_4, \qquad \Gamma_{b,0} = -g_4 + g_3,$$

$\Gamma > 0$ is needed for SC

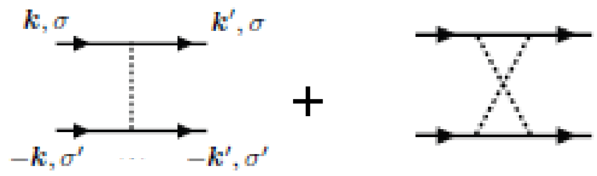
$$\Gamma_{a,0} = -g_3 - g_4,$$
$$\Gamma_{b,0} = g_3 - g_4,$$
$\Gamma > 0$ is needed for SC

Do Kohn-Luttinger analysis:

As before, consider Hubbard U

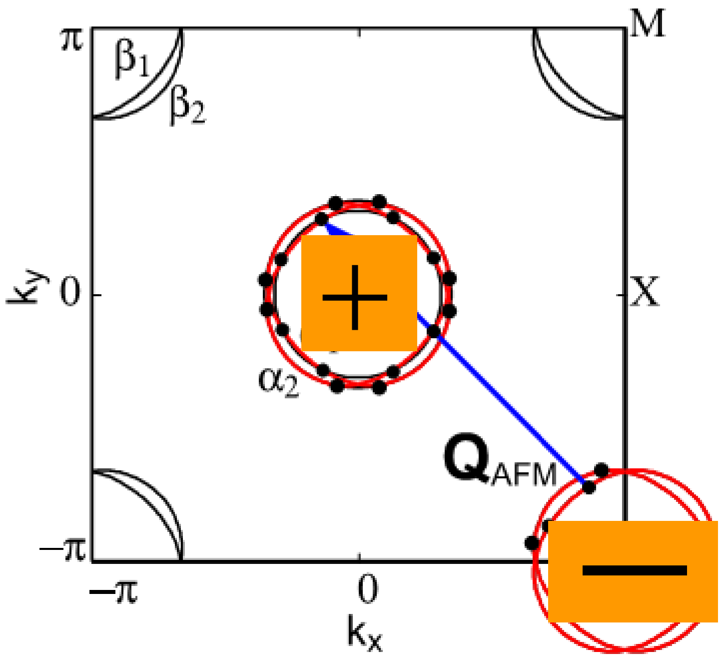
To first order in U, $g_4=g_3=U$, and we only have a repulsive s-wave component $\Gamma_{a,0} < 0$, $\Gamma_{b,0} = 0$

To order U^2

$\Gamma_0 =$ 

Inter-pocket repulsion g_3 exceeds intra-pocker repulsion g_4, and $\Gamma_{b,0}$ becomes positive, i.e., the system is unstable towards a superconductivity with $\Delta = \Delta_b\,(1,-1)$.

Agterberg, Barzykin, Gorkov, Mazin, Kuroki, ……………….



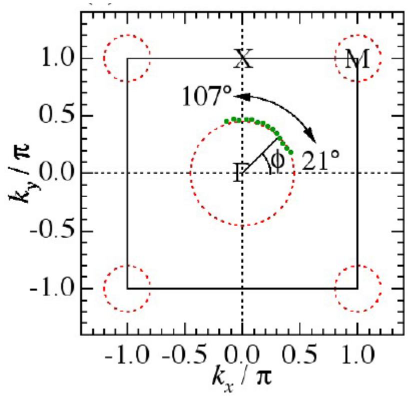
sign-changing s-wave gap (s^{+-})

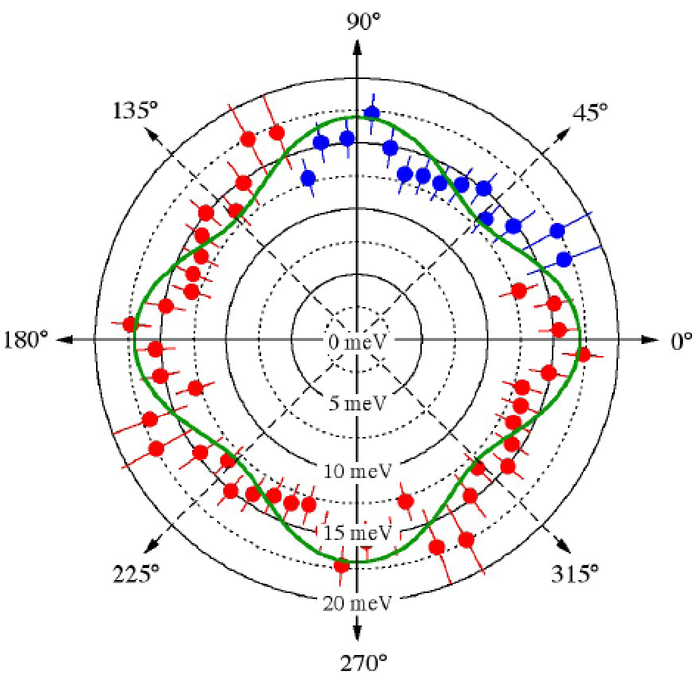
S-wave

Photoemission in 1111 and 122 FeAs

Data on the hole Fermi surfaces

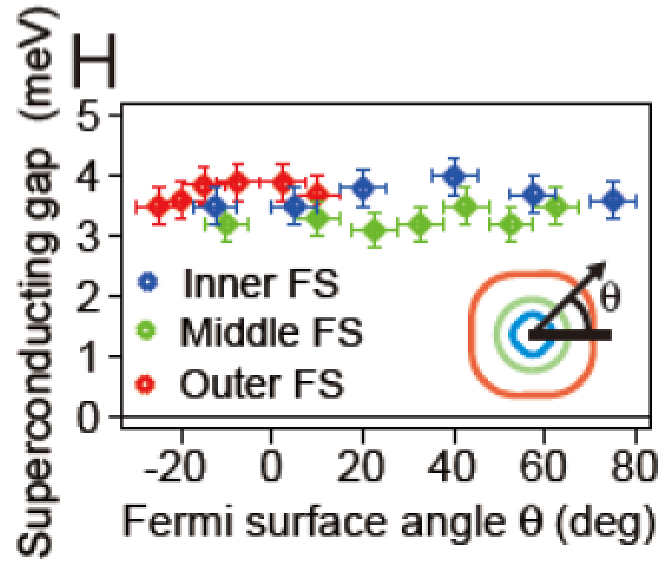
$NdFeAsO_{1-x}F_x$





T. Kondo et al.

$BaFe_2(As_{1-x}P_x)_2$



T. Shimojima et al

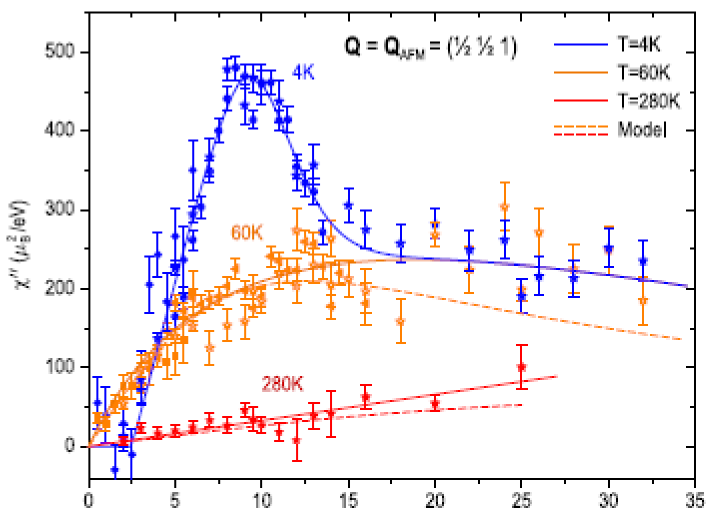
laser ARPES

Almost angle-independent gap
(consistent with s-wave)

s+- gap

Neutron scattering – resonance peak below 2D

$BaFe_{1.85}Co_{0.15}As_2$ ($T_c = 25$ K)



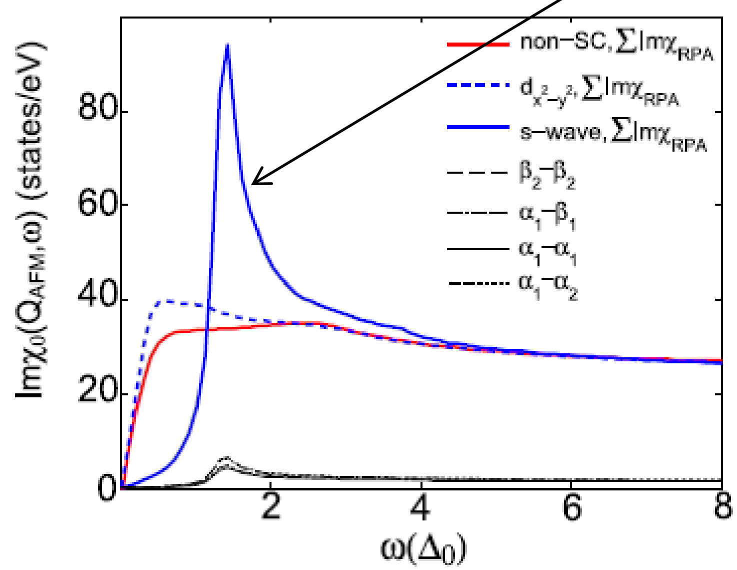
D. Inosov et al

s_{+-} gap

Theorists say :

one needs $\Delta_{k+\pi} = - \Delta_k$

The "plus-minus" gap is the best candidate



Eremin & Korshunov Scalapino & Maier…

Summary of Kohn-Luttinger physics:

At weak coupling, a fermionic system may undergo a superconducting instability, despite that the interaction is repulsive. The instability is never an ordinary s-wave

d-wave ($d_{x^2-y^2}$) pairing in the cuprates
d+id ($d_{x^2-y^2} + d_{xy}$) in doped graphene
s+- in Fe-pnictides

This story is a little bit too good to be true.

In all three cases, we assumed that bare interaction is a Hubbard U, in which case, in a relevant channel $\Gamma = 0$ to order U and becomes positive (attractive) to order U^2

In reality, to order U, $\Gamma = -U_{small} + U_{large}$

small (large) is a momentum transfer

For any realistic interaction, $U_{small} > U_{large}$

Then bare $\Gamma < 0$, and the second order term has to overcome it

Houston, we have a problem

One possibility is to abandon weak coupling (next lecture – spin fluctuation induced pairing)

Another is to keep couplings as weak, but see whether we can additionally enhance KL terms (this is what we will do now)

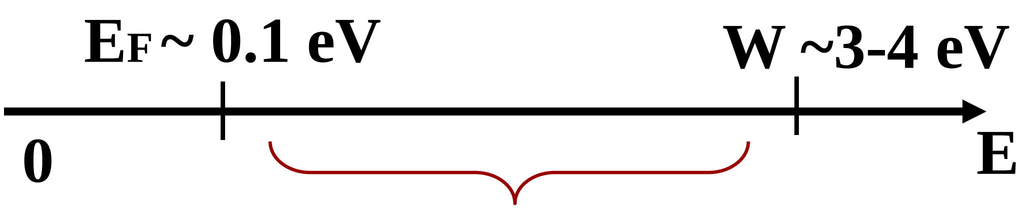
The idea is that, if superconductivity competes with other potential instabilities, like SDW or CDW, there may be additional enhancement of the pairing interaction at large momentum transfer, and simultaneous reduction (and even sign change) of the pairing interaction at a small momentum transfer

$$\Gamma_{a,0} = -g_3 - g_4,$$
$$\Gamma_{b,0} = g_3 - g_4,$$
$$\Gamma > 0 \text{ is needed for SC}$$

Consider Fe-pnictides as an example

g_3 and g_4 are bare interactions, at energies of a bandwidth

For SC we need interactions at energies smaller than the Fermi energy



Couplings flow due to renormalizations in particle-particle and particle-hole channels

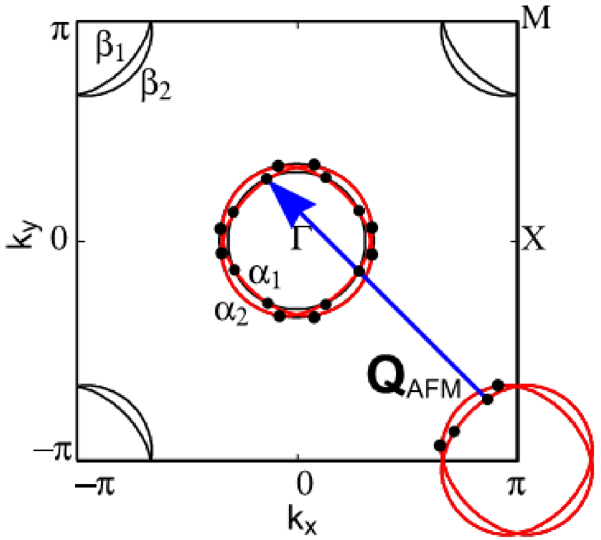
Suppose that hole and electron pockets are identical

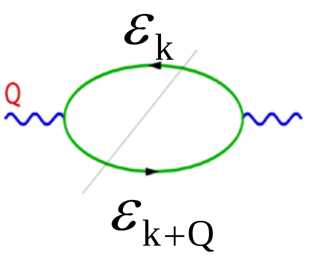
$$\varepsilon_k^h = -\varepsilon_{k+Q}^e$$

Renormalizations in particle-particle and particle-hole channels are both logarithmically singular

particle-particle channel – Cooper logarithm

particle-hole cannel – logarithm due to nesting



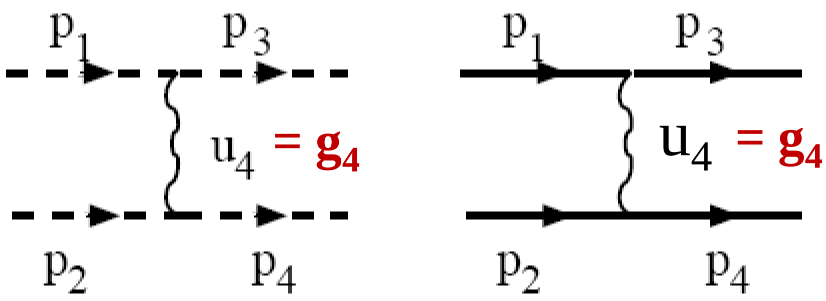


$$= \iint_T \frac{d\omega\, d\varepsilon_k}{\omega^2 + \varepsilon_k^2} = \log \frac{E_F}{T}$$

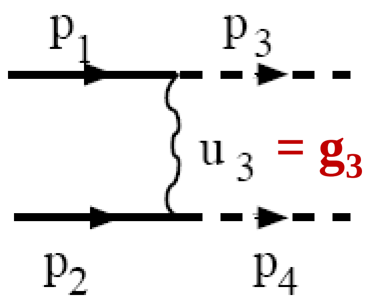
Then we have to treat particle-particle (SC) and particle-hole channels on equal footings

Introduce all relevant couplings between low-energy fermions

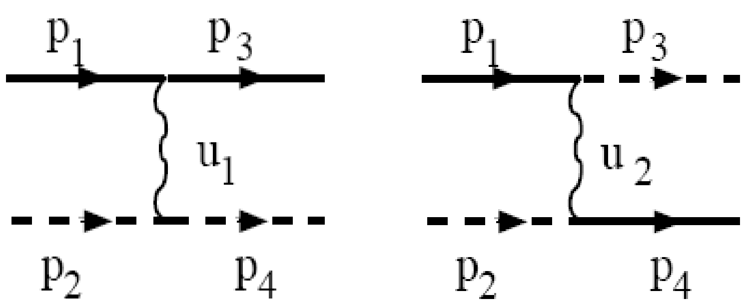
With apologies, I will label interactions as u instead of g



Intra-pocket repulsion



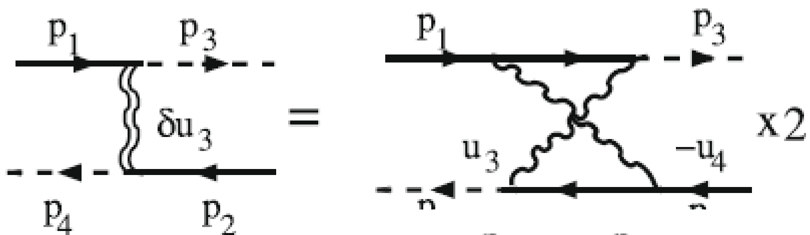
Inter-pocket repulsion



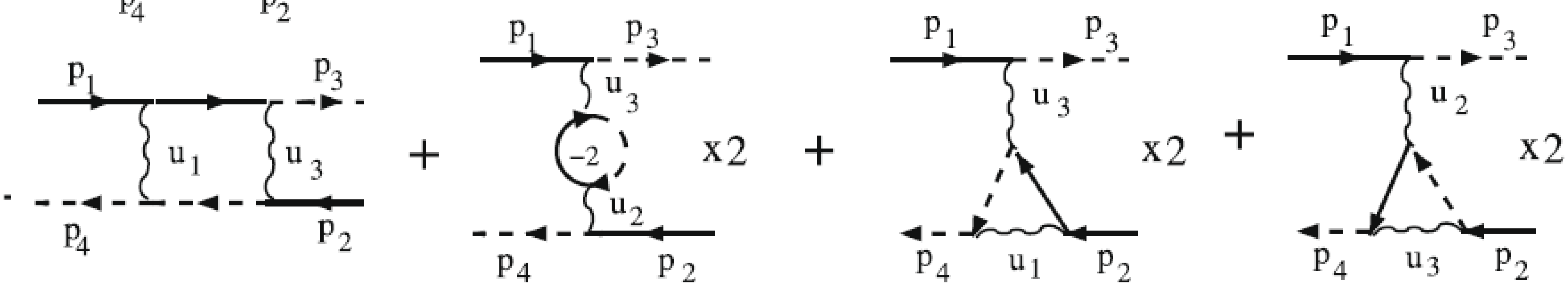
Inter-pocket forward and backward scattering

We need enhancement of u_3 relative to u_4 for superconductivity

Renormalization of u_3

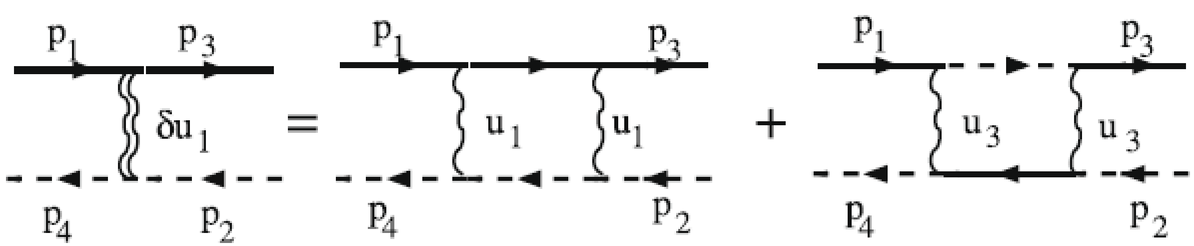


Particle-particle channel,
Cooper logarithm



Kohn-Luttinger diagrams, "nesting logarithms"

Renormalization of u_1



Also contains "nesting logaritms"

$$\dot{u}_1 = u_1^2 + u_3^2$$

$$\dot{u}_2 = 2u_2(u_1 - u_2)$$

$$\dot{u}_3 = u_3(4u_1 - 2u_2 - 2u_4 \quad)$$

$$\dot{u}_4 = -u_3^2 - u_4^2$$

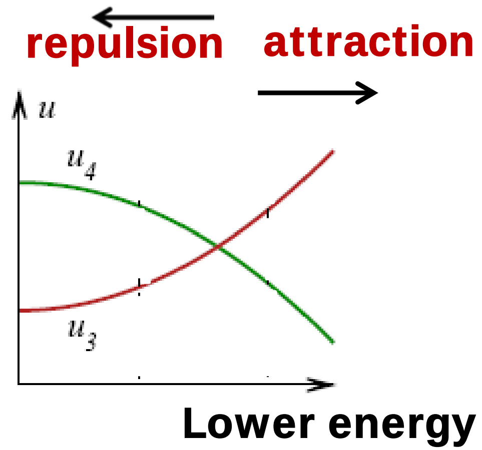
One-loop parquet RG

$$\dot{u}_1 = u_1^2 + u_3^2$$

$$\dot{u}_2 = 2u_2(u_1 - u_2)$$

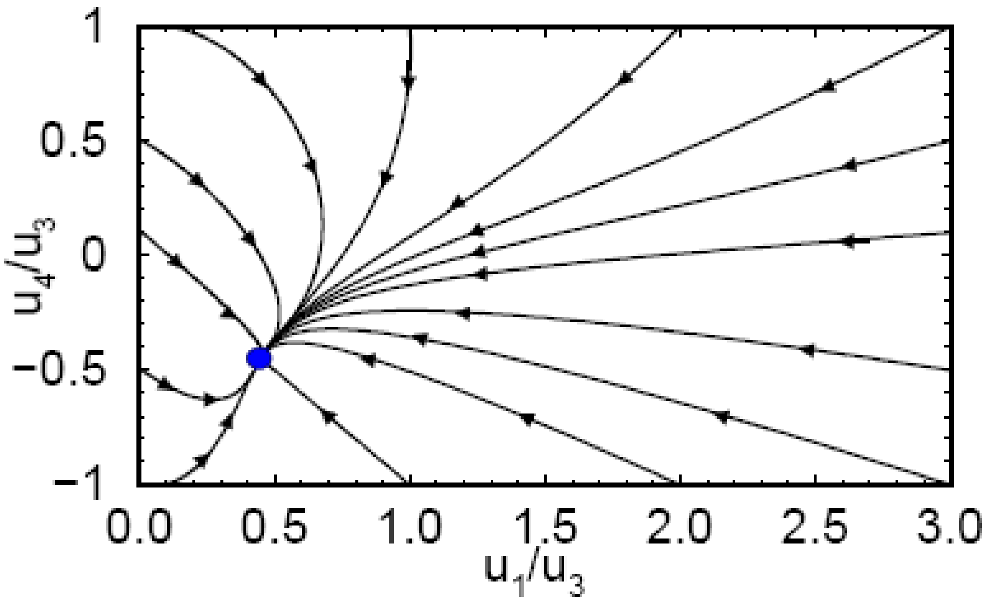
$$\dot{u}_3 = 2u_3(2u_1 - u_2 - u_4)$$

$$\dot{u}_4 = -u_3^2 - u_4^2$$



One-loop parquet RG

$$\dot{u}_1 = u_1^2 + u_3^2$$
$$\dot{u}_2 = 2u_2(u_1 - u_2)$$
$$\dot{u}_3 = 2u_3(2u_1 - u_2 - u_4)$$
$$\dot{u}_4 = -u_3^2 - u_4^2$$



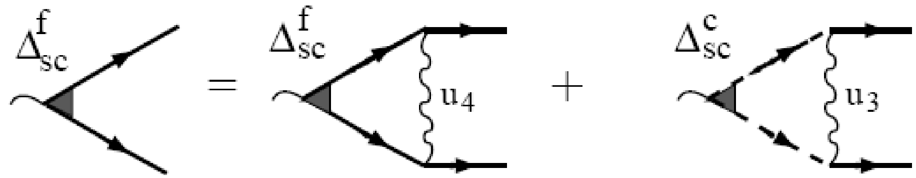
The fixed point: the pair hopping term u_3 is the largest

$$u_1 = -u_4 = \frac{|u_3|}{\sqrt{5}}, u_2 \propto |u_3|^{1/3}$$

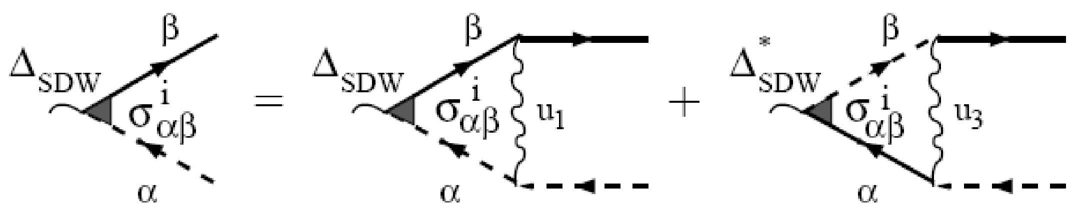
Over-screening: intraband interaction u_4 changes sign and becomes attractive below some scale.

We can re-write parquet RG equations as equations for density-wave and superconducting vertices

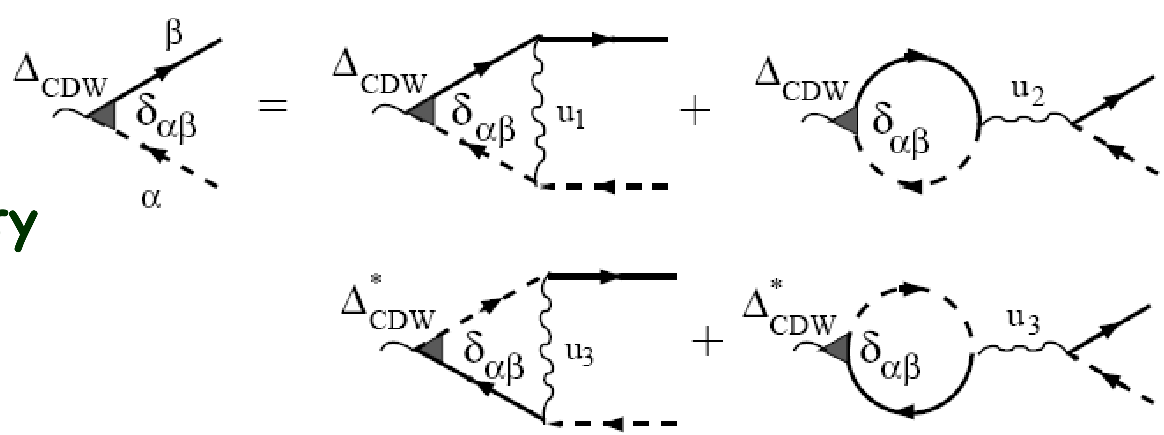
Super-conductivity



Spin-density wave



Charge-density wave

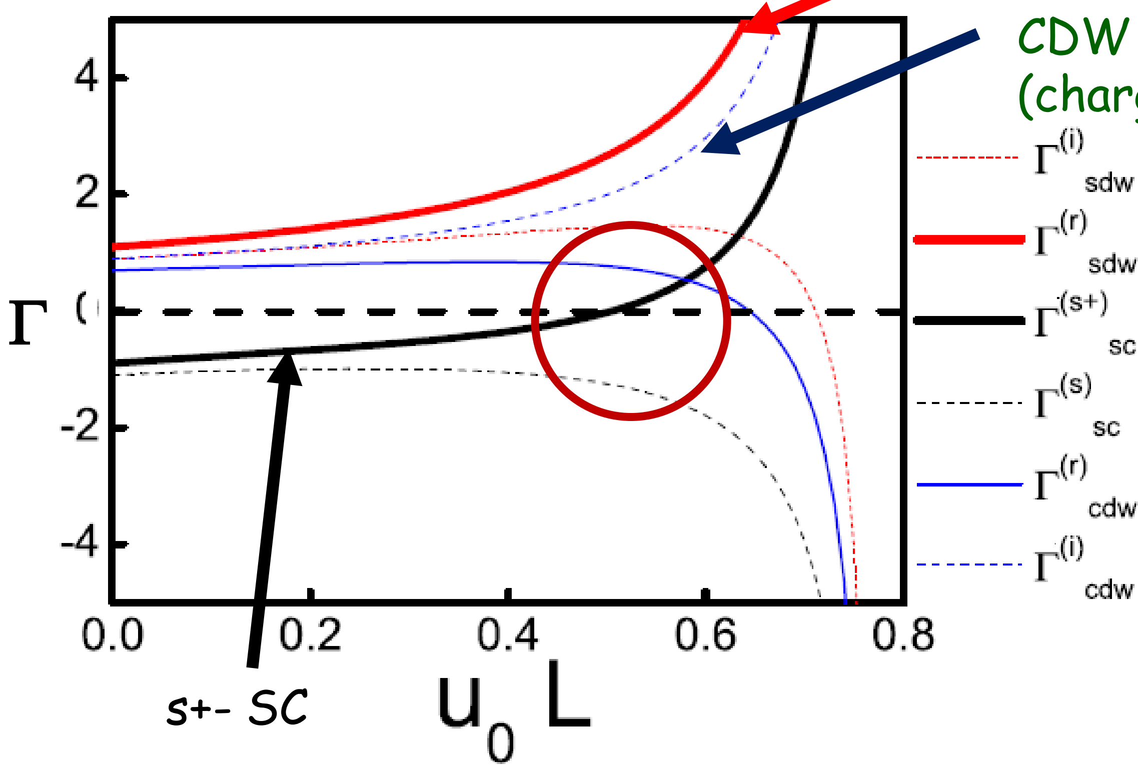


One-loop RG Flow – all channels

SDW with real order parameter

Flow of the vertices

CDW with imaginary order parameter (charge current)



O(6) fixed point:
3 for SDW,
2 for SC,
1 for CDW

At some scale, generated by the system, s+- SC vertex changes sign and becomes attractive

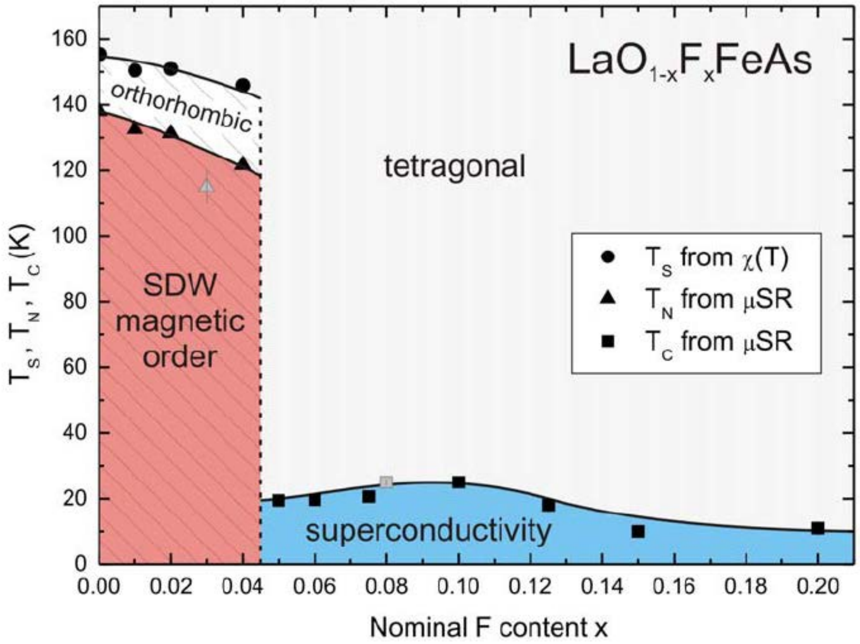
Lower boundary for parquet RG is the Fermi energy, E_F

Below E_F – decoupling between SDW and SC channels

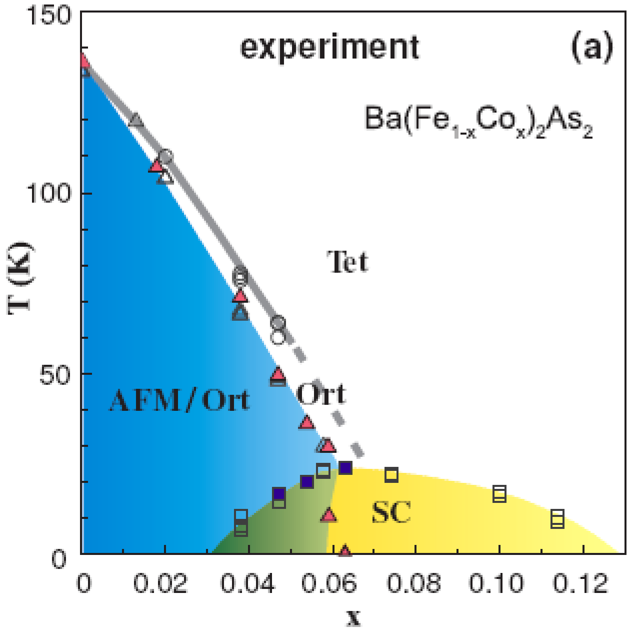
$$\frac{d\Gamma_j}{dL} = \Gamma_j^2$$

$$\text{Boundary values: } u_i\,(E \sim E_F) = u_i^R$$
$$\Gamma_{SDW} = u_1^R + u_3^R,\ \ \Gamma_{SC} = u_3^R - u_4^R$$

Whichever vertex is the larger by magnitude at E_F, wins



Perfect nesting – SDW wins

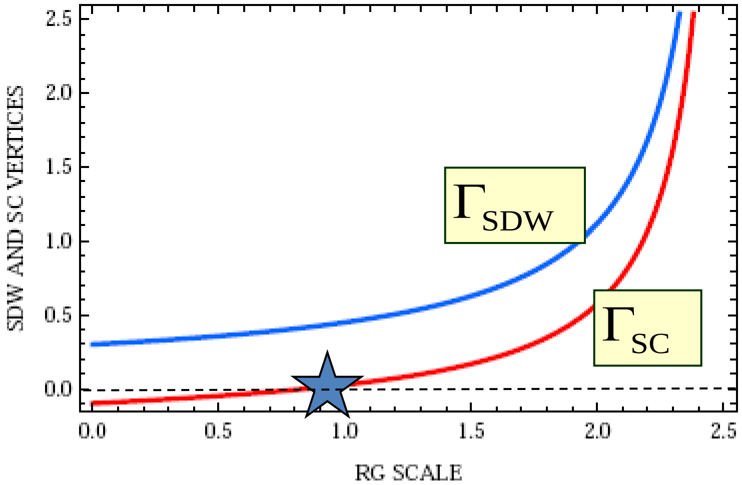


Non-perfect nesting –SDW vertex remains the strongest, but the SDW instability is cut, and s^{+-} SC wins

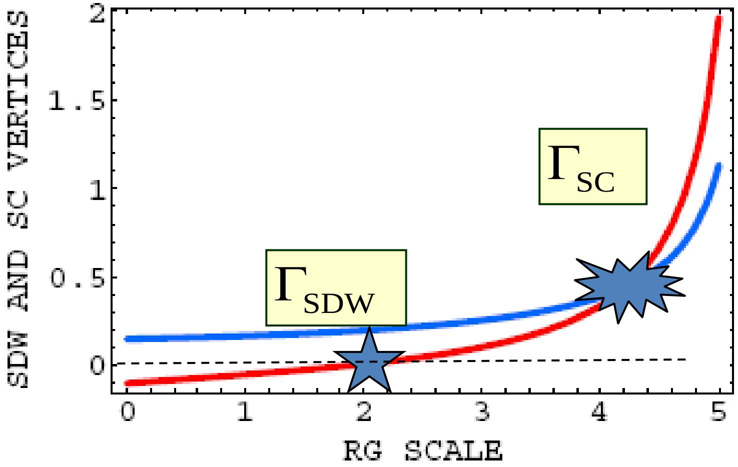
It is essential that Γ_{SC} is already attractive

In real systems, there are 2-3 hole and 2 electron Fermi surfaces

1 hole and 1 electron FSs

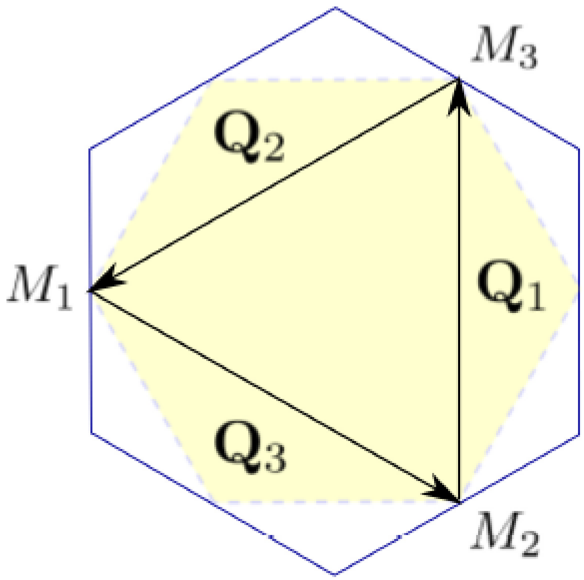


2 hole and 2 electron FSs



**SC vertex can overshoot SDW vertex,
in which case SC becomes the leading instability**

A very similar behavior in doped graphene



Because of van-Hove points

- superconducting susceptibility gets an extra boost:

$$\Pi_{pp}(0) \propto \log^2 \frac{\Lambda}{T}$$

Because of nesting and van-Hove points

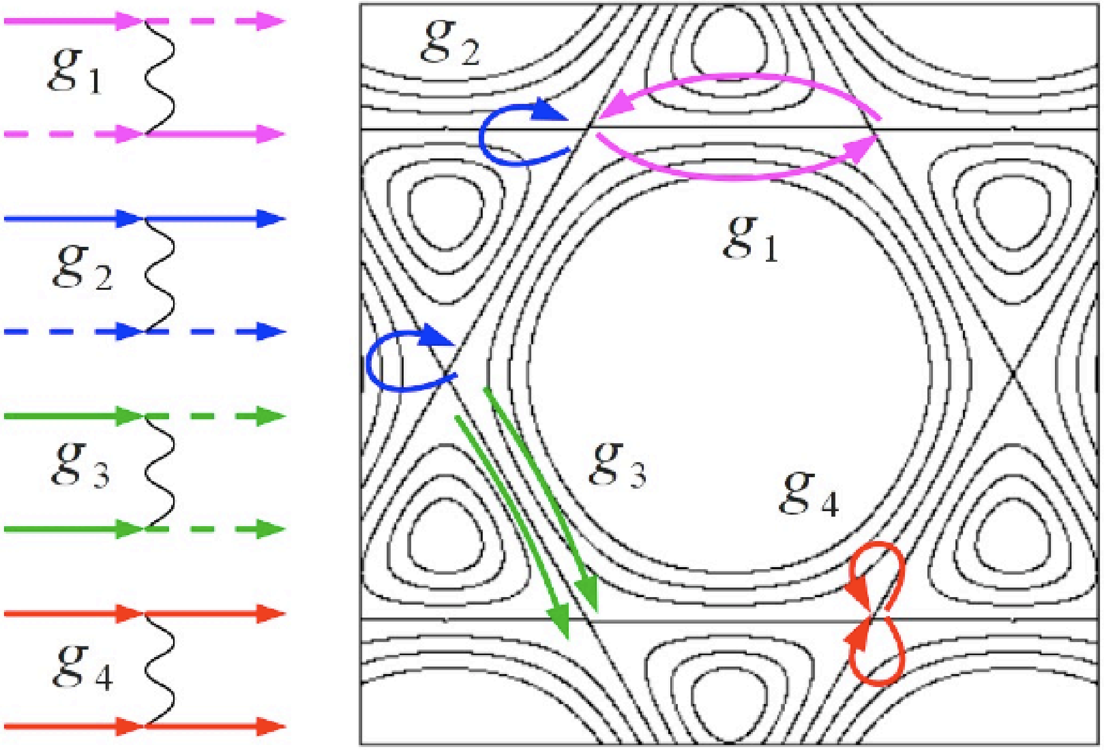
- density-wave susceptibilities at Q_i become equivalent to SC susceptibility

$$\Pi_{ph}(Q_i) \propto \log^2 \frac{\Lambda}{T}$$

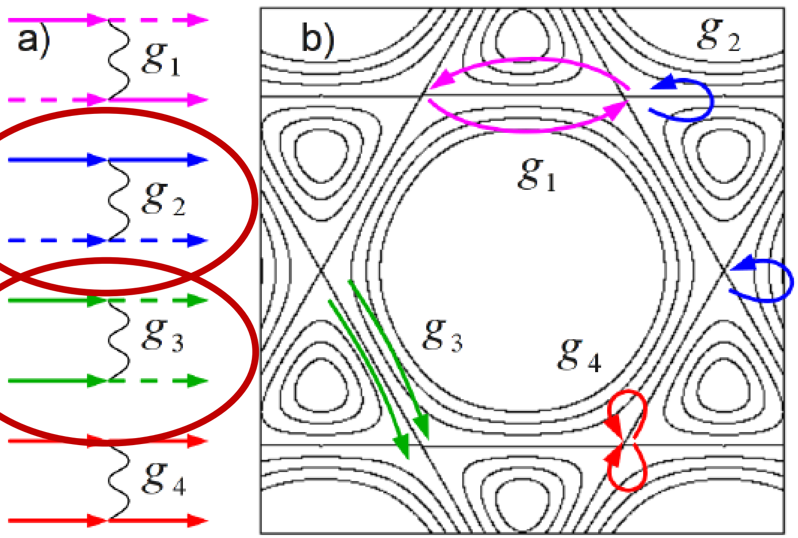
Like before, we introduce all possible interactions between low-energy fermions

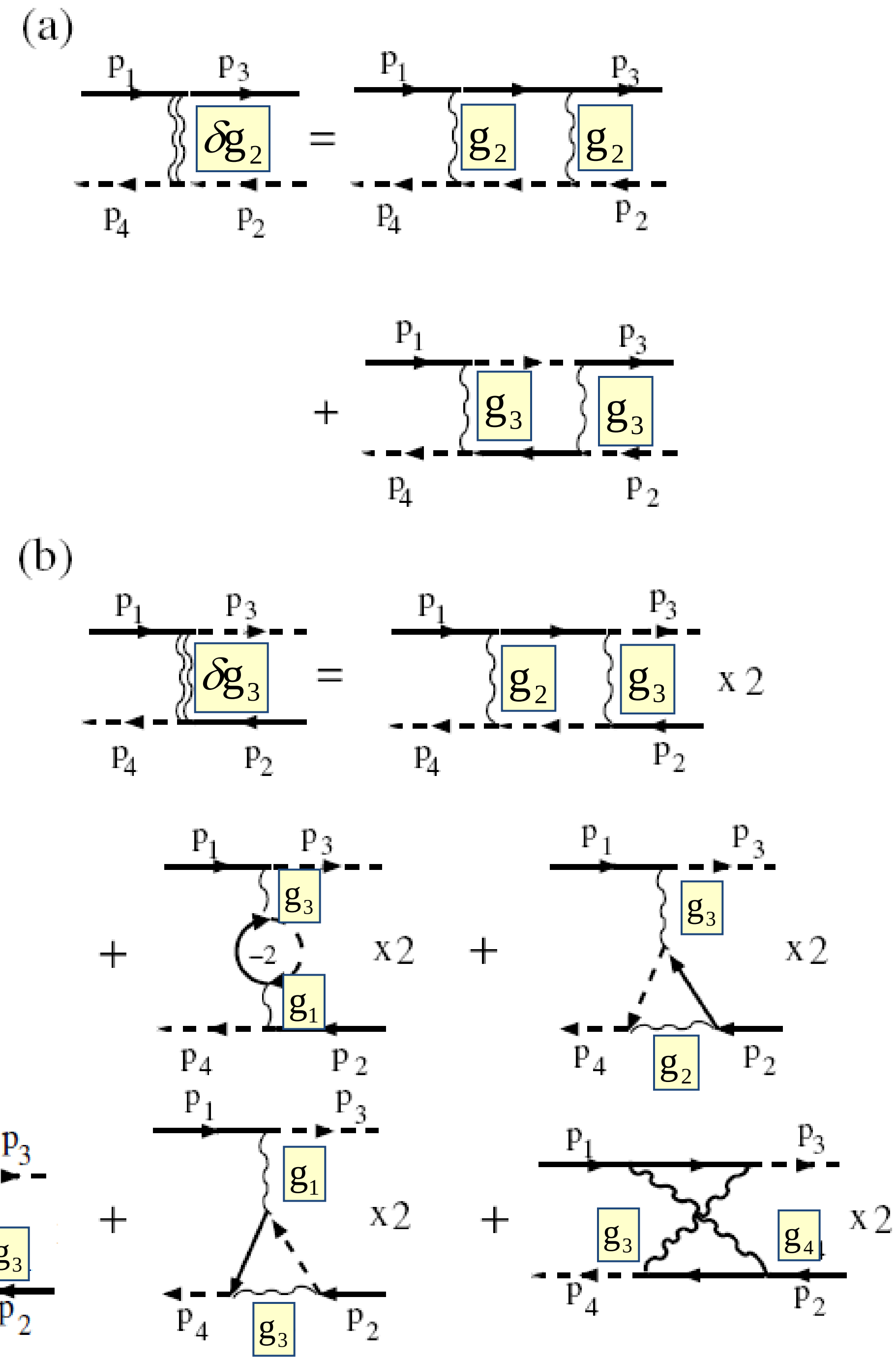
$$H_{two-particle}=\sum_{\alpha,\beta=1}^{3}\frac{g_1}{2}\psi_\alpha^+\psi_\beta^+\psi_\alpha\psi_\beta+\frac{g_2}{2}\psi_\alpha^+\psi_\beta^+\psi_\beta\psi_\alpha+\frac{g_3}{2}\psi_\alpha^+\psi_\alpha^+\psi_\beta\psi_\beta$$

$$+\sum_{\alpha=1}^{3}\frac{g_4}{2}\psi_\alpha^+\psi_\alpha^+\psi_\alpha\psi_\alpha$$



RG equations (perfect nesting)





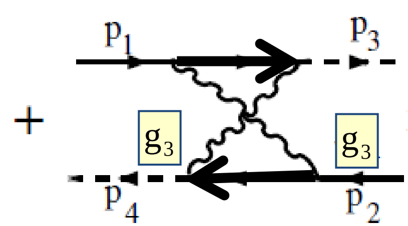
$$\dot{g}_2 = g_2^2 + g_3^2$$

$$\dot{g} = dg/d\,(\log(\Lambda/E))^2$$

$$\dot{g}_3 = g_3\left(4g_2 - 2g_1 - 2g_4 - g_3^2\right)$$

$$\dot{g}_1 = 2\,g_1\left(g_2 - g_1\right)$$

$$\dot{g}_4 = -2g_3^2 - g_4^2$$



all 3 patches are involved

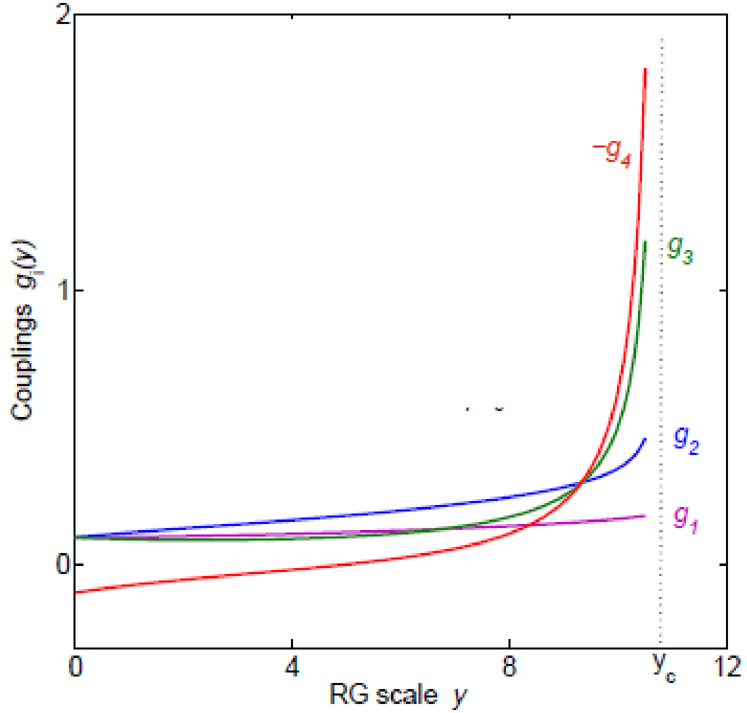
General RG equations

$$y = \Pi_{\rm pp}(\mathbf{k}=0, E) = \frac{\nu_0}{4}\ln^2\frac{\Lambda}{E}$$

$$\frac{dg_1}{dy} = 2 \quad g_1(g_2 - g_1), \qquad \frac{dg_2}{dy} = \quad (g_2^2 + g_3^2),$$

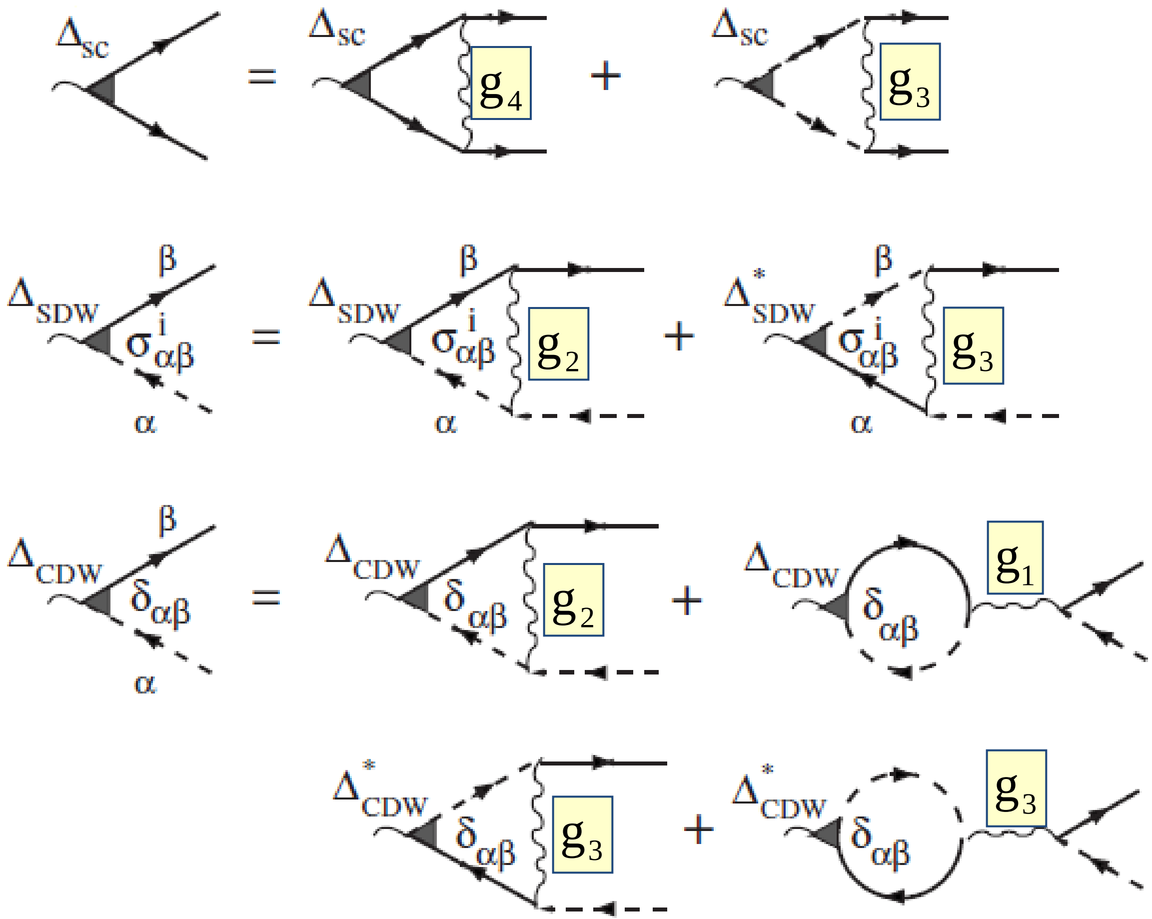
$$\frac{dg_3}{dy} = -(n-2)g_3^2 - 2g_3g_4 + 2 \quad g_3(2g_2 - g_1), \qquad \frac{dg_4}{dy} = -(n-1)g_3^2 - g_4^2.$$

n=3 is the # of patches (n=2 for the cuprates)



Inter-patch pairing interaction g_3 again becomes the largest one

SDW, CDW, and SC vertices

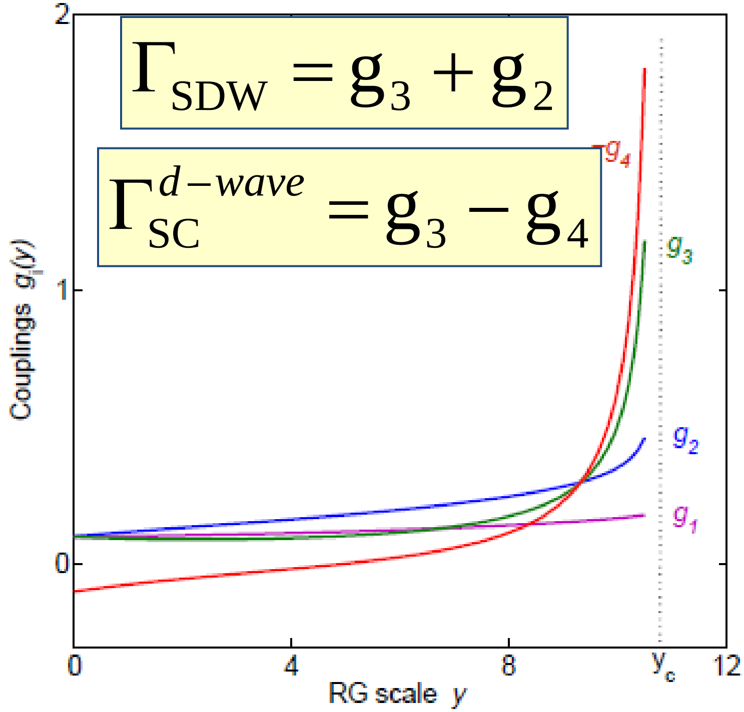


$$\Delta_j = \Delta^0_j \left(1 - \Gamma_j \log^2 \frac{\Lambda}{E}\right)$$

$$\Gamma_{SDW} = g_3 + g_2$$

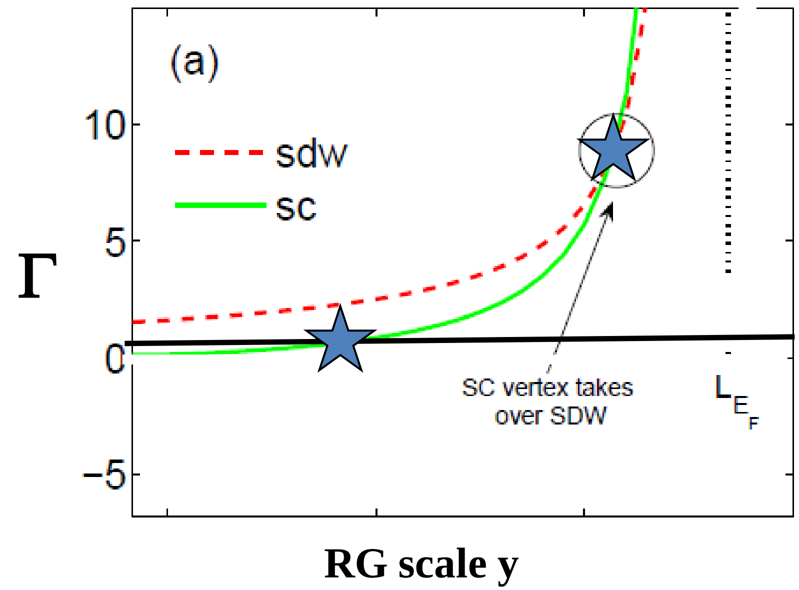
$$\Gamma^{d-wave}_{SC} = g_3 - g_4$$

Need $\Gamma > 0$ for instability



$$\Gamma_{SDW} = g_3 + g_2$$

$$\Gamma_{SC}^{d-wave} = g_3 - g_4$$



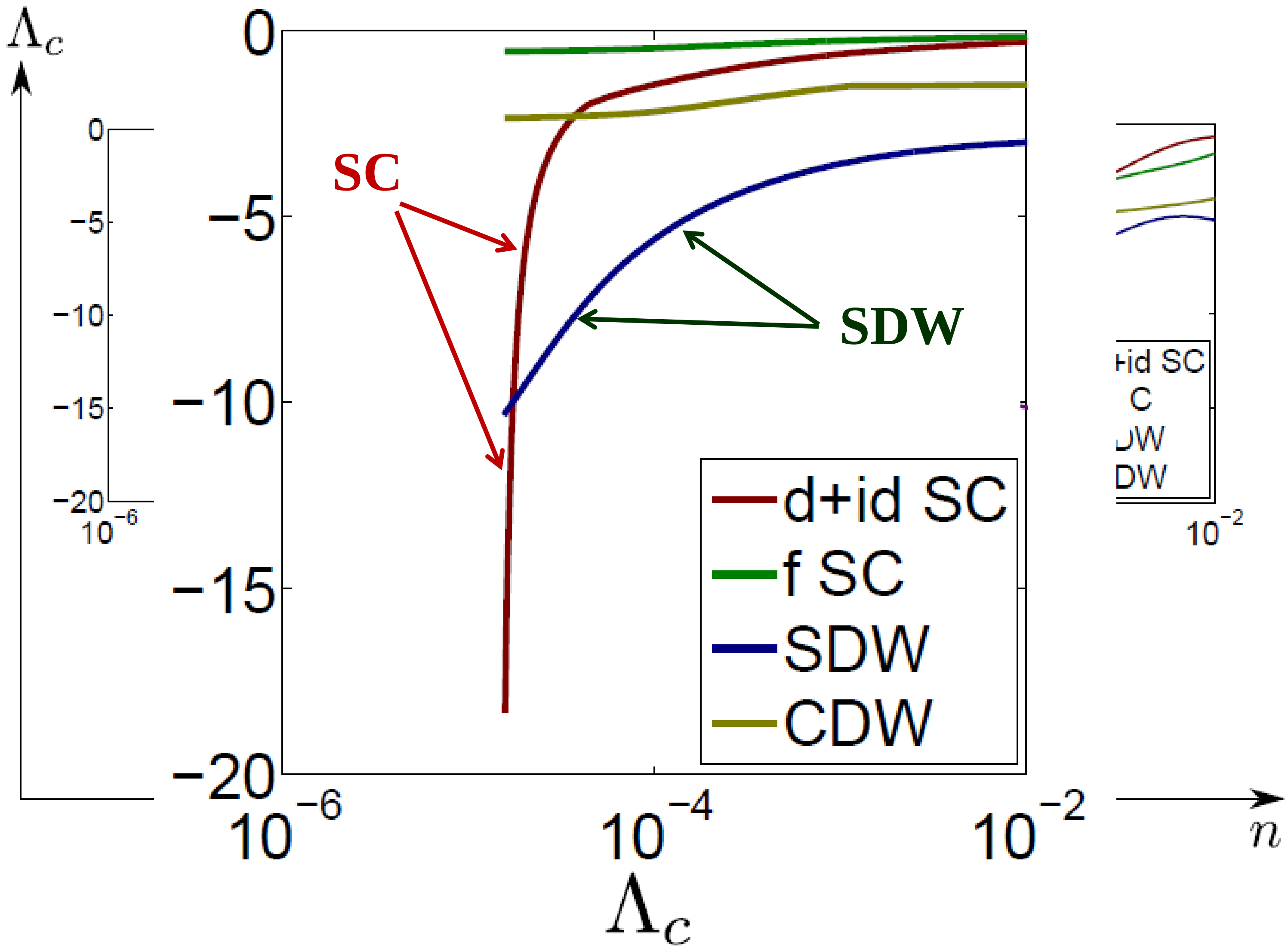
- **The SDW vertex is the largest one at intermediate energies**

- **Interaction with SDW channel pushes SC vertex up, and $\Gamma^{d\text{-wave}}_{SC}$ changes sign and becomes attractive**

- **The superconducting vertex eventually takes over and becomes the leading instability at low energies**

Functional RG – the same result

Thomale et al



Conclusions:

The issue is the pairing by electron-electron interaction

I. Kohn-Luttinger mechanism:

For on-cite Hubbard interaction

p-wave pairing for isotropic dispersion
d-wave ($d_{x^2-y^2}$) pairing in the cuprates
d+id ($d_{x^2-y^2} + d_{xy}$) in doped graphene
s+- in Fe-pnictides

II. If first-order (bare) interaction in these channels is repulsive, SC is still possible when fluctuations in the density-wave channel are comparable to SC fluctuations (SC vertex is pushed up due to interaction with SDW)

THANK YOU