Fluctuations in Systems without Quasiparticles (SYK)

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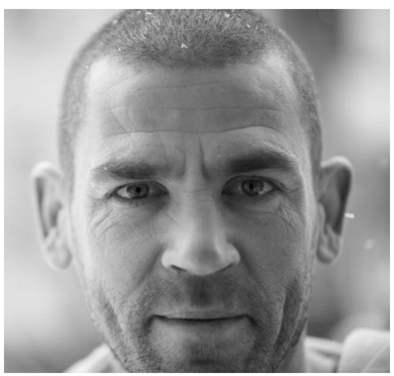


Alexander Altland
Univ. zu Köln

Dmitry Bagrets
Univ. zu Köln

Nucl. Phys. B 911, 191, 2016

Phys. Rev. Lett. 123, 106601, 2019

Phys. Rev. Lett. 123, 226801, 2019

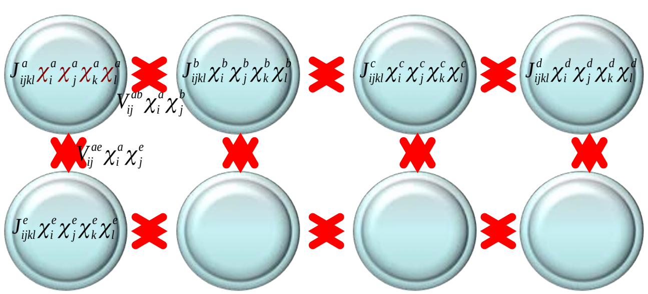
Florida Winter School, Jan. 2020

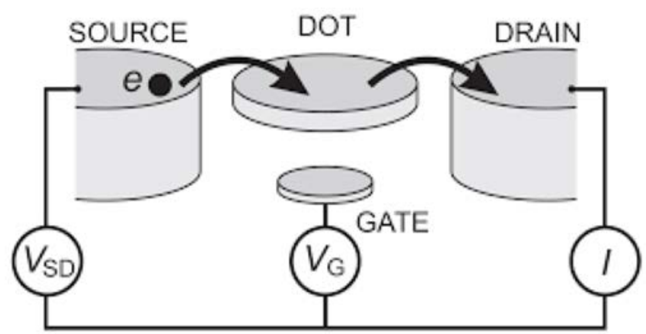
Why SYK?

Exactly solvable model (in more than one sense) of a system without quasiparticles (non-Fermi liquid).

Relation to holography and gravitation (*may be* teaches us new techniques, *may be* solves problems in gravity).

A stable fixed point, describing real life phenomena: bulk correlated matter or transport in quantum dots.





Outline:

Fluctuations in SYK model (Schwarzian FT).

SYK matter (Schwarzian RG).

SYK superconductivity   (extra degrees of freedom).

Sachdev-Ye-Kitaev model

$$\hat{H} = \frac{1}{4!}\sum_{ijkl}^{N} J_{ijkl}\,\chi_i \chi_j \chi_k \chi_l$$

$$\{\chi_i, \chi_j\} = \delta_{ij}$$

- Couplings J's are quenched random Gaussian variables:

$$\langle J_{ijkl} \rangle = 0; \quad \langle (J_{ijkl})^2 \rangle = 3!\, J^2 / N^3$$

Sachdev-Ye model

S. Sachdev, J. Ye, PRL 69 (1992).

$$\hat{H} = \frac{1}{2}\sum_{ij,kl}^{N/2} J_{ij;kl} c_i^+ c_j^+ c_k c_l - \mu \sum_i c_i^+ c_i$$

$$\left\{ c_i^+, c_j \right\} = \delta_{ij}$$

- spinless fermions

Couplings J's are quenched random Gaussian (either REAL or COMPLEX) variables:

$$\left\langle \left| J_{ijkl} \right|^2 \right\rangle = \frac{J^2}{N^3}$$

Volume 34B. number 4 PHYSICS LETTERS 1 March 1971

TWO-BODY RANDOM HAMILTONIAN AND LEVEL DENSITY

O. BOHIGAS and J. FLORES *
Institut de Physique Nucléaire, Division de Physique Théorique ‡ , 91 - *Orsay* - *France*

Received 22 December 1970

Volume 33B, number 7 PHYSICS LETTERS 7 December 1970

VALIDITY OF RANDOM MATRIX THEORIES FOR MANY-PARTICLE SYSTEMS *

J. B. FRENCH
Department of Physics and Astronomy, University of Rochester, Rochester, New York, USA

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and Department of Physics and Astronomy, University of Rochester, Rochester, New York, USA

Received 19 October 1970

VOLUME 70, NUMBER 21 PHYSICAL REVIEW LETTERS 24 MAY 1993

Gapless Spin-Fluid Ground State in a Random Quantum Heisenberg Magnet

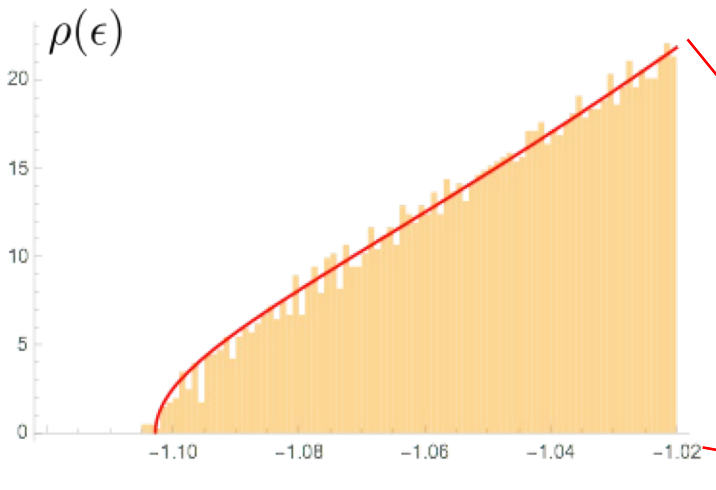
Subir Sachdev and Jinwu Ye
Departments of Physics and Applied Physics, P.O. Box 2157, Yale University, New Haven, Connecticut 06520
(Received 22 December 1992)

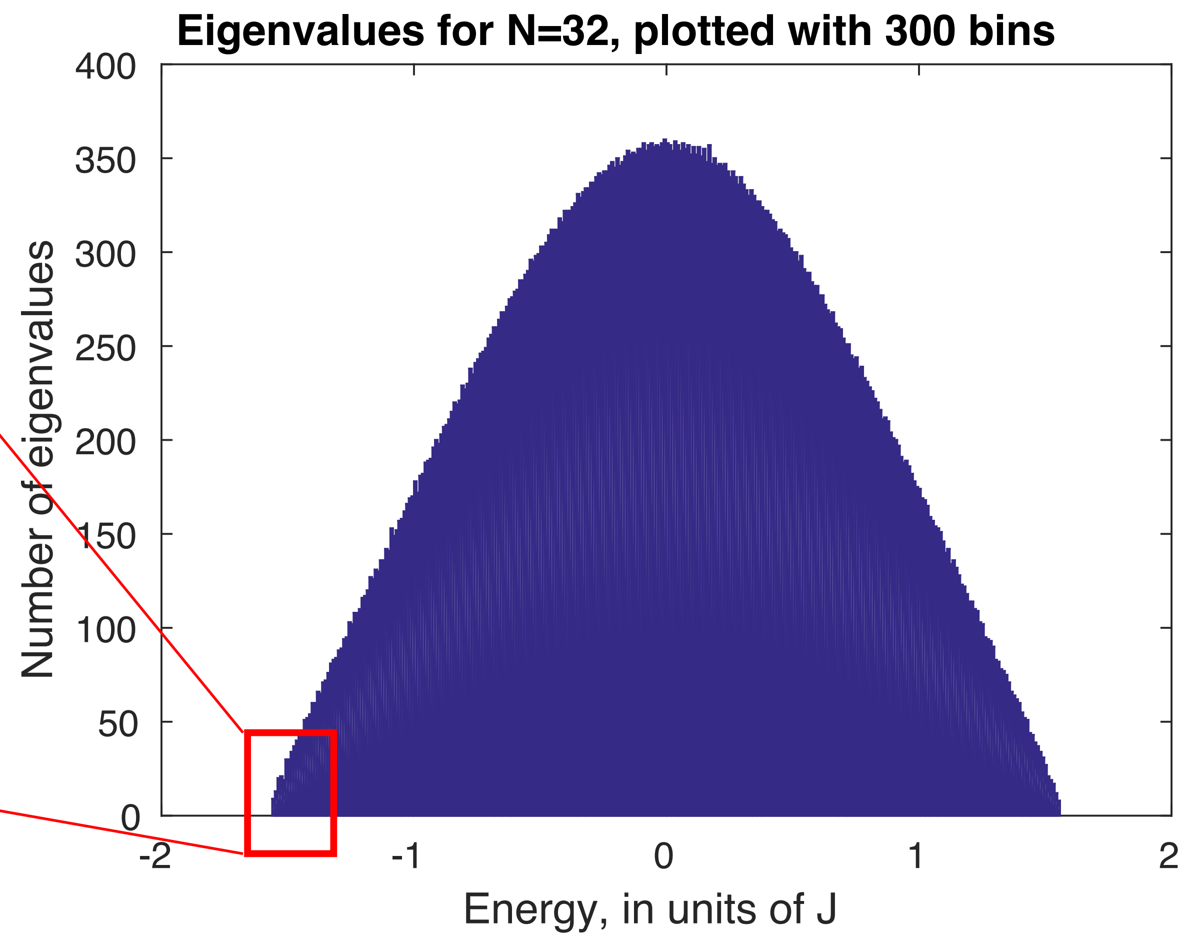
We examine the spin-S quantum Heisenberg magnet with Gaussian-random, infinite-range exchange interactions. The quantum-disordered phase is accessed by generalizing to SU(M) symmetry and studying the large M limit. For large S the ground state is a spin glass, while quantum fluctuations produce a spin-fluid state for small S. The spin-fluid phase is found to be generically gapless—the average, zero temperature, local dynamic spin susceptibility obeys $\bar{\chi}(\omega) \sim \ln(1/|\omega|) + i(\pi/2)\text{sgn}(\omega)$ at low frequencies.

A. Kitaev, talks at KITP, Spring 2015

Many-Body Spectrum

$2^{N/2}$ eigenvalues

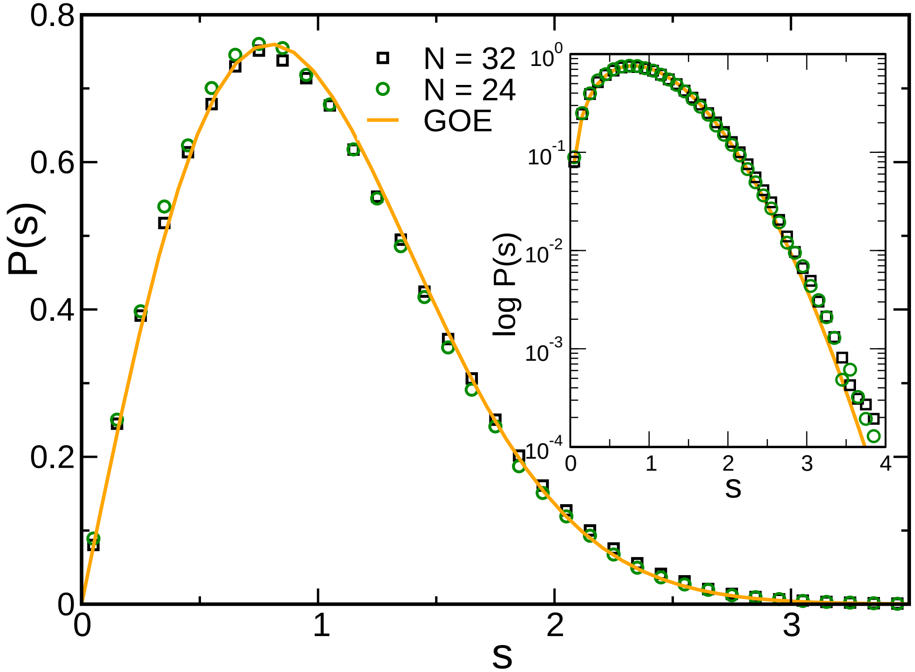




J. Maldacena & D. Stanford '2015

Many-Body Level Statistics

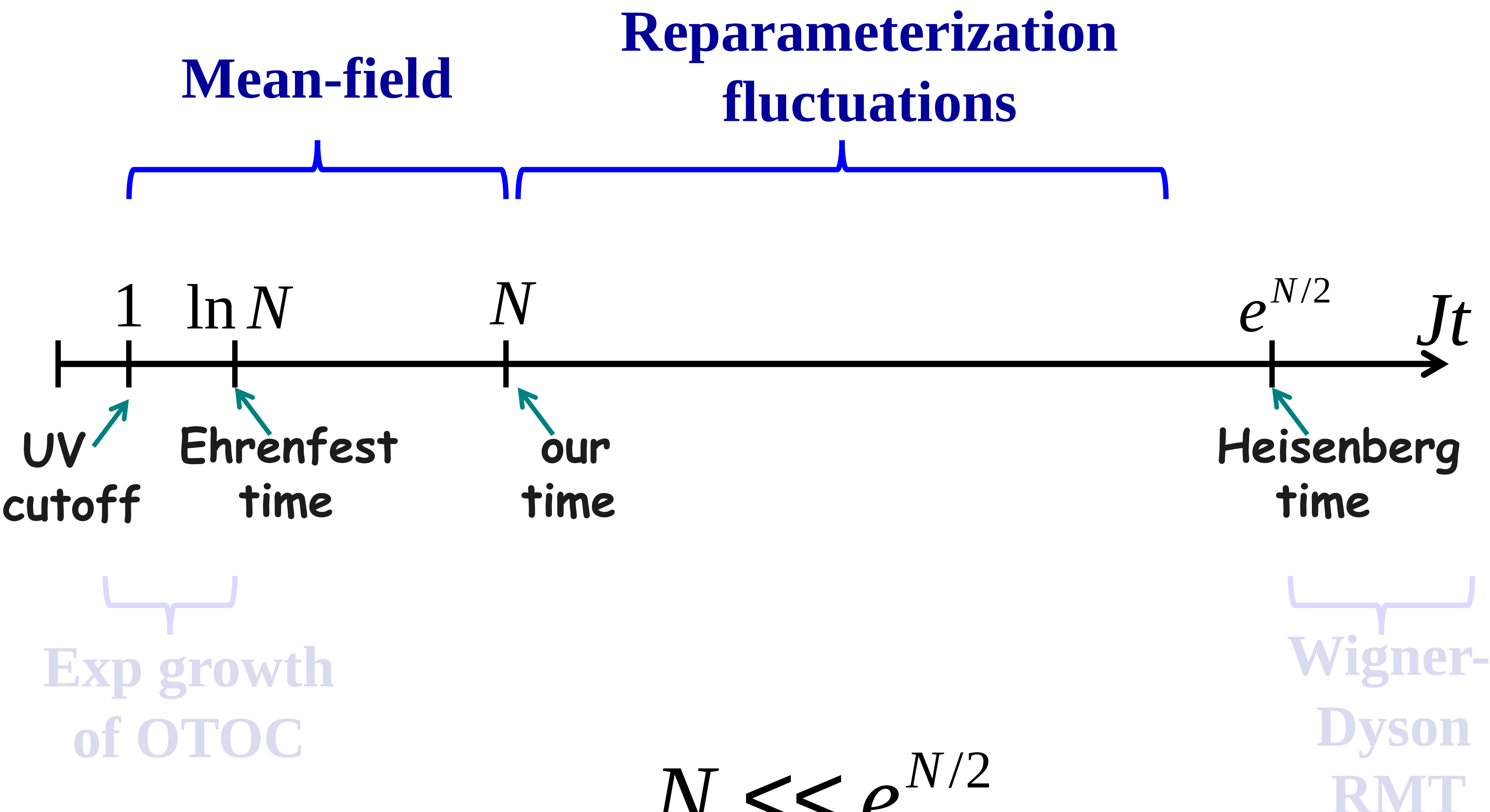
A. Garcia-Garcia, J. Verbaarschot, 2017



$$\Delta E_n = E_n - E_{n+1}$$

Many-Body delocalized ?

Time Scales



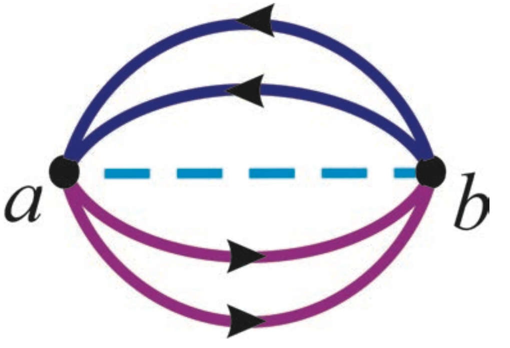
$$N << e^{N/2}$$

Effective action

S. Sachdev '2015; J. Maldacena & D. Stanford '2015

- **(R-times) replicated Matsubara action**

$$\left\langle \exp\left[-\sum_{a=1}^{R}\int \hat{H}^{a} d\tau\right]\right\rangle = \exp\left[\frac{NJ^{2}}{8}\sum_{a,b}^{R}\int\left[G_{\tau\tau'}^{ab}\right]^{4} d\tau d\tau'\right]$$



2-point Green's function: $G_{\tau\tau'}^{ab} = -\frac{1}{N}\sum_{i}\chi_{i}^{a}(\tau)\chi_{i}^{b}(\tau')$

Effective action

S. Sachdev '2015; J. Maldacena & D. Stanford '2015

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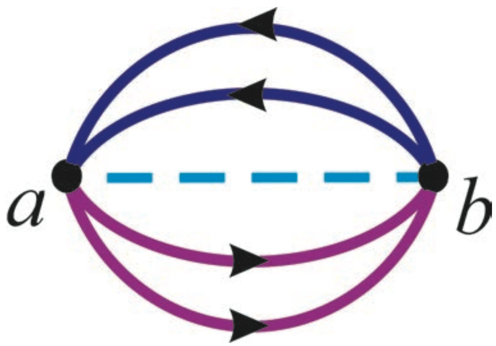


2-point Green's function: $G_{\tau\tau'}^{ab} = -\frac{1}{N}\sum_{i}\chi_i^a(\tau)\chi_i^b(\tau')$

- Resolution of identity

$$1 = \int \delta\left(NG_{\tau\tau'}^{ab} + \sum_i \chi_i^a(\tau)\chi_i^b(\tau')\right) DG = \int \exp\left[\frac{1}{2}\mathrm{tr}\left(N\Sigma \bullet G - \Sigma \bullet \sum_i \chi_i^T \otimes \chi_i\right)\right] D(G,\Sigma)$$

Effective action

S. Sachdev '2015; J. Maldacena & D. Stanford '2015

- integrating out Majoranas …

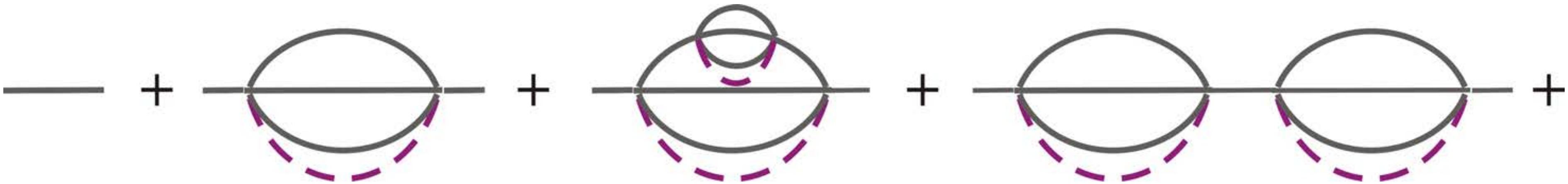
Self-energy

$$-S[G,\Sigma]=\frac{N}{2}\left(\mathrm{tr}\ln\left(\partial_{\tau}+\Sigma\right)+\frac{J^{2}}{4}\int\left[G_{\tau\tau'}^{ab}\right]^{4}d\tau d\tau'+\int\Sigma_{\tau'\tau}^{ba}G_{\tau\tau'}^{ab}d\tau d\tau'\right)$$

$N\rightarrow\infty$ classical limit – saddle point equations:

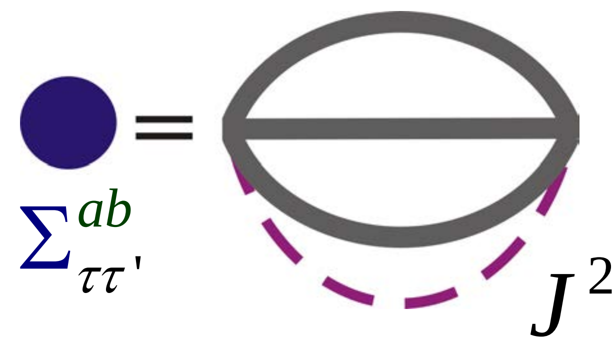
$$\delta S/\delta G=0 \qquad \delta S/\delta\Sigma=0$$

Mean-field solution: $N \to \infty$



- **Self-consistent Dyson equation** (S. Sachdev, J. Ye '1993)

$$-(\cancel{\partial_\tau} + \Sigma)\bullet G = 1, \quad \Sigma^{ab}_{\tau\tau'} = J^2\left[G^{ab}_{\tau\tau'}\right]^3$$



- **Mean-field solution (T=0)**

$$\bar{G}^{ab}_{t-t'} \propto -\frac{\delta^{ab}}{\sqrt{J}}\frac{1}{|t-t'|^{1/2}} \quad \to \quad \frac{\delta^{ab}}{\sqrt{J}}\frac{1}{|\varepsilon|^{1/2}}$$

- is of conformal form with the scaling dimension $\Delta = 1/4$

Symmetries

$$S = \int d\tau \left[\sum_j \chi_j \partial_\tau \chi_j - \frac{1}{4!} \sum_{ijkl} J_{ijkl} \chi_i \chi_j \chi_k \chi_l \right]$$

$$\tau \to h(\tau) \qquad\qquad h(\tau) \in \mathrm{Diff}(\mathrm{S}_1)$$

$$\chi_j(\tau) \to [h'(\tau)]^\Delta \chi_j(h(\tau)) \qquad\qquad \Delta = \frac{1}{4}$$

$$G_{\tau_1 - \tau_2} \to G_{\tau_1, \tau_2}[h] = \left[\frac{h'(\tau_1) h'(\tau_2)}{[h(\tau_1) - h(\tau_2)]^2} \right]^\Delta$$

Still a solution of Dyson equation: $-(\partial_\tau + \Sigma) \bullet G = 1, \quad \Sigma^{ab}_{\tau\tau'} = J^2 \left[G^{ab}_{\tau\tau'} \right]^3$

SL(2,R)

$$G_{\tau_1-\tau_2} \to G_{\tau_1,\tau_2}[h] = \left[\frac{h'(\tau_1)h'(\tau_2)}{[h(\tau_1)-h(\tau_2)]^2}\right]^{\Delta}$$

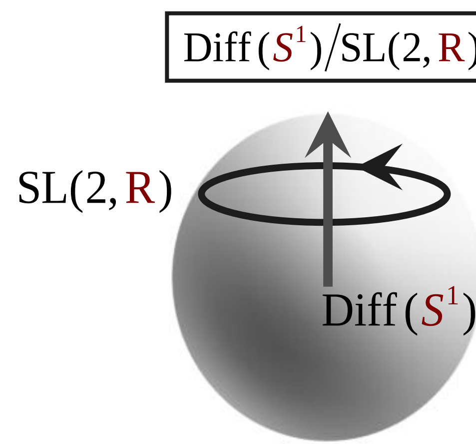
An additional exact symmetry of the averaged theory:

$$G_{\tau_1,\tau_2}[h] \equiv G_{\tau_1,\tau_2}[g \circ h] \qquad g \circ h = g\big(h(\tau)\big)$$

$$g(h) = \frac{ah+b}{ch+d}$$

Mobius transformation

Symmetry space of the averaged theory: $h(\tau) \in \dfrac{\mathrm{Diff}(\mathrm{S}_1)}{\mathrm{SL}(2,\mathrm{R})}$



Goldstone action

J. Maldacena & D. Stanford '2015

- **Schwarzian action of reparametrizations**

$$S_0[h] = -m \int_0^\beta d\tau \{h, \tau\}$$

at τ>m fluctuations are strong

- the Schwarzian derivative is defined by

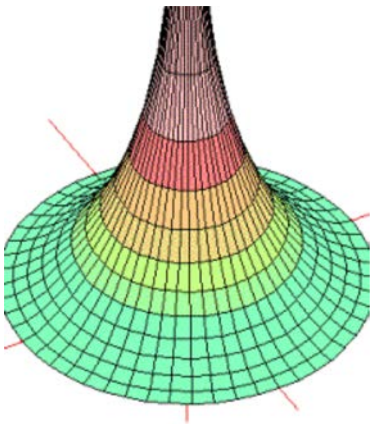
$$\{h,\tau\} \equiv \frac{h'''}{h'} - \frac{3}{2}\left(\frac{h''}{h'}\right)^2, \qquad m \propto N/J$$

- it respects the coset structure versus H=SL(2,R)

$$\{g \circ h, \tau\} = \{h, \tau\} \quad \text{if} \quad g(\tau) = \frac{a\tau + b}{c\tau + d} \in \mathrm{SL(2,R)}$$

Dilaton (Jackiw 85-Teitelboim 83) gravity

$$I = -\frac{\phi_0}{16\pi G}\left[\int \sqrt{g}R + 2\int_{bdy} K\right] - \frac{1}{16\pi G}\left[\int d^2x\phi\sqrt{g}(R+2) + 2\int_{bdy}\phi_b K\right]$$



The top half of a pseudo-sphere

AdS$_2$ metric

$$K = \frac{t'(t'^2 + z'^2 + zz'') - zz't''}{(t'^2+z'^2)^{\frac{3}{2}}} = 1 + \epsilon^2 \mathrm{Sch}(t,u)$$

$$I = -\frac{1}{8\pi G}\int du\phi_r(u)\mathrm{Sch}(t,u) \qquad (3.15)$$

We see that the zero modes get an action detemined by the Schwarzian. Here $\phi_r(u)$ is an external coupling and $t(u)$ is the field variable.

Green function

Q: What is the IR limit of Green's function?

$$G(\tau_1-\tau_2)\propto \int_{G/H} Dh\times\frac{\left[h'(\tau_1)\right]^{1/4}\left[h'(\tau_2)\right]^{1/4}}{\left|h(\tau_1)-h(\tau_2)\right|^{1/2}}\times e^{-S_0[h]}$$

- average the mean-field result over Goldstone modes

• **Phase representation (measure is flat!)**

$$S_0[\varphi]=\frac{M}{2}\int_{-\infty}^{+\infty}\left[\varphi'(\tau)\right]^2 d\tau,\quad h'(\tau)=e^{\varphi(\tau)}$$

non-compact phase

Green's function

$$h'(\tau) = e^{\varphi(\tau)}$$

$$\left[h(\tau_1) - h(\tau_2)\right]^{-1/2} = \left[\int_{\tau_1}^{\tau_2} e^{\varphi(\tau)} d\tau\right]^{-1/2} = \int_0^{+\infty} \frac{d\alpha}{\sqrt{\alpha}} \; e^{-\alpha \int_{\tau_1}^{\tau_2} \exp[\varphi(\tau)] d\tau}$$

$$G_{\tau_1 - \tau_2} \propto \int_0^{+\infty} \frac{d\alpha}{\sqrt{\alpha}} \int_{G/H} D\varphi[\tau] \; e^{\frac{1}{4}\varphi(\tau_1)} e^{\frac{1}{4}\varphi(\tau_2)} e^{-\frac{M}{2}\int_{-\infty}^{+\infty}[\varphi'(\tau)]^2 d\tau - \alpha \int_{\tau_1}^{\tau_2} \exp[\varphi(\tau)] d\tau}$$

Schwarzian

Liouville potential

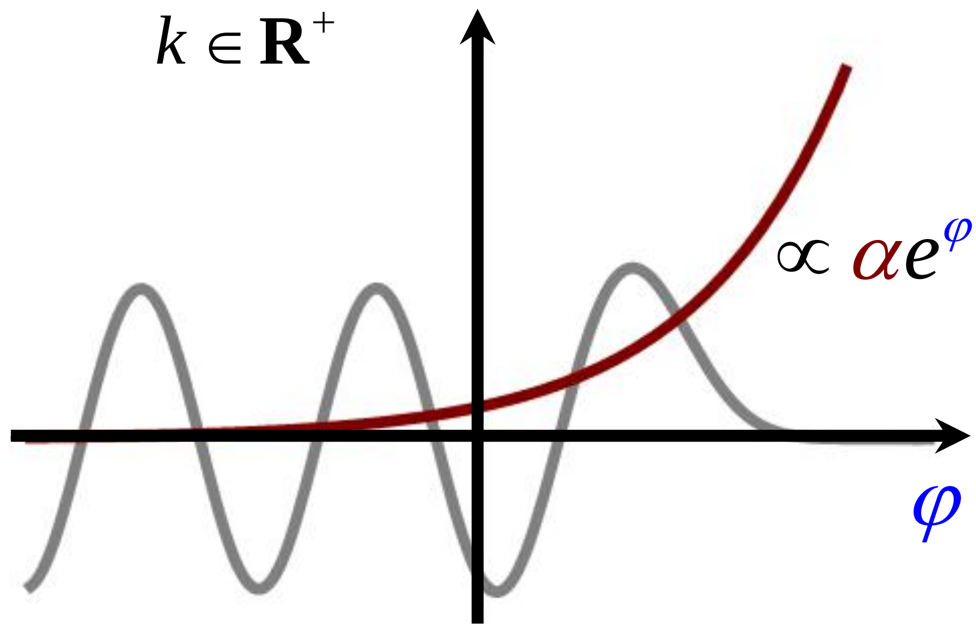
Liouville QM

- **Effective Hamiltonian**

$$\hat{H} = -\frac{\partial_\varphi^2}{2M} + \alpha e^{\varphi},$$

"effective mass"

$$\langle \varphi | k \rangle \propto K_{2ik}\left(\sqrt{8M\alpha}\, e^{\varphi/2}\right)$$



- **Spectral decomposition of the Green's function**

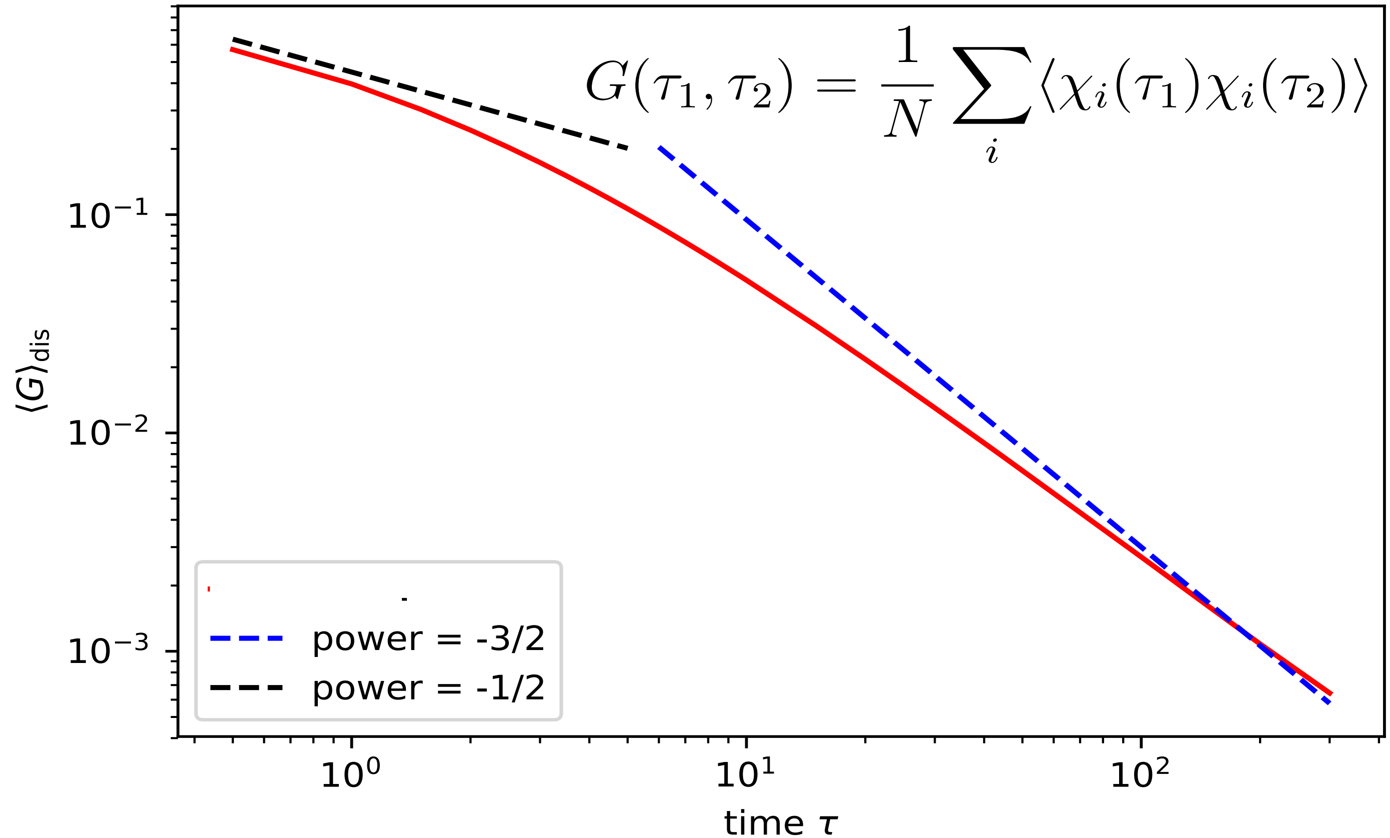
$$G(\tau) \propto \int_0^{+\infty} \frac{d\alpha}{\sqrt{\alpha}} \sum_k \langle 0 | e^{\frac{1}{4}\varphi} | k_\alpha \rangle e^{-|\tau| k^2/2M} \langle k_\alpha | e^{\frac{1}{4}\varphi} | 0 \rangle$$

Green Function

Bagrets, Altland, A.K., 2016
H. Verlinde, et al, 2017
M. Berkooz, et al, 2018

$$G(\tau) \propto \begin{cases} |\tau|^{-1/2}, & \tau < m \\ m|\tau|^{-3/2}, & \tau > m \end{cases}$$

$$m = \frac{N}{J}$$



Lessons from SYK model:

On the mean-field level: $N \to \infty$ $\qquad \bar{G}^{ab}_{t-t'} \propto -\frac{\delta^{ab}}{\sqrt{J}} \frac{1}{|t-t'|^{1/2}}$

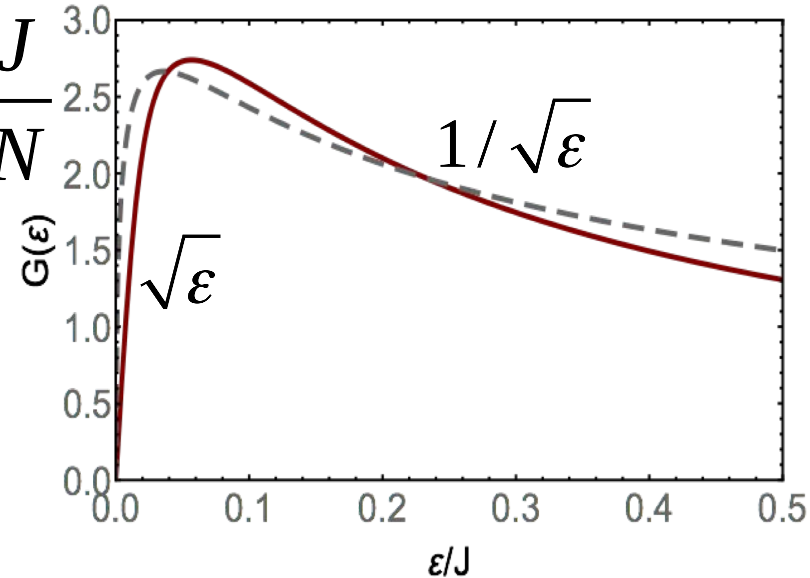
fermion dimensions: $\Delta = \frac{1}{4}$

Goldstone collective modes $\boxed{\mathrm{Diff}(S^1)/SL(2,\mathrm{R})}$ change

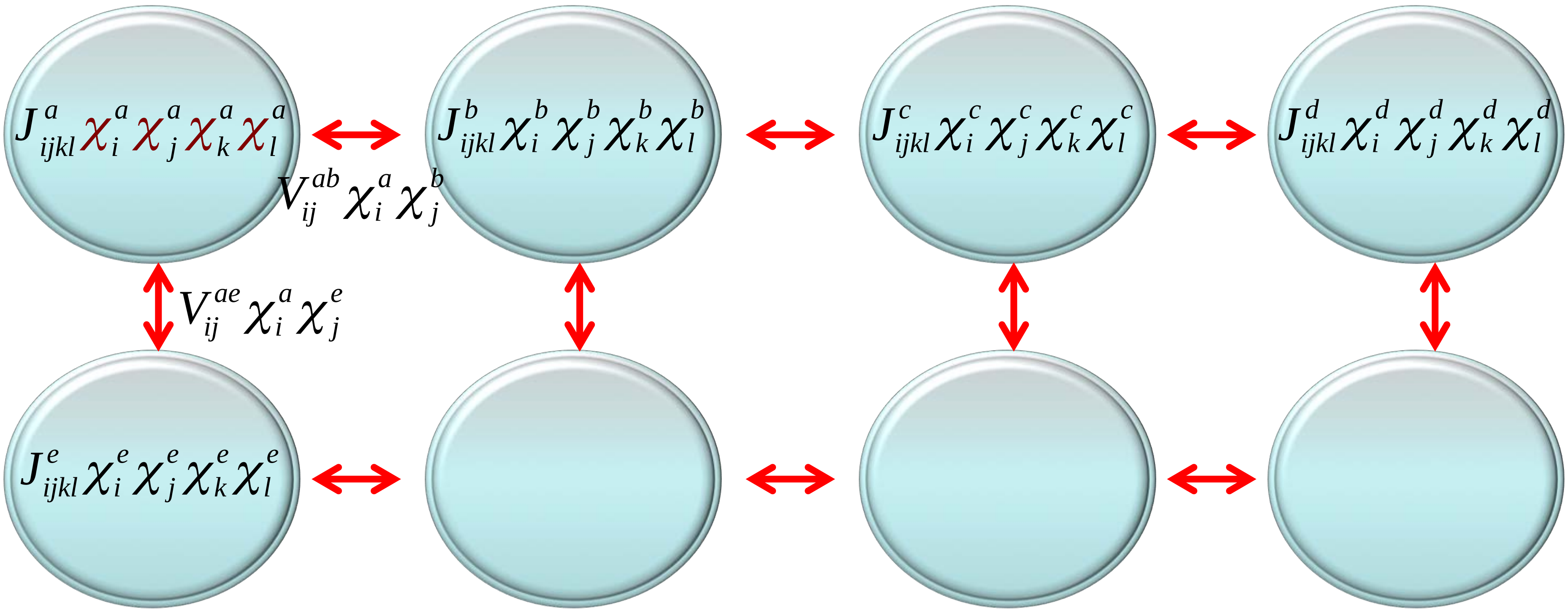
fermion dimensions in IR limit: $\Delta = \frac{1}{4} \Rightarrow \frac{3}{4}$ below an

emergent new energy scale $\varepsilon = \frac{J}{N}$

An emergent "zero-bias anomaly" indicates onset of the insulating behavior.



SYK Matter



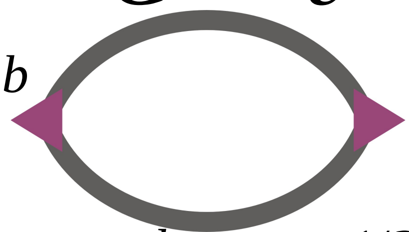
$$H = \frac{1}{4!}\sum_a \sum_{ijkl}^{N} J^a_{ijkl}\eta^a_i\eta^a_j\eta^a_k\eta^a_l + \frac{i}{2}\sum_{\langle ab\rangle}\sum_{ij}^{N} V^{ab}_{ij}\eta^a_i\eta^b_j$$

From Strange Metal to Fermi Liquid:

Balents et al '2017

$G^{a} \propto \varepsilon^{-1/2}$

(thermal) conductivity $\sigma \propto V_{ij}^{ab}$



$V_{ji}^{ba} \propto \frac{1}{T}$

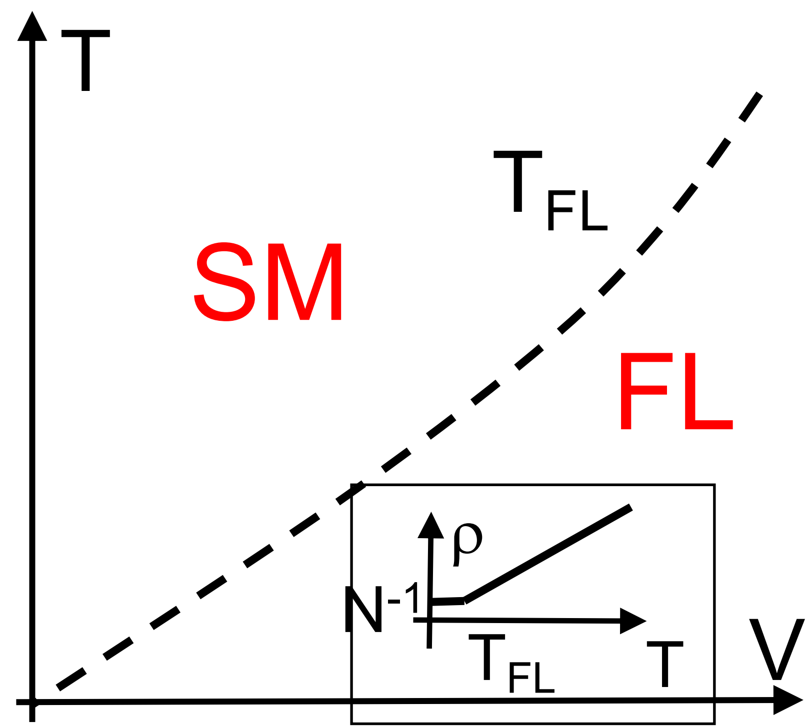
T-linear resistivity

$G^{b} \propto \varepsilon^{-1/2}$

If $\Delta = \frac{1}{4}$ then $V_{ij}^{ab}\chi_i^a\chi_j^b$

is a **relevant** perturbation.

Thus **strange metal** is destroyed at small T and crosses over to usual **Fermi Liquid.**

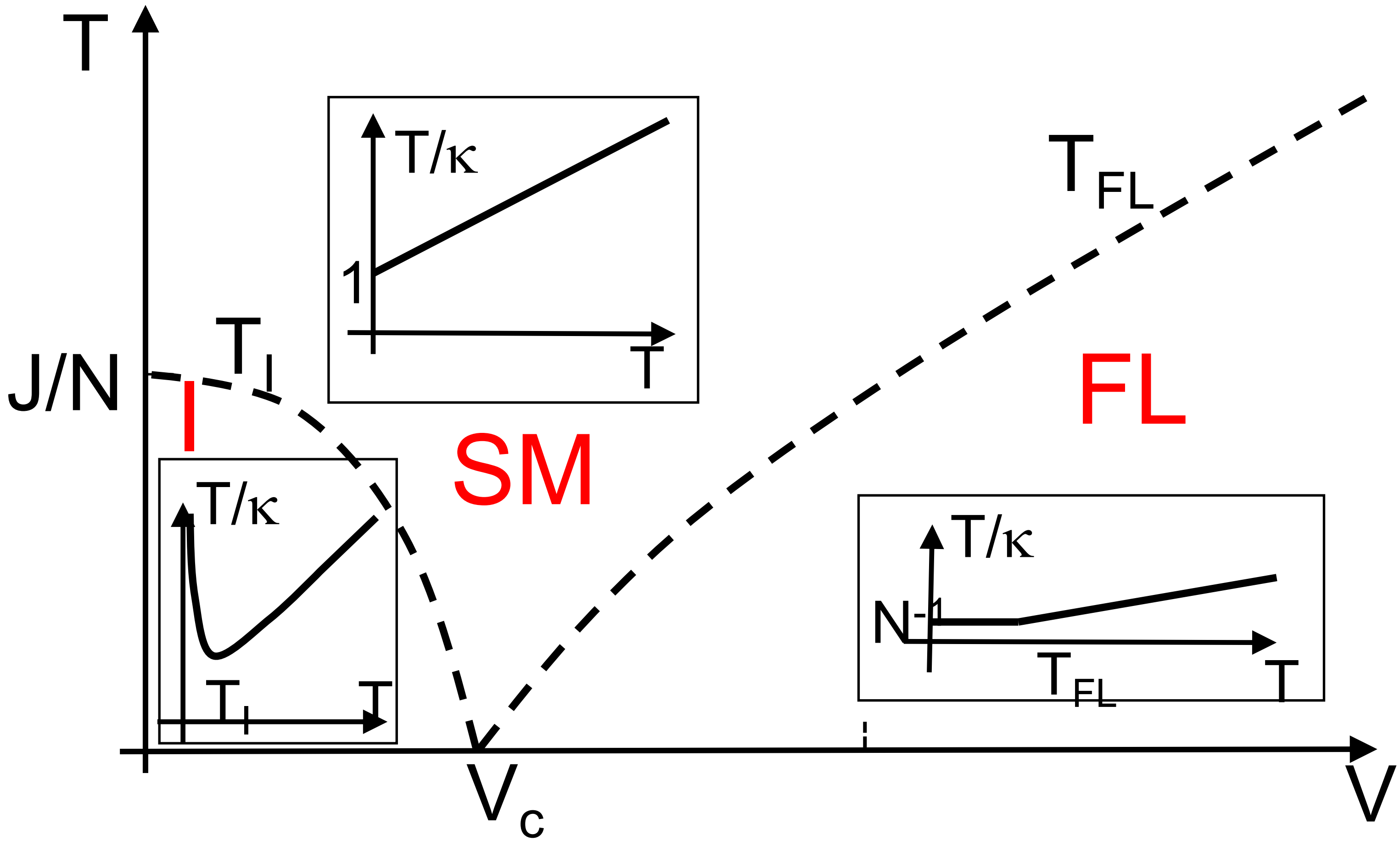


However! Goldstones: $\Delta = \frac{1}{4} \Rightarrow \frac{3}{4}$

Altland, Bagrets, AK '2019

If $[\chi] = \frac{1}{4} \Rightarrow \frac{3}{4}$ then $V_{ij}^{ab} \chi_i^a \chi_j^b$ is an **irrelevant** perturbation.

Thus the **strange metal** crosses over to an **insulator.**



RG Treatment of Schwarzian Theory

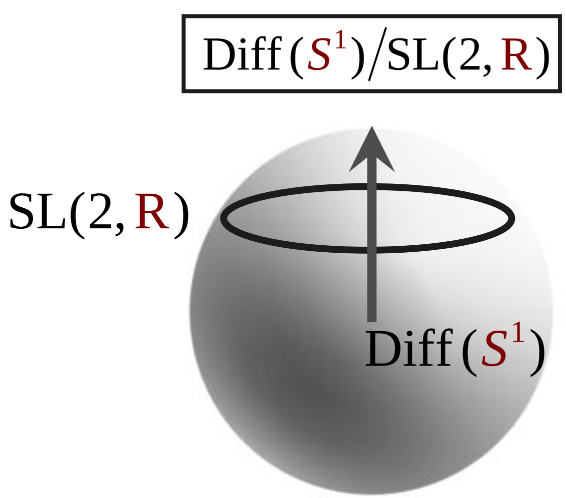
$$S_0[h] = -m \sum_a \int d\tau \, \{h^a, \tau\},$$

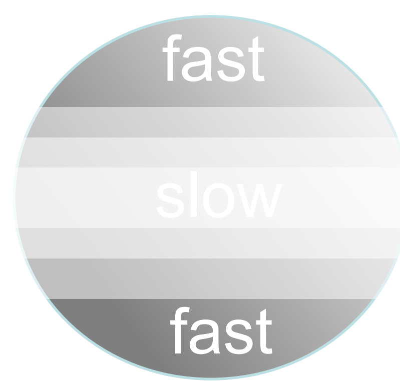
$$S_{\mathrm{T}}[h] = -w \sum_{\langle ab \rangle} \iint d\tau_1 d\tau_2 \left(\frac{h_1'^a h_2'^a}{[h_1^a - h_2^a]^2} \times \frac{h_1'^b h_2'^b}{[h_1^b - h_2^b]^2} \right)^{1/4},$$

bare values $m \propto N/J$ and $w \propto NV^2/J$

Both terms in the action are SL(2,R) invariant.

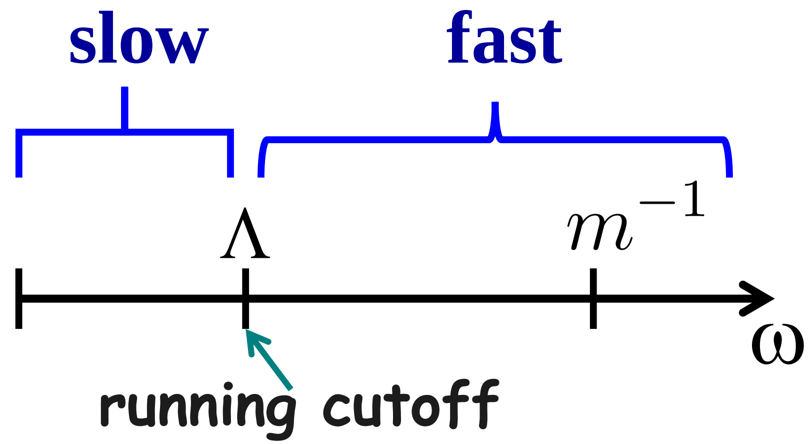
RG:





RG Treatment of Schwarzian Theory

$$h(\tau) = f(s(\tau)) \equiv (f \circ s)(\tau)$$



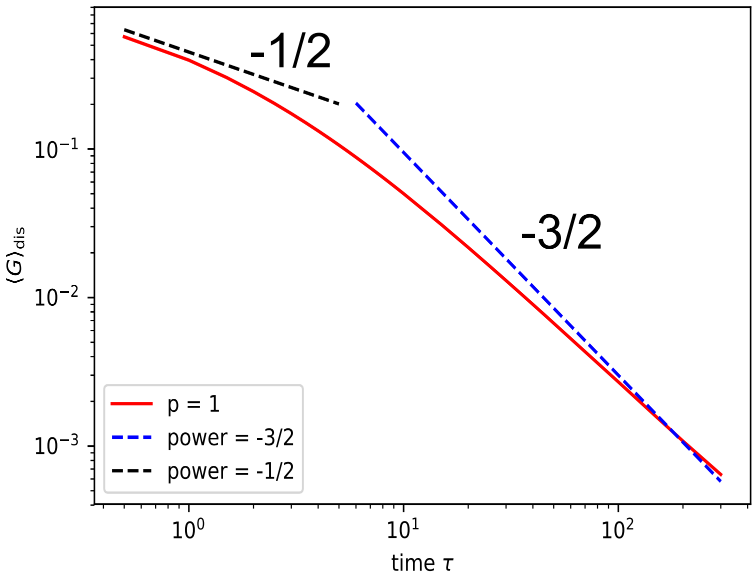
Schwarzian chain rule:

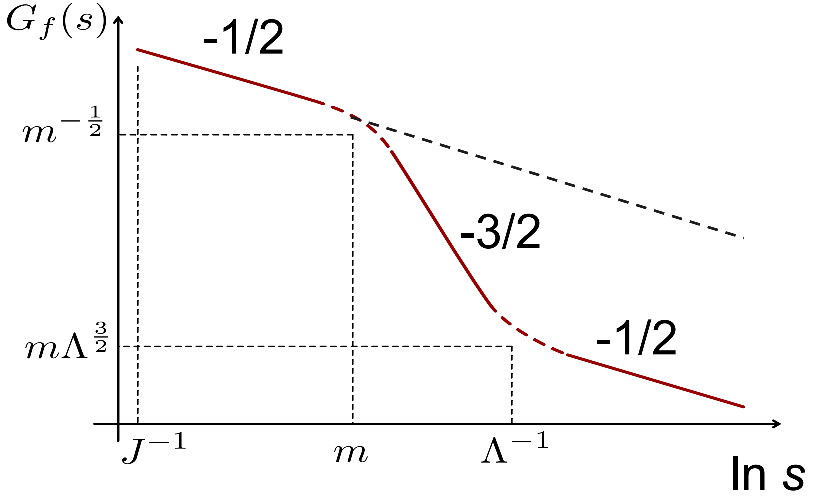
$$\{f \circ s, \tau\} = \underbrace{(s')^2\{f, s\}}_{\text{fast action}} + \underbrace{\{s, \tau\}}_{\text{slow action}}$$

$$G_{\tau_1,\tau_2}[f \circ s] = G_{s_1,s_2}[f](s_1' s_2')^{1/4}$$

$$\langle S_{\mathrm{T}}[f \circ s]\rangle_f \propto \langle G_{s_1,s_2}[f^a]\rangle_{f^a} \times \langle G_{s_2,s_1}[f^b]\rangle_{f^b}$$

Fast Averaged Green Function $\langle G_{s_1,s_2}[f]\rangle_f$





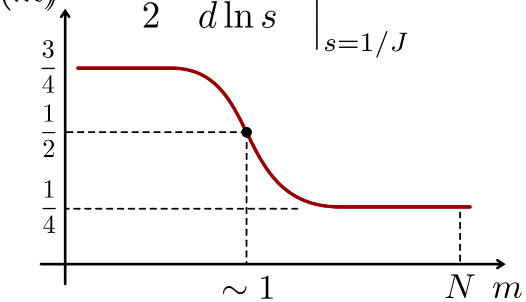
$$\left(\frac{s_1' s_2'}{[s_1 - s_2]^2}\right)^{\Delta} \approx \frac{1}{[\tau_1 - \tau_2]^{2\Delta}} + \frac{\Delta}{6}\frac{\{s(\tau),\tau\}}{[\tau_1 - \tau_2]^{2\Delta-2}} + \dots$$

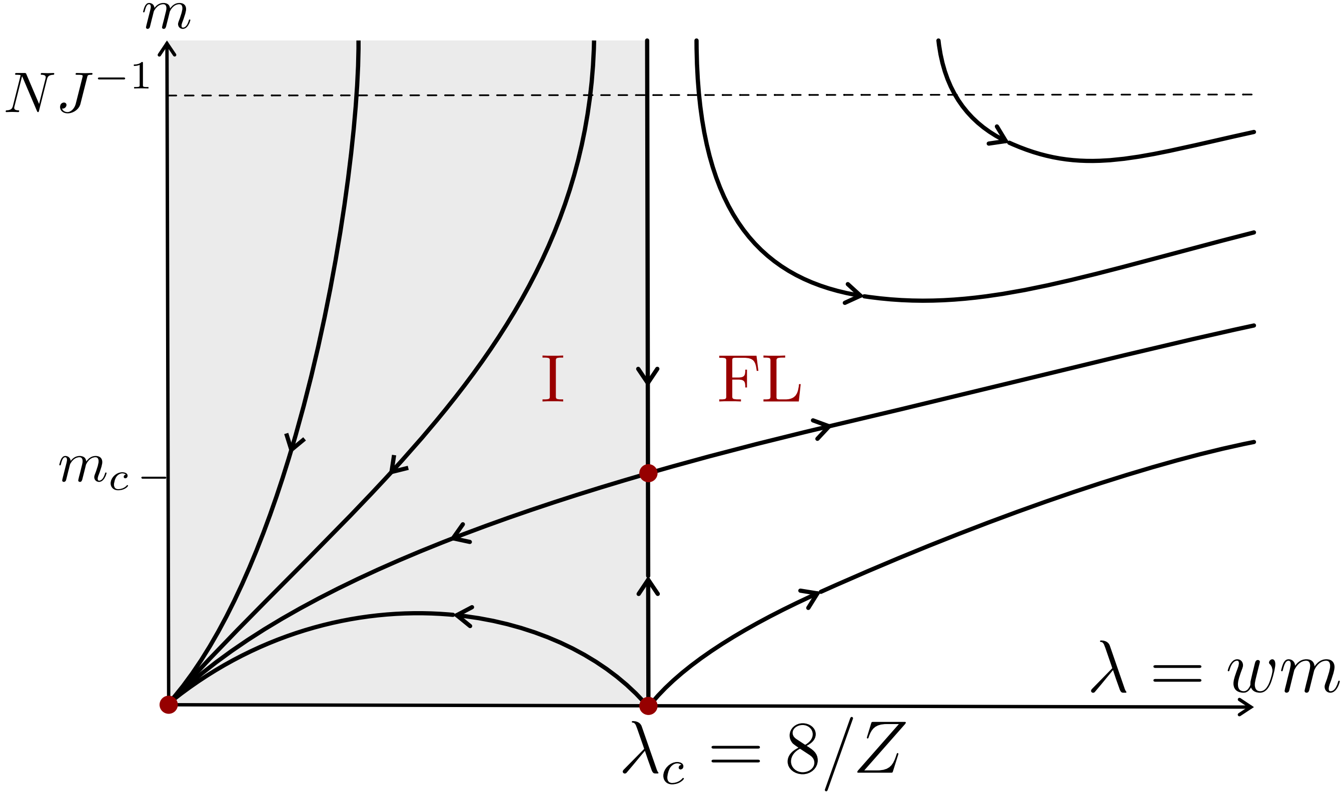
RG Treatment of Schwarzian Theory

Altland, Bagrets, A.K., PRL 2019

$$\frac{d\ln m}{dl} = -1 + \frac{Z}{4} m w \left(2\Delta_\psi(m) - \tfrac{1}{2}\right)$$
$$\frac{d\ln w}{dl} = 1 - 2\left(2\Delta_\psi(m) - \tfrac{1}{2}\right).$$

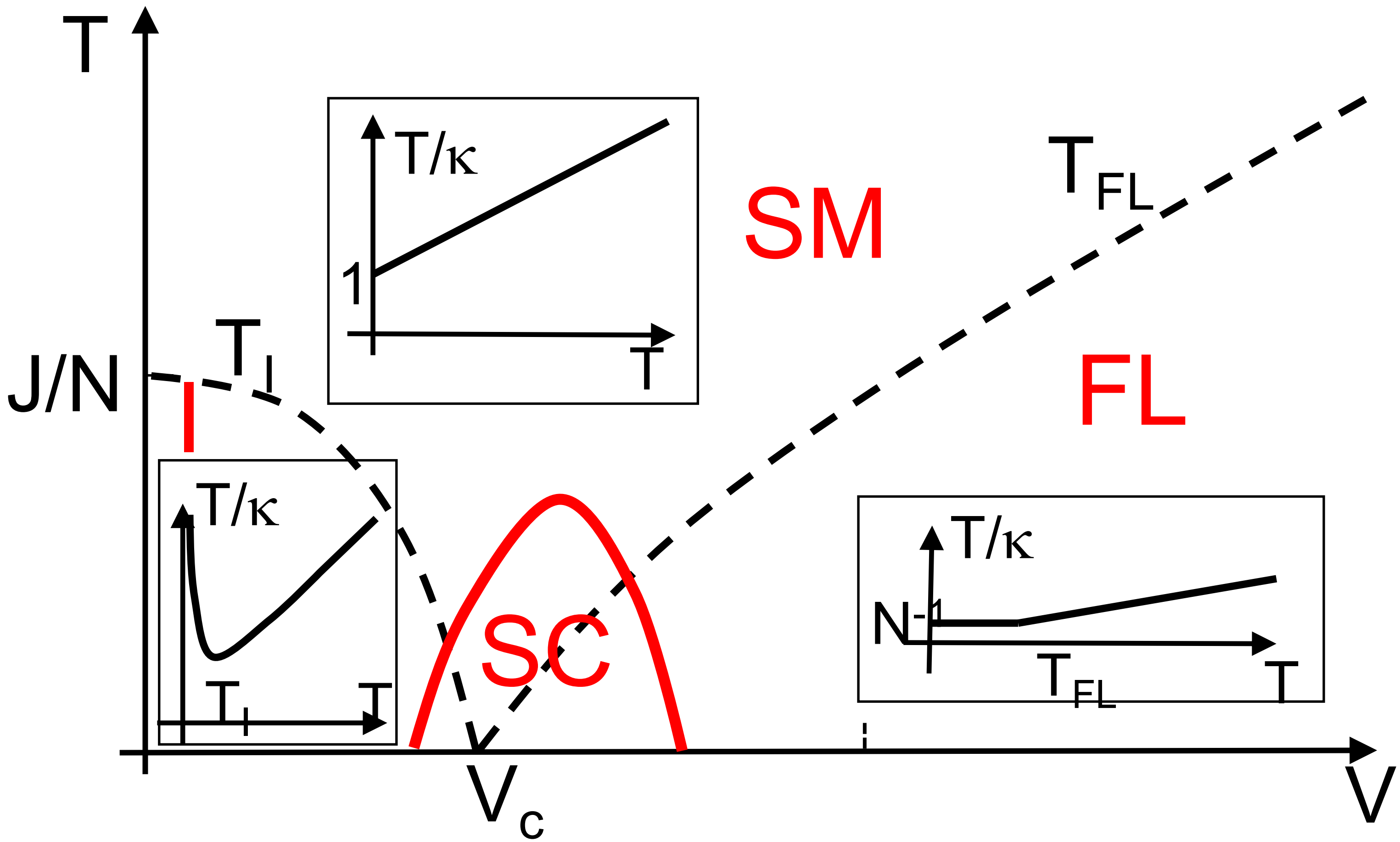
$$\boldsymbol{\Delta_\psi(m)} = -\frac{1}{2}\frac{d\ln G(s)}{d\ln s}\bigg|_{s=1/J}$$





Z – array coordination number

Metal Insulator Transition



SYK Superconductivity

E. Berg, et al, 2018,
E-A. Kim, et al, 2019
J. Schmalian et al, 2019
H.Wang, A.K. et al,2020

$$H = \sum_{i<j;k<l} J_{ij;kl} \sum_{\sigma,\sigma'} c^\dagger_{i\sigma} c^\dagger_{j\sigma'} c_{k\sigma'} c_{l\sigma} - U \sum_i c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow} c_{i\uparrow}$$

real Gaussian

spin 1/2

on-site attraction

Off-Diagonal Long Range Order ODLRO

$< c_{i\uparrow}^{\dagger} c_{i\downarrow}^{\dagger} > = 0$ in any finite size system

Local pair-density matrix: $\rho_{ij} = \langle b_i^{\dagger} b_j \rangle$

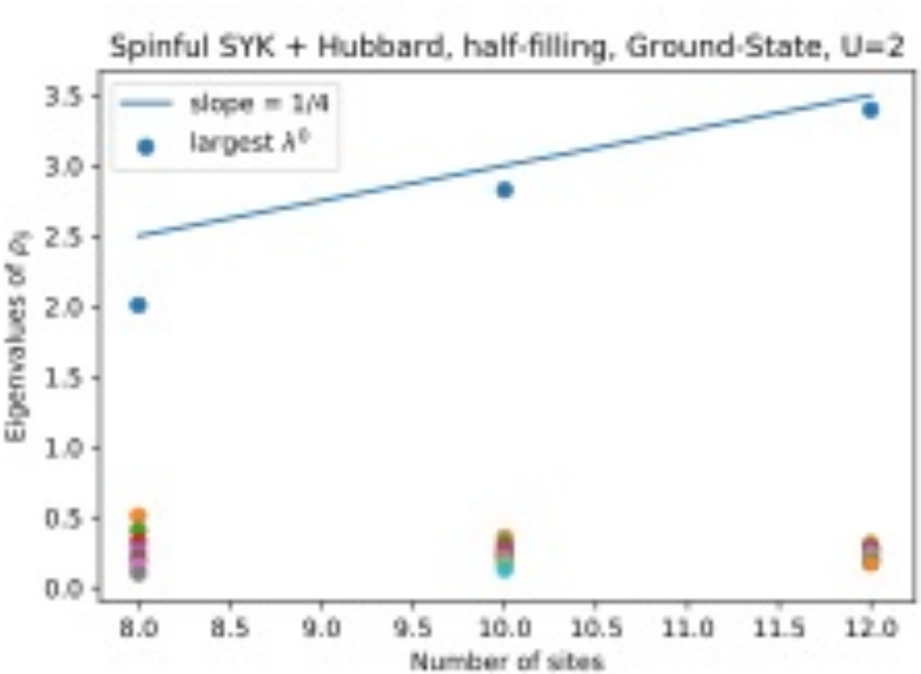
positive-definite N*N matrix $b_i^{\dagger} = c_{i\uparrow}^{\dagger} c_{i\downarrow}^{\dagger}$

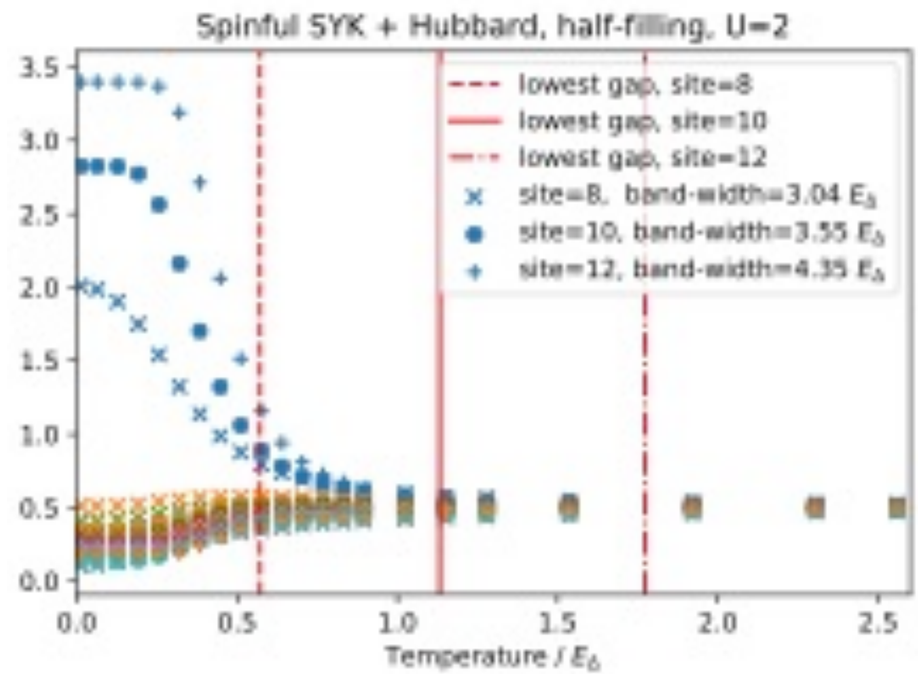
Tr = # of local fermion pairs < (# of fermions)/2

ODLRO= largest eigenvalue of ρ_{ij} scales with N

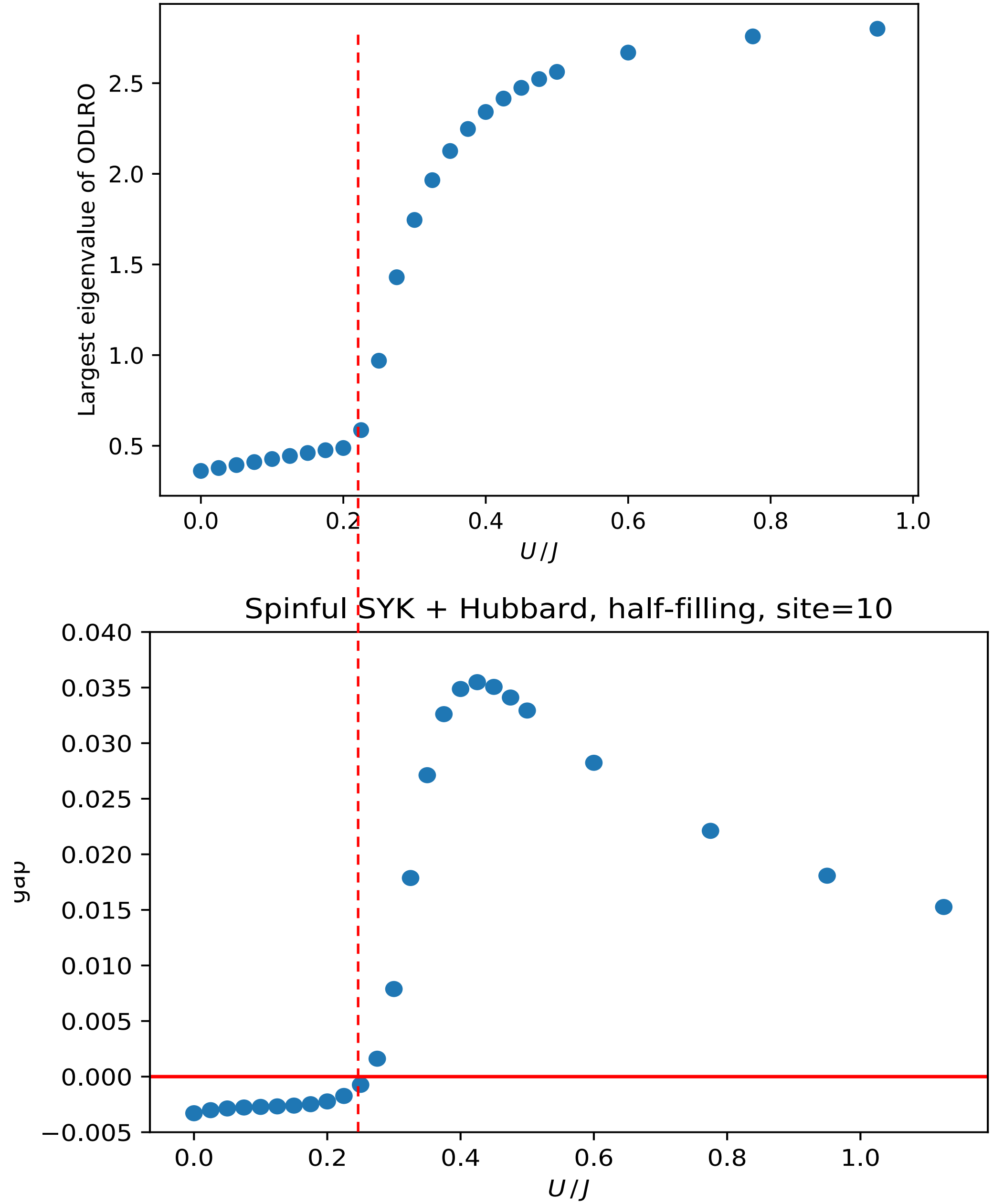
Off-Diagonal Long Range Order

spectrum of $\rho_{ij} = \langle b_i^\dagger b_j \rangle$





Off-Diagonal Long Range Order



Almost done ☺

SYK model is exactly solvable

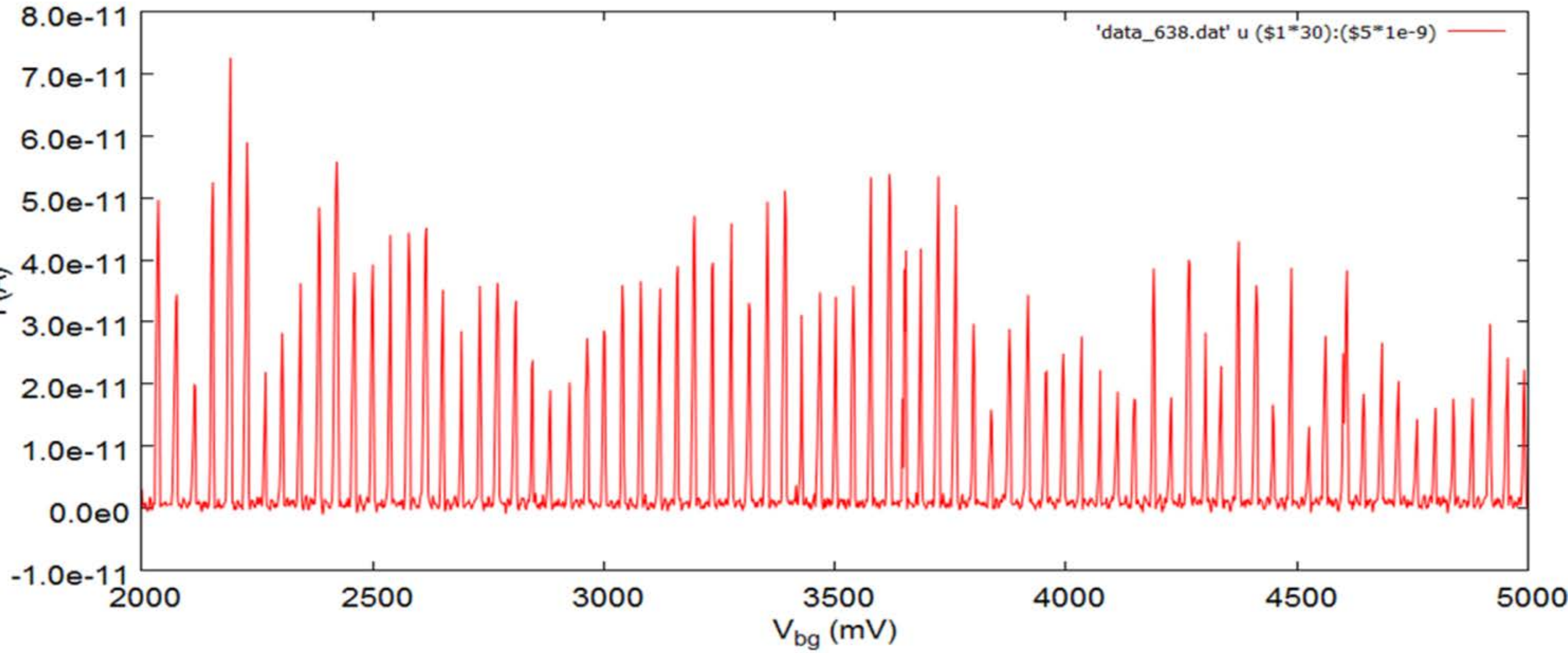
$N \to \infty, \quad \varepsilon\text{-fixed};$ $\quad\quad N \to \infty, \quad \varepsilon \to 0, \; N\varepsilon\text{-fixed}$

SYK matter exhibits distinct observable signatures of the non-Fermi liquid fixed point.

SYK can be superconducting.

Complex SYK: Nano-Transport

$$\hat{H} = \sum_{i}^{N} \epsilon_i c_i^\dagger c_i + \frac{1}{2} E_C \, \hat{n}^2 + \sum_{ijkl}^{N} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$



Symmetries

$$c_j(\tau) \to [h'(\tau)]^{\Delta} \, c_j(h(\tau)) \, e^{i\phi(\tau)} \qquad \Delta = \frac{1}{4}$$

$\phi(\tau)$ compact U(1) phase

$$\tau \to h(\tau) \qquad h(\tau) \in \frac{\mathrm{Diff}(\mathrm{S}_1)}{\mathrm{SL}(2, \mathrm{R})}$$

$$G_{\tau_1 - \tau_2} \to G_{\tau_1, \tau_2}[\phi, h] = e^{i\phi(\tau_1)} \left[\frac{h'(\tau_1) h'(\tau_2)}{[h(\tau_1) - h(\tau_2)]^2} \right]^{\Delta} e^{-i\phi(\tau_2)}$$

Still a solution of Dyson equation: $-(\not{\partial_\tau} + \Sigma) \bullet G = 1, \quad \Sigma^{ab}_{\tau\tau'} = J^2 \left[G^{ab}_{\tau\tau'} \right]^3$

Soft mode action

$$S_0[\phi, h] = \int d\tau \left[\tfrac{1}{2} E_C^{-1} \dot{\phi}^2 - m\{h, \tau\} \right]$$

$$D(\tau_1 - \tau_2) \equiv \left\langle e^{-i\phi(\tau_1)} e^{i\phi(\tau_2)} \right\rangle_\phi = e^{-E_C|\tau_1 - \tau_2|/2}$$

Tunneling:

$$H_T = \sum_{i,k} V_{ik} c_i^\dagger d_k + h.c.$$

dot lead

$$\langle |V_{ik}|^2 \rangle \equiv v^2$$

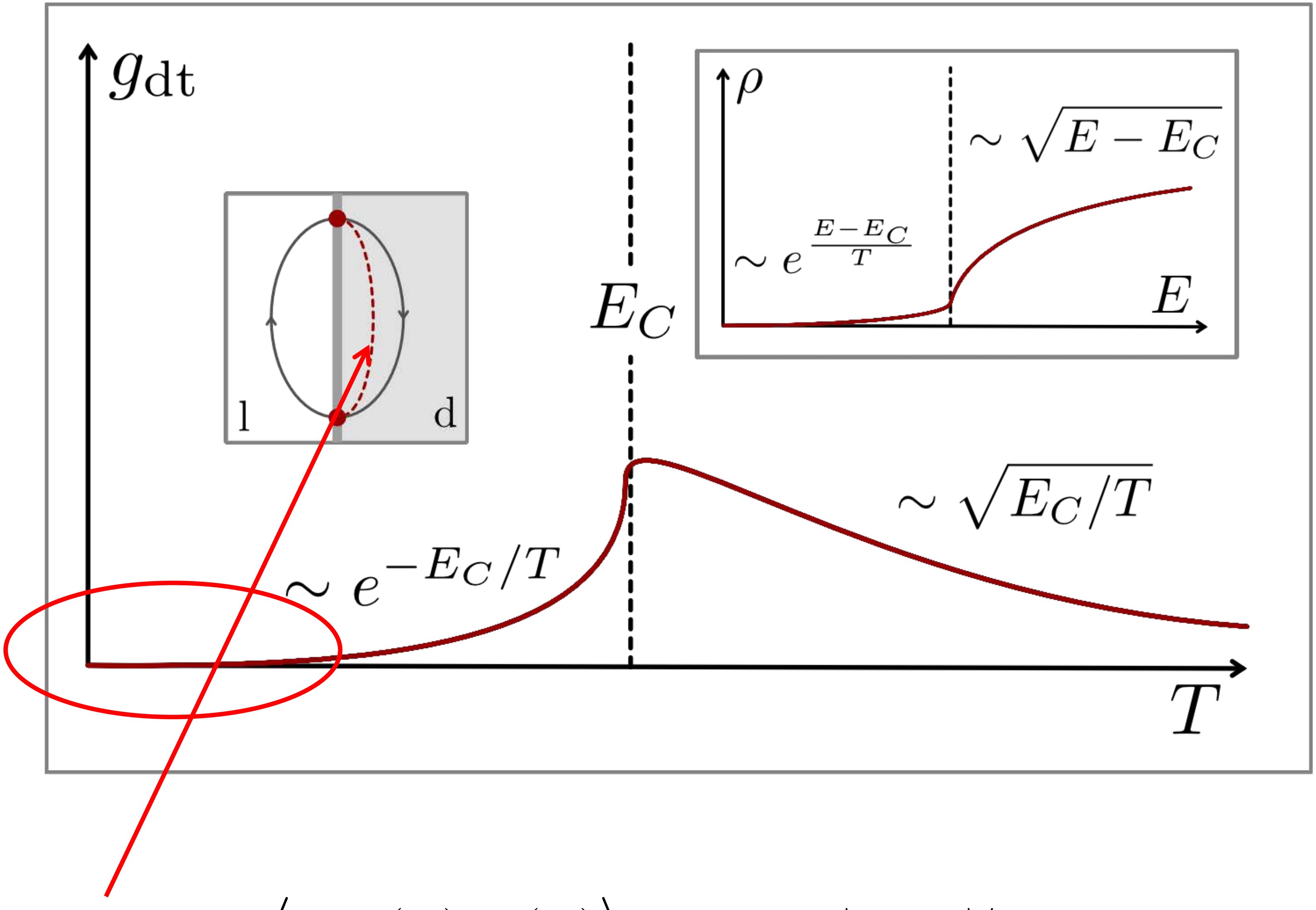
$$S_T[\phi, h] = -g_0 T \iint d^2\tau \frac{e^{-i\phi(\tau_2)} G_{\tau_2,\tau_1}[h] e^{i\phi(\tau_1)}}{\sin(\pi T(\tau_1 - \tau_2))}$$

$$g_0 \propto \nu v^2 N/J$$

dimensionless conductance

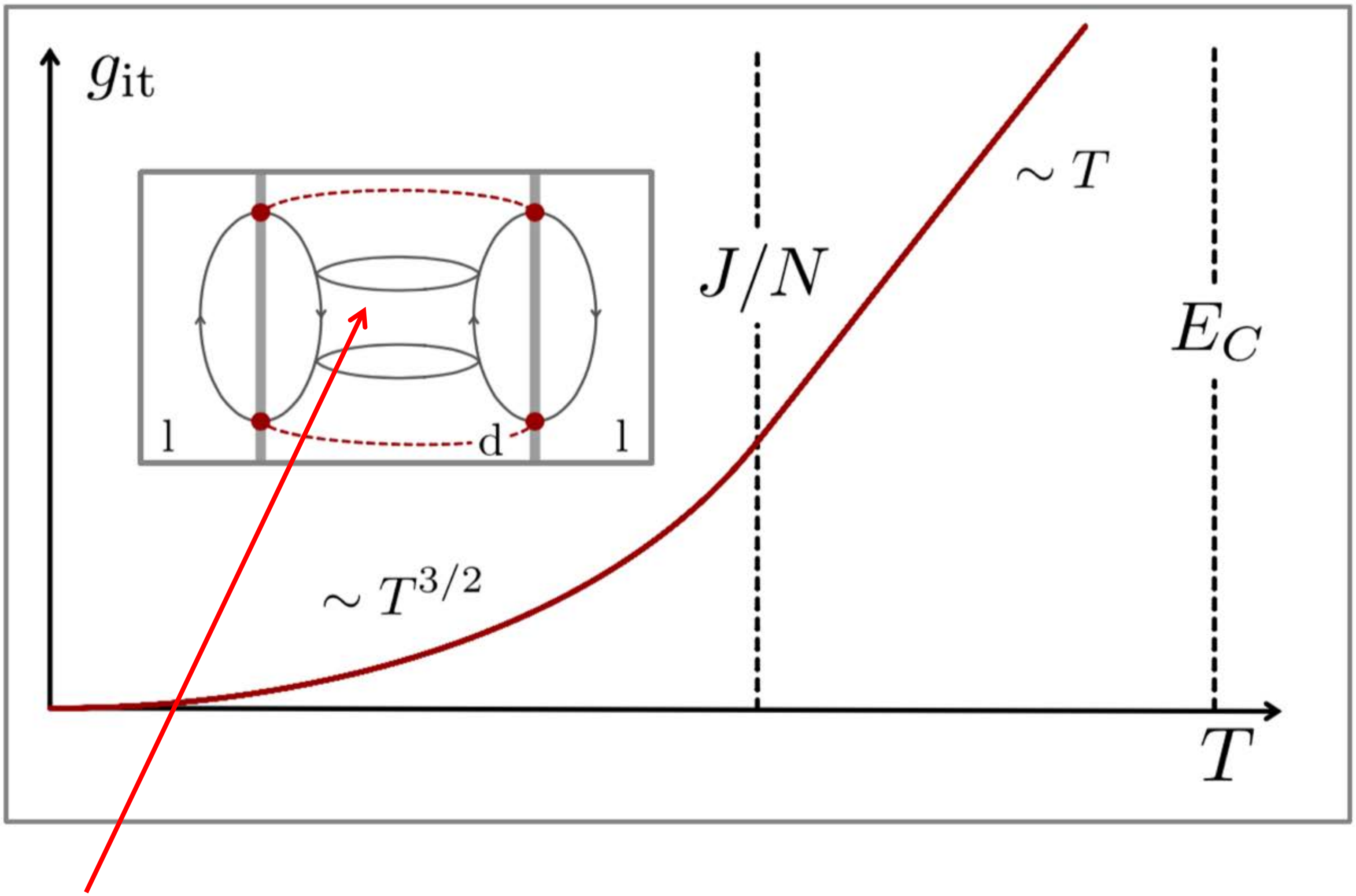
cf. Ambegoakar, Eckern, Schon, 1982

Coulomb Blockade



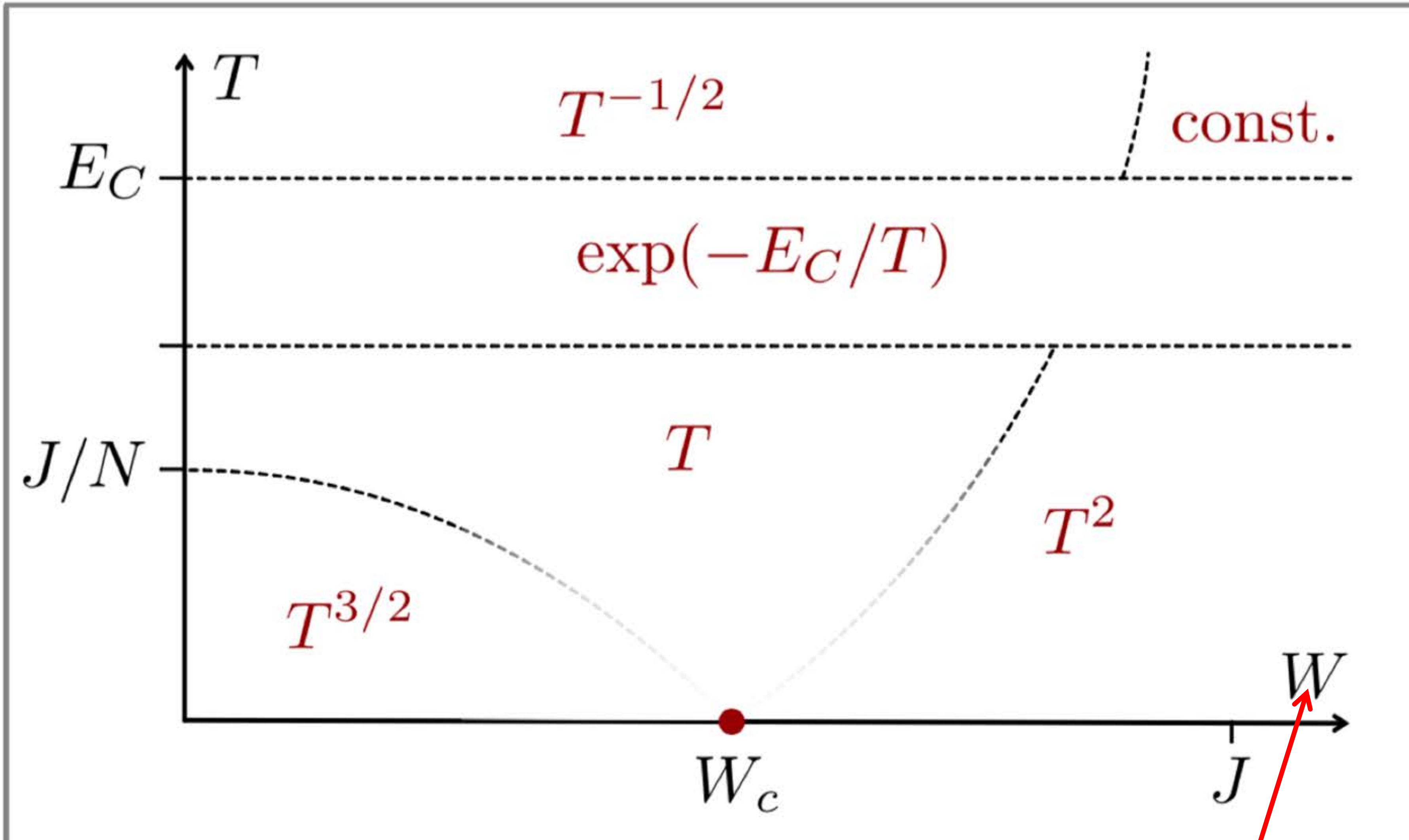
$$D(\tau_1 - \tau_2) \equiv \left\langle e^{-i\phi(\tau_1)} e^{i\phi(\tau_2)} \right\rangle_\phi = e^{-E_C|\tau_1 - \tau_2|/2}$$

Inelastic Cotunneling



SYK four-point function

Stability against kinetic energy



Kinetic energy bandwidth

Lunkin, Feigelman, Tikhonov, 2018

Almost done ☺

SYK model is exactly solvable

$N \to \infty, \quad \varepsilon\text{-fixed};$ $\quad$ $N \to \infty, \quad \varepsilon \to 0, \; N\varepsilon\text{-fixed}$

SYK arrays and dots exhibit distinct observable signatures of the non-Fermi liquid fixed point.

SYK fixed point is locally stable against perturbations, such as inter-dot tunneling and intra-dot kinetic energy.