Superconductivity out of a non-Fermi liquid

Andrey Chubukov

University of Minnesota

School on Superconductivity, Tallahassee, FL, January 8, 2024

Two lessons from previous lecture:

1. Superconductivity develops even when fermionic self-energy diverges

2. Superconducting Tc saturates at a finite value when ω_D vanishes

Are phonons always responsible for superconductivity?

New era began in 1986: cuprates

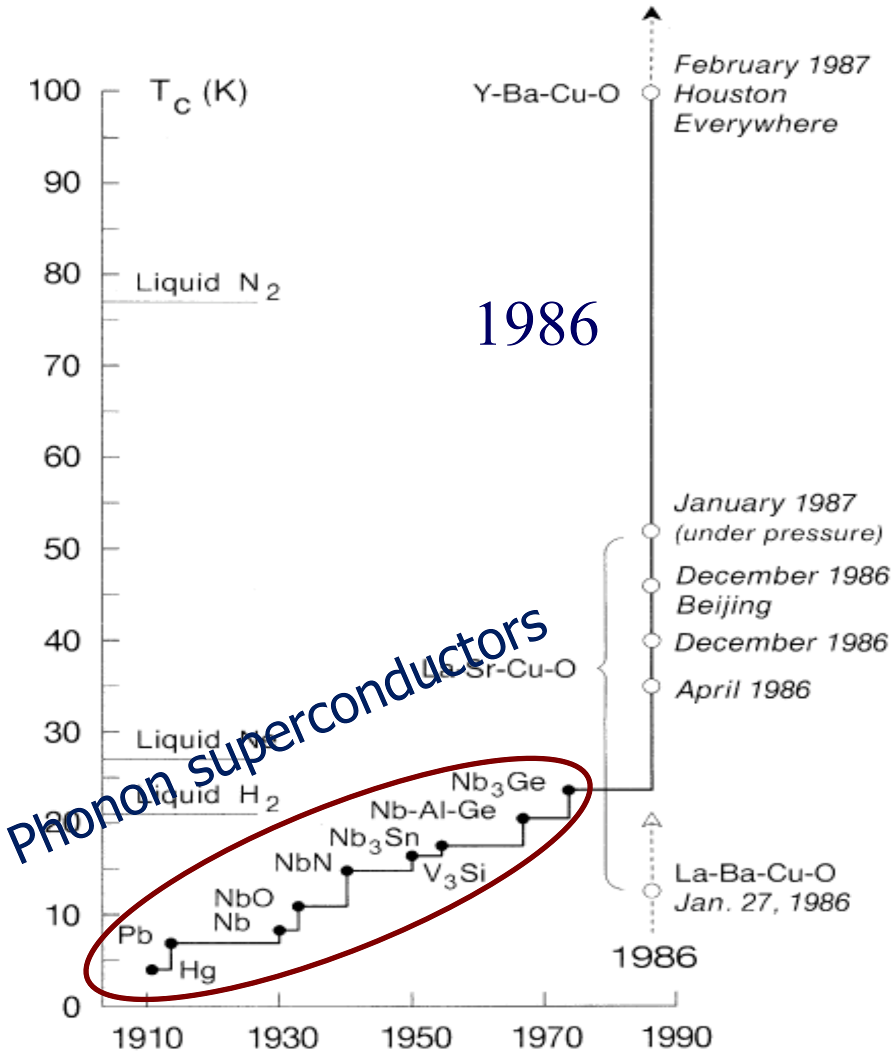
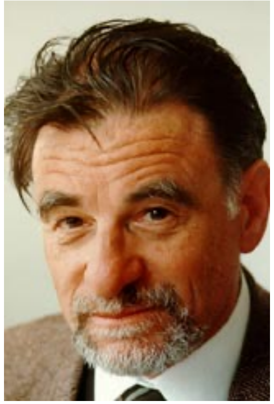


Fig. 1. Evolution of the superconductive transition temperature subsequent to the discovery of the phenomenon.



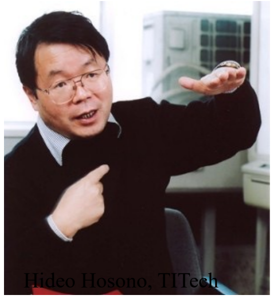
Alex Muller and Georg Bednortz

Nobel prize, 1987

New breakthrough in 2008: Fe-pnictides

$LaFeAsO_{1-x}F_x$, $Tc = 26K$

$SmFeAsO_{1-x}F_x$, $Tc = 43K$



Hideo Hosono

nature International weekly journal of science

Letter

Nature 453, 761-762 (5 June 2008) | doi:10.1038/nature07045

Superconductivity at 43 K in SmFe

X. H. Chen[1 (#a1)], T. Wu[1 (#a1)], G. Wu[1 (#a1)], R. H

1. Hefei National Laboratory for Physical Sciences at Microsca

Correspondence to: X. H. Chen[1 (#a1)] Correspondence and reque

Since the discovery of high-transition-tem been devoted to exploring the origins of th

nature International weekly journal of science

go | Advanced search

nature International weekly journal of science

Letter

nature International weekly journal of science

Search | This journal | go | Advanced search

3/nature07081; Received 23 April 2008; Accepted 5 May 2008; Published online 4 June 2008

onductor $SmFeAsO_{0.85}F_{0.15}$

H. Liu[2 (#a2)], X. H. Chen[2 (#a2)] & C. L. Chien[1 (#a1)]

pkins University, Baltimore, Maryland 21218, USA

t Microscale and Department of Physics, University of Science and Technology of China, Hefei, Anhui 230026, China

nature International weekly journal of science

Letter

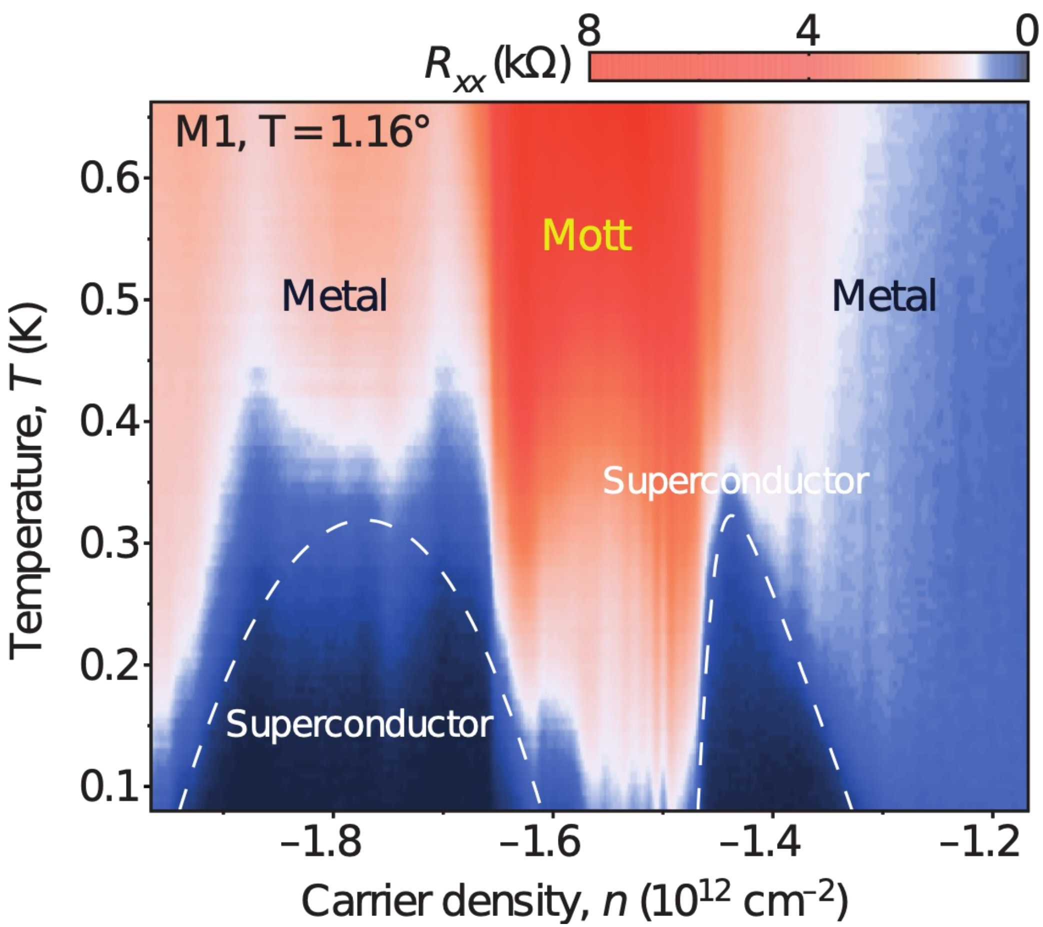
Nature 459, 64-67 (7 May 2009) | doi:10.1038/nature07981; Received 4 November 2008; Accepted 13 March 2009

A large iron isotope effect in $SmFeAsO_{1-x}F_x$ and $Ba_{1-x}K_xFe_2As_2$

R. H. Liu[1], T. Wu[1], G. Wu[1], H. Chen[1], X. F. Wang[1], Y. L. Xie[1], J. J. Ying[1], Y. J. Yan[1], Q. J. Li[1], B. C. Shi[1], W. S. Chu[2,3], Z. Y. Wu[2,3] & X. H. Chen[1]

1. Hefei National Laboratory for Physical Sciences at Microscale and Department of Physics, University of Science and Technology of China, Hefei, Anhui 230026, China
2. Beijing Synchrotron Radiation Facility, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
3. National Synchrotron Radiation Laboratory, University of Science and Technology of China, Hefei 230026, China

Twisted bilayer graphene, Bernal bilayer graphene…



Cao et al, Nature (2018)

Superconductivity in other materials

Heavy fermion materials and their likes

Ruthenates

Titanates

Nickelades

Kagome materials

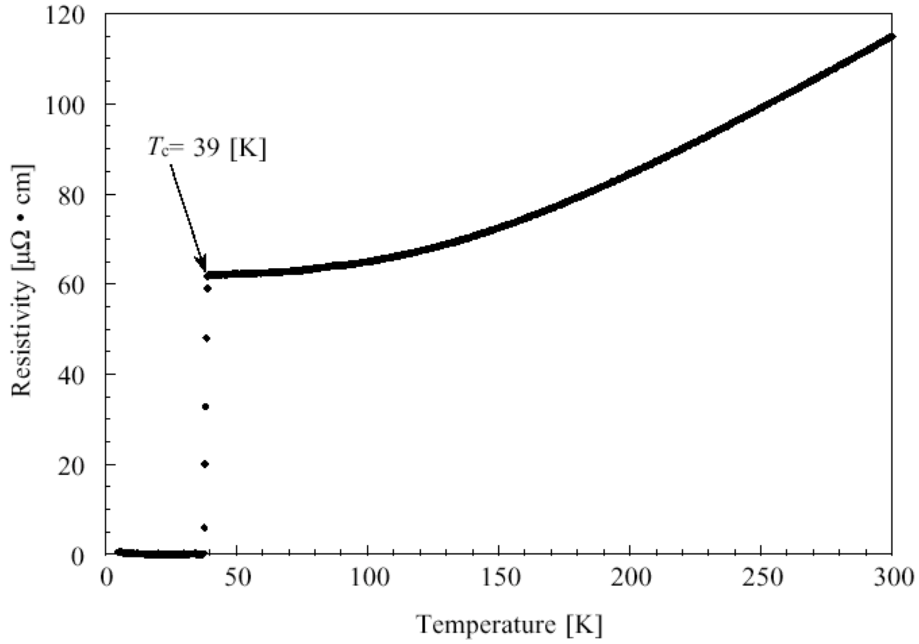
Irridates, Kitaev materials

+ ...

Is only high Tc relevant? No

MgB_2: A phonon Superconductor at 40 K

T_c=39 K Akimitsu et al (2001)



Superconductors with highest Tc are electron-phonon ones under high pressure:

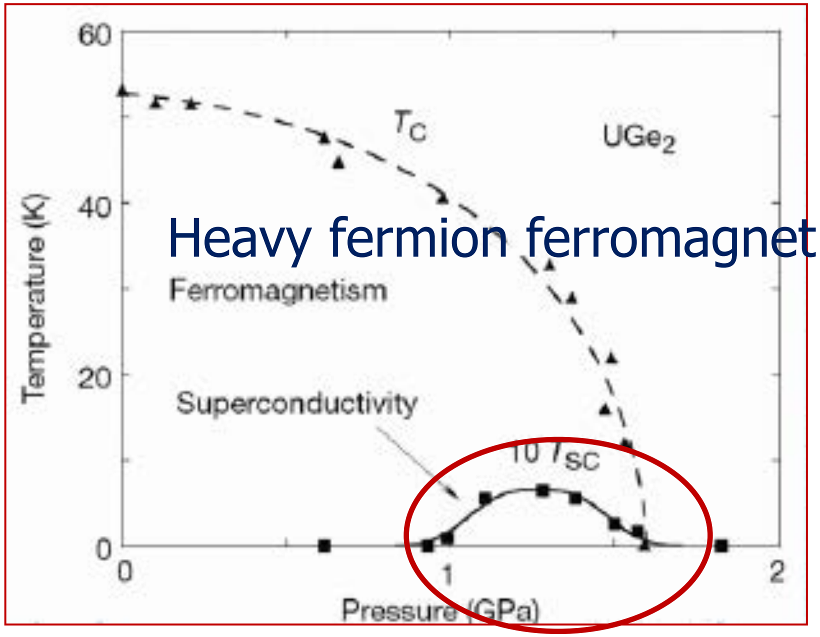
hydrogen sulfide (H_3S): Tc =203K, La and Y-hydrates:Tc =250K

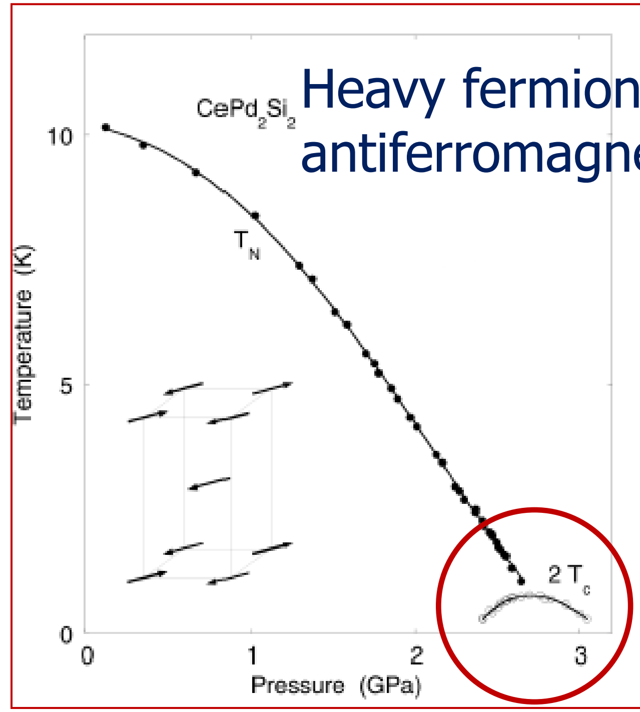
Then what is relevant?

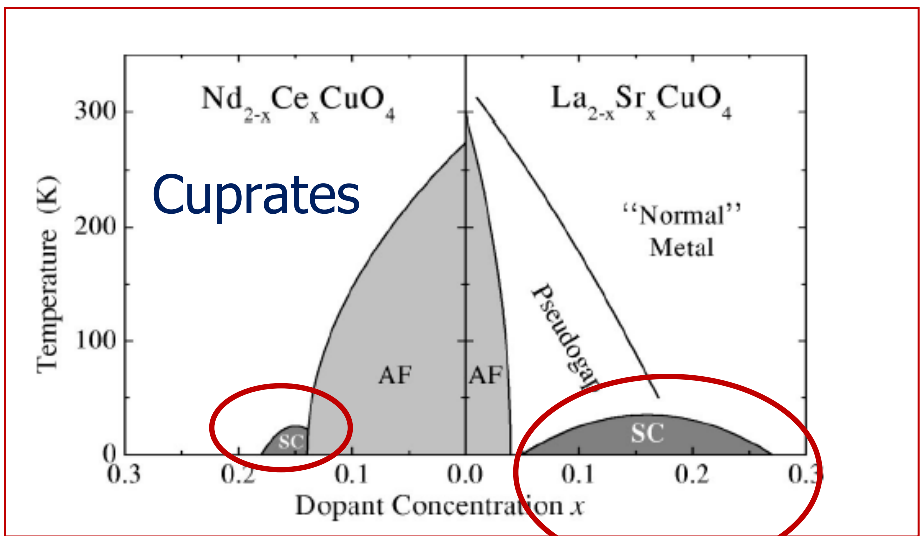
In Cuprates, Fe-pnictides, as well as in
Ruthenates (Sr_2RuO_4) , Titanates ($SrTiO_3$ ..)
Heavy fermion materials ($CeIn_5$, UPl_3, $CePd_2Si_2$ ….),
Organic superconductors ($(BEDT\text{-}TTF)_2\text{-}Cu[N(CN)_2]Br$…)
…
electron-phonon interaction most likely is <u>NOT</u> responsible for the pairing, either by symmetry reasons, or because it is just too weak (Tc would be 1K in Fe-pnictides)

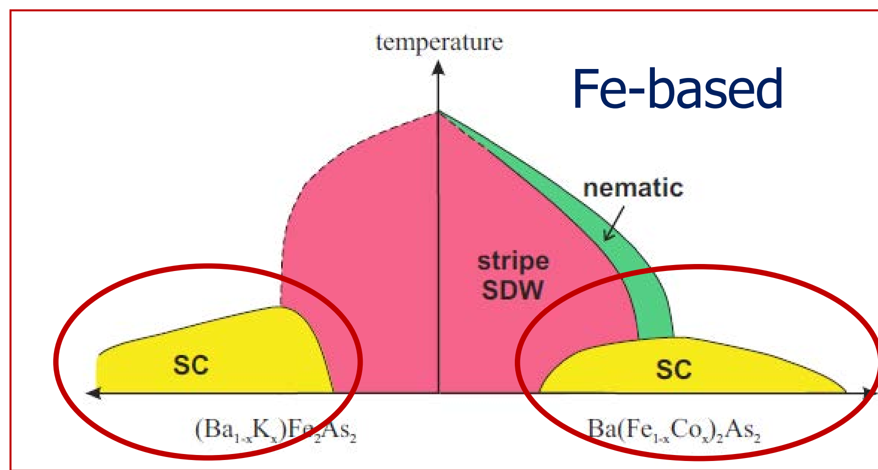
If so, then the pairing must somehow come from electron-electron interaction

In many systems, superconductivity occurs near a point where charge or spin order is about to emerge



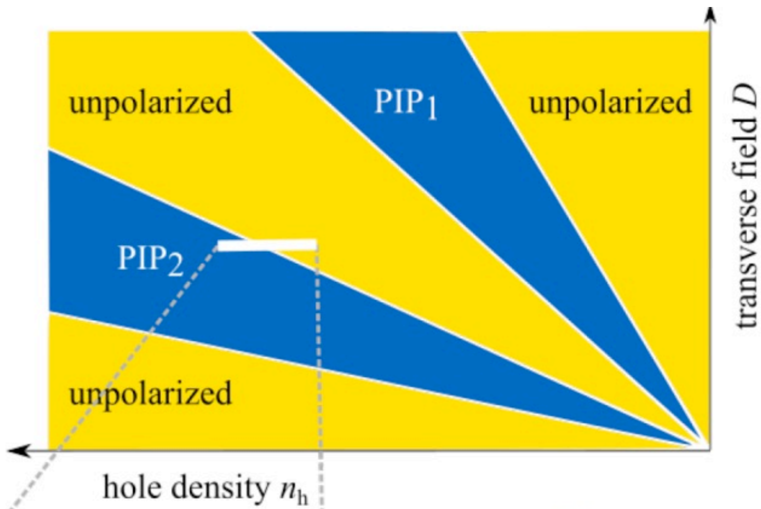






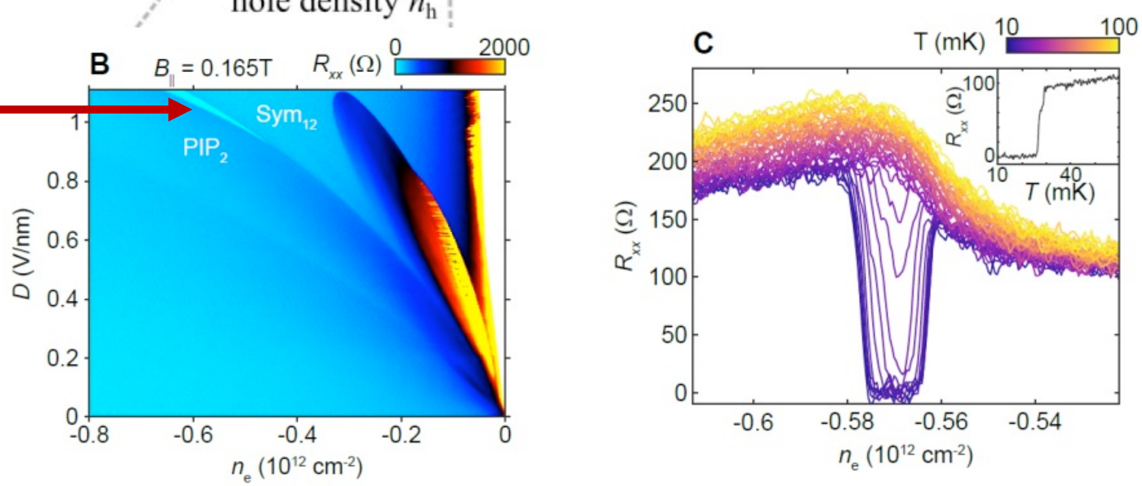
Superconductivity near an onset of one of the ordered states in biased Bernal bilayer graphene

a)



Zhou, Young et al, Science (2022)

Vanishing R_{xx}



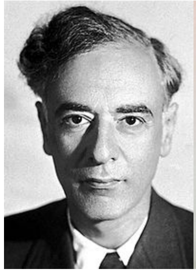
Near every electronic instability, one can identify a soft boson (the one which will condense on the other side of the transition) and treat electron-electron interaction as mediated by a soft boson

Examples: ferro/anti-ferro magnetic fluctuations near a spin order, nematic fluctuations near an order breaking lattice rotation …

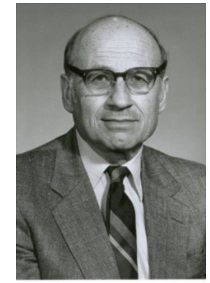
Phonons were selected by BCS/Eliashberg for a reason: electron-electron interaction (screened Coulomb interaction) is repulsive.

Not so fast….

Screened Coulomb potential

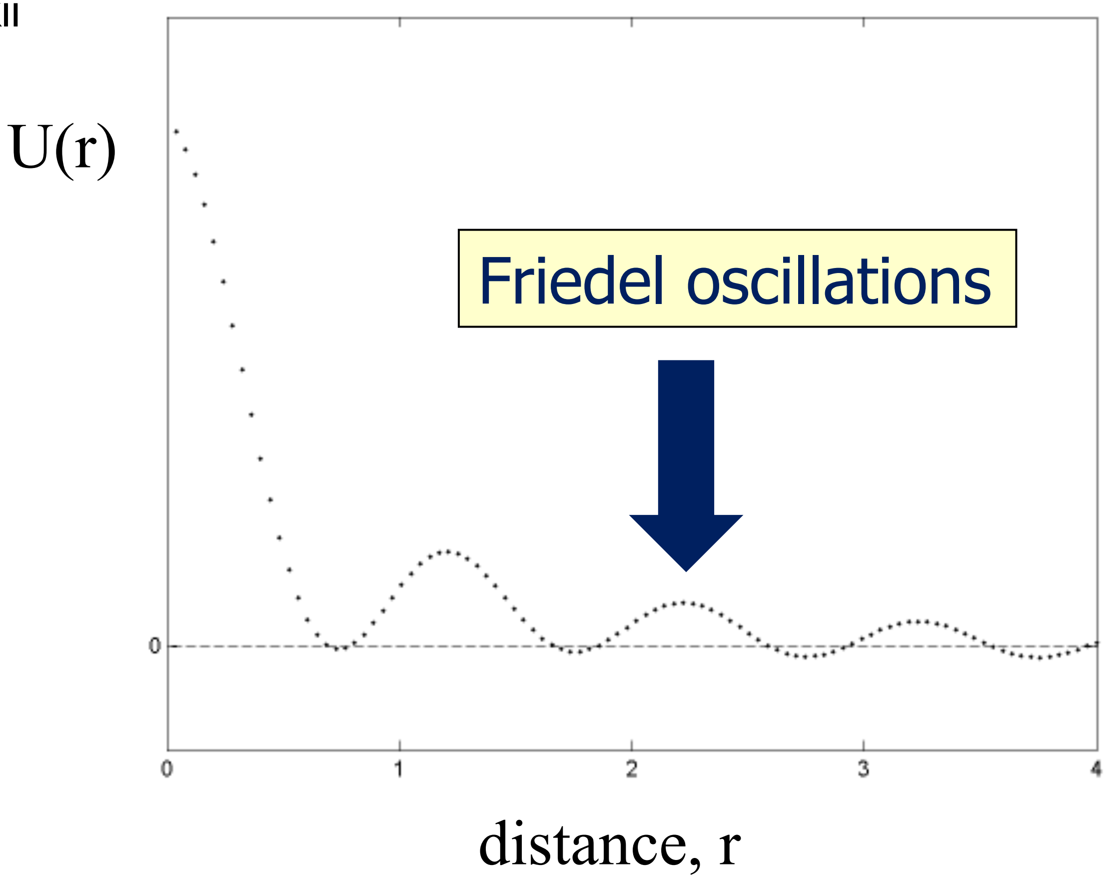



Landau Pitaevskii




Kohn Luttinger

P.W. Anderson
(with Morel)



At large distances, screened Coulomb interaction oscillates an occasionally becomes over-screened [$U(r) = \cos(2k_F r)/r^3$]

Oscillations increase as the system approaches
a non-superconducting instability

This often selects non-s-wave superconductivity:
p-wave pairing by ferromagnetic fluctuations,
d-wave pairing by antiferromagnetic fluctuations,
s+- pairing by stripe magnetic fluctuations and so on

However, once this is taken into consideration, one ends up
with the dynamical pairing, similar to electron-phonon case.

There is a difference, though: there is only one velocity for collective mode-mediated pairing – the Fermi velocity.

At a first glance, one cannot then treat bosons as slow compared to fermions, as we did in Eliashberg theory for el-ph interaction.

At a second glance, collective bosons are actually slow because they are Landau overdamped. Then Eliashberg-type treatment is possible, but bosonic polarization must be included.

Which parameter controls vertex corrections?

None. Vertex corrections are generally of order one, but in most cases are very small numerically.

There is another difference, which acts "in favor" of pairing mediated by overdamped collective modes.

For el-phonon interaction, fermionic self-energy diverges when Debye frequency vanishes. One has to keep ω_D finite

The spectrum of collective excitations generally has strong momentum dependence

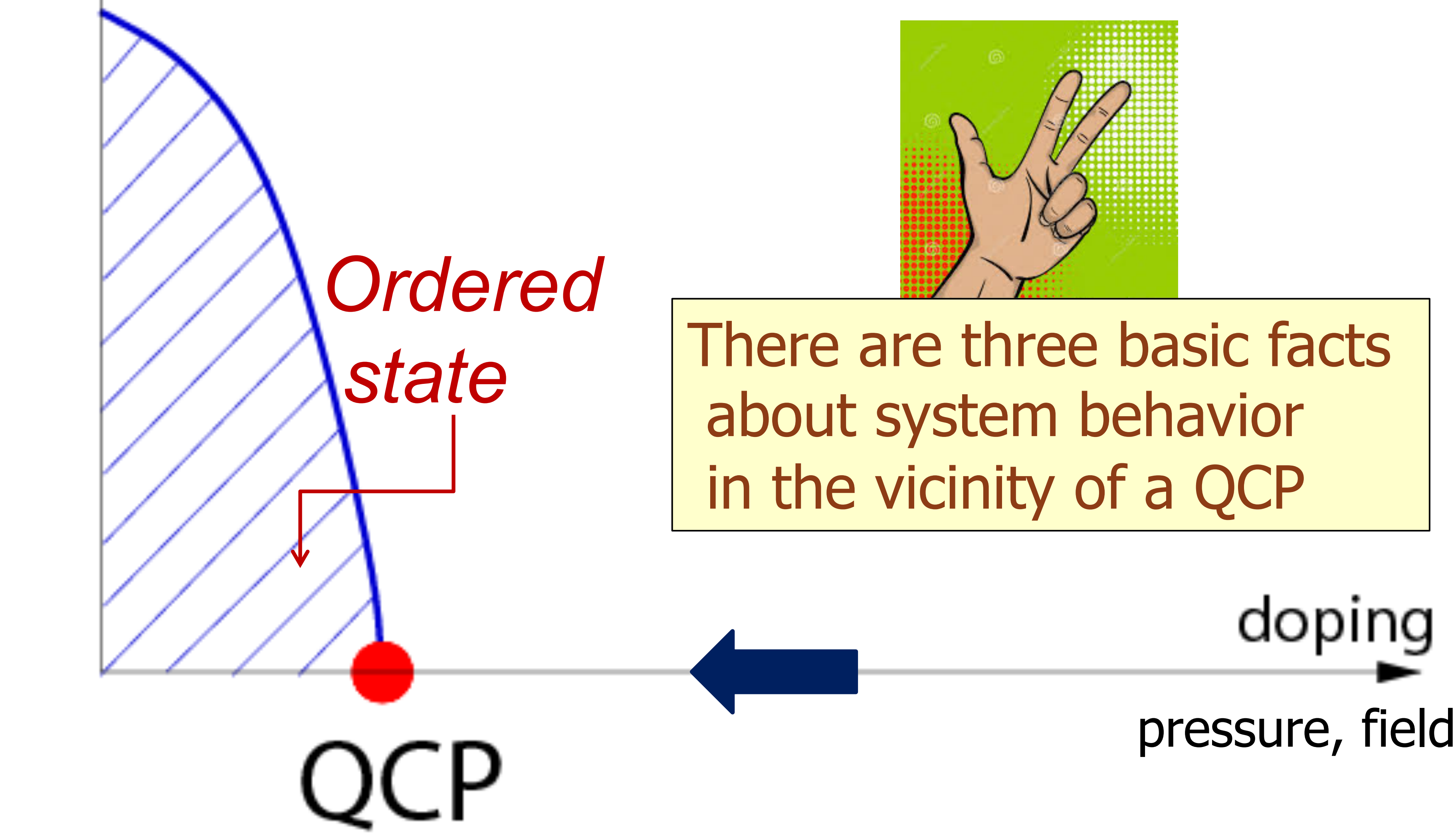
$$\chi(q,\Omega_m) = \frac{1}{m^2 + q^2 + \alpha \frac{|\Omega_m|}{q}}$$

Near a ferromagnetic or a nematic transition

As a result, when bosonic mass vanishes, quasiparticle self-energy acquires a non-Fermi liquid form ($\Sigma(\omega)$ \sim $\omega^{2/3}$ at a nematic QCP in 2D), but does not diverge. Then one can do analysis right at the critical point, without a fear of having to deal with divergencies.

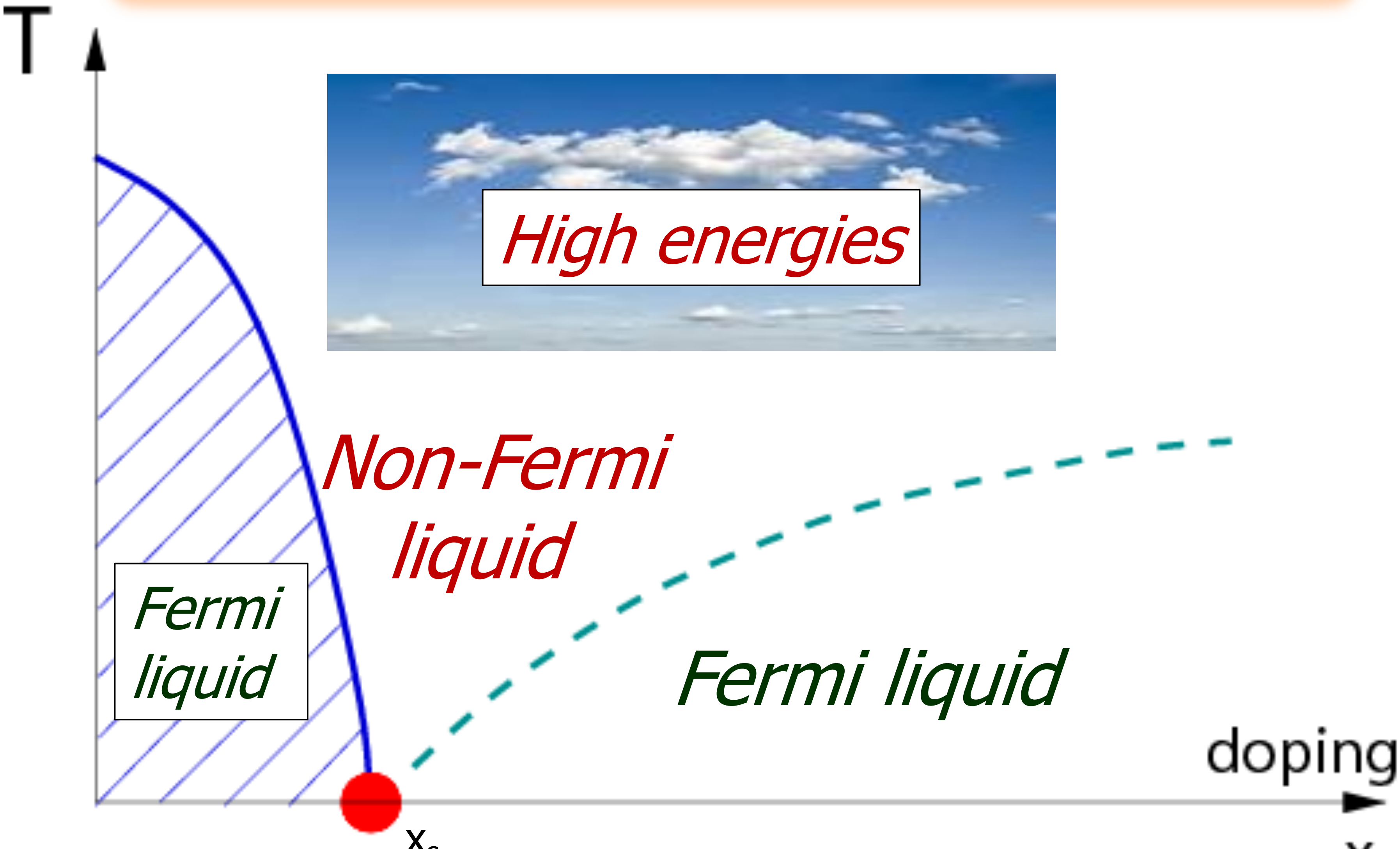
The set:

Consider a situation when a system of itinerant fermions approaches an instability towards some electronic order



There are three basic facts about system behavior in the vicinity of a QCP

Basic fact #1: interaction, mediated by a gapless boson, destroys Fermi liquid behavior



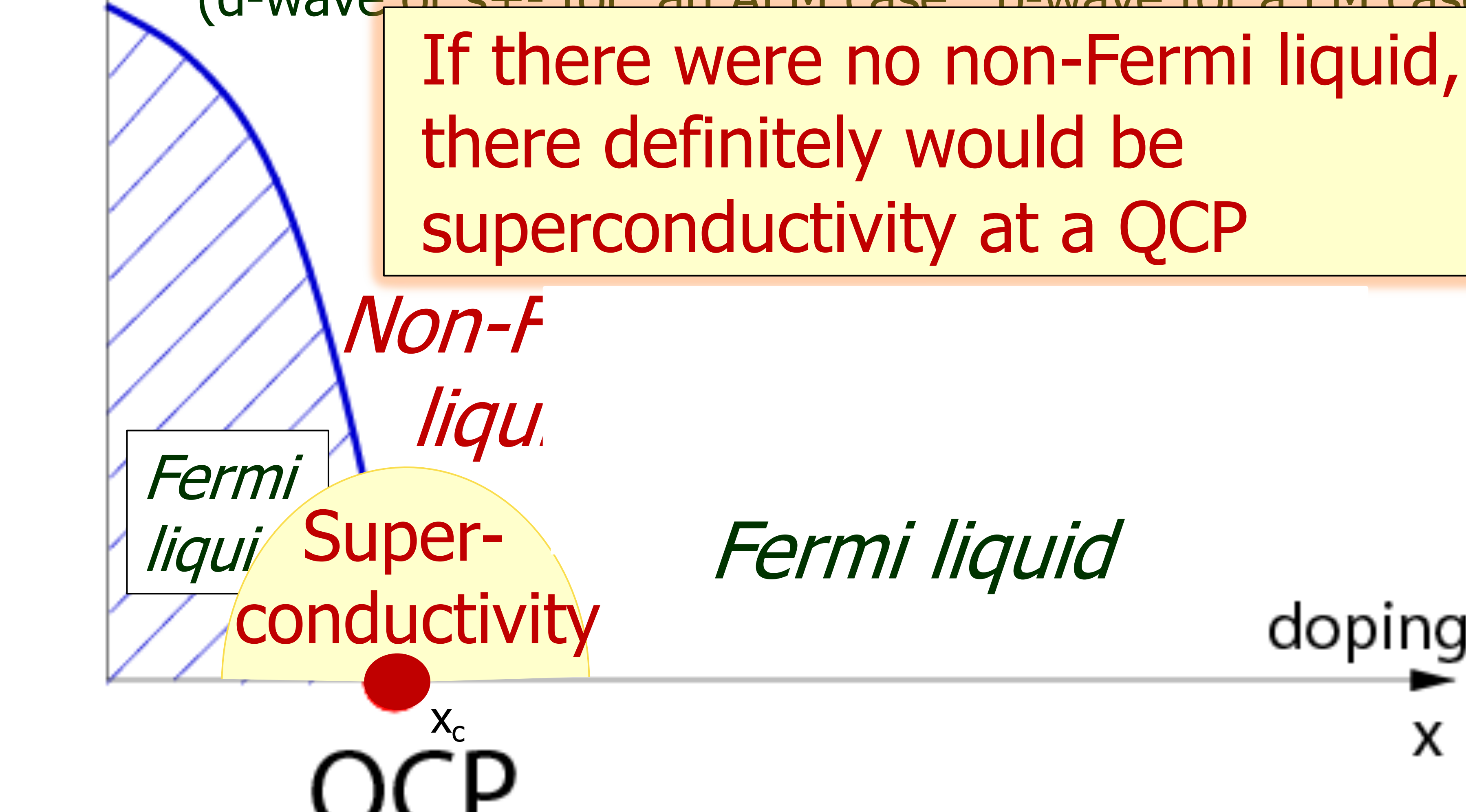
$$\Sigma'(\omega) \sim \Sigma''(\omega) \propto \omega^{2/3}$$

Basic fact #2: the same interaction mediates pairing

Interaction, mediated by a soft boson, gives rise to an attraction in at least one pairing channel

(d-wave or s+- for an AFM case, p-wave for a FM case)

If there were no non-Fermi liquid, there definitely would be superconductivity at a QCP



Basic fact #3: competition

Pairing wants to make electrons coherent,
Non-Fermi liquid tends to keep them incoherent

It is not clear a'priori that superconductivity out of a non-Fermi liquid actually develops

This competition is built into Eliashberg theory
(coupled equations for fermionic self-energy and pairing vertex)

Let's apply Eliashberg theory to electronic mediated pairing

Let's focus on system behavior right at a QCP

The equations are formally the same as in the el-phonon case

Pairing vertex

$$\Phi(\Omega_m) = \pi T \sum_{\omega \neq \Omega} \frac{\Phi(\omega_m)}{\sqrt{(\omega_m + \Sigma(\omega_m))^2 + \Phi^2(\omega_m)}} \chi_L(\omega_m - \Omega_m)$$

Self-energy

$$\Sigma(\Omega_m) = \pi T \sum_{\omega \neq \Omega} \frac{\omega_m + \Sigma(\omega_m)}{\sqrt{(\omega_m + \Sigma(\omega_m))^2 + \Phi^2(\omega_m)}} \chi_L(\omega_m - \Omega_m)$$

Superconducting gap

$$\Delta(\Omega_m) = \Omega_m \frac{\Phi(\Omega_m)}{\Omega_m + \Sigma(\Omega_m)}$$

$\chi_L(\Omega)$ is a propagator of collective excitations, integrated along the Fermi surface

The term with $\omega_m = \Omega_m$ is eliminated by the same reason as for el-phonon case

Examples:

Near a nematic transition in 2D

$$\chi(q,\Omega_m) = \frac{1}{m^2 + q^2 + \alpha \frac{|\Omega_m|}{q}}$$

$$\chi_L(\Omega_m) = \int \chi(q,\Omega_m)\, dq \propto \Omega_m^{-1/3}, \quad m = 0$$

P-A Lee, Bonesteel, McDonald, Nayak, Millis, Altshuler, Ioffe, Metlitski, Sachdev, Senthil, Berg, Klein Kivelson, Fradkin, Oganesyan, Lederer, Fernandes, Trebst, Metzner, Pepin, Efetov, Maslov, Raghu…

Near an AFM transition in 2D (hot spot model)

$$\chi(q,\Omega_m) = \frac{1}{m^2 + (q - Q)^2 + \alpha|\Omega_m|}$$

$$\chi_L(\Omega_m) = \int \chi(q,\Omega_m)\, dq \propto \Omega_m^{-1/2}, \quad m = 0$$

Millis, Sachdev, Varma, Finkelstein, Schmalian, Metlitski, Sachdev, Y. Wang, Efetov, Pepin, Zaanen, Tremblay, Berg, Fernandes, Tsvelik, S-S Lee, Di Castro, Castellani, Grilli, Caprara..(CDW), Georges…

Anisotropic 3D systems

$$\chi_L(\Omega_m) = \int \chi(q,\Omega_m)\, dq \propto \Omega_m^{-a}, \quad a \ll 1$$

Son, Raghu, Torroba, Senthil, Mross, Metlitski, Sachdev, Moon, Schmalian….

I will combine all these cases into

$$\chi_L(\Omega) = (g/\Omega)^{\gamma}$$

and will keep γ as a continuous parameter

The γ-model

For these models,
the normal state is a non-Fermi liquid

$$\Sigma(\omega_m) \sim (\omega_m)^{1-\gamma}$$

Self-energy does not diverge for $\gamma < 1$

Let's solve for T_c and check whether it is zero or finite.

$$\Phi(\Omega_m) = \pi T \sum_{\omega \neq \Omega} \frac{\Phi(\omega_m)}{\sqrt{(\omega_m + \Sigma(\omega_m))^2 + \Phi^2(\omega_m)}} \chi_L(\omega_m - \Omega_m)$$

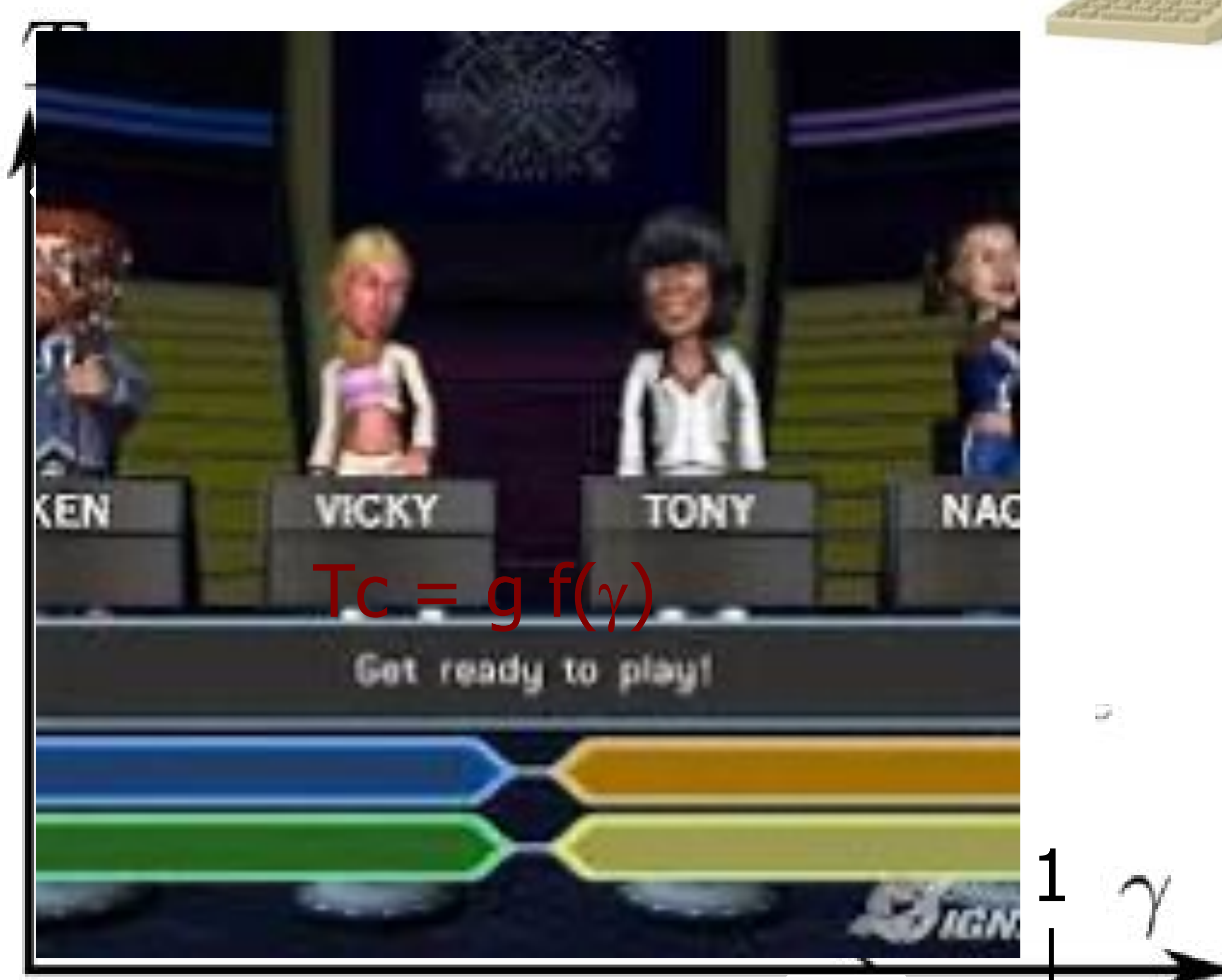
$$\Sigma(\Omega_m) = \pi T \sum_{\omega \neq \Omega} \frac{\omega_m + \Sigma(\omega_m)}{\sqrt{(\omega_m + \Sigma(\omega_m))^2 + \Phi^2(\omega_m)}} \chi_L(\omega_m - \Omega_m)$$

Treat Φ as infinitesimally small,

$\chi_L = (g/|\Omega_m|)^\gamma$, $\Sigma(\Omega_m) \sim |\Omega_m|^{1-\gamma}$

Ask a friend

Ask a computer



Is this superconductivity associated with the Cooper logarithm?

A way to detect Tc analytically (and check Cooper logs) is to depart from the normal state and analyze the pairing susceptibility. It should diverge at Tc and become negative at T <Tc

BCS: interaction is frequency independent,

$$\Phi\,(T) = \pi T\,\lambda\,\sum_{\omega} \frac{\Phi(T)}{|\omega_m|} + \Phi_0$$

The kernel is marginal, iterations yield Cooper logarithms

$$\Phi(T) = \Phi_0\left(1 + \lambda \log\frac{w_0}{T} + \lambda^2 \log^2\frac{w_0}{T} + \ldots\right) = \frac{\Phi_0}{1 - \lambda \log\frac{w_0}{T}}$$

Pairing susceptibility $\Phi(T)/\Phi_0$ diverges at $T_c \sim w_0\, e^{-1/\lambda}$ and is negative at smaller T

Let's now depart from a non-FL normal state ($\Sigma(\omega_m) \sim (\omega_m)^{1-\gamma}$)

$$\Phi\,(\Omega_m) = \pi T\,(1-\gamma)\,\sum_{\omega \neq \Omega} \frac{\Phi(\omega_m)}{|\omega_m|^{1-\gamma}}\,\frac{1}{|\omega_m - \Omega_m|^{\gamma}}\,\frac{1}{1+|\omega_m/g|^{\gamma}} + \Phi_0$$

$0 < \gamma < 1$

$1 + \omega/\Sigma(\omega)$

self-energy

$\Sigma(\omega) \sim \omega^{1-\gamma}$

interaction

$\chi(\Omega) \sim 1/|\Omega|^{\gamma}$

If we sum up logarithms, we get, even at T=0

$$\Phi(\Omega_m) = \Phi_0 \left(1 + (1-\gamma)\log\frac{g}{|\Omega_m|} + \frac{(1-\gamma)^2}{2}\log^2\frac{g}{|\Omega_m|} + \ldots \right) = \Phi_0 \left(\frac{g}{|\Omega_m|}\right)^{1-\gamma}$$

No indication of pairing instability.

Pairing susceptibility remains positive and finite at T=0.

Did our best friend betray us?

You do not have a weak coupling limit
to justify logarithmic approximation

Let's then go beyond summing up the logarithms and look more carefully into the low-frequency region:

$$\Phi(\Omega) = \frac{1-\gamma}{2} \int d\omega \frac{\Phi(\omega)}{|\omega-\Omega|^{\gamma}|\omega|^{1-\gamma}} + \Phi_0$$

The solution is a power-law $\Phi(\Omega) \sim |\Omega|^{-\alpha}$

The summation of the logarithms gives a real $\alpha = 1-\gamma$

$$\Phi(\Omega_m) = \Phi_0 \left(\frac{g}{|\Omega_m|}\right)^{1-\gamma}$$

The actual solution for any γ has two complex exponents:

$\alpha = \gamma/2 + \iota\,\beta$ and $\alpha = \gamma/2 - \iota\,\beta$

$$\Phi(\Omega) = \frac{\Phi_0}{|\Omega|^{\gamma}/2}\left(C_1\,|\Omega|^{i\beta} + C_1^*\,|\Omega|^{-i\beta}\right) \sim \frac{\Phi_0}{|\Omega|^{\gamma}/2} \cos(\beta \log|\Omega| + \phi)$$

H. Liu, McGrrevy, Vegh, Cortez et al, Zaanen, Klebanov et al, Tsvelik, Torroba, Raghu, H. Wang, Esterlis, Schmalian, Y. Wang, Sachdev, Patel…

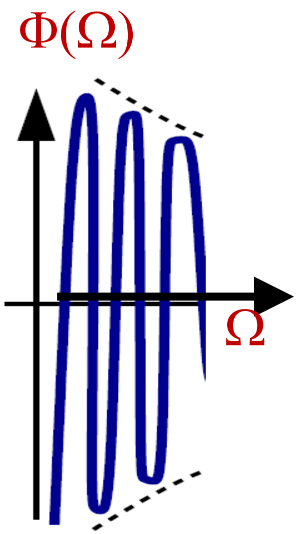
Small Ω

$$\Phi(\Omega) = \frac{1}{|\Omega|^{\gamma/2}}\left(C_1 |\Omega|^{i\beta} + C_1^* |\Omega|^{-i\beta}\right)$$

Inconsistent

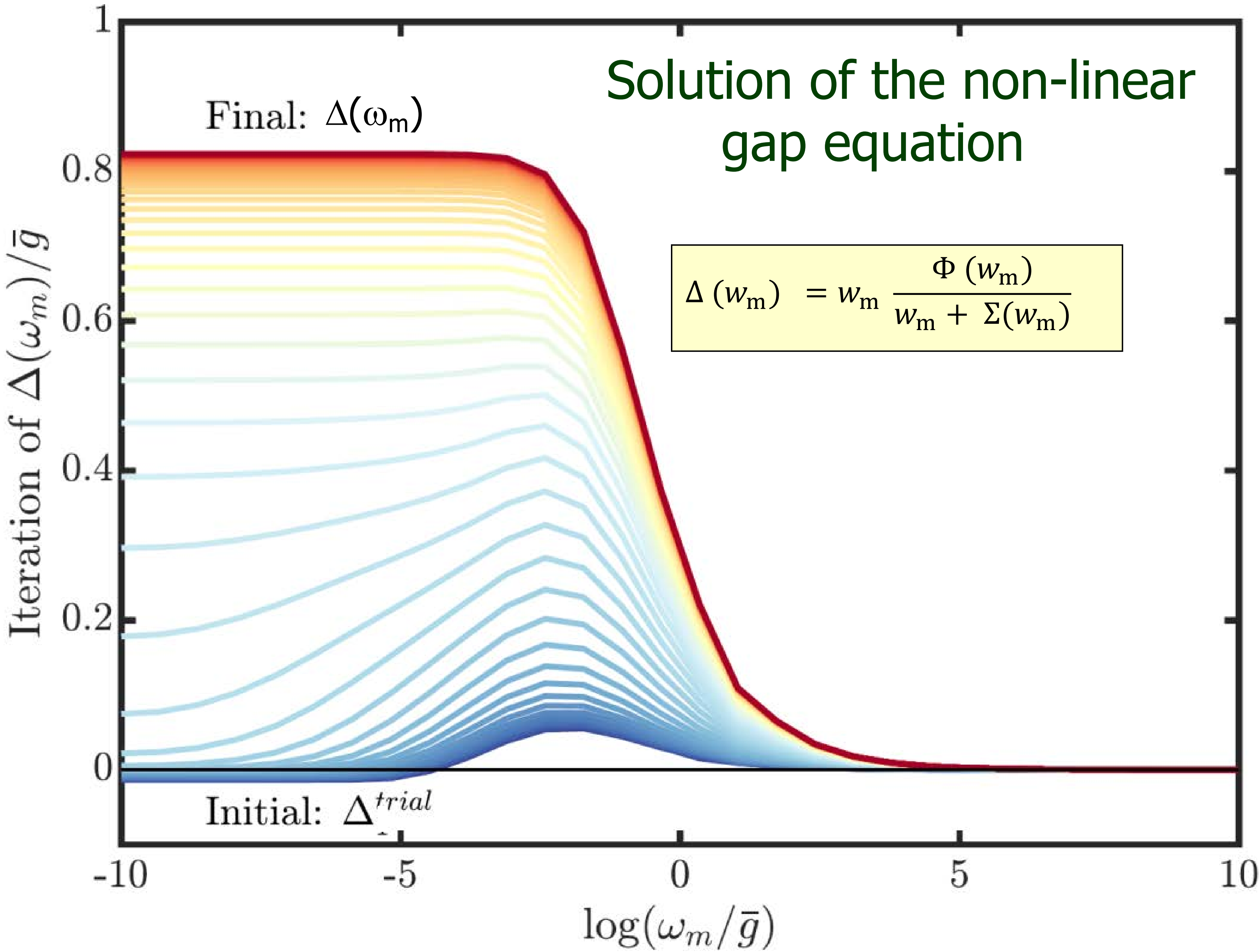
Large Ω

$\Phi(\Omega) = \Phi_0 +$ positive corrections



Perturbative expansion breaks down, and one needs a finite $\Phi(\omega)$ (i.e., a finite pairing gap) to modify the low-frequency behavior and match with the high-frequency one

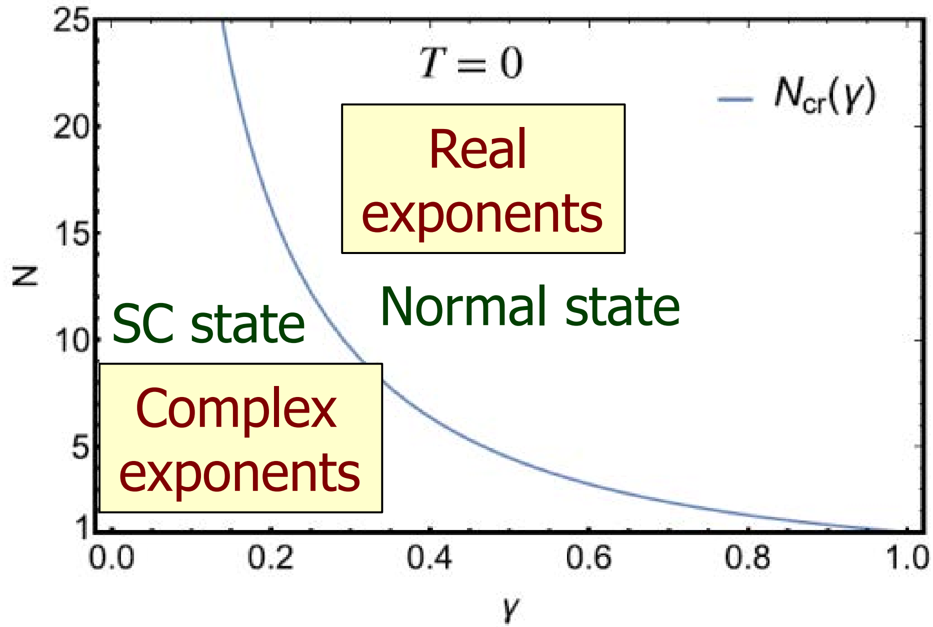
This is non-BCS pairing mechanism: complex exponents instead of Cooper logarithm



Solution of the non-linear gap equation
Final: $\Delta(\omega_m)$
$\Delta(w_m) = w_m \frac{\Phi(w_m)}{w_m + \Sigma(w_m)}$
Initial: Δ^{trial}
Iteration of $\Delta(\omega_m)/\bar{g}$
$\log(\omega_m/\bar{g})$
1
0.8
0.6
0.4
0.2
0
-10
-5
0
5
10

One can see all this more clearly by extending the model to non-equal interactions in the particle-hole and particle-particle channels (rescale pp interaction by 1/N)

$$\Phi(\Omega) = \frac{1-\gamma}{2N} \int d\omega \frac{\Phi(\omega)}{|\omega-\Omega|^{\gamma}|\omega|^{1-\gamma}} + \Phi_0$$



At $N = N_{cr}$,

$$\Phi(\Omega) = \frac{1}{|\Omega|^{\gamma/2}}(1 + a \; log|\Omega|)$$

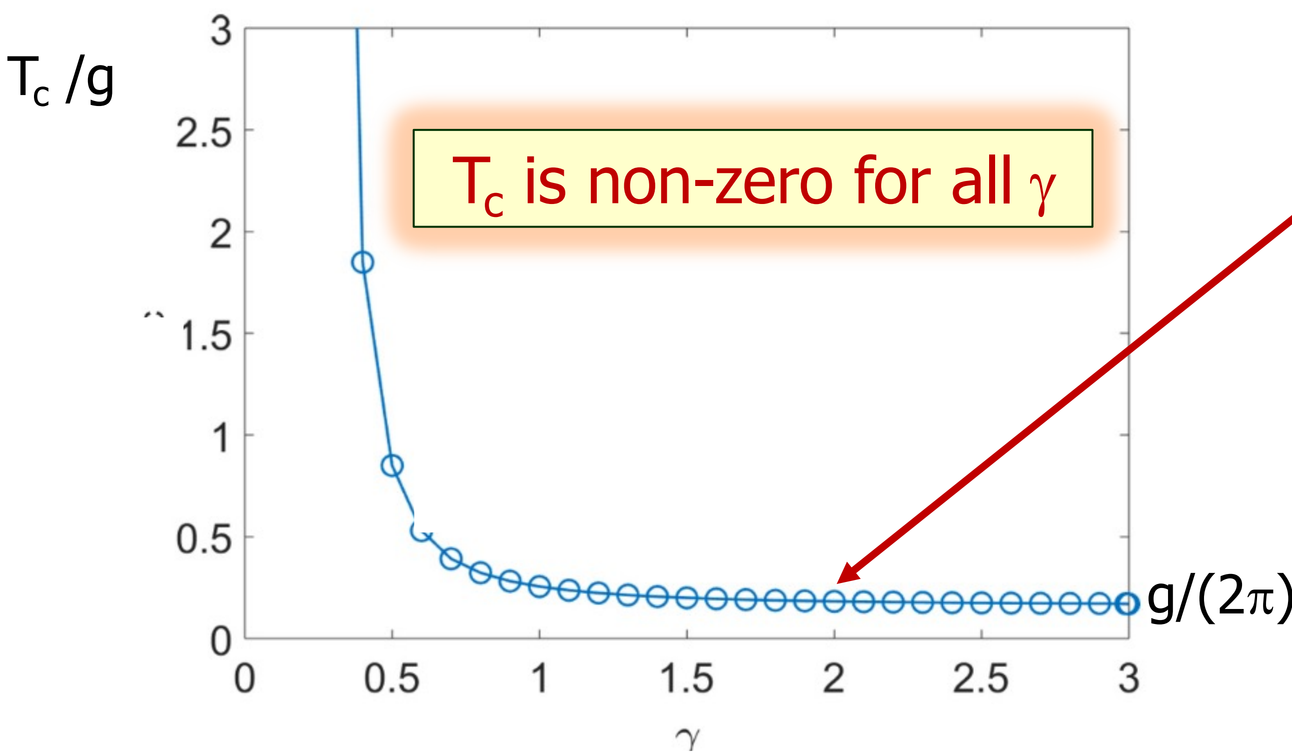
Let’s connect pairing by collective modes and el-phonon superconductivity at vanishing ω_D

For el-phonon case

$$\chi_L(\Omega_m) = \int \chi(q,\Omega_m)\, dq \propto 1/\Omega_m^2$$

$$\chi_L(\Omega) = (g/\Omega)^\gamma$$

$\gamma=2$



Karakozov et al, Marsiglio, Carbotte, Combescot; Metlitski et al, Mross et al; Y. Wang et al, Torroba et al, Y. Wu et al ….

There are model systems for $\gamma>1$: dispersion-less fermions interacting with an Einstein phonon via a random Yukawa coupling (Yukawa SYK model)

$$\boxed{0<\gamma<2}$$

Depending on the number of fermionic and bosonic flavors

Esterlis, Schmalian, Y. Wang, Classen…

Dynamical vortices

Let's compare interaction on the Matsubara axis and on the real axis

Matsubara axis

$\chi_L(\Omega_m) = (g/|\Omega_m|)^\gamma$

Attractive for all γ

Real axis

$\chi_L(\Omega) = (g/|\Omega|)^\gamma e^{i\pi\gamma/2 \,\mathrm{sgn}\,\Omega}$

Real axis

$\mathrm{Re}\,\chi_L(\Omega) = (g/|\Omega|)^\gamma \cos(\pi\gamma/2)$

attractive for $\gamma < 1$
repulsive for $\gamma > 1$

$\mathrm{Im}\,\chi_L(\Omega) = (g/|\Omega|)^\gamma \sin(\pi\gamma/2)\,\mathrm{sgn}\,(\Omega)$

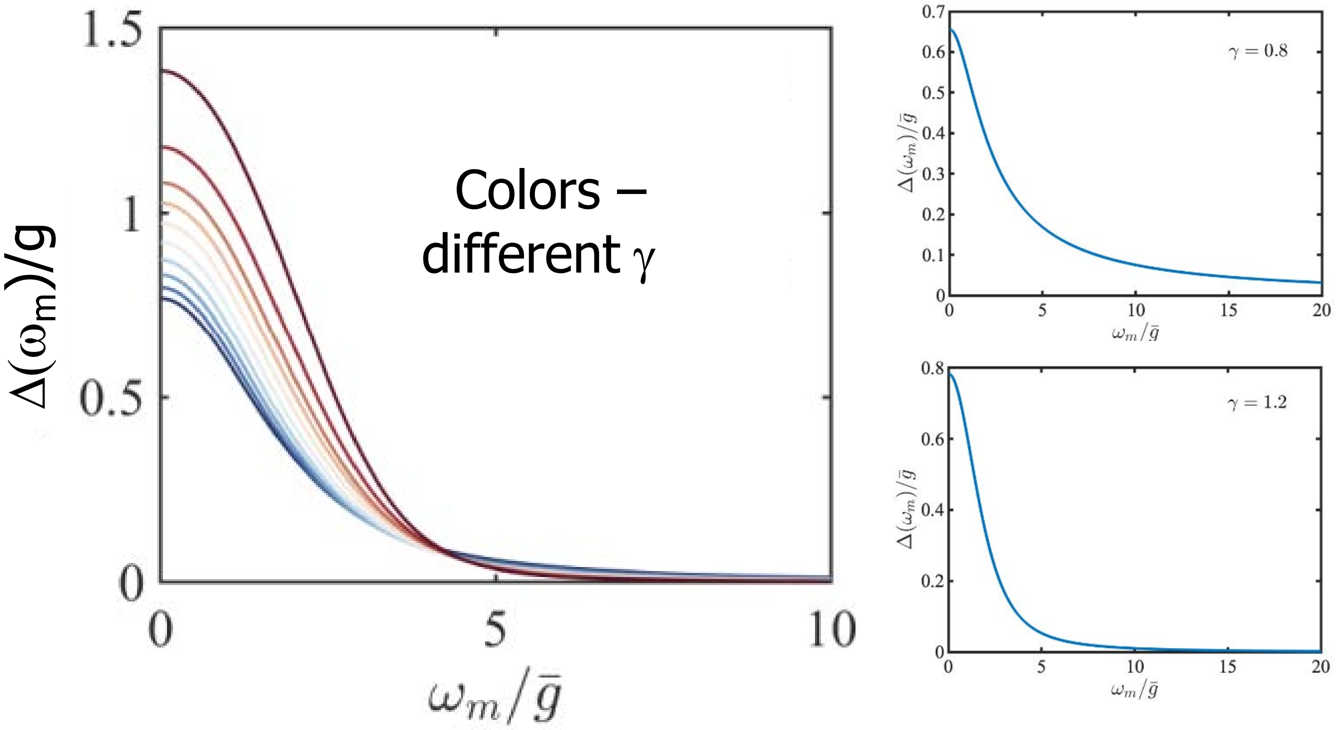
vanishes at $\gamma = 2$

At $\gamma=2$, interaction is purely attractive on the Matsubara axis and purely repulsive on the real axis (el-ph. case at $\omega_D=0$)

Carbotte, Marsiglio; R. Combescot

Matsubara axis:

Solve the non-linear gap equation at T=0



A rather conventional behavior:

- gap function $\Delta(\omega_m)$ tends to a finite value at $\omega=0$
- gap magnitude scales with g

Nothing special happens at $\gamma=2$

Convert the gap equation onto the real axis and solve

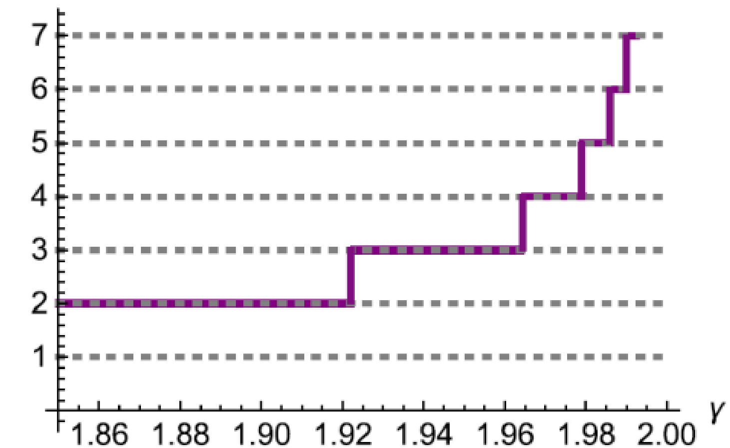
Carbotte, Marsiglio, Combescot,
Karakozov, Maksimov, Mikhailovsky

$1<\gamma<2$

On the Matsubara axis

Number of 2π phase slips increases as γ approaches 2

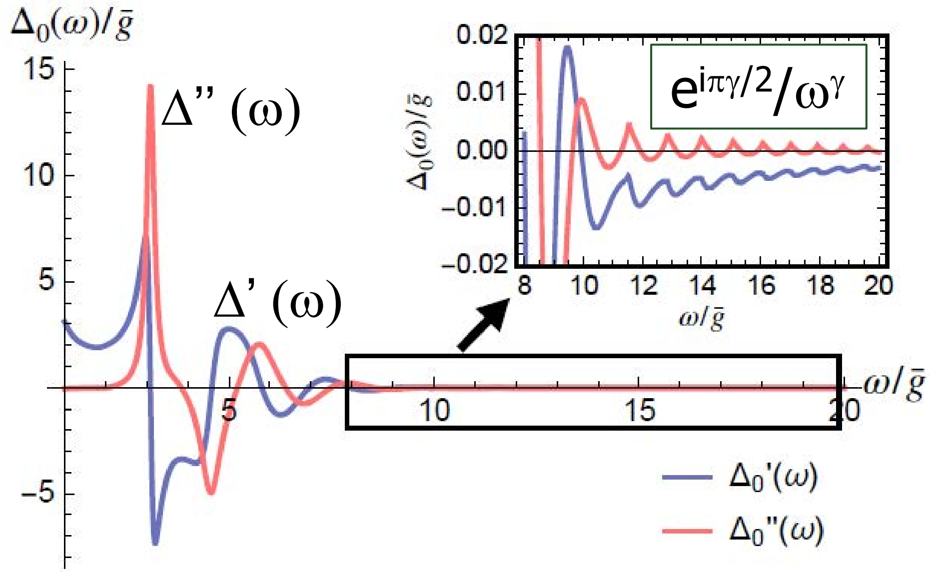
Sign-preserving

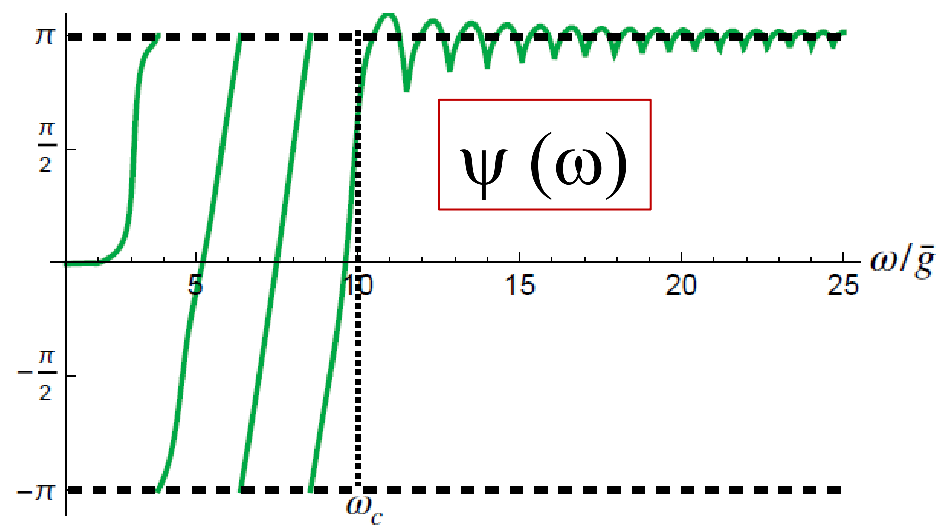


$$\Delta(\omega) = |\Delta(\omega)| e^{i\psi(\omega)}$$

There appear 2π phase slips!

On the real axis

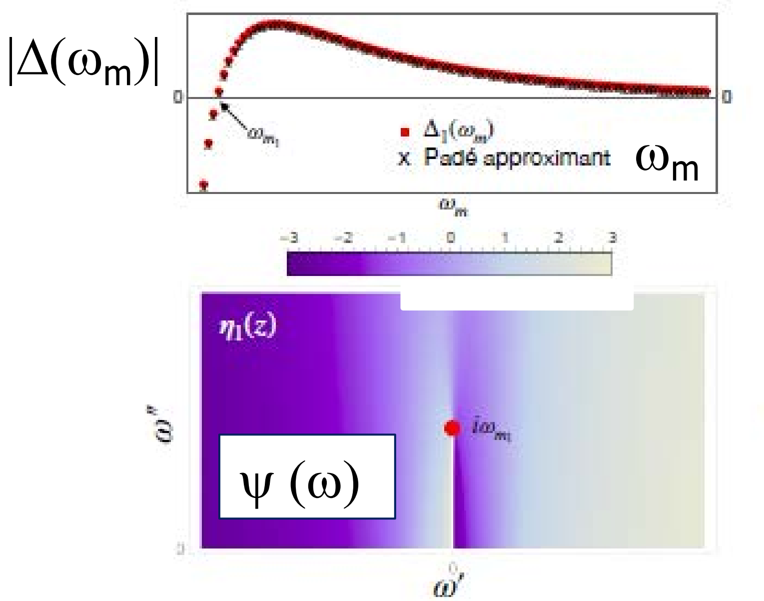




If there are m 2π phase slips on the real axis, and the gap function is purely real on the Matsubara axis, there must be m 2π vortices in the upper frequency half-plane.

A dynamical vortex: a point in the upper frequency half-plane, where the gap function vanishes.

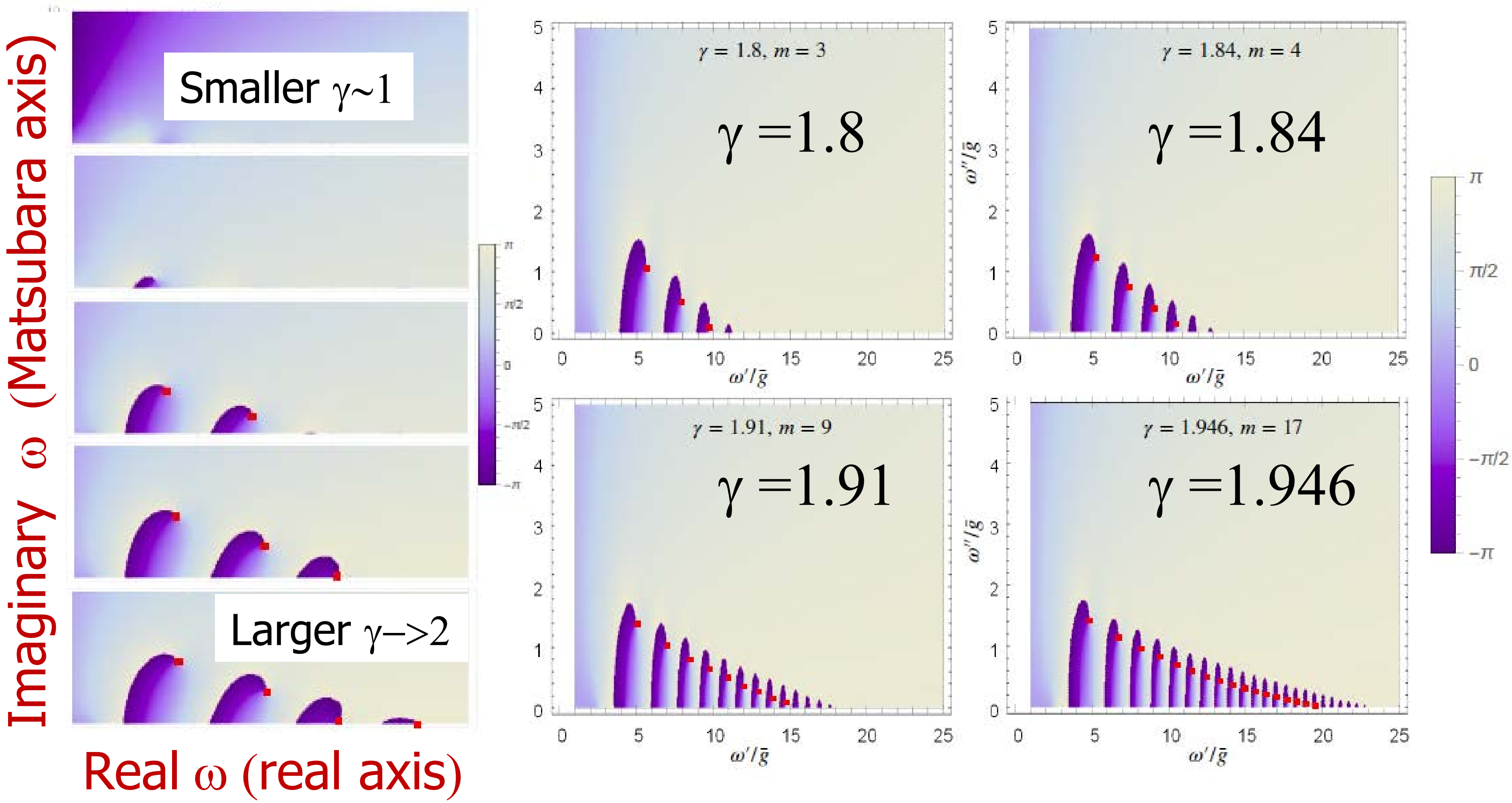
$$\Delta(\omega) = |\Delta(\omega)|e^{i\psi(\omega)}$$



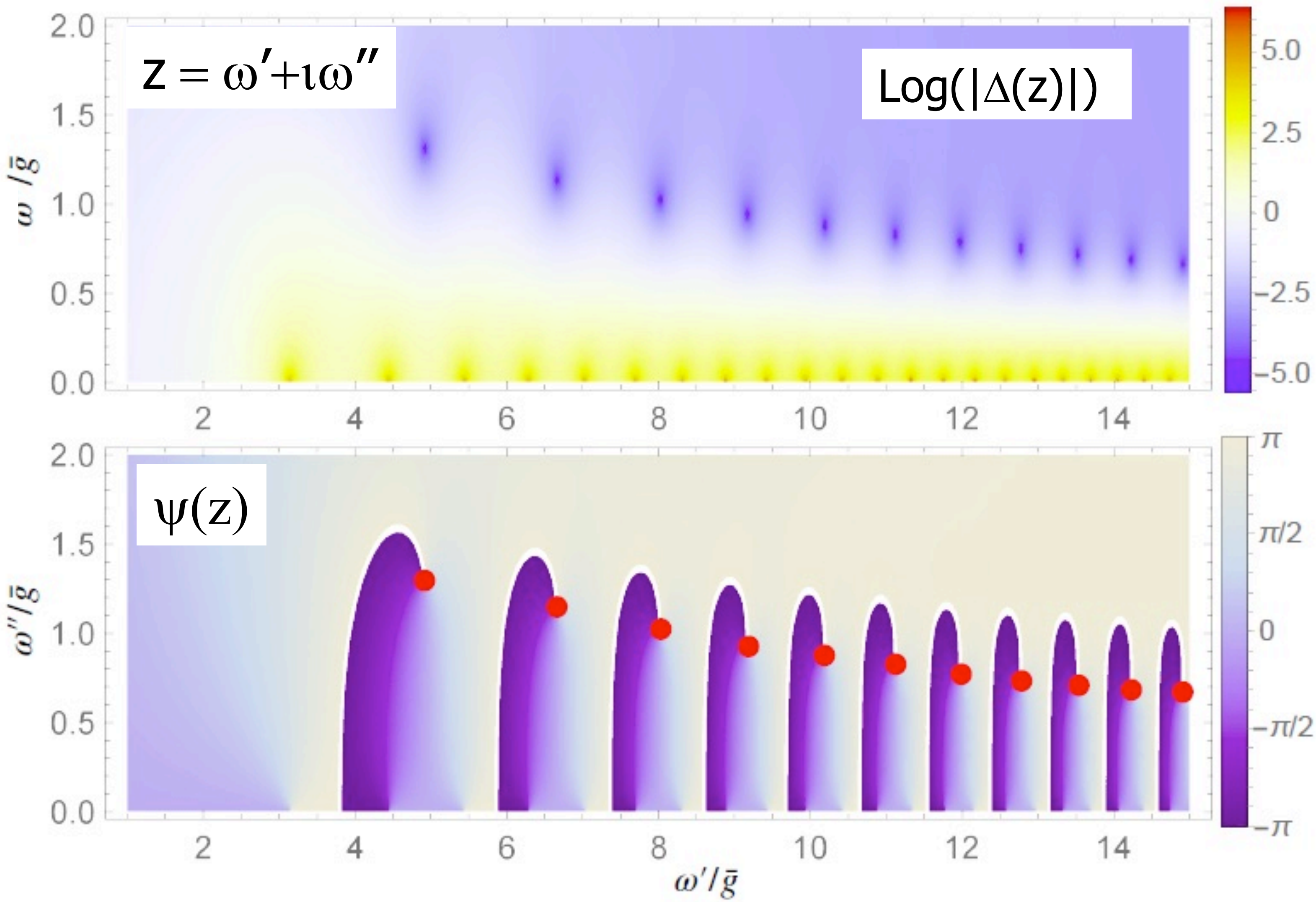
Phase $\Psi(\omega)$ winds up by 2π around a vortex

As γ increases, dynamical vortices penetrate into the upper half-plane, one-by-one

Plots of the phase ψ of the order parameter $\Delta\, e^{i\psi}$

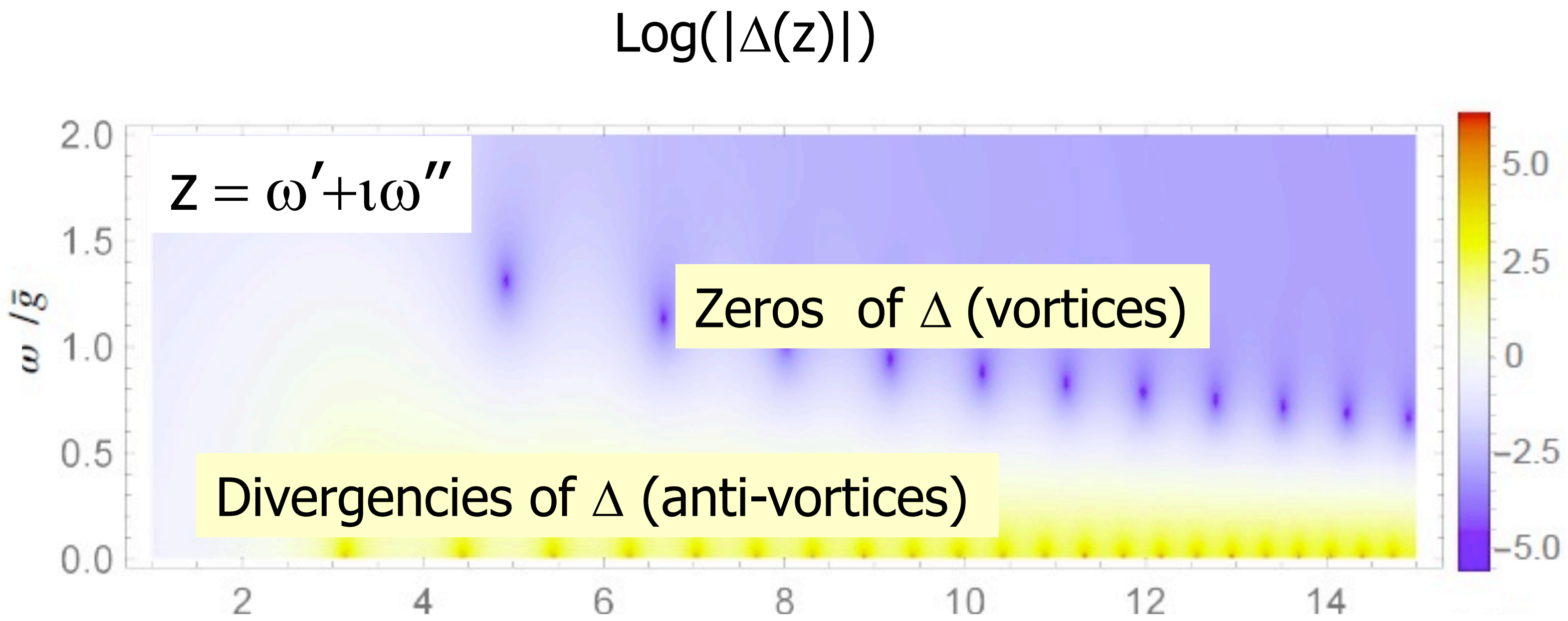


γ=2. Number of vortices becomes infinite, and they penetrate up to infinite frequency.



The gap function develops an essential singularity at ω = infinity

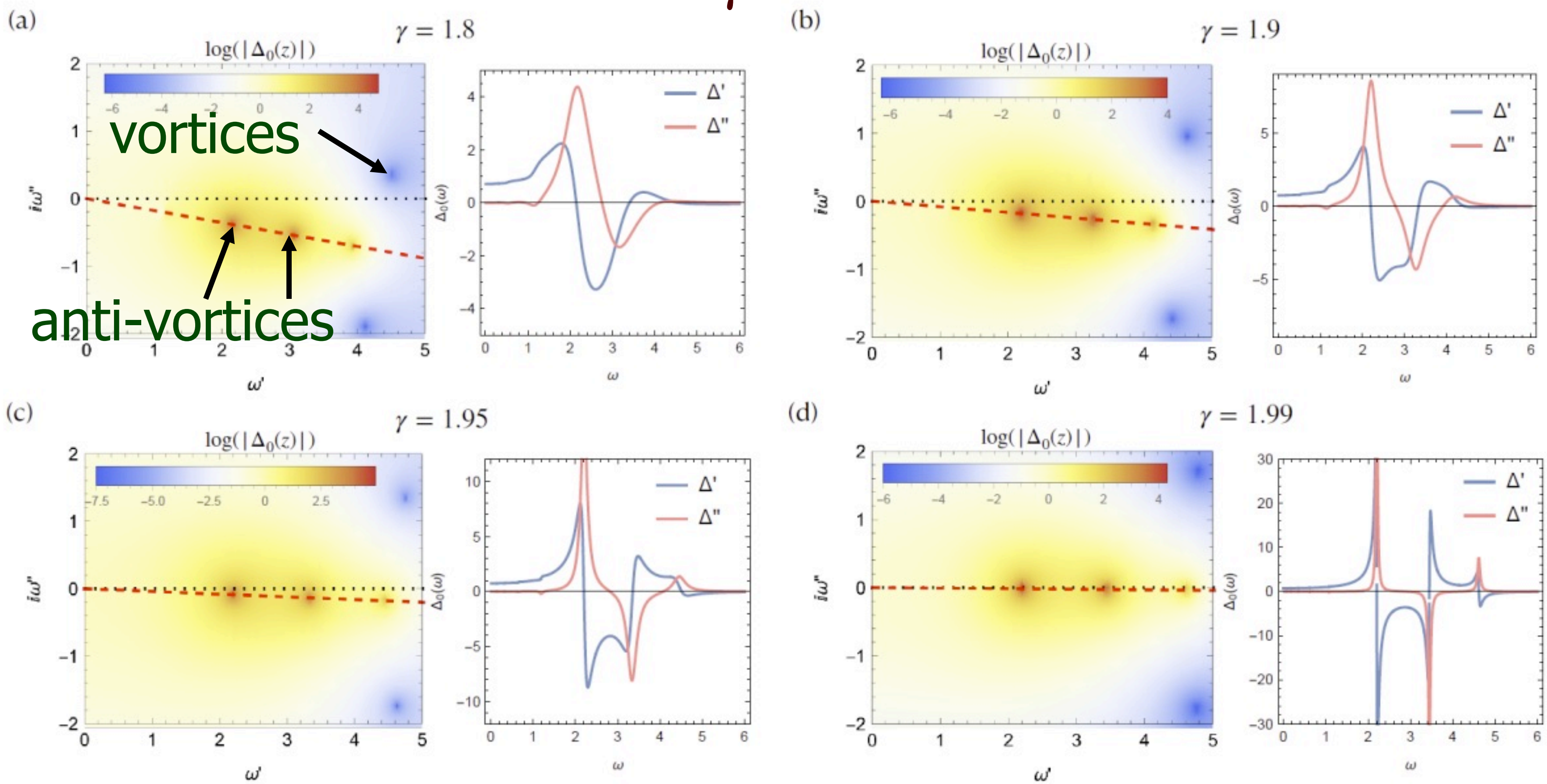
There is more at γ=2



Vortices – zeros of $\Delta(z)$, $z = \omega' + i\,\omega''$
Anti-vortices – poles of $\Delta(z)$

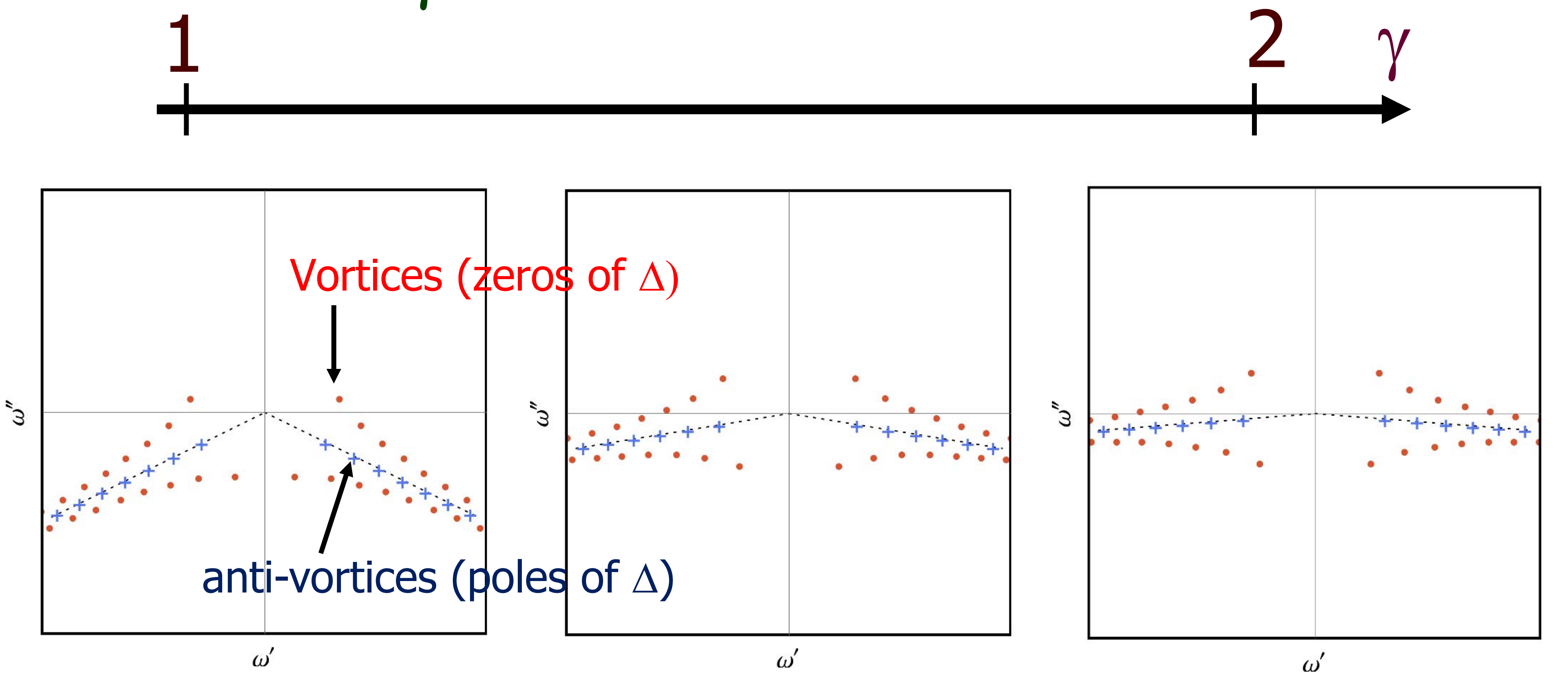
Pade approximants

$1 < \gamma < 2$



The gap function is analytic in the upper half-plane of $z = \omega' + i\,\omega''$ (as the physical function must be).
Poles (anti-vortices) are in the lower half-plane.

As γ increases towards 2

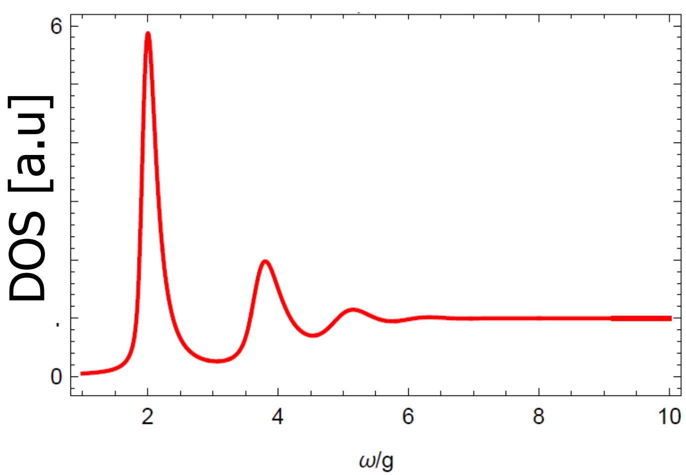


As the line of anti-vortices moves towards the real axis, vortices are pushed into the upper ½ plane of frequency

A highly non-trivial behavior of an el-phonon superconductor along real frequency axis in the limit of vanishing ω_D

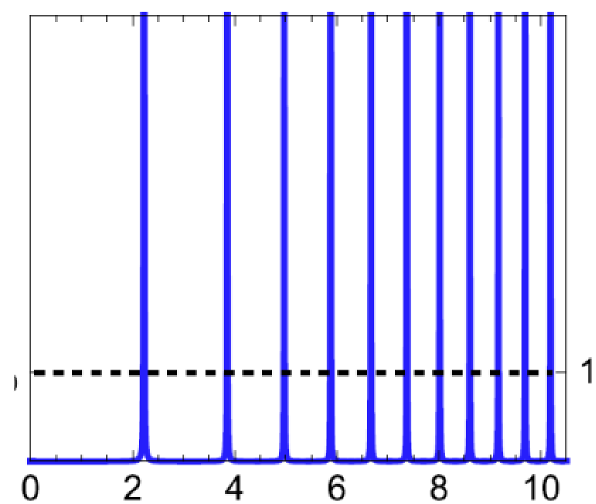
The density of states:

$\gamma<2$



DOS = gapped continuum

$\gamma=2$



DOS = a set o bound states

Conclusions:

- Pairing out of a Non-FL is fundamentally different from BCS

  Pairing develops in most cases, but the reason is complex exponents rather than Cooper logarithms

- As the exponent γ increases, dynamical vortices penetrate into the upper ½ plane, one by one.

The $\gamma=2$ model (el-phonon interaction at vanishing ω_D) is a critical one: the number of vortices is infinite, and there is an essential singularity at an infinite frequency.

THANK YOU