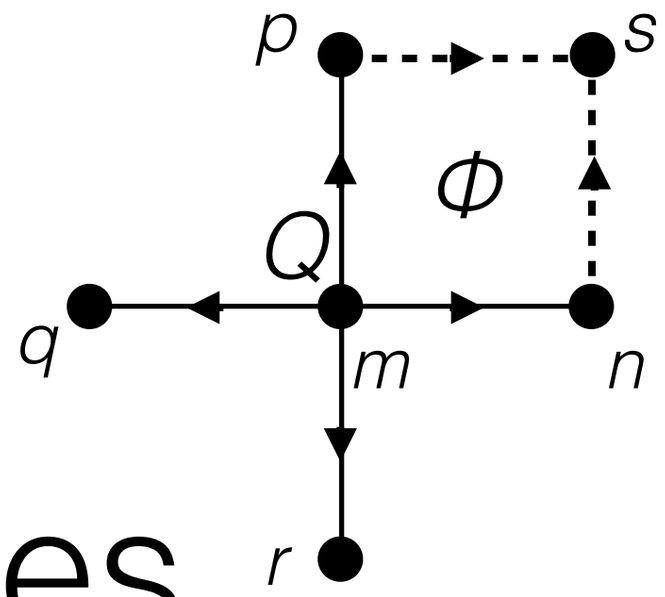
Spin liquids I and II: an introduction to lattice gauge theories



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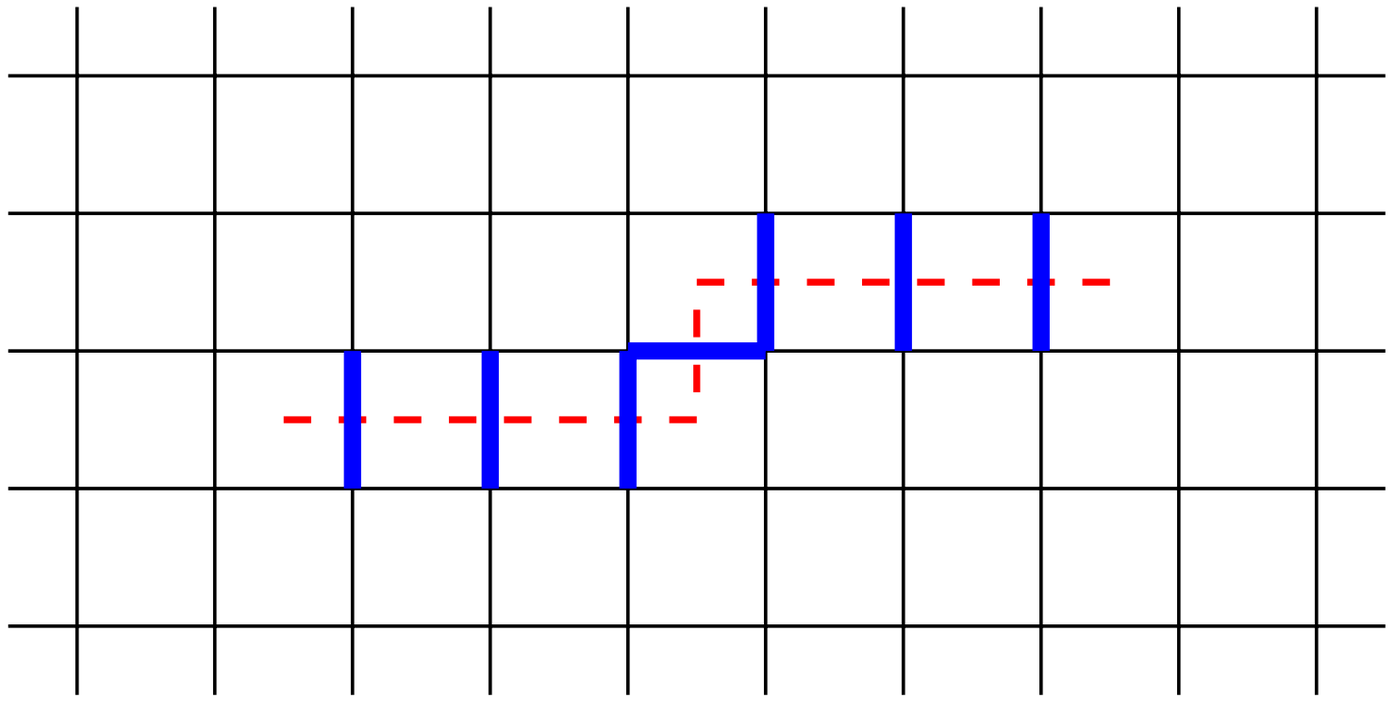
U.S. DEPARTMENT OF ENERGY



Spin liquids 40 years ago

This is nearly 20% lower than the spin-wave energy (11) of the Néel state. It seems almost certain that it represents the energy of a qualitatively different state.

Let us make some brief comments about the nature of this state. A disclaimer is in order: we really know very little about it. On the other hand, there are a few very basic things which can be said. We note that wherever two bonds are parallel neighbors, such as (12) and (34) in Fig. 3a, either $(S_1 \cdot S_2)$ or $(S_3 \cdot S_4)$ provides a matrix element to the degenerate configuration (23)(41),

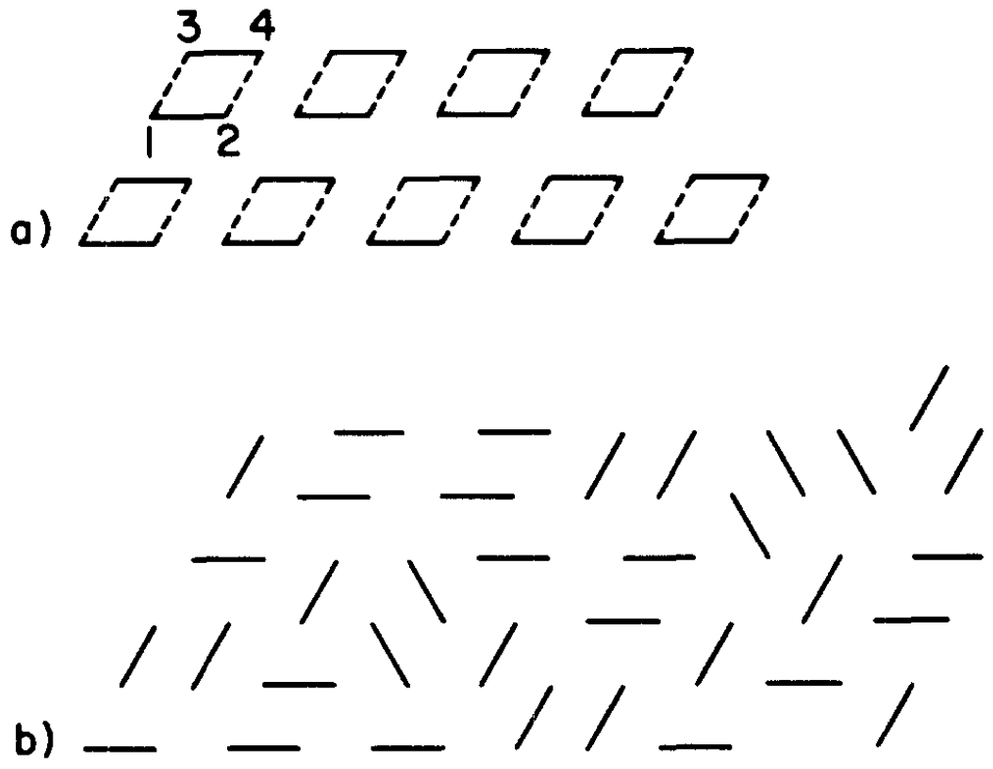
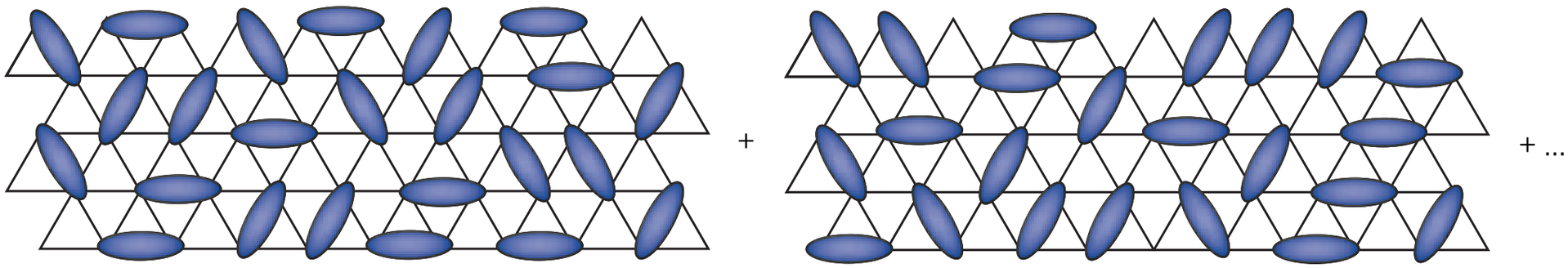


FIG. 3

Random arrangements of pair bonds on a triangle lattice. (a) Shows a regular arrangement with $2^{N/4}$ alternative distinct pairings ("rhombus" approximation). (b) An arbitrary arrangement.

P.W. Anderson, Mat. Res. Bull. **8**, 153 (1973).

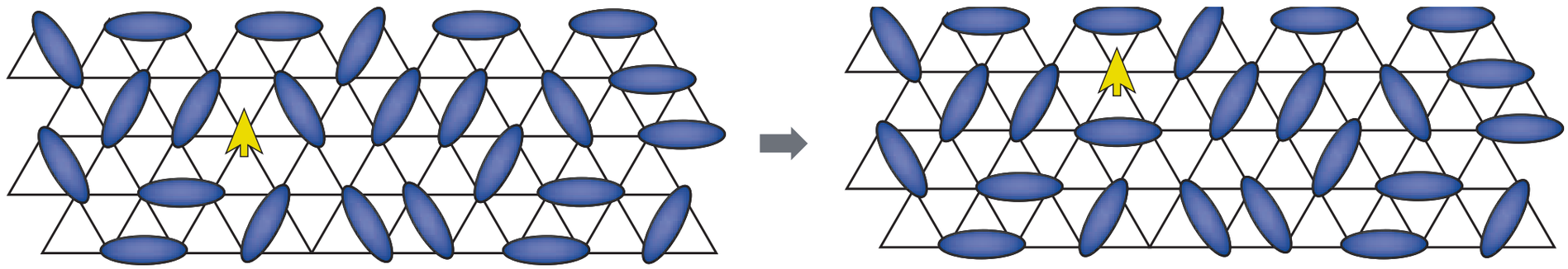
Spin liquids today



whether these excitations are localized or itinerant. Elastic and inelastic neutron-scattering measurements, especially on single crystals, provide crucial information on the nature of correlations and excitations, and these could perhaps uncover spinons. All told, this is a powerful arsenal of experimental tools, but the task is extremely challenging. At the heart of the problem is that there is no single experimental feature that identifies a spin-liquid state. As long as a spin liquid is characterized by what it is not — a symmetry-broken state with conventional order — it will be much more difficult to identify conclusively in experiments.

L. Balents, Nature **464**, 199 (2010).

Spin liquids today



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L. Balents, Nature **464**, 199 (2010).

Lattice gauge theories

Lattice gauge theories were studied in the 1970s by particle theorists as toy models of confinement of particles with fractional quantum numbers (quarks).

They were adopted in condensed matter physics for theory of quantum spin liquids. They are toy models of de-confinement of quasiparticles with fractional quantum numbers (spinons).

J. Kogut and L. Susskind, Phys. Rev. D **11**, 395 (1975).
E. Fradkin and L. Susskind, Phys. Rev. D **17**, 2637 (1978).
A.M. Polyakov, *Gauge Fields and Strings* (CRC, 1987).
R. Moessner, S.L. Sondhi, and E. Fradkin, Phys. Rev. B **65**, 024504 (2001).
E. Fradkin, *Field Theories of Condensed Matter Physics* (Cambridge, 2013).

U(1) gauge theory

Bears close resemblance to electromagnetism (E&M), making it easier to build the theory by analogy with a familiar subject.

Relevant to some quantum spin models: quantum spin ice on the pyrochlore lattice and Heisenberg model on the kagome lattice.

M. Hermele, M.P.A. Fisher, and L. Balents, Phys. Rev. B **69**, 064404 (2004).
L. Savary and L. Balents, Phys. Rev. Lett. **108**, 037202 (2012).
S.B. Lee, S. Onoda, and L. Balents, Phys. Rev. B **86**, 104412 (2012).
Z.H. Hao and O. Tchernyshyov, Phys. Rev. B **87**, 214404 (2013).
Z.H. Hao, A.G.R. Day, and M.J.P. Gingras, Phys. Rev. B **90**, 214430 (2014).

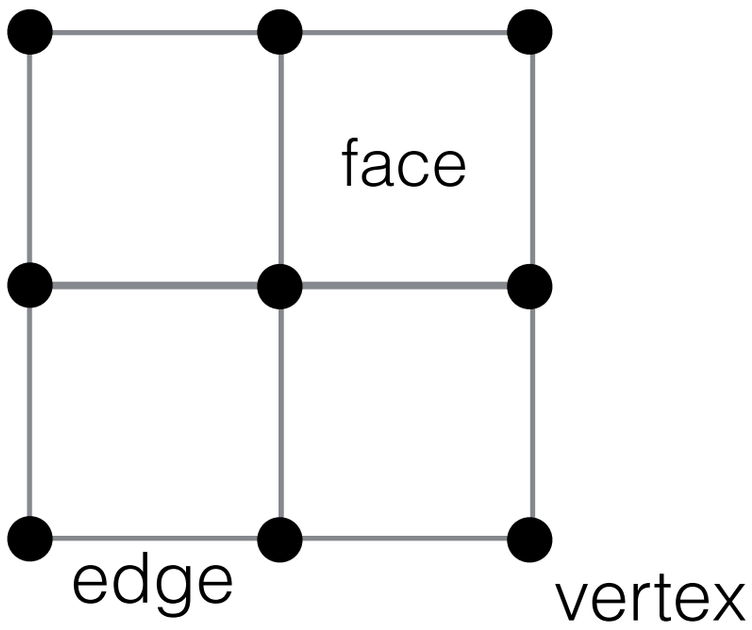
Z_2 lattice gauge theory

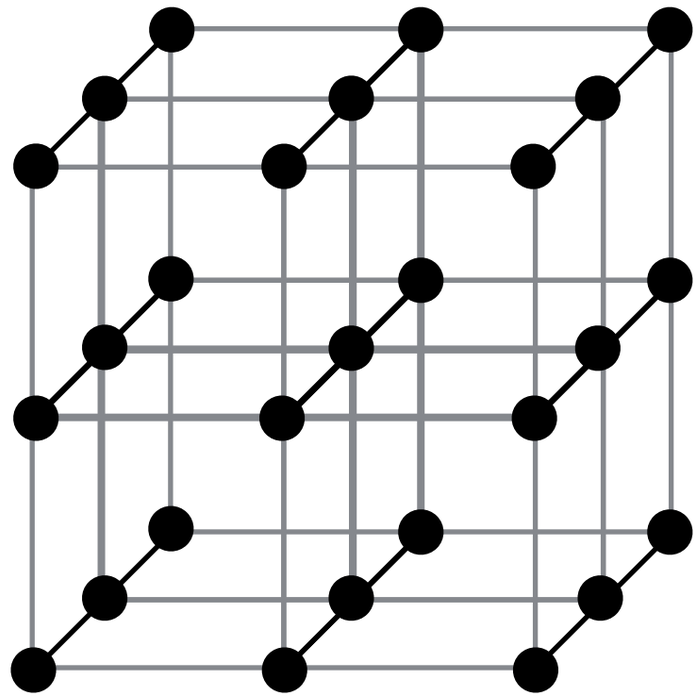
(tomorrow)

The simplest gauge theory ever: uses binary arithmetics!

Relevant to some quantum spin models: Heisenberg model on the square and kagome lattices.

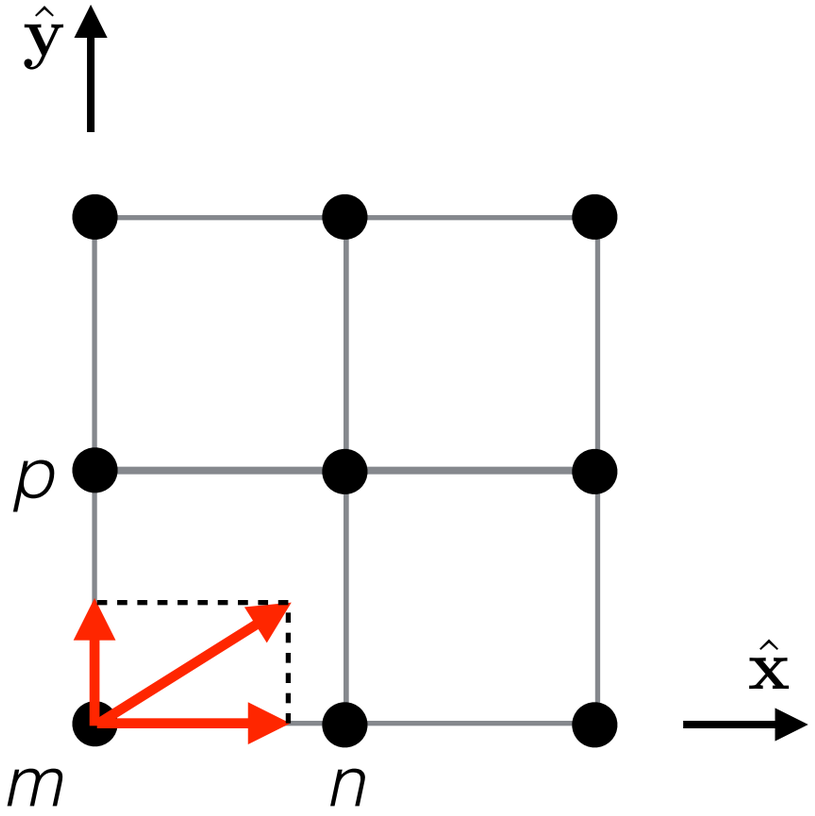
G. Misguich, D. Serban, and V. Pasquier, Phys. Rev. Lett. **89**, 137202 (2002).
H.C. Jiang, H. Yao, and L. Balents, Phys. Rev. B **86**, 024424 (2012).
Y. Wan and O. Tchernyshyov, Phys. Rev. B **87**, 104408 (2013).
H.J. Ju and L. Balents, Phys. Rev. B **87**, 195109 (2013).





For simplicity I will work with hypercubic lattices (square in d=2, cubic in d=3). In principle, gauge theory can be defined on any lattice, regular or not.

In quantum spin ice, the gauge field lives on a diamond lattice.



$$A_{mn} \approx \mathbf{A} \cdot \hat{\mathbf{x}}$$

$$A_{mp} \approx \mathbf{A} \cdot \hat{\mathbf{y}}$$

$$A_{mn} = -A_{nm}$$

In E&M, the gauge potential **A**(**r**) is a vector field.

Lattice gauge variables A_{mn} live on edges of the lattice and are labeled by the two vertices of the edge (here *mn*).

They can be thought of as the projection of the vector gauge field onto the link direction.

Regular U(1) gauge theory

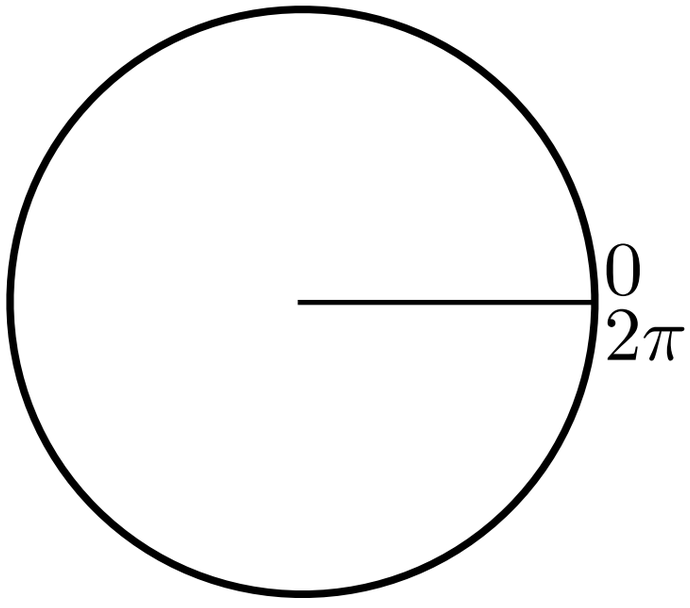
$$-\infty < A < \infty$$

$-\infty$ ∞

In a regular U(1) gauge theory, gauge variable A_{mn} takes on values on a straight line.

No restrictions on possible values of electric charge.

Compact U(1) gauge theory

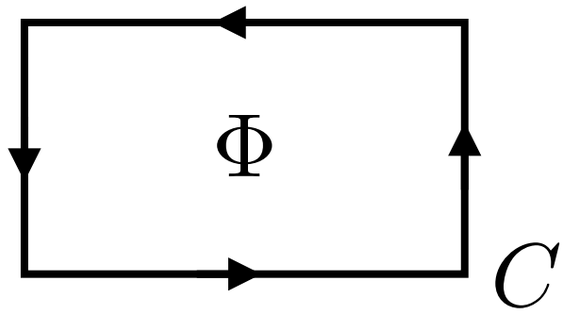


$$0 \leq A \leq 2\pi$$

$$U(A + 2\pi) = U(A)$$

In a compact U(1) gauge theory, gauge variable A_{mn} takes on values on a circle (a compact manifold). In other words, it is an angular variable.

Any physical variable (e.g., potential energy *U*) must be a periodic function of *A*. Yields quantized values of charge.

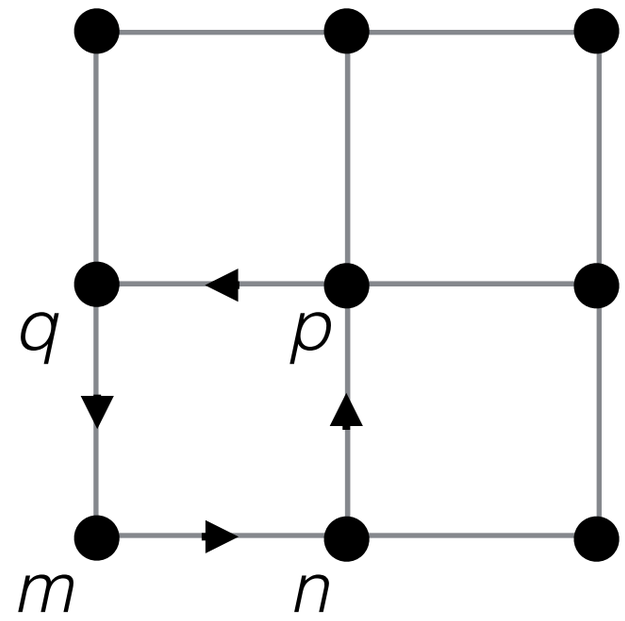


$$\mathbf{E} = -\dot{\mathbf{A}}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\Phi_C = \oint_C \mathbf{A} \cdot d\mathbf{r}$$

In E&M, the gauge field **A** is not a physical variable. Physical fields **E** and **B** and magnetic flux Φ can be obtained from it.



$$E_{mn} = -\dot{A}_{mn}$$

$$\Phi_{mnpq} = A_{mn} + A_{np} + A_{pq} + A_{qn}$$

Similar recipes work on a lattice for electric flux *E* and magnetic flux Φ.

Next steps

- Define kinetic energy $U(A)$ and potential energy $T(\dot{A})$.
- Construct the Lagrangian $L(A,\dot{A}) = T(\dot{A}) - U(A)$.
- Deduce momenta: $p = \partial L/\partial \dot{A} = \partial T/\partial \dot{A}$.
- Quantize the theory: $[p,A] = -i\hbar$.
- Construct the Hamiltonian: $H(A,p) = p\dot{A} - L$.
- Identify conserved quantities.
- Determine the quantum ground state.

Kinetic energy and canonical momentum

E&M:

$$T = \int d^3r \, \frac{\epsilon_0 \dot{A}^2}{2} = \int d^3r \, \frac{\epsilon_0 E^2}{2}$$

ε_0 is “vacuum permittivity.”

Conjugate momentum:

$$\boldsymbol{\pi}(\mathbf{r}) = \frac{\delta T}{\delta \dot{\mathbf{A}}(\mathbf{r})} = \epsilon_0 \dot{\mathbf{A}}(\mathbf{r}) = -\epsilon_0 \mathbf{E}(\mathbf{r})$$

Lattice gauge theory:

$$T = \sum_{\text{edges}} \frac{I \dot{A}_{mn}^2}{2} = \sum_{\text{edges}} \frac{p_{mn}^2}{2I}$$

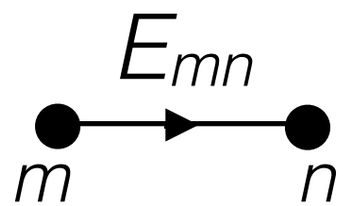
I is “moment of inertia” of *A*.

Conjugate momentum:

$$p_{mn} = \frac{\partial T}{\partial \dot{A}_{mn}} = I \dot{A}_{mn} = -E_{mn}$$

$E_{mn} = -I \dot{A}_{mn}$ for convenience

Quantization



$$p = \frac{\partial T}{\partial \dot{A}} = I\dot{A} = -E \qquad [p, A] = -i \qquad [E, A] = i$$

Quantum state of A is given by the wavefunction $\psi(A)$.

E operator acts on it thus: $E\psi(A) = i\frac{d}{dA}\psi(A)$

Take periodic wavefunctions: $\psi(A + 2\pi) = \psi(A)$

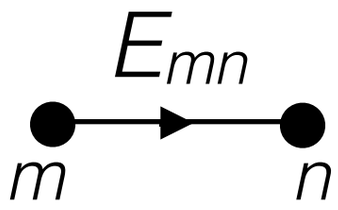
Convenient basis: $\psi_m(A) = e^{-imA}/\sqrt{2\pi},\ m = 0, \pm1, \pm2, \ldots$

$$E\psi_m(A) = m\psi_m(A)$$

Electric field is quantized: $E = 0, \pm1, \pm2, \ldots$

We use units such that $\hbar = 1$.

Quantization



$$p = \frac{\partial T}{\partial \dot{A}} = I\dot{A} = -E \qquad [p, A] = -i \qquad [E, A] = i$$

$$\psi_m(A) = e^{-imA}/\sqrt{2\pi}$$

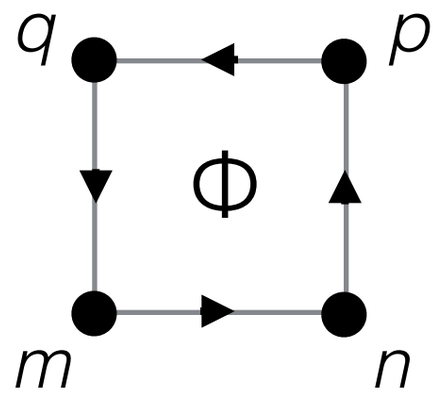
$$E\psi_m(A) = m\psi_m(A)$$

$$e^{\pm iA}\psi_m(A) = \psi_{m\mp 1}(A)$$

$$e^{\pm iA}Ee^{\mp iA} = E \pm 1$$

e^{+iA} and e^{-iA} are lowering and raising operators for E.

Potential energy



E&M:

$$U = \int d^3r \, \frac{B^2}{2\mu_0} = \int d^3r \, \frac{(\nabla \times \mathbf{A})^2}{2\mu_0}$$

Lattice gauge theory:

U must be 2*π*-periodic in *A* and depend on fluxes

$$\Phi_{mnpq} = A_{mn} + A_{np} + A_{pq} + A_{qn}$$

E.g., $U = -\sum_{\text{faces}} \lambda \cos \Phi_{mnpq}$

If Φ values are small,

$$U = \text{const} + \sum_{\text{faces}} \frac{\lambda \Phi_{mnpq}^2}{2}$$

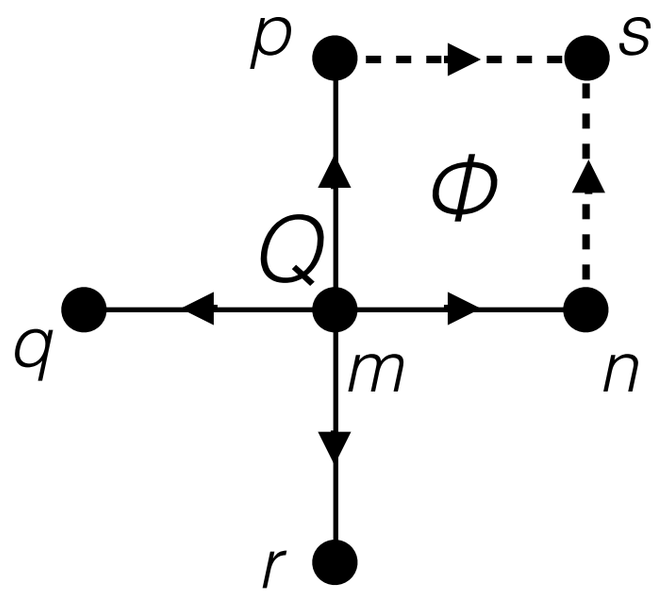
Compare

Conserved charges

$$H = \sum_{\text{edges}} \frac{E_{mn}^2}{2I} - \sum_{\text{faces}} \lambda \cos \Phi_{mnpq}, \quad [E_{mn}, A_{mn}] = i.$$

Electric charge = net electric flux:

$$Q_m = E_{mn} + E_{mp} + E_{mq} + E_{mr}$$



Electric charges are quantized:

$$Q_m = 0, \pm 1, \pm 2 \ldots$$

Electric charges are constants of motion:

$$[Q_m, H] = 0$$

States split into different charge sectors.

Properties of ground states

$$H = \sum_{\text{edges}} \frac{E_{mn}^2}{2I} - \sum_{\text{faces}} \lambda \cos \Phi_{mnpq}$$

The ground state depends on the product I/λ.

We will explore the nature of the ground state(s) in the two limits: $I/\lambda \ll 1$ and $I/\lambda \gg 1$.

We will work in the vacuum sector (no charges) and in the sector with two probe charges $+Q$ and $-Q$.

We will see that electric charges are confined in one limit but not in the other.

Ground state for $\lambda \ll 1/I$

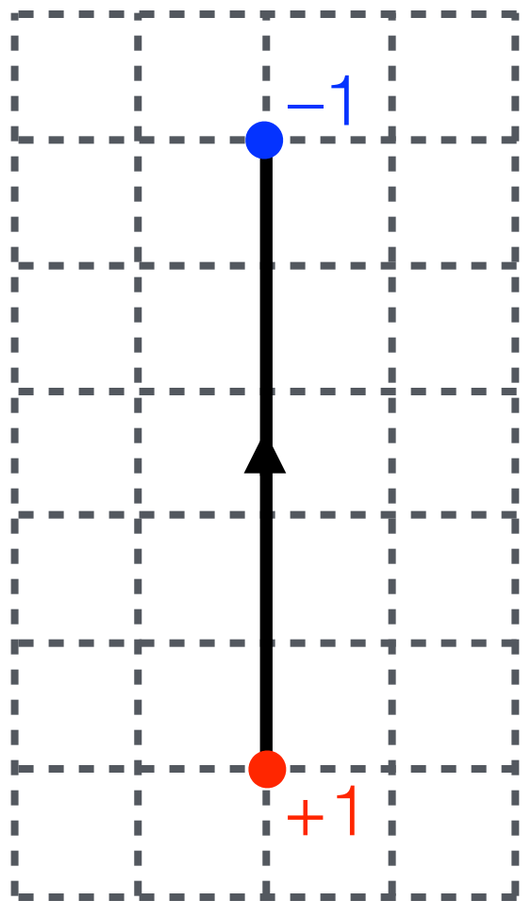
$$H = H_0 + H_1, \quad H_0 = \sum_{\text{edges}} \frac{E_{mn}^2}{2I}, \quad H_1 = -\sum_{\text{faces}} \lambda \cos \Phi_{mnpq}$$

Neglect the weak magnetic term.

No-charge sector: $E_{mn} = 0$ everywhere.

Sector with two minimal charges $Q = \pm 1$: ground state with a minimal electric flux line connecting the charges.

Energy grows linearly with the distance. Electric charges are confined.



$$\epsilon(\ell) = \ell/2I$$

Ground state for $\lambda \ll 1/I$

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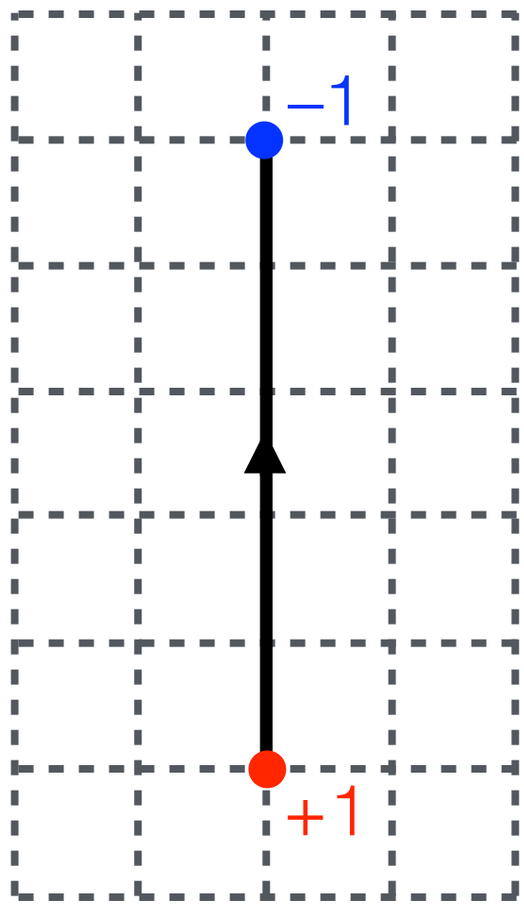
Treat the magnetic term as a perturbation.



It induces quantum fluctuations of the electric flux line connecting the charges.

Tension σ of the electric flux line is reduced by quantum fluctuations.

$$\sigma = \frac{1}{2I} - CI\lambda^2 + \ldots, C > 0$$

$$\epsilon(\ell) = \sigma\ell$$

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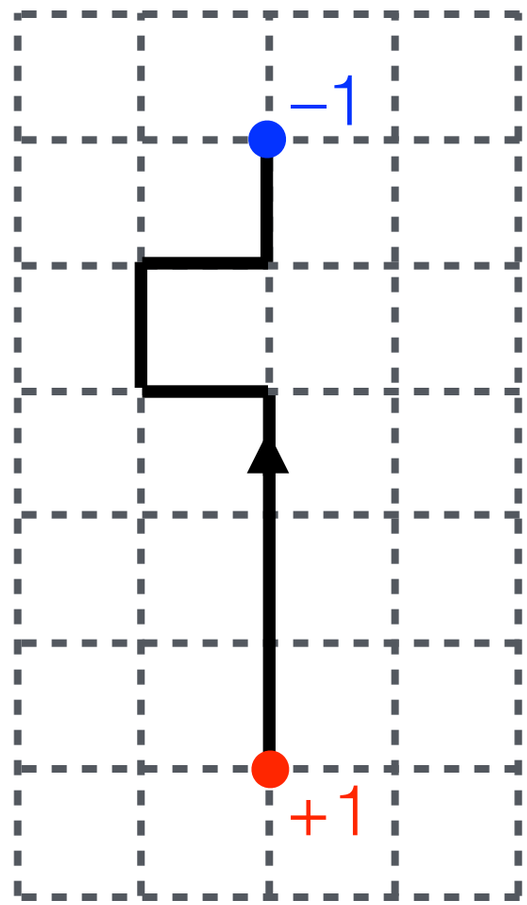

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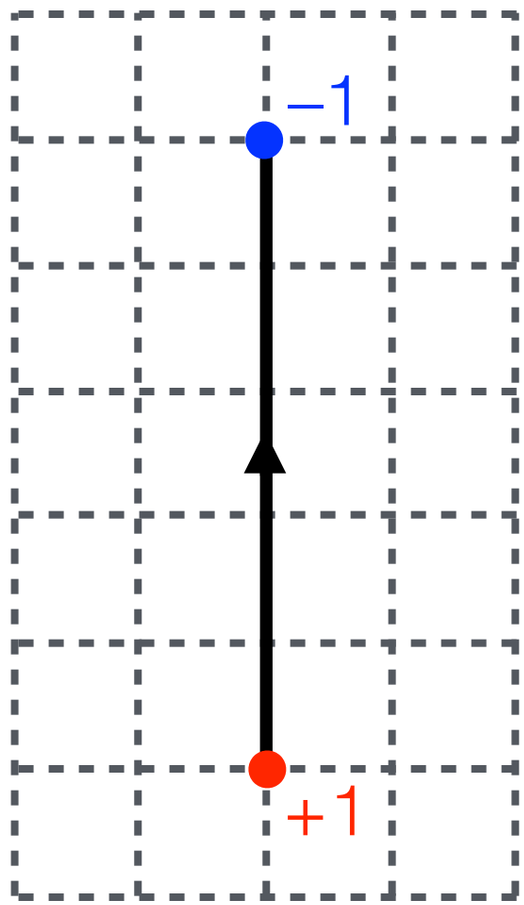

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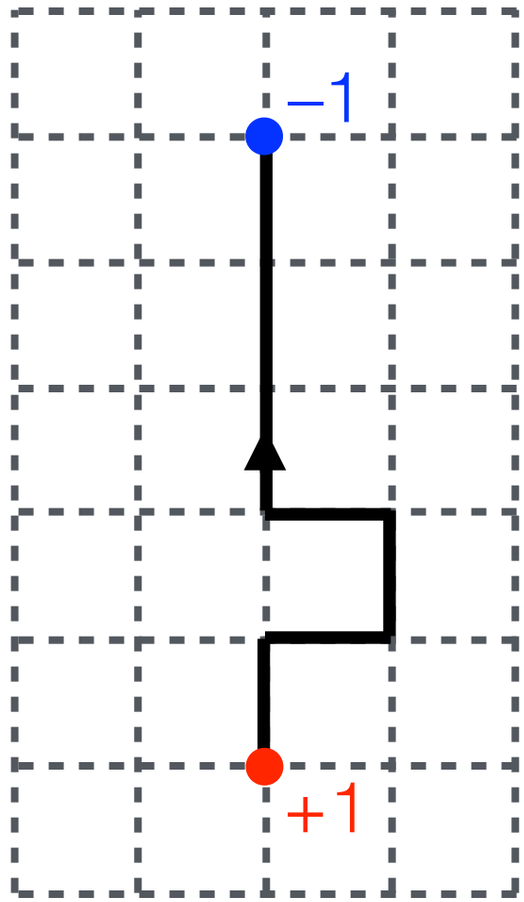

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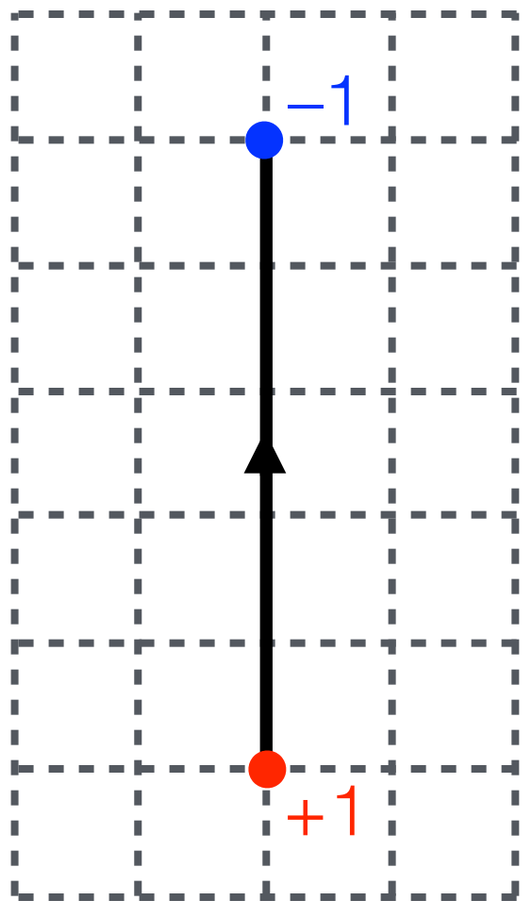

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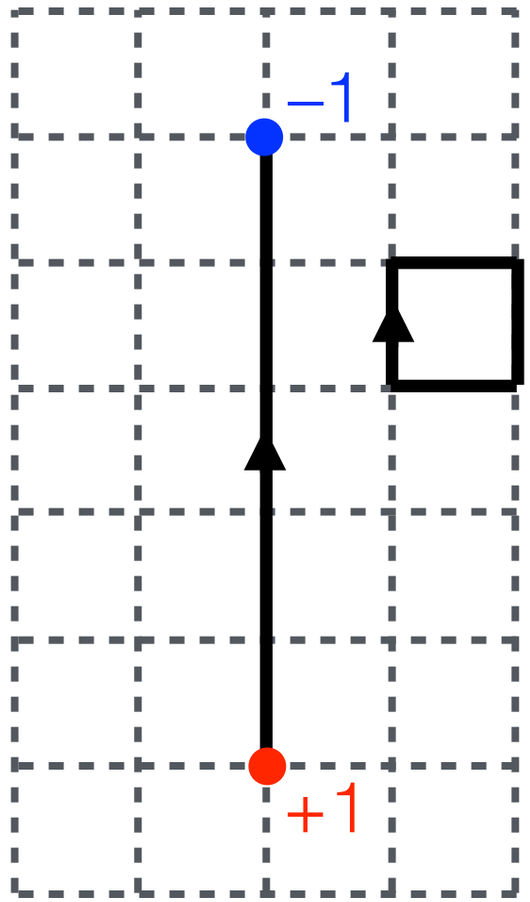

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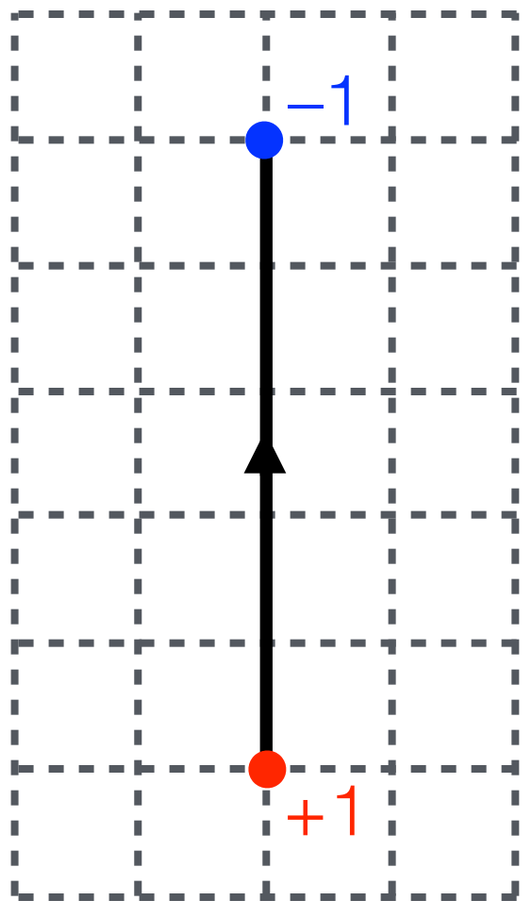

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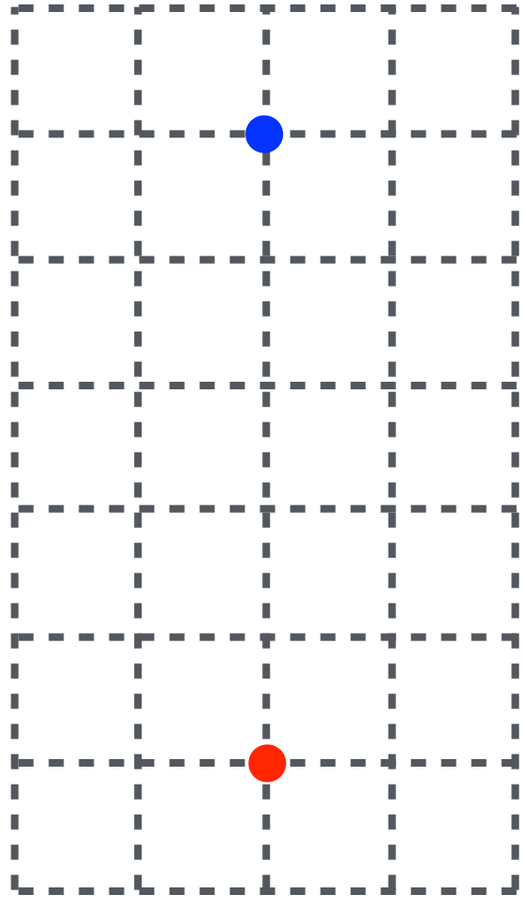
Ground state for $\lambda \gg 1/I$

$$H = H_0 + H_1, \quad H_0 = -\sum_{\text{faces}} \lambda \cos \Phi_{mnpq}, \quad H_1 = \sum_{\text{edges}} \frac{E_{mn}^2}{2I}$$

Neglect the weak electric term. The magnetic term is minimized if all $\Phi = 0$.

This condition is independent of the charge sector (Φ and Q commute).

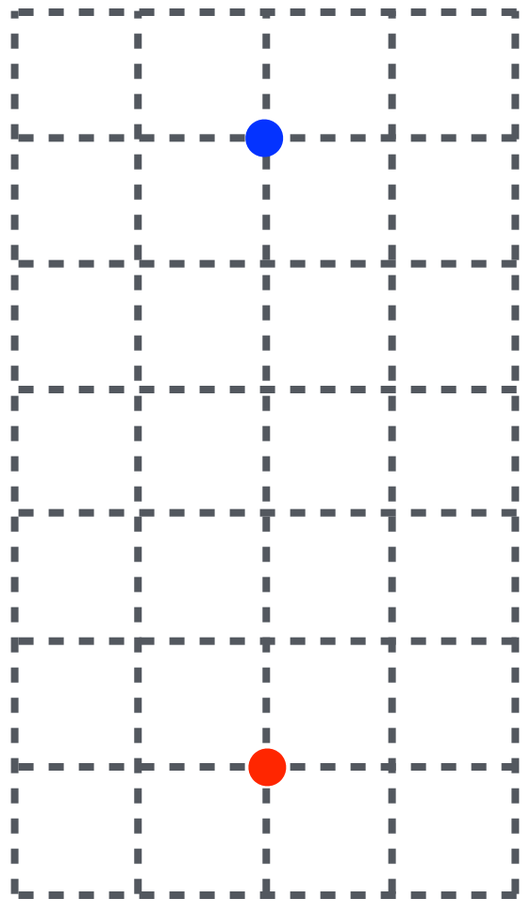
Energy of two charges does not depend on the distance between them.
Electric charges are not confined.



Ground state for $\lambda \gg 1/I$

$$H = H_0 + H_1, \quad H_0 = -\sum_{\text{faces}} \lambda \cos \Phi_{mnpq}, \quad H_1 = \sum_{\text{edges}} \frac{E_{mn}^2}{2I}$$

Treat the electric term as a perturbation inducing small fluctuations of Φ around 0.

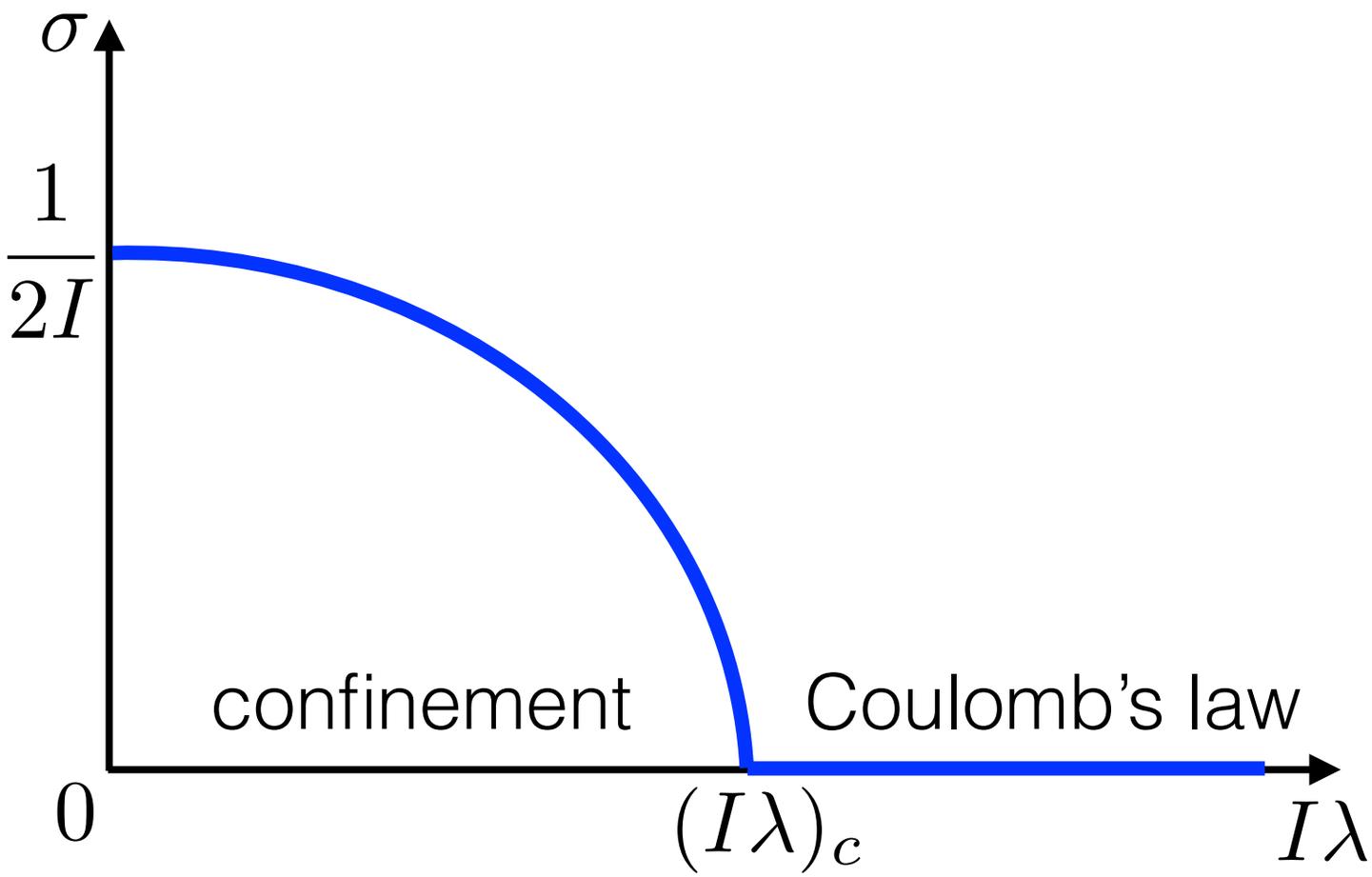


$$H \approx \sum_{\text{edges}} \frac{E_{mn}^2}{2I} + \sum_{\text{faces}} \frac{\lambda \Phi_{mnpq}^2}{2} + \text{const}$$

This yields regular (non-compact) E&M with a speed of light $c = \sqrt{\lambda / I}$.

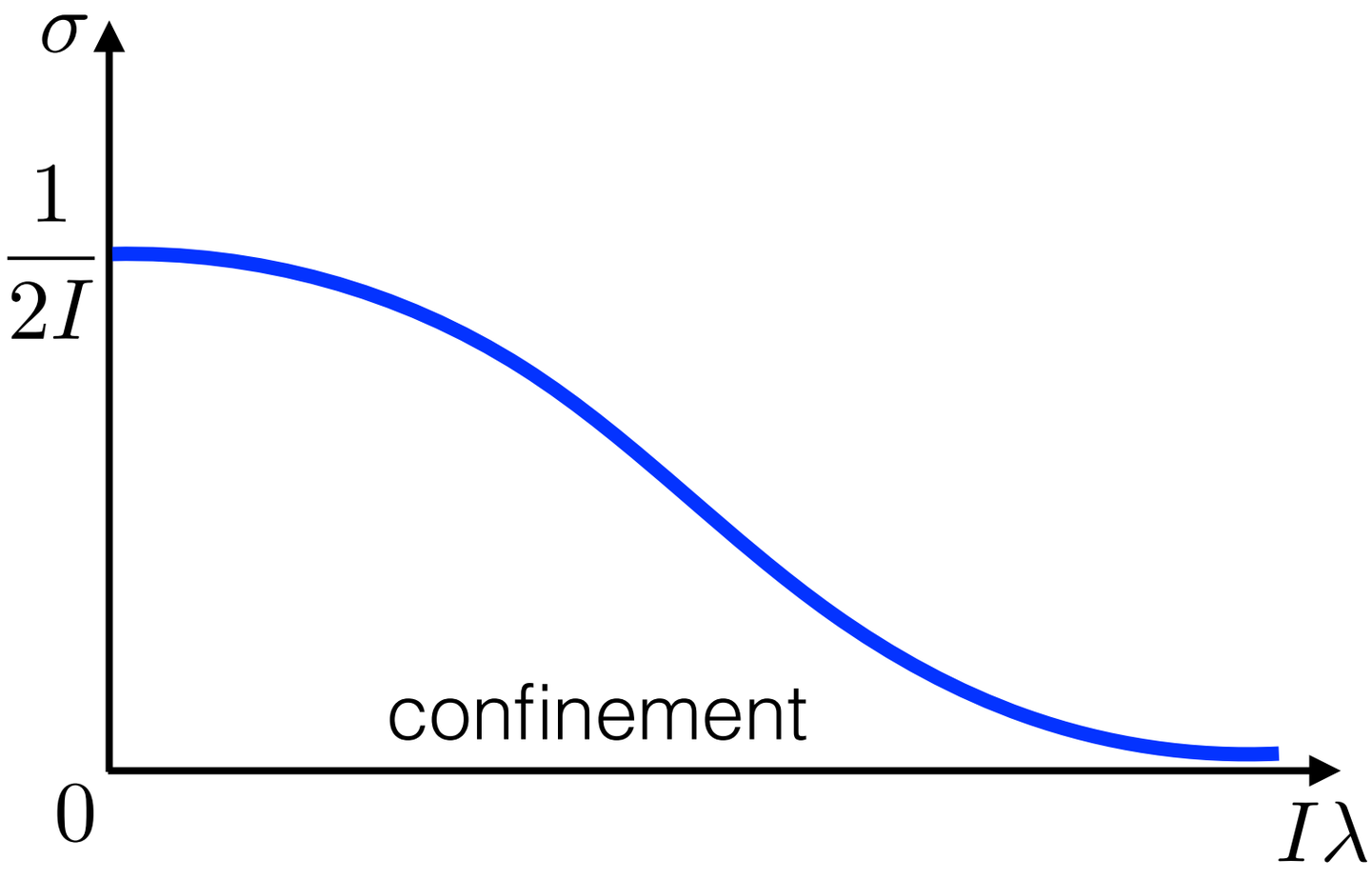
Coulomb's law, $\epsilon(\ell) = -\dfrac{Q^2}{2I\ell}$, no confinement!

String tension in d=3



Two distinct phases of matter: confined and deconfined. String tension can be used as an order parameter whose presence or absence determines which phase we are in.

String tension in d=2



One confining phase of matter for all couplings $I\lambda < \infty$.

Quantization revisited

We assumed that $\psi(A + 2\pi) = \psi(A)$

because physical variables are 2π-periodic in A.

$$\psi_m(A) = e^{-imA}/\sqrt{2\pi},$$

$$m = 0, \pm 1, \pm 2, \ldots$$

$$E\psi_m(A) = m\psi_m(A)$$

But physical quantities are bilinear in ψ and ψ^*.
Hence a more relaxed boundary condition:

$$\psi(A + 2\pi) = e^{i\theta}\psi(A), \quad \psi^*(A + 2\pi) = e^{-i\theta}\psi^*(A)$$

$$\theta = \text{const}$$

Quantization revisited

Antiperiodic boundary conditions:

$$\psi(A + 2\pi) = -\psi(A)$$

$$\psi_\nu(A) = e^{-i\nu A}/\sqrt{2\pi}$$

$$E\,\psi_\nu(A) = \nu\,\psi_\nu(A)$$

$$\nu = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots$$

Electric field is quantized to half-integer values.

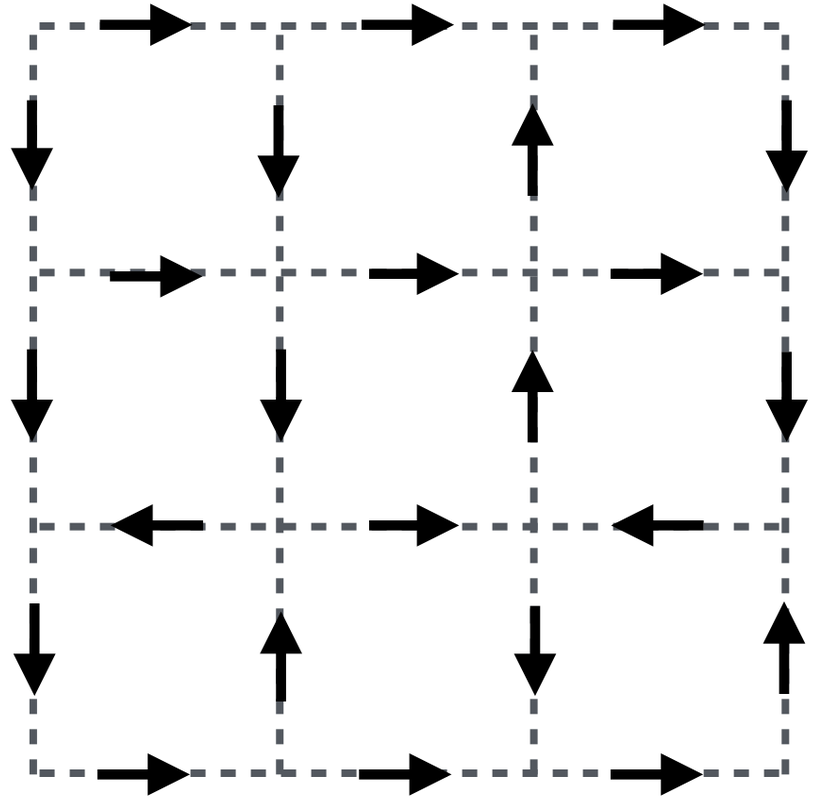
Ground state for $\lambda \ll 1/I$

$$H = H_0 + H_1, \quad H_0 = \sum_{\text{edges}} \frac{E_{mn}^2}{2I}, \quad H_1 = -\sum_{\text{faces}} \lambda \cos \Phi_{mnpq}$$

Neglect the weak magnetic term.

No-charge sector:
Q_m=0 everywhere.
E_{mn} = ±1/2 everywhere.

Classical spin ice states!



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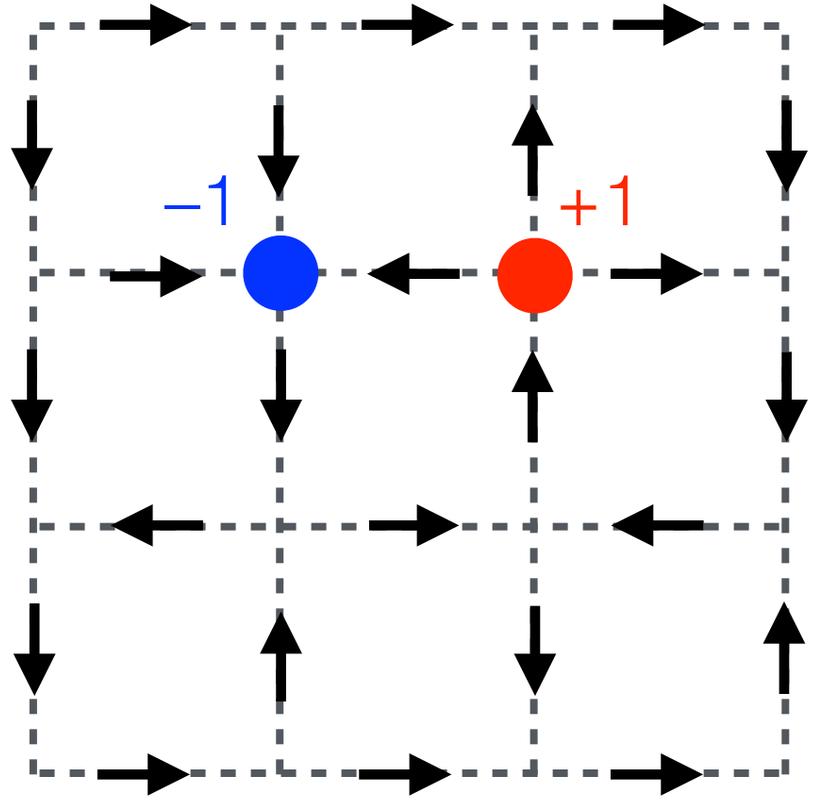
Neglect the weak magnetic term.

Sector with two probe charges:
Q_m=0, except for two $Q_m = \pm 1$.
$E_{mn} = \pm 1/2$ everywhere.

“Magnetic monopoles” of spin ice become unit electric charges.



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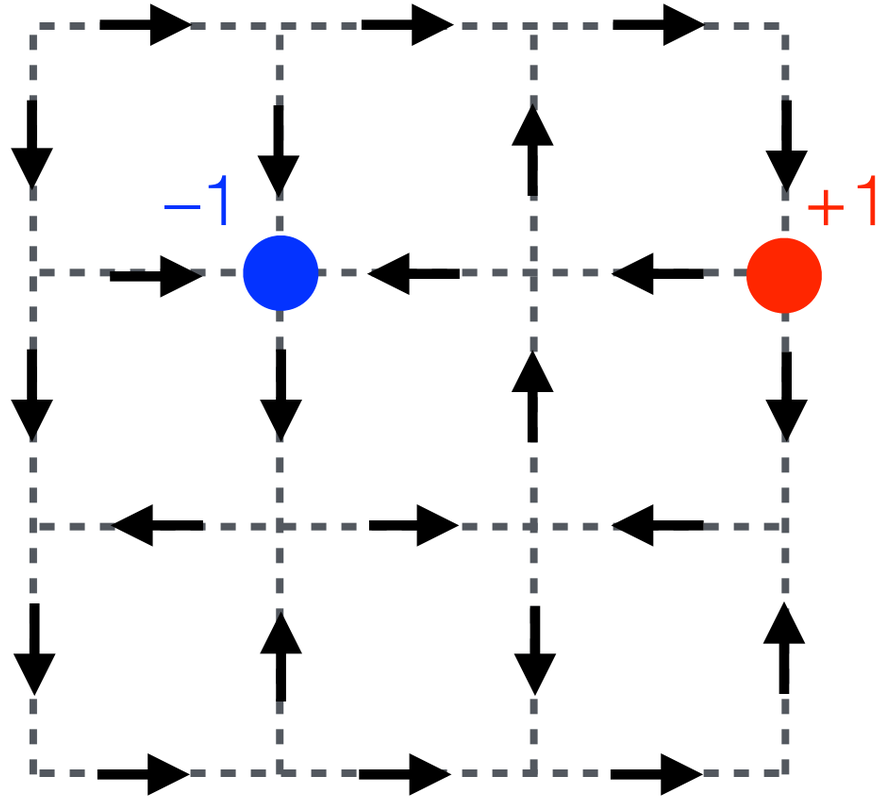

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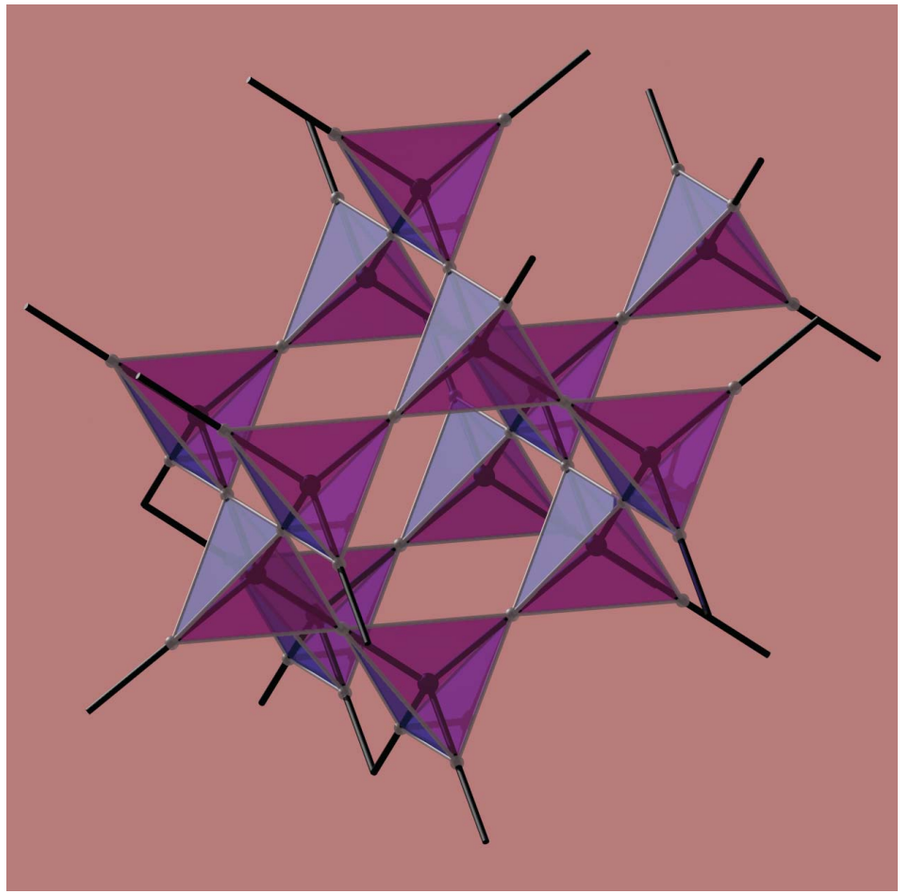
This construction also works on the pyrochlore lattice. The gauge field A lives on edges of the diamond lattice whose vertices are centers of tetrahedra.



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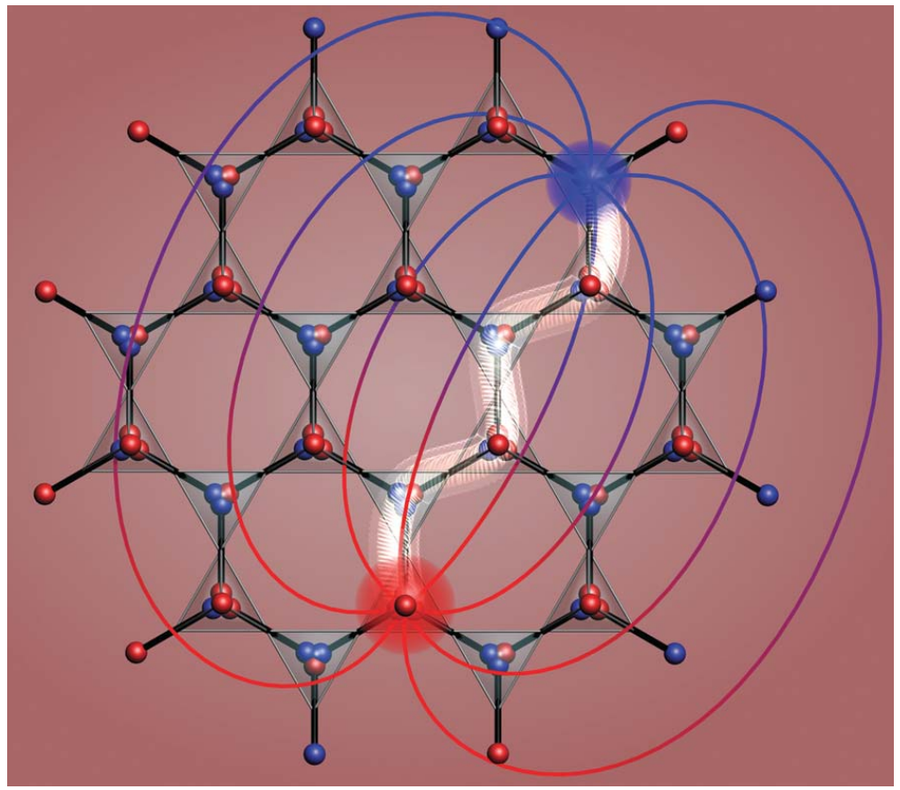

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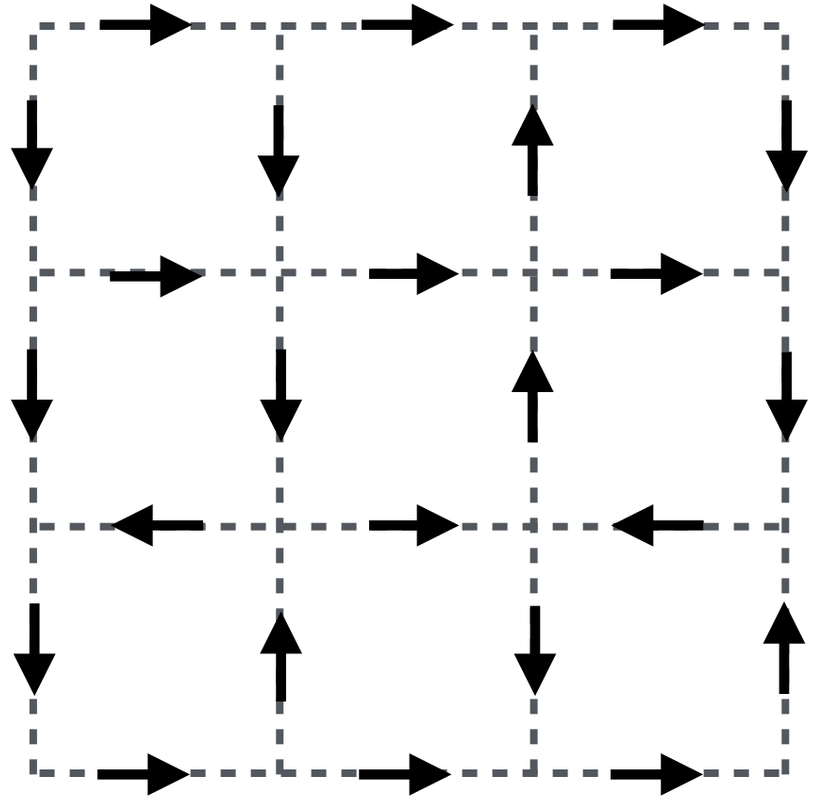
Treat the weak magnetic term as a perturbation.

Operators $e^{\pm i\Phi}$ increment E on all edges around a face by ±1.

Quantum spin ice.

L. Savary and L. Balents (2012).
S.B. Lee, S. Onoda, and L. Balents (2012).



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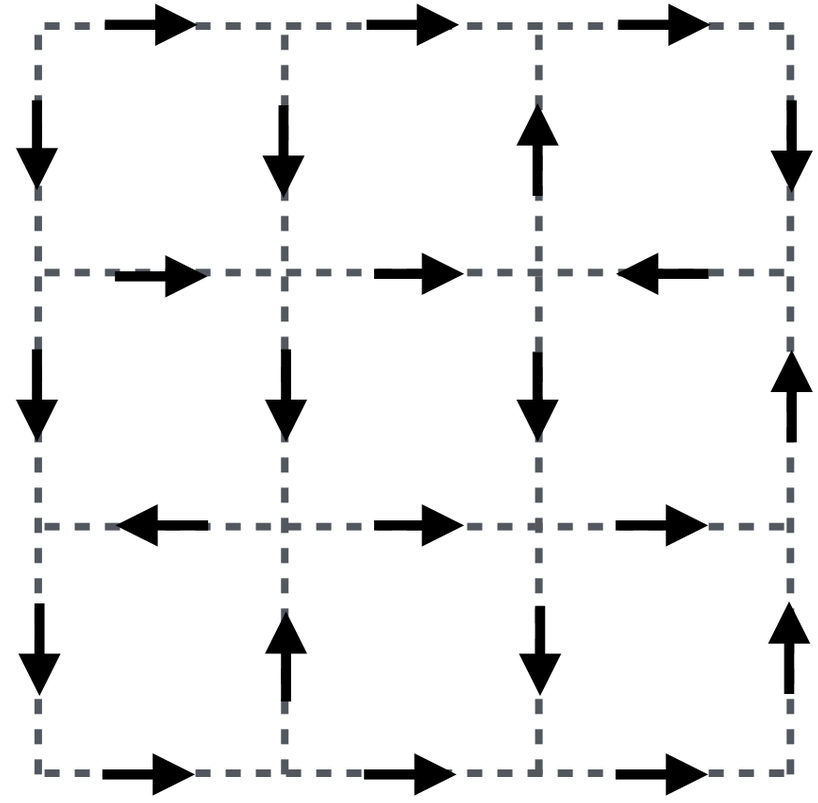

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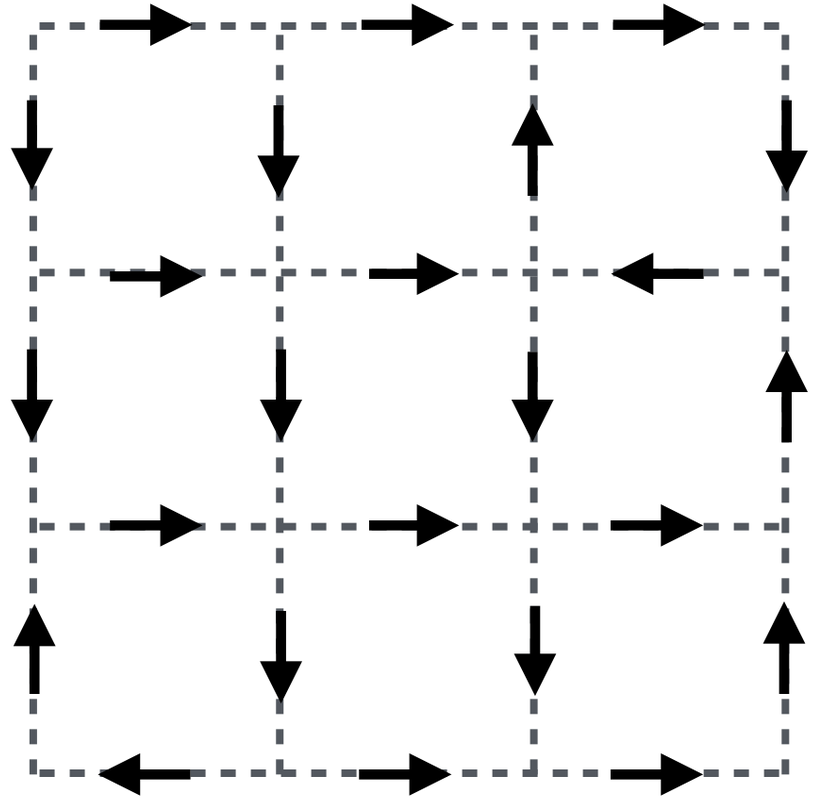

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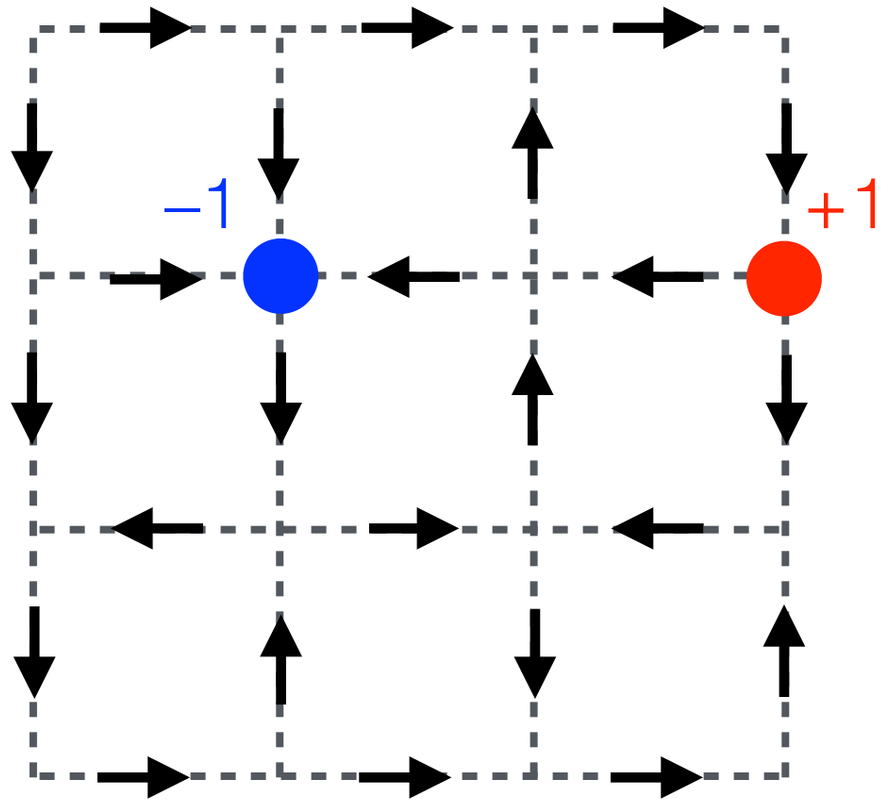
“Magnetic monopoles” become mobile.

Need to add matter particles:

$$H_2 = -t \sum_{\text{edges}} b_m^\dagger e^{iA_{mn}} b_n + \text{H.c.}$$



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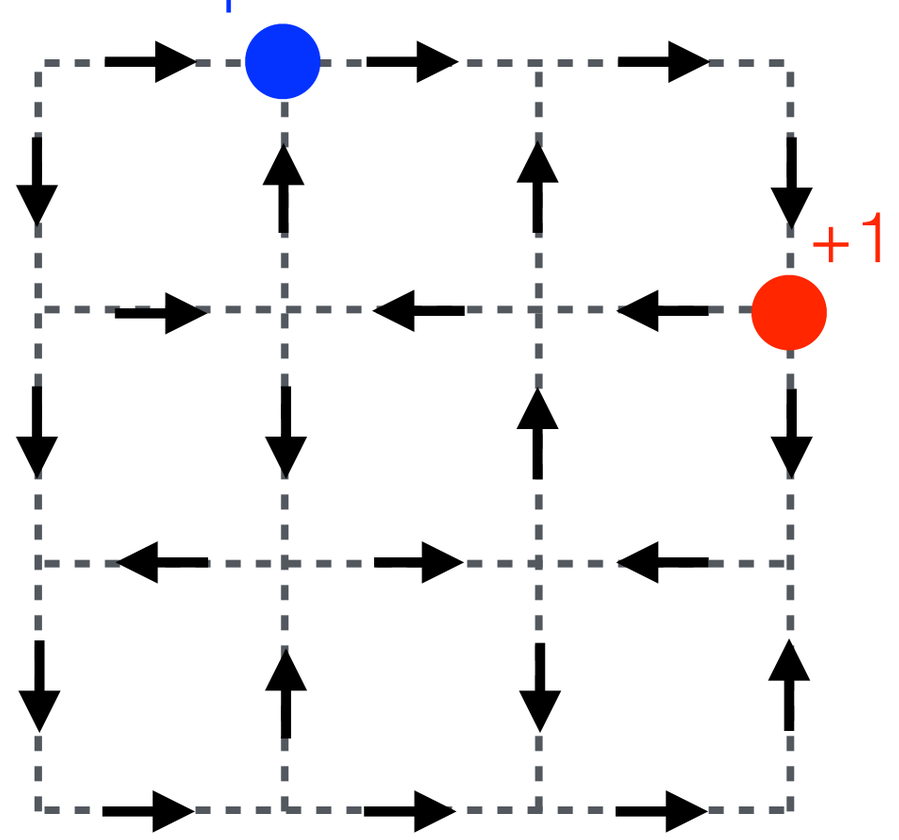


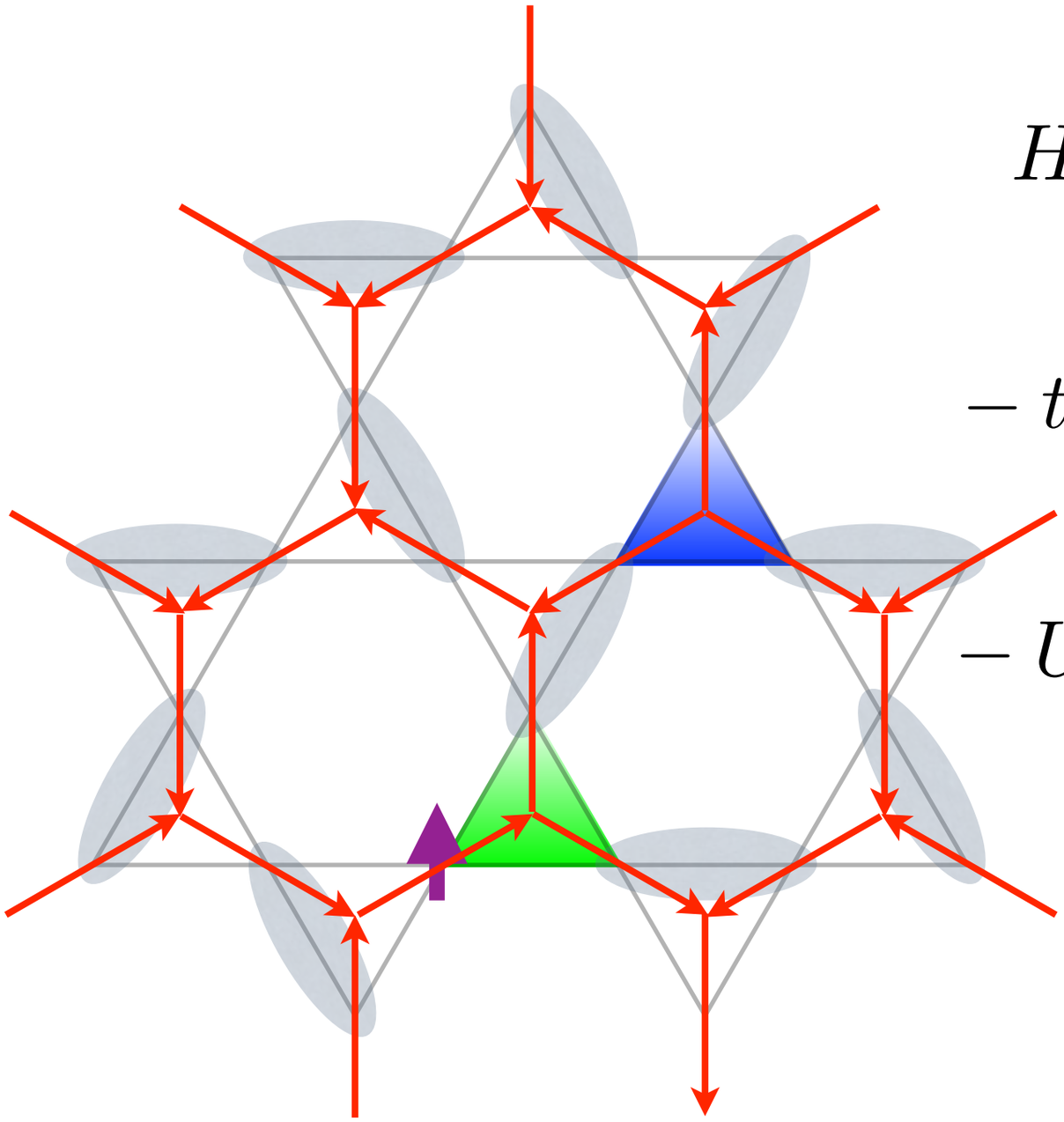
U(1) gauge theory on kagome

$E = \pm 1/2$

$Q = -1/2$

$Q = +1$

$Q = +2$



$$H = \sum_{\langle ij \rangle} \frac{E_{ij}^2}{2\epsilon}$$

$$- t \sum_{\langle ij \rangle} a_{i\sigma}^\dagger e^{iA_{ij}} a_{j\sigma}$$

$$- U \sum_i a_{i\uparrow}^\dagger a_{i\downarrow}^\dagger a_{i\downarrow} a_{i\uparrow}.$$

$$t = J/2$$

$$U = 3J/4$$

$$\epsilon = \infty$$

Z.H. Hao and O. Tchernyshyov, Phys. Rev. B **87**, 214404 (2013).

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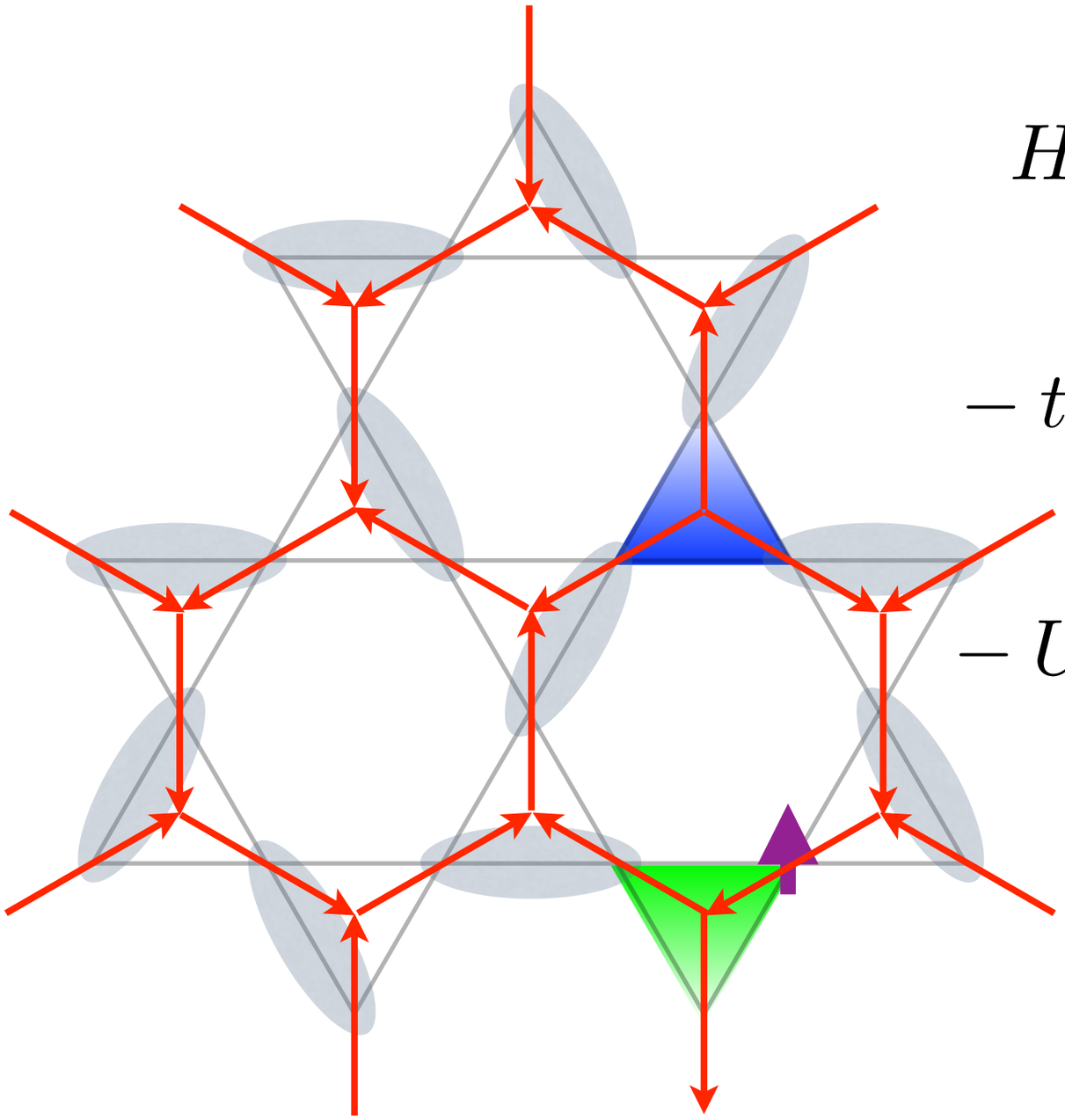

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