Counterflow Ordering

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Winter School on New Frontiers in Superconductivity

The National High Magnetic Field Lab, Tallahassee, January 8 - 12, 2024

Motivating experiment

Nature Physics 17, 1254 (2021)

$Ba_{1-x}K_xFe_2As_2$

ARTICLES
https://doi.org/10.1038/s41567-021-01350-9

nature physics

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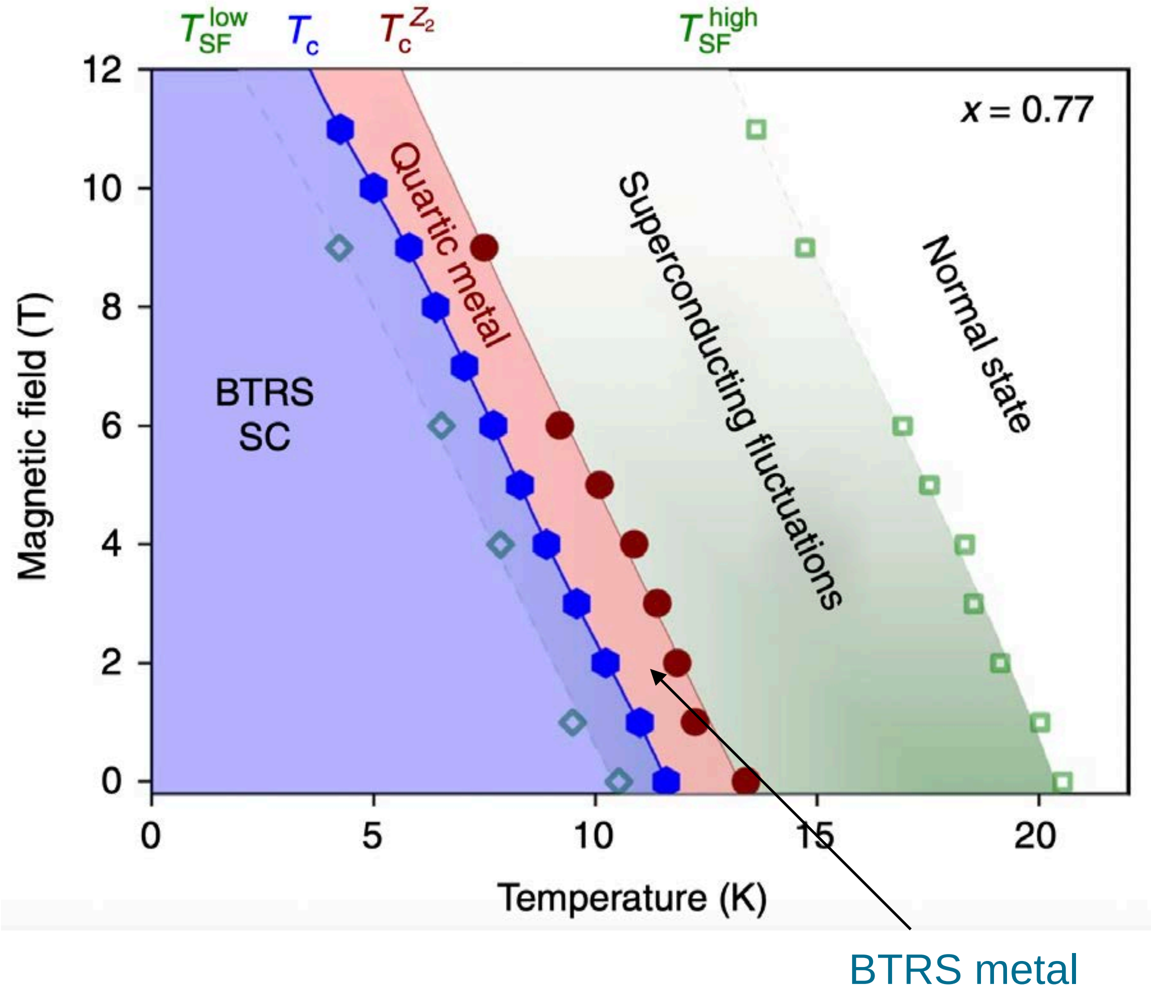
State with spontaneously broken time-reversal symmetry above the superconducting phase transition

Vadim Grinenko[1,2,3] ✉, Daniel Weston[4], Federico Caglieris[2], Christoph Wuttke[2], Christian Hess[2,5], Tino Gottschall[6], Ilaria Maccari[4], Denis Gorbunov[6], Sergei Zherlitsyn[6], Jochen Wosnitza[1,6], Andreas Rydh[7], Kunihiro Kihou[8], Chul-Ho Lee[8], Rajib Sarkar[1], Shanu Dengre[1], Julien Garaud[9], Aliaksei Charnukha[2], Ruben Hühne[2], Kornelius Nielsch[2], Bernd Büchner[1,2], Hans-Henning Klauss[1] and Egor Babaev[4] ✉

The most well-known example of an ordered quantum state—superconductivity—is caused by the formation and condensation of pairs of electrons. Fundamentally, what distinguishes a superconducting state from a normal state is a spontaneously broken symmetry corresponding to the long-range coherence of pairs of electrons, leading to zero resistivity and diamagnetism. Here we report a set of experimental observations in hole-doped $Ba_{1-x}K_xFe_2As_2$. Our specific-heat measurements indicate the formation of fermionic bound states when the temperature is lowered from the normal state. However, when the doping level is $x \approx 0.8$, instead of the characteristic onset of diamagnetic screening and zero resistance expected below the superconducting phase transition, we observe the opposite effect: the generation of self-induced magnetic fields in the resistive state, measured by spontaneous Nernst effect and muon spin rotation experiments. This combined evidence indicates the existence of a bosonic metal state in which Cooper pairs of electrons lack coherence, but the system spontaneously breaks time-reversal symmetry. The observations are consistent with the theory of a state with fermionic quadrupling, in which long-range order exists not between Cooper pairs but only between pairs of pairs.

V. Grinenko et al., Nature Physics 17, 1254 (2021)



BTRS = broken time-reversal symmetry

Counterflow ordering: Composite order parameters

Symmetry of the Hamiltonian: $[\mathrm{U}(1)]^N$

Order parameters: $\langle \psi_\alpha \rangle = 0\,, \qquad \langle \psi_\alpha \psi_\beta^* \rangle \neq 0\,, \qquad \alpha, \beta = 1, 2, 3, \ldots N\,.$

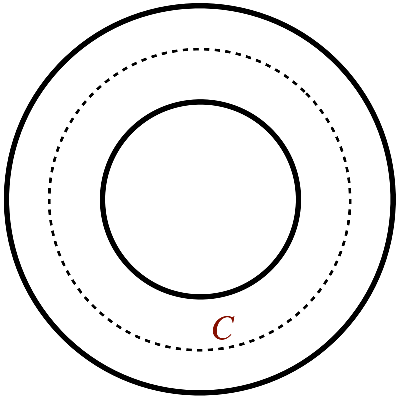
The fields themselves can be composite, but this is not relevant for our discussion.—Think of He-4 consisting of 6 fermions.

As opposed to individual phases, relative phases are ordered:

$$\psi_\alpha = |\psi_\alpha|\, e^{i\theta_\alpha}\,, \qquad \Phi_{\alpha\beta} = \theta_\alpha - \theta_\beta\,, \qquad \langle e^{i\Phi_{\alpha\beta}} \rangle \neq 0\,.$$

Counterflow supercurrents:

$$\oint_C d\mathbf{l} \cdot \nabla \Phi_{\alpha\beta} = 2\pi \times \text{integer}$$



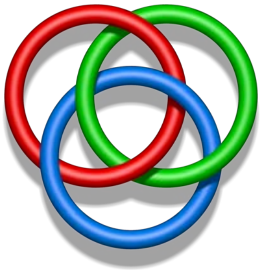
Two scenarios leading to supercounterfluidity

E. Babaev, arXiv:cond-mat/0201547

A. Kuklov and BS, PRL 90, 100401 (2003)

1. Finite-temperature proliferation of composite vortices (Babaev, 2002; we will discuss this scenario a bit later).

2. SCF ground state: super-exchange on top of the multi-component Mott state (Kuklov and BS, 2002):



PHYSICAL REVIEW LETTERS **127,** 255303 (2021)

Borromean Supercounterfluidity

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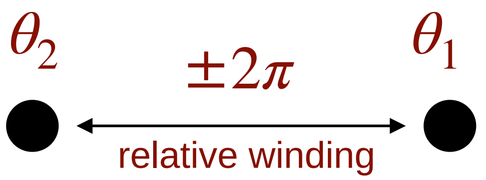
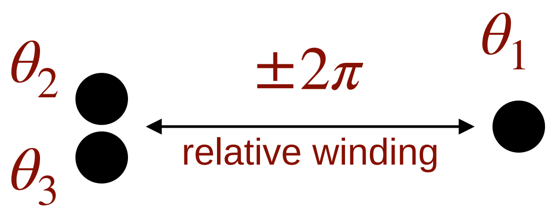
(Received 6 September 2021; accepted 12 November 2021; published 15 December 2021)

We demonstrate microscopically the existence of a new superfluid state of matter in a three-component Bose mixture trapped in an optical lattice. The superfluid transport involving coflow of all three components is arrested in that state, while counterflows between any pair of components are dissipationless. The presence of three components allows for three different types of counterflows with only two independent superfluid degrees of freedom.

$N \geq 3$ is very different from $N = 2$ in terms of topological excitations



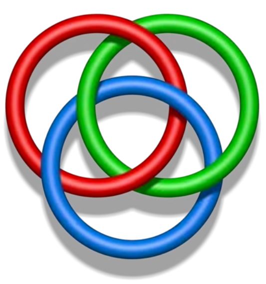
Only one type of elementary vortices.



N types of elementary vortices.

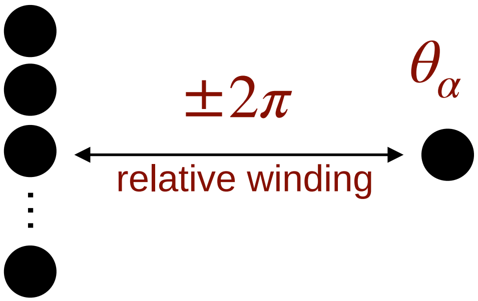
Borromean ordering: the number of superfluid modes is larger than the number of ordered phase variables.

Somehow all the N phases are relevant despite being not uniquely defined!



Borromean rings

Symmetric way of specifying N elementary vortices in Borromean SCF



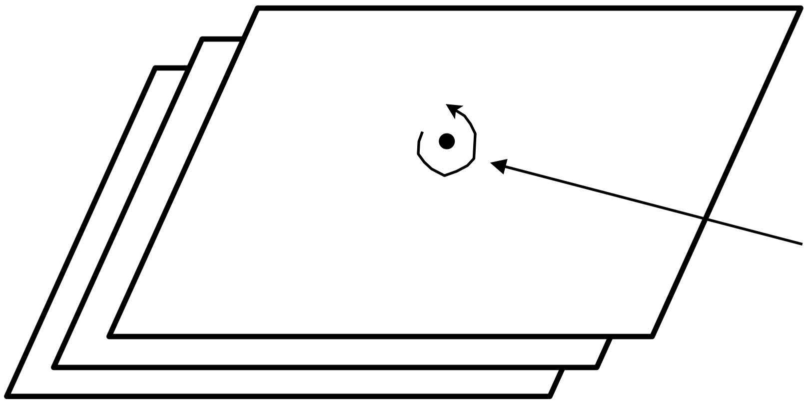
$$\forall \beta \neq \alpha : \quad \oint d\mathbf{l} \cdot \nabla \Phi_{\alpha\beta} = \pm 2\pi , \quad \underline{\forall \beta \neq \alpha, \forall \gamma \neq \alpha : \oint d\mathbf{l} \cdot \nabla \Phi_{\beta\gamma} = 0}$$

This part exists only for N > 2.

Finite-temperature proliferation of composite vortices

Stack of 2D superconductors

E. Babaev, 2002



Global (=composite) vortex: goes through all the N layers.

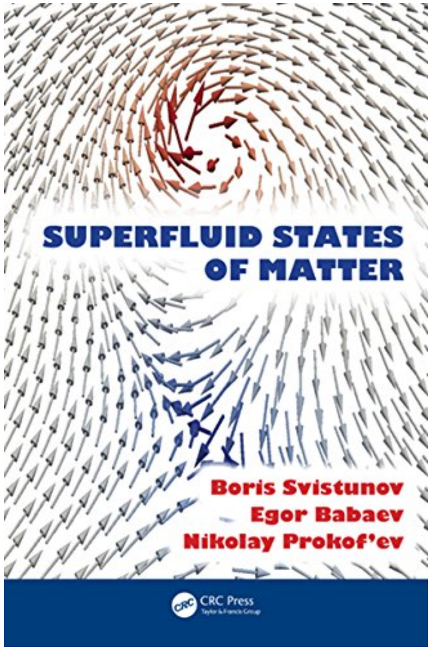
Proliferation of “shared” (=composite) vortices at any finite temperature.

Due to the long-range coupling via vector potential, vortices of the same sign in different layers attract each. At low enough temperature, they are bound into a composite vortex, which costs only a finite energy.

Another example is a slightly doped lattice bosonic supercounterfluid at a finite temperature, when the net flow becomes normal.

Kuklov, Prokof'ev, and BS, PRL 2003

More examples can be found in Chapter 6 of our book:



Hydrodynamics of Borromean Counterfluids

E. Babaev and BS, arXiv:2311.04340

Egor Babaev
KTH

Let us try to guess the form of the ground-state hydrodynamic Hamiltonian based on the effective theory of finite-T of proliferation of composite vortices.

Finite-T proliferation of composite vortices: free-energy argument

J. Smiseth, E. Smørgrav, E. Babaev, and A. Sudbø, PRB 71, 214509 (2005)

E. Blomquist, A. Syrwid, and E. Babaev, PRL 127, 255303 (2021)

$$F = \frac{\rho_*}{2}\left(\sum_\alpha \nabla\theta_\alpha\right)^2 + \frac{\Lambda}{2}\sum_{\alpha,\beta}\left(\nabla\theta_\alpha - \nabla\theta_\beta\right)^2 ,$$

$$\rho_* \ll \Lambda$$

Can we generalize this to the ground-state hydrodynamic counterflow Hamiltonian?

Sounds weird:

(i) Why would the individual phases have hydrodynamic meaning in the absence of corresponding order?

(ii) What would arrest the net flow ?!

On the other hand, old good London-Ginzburg-Landau theory provides us with a hope in the form of Anderson effect.

Hydrodynamic Hamiltonian

$$\mathscr{H} = \frac{1}{2}\sum_{\alpha,\beta}\kappa_{\alpha\beta}\,\eta_\alpha\eta_\beta + \frac{1}{2}\sum_{\alpha<\beta}\Lambda_{\alpha\beta}\left(\nabla\theta_\alpha - \nabla\theta_\beta\right)^2 , \qquad \eta_\alpha = n_\alpha - \bar{n}_\alpha$$

$$\dot{\eta}_\alpha = -\sum_\beta \Lambda_{\alpha\beta}\left(\Delta\theta_\alpha - \Delta\theta_\beta\right) , \qquad \dot{\theta}_\alpha = -\sum_\beta \kappa_{\alpha\beta}\,\eta_\beta$$

Hamiltonian equations of motion

Summing up all the equations for $\dot{\eta}_\alpha$ we get:

$$\frac{\partial}{\partial t}\sum_{\alpha=1}^{N}\eta_\alpha(\mathbf{r},t) = 0$$

consistent with

$$\sum_{\alpha=1}^{N}\eta_\alpha(\mathbf{r},t) \equiv 0$$

Equations of motion for densities have the form of continuity equations:

$$\dot{\eta}_\alpha + \nabla\cdot\mathbf{j}_\alpha = 0 , \qquad \mathbf{j}_\alpha = \sum_\beta \Lambda_{\alpha\beta}\left(\nabla\theta_\alpha - \nabla\theta_\beta\right) , \qquad \sum_\alpha \mathbf{j}_\alpha = 0$$

Compact-gauge invariance

$$\forall \alpha : \quad \theta_\alpha(\mathbf{r}) \to \theta_\alpha(\mathbf{r}) + \phi(\mathbf{r})$$

Compactness: As opposed to the standard gauge transformation, the field $\phi(\mathbf{r})$ is not single-valued.

The constrain $\frac{\partial}{\partial t} \sum_{\alpha=1}^{N} \eta_\alpha(\mathbf{r}, t) = 0$ emerges as a local Noether's constant of motion.

Modular arithmetic of topological charges

Signature feature of the compact-gauge invariant theory

Gauge redundant way of specifying topological charges via individual winding numbers for each component:

$$(m_1, m_2, \ldots, m_N)$$

- Respects symmetry between components.
- Convenient for addition.

Addition/equivalence is modulo $(1, 1, \ldots, 1)$.

Characteristic examples for the *N=3* case:

$$(1,0,0) + (0,1,0) + (0,0,1) = (1,1,1) \equiv (0,0,0)$$

A system of three different elementary vortices of the same sense is topologically trivial.

$$(1,0,0) + (0,1,0) = (1,1,0) \equiv (0,0,-1)$$

A system of two different elementary vortices of the same sense is topologically equivalent to the third elementary vortex of the opposite sense.

Borromean metals and insulators

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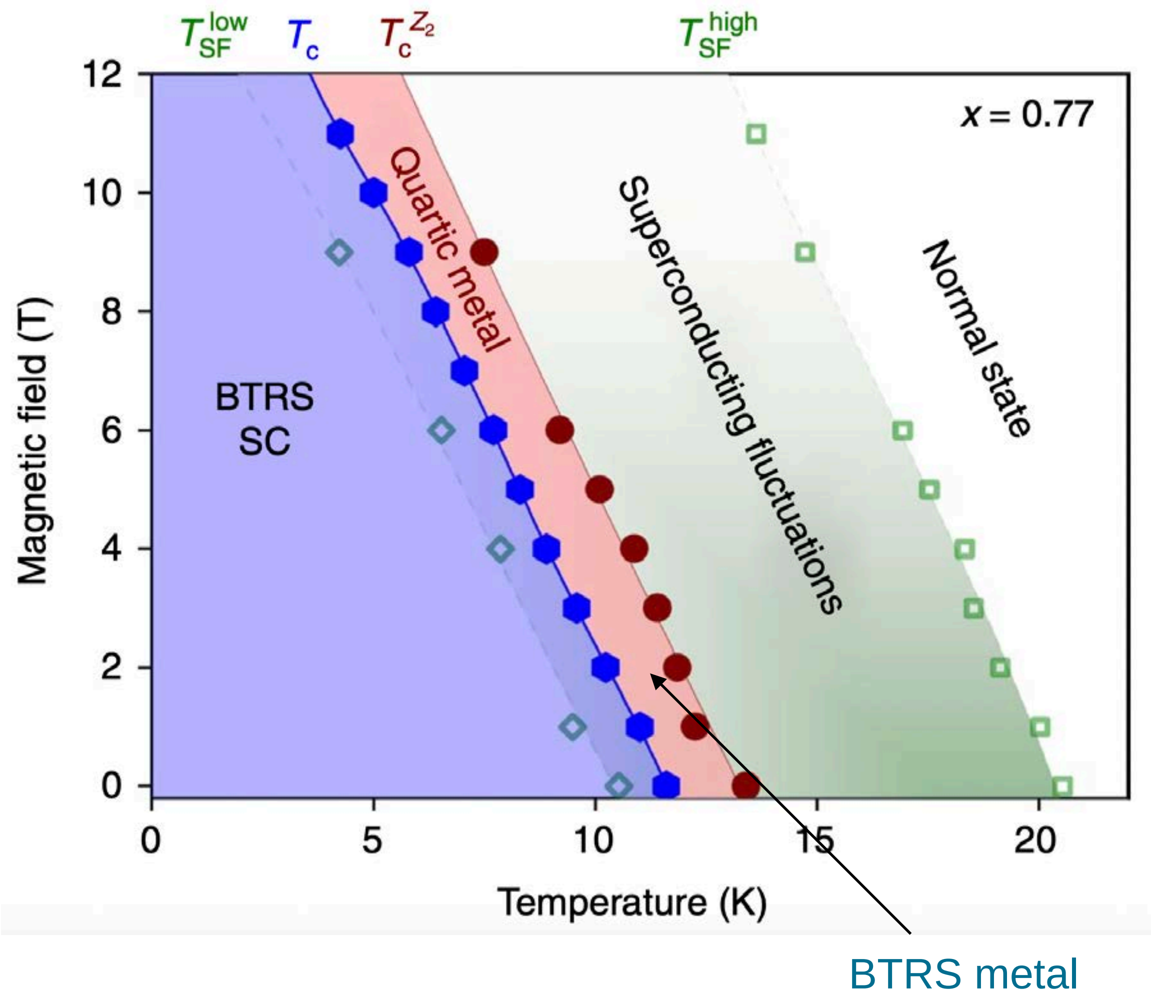

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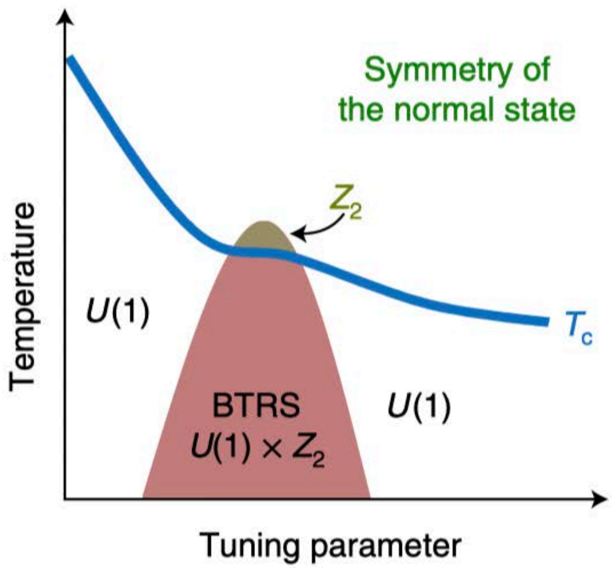
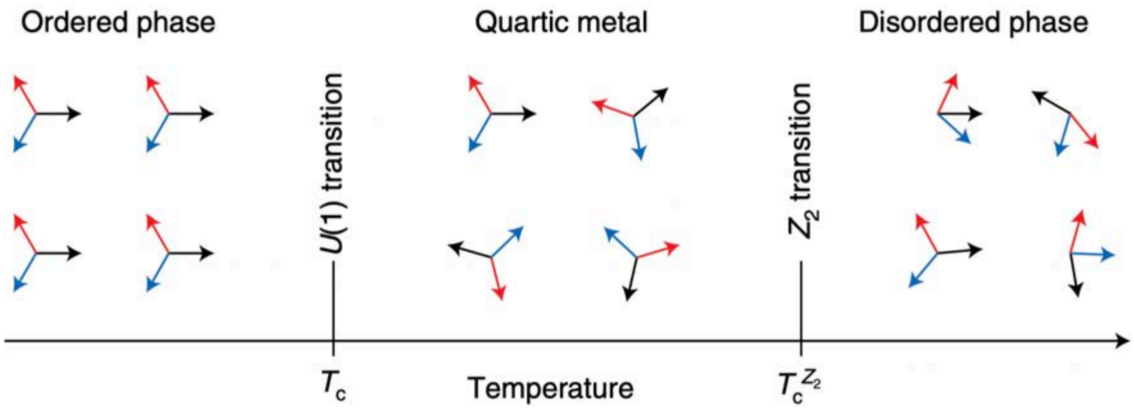
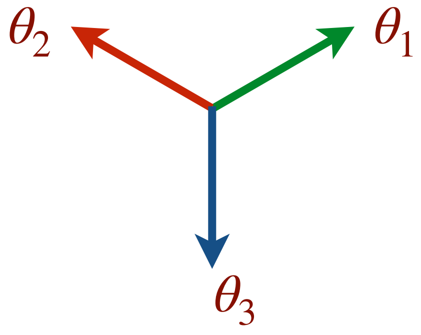




Minimal microscopic model for Borromean insulating ground state with broken time-reversal symmetry

Augment 3-component, $[U(1)]^3$-symmetric Bose-Hubbard Hamiltonian with weak frustrating intercomponent Josephson couplings:

$$H \to H + J_0 \sum_{j,\alpha\neq\beta} a^\dagger_{j\alpha} a_{j\beta}$$

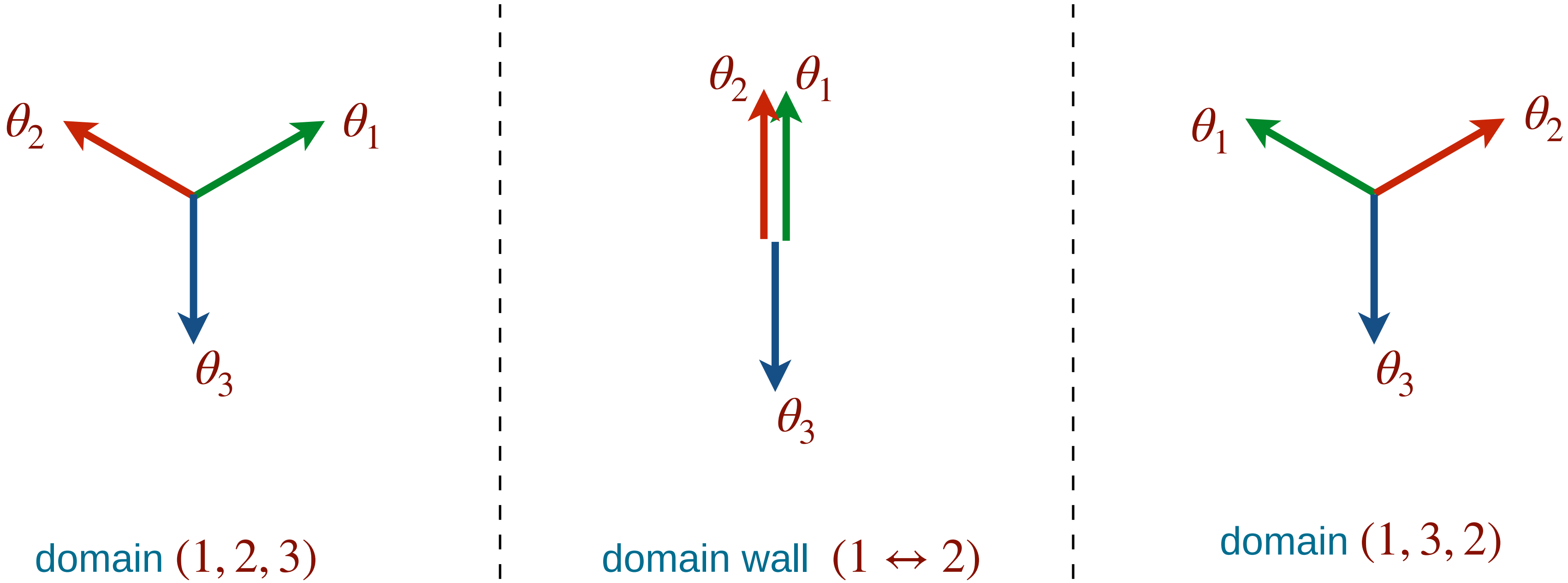


Order parameter: $\mathrm{Im}\,\langle\, a_\alpha\, a^\dagger_\beta \,\rangle \neq 0\,, \qquad \alpha,\beta \,=\, 1,2,3\,.$

Hydrodynamic theory has to account for the qualitative difference between BTRS Borromean insulator/metal and a generic normal BTRS state:

Existence of very specific multiple domain walls and corresponding domain-wall vortices (sort of topological ordering).

Multiple domain walls

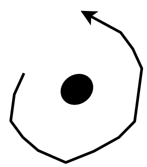


The three domain walls: $(1 \leftrightarrow 2), (1 \leftrightarrow 3), (2 \leftrightarrow 3)\,.$

At finite T: Garaud, Carlström, Babaev, and Speight, PRB 87, 014507 (2013)

Domain wall vortices

domain wall $(1 \leftrightarrow 3)$



vortex (1): phase 1 does a 2π twist
with respect to the other two components

domain wall $(1 \leftrightarrow 2)$

At finite T: Garaud, Carlström, Babaev, and Speight, PRB 87, 014507 (2013)

Hydrodynamic Hamiltonian for Borromean insulators

Off-diagonal intercomponent couplings

$$\mathscr{H} \to \mathscr{H} - \frac{1}{2}\sum_{\alpha,\beta} \mathscr{Q}_{\alpha\beta}(\theta_\alpha - \theta_\beta)$$

$\mathscr{Q}_{\alpha\beta}(\theta) = \mathscr{Q}_{\beta\alpha}(\theta) = \mathscr{Q}_{\alpha\beta}(-\theta)$ are 2π-periodic functions.

$$\dot{\eta}_\alpha + \nabla \cdot \mathbf{j}_\alpha = \mathscr{J}_\alpha , \qquad \mathscr{J}_\alpha = \sum_\beta \mathscr{W}_{\alpha\beta}(\theta_\alpha - \theta_\beta) , \qquad \mathscr{W}_{\alpha\beta}(\theta) = \frac{\partial \mathscr{Q}_{\alpha\beta}(\theta)}{\partial \theta}$$

$$\sum_\alpha \mathscr{J}_\alpha = 0$$

General form of the counterflow Hamiltonian

Works for both Borromean supercounterfluids and Borromean insulators

$$\mathscr{H} \equiv \mathscr{H}(\{\eta\}, \{\mathbf{w}\}, \{\theta\}, \{\mathbf{v}\})$$

$$\{\eta\} \equiv (\eta_1, \ldots \eta_N), \quad \{\mathbf{w}\} \equiv (\mathbf{w}_1, \ldots \mathbf{w}_N), \quad \{\theta\} \equiv (\theta_1, \ldots \theta_N), \quad \{\mathbf{v}\} \equiv (\mathbf{v}_1, \ldots \mathbf{v}_N)$$

placeholders for gradients of densities

placeholders for phase gradients

$$\sum_\alpha \frac{\partial \mathscr{H}}{\partial \theta_\alpha} = 0, \qquad \sum_\alpha \frac{\partial \mathscr{H}}{\partial \mathbf{v}_\alpha} = 0$$

conditions of compact-gauge invariance

$$\dot{\eta}_\alpha + \nabla \cdot \mathbf{j}_\alpha = \mathscr{J}_\alpha,$$

$$\mathbf{j}_\alpha = \frac{\partial \mathscr{H}}{\partial \mathbf{v}_\alpha} \qquad (\{\mathbf{v}\} \to \{\nabla\theta\})$$

$$\mathscr{J}_\alpha = \frac{\partial \mathscr{H}}{\partial \theta_\alpha} \qquad (\{\mathbf{v}\} \to \{\nabla\theta\})$$

$$\boxed{\sum_\alpha \mathbf{j}_\alpha = 0 \qquad \sum_\alpha \mathscr{J}_\alpha = 0}$$

Net flow is arrested.

Conclusions

Borromean counterfluids: N-1 phonon modes, N elementary vortices/supercounterflow states.

Borromean insulator: A ground state with broken time reversal symmetry and sort of "topological-like" order manifesting itself in multiple domain walls.

Hydrodynamic description of Borromean states:
Compact-gauge-symmetric N-component hydrodynamic Hamiltonian.

Bulk of a multi-band ($N > 2$) superconductor as a Borromean insulator: Gauge invariance arrests the net flow (Anderson effect) and the inter-band Josephson coupling does the rest.