Entanglement Dynamics in Hybrid quantum Circuits

Maglab Theory Winter School. Jan. 2022

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Entanglement Dynamics in Hybrid Quantum Circuits

"Monitored"

References: Pedagogical Book chapter / Review:
A.C. Potter and RV, 2111.08018

Monitored circuits:
Li, Chen, Fisher 1808.06134
Skinner, Ruhman, Nahum 1808.05953
1st papers

Gullans, Huse 1905.05195, 1910.00020
Bao, Choi, Altman 1908.05124 + Qi
1908.04305
Jian, You, Vasseur, Ludwig 1908.08051
Replica Stat. Mech. approach

Zabalo et al, 1911.00008
⋮

+ Many more!

Related papers on stat mech approach:

Random circuits (See Adam's lecture):

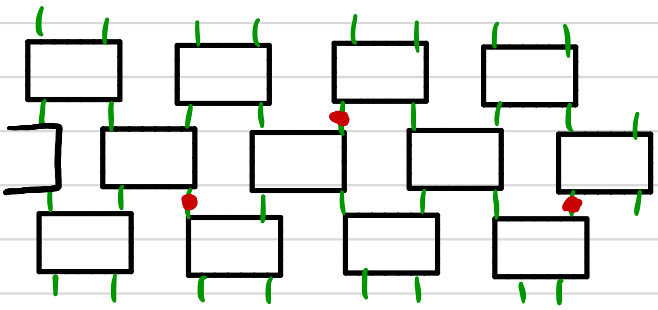
Nahum, Ruhman, Vijay, Haah 1608.06950
Nahum, Vijay, Haah 1705.08975
Zhou, Nahum 1804.09737

Random tensor networks
Hayden et al 1601.01694
Vasseur, Potter, You, Ludwig 1807.07082
Nahum, Roy, Skinner, Ruhman 2009.11311

Replica stat mech

Measurement-induced transition

Chaotic dynamics (entanglement growth) vs local projective measurements.



- $\mathcal{H} = (\mathbb{C}^d)^N$

  qudit on each site

- Model quantum chaotic dynamics by random quantum circuit

(not necessary, but makes problem tractable)

• = Random projective measurement with proba p.

projector onto $|m\rangle$

$$|\psi\rangle \longrightarrow |\psi_m\rangle = \hat{\Pi}_m |\psi\rangle$$

$$p_m = |\langle m|\psi\rangle|^2 = \langle\psi|\hat{\Pi}_m|\psi\rangle = \text{Born Probability}$$

Quantum trajectories vs Quantum channel

Trace out measurement outcomes: $\rho = \sum_{\{m\}} |\psi_m\rangle\langle\psi_m|$

Linear quantity: $\langle O\rangle$, average over measurement outcomes:

$$\overline{\langle O\rangle} = \sum_{\{m\}} \cancel{p_m} \frac{\langle\psi_m|O|\psi_m\rangle}{\cancel{\langle\psi_m|\psi_m\rangle}} = \mathrm{Tr}(\rho O)$$

sum over quantum trajectories

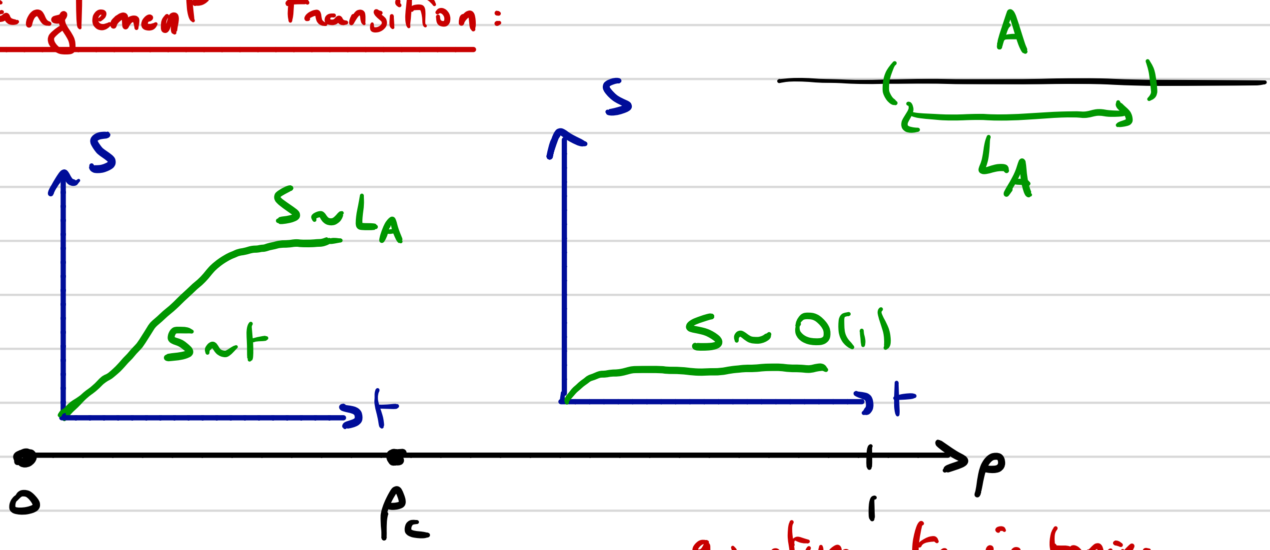
Nonlinear quantity:

$$\overline{\langle O\rangle^2} = \sum_{\{m\}} \cancel{p_m} \frac{\langle\psi_m|O|\psi_m\rangle^2}{\langle\psi_m|\psi_m\rangle^{\cancel{2}}} ,$$

can't be expressed in terms of ρ!

Claim: Interesting phase structure, transitions, in quantum trajectories $|\psi_m\rangle$, invisible in ρ! ($\rightarrow$ post selection issue)

<u>Entanglement transition:</u>



quantum trajectories

$$\boxed{S_n = \mathbb{E}_{\text{circuits}} \sum_{\{m\}} p_m \frac{1}{1-n} \log\left[\frac{\mathrm{tr}\,\rho_{A,m}^n}{(\mathrm{tr}\,\rho_m)^n}\right]}$$

Average over Haar unitaries, measurement locations

$$\rho_m = |\psi_m\rangle\langle\psi_m| , \qquad \rho_{A,m} = \mathrm{tr}_{\bar{A}}\, \rho_m$$

Lots of exciting recent results: different observables and interpretations of the transition (purification, QEC, ancilla probes), Experiment from Monroe group (decoding problem), New phases (topological and symmetry breaking) stabilized by non-unitary dynamics from symmetries / competing measurements...

Here focus on entanglement transition (universality class? criticality?)

↳ Stat. Mech. Approach + Replica trick

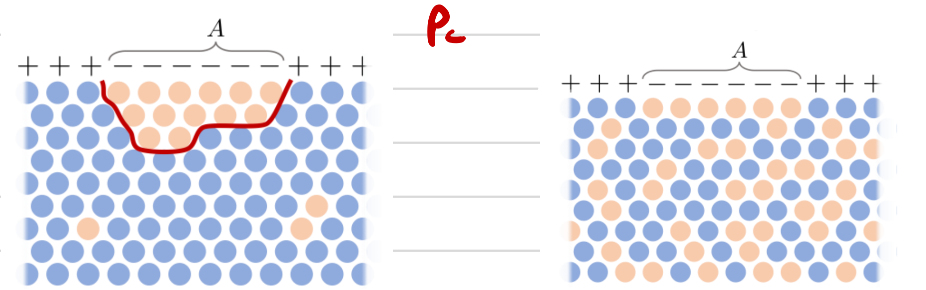
→ What do we know?:

- Exact Mapping onto (replica) 2D Stat. Mech. Model
- Qualitative picture:

$$S_A = \Delta F \,(\text{insert DW})$$

All Renyi Entropies have the same p_c

$\Delta F \propto L_A$ —— p_c —— $\Delta F \propto \text{Const.}$ →



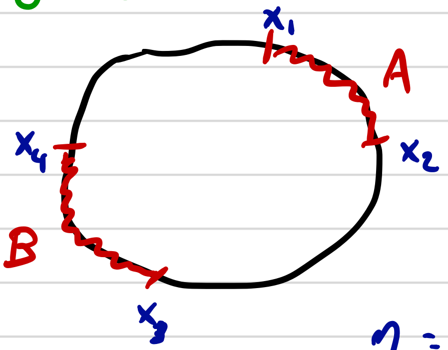
(Fig from Bao et al.)

- Critical point = CFT in 2d with $c=0$ } non unitary field theory
  can write down a Lagrangian etc.

- Conformal invariance at transition : $\boxed{z=1}$
- $S_A \sim \log L_A$ at criticality

  $S_A \sim \log \langle \phi_{Bcc} \phi_{Bcc} \rangle$ in CFT

Conformal invariance : (seen numerically for Clifford + Measurements)



$$I_{AB} = S_A + S_B - S_{A\cup B}$$

$$= f(\eta)$$

$$\eta = \frac{x_{12}\, x_{34}}{x_{13}\, x_{24}} \qquad x_{ij} = \frac{L}{\pi} \sin\left(\frac{\pi}{L}|x_i - x_j|\right)$$

- $d \to \infty$: mapping onto Percolation

Replica stat. mech. approach :

- Replica trick: $\log x = \lim_{k\to 0} \frac{x^k - 1}{k}$

$$S_A^{(n)} = \lim_{k\to 0} \mathbb{E}_{\text{circuits}} \sum_{\{m\}} \frac{p_m}{(1-n)k} \left[\underbrace{(\mathrm{tr}\, \rho_{A,m}^n)^k - (\mathrm{tr}\, \rho_m^n)^k}_{\text{"easy" to average if } n,k \text{ integers}} \right]$$

$$= \lim_{k\to 0} \frac{1}{(1-n)k} (Z_A - Z_0) = \lim_{k\to 0} \frac{1}{(n-1)k} (F_A - F_0)$$

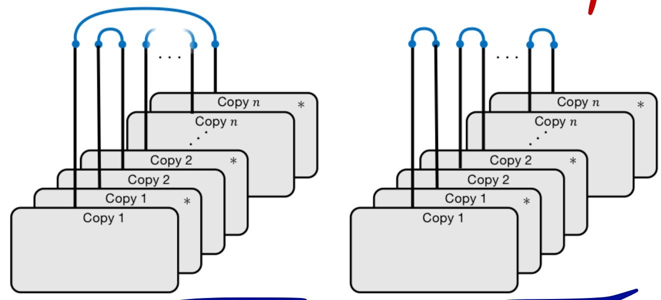
$F = -\log Z$

$\Rightarrow$ Need to average : $\rho^{\otimes Q}$, $Q = nk + 1$

Born proba

with top boundary contraction:

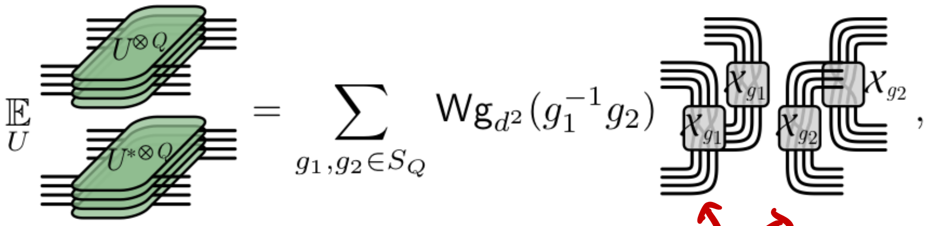
(Fig from Bao et al)



$\otimes K$ times in region A — in $\bar{A}$

Different "boundary conditions" (top layer) in Z_A, Z_0

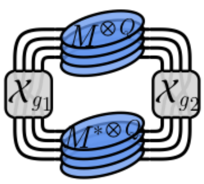
• Haar average



permutations ("Schur-Weyl" duality)

→ degrees of freedom: $g \in S_Q$, permutations
"spins"

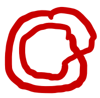
• Contracting unitaries:



$= \mathrm{Tr}\, \mathcal{X}_{g_1} M^{\otimes Q} \mathcal{X}_{g_2} M^{\dagger \otimes Q}$,

measurement

if no measurement: $\mathrm{Tr}\left[\mathcal{X}_{g_1^{-1} g_2}\right] = d^{\,c(g_1^{-1} g_2)}$ # of cycles

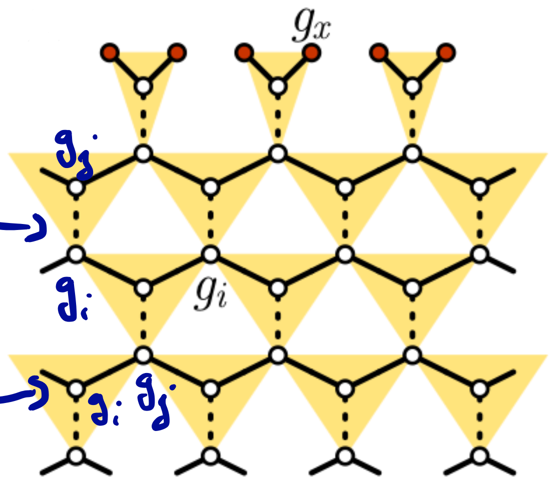
$Q = 2$:  $= d^2$  $= d^1$

if measurement: all replicas are forced to agree : weight $d' = d$

⟹ Stat mech model:

$W_g(g_i^{-1} g_j)$

$p\,d + (1-p)\,d^{C(g_i^{-1} g_j)}$



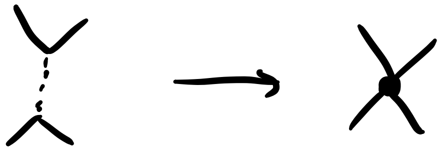
$$\boxed{S_n = \lim_{K\to 0} \frac{1}{K(n-1)} \left(F_A - F_0 \right)}$$

≠ boundary conditions in A

→ Explains most qualitative features of transition, entanglement scaling etc. Volume law phase: "Spontaneous Symmetry breaking"

$S_Q \times S_Q$ $\qquad g_i \longrightarrow g_L^{-1} g_i g_R$

→ Replica limit tricky in general ($K \to 0$, $Q \to 1$) except $d \to \infty$



<u>Large onsite Hilbert space dimension limit</u> : $d \to \infty$

$$d^{C(g)} \sim d^{Q}\,\delta_{g,1} \qquad \text{as } d \to \infty$$

$$W_g(g_i^{-1} g_j) \underset{d\to\infty}{\sim} \delta_{g_i, g_j} \quad (\text{up to } d^Q \text{ factors})$$

square lattice

$$\boxed{Z_{d\to\infty} = \sum_{\{g_i \in S_Q\}} \prod_{\langle i,j\rangle} \left((1-p)\,\delta_{g_i, g_j} + p\right)}$$

(For the measurement transition $Q = nm+1 \to 1$ instead of 0)

Enlarged symmetry: $S_{Q!}$ (permutation of all g_i's)

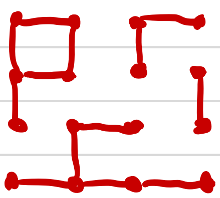
- This is a $Q!$-Potts model. $Q! \to 1$ corresponds to percolation (c = 0 CFT)

Expand product: Fortuin-Kasteleyn clusters (FK)

$\bullet\!\!-\!\!\!-\!\!\bullet \;= (1-p)\,\delta_{g_i, g_j}$

$\bullet \quad \bullet \;= p$

$\sum_{g_i}$: $Q!$ per cluster
all spins the same



$$Z = \sum_{\text{clusters}} (1-p)^{\#\,\text{links}}\; p^{\#\,\text{empty links}}\; \underbrace{(Q!)}_{1}^{\#\,\text{clusters}}$$

in replica limit

$$\boxed{p_c = 1/2}$$

$$S_Q \subset S_{Q!}$$

$1/d$ connections? Hard! $S_{Q!} \longrightarrow S_Q \times S_Q$

$$\mathcal{L} = \underset{\text{Potts}}{\mathcal{L}[\phi_a]} + \sum_{a,b \in S_Q} C(a^{-1}b)\, \phi_a \phi_b$$

: Relevant! IR fixed point?

class function