Schrödinger Cats, Maxwell's Demon and Quantum Error Correction

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<u>Theory</u> SMG Liang Jiang Leonid Glazman <u>M. Mirrahimi</u> **

Shruti Puri Yaxing Zhang Victor Albert** Kjungjoo Noh** Richard Brierley Claudia De Grandi Zaki Leghtas Juha Salmilehto Matti Silveri Uri Vool Huaixui Zheng Marios Michael +.... In the first lecture we learned:

An arbitrary quantum channel (CPTP map) is the most general possible operation on a quantum system.

Therefore <u>if quantum error correction is possible</u>, it can be performed via a quantum channel.

$$\rho'_{\text{sys}} = \sum_{k=1}^{d^2} E_k \rho_{\text{sys}} E_k^{\dagger} \quad \text{`error map'}$$
$$\rho_{\text{sys}} = \sum_{k=1}^{d^2} R_k \rho'_{\text{sys}} R_k^{\dagger} \quad \text{`recovery map'}$$

Under what conditions is recovery possible?

Let the system ('sys') be N physical qubits. A logical qubit encoded in sys consists of two orthogonal 'words' in the Hilbert of sys

$$\operatorname{code} = \operatorname{span} \left\{ |W_0\rangle, |W_1\rangle \right\}$$
$$P_{\operatorname{code}} = |W_0\rangle \langle W_0| + |W_1\rangle \langle W_1|$$

Knill-Laflamme condition

A recovery map for a set of errors $\{E_1, E_2, ..., E_N\}$ exists if $P_{\text{code}} E_i^{\dagger} E_j P_{\text{code}} = \alpha_{ij} P_{\text{code}}$

where α is a Hermitian matrix.

Equivalently
$$\langle W_{\mu} | E_i^{\dagger} E_j | W_{\nu} \rangle = \alpha_{ij} \delta_{\mu\nu}$$

where α_{ij} is state-independent (i.e. independent of μ, ν

In learning the error, we learn nothing about the stored information.

Knill-Laflamme condition

A recovery map for a set of errors $\{E_1, E_2, ..., E_N\}$ exists if

$$P_{\text{code}} E_i^{\dagger} E_j P_{\text{code}} = \alpha_{ij} P_{\text{code}}$$

where α is a Hermitian matrix.

"Proof:" Let
$$S\alpha S^{\dagger} = d$$
 diagonalize α . Let $K = SE$.
 $P_{\text{code}}K_i^{\dagger}K_jP_{\text{code}} = d_{ij}P_{\text{code}}$

Different error states $K_j | W_{\mu} \rangle$ are orthogonal and hence identifiable by measurement of the projector (which does <u>not</u> tell us about the stored quantum information)

$$\Pi_j = \frac{K_j P_{\text{code}} K_j^{\dagger}}{d_{jj}}, \quad (\Pi_j)^2 = \Pi_j$$

Given knowledge of which error occurred, there exists a unitary map from the error state back to the original state in the code space.

Errors can be <u>non-unitary</u> (increase entropy) But Knill-Laflamme condition says we can correct them with a <u>unitary</u>, if the choice of unitary is conditioned on measurement result.

$$\Pi_{j} = \frac{K_{j} P_{\text{code}} K_{j}^{\dagger}}{d_{jj}}, \quad (\Pi_{j})^{2} = \Pi_{j}$$

Next up: Quantum Error Correction Codes for Bosonic Modes (microwave photons)

Photons are excitations of harmonic oscillators (e.g. one mode of a microwave resonator)

We will use superpositions of photon Fock states (number states) as quantum code words.

Example: Binomial Code (aka 'kitten code')



M. Michael et al., PRX 6, 031006 (2016)



Experimentally realistic error model:

Amplitude damping of harmonic oscillator
 No intrinsic dephasing (frequency noise)

Quantum channel for damped oscillator:

0 photon loss 1 photon loss

$$\rho(t + \delta t) = E_0 \rho(0) E_0^{\dagger} + E_1 \rho(0) E_1^{\dagger} + \dots$$

$$E_1 = \sqrt{\kappa \delta t} a$$

$$E_1^{\dagger} E_1 = \kappa \delta t \hat{n}$$

Sanity check: probability of 1 photon loss: $p_1 = \text{Tr}\left\{E_1 \rho E_1^{\dagger}\right\} = \kappa \delta t \langle \hat{n} \rangle$

Quantum channel for damped oscillator:



Sanity check: probability of 1 photon loss: $p_1 = \operatorname{Tr}\left\{E_1 \rho E_1^{\dagger}\right\} = \kappa \delta t \langle \hat{n} \rangle$

What is
$$E_0$$
? Completeness says: $E_0^{\dagger}E_0 = 1 - E_1^{\dagger}E_1$ Exact answer:
Guess: $E_0 = U\sqrt{1 - E_1^{\dagger}E_1} \approx 1 - \frac{\kappa}{2}\delta t \ a^{\dagger}a; \ U = I$ $E_0 = \exp\left\{-\frac{\kappa}{2}\delta t \ \hat{n}\right\}$
 $\rho(t + \delta t) = \rho(t) + \kappa\delta t \left\{a\rho(t)a^{\dagger} - \frac{1}{2}[a^{\dagger}a\rho(t) + \rho(t)a^{\dagger}a]\right\}$ Lindblad master equation

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equation



[Master Equation]

Quantum trajectory interpretation:

Toss a coin to randomly select

$$\rho(t+\delta t) = \frac{E_0 \rho(0) E_0^{\dagger}}{p_0} \quad \text{with probability} \quad p_0 = \text{Tr}\left\{E_0 \rho(0) E_0^{\dagger}\right\}$$

or select

$$\rho(t+\delta t) = \frac{E_1 \rho(0) E_1^{\dagger}}{p_1} \quad \text{with probability} \quad p_1 = \text{Tr}\left\{E_1 \rho(0) E_1^{\dagger}\right\}$$

Each Monte Carlo run simulates an actual experimental history (and gives a pure state). Ensemble averaging gives same result as master equation evolution—density matrix becomes impure.

What if, instead of Monte Carlo data, we had real data?



The measurement record tells us exactly the state of the environment. Hence the resonator cannot be entangled with it.

'<u>Conditional</u> density matrix' $\rho_{c}(t + \delta t)$ must be in a pure state(!)

What if, instead of Monte Carlo data, we had real data?



'no click'
$$\rho_{\rm c}(t+\delta t) = \frac{E_0\rho(0)E_0^{\dagger}}{p_0}$$
 with probability $p_0 = \operatorname{Tr}\left\{E_0\rho(0)E_0^{\dagger}\right\}$
'click' $\rho_{\rm c}(t+\delta t) = \frac{E_1\rho(0)E_1^{\dagger}}{p_1}$ with probability $p_1 = \operatorname{Tr}\left\{E_1\rho(0)E_1^{\dagger}\right\}$

Conditional density matrix is pure: $\rho_{\rm c}(t + \delta t) = |\psi_{\rm c}\rangle\langle\psi_{\rm c}|$

If detector clicks:
$$|\psi_{c}\rangle = \frac{E_{1}|\psi_{0}\rangle}{\sqrt{\langle\psi_{0}|E_{1}^{\dagger}E_{1}|\psi_{0}\rangle}} = \frac{a|\psi_{0}\rangle}{\sqrt{\langle\psi_{0}|a^{\dagger}a|\psi_{0}\rangle}}$$

But what happens if the detector does not click?

Example 2:
$$|\psi_0\rangle = \sqrt{0.99} |0\rangle + \sqrt{0.01} |100\rangle; \langle \psi_0 | \hat{n} | \psi_0 \rangle = 1$$

Click: $|\psi_{c}\rangle = |99\rangle$ $\langle\psi_{c}|\hat{n}|\psi_{c}\rangle = 99$ (!!!)

No click: $|\psi_{c}\rangle \propto \sqrt{0.99} |0\rangle + \sqrt{0.01} \exp\left(-\frac{\kappa}{2} \delta t \ 100\right) |100\rangle$

State changes even though no click!! $\langle \psi \downarrow c \mid n \mid \psi \downarrow c \rangle < 1.$

But what happens if the detector does not click?

Sir Arthur Conan Doyle: Silver Blaze

Scotland Yard Detective: "Is there any other point to which you wish to draw my attention?

Sherlock Holmes: "To the curious incident of the dog during the night."

Detective: "The dog did nothing in the night-time."

Holmes: "That was the curious incident."

The quantum state changes even if the dog does not bark.

If we keep track of when the dog barks and doesn't bark, the system state remains pure. We will use this for QEC.

One scheme involves some remarkable properties of coherent states and Schrödinger cat states. (Mazyar Mirrahimi)

Quick review of microwave resonators and photonic states

Circuit QED
$$\begin{array}{c} \varphi = & \varphi = &$$

Coherent state is closest thing to a classical sinusoidal RF signal

$$\psi(\Phi) = \psi_0(\Phi - \alpha)$$



Experimental setup: superconducting qubit (artificial twolevel atom) in a superconducting microwave resonator



$$H = \omega_r a^{\dagger} a + \omega_q \left| e \right\rangle \left\langle e \right| + 2\chi a^{\dagger} a \left| e \right\rangle \left\langle e \right|$$

resonator

qubit

dispersive coupling

Photon number distribution in a coherent state (measured via quantized light shift of qubit transition frequency)



We will use Schrödinger cat states of cavity photons

(normalization is only approximate)

Parity of Cat States

$$P = e^{i\pi a^{\dagger}a} = (-1)^n \longrightarrow P = \sum_n p_n (-1)^n$$

Coherent state:

$$|\psi\rangle = |\alpha = 2\rangle$$

Mean photon number: 4

Even parity cat state: $|\psi\rangle = |\alpha\rangle + |-\alpha\rangle$ Only photon numbers: 0, 2, 4, ...

$\hat{P}|\psi\rangle = +|\psi\rangle$

Odd parity cat state: $|\psi\rangle = |\alpha\rangle - |-\alpha\rangle$ Only photon numbers: 1, 3, 5, ...

 $\hat{P}|\psi\rangle = -|\psi\rangle$

Photon number



Schoelkopf Lab

Key enabling technology: ability to make nearly ideal measurement of photon number parity (without measuring photon number!)

$$\hat{P} = (-1)^{a^{\dagger}a} = \sum_{n=0}^{\infty} |n\rangle (-1)^n \langle n|$$

We learn whether *n* is even or odd without learning the value of *n*.

Use dispersive $e^{-iVt} \rightarrow e^{-i\pi|e\rangle\langle e|a^{\dagger}a} = |g\rangle\langle g|+|e\rangle\langle e|\hat{P}$ coupling:

Measurement is 99.8% QND. (Can be repeated <u>hundreds</u> of times.)

If we can measure parity, we can perform complete state tomography (measure Wigner function)

Wigner Function of a Cat State

Vlastakis, Kirchmair, et al., Science (2013)

Interference fringes prove cat is coherent (even for sizes > 100 photons)



Using Schrödinger cat states to store and correct quantum information The magic of coherent states:

Lose a photon and stay in same state!

$$|\alpha\rangle = \exp(a^{\dagger} - \alpha) |0\rangle = e^{-\frac{1}{2}|\alpha|^{2}} \sum_{n=0}^{\infty} \frac{\alpha^{n}}{\sqrt{n!}} |\alpha| \alpha\rangle = \alpha |\alpha\rangle$$
$$\frac{E_{1} |\alpha\rangle \langle \alpha | E_{1}^{\dagger}}{p_{1}} = |\alpha\rangle \langle \alpha |$$

$$\langle \alpha | \hat{n} | \alpha \rangle = | \alpha |^2$$
 but $a^{\dagger} a | \alpha \rangle \neq \alpha * \alpha | \alpha \rangle$

Nevertheless
$$E_0 | \alpha \rangle = \exp\{-\frac{\kappa \delta t}{2}\hat{n}\} | \alpha \rangle = Z(t) | e^{-\frac{\kappa}{2}t} \alpha \rangle$$

Hence, damping comes <u>not</u> from loss of photons but from the 'no click' events (when the dog does not bark)!

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Driven damped oscillator: $V = i \left(\acute{U} a - \acute{U} a^{\dagger} \right)$

$$\frac{d}{dt}\rho(t) = i[V,\rho(t)] + \kappa \left\{ a\rho(t)a^{\dagger} - \frac{1}{2}[a^{\dagger}a\rho(t) + \rho(t)a^{\dagger}a] \right\}$$

Lindblad master equation

Define:
$$b = a - \frac{2}{\kappa}$$

$$\frac{d}{dt}\rho(t) = \kappa \left\{ b\rho(t)b^{\dagger} - \frac{1}{2} [b^{\dagger}b\rho(t) + \rho(t)b^{\dagger}b] \right\}$$
$$\rho(\infty) \rightarrow |\alpha\rangle \langle \alpha|, \qquad \alpha = \frac{2\acute{U}}{\kappa}$$

Energy is being stochastically dumped into the bath, yet the dissipation stabilizes a pure state (coherent state)!

Can we extend this idea to use dissipation to autonomously stabilize a manifold of 2 states?

Two-photon pumping: $V = i\lambda^2 \left[a^{\dagger 2} - a^2 \right]$

Bath engineering

Two-photon damping: $E_2 = \kappa_2 \delta t \left[a^{\dagger 2} - a^2 \right]$

$$b = (a^2 - \alpha^2)$$

$$\frac{d}{dt}\rho(t) = \kappa \left\{ b\rho(t)b^{\dagger} - \frac{1}{2} [b^{\dagger}b\rho(t) + \rho(t)b^{\dagger}b] \right\}$$

Stable manifold: (can be used as a qubit)

span {
$$|+\alpha\rangle$$
, $|-\alpha\rangle$ }

Devoret group: arXiv:1705.02401

Stable manifold using two-photon pumping and two-photon damping

Expt'l Wigner functions of cat states



Recall that if we can keep track of the state of the bath the system state remains pure.



The measurement record tells us exactly the state of the environment. Hence the resonator cannot be entangled with it.

<u>'Conditional</u> density matrix' $\rho_{c}(t + \delta t)$ must be in a pure state(!)

Problem: We don't actually have a PMT for microwave photons sitting outside the cavity. (Also some of the photons are lost internally.)

Solution: We <u>are</u> able to measure the photon number <u>parity</u>.

We said before that photon loss leaves coherent state invariant, but actually there is an important phase:

$$a |+\alpha\rangle = +\alpha |+\alpha\rangle, \qquad a |-\alpha\rangle = -\alpha |-\alpha\rangle$$

This is necessary to achieve:

even

$$a\left[\frac{1}{\sqrt{2}}\left\{|+\alpha\rangle+|-\alpha\rangle\right\}\right] = \frac{1}{\sqrt{2}}\left\{|+\alpha\rangle-|-\alpha\rangle\right\}$$
code word error word

even

$$a\left[\frac{1}{\sqrt{2}}\left\{|+\alpha\rangle+|-\alpha\rangle\right\}\right] = \frac{1}{\sqrt{2}}\left\{|+\alpha\rangle-|-\alpha\rangle\right\}$$
code word error word

We therefore need a second code word with even parity.

Encode information in two (nearly) orthogonal even-parity code "words"

$$|\psi\rangle = \psi_{\uparrow} |W_{1}\rangle + \psi_{\downarrow} |W_{2}\rangle$$
 code word Wigner functions:

$$|W_{1}\rangle = |\alpha\rangle + |-\alpha\rangle$$

$$|W_{2}\rangle = |i\alpha\rangle + |-i\alpha\rangle$$

$$|W_{1}\rangle$$

$$|W_{1}\rangle$$

$$|W_{2}\rangle = |i\alpha\rangle + |-i\alpha\rangle$$

Store a *qubit* as a *superposition* of two cats of same *parity*

Photon loss flips the parity which is the <u>error syndrome</u> we can measure (and repeat hundreds of times).

Coherent states are eigenstates of photon destruction operator. $a |\alpha\rangle = \alpha |\alpha\rangle$

Effect of photon loss on code words:

$$a | W_1 \rangle = a (|\alpha\rangle + |-\alpha\rangle) \rightarrow (|\alpha\rangle - |-\alpha\rangle) \quad \text{(if α real)}$$

$$a^2 | W_1 \rangle = a^2 (|\alpha\rangle + |-\alpha\rangle) \rightarrow (|\alpha\rangle + |-\alpha\rangle) = | W_1 \rangle$$

$$a | W_2 \rangle = a (|i\alpha\rangle + |-i\alpha\rangle) \rightarrow i (|i\alpha\rangle - |-i\alpha\rangle)$$

$$a^2 | W_2 \rangle = a^2 (|i\alpha\rangle + |-i\alpha\rangle) = (i)^2 (|i\alpha\rangle + |-i\alpha\rangle) = -| W_2 \rangle$$

After loss of 4 photons cycle repeats:

$$a^{4}\left(\xi_{1}\left|W_{1}\right\rangle+\xi_{2}\left|W_{2}\right\rangle\right)\rightarrow\left(\xi_{1}\left|W_{1}\right\rangle+\xi_{2}\left|W_{2}\right\rangle\right)$$

We can recover the state if we know: (via monitoring parity jumps)



We can recover the state if we know: (via monitoring parity jumps)



Amplitude damping is deterministic (independent of the number of parity jumps!)

$$|W(t)\rangle = |e^{-\kappa t/2}\alpha\rangle \pm |-e^{-\kappa t/2}\alpha\rangle$$

Maxwell Demon takes this into account 'in software.' (We don't yet have 4-photon pumping/damping working.)

QUANTUM ERROR CORRECTION Keeping your Cat Alive



"I have good news and bad news"

2016: First true Error Correction Engine that works



MAXWELL'S DEMON



- Single system performs all measurement, control, & feedback (latency ~ 200 nanoseconds)
- \sim 15% of the latency is the time it takes signals to move at the speed of light from the quantum a manutar ta tha a pretrallar and haald



A prototype quantum computer being prepared for cooling close to absolute zero.

Implementing a Full QEC System: Debugger View



(This is all real, raw data.) Ofek, et al., Nature **536**, 441–445 (2016).

Process Fidelity: Uncorrected Transmon



System's Best Component



Process Fidelity: Cats without QEC



Process Fidelity: Cats with QEC



Only High-Confidence Trajectories



We are on the way!



Achieved goal of reaching "break-even" point for error correction. Now need to surpass by 10x or more.

M. Devoret and RS, Science (2013)

Experiment:

'Extending the lifetime of a quantum bit with error correction in superconducting circuits,'

Ofek, et al., Nature **536**, 441–445 (2016).

Theory:

'cat codes' Leghtas, Mirrahimi, et al., PRL **111**, 120501(2013).

'kitten codes'

M. Michael et al., Phys. Rev. X 6, 031006 (2016). 'New class of error correction codes for a bosonic mode'

Thanks for listening!

QuantumInstitute.yale.edu

Extra slides

Storing information in quantum states sounds great...,

but how on earth do you build a quantum computer?

ATOM vs CIRCUIT



How to Build a Qubit with an Artificial Atom...



Superconducting integrated circuits are a promising technology for <u>scalable</u> quantum computing

Josephson junction:

The "transistor of quantum computing"

Provides <u>anharmonic</u> energy level structure



Enormous transition dipole moment

The first electronic quantum processor (2009)

Executed Deutsch-Josza and Grover search algorithms



Lithographically produced integrated circuit with semiconductors replaced by superconductors.



Michel Devoret

Rob Schoelkopf

DiCarlo et al., Nature 460, 240 (2009)

Scale then correct



Surface/Toric Code (readout wires <u>not</u> shown)

- Large, complex:
 - <u>Non-universal</u> (Clifford gates only)
 - Measurement via many wires
 - Difficult process tomography
- Large part count
- Fixed encoding

Correct then scale



Cat Code Photonic Qubit hardware shortcut (readout wire shown)

- Precision:
 - <u>Universal</u> control (all possible gates)
 - Measurement via single wire
 - Easy process tomography
 - Long-lived cavities
 - Fault-tolerant QEC
- Reduced part count
- Flexible encoding

All previous attempts to overcome the factor of *N* and reach the 'break even' point of QEC have failed.

With major technological advances a 49-transmon 'surface code' might conceivably break even at $T_2 = 40 \mu s$. [O'Brien et al. arXiv:1703.04136]

However to date, good repeatable (QND) (weight 4) error syndrome measurements have <u>not</u> been demonstrated [Takita et al., *Phys. Rev. Lett.* **117**, 210505 (2016)]

'Scale up and then error correct' seems almost hopeless.

We need a simpler and better idea...

'Error correct and then scale up!'

"Hardware-Efficienct Bosonic Encoding"

Leghtas, Mirrahimi, et al., PRL **111**, 120501(2013).

Replace 'Logical' qubit with this:





- Cavity has longer lifetime (~ms)
- Large Hilbert space
- Single dominant error channel photon loss: $\Gamma = \kappa \langle \hat{n} \rangle$
- Single readout channel

earlier ideas: Gottesman, Kitaev & Preskill, PRA 64, 012310 (2001) Chuang, Leung, Yamamoto, PRA 56, 1114 (1997)

Photonic Code States

Can we find novel (multi-photon) code words that can store quantum information even if some photons are lost?

Ancilla transmon coupled to resonator gives us <u>universal</u> <u>control</u> to make 'any' code word states we want.

