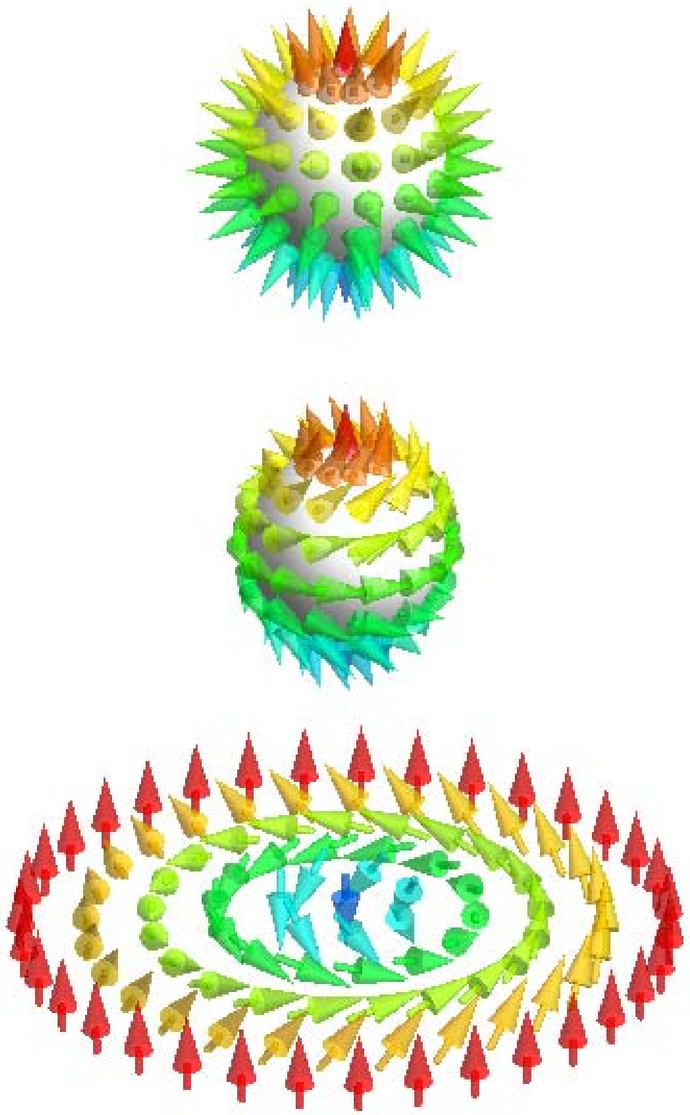
Skyrmions in Chiral Magnets

Achim Rosch, Institute for Theoretical Physics, Cologne, Germany

- magnets & topology
- Berry phases
- experimental realization of emergent electric and magnetic fields
- electric manipulation of magnetic structures
- spintronics and `skyrmionics`



lecture 1:
skyrmions in chiral magnets

- effective field theory for chiral magnets
- Berry phases and emergent electromagnetic fields
- experiments
- selected examples

lecture 2:
skyrmions & magnetic monopoles

- skyrmion as a particle:
  effective mass, screening, dynamics
- changing topology:
  emergent magnetic monopoles

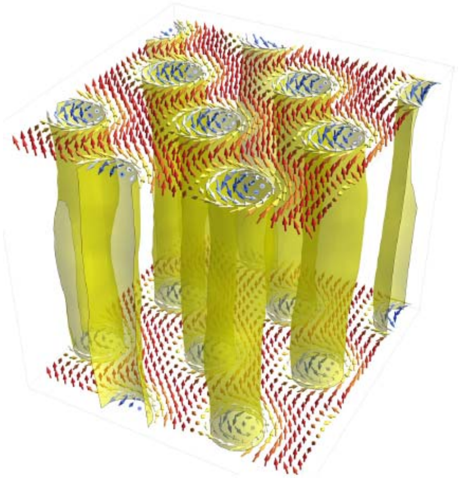
theory @ Cologne, Germany
Markus Garst, Stefan Buhrandt, Karin Everschor,
Robert Bamler, Christoph Schütte, Johannes Waizner,
Jan Müller, A. R.

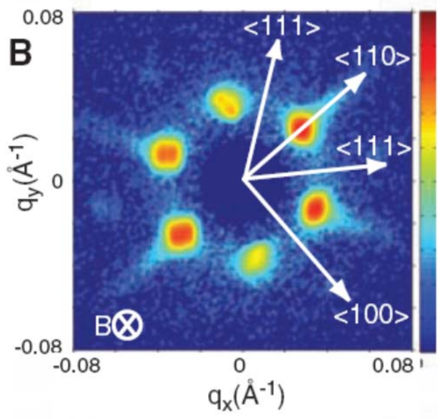
theory @ Tokyo, Japan
Naoto Nagaosa and coworkers

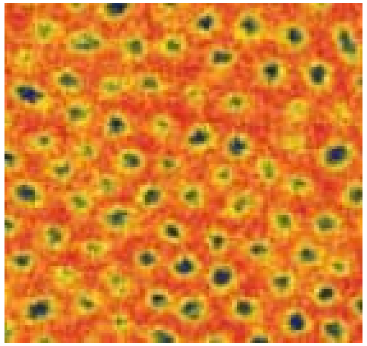
theory @ FZ Jülich (ab initio)
Frank Freimuth, Yuriy Mokrousov

experiments @ TU Munich, Germany
Ch. Pfleiderer, P. Böni, A. Bauer, A. Chacon,
T. Schulz, R. Ritz, M. Halder, M. Wagner,
C. Franz, F. Jonietz, M. Janoschek,
S. Mühlbauer, …

experiments @ TU Dresden, Germany
P. Milde, D. Köhler, L. Eng
+ Jan Seidel, University of New South Wales, Sydney

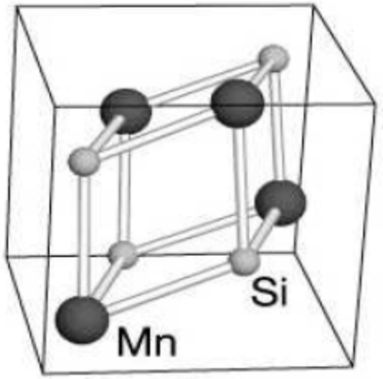


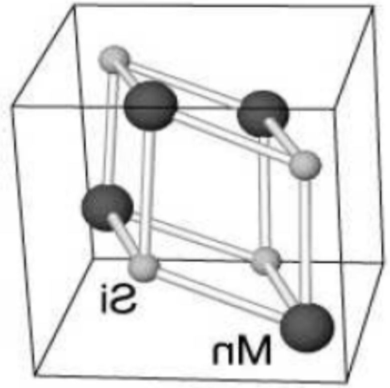




chiral magnets: e.g. MnSi

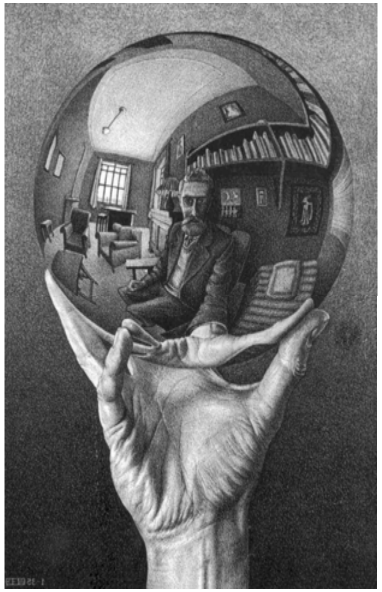
cubic but no inversion symmetry





left handed

right handed



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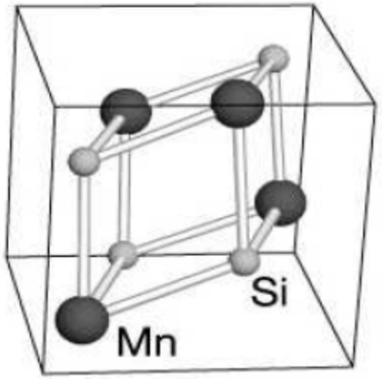


left handed

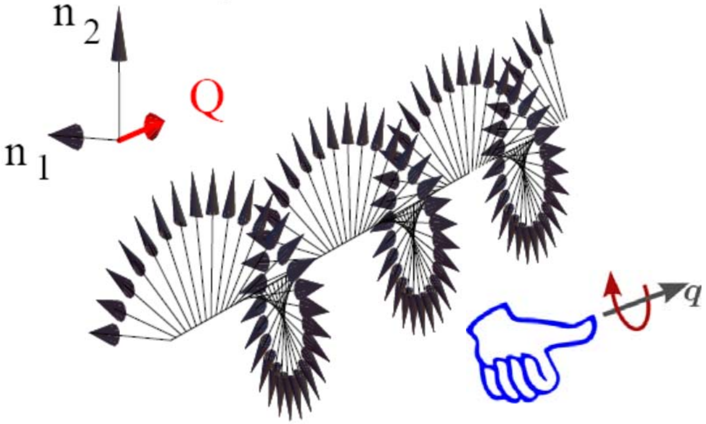
magnetic structures like to twist

(Dzyaloshinsky-Moriya interactions)

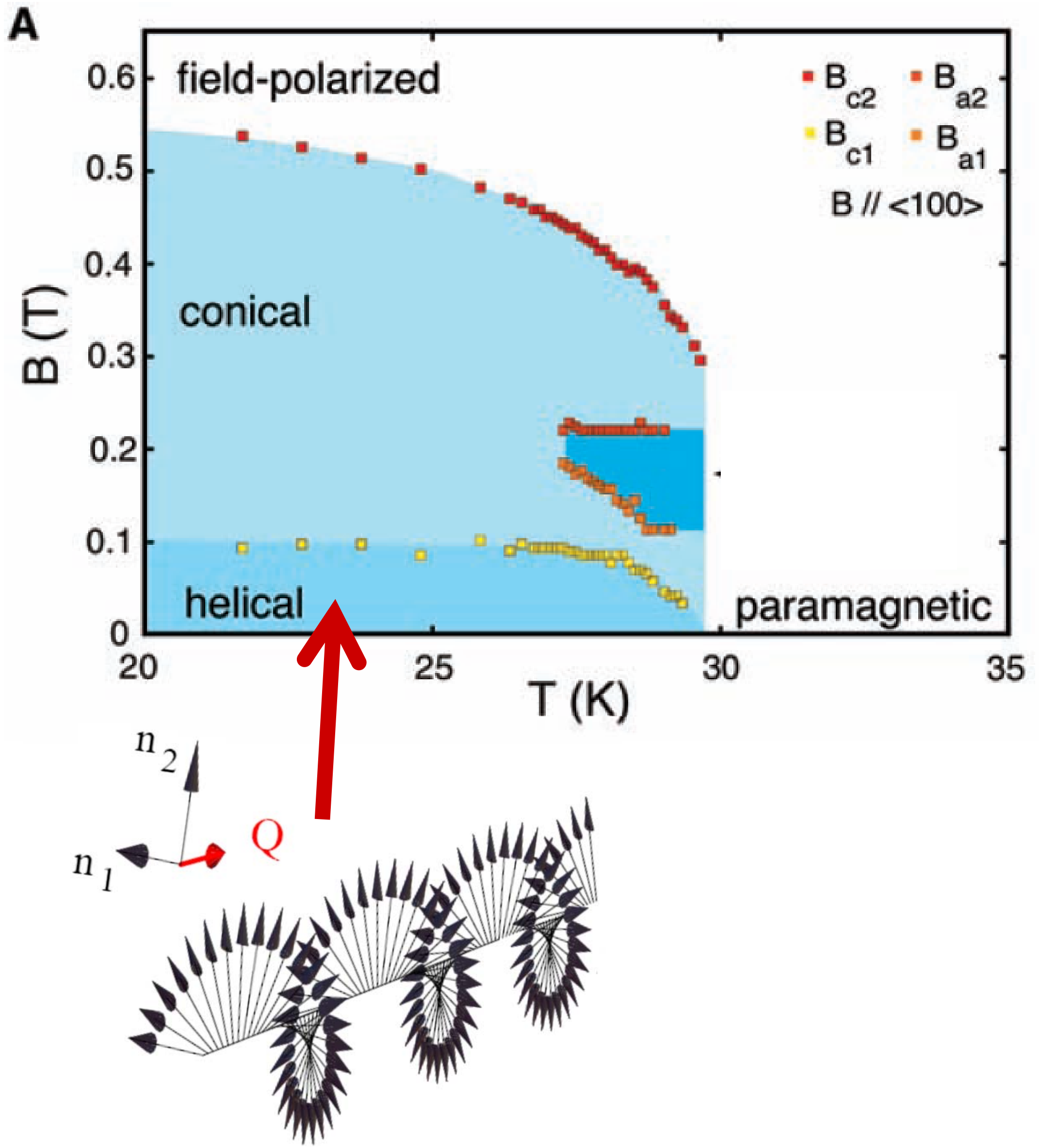
$$\int \vec{M} \cdot (\nabla \times \vec{M})$$

often: forbidden by inversion symmetry

here: allowed (crystal symmetry + by relativistic effects)



generic phase diagram of cubic magnets without inversion symmetry, here: MnSi



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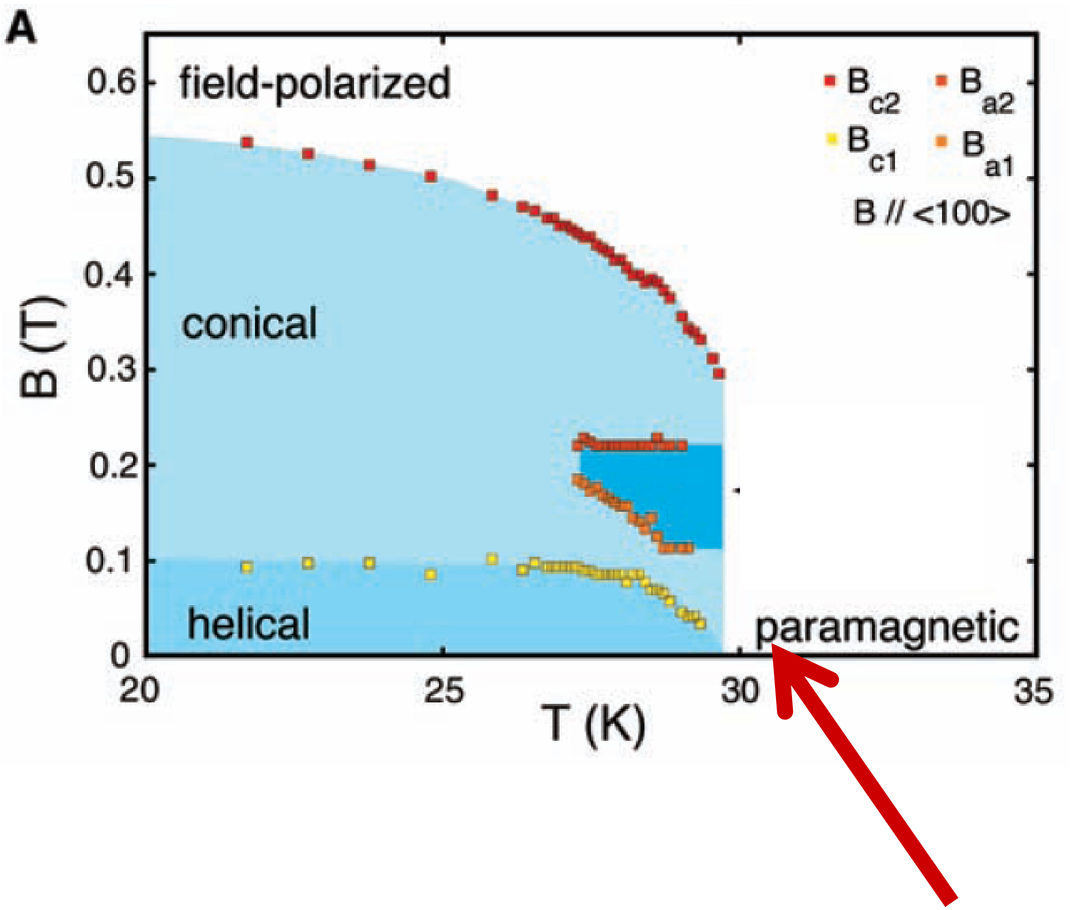
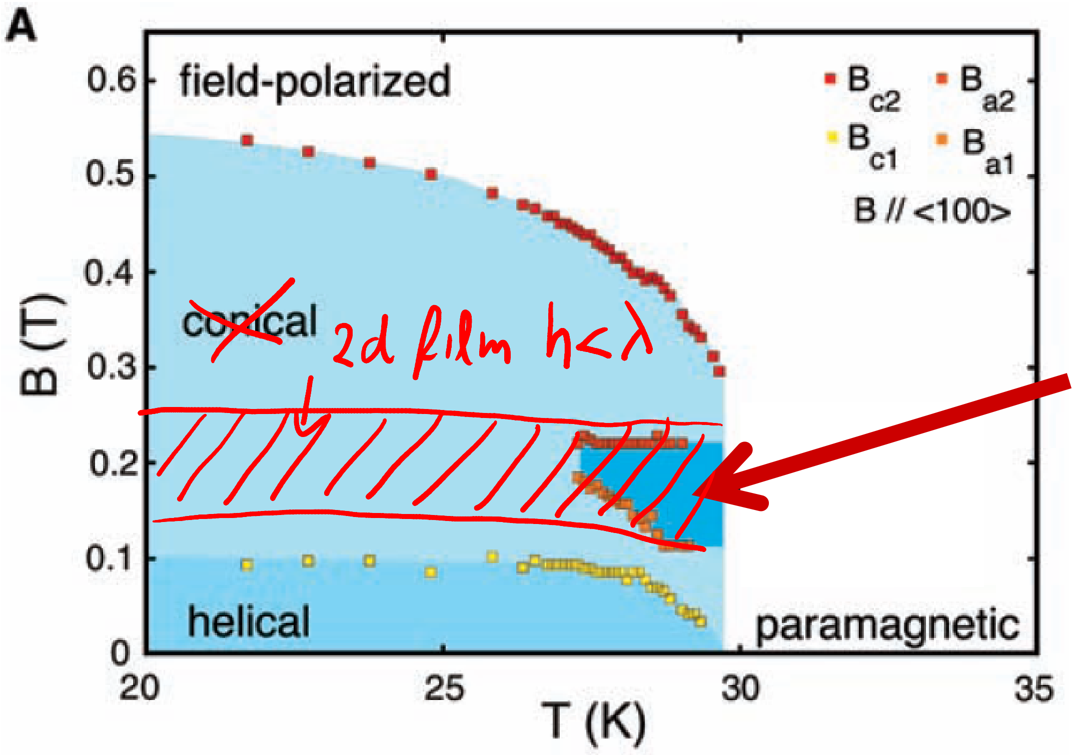
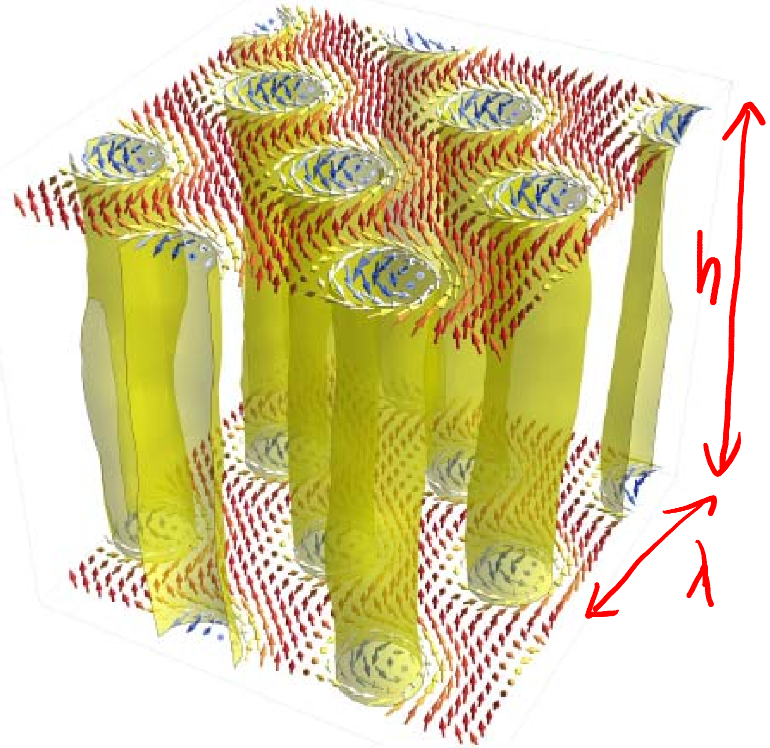


fluctuation induced first order transition

generic phase diagram of cubic magnets without inversion symmetry, here: MnSi



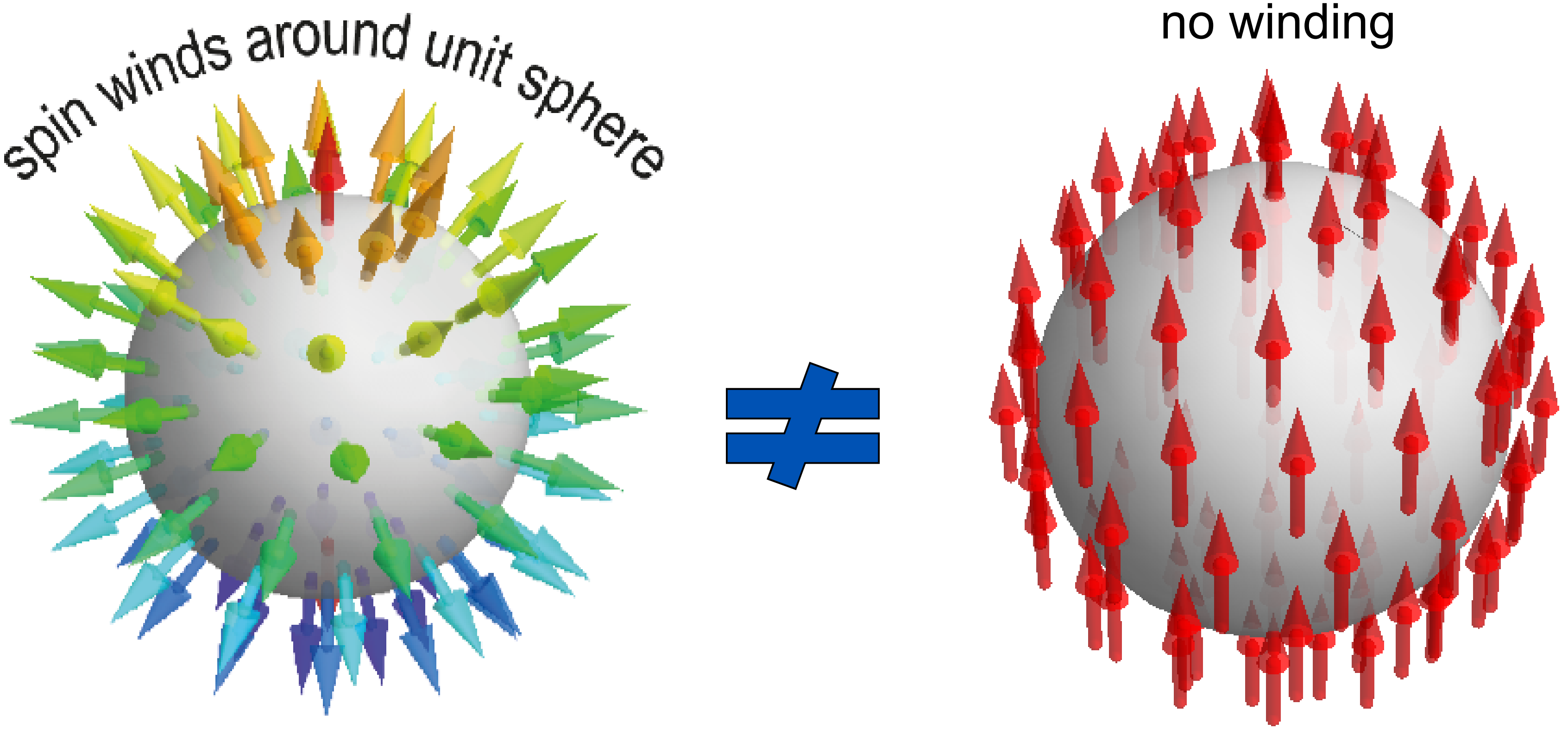


- lattice of magnetic whirls (skyrmion lattice, 2009)
- whirl-lines $\|\mathbf{B}$
  hexagonal lattice $\perp \mathbf{B}$
- length scale in MnSi: $200\,\text{Å}$

Mühlbauer, A.R. et al. , Science (2009)

hedgehog

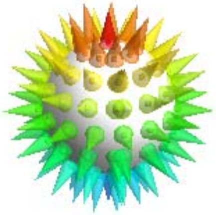
spin configuration $\Pi_2(S_2) = \mathbb{Z}$

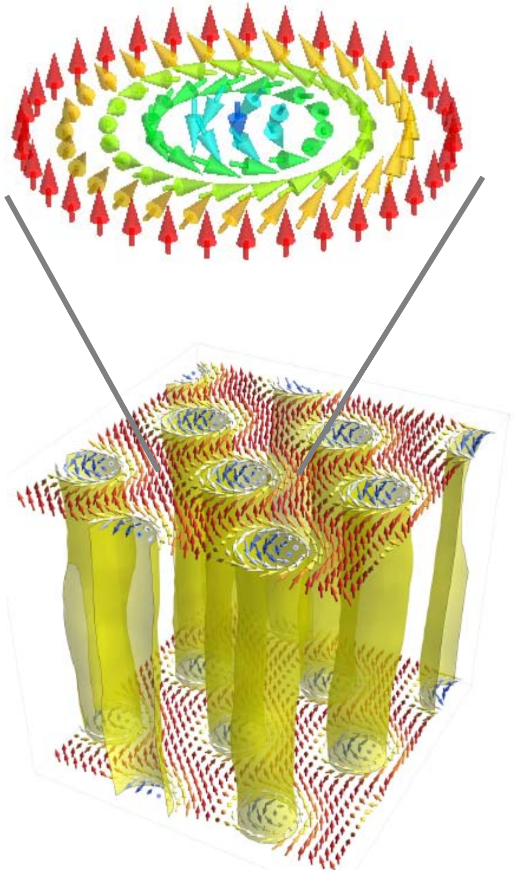


cannot smoothly be transformed into each other

⇔ hedgehog is topologically stable

hedgehog spin configuration

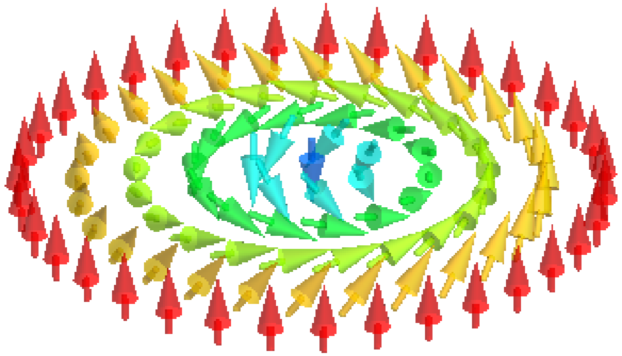


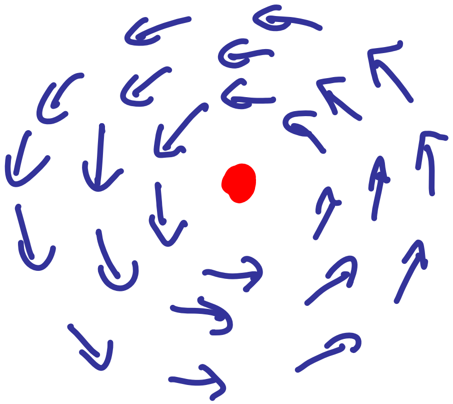


skyrmion

lattice of skyrmion lines
in 3d

comparison skyrmion vs. vortex





skyrmion:

- trivial at spatial infinity

- mapping of 2d real space to order parameter space $\Pi_2(S_2) = \mathbb{Z}$

$$\int \frac{dxdy}{4\pi}\, \hat{n} \cdot (\partial_x \hat{n} \times \partial_y \hat{n}) = -1$$

- smooth everywhere
- topologically quantized only as long as order parameter finite
- protected by **finite** energy barrier

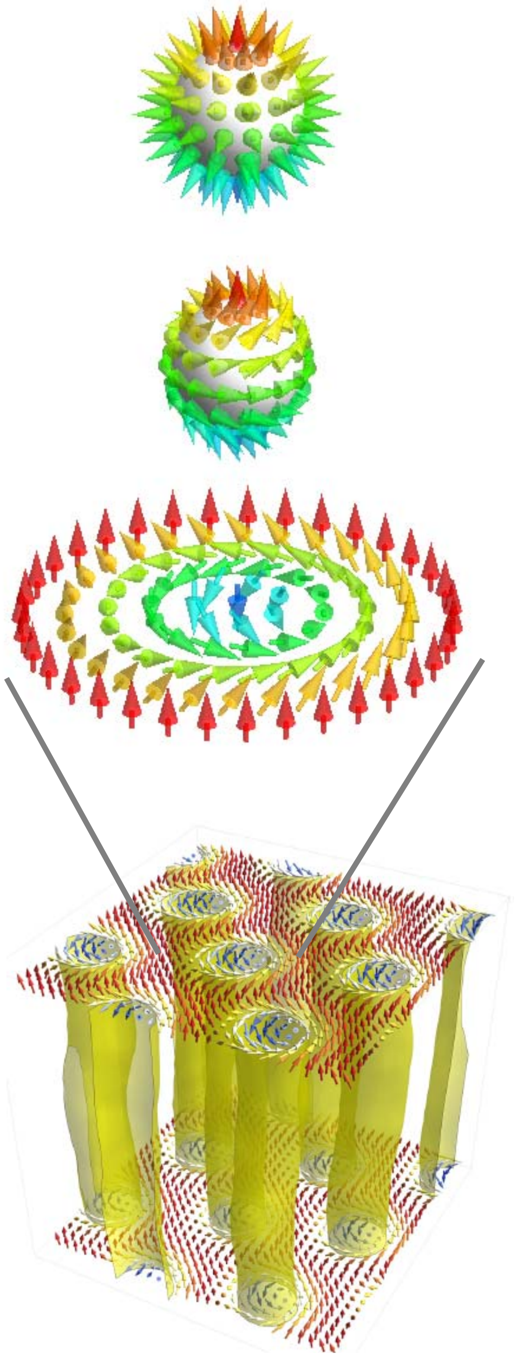
vortex:

- winding far away from vortex core
- mapping of points at infinity (1d) to order parameter space, e.g. superconductor or xy magnet:

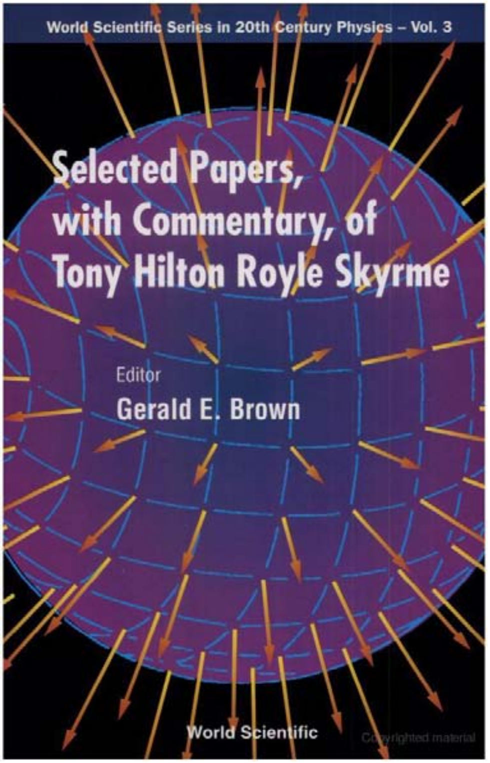
$$\Pi_1(S_1) = \mathbb{Z} \qquad \oint_{r=\infty} d\mathbf{r}\, \frac{d\phi}{d\mathbf{r}} = 2\pi n$$

- singular vortex core
- protected by **infinite** energy barrier

skyrmions in chiral magnets



1922-1987



World Scientific Series in 20th Century Physics – Vol. 3

Selected Papers, with Commentary, of Tony Hilton Royle Skyrme

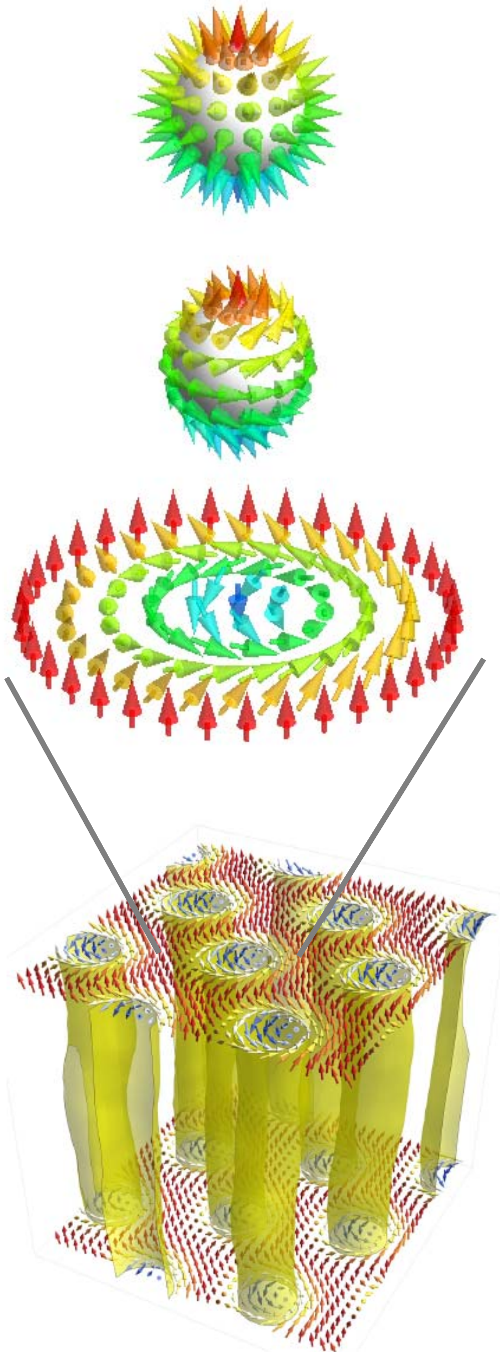
Editor
Gerald E. Brown

World Scientific

Skyrme (1962):

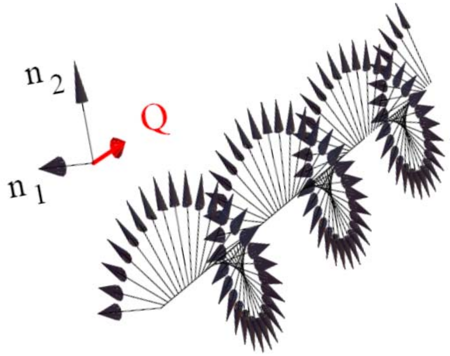
quantized topological defects in non-linear σ-model (d=3) for pions are baryons, i.e. spin-1/2 **fermions**

skyrmions in chiral magnets:



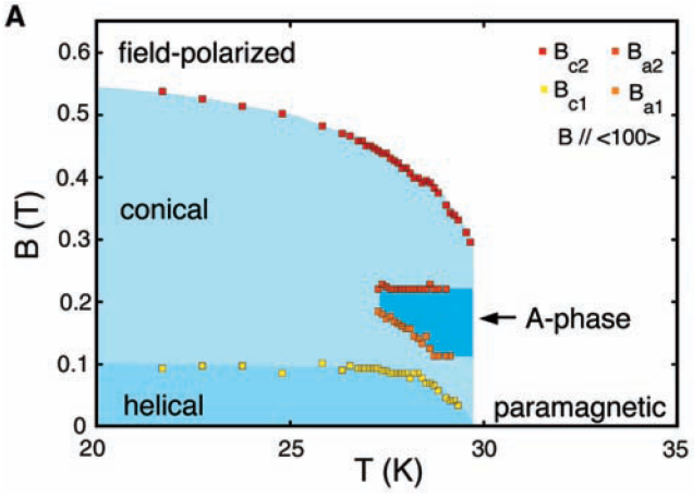
- Bogdanov, Yablonskii (1989): skyrmions energetically metastable in cubic **magnets without inversion symmetry**,
- skyrmions in **quantum Hall systems** close to $\nu=1$
  (Sondhi et al. 1993), lattices (Brey, Fertig, Cote, McDonald 1995, Timm Girvin, Fertig 1998, Green 2000)
  Destrat et al 2002, Gervais *et al.* 2005, Galais *et al.* 2008
- magnetic bubble domains: textures from dipolar interactions
- 2009: experimental discovery in MnSi
  Mühlbauer, A.R. et al. , Science (2009)

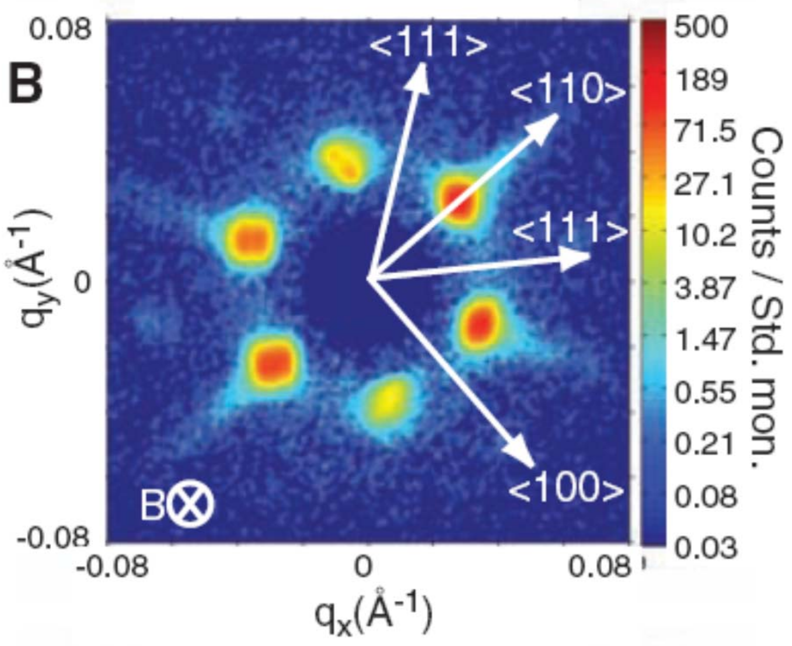
discovery of skyrmion lattice in MnSi

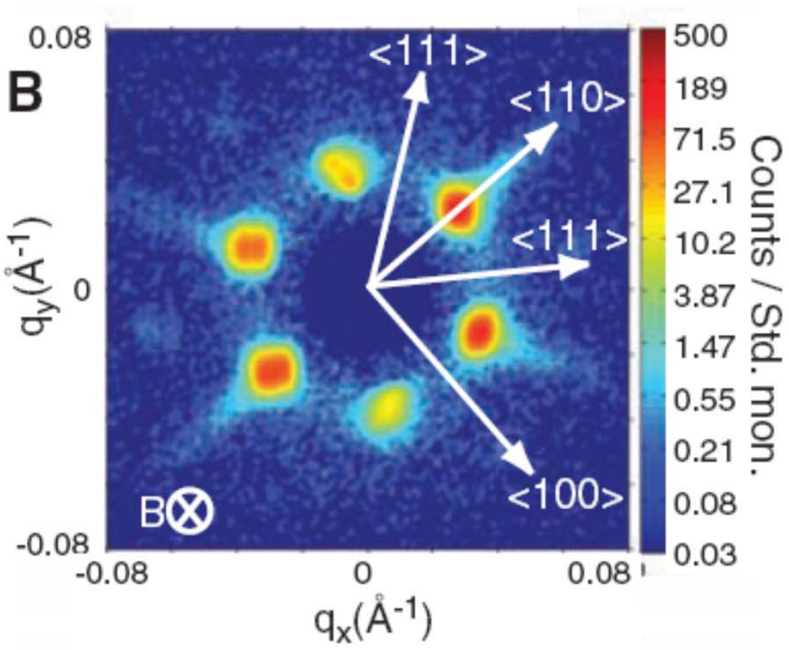


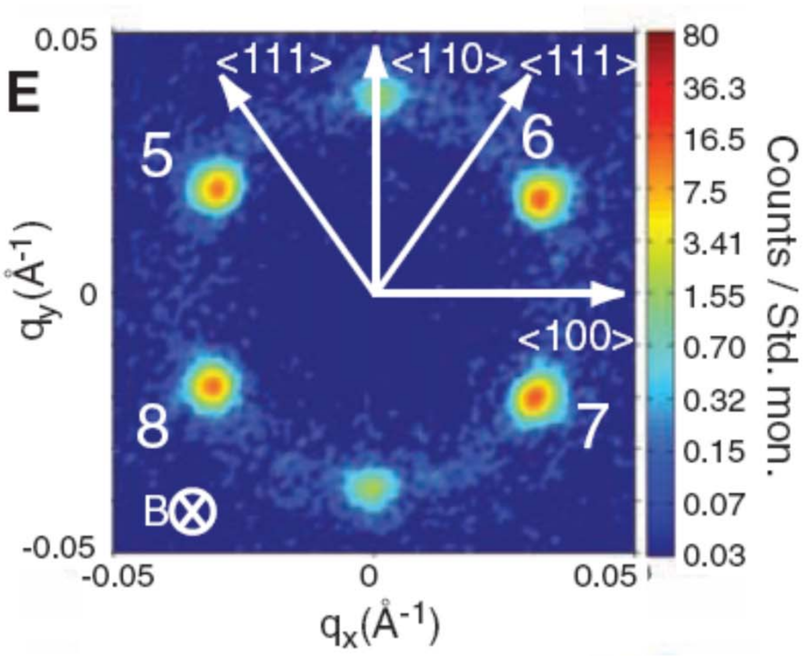
- original idea: manipulate helices by electric currents ☹
- surprise: previously unidentified “A-phase“ in MnSi sensitive to currents
- neutron scattering: measures Fourier components $|\Phi_{\vec{q}}|^2$ of magnetic structure

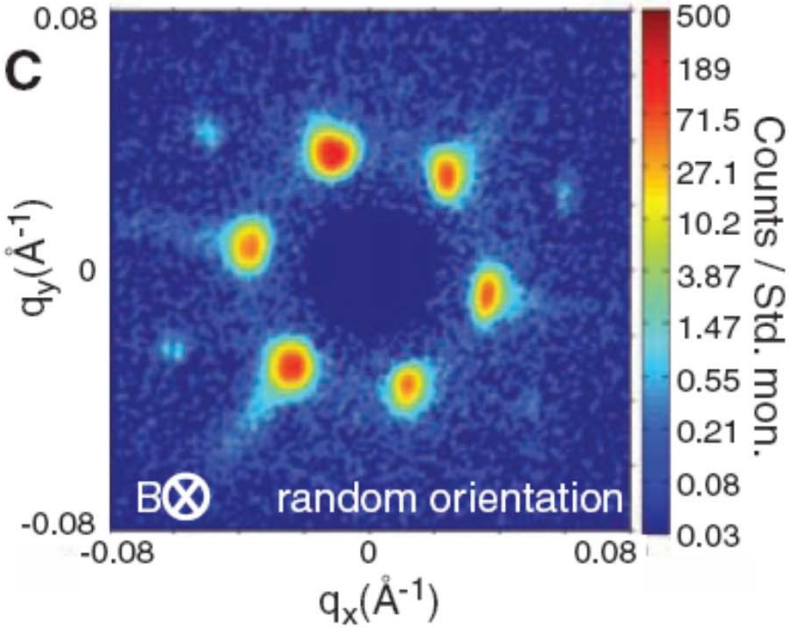
  in plane perpendicular to B:











first neutron scattering experiments:
6-fold symmetry in plane perpendicular to ***B*** for all orientations of ***B***

spins-crystal formed **independent**
from
underlying atomar crystal

Mühlbauer, Binz, Jonietz, Pfleiderer, Rosch, Neubauer, Georgii, Böni, Science (2009)

theory of skyrmion formation in cubic chiral magnets:

controlled by weakness of relativistic spin-orbit $\lambda_{\mathrm{SO}} \sim \alpha \ll 1$

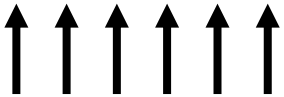
(Bak, Jensen 1980, Nakanishi et al. 1980)

$$\int \vec{M}\cdot(\nabla\times\vec{M})$$

$$F = F_{FM}(\vec{\Phi}^2) + \vec{q}^2|\vec{\Phi}_{\vec{q}}|^2 + \mathbf{k_h}\vec{\mathbf{q}}\cdot(\vec{\mathbf{\Phi}}_{\vec{\mathbf{q}}}\times\vec{\mathbf{\Phi}}^*_{\vec{\mathbf{q}}}) + \ldots$$

$O(\lambda_{\mathrm{SO}}^0)$: locally (itinerant) **ferromagnet**

below transition temperature: ferromagnetic order



energy cost to twist with wave vector *q*: $\vec{q}^2$

but: ***energy gain linear in q***

⇨ ferromagnetic state unstable

theory of skyrmion formation in cubic chiral magnets:

controlled by weakness of relativistic spin-orbit $\lambda_{\mathrm{SO}} \sim \alpha \ll 1$

(Bak, Jensen 1980, Nakanishi et al. 1980)

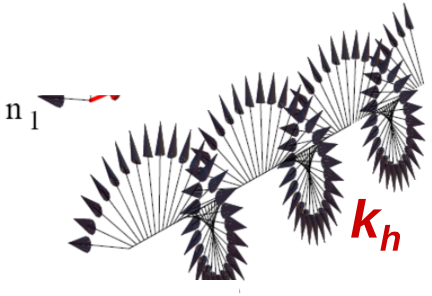
$$\int \vec{M} \cdot (\nabla \times \vec{M})$$

$$\parallel$$

$$F = F_{FM}(\vec{\Phi}^2) + \vec{q}^2 |\vec{\Phi}_{\vec{q}}|^2 + \mathbf{k_h} \vec{\mathbf{q}} \cdot (\vec{\mathbf{\Phi}}_{\vec{\mathbf{q}}} \times \vec{\mathbf{\Phi}}^*_{\vec{\mathbf{q}}}) + \ldots$$

$O(\lambda_{\mathrm{SO}}^0)$: locally (itinerant) **ferromagnet**

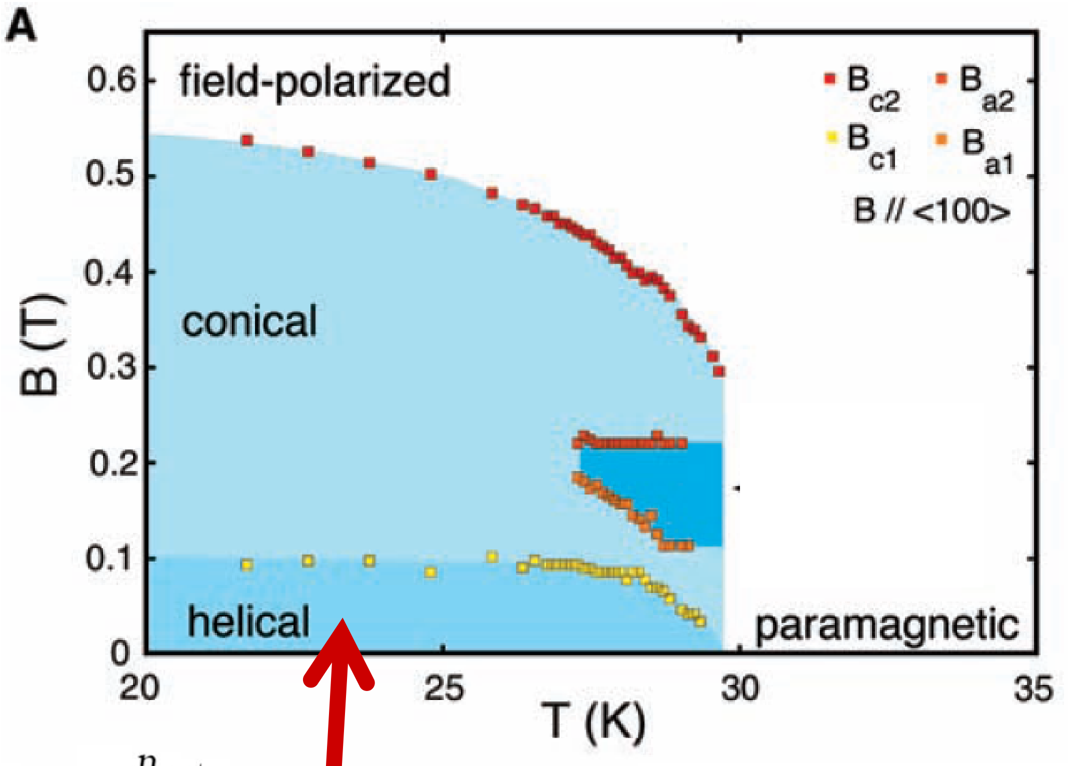
$O(\lambda_{\mathrm{SO}}^2)$: ferromagnet instable to twists
e.g., helical state
long pitch of $O(1/\lambda_{\mathrm{SO}})$

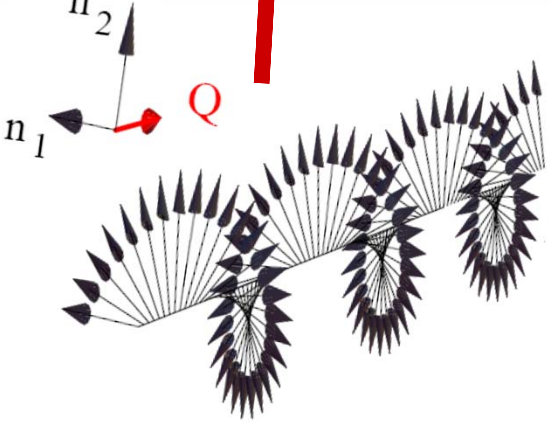


nominally same order of magnitude:
dipol-dipol interactions, in practice: almost no effect

$O(\lambda_{\mathrm{SO}}^4)$: terms breaking rotational symmetries,
e.g. preferential direction of helix

$\Big\}$ small due to cubic symmetry





explained:
phase diagram at B=0

small finite B:
Helix orients parallel to magnetic field (conical phase)

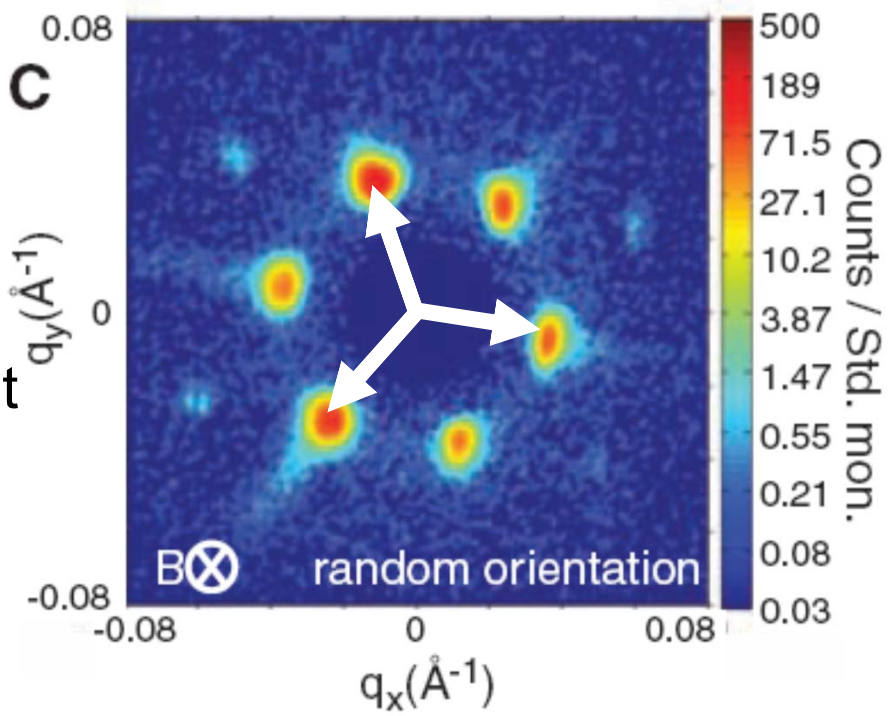
large B: field polarized state

why skyrmion phase?

Why spin-crystal stabilized by finite field?

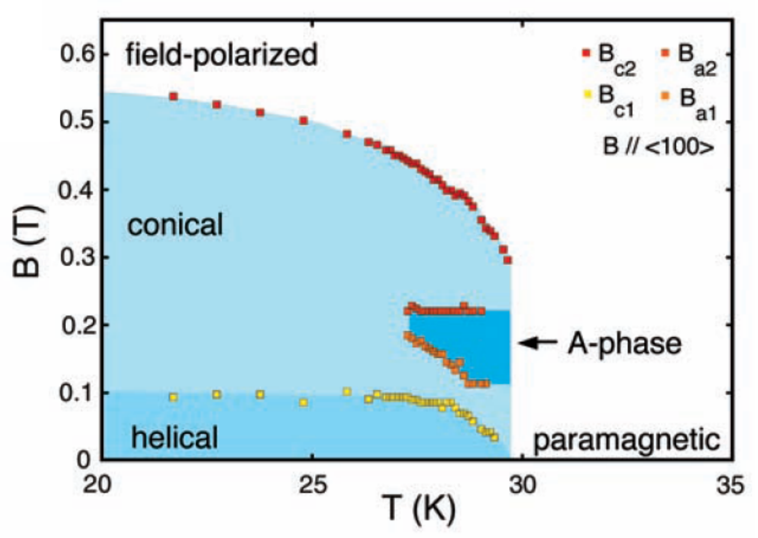
- preformed helices with ordering vector ***Q***
- Interactions in presence of finite magnetization **M**

$$\Phi^4 = \sum_{q_1,q_2,q_3} (\vec{\mathbf{M}}\vec{\Phi}_{\vec{q}_1})(\vec{\Phi}_{\vec{q}_2}\vec{\Phi}_{\vec{q}_3})\delta(\vec{q}_1+\vec{q}_2+\vec{q}_3)+\dots$$

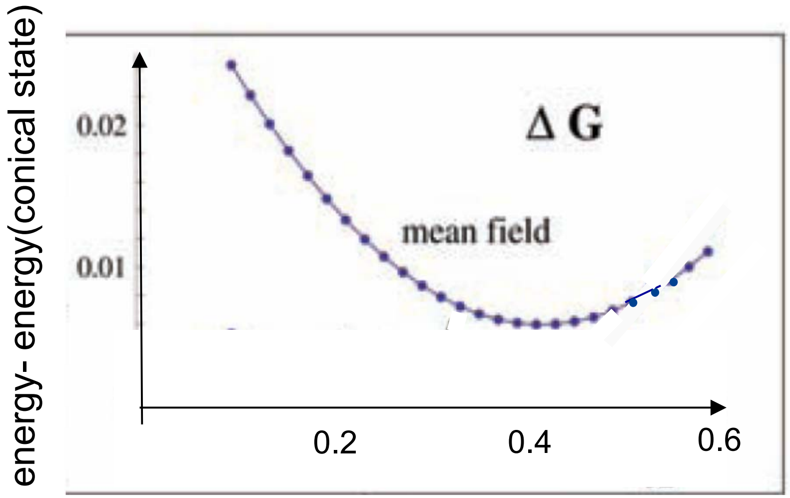


- **energy gain** if 3 ***q*** vectors add to zero
- relative phase defines magnetic structure
  calculation: skyrmion state is best
- Is this energy gain sufficient ?

theory of skyrmion formation



- easy to prove:
  within Ginzburg-Landau mean-field theory:
  helix parallel to B (conical state)
  only true mean-field ground state
- but: spin crystal very close
  in energy



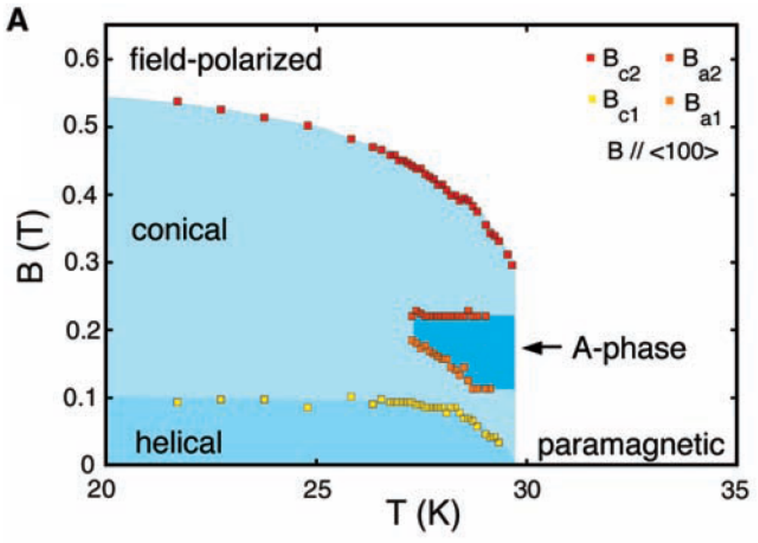
corrections to mean field?
Thermal fluctuations

$$\Phi = \Phi_0 + \delta\Phi$$

$$S \approx \beta F_0 + \frac{\beta}{2}\delta\Phi \left.\frac{\partial^2 F}{\partial\Phi\partial\Phi}\right|_{\Phi=\Phi_0} \delta\Phi$$

$$e^{-\beta F} = \int D[\Phi]\, e^{-S}$$

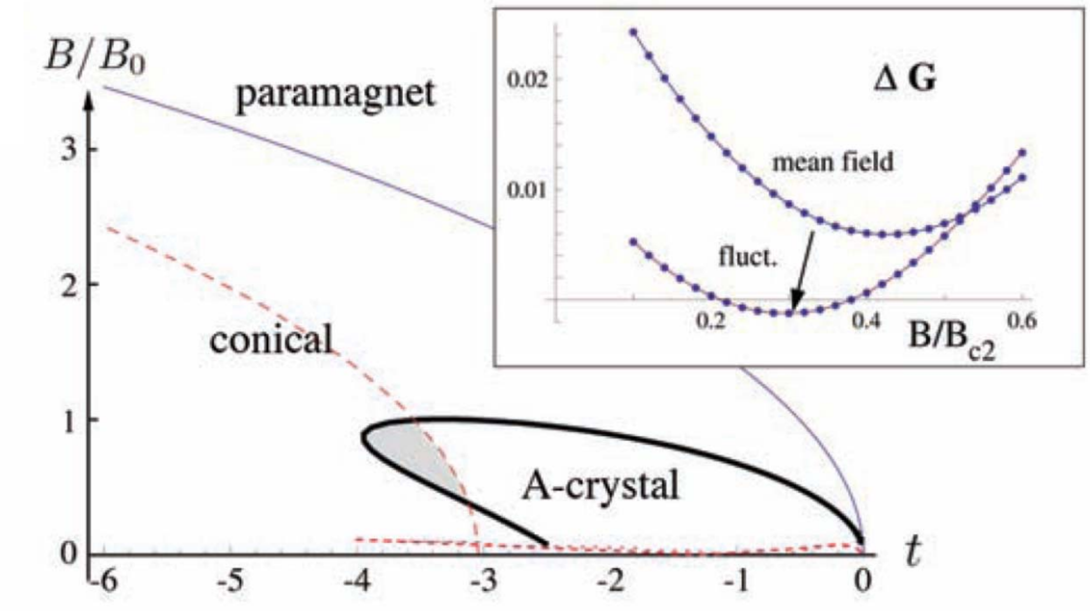
theory of skyrmion formation

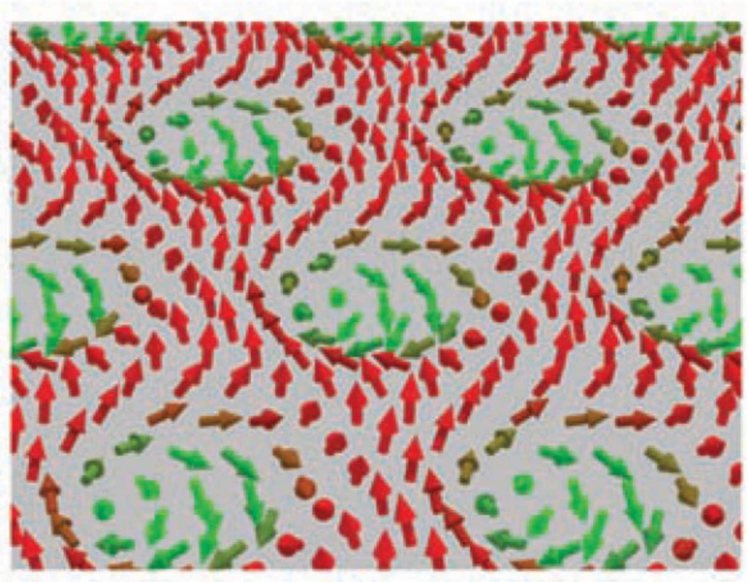


spin crystal stabilized by thermal fluctuations

$$F \approx F_0 + \mathrm{tr}\log \frac{\partial^2 F}{\partial\Phi\partial\Phi}$$

fluctuation driven 1st order transition but spin crystal lattice stabilized in regime (grey area) where fluctuations still „small“





theory of skyrmion formation:

$$F = F_{FM}(\vec{\Phi}^2) + \vec{q}^2|\vec{\Phi}_{\vec{q}}|^2 + \mathbf{k_h \vec{q}} \cdot (\vec{\mathbf{\Phi}}_{\vec{\mathbf{q}}} \times \vec{\mathbf{\Phi}}^*_{\vec{\mathbf{q}}}) + \ldots$$

mean field theory: skyrmion lattice **never** stable in cubic bulk system

in 3d: magnetic whirls stabilitzed by thermal fluctuations

Mühlbauer, A.R. et al. , Science (2009)

in 2d films: stable (already within mean-field theory)

down to T=0 Nagaosa et al. 2010

theory of skyrmion formation

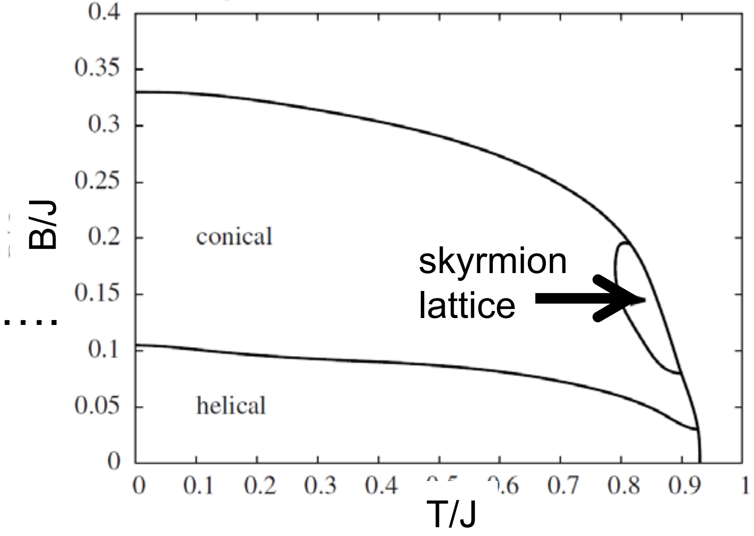
theory: **generic** phase for all cubic magnets without inversion symmetry (for weak spin-orbit)
confirmed by classical Monte Carlo calculations

experiment: **always** observed (B20 compounds)

many different systems:

MnSi, $Fe_xMn_{1-x}Si$, FeGe,$Fe_xCo_{1-x}Si$, Cu_2OSeO_3 ….

- metals, semiconductors, insulators
- thin films & bulk systems
- low T up to room-temperature
- from 10 to 1000 nm



Monte Carlo: S. Buhrandt, L. Fritz

also possible: nano-skyrmions in monolayer magnetic films
(e.g. Fe on Ir) Heinze, Wiesendanger et al. 2011

theory of skyrmion formation:

universality of low-energy theory of ferromagnet
+ smallness of relativistic effects
+ some luck (structure of non-perturbative effects)

⇨ **quantitative** description on level of a few % based on only few measured parameters of

- phase diagram, thermodynamics
- fluctuation induced 1st order transitions
- details of structure of skyrmion lattice

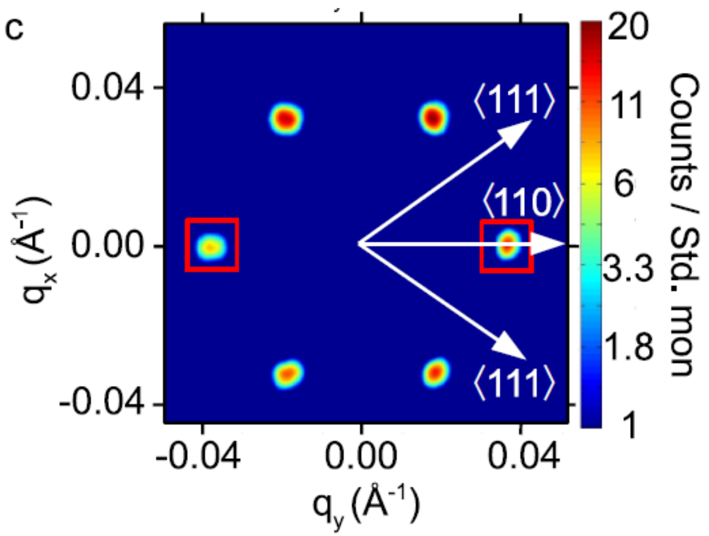
not covered
by these lectures

- magnetic excitation spectra (neutron scattering, FM resonance,…)

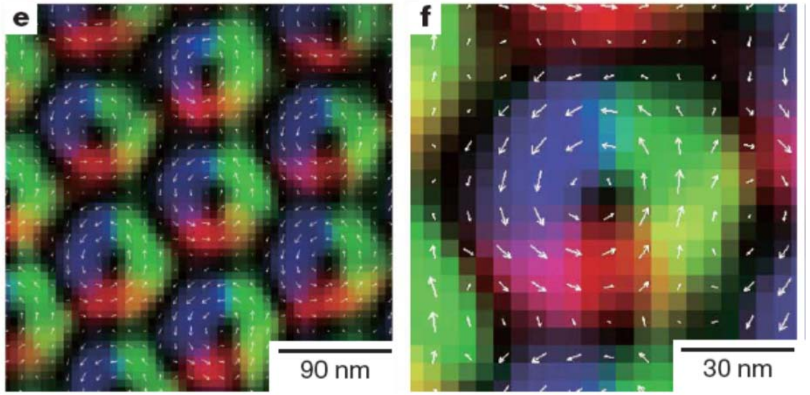
not understood:

- novel Berry phase effects
- exotic high-pressure phase in MnSi
- disorder & nonequilibrium effects, metastability,…

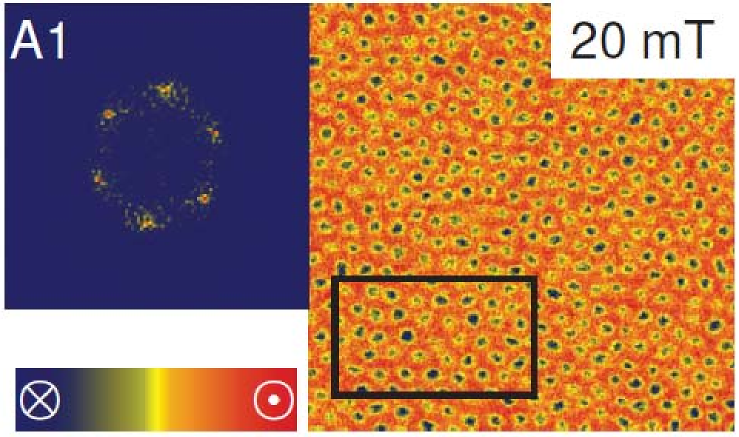
imaging skyrmions in chiral magnets



neutron scattering (here MnSi)
Pfleiderer, Böni, et al., 2009-2012



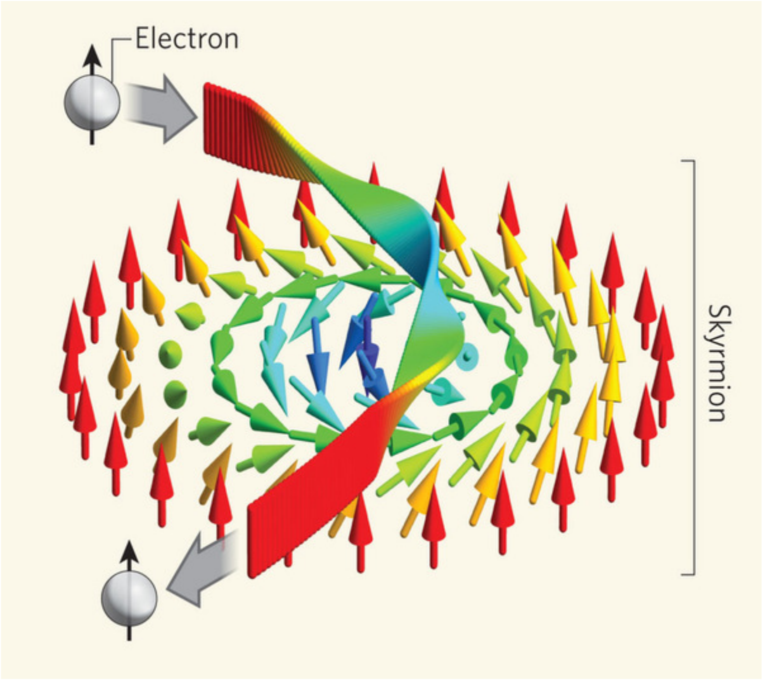
Lorentz transmission electron microscopy
(here: $Fe_{0.5}Co_{0.5}Si$ film)
Tokura group, 2010



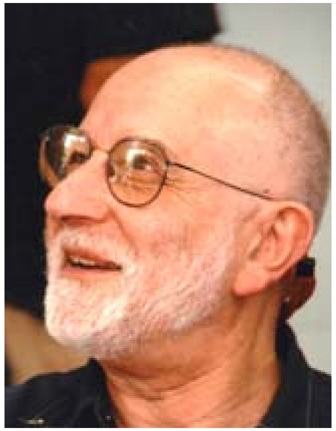
magnetic force microscopy
(here: surface of $Fe_{0.5}Co_{0.5}Si$)
Milde, Köhler, Seidel, Eng 2013

- coupling of skyrmions to electric currents?

⇨ emergent electrodynamics



Berry phases



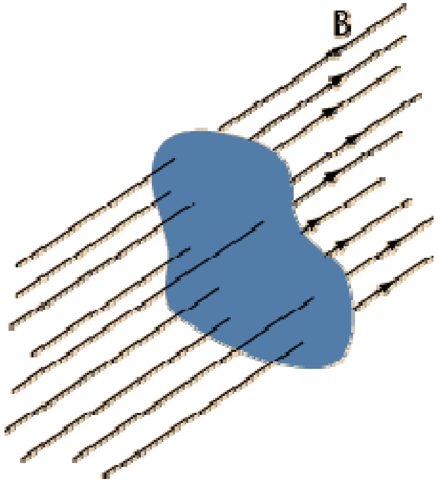
slowly changing quantum system: $H(t) = H(\vec{\lambda}(t))$

⇨ system remains in ground state

$$H(\vec{\lambda})|\vec{\lambda}\rangle = E_\lambda|\vec{\lambda}\rangle$$

but: wave function picks up a phase $|\Psi(t)\rangle = e^{i\phi(t)}|\vec{\lambda}(t)\rangle$

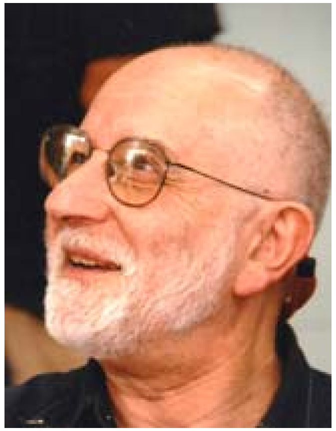
$$\phi = -\int_0^t \frac{E(\lambda(t'))}{\hbar}dt' + i\underbrace{\int_{\lambda_i}^{\lambda_f} \langle\vec{\lambda}|\frac{d}{d\lambda_i}|\vec{\lambda}\rangle d\lambda_i}_{\text{Berry phase, geometric property}}$$



example: magnetic field produces Aharonov-Bohm phase

$$\oint \frac{e}{\hbar}A(r)dr_i = 2\pi\frac{\Phi}{\Phi_0}$$

Berry phase of a spin S

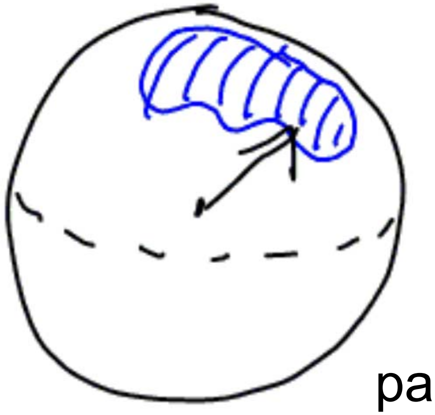


consider: $H(t) = -\vec{B}(t)\vec{S}$

spin direction follows field orientation $\hat{\mathbf{n}}$

$$|\Psi(t)\rangle = e^{i\phi(t)}|\hat{\mathbf{n}}(t)\rangle \qquad \phi = i\int \langle\hat{\mathbf{n}}|\frac{d}{d\hat{n}_i}|\hat{\mathbf{n}}\rangle d\hat{\mathbf{n}}_i$$

geometric phase of spin = spin size * area on unit-sphere enclosed by spin



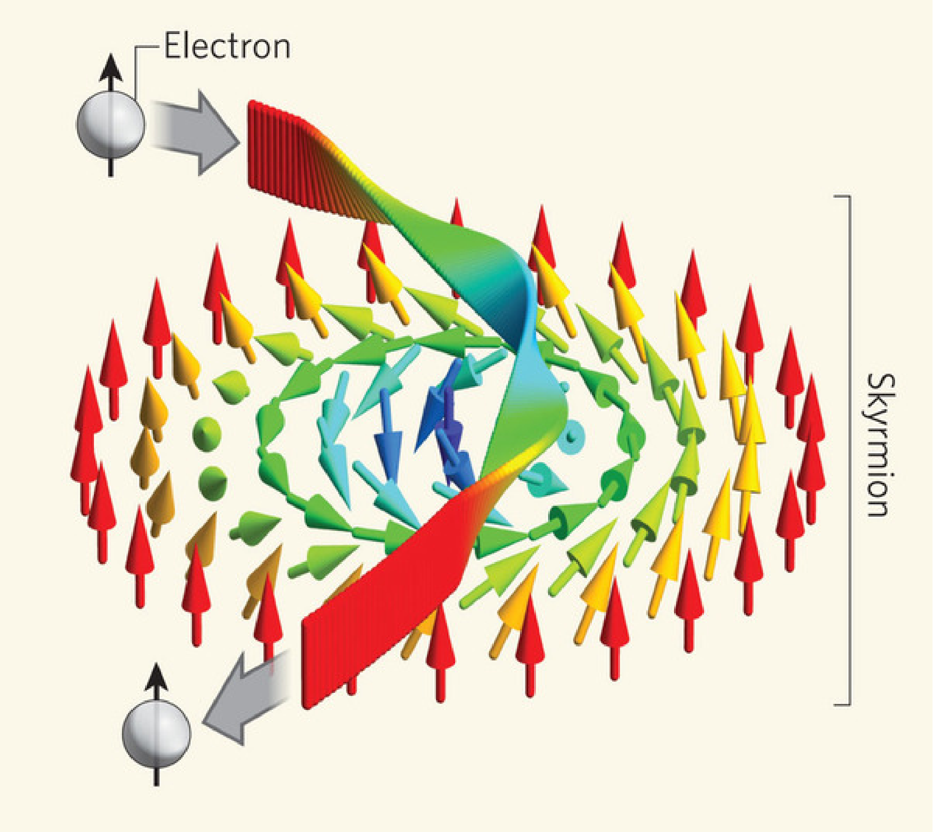
$$S_B = \hbar s \int dt\, \underbrace{\vec{A}(\hat{n})}\partial_t \hat{n}$$

monopole vector field counts area on surface

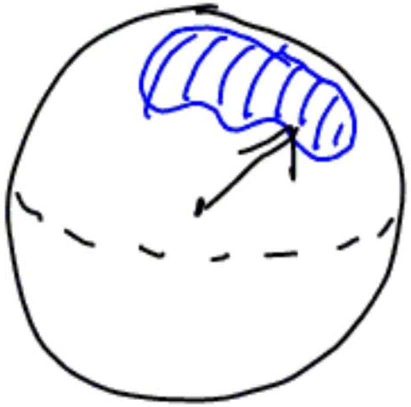
path-integral of spin: $\int D[\hat{n}]e^{iS_B/\hbar}$

surface on sphere: defined modulo 4 π ⇨ spin has to be half integer

coupling of electrons to skyrmions by Berry phases



geometric phase of spin
=
spin size * area on unit-sphere enclosed by spin



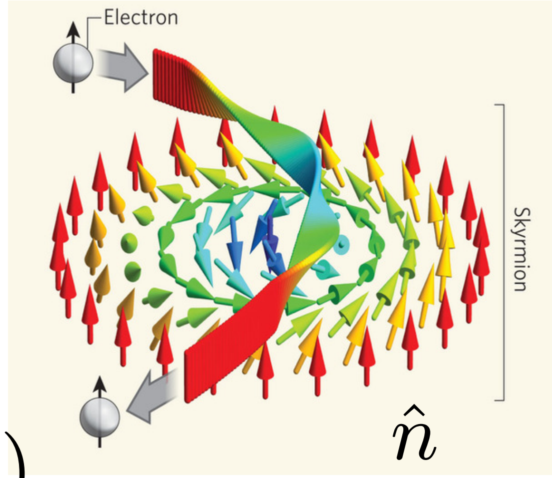
electron spin follows magnetic texture

➡ Berry phase proportional to winding number

Berry phase as Aharonov Bohm phase ➡ emergent electrodynamics

Volovik 87

microscopic derivation



electron spins follows adiabatically direction of background magnetization $\hat{n}$

choose local spin quantization action parallel to $\hat{n}$ by unitary transformation $U(\hat{n})$

rewrite action in new spinless fermion: $\mathbf{d}^\dagger = U^\dagger(\hat{n}) c^\dagger U(\hat{n})$

note: ***U*** not unique, U(1) Gauge degree of freedom

to do: gradient expansion of $\int c^\dagger_{\sigma k}(\partial_\tau + \epsilon_k) c_{\sigma k}$

$$S_B = \int \mathbf{j}^e_\mu \mathbf{A}^\mu_e \, d^3r \, dt$$

$$\mathbf{A}^\mu_e = U^\dagger \partial^\mu U$$

comoving quasiparticles couple to new **emergent electrodynamics**

Volovik 87

generalized Berry phases in phase space

- 7-dimensional phase space: $x^\mu = (t, \mathbf{R}, \mathbf{k})$
- electronic eigenstates in band *m* for constant direction of magnetization $\hat{\mathbf{n}}$: $|\hat{\mathbf{n}}, \mathbf{k}, m\rangle$
  (includes spin-orbit coupling effects in band structure)
- for slowly varying $\hat{\mathbf{n}}$ use eigenstates $|\mathbf{x}, m\rangle = |\hat{\mathbf{n}}(\mathbf{R}, t), \mathbf{k}, m\rangle$
  and express $\Psi^\dagger_\alpha(\mathbf{R}) = \sum_{n,\mathbf{k}} \langle \mathbf{R}, \alpha | \hat{\mathbf{n}}(\mathbf{R}, t), \mathbf{k}, m\rangle \, d^\dagger_{\mathbf{x},m}$
- Berry potential in phase space

$$A_\mu^{n,m} = \langle \mathbf{x}, n | \frac{d}{dx^\mu} | \mathbf{x}, m \rangle$$

generalized Berry phases in phase space

$$A^{n,m}_{\mu}$$

band indices,
possibly non-abelian
here: n = m, abelian

phase space & time (7-dimensional)
$\mu = t, x, y, z, p_x, p_y, p_z$

Berry curvature in phase space: semiclassics

$$\begin{pmatrix} \partial_t \mathbf{R} \\ \partial_t \mathbf{p} \end{pmatrix} = \begin{pmatrix} \partial_{\mathbf{p}} H \\ -\partial_{\mathbf{R}} H \end{pmatrix} + \begin{pmatrix} -\Omega^{tp} \\ \Omega^{tR} \end{pmatrix} + \begin{pmatrix} -\Omega^{pR}\,\partial_t \mathbf{R} & - & \Omega^{pp}\,\partial_t \mathbf{p} \\ \Omega^{Rp}\,\partial_t \mathbf{p} & + & \Omega^{RR}\,\partial_t \mathbf{R} \end{pmatrix}$$

$$\Omega_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{array}{cc} \begin{array}{ccccccc} \text{time} & & \text{position} & & & \text{momentum} & \end{array} & \\ \left(\begin{array}{ccccccc} 0 & . & . & . & . & . & . \\ . & 0 & . & . & . & . & . \\ . & . & 0 & . & . & . & . \\ . & . & . & 0 & . & . & . \\ . & . & . & . & 0 & . & . \\ . & . & . & . & . & 0 & . \\ . & . & . & . & . & . & 0 \end{array}\right) & \begin{array}{c} \text{time} \\ \\ \text{position} \\ \\ \\ \text{momen-} \\ \text{tum} \end{array} \end{array}$$

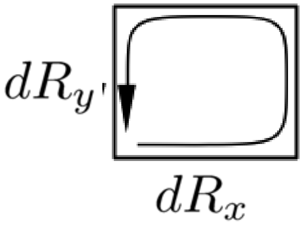
Berry curvature in 7 dimensions: 21 independent components

Xiao, Shi, Niu (2005)

Berry curvature in phase space: semiclassics

$$\begin{pmatrix}\partial_t \mathbf{R}\\ \partial_t \mathbf{p}\end{pmatrix} = \begin{pmatrix}\partial_{\mathbf{p}} H\\ -\partial_{\mathbf{R}} H\end{pmatrix} + \begin{pmatrix}-\Omega^{tp}\\ \Omega^{tR}\end{pmatrix} + \begin{pmatrix}-\Omega^{pR}\,\partial_t\mathbf{R} & - & \Omega^{pp}\,\partial_t\mathbf{p}\\ \Omega^{Rp}\,\partial_t\mathbf{p} & \boxed{+} & \boxed{\Omega^{RR}\,\partial_t\mathbf{R}}\end{pmatrix}$$

$$\Omega_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{array}{cccccccc} \text{time} & & \text{position} & & & \text{momentum} & & \\ 0 & . & . & . & . & . & . & \text{time} \\ . & 0 & . & . & . & . & . & \\ . & . & 0 & . & . & . & . & \text{position} \\ . & . & . & 0 & . & . & . & \\ . & . & . & . & 0 & . & . & \\ . & . & . & . & . & 0 & . & \text{momen-tum} \\ . & . & . & . & . & . & 0 & \end{array}$$



position space curvature:
emergent magnetic field

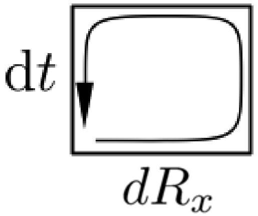
$$\Omega^{RR}\,\partial_t\mathbf{R} = \mathbf{v} \times \mathbf{B}^e$$

Lorentz force

Berry curvature in phase space: semiclassics

$$\begin{pmatrix} \partial_t \mathbf{R} \\ \partial_t \mathbf{p} \end{pmatrix} = \begin{pmatrix} \partial_{\mathbf{p}} H \\ -\partial_{\mathbf{R}} H \end{pmatrix} + \begin{pmatrix} -\Omega^{tp} \\ \boxed{\Omega^{tR}} \end{pmatrix} + \begin{pmatrix} -\Omega^{pR}\, \partial_t \mathbf{R} & - & \Omega^{pp}\, \partial_t \mathbf{p} \\ \Omega^{Rp}\, \partial_t \mathbf{p} & + & \Omega^{RR}\, \partial_t \mathbf{R} \end{pmatrix}$$

$$\Omega_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{array}{cc} \begin{array}{ccccccc} \text{time} & & \text{position} & & & \text{momentum} & \end{array} & \\ \begin{pmatrix} 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} & \begin{array}{c} \text{time} \\ \\ \text{position} \\ \\ \\ \text{momen-} \\ \text{tum} \end{array} \end{array}$$



time/space curvature:

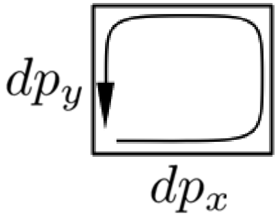
emergent electric field

$$\Omega^{tR} = \mathbf{E}^e$$

Berry curvature in phase space: semiclassics

$$\begin{pmatrix} \partial_t \mathbf{R} \\ \partial_t \mathbf{p} \end{pmatrix} = \begin{pmatrix} \partial_{\mathbf{p}} H \\ -\partial_{\mathbf{R}} H \end{pmatrix} + \begin{pmatrix} -\Omega^{tp} \\ \Omega^{tR} \end{pmatrix} + \begin{pmatrix} -\Omega^{pR}\,\partial_t \mathbf{R} & - & \boxed{\Omega^{pp}\,\partial_t \mathbf{p}} \\ \Omega^{Rp}\,\partial_t \mathbf{p} & + & \Omega^{RR}\,\partial_t \mathbf{R} \end{pmatrix}$$

$$\Omega_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{array}{cccccccc} \text{time} & & \text{position} & & & \text{momentum} & & \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \text{time} \\ \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \text{position} \\ \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \text{momentum} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \end{array}$$



momentum curvature:
anomalous velocity

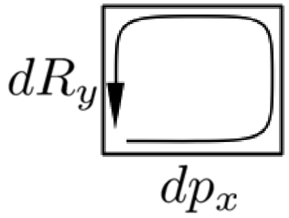
⇨

anomalous Hall effect in magnets
topology of band structure
(topological insulators)

Berry curvature in phase space: semiclassics

$$\begin{pmatrix}\partial_t \mathbf{R}\\ \partial_t \mathbf{p}\end{pmatrix} = \begin{pmatrix}\partial_{\mathbf{p}} H\\ -\partial_{\mathbf{R}} H\end{pmatrix} + \begin{pmatrix}-\Omega^{tp}\\ \Omega^{tR}\end{pmatrix} + \left(\boxed{\begin{matrix}-\Omega^{pR}\,\partial_t\mathbf{R}\\ \Omega^{Rp}\,\partial_t\mathbf{p}\end{matrix}} \begin{matrix}-\\+\end{matrix} \begin{matrix}\Omega^{pp}\,\partial_t\mathbf{p}\\ \Omega^{RR}\,\partial_t\mathbf{R}\end{matrix}\right)$$

$$\Omega_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{array}{ccccccccc} & \text{time} & & \text{position} & & & \text{momentum} & & \\ \left(\begin{matrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{matrix}\right. & \begin{matrix} \cdot \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{matrix} & \begin{matrix} \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{matrix} & \begin{matrix} \cdot \\ \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ \cdot \end{matrix} & \begin{matrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \end{matrix} & \begin{matrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \cdot \end{matrix} & \left.\begin{matrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{matrix}\right) & \begin{matrix} \text{time} \\ \\ \text{position} \\ \\ \\ \text{momen-} \\ \text{tum} \end{matrix} \end{array}$$



mixed momentum/
position curvature

⇨

change Hall angle
skyrmion charge
origin of DM interaction

Berry curvature in phase space: semiclassics

Xiao, Shi, Niu (2005)

modified Poisson brackets (or commutators) $\mathbf{x} = (\mathbf{R}, \mathbf{p})$

$$\{x_i, x_j\} = \left(\begin{pmatrix} \Omega^{RR} & \Omega^{Rp} \\ \Omega^{pR} & \Omega^{pp} \end{pmatrix} - \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} \right)^{-1}_{ij}$$

⇨ modified density of state in phase space & shifts of energies

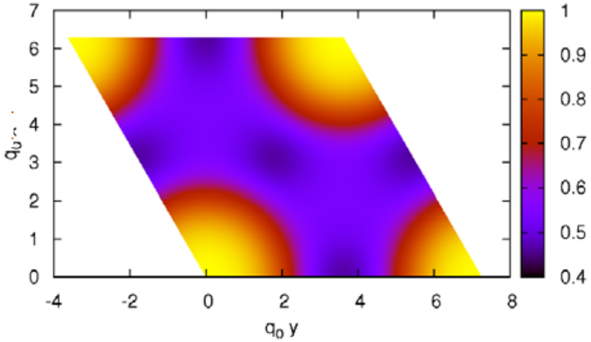
$$\frac{1}{(2\pi\hbar)^3} \to \frac{1}{(2\pi\hbar)^3} \left(1 - \sum_{i=x,y,z} \Omega^{Rp}_{ii} + \mathcal{O}(\Omega^2) \right)$$

$$\delta\epsilon_n(\mathbf{x}) = -\mathrm{Im}\left[\frac{\partial \langle \mathbf{x}, n|}{\partial R_i} (\epsilon_n^{(0)}(\mathbf{x}) - H(\mathbf{x})) \frac{\partial |\mathbf{x}, n\rangle}{\partial k_i} \right]$$

⇨ 1. Dzyloshinskii Moriya interaction = Berry curvature effect

2. Charge of skyrmion
(3.4*e* per skyrmion in MnSi
ignoring screening)

$D = -4.1 \text{ meV}\text{Å}$



3. Corrections to Hall effect,
emergent magnetic fields of unknown size

Bamler, Freimuth, Moukrosov, Rosch, 2013

from now on: only Berry phases in space & time

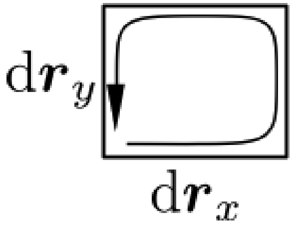
⇨ **emergent electro-magnetic fields**

emergent electrodynamics & topological quantization

- effective electric charge:
  spin parallel/antiparallel to local magnetization $\mathbf{q}^{\mathbf{e}}_{\downarrow/\uparrow} = \mp\frac{1}{2}$

- **emergent magnetic & electric fields**:

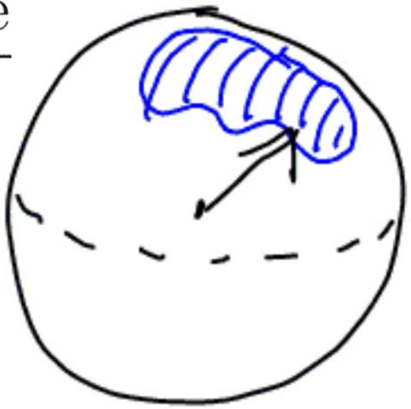
  Berry phase for loops
  in space



$$\mathbf{B}^{\mathbf{e}}_{\mathbf{i}} = \frac{\hbar}{2}\epsilon_{ijk}\hat{n}\cdot(\partial_j\hat{n}\times\partial_k\hat{n})$$

interpretation: Berry phase written as Aharonov Bohm phase

$$2\pi\frac{\int \mathbf{B^e}_z\, dr_x\, dr_y}{\mathbf{\Phi_0}} = \frac{\text{area on unit sphere}}{2}$$



emergent electrodynamics & topological quantization

- effective electric charge:
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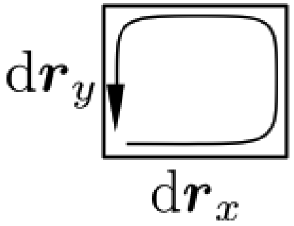
$$\mathbf{q}^{\mathbf{e}}_{\downarrow/\uparrow} = \mp\frac{1}{2}$$

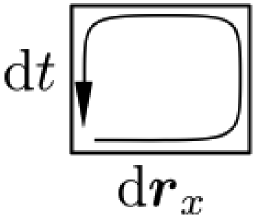
- **emergent magnetic & electric fields**:

Berry phase for loops
in space



$$\mathbf{B}^{\mathbf{e}}_{\mathbf{i}} = \frac{\hbar}{2}\epsilon_{ijk}\hat{n}\cdot(\partial_j\hat{n}\times\partial_k\hat{n})$$

Berry phase for loops
in space-time



$$\mathbf{E}^{\mathbf{e}}_{\mathbf{i}} = \hbar\,\hat{n}\cdot(\partial_i\hat{n}\times\partial_t\hat{n})$$

- **topological quantization**:

winding number -1 ⟺ one flux quantum per skyrmion

measure skyrmion-winding number by **topological Hall** effect

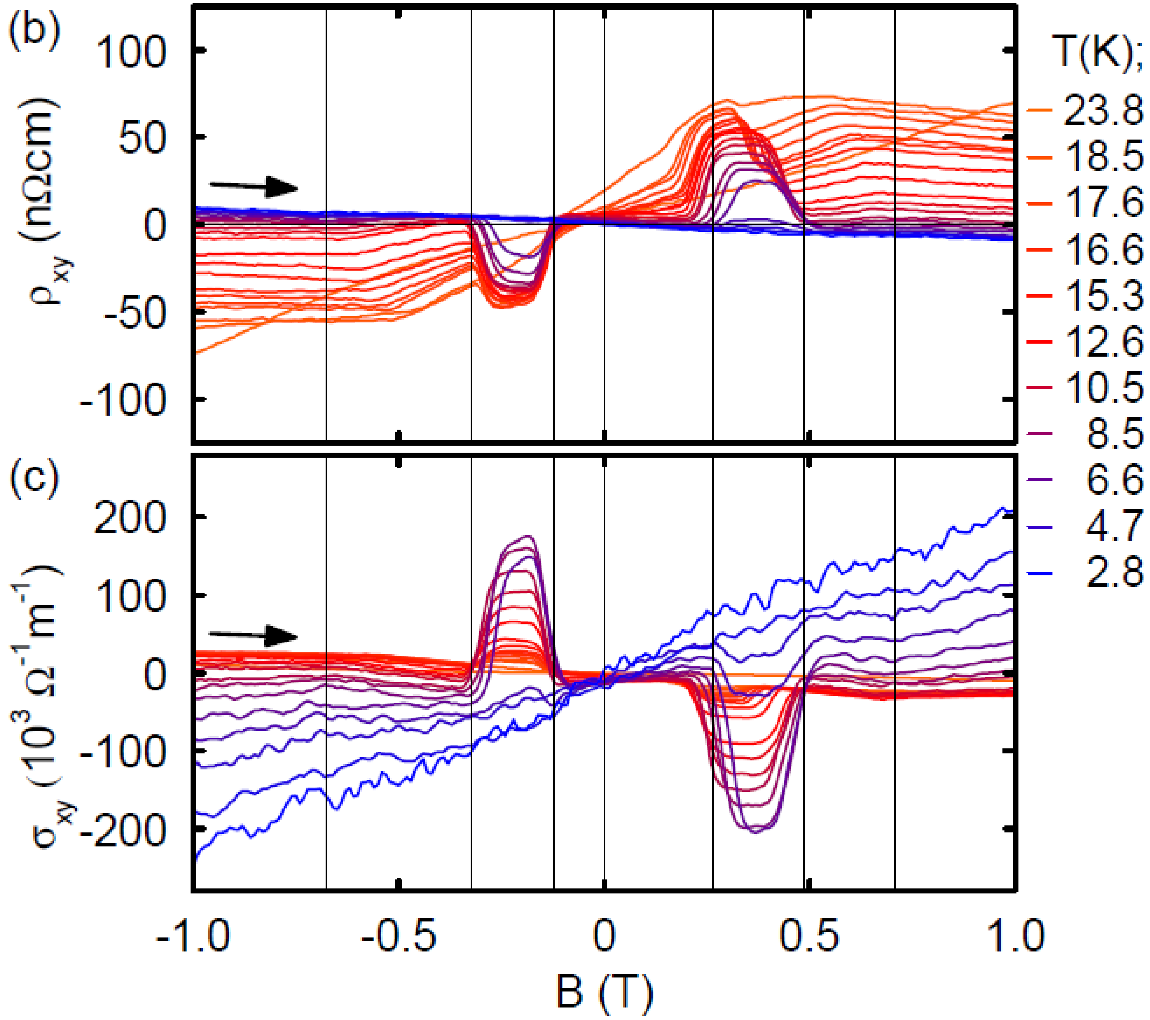
one flux quantum of emergent magnetic flux per unit cell:

in MnSi

$$\mathbf{B^e} \sim -12\,T$$

Ritz et al. (2013)
A. Neubauer, et al. PRL (2009)

possible: 100 x larger fields



MnSi under pressure (7kbar) for various temperatures

emergent **Faraday‘s law of induction**

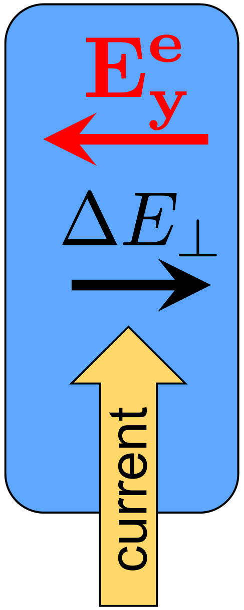
moving magnetic field ➡ electric field

$$\mathbf{E}^{\mathbf{e}} = -\mathbf{v}_{\mathbf{d}} \times \mathbf{B}^{\mathbf{e}}$$

➡ detect skyrmion motion

measuring skyrmion motion & emergent **Faraday law**

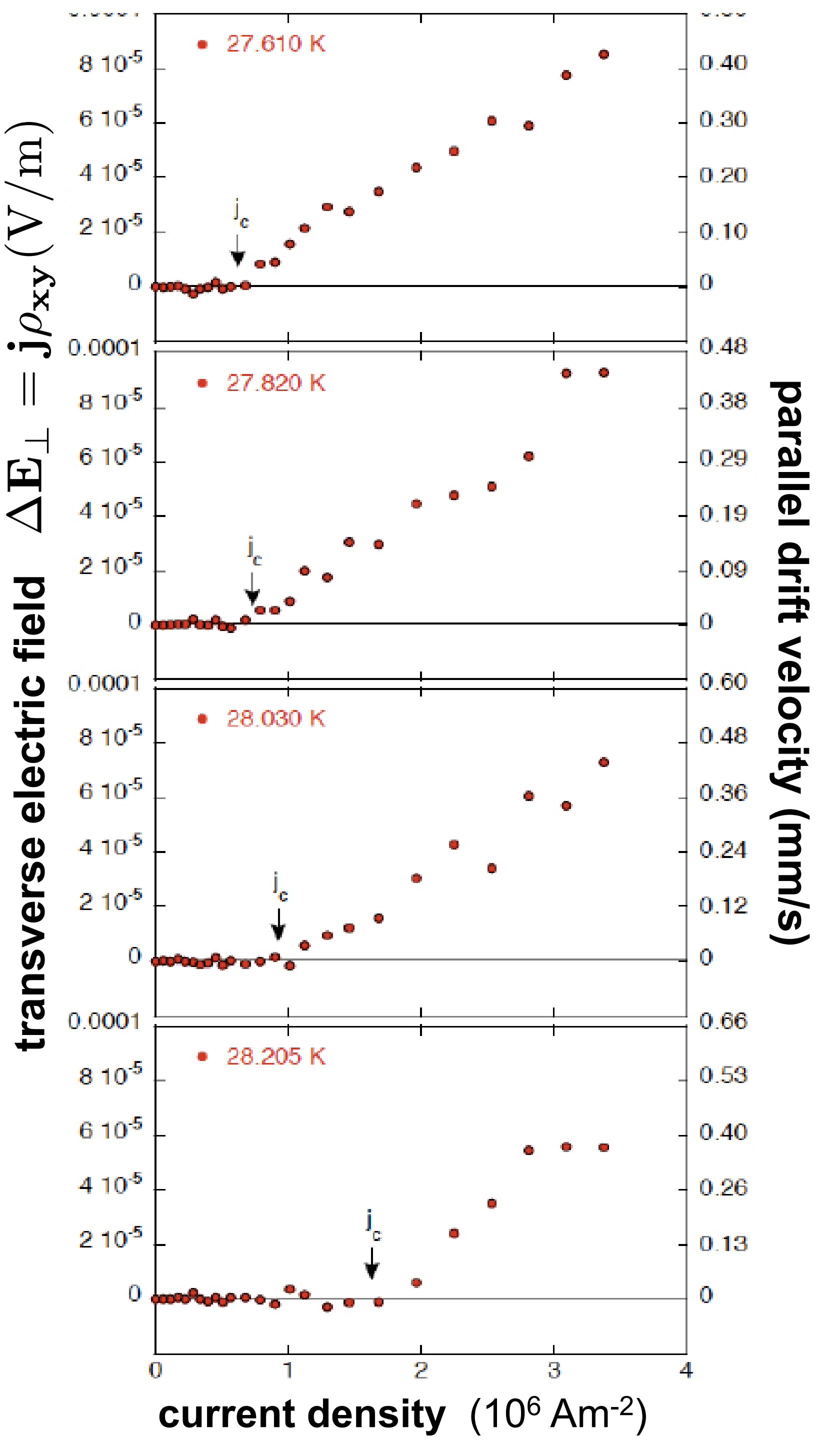
moving skyrmions ⇨ emergent electric field $\mathbf{E^e} = -\mathbf{v_d} \times \mathbf{B^e}$



extra „real" electric field compensates emergent field

$$\Delta E_\perp \approx -\tilde{P}\,\mathbf{E^e_y}$$

conversion factor:
effective spin polarization $\tilde{P} = \frac{\langle\langle j, \mathbf{j^e} \rangle\rangle}{\langle\langle j, j \rangle\rangle}$



skyrmions start to move above ultrasmall critical current density ~ **10^6 Am^{-2}**

critical current **5-6 orders of magnitude smaller** than in typical spin-torque experiments

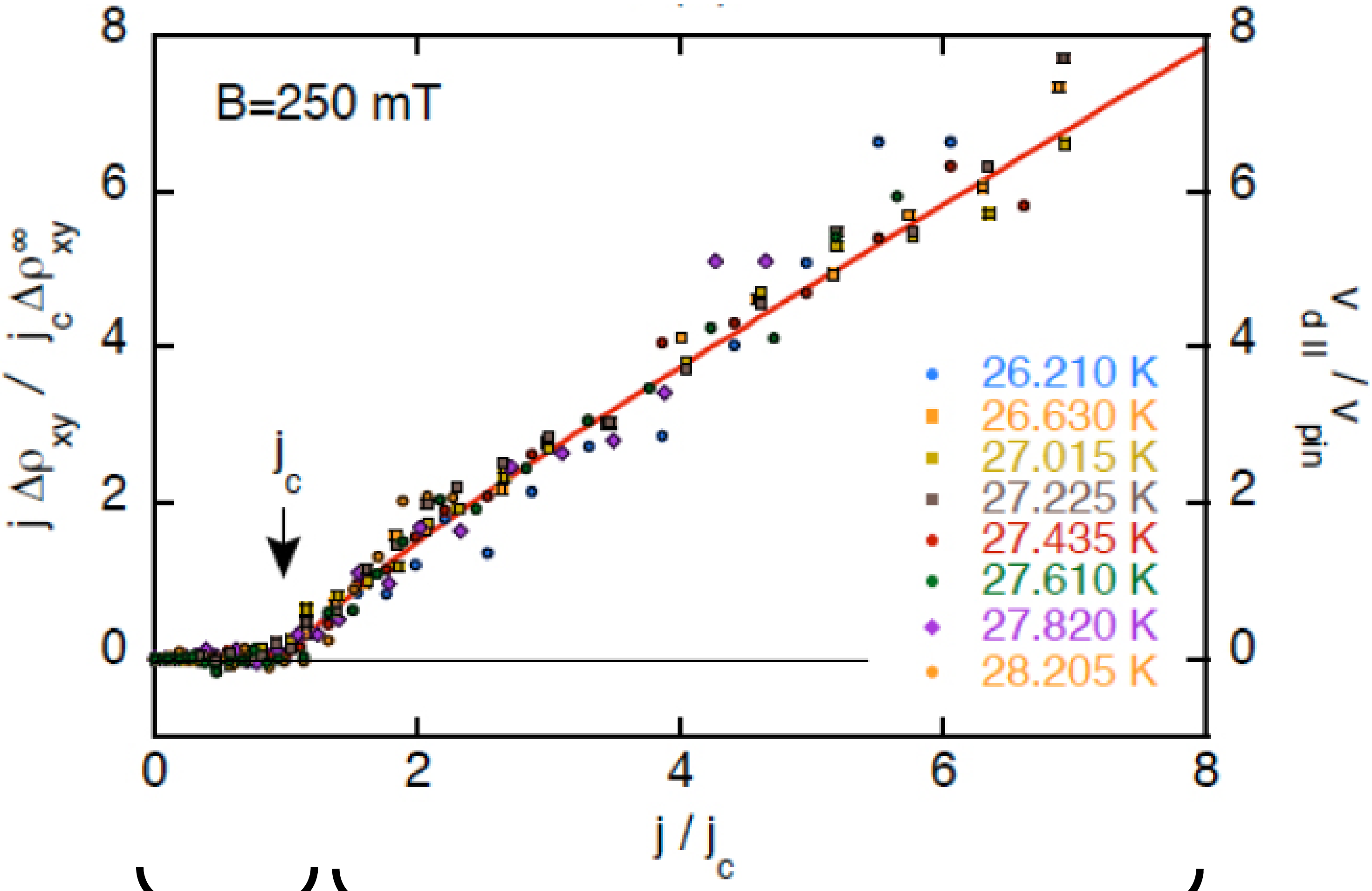
velocity: comparable to drift velocity of electrons

$$v_{\rm drift} \sim \frac{j}{en} \sim 0.16\,\frac{\rm mm}{s}\,\frac{j}{10^6\,Am^2/s}$$

Jonietz, Pfleiderer, A.R., *et al.* (2010)

Schulz, Pfleiderer, A.R., *et al.* (2012)

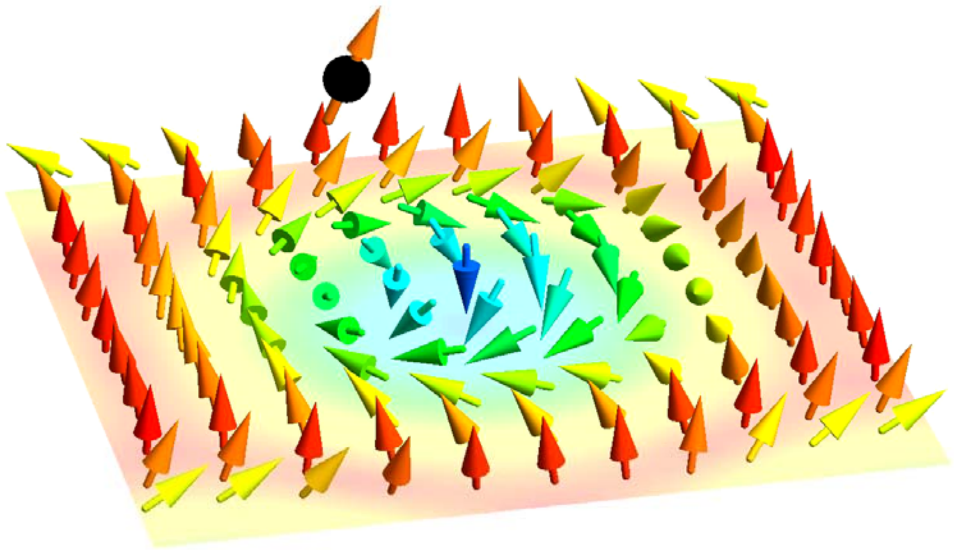
scaling plot



skyrmions
„pinned“ by
weak disorder

skyrmion depin and
follow electron drift

coupling currents to magnetism



counter force to

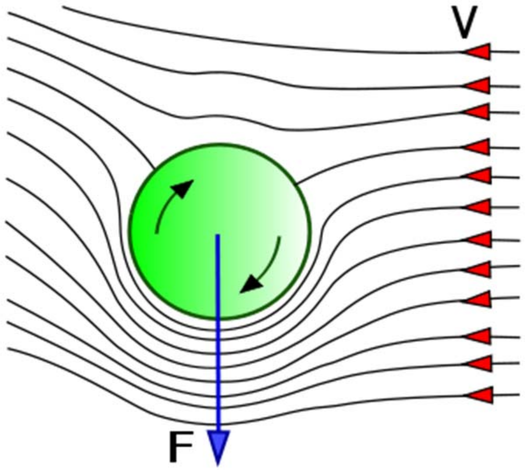
emergent Lorentz force

alternative point of view:

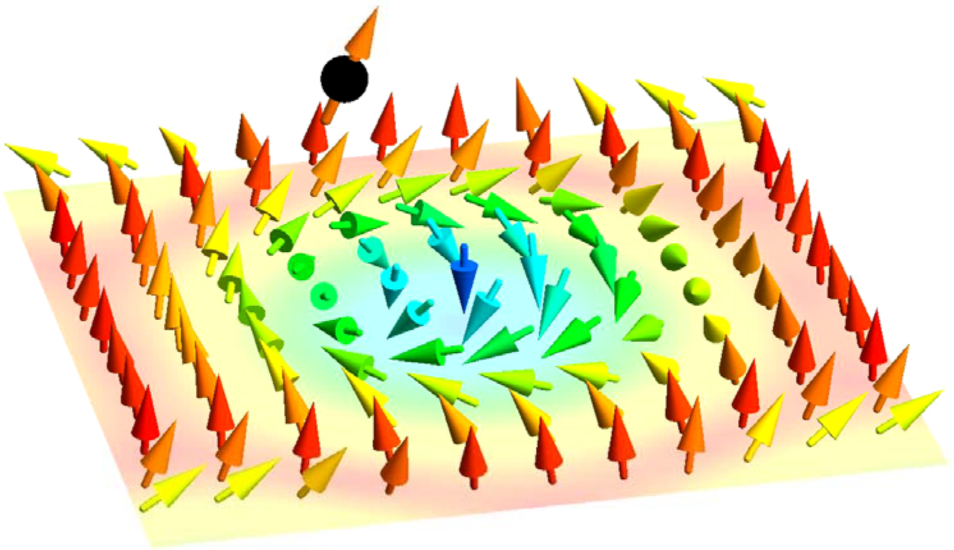
Skyrmion lattice = **rotating spin-supercurrents** $j_i \sim \boldsymbol{M} \times \nabla_i \boldsymbol{M}$

in presence of charge current:
extra **dissipative spin current**

Interplay: **Magnus force**



coupling currents to magnetism



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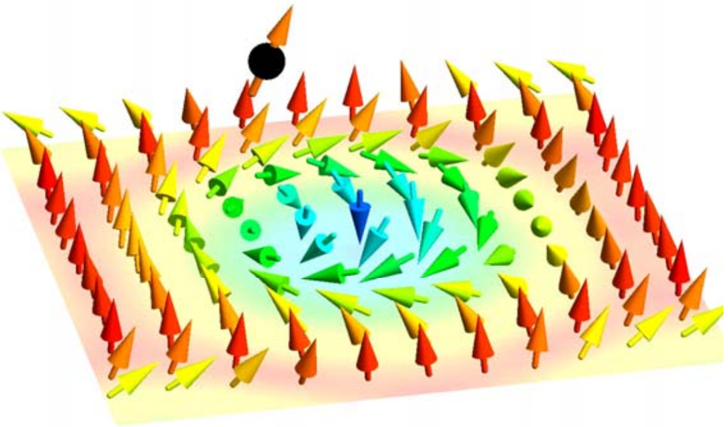
Interplay: **Magnus force**

Roberto Carlos 1997

Why ultrasmall critical current densities?



- very **efficient Berry-phase coupling**
  (gyromagnetic coupling by adiabatic spin transfer torques
  Magnus force:

$$\vec{\mathcal{G}} \times \left(\dot{\vec{R}} - \vec{v}_s\right)$$

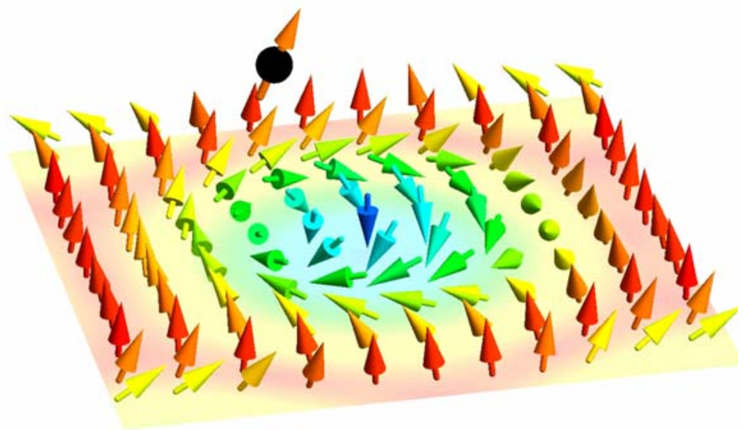
“gyrocoupling“

skyrmion velocity

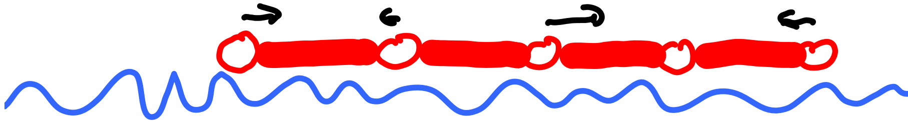
electronic drift-velocity (spin-current /magnetization)

$$\mathcal{G} = 4\pi M \frac{\hbar}{a^2} \sim \frac{\text{flux quantum}}{\text{skyrmion size}} \times \frac{\text{spins}}{\text{skyrmion}} \sim 100.000\,\mathrm{T}/e$$

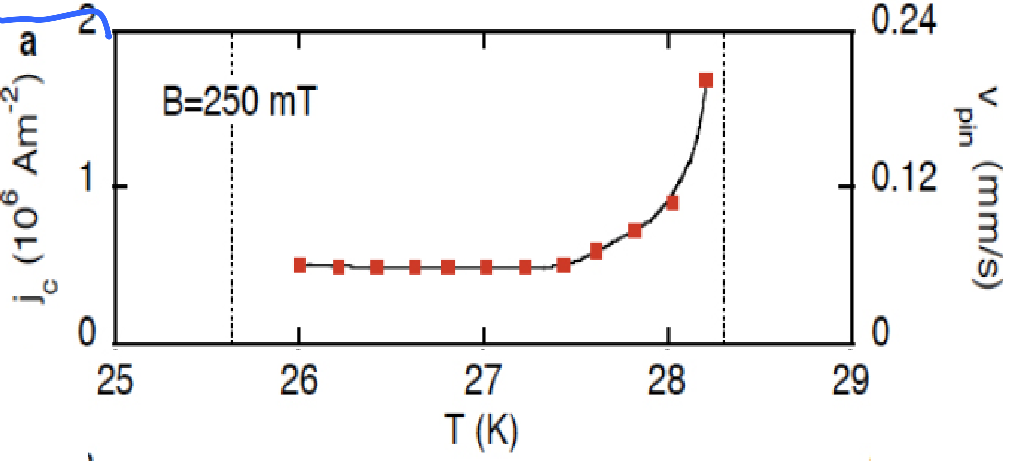
Why ultrasmall critical current densities?



- very **efficient Berry-phase coupling**
  (gyromagnetic coupling by adiabatic spin transfer torques
- very **weak pinning** due to very smooth magnetic structure
  (single point defect: potential << k_B T)
- „**collective pinning**“: partial cancellation of pinning forces due to rigidity of skyrmion lattice



upturn in critical current close to Tc:
softer lattice adjust better to disorder



validity of description by emergent electromagnetic fields

- adiabatic limit:
  time to cross skyrmion >> 1 / band-splitting

  valid as spin orbit interactions are weak
  ⇨ skyrmion radius $\mathbf{R_S}$ large, $\mathbf{R_S} \sim 1/\lambda_{\text{SO}}$

- spin-flip scattering small
- validity of real-space picture:
  Umklapp scattering from skyrmion lattice can be ignored
  if no-spin-flip scattering rate > size of minigaps

$$\ell_{\text{no spin-flip}} < \mathbf{R_s} < \ell_{\text{spin-flip}}$$

ab initio calculation of Berry phase effects:

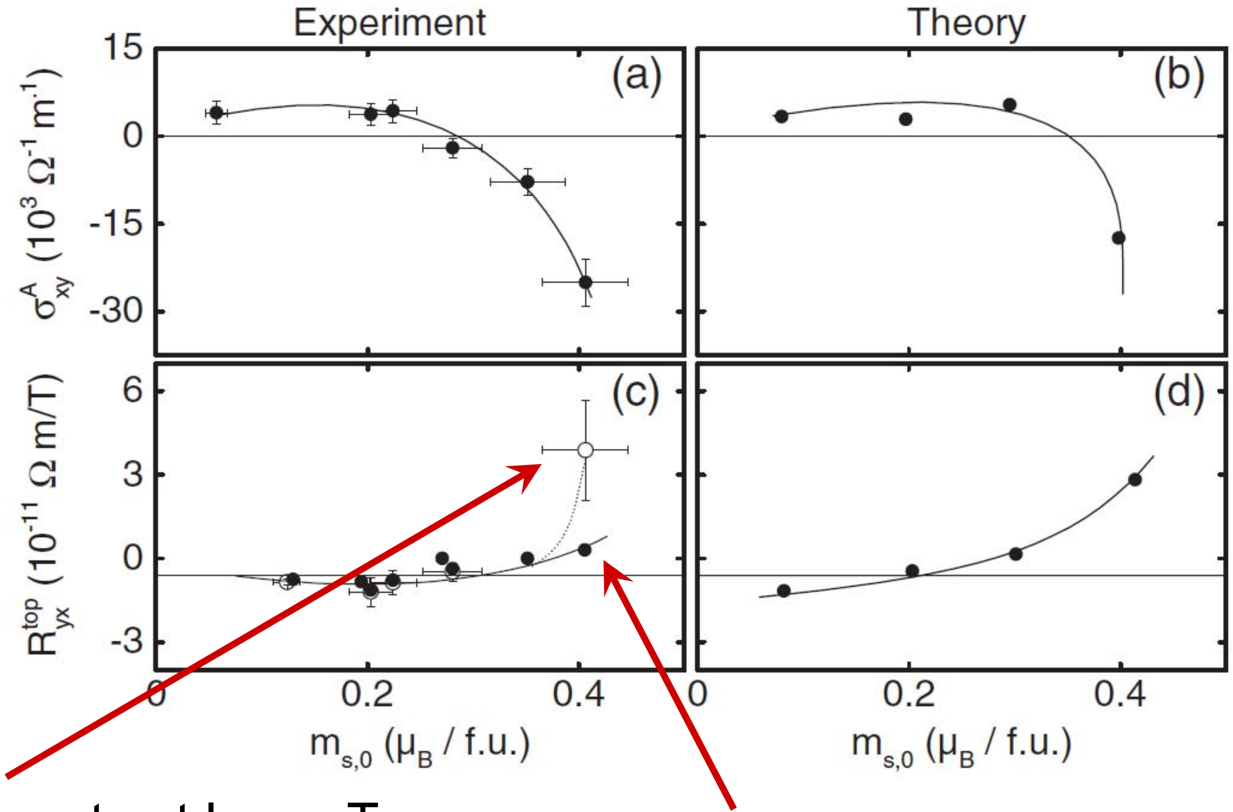
Real-Space and Reciprocal-Space Berry Phases in the Hall Effect of $\mathrm{Mn}_{1-x}\mathrm{Fe}_x\mathrm{Si}$

C. Franz,[1] F. Freimuth,[2] A. Bauer,[1] R. Ritz,[1] C. Schnarr,[1] C. Duvinage,[1] T. Adams,[1] S. Blügel,[2]
A. Rosch,[3] Y. Mokrousov,[2] and C. Pfleiderer[1]

(PRL, 2014)

anomalous Hall effect:
momentum-space Berry phase

topological Hall effect:
real-space Berry phase

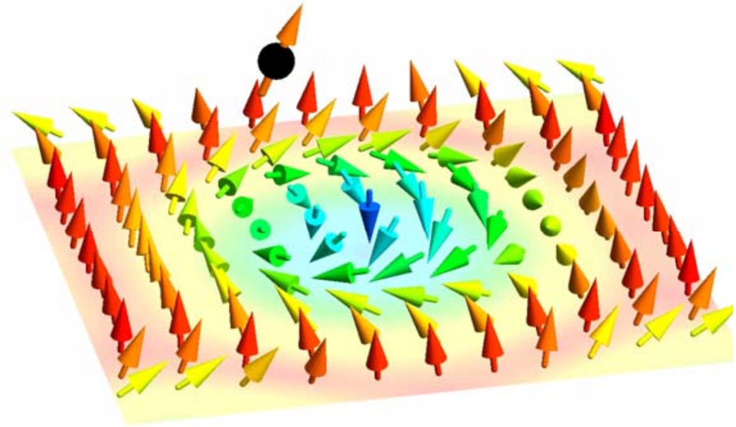


measurements at lower T
in metastable phase

large deviation in pure MnSi
strong spin-flip scattering !

Conclusions Part I

- skyrmion lattices:
  universal phase in cubic chiral magnets
- driven by weak spin-orbit interactions
- magnetic crystal indepent of atomic structu
- extremely easy to manipulate by ultrasmal
  currents
- best described by emergent electric
  and magnetic field
- super-efficient Berry phase coupling
  + weak pinning



Coupling magnetism to current:spintronics

some goals

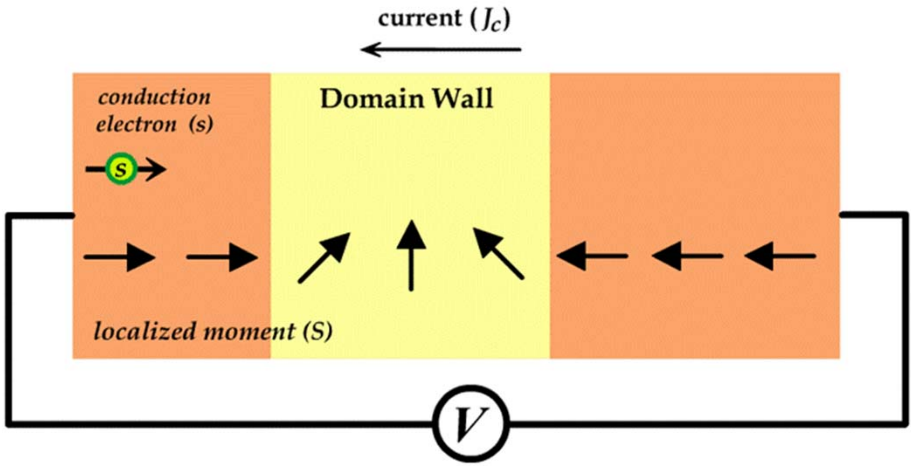
- modify electric currents by sp n structure
- manipulate magnetic structures urr

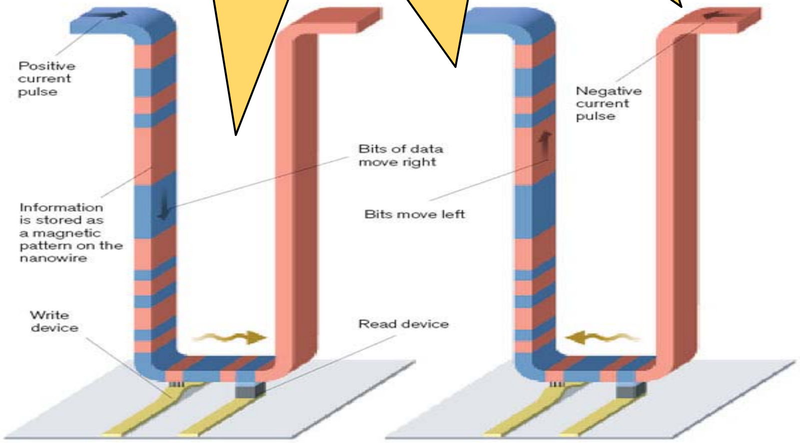
some hopes:

- build superfast non-volatile co
- superfast low power-dissipation tra

skyrmions ???

spin transfer torque: Maekawa

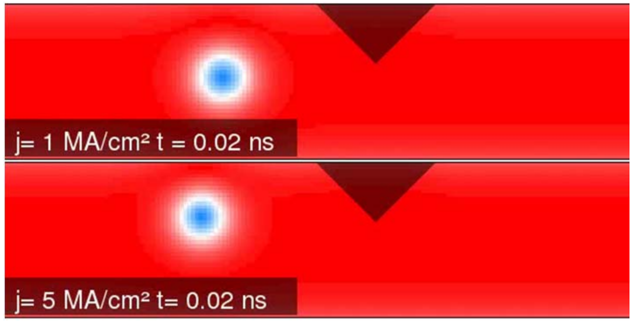




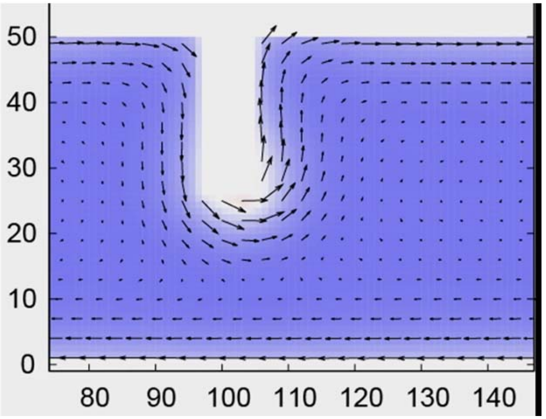
racetrack memory (Parkins)

selected recent developments with magnetic skyrmions:

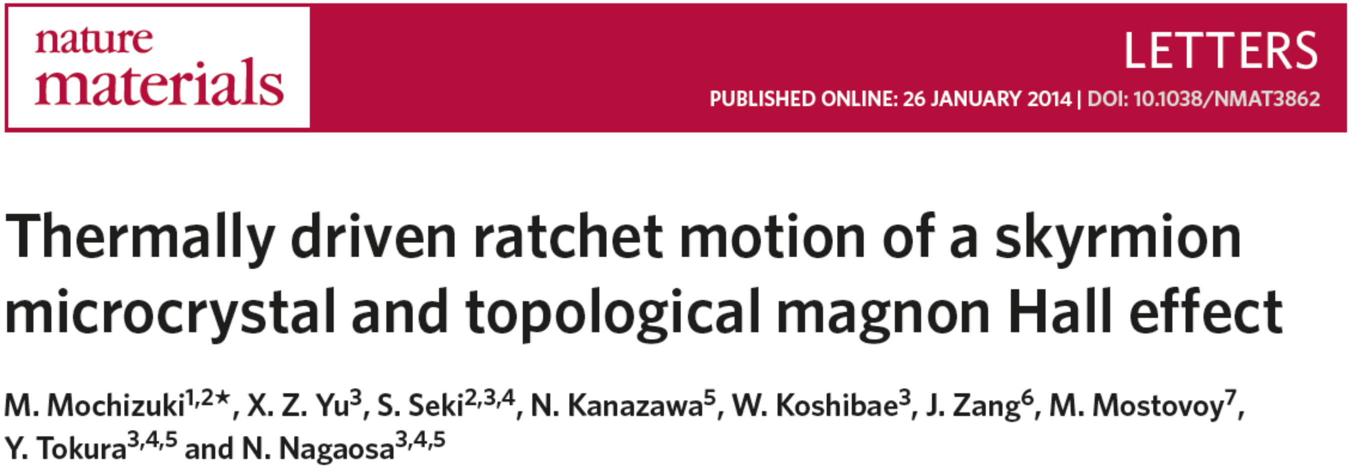
- skyrmions in nano wires
  (Tokura group, Nanoletter 2013)
- driving skyrmions near room temperature
  (Tokura group, Nat. Comm. 2012)
- elements for a future ‟skyrmionics“
  (Fert group Nature Nanot. 2013, Nagaosa group,
  Nature Comm. 2013, …)
- multiferroic skyrmions & electrical manipulation (Tokura group, Science 2012, Nature Comm. 2013)
- skyrmion molecules driven by currents in a bilayer manganite (Tokura group, Nature Comm. 2014)
- skyrmion lattice rotates when observed by electron microscope
  (Nagaosa/Tokura groups, Nature Mat. 2014)
- writing and reading single nanoskyrmions with magnetic scanning tunneling microscope
  (Wiesendanger group, Science 2013, Nature Physics 2011)



Fert group. 2013



Nagaosa group. 2013



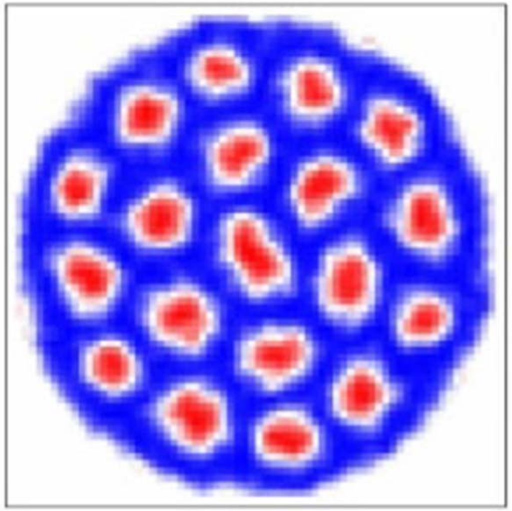
nature materials

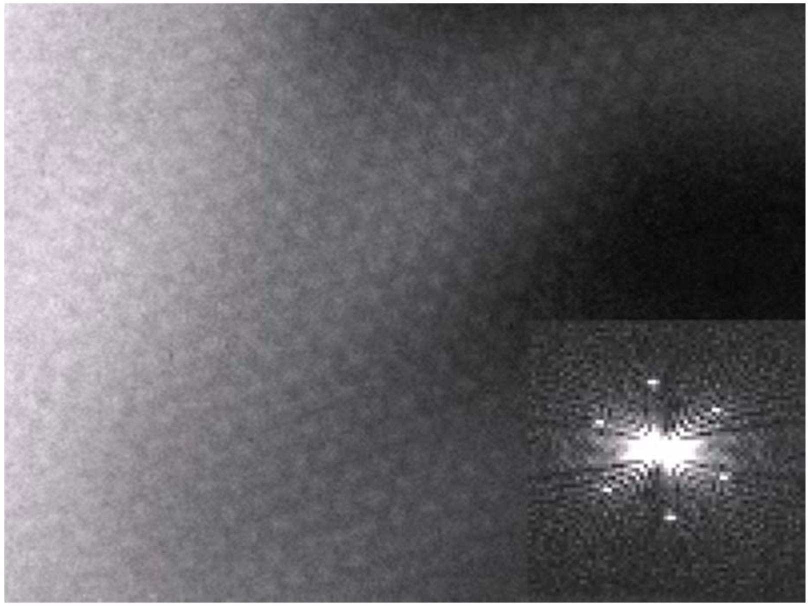
LETTERS

PUBLISHED ONLINE: 26 JANUARY 2014 | DOI: 10.1038/NMAT3862

Thermally driven ratchet motion of a skyrmion microcrystal and topological magnon Hall effect

M. Mochizuki[1,2*], X. Z. Yu[3], S. Seki[2,3,4], N. Kanazawa[5], W. Koshibae[3], J. Zang[6], M. Mostovoy[7], Y. Tokura[3,4,5] and N. Nagaosa[3,4,5]



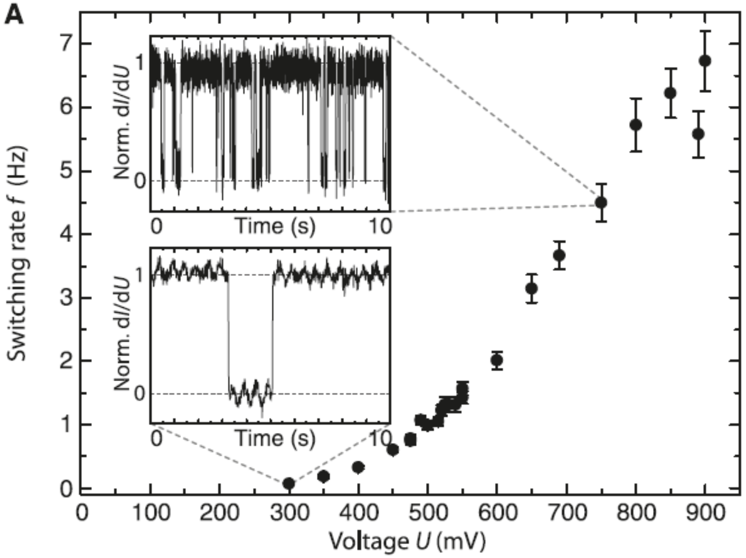


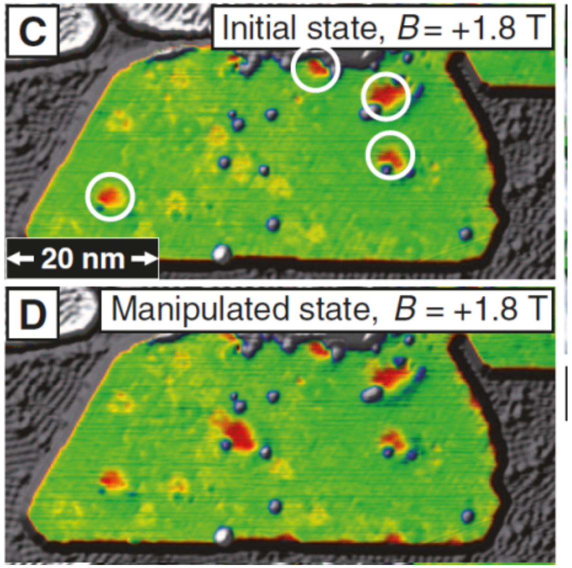
skyrmion lattice rotates when
viewed with electron microscope
origin: magnon-heat currents

Writing and Deleting Single Magnetic Skyrmions

Niklas Romming, Christian Hanneken, Matthias Menzel, Jessica E. Bickel,* Boris Wolter,
Kirsten von Bergmann,† André Kubetzka,† Roland Wiesendanger

- nanoskyrmions on FePd layers on Ir 111 surface (use spin-orbit interactions at surfaces)
- imaging by magnetic STM
- Write and delete skyrmions by the shot-noise of electrons tunneling into the sample





Interesting ?

topological quantization & Berry phases

experimentally detected emergent electromagnetic fields

coupling of magnetism and currents

open questions: classical and quantum dynamics
phase-space Berry phase effects
effects of disorder & pinning
exotic liquid states
….

Applications ?

topological stability ⇨ memory devices

efficient coupling to currents, Berry-phase detection

first ideas/experiments on skyrmions in nanostructures

logic devices ?

“Skyrmionics in sight“ (editorial Nature Nanotechnology)