Weak coupling theories of unconventional superconductivity (II)

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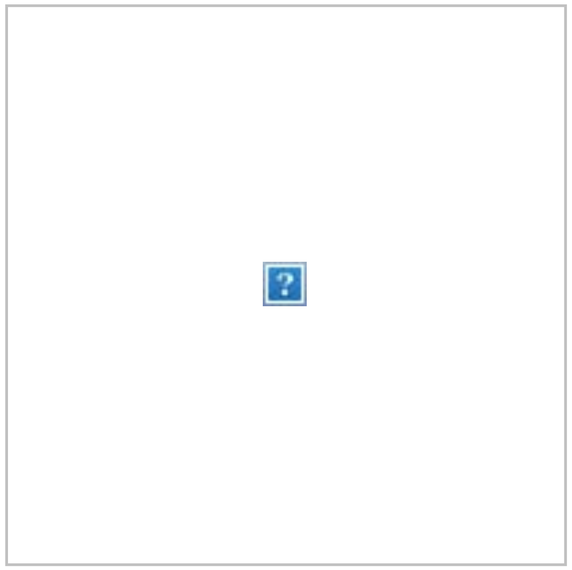
Collaborators



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Acknowledgments

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References

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Superconductivity in the repulsive Hubbard model: An asymptotically exact weak-coupling solution

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PHYSICAL REVIEW B 83, 094518 (2011)

Superconductivity from repulsive interactions in the two-dimensional electron gas

S. Raghu[1,2] and S. A. Kivelson[1]

PHYSICAL REVIEW B 85, 024516 (2012)

Effects of longer-range interactions on unconventional superconductivity

S. Raghu,[1,2] E. Berg,[3] A. V. Chubukov,[4] and S. A. Kivelson[1]

PHYSICAL REVIEW B 86, 094506 (2012)

Optimal T_c of cuprates: The role of screening and reservoir layers

S. Raghu, R. Thomale, and T. H. Geballe

I. Overview and Motivation

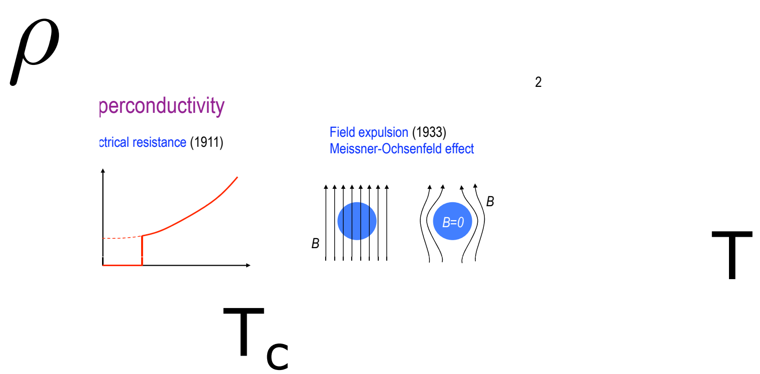
Conventional superconductivity

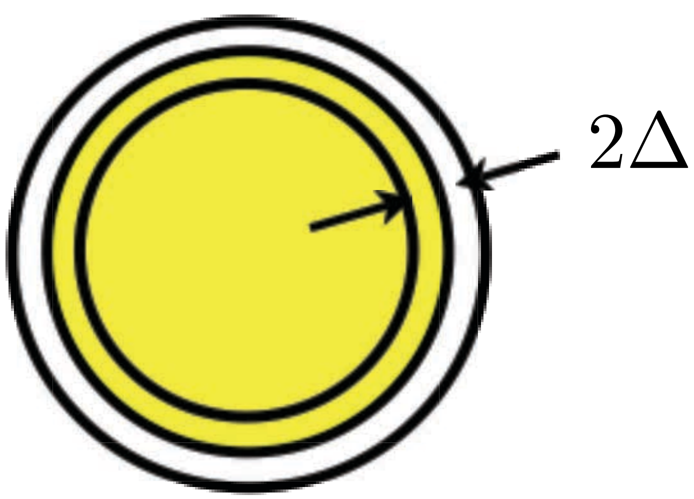
e.g. Pb, Hg

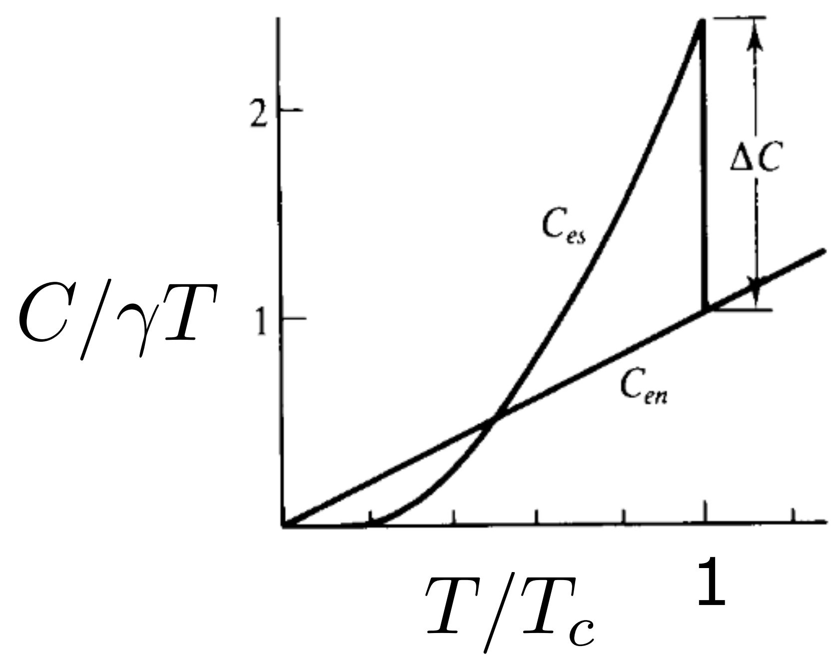
Zero dc resistance below T_c.

Electrons on the Fermi surface form Cooper pairs.

Gap to electronic excitations opens at the Fermi energy.







Properties of conventional superconductors

Pairing symmetry is s-wave: constant, momentum-independent gap function on the Fermi surface.

Insensitive to weak non-magnetic disorder.

No competing magnetic phases.

Exponentially activated specific heat, spin susceptibility, etc.

No spontaneous currents across Josephson junctions.

Normal state is a well-behaved Landau Fermi liquid.

Effective field theory of the Fermi liquid

(Polchinski 1984, Shankar 1994.)

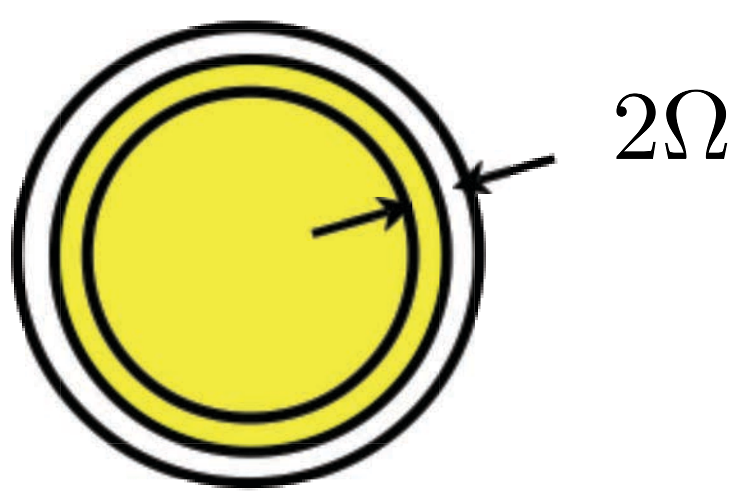
Non-interacting Fixed point action, S_0:

$$S_0 = \int_{-\Omega}^{\Omega} \frac{d\omega}{2\pi} \int \frac{d\vec{k}}{(2\pi)^d} \bar{\psi} \left[i\omega + v_f(\hat{k})k \right] \psi$$

$$S_{int} = \int \prod_{i=1}^{3} \frac{d\omega_i d\vec{k}_i}{(2\pi)^{d+1}} \bar{\psi}(1)\bar{\psi}(2)\psi(3)\psi(4)g(1234)$$

$$\vec{k}_4 = \vec{k}_1 + \vec{k}_2 - \vec{k}_3$$
$$\omega_4 = \omega_1 + \omega_2 - \omega_3$$

$$|\omega_i| < \Omega << E_F$$



Dispersion can be linearized about the FS.
Corrections are *small* irrelevant terms.

Effective field theory of the Fermi liquid

(Polchinski 1984, Shankar 1994.)

Renormalization group (RG) procedure

$\Omega \to \Omega e^{-\ell}, \ell > 0$ Rescale energy, fields to preserve S_0.

Generically, the interaction g(1234) is irrelevant except when

(I) $\vec{k}_1 = \vec{k}_3, \vec{k}_2 = \vec{k}_4$ $g(1212) \equiv F(12)$ Forward scattering

(II) $\vec{k}_2 = -\vec{k}_1, \vec{k}_3 = -\vec{k}_4$ $g(1\bar{1}2\bar{2}) \equiv V(12)$ Cooper channel

Infinitessimally thin energy shells are integrated out and the flow of the couplings are determined.

Effective field theory of the Fermi liquid

(Polchinski 1992, Shankar 1994.)

F(12) and V(12) are both marginal. They are also small in the weak-coupling limit. RG done perturbatively to one-loop order produces

$$\frac{dF}{d\ell} = 0$$

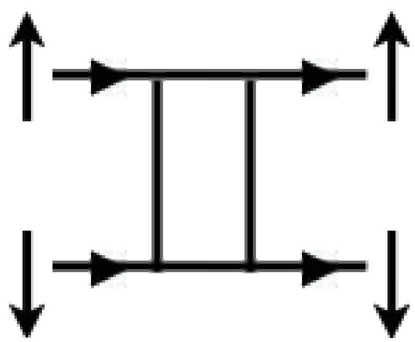
Forward scattering remains marginal (Landau Fermi liquid parameters).

$$\frac{dV}{d\ell} = -\rho V^2 \qquad\qquad V = \frac{V_0}{1 + \rho V_0 \log\left(\Omega_0/\Omega\right)}$$

In the Cooper channel, repulsive interactions ($V_0 > 0$) weaken whereas attractive interactions ($V_0 < 0$) grow at low energies, producing the BCS instability. This is the only way to destroy a Fermi liquid in the weak-coupling limit.

Alternative perspective: partial summation of ``ladder" diagrams

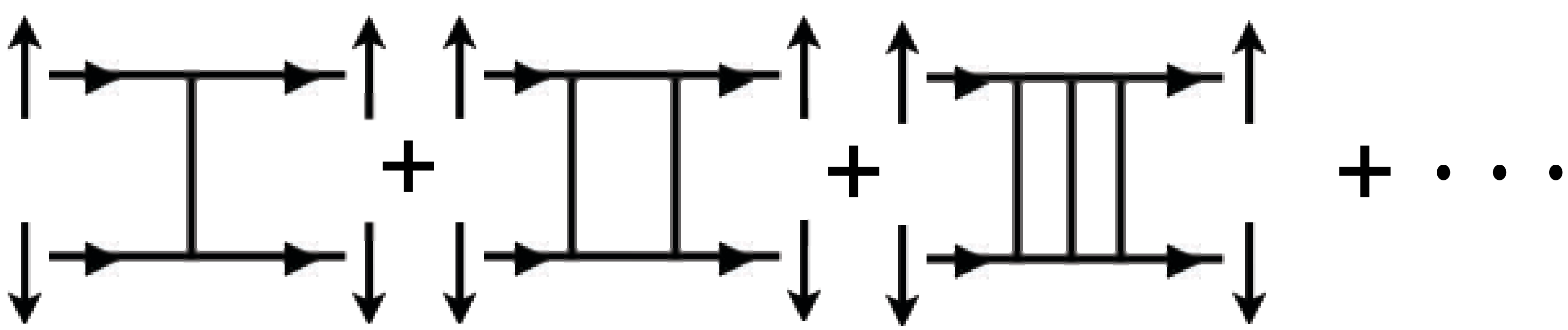
Multiple scattering processes cannot be neglected:



$$= \rho V_0 \log \left[\Omega_0 / \Omega \right]$$

diverges logarithmically
at low energy (i.e. close to FS).

The entire sum of "ladders" must be taken into account:



$$= \frac{V_0}{1 + \rho V_0 \log \left[\Omega_0 / \Omega \right]}$$

Conventional pairing mechanism

2 well-separated energy scales: $1)\ \omega_D,\ 2)\ E_F \quad \omega_D << E_F$

$$g(1234) = g_c - g_p \Theta(\omega_D - |\omega_2 - \omega_3|)$$

g_c: instantaneous screened Coulomb interaction: short-ranged and repulsive.

g_p: retarded attractive interaction due to electron-phonon coupling.

Pairing occurs due to the separation in time-scales between the instantaneous Coulomb and retarded electron-phonon interaction.

At high frequencies, electron-phonon coupling is not renormalized. Therefore, it can be estimated from microscopic calculations and experiments.

Conventional pairing mechanism

2 step RG treatment

Step 1: $\omega_D < \Omega << E_F$

g_p is unaffected, and g_c becomes weaker under RG.

$$g_p(\omega_D) = g_p \qquad g_c(\omega_D) = g^* = \frac{g_{c,0}}{1 + \rho g_{c,0} \log\left(\Omega_0/\omega_D\right)}$$

$$\Omega_0/\omega_D \sim (M/m)^{1/2}$$

Step 2: $\Omega < \omega_D$

$$g_{eff} = g^* - g_p \equiv \left(\mu^* - \lambda\right)/\rho$$

$$\rho g_{eff}(\Omega) = \frac{\mu^* - \lambda}{1 + (\mu^* - \lambda)\log\left(\omega_D/\Omega\right)}$$

If $g_{eff} < 0$, there is a BCS instability below $T_c \sim \omega_D e^{-1/(\lambda - \mu^*)}$

Unconventional superconductivity

A number of materials do not obey this simple physical picture.

Such systems occur when electrons are more tightly bound to the atom producing narrow bandwidths.

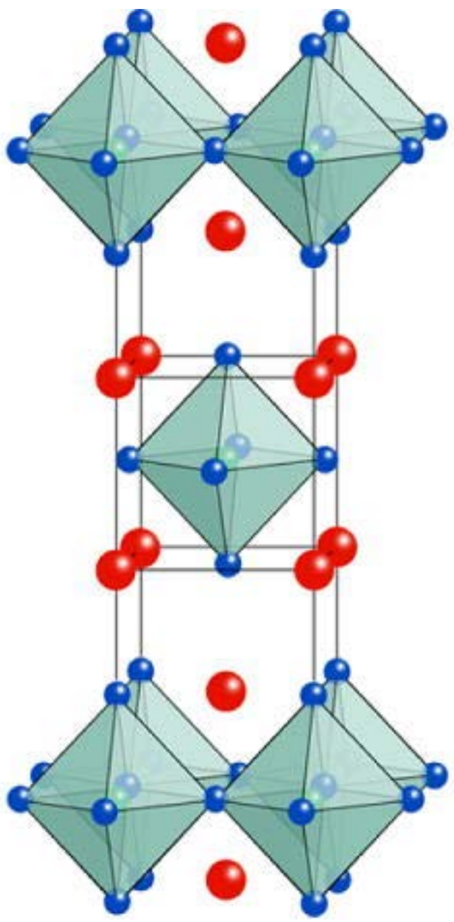
Interaction effects also enhance the electron's effective mass, and produces "heavy" quasiparticles.

In this case, the separation in scale of Debye and Fermi temperature is less sharp.

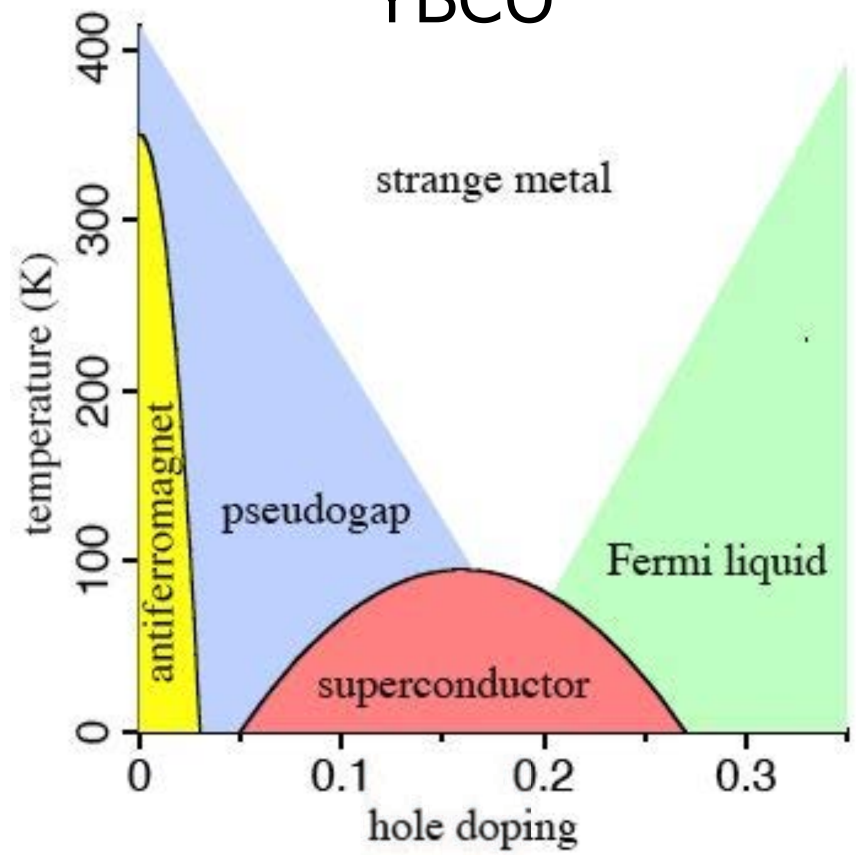
There can be new bosons, e.g. spin fluctuations, or even emergent bosons, that replace the role of phonons in mediating pairing. It's even possible that a sharp bosonic 'glue' isn't even present in these materials.

Unconventional superconductivity

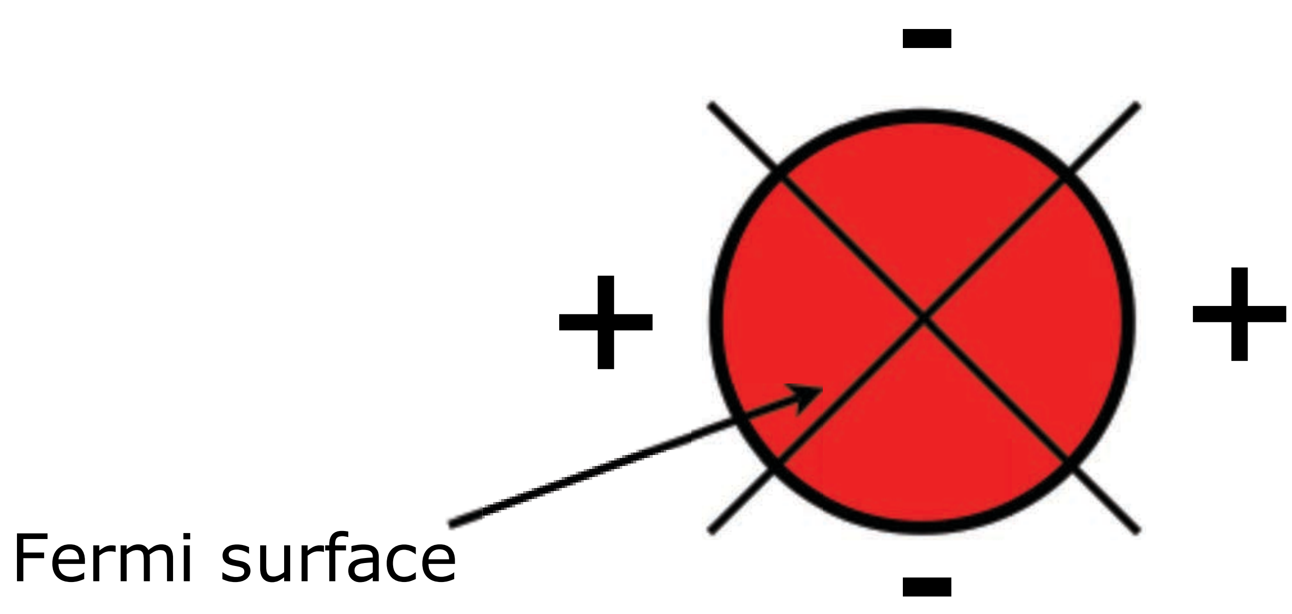
e.g. Cuprates



YBCO



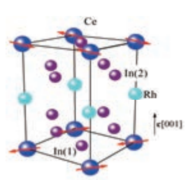
Antiferromagnetic ground state gives way to d-wave superconductivity with doping.



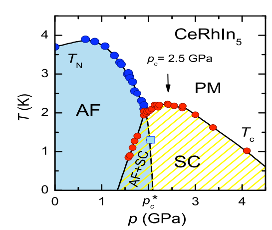
d-wave superconductivity

Unconventional superconductivity

e.g. 115 systems



$CeRhIn_5$



J. Flouquet (2009)

Antiferromagnetism competes with superconductivity.

The same electronic correlations which give rise to magnetism are likely to play a role in mediating superconductivity.

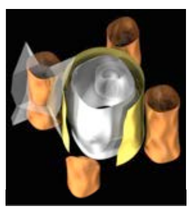
Spin triplet superconductivity

Sr_2RuO_4

UPt_3

140

)



de Haas-van Alphen

R. Joynt and L. Taillefer: Superconducting phases of UPt_3 239

(i.e., parallel to the a^* axis). In terms of reciprocal space, we have $a \| \Gamma K$ and $b \| \Gamma M$. The nearest U-U distance is between atoms in adjacent layers, equal to 4.132 Å. Correspondingly, as we shall see in Sec. II.C, the conductivity is greatest along the c axis. The volume of the unit cell is 140.96 Å^3, the molar volume $V_m = 42.43 \times 10^{-6}$ m^3/mol U, the mass density 1.940×10^4 kg/m^3, and molar weight 823.3 g. The mean atomic volume is

Fermi surface

Likely to be a p_x+ip_y superconductor which is odd under parity and time-reversal.

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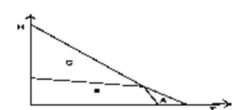
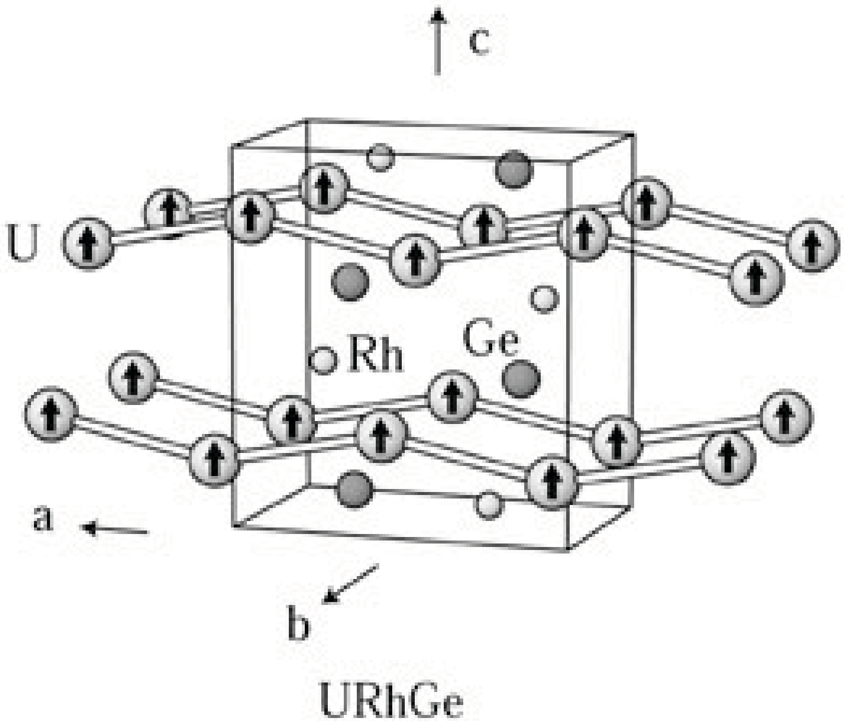


Unconventional superconductivity is defined as a superconducting state in which the order parameter also breaks the crystal symmetry. The order parameter of an unconventional superconductor may also have more than one component, but this is not part of the definition.

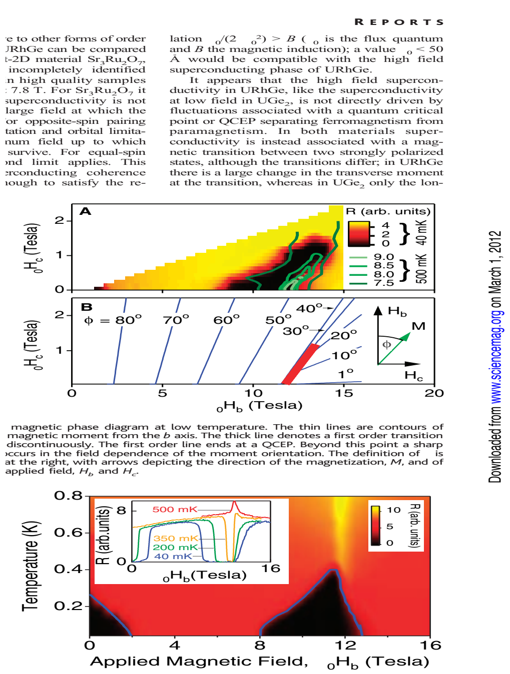
The superfluidity of ^{3}He is both unconventional (the order parameter breaks rotation symmetry) and multicomponent (for a review, see Leggett, 1975). As a result, it became a paradigm for research in heavy-fermion su-

UPt_3 : a hexagonal system with multiple superconducting phases, f-wave pairing.

Exotic superconductivity



URhGe: an orthorhombic ferromagnetic superconductor



REPORTS

e to other forms of order URhGe can be compared t-2D material $Sr_3Ru_2O_7$, incompletely identified n high quality samples 7.8 T. For $Sr_3Ru_2O_7$ it superconductivity is not large field at which the or opposite-spin pairing tation and orbital limita- num field up to which survive. For equal-spin ond limit applies. This erconducting coherence nough to satisfy the re-

lation $_0/(2 \; _0^2) > B$ ($_0$ is the flux quantum and B the magnetic induction); a value $_0 < 50$ Å would be compatible with the high field superconducting phase of URhGe.

It appears that the high field superconductivity in URhGe, like the superconductivity at low field in UGe_2, is not directly driven by fluctuations associated with a quantum critical point or QCEP separating ferromagnetism from paramagnetism. In both materials superconductivity is instead associated with a magnetic transition between two strongly polarized states, although the transitions differ; in URhGe there is a large change in the transverse moment at the transition, whereas in UGe_2 only the lon-

magnetic phase diagram at low temperature. The thin lines are contours of magnetic moment from the b axis. The thick line denotes a first order transition discontinuously. The first order line ends at a QCEP. Beyond this point a sharp occurs in the field dependence of the moment orientation. The definition of is at the right, with arrows depicting the direction of the magnetization, M, and of applied field, H_b and H_c.

Re-entrant superconductivity!

High magnetic field SC - twice as large T_c as low magnetic field SC!

F. Levy et al., (2005)

Properties of unconventional superconductors

$$\boxed{\langle\Delta(\hat{k})\rangle_{F.S} \ll \Delta_{max}(\hat{k})}$$

Often exhibit coexistence and competition with magnetism.

Typically exhibit phase diagrams that depend in a *non-monotonic* fashion on pressure or doping.

Some spontaneously break time-reversal symmetry (e.g. p+ip, d+id pairing).

Repulsive interactions among electrons (on the scale of the bandwidth) are likely to be the origin of unconventional pairing.

Paradigmatic lattice model of electron interactions: Hubbard model.

Hubbard model

$$H = H_0 + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$H_0 = -t \sum_{\langle ij \rangle, \sigma} \left(c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right) - t' \sum_{\langle\langle ij \rangle\rangle, \sigma} \left(c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right)$$

Proposed to be the minimal model for the Cuprates (Anderson 1987).

Model cannot be solved for arbitrary U in d>1.

Monte Carlo methods - Fermion sign problem.

We can consider the weak coupling limit, U/W<<1, and establish in a well-controlled fashion that the ground state in d>1 is generically an unconventional superconductor.

Summary of results

$$T_c \sim W \exp\left\{-\alpha_2 \left(t/U\right)^2 - \alpha_1 \left(t/U\right) - \alpha_0\right\} \times \left[1 + \mathcal{O}(U/t)\right]$$

$$\sim W \exp\left\{-1/\left[\rho V_{eff}\right]\right\} \times \left[1 + \mathcal{O}(U/t)\right]$$

$$\boxed{\alpha_2 = \lim_{U \to 0} \left\{\left(U/t\right)^2 \ln\left[W/T_c\right]\right\}}$$

We have derived an explicit expression for α_2.

From α_2, the pairing symmetry of the superconductor, and explicit pair wave functions are obtained.

We have obtained an explicit prescription for computing the correction term α_1.

How to compute α_2

Electron bandstructure and concentration are basic inputs.

$$\chi(\vec{k}) = -\int \frac{d^d q}{(2\pi)^d} \frac{f(\epsilon_{\vec{k}+\vec{q}}) - f(\epsilon_{\vec{q}})}{\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{q}}}$$

non-interacting susceptibility

$$\int \frac{d\hat{q}}{S_F} g^{s(t)}_{\hat{k},\hat{q}} \psi^{(n)}_{s(t),\hat{q}} = \lambda \psi^{(n)}_{s(t),\hat{k}}$$

eigenvalue problem on the Fermi surface

Singlet channel:

$$g^{s}_{\hat{k}\hat{q}} = \rho U^2 \sqrt{\frac{\bar{v}_f}{v_f(\hat{k})}} \left[\chi(\hat{k}+\hat{q}) + c_1\right] \sqrt{\frac{\bar{v}_f}{v_f(\hat{q})}}$$

$$c_1 \sim \mathcal{O}(t/U) \gg 1$$

How to compute α_2

Triplet channel:

$$g^t_{\hat{k}\hat{q}} = -\rho U^2 \sqrt{\frac{\bar{v}_f}{v_f(\hat{k})}} \left[\chi(\hat{k}-\hat{q})\right] \sqrt{\frac{\bar{v}_f}{v_f(\hat{q})}}$$

In both cases, α_2 is related to the most negative eigenvalue of $g^{s(t)}$:

$$\alpha_2 = |\lambda_0|^{-1} \left(U/t\right)^2$$

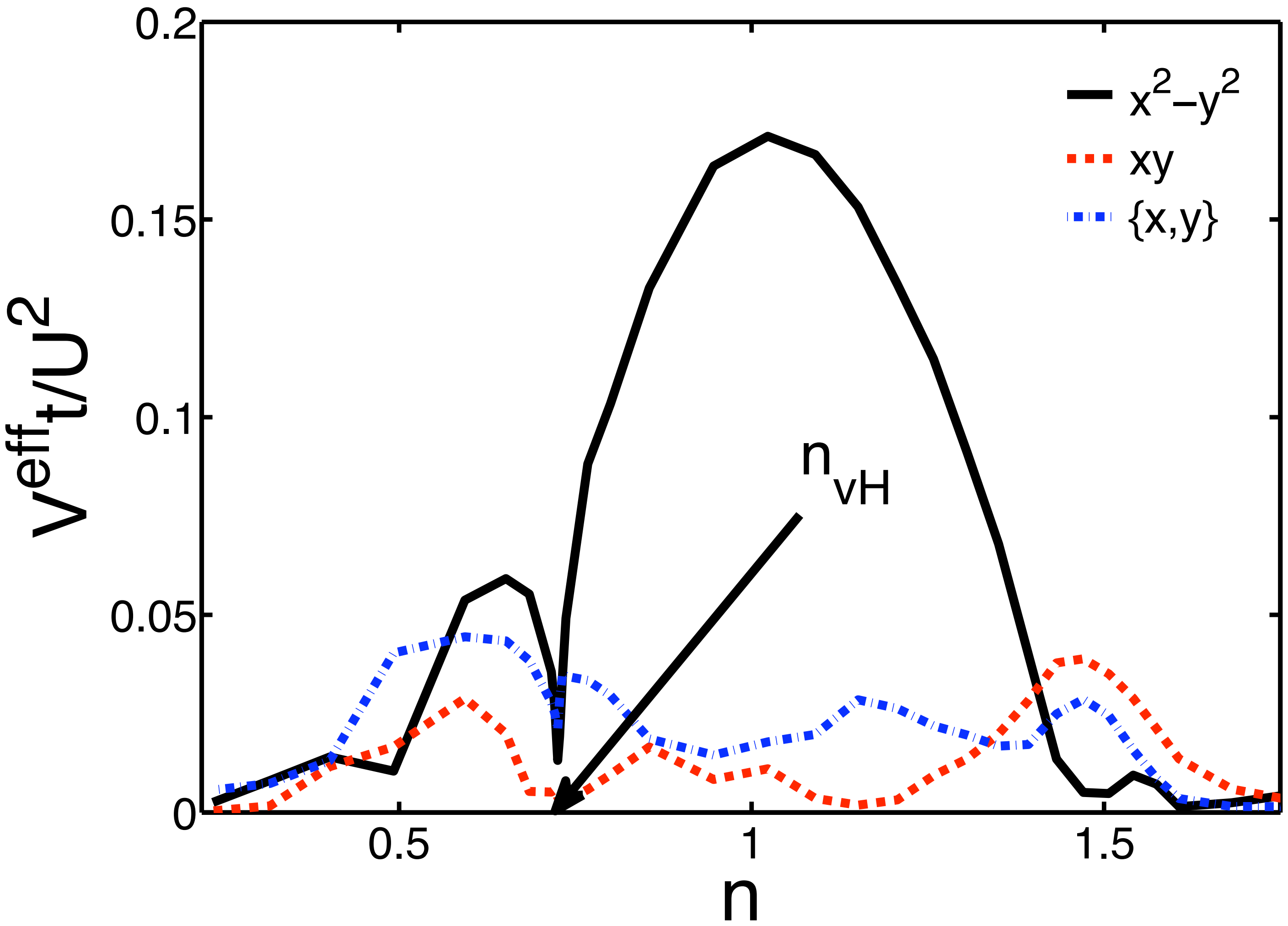
The gap function is related to the associated eigenfunction:

$$\Delta_{s(t)}(\hat{k}) \sim T_c \sqrt{\frac{v_f(\hat{k})}{\bar{v}_f}} \psi^{(0)}_{s(t)}(\hat{k})$$

II. Survey of results

Tetragonal lattice systems

square lattice t'=0.3



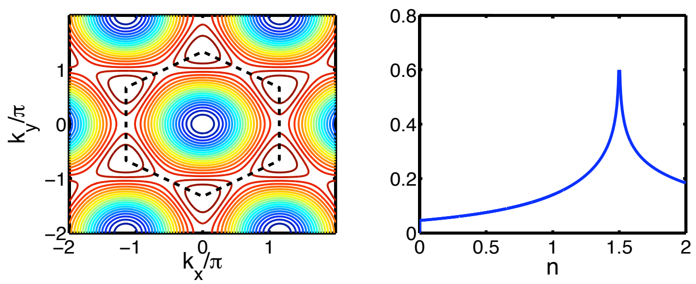
$d_{x^2-y^2}$ pairing is dominant near half-filling.
p-wave and d_{xy} pairing occur for dilute concentrations. Suppression of d-wave pairing strength occurs near the van Hove filling when t'> t/2e, and enhancement for t'<t/2e.

Hexagonal lattice systems

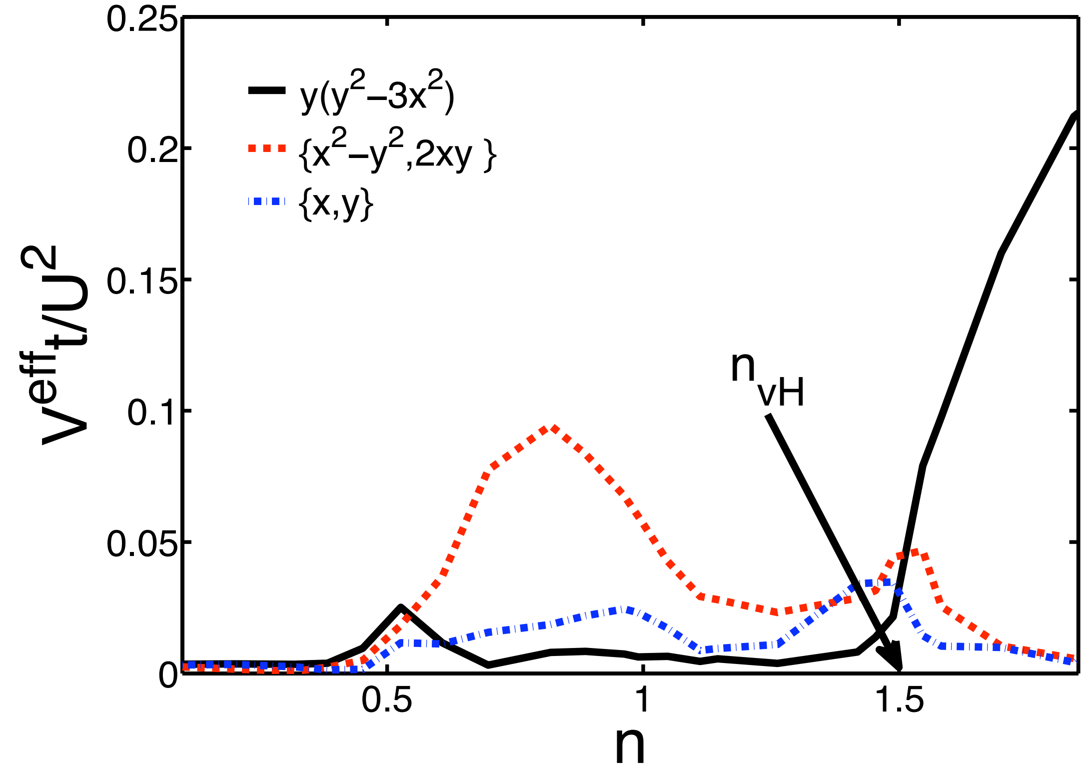
Energy contours

f-wave regime

d+id regime



Triangular lattice



Lifshitz transition at the van Hove filling separates the d+id and f-wave pairing regimes.

f-wave state: fully gapped, alternating sign, reminiscent of S_{+-} state of Pnictide superconductors.

Similar phases occur on a honeycomb lattice.

3D Lattice systems

We have studied the Hubbard Model on the simple cubic, BCC, FCC, diamond lattices.

We find p-wave, d-wave, and f-wave solutions.

p-wave solutions occur at dilute concentrations and are identical to those found in the continuum limit (i.e. the Kohn-Luttinger instability) with a spherical Fermi surface.

d-wave solutions are both doubly degenerate (e_g) and triply degenerate (t_{2g}). The former is directly related to the d-wave state found on the square lattice near half-filling.

In general, pairing strengths are considerably lower in 3d than in 2d.

III. RG treatment

Application to the Hubbard model

In contrast to Polchinski/Shankar problem, when studying the Hubbard model, we consider electron states with energies O(W) above and below E_F.

The dispersion cannot be linearized about the Fermi surface - the higher order terms are irrelevant but not small!

Frequency dependence of the interaction vertices cannot be ignored.

It is not clear how Shankar/Polchinski's results can be applied straightforwardly to the Hubbard model.

We solve this problem by taking a two-stage renormalization group approach.

RG strategy

Stage 1: Perturbation theory in U/t.

Integrate out all states down to an unphysical cutoff Ω_0.

$$We^{-1/\rho|U|} \ll \Omega_0 \ll U^2/t$$

This produces a k-dependent *effective* interaction in the Cooper channel.

Stage 2: Polchinski/Shankar RG treatment of action obtained above.

RG flow of the 2-particle scattering amplitude in the Cooper channel
is marginally relevant -> leads to BCS instability.

T_c : energy scale where the flows diverge. Independent of Ω_0.

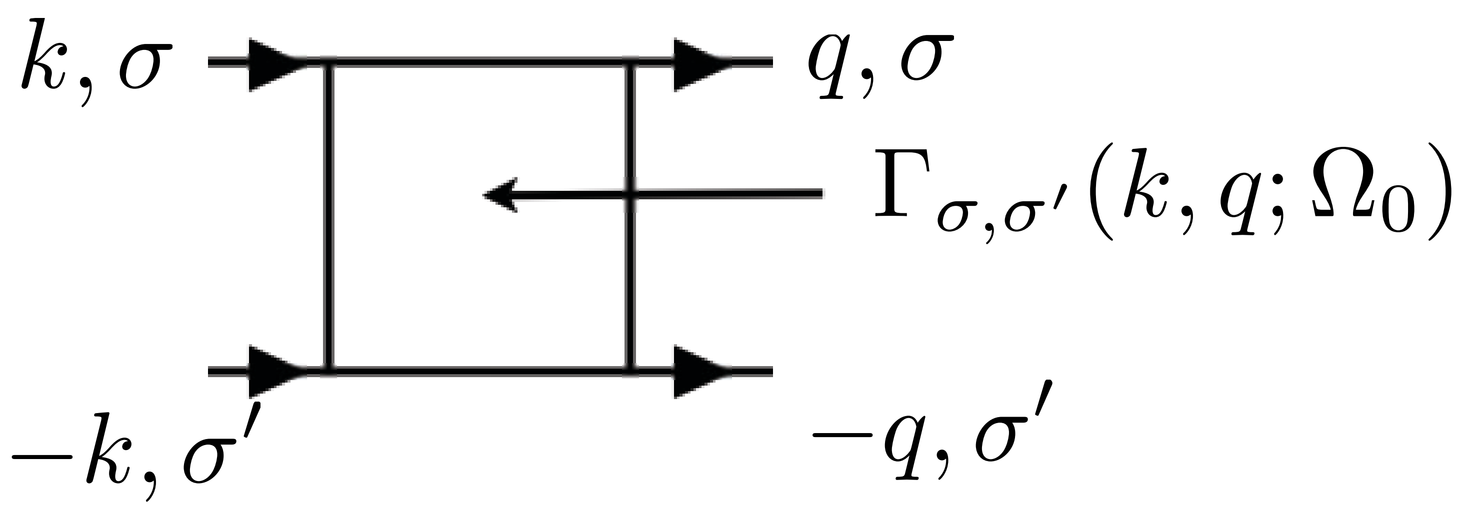
Stage 1: perturbation theory in U/t

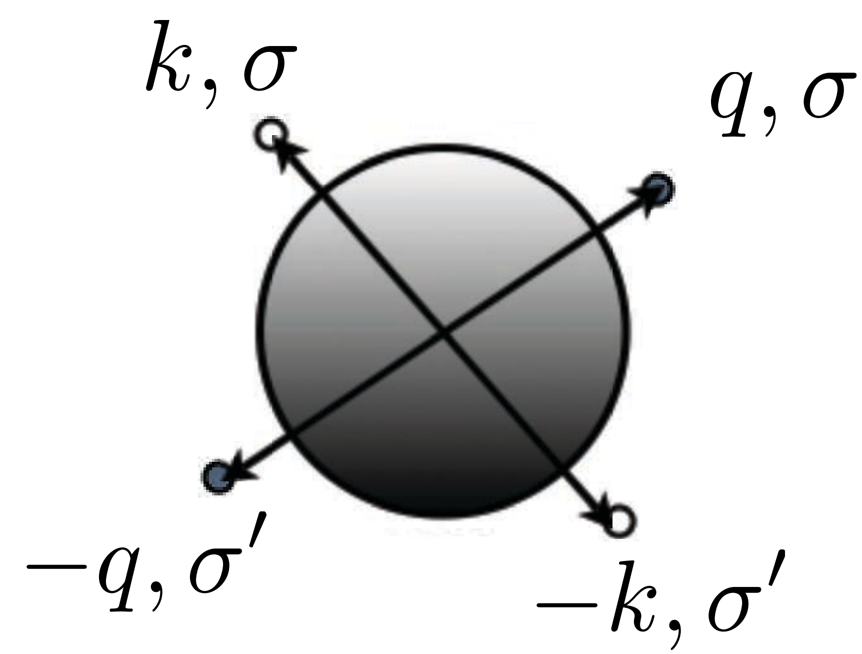
$$S_0 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^d k}{(2\pi)^d} \bar{\psi}_{k\sigma} \left(i\omega - \epsilon_{\vec{k}} + \mu \right) \psi_{k\sigma}$$

$$S = S_0 + S_1$$

$$S_1 = \prod_{i=1}^{3} \int_{-\infty}^{\infty} \frac{d\omega_i}{2\pi} \frac{d^d k_i}{(2\pi)^d} U \bar{\psi}_1 \bar{\psi}_2 \psi_3 \psi_{1+2-3}$$

Integrate out states above $\Omega_0$perturbatively in U/t.





$\Gamma_{\sigma,\sigma'}$: two particle scattering amplitude with zero CM momentum.

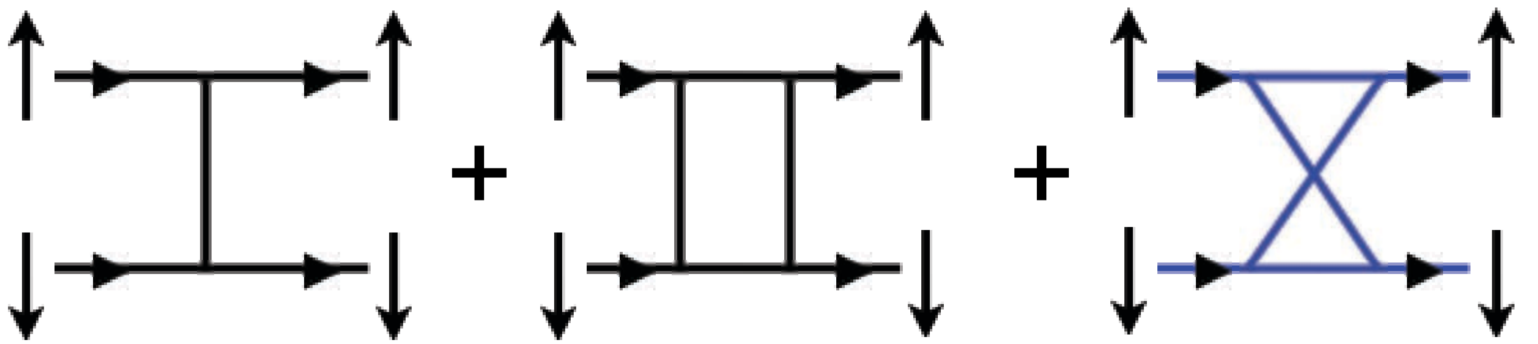
Stage 1: perturbation theory in U/t

Singlet channel: $\Gamma_s = \frac{1}{2}\left[\Gamma_{\uparrow,\downarrow} + \Gamma_{\downarrow,\uparrow}\right]$

Triplet channel: $\Gamma_t = \Gamma_{\uparrow,\uparrow}$

These do not mix in a system with inversion symmetry.

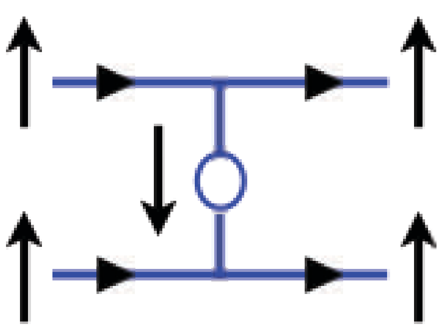
$\Gamma_s =$



$+ \cdots$

$$= U + U^2 \log\left[\frac{W}{\Omega_0}\right] + U^2\chi(k+q;\Omega_0) + \mathcal{O}(U^3)$$

$\Gamma_t =$



$+ \cdots$

$$= -U^2\chi(k-q;\Omega_0) + \mathcal{O}(U^3)$$

Stage 1: perturbation theory in U/t

After doing perturbation theory, momentum-dependent effective interactions are found. What about frequency dependence?

$$U^2\chi(k+q;\Omega_0) = U^2\chi(k+q) + \mathcal{O}(\Omega_0)$$

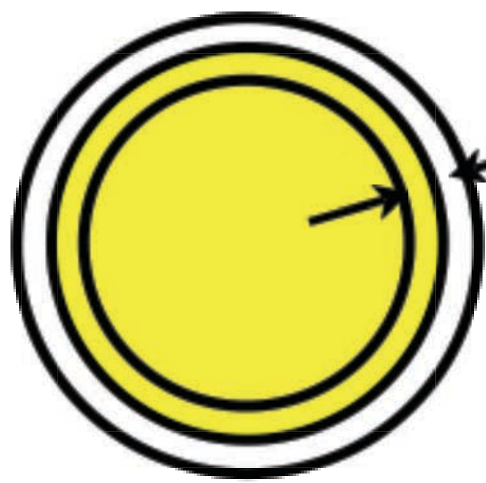
Since $\chi(q) \sim 1/t$, the frequency-dependent correction is small if

$$\Omega_0 << U^2/t$$

By choosing the initial cutoff judiciously, we are justified in neglecting frequency dependence so long as the zero frequency limit of the effective interaction is non-singular.

End product of first stage

From stage 1, we obtain an effective action involving electrons within a width $2\Omega_0$ about the Fermi surface with k-dependent interactions.



This is just the action for a Landau Fermi liquid studied by Shankar and Polchinski .

$$S \to \int_{-\Omega_0}^{\Omega_0} \frac{d\omega}{2\pi} \int \frac{d^d k}{(2\pi)^d} \bar{\psi}_{k\sigma} \left(i\omega - \epsilon_{\vec{k}} + \mu\right) \psi_{k\sigma}$$

$$+ \int_{-\Omega_0}^{\Omega_0} \frac{d\omega_1 d\omega_2}{(2\pi)^2} \int \frac{d^d k_1 d^d k_2}{(2\pi)^{2d}} \Gamma_{\sigma\sigma'}(k_1, k_2) \bar{\psi}_{k_1\sigma} \bar{\psi}_{-k_1\sigma'} \psi_{-k_2\sigma'} \psi_{-k_2\sigma}$$

Stage 2: RG analysis

Forward scattering amplitudes are constant and finite. Only relevant couplings occur in the Cooper channel.

RG flows obtained as cutoff is lowered.

$$\Omega = \Omega_0 e^{-\ell}$$

Define

$$g_{\hat{k},\hat{q}} = \rho \sqrt{\frac{\bar{v}_F}{v_F(\hat{k})}} \Gamma(\hat{k},\hat{q}) \sqrt{\frac{\bar{v}_F}{v_F(\hat{q})}}$$

Hermitian matrix

RG flow eq.

$$\boxed{\frac{dg}{d\ell} = -g \star g}$$

$$(a \star b)_{\hat{k},\hat{q}} = \int \frac{d\hat{p}}{S_F} a_{\hat{k},\hat{p}} b_{\hat{p},\hat{q}}$$

Stage 2: RG analysis

RG flow in the diagonal basis: $$\frac{d\lambda_n}{d\ell} = -\lambda_n^2$$

$$\Rightarrow \lambda_n(\Omega) = \frac{\lambda_n(\Omega_0)}{1 + \lambda_n(\Omega_0) \log\left[\Omega_0/\Omega\right]}$$

When the most negative e.v. grows to be O(1), the superconducting transition occurs.

$$\boxed{T_c \sim \Omega^* = \Omega_0 e^{-1/|\lambda_0|}}$$

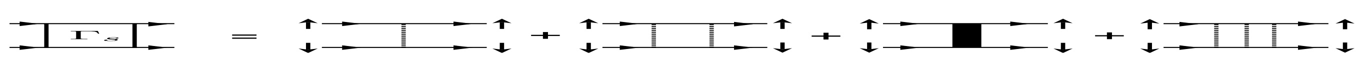
There is a problem with this expression: the arbitrary initial cutoff appears in it which cannot be physical.

A careful analysis of perturbation expansion shows that this is not the case.

Cutoff independence of results

The basic idea: the most negative eigenvalue has an **implicit** dependence on Ω_0 and must be taken into account.

Simple illustration: negative U Hubbard model - spin singlet pairing.

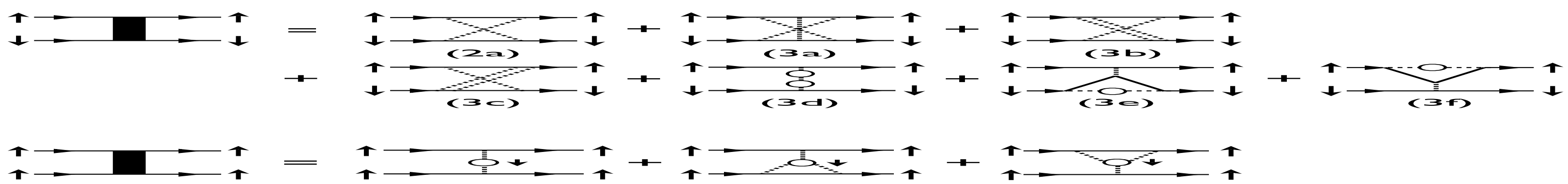


$$
\begin{aligned}
|\lambda_0| &= \rho|U|\left[1+\rho|U|\log(W/\Omega_0)\right] \\
T_c &\sim \Omega_0 e^{-1/\rho|U|} e^{\log(W/\Omega_0)} = W e^{-1/\rho|U|}
\end{aligned}
$$

In a similar fashion, the cutoff independence of the repulsive U case may be established.

Cutoff independence of results

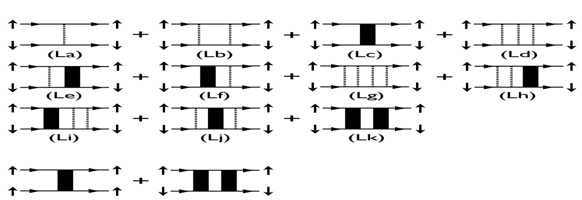
Consider the spin-triplet channel. To $O(U^3)$,



$$\equiv g_t^{(3)}$$

Diagonalizing this, we will find again that $T_c \sim \Omega^* = \Omega_0 e^{-1/|\lambda_0|}$

However, a careful perturbation analysis leads us to consider



$$= g_t^{(3)} + g_t^{(3)} \star g_t^{(3)} \log(W/\Omega_0)$$

This quantity has precisely the log divergence needed to eliminate the cutoff dependence, as in the negative U case. The analysis in the singlet channel is more technical but conceptually identical.

Summary of weak-coupling perspective

The bare repulsive interaction U>0 prevents onsite pairing. States with sign-changing order parameters are unaffected by it.

Effective interaction in the Cooper channel: obtained by integrating out modes away from the Fermi energy.

$$\Gamma(\hat{k}, \hat{k}') = U + a_2(\hat{k}, \hat{k}')U^2 + \cdots$$

$O(U^2)$ interactions: induced by particle-hole fluctuations. They are *non-local*, and *attractive* in unconventional pairing channels.

By integrating out high energy modes, we produce an effective action with states only in a narrow energy shell around E_F.

This is precisely the effective field theory studied by R. Shankar and J. Polchinski.

Discussion

We have shown, using well-controlled methods that the ground state of the Hubbard model at small U is an unconventional superconductor.

First analysis of superconductivity from bare repulsive interactions: Kohn and Luttinger (KL). Non-analyticities occur at T=0 in $\chi(2k_F)$due to sharpness of the Fermi surface. They give rise to Friedel oscillations which mediate attraction among electrons. This gives rise to p-wave pairing in a 3D electron gas.

Our results are identical to Kohn-Luttinger theory in the limit of dilute electron concentrations in 3d.

"Spin fluctuation exchange" mechanisms are extrapolations of the present results into a more strong coupling regime. However, these approaches are uncontrolled in this limit.

Discussion

While we do obtain d-wave pairing on a square lattice, there are important discrepancies between our results and the physics of real materials.

T_c is an emergent scale exponentially smaller than bare energy scales. Above it, the system is a well-behaved Fermi liquid. There are no bad metals, non-Fermi liquids, etc.

Induced pairing interactions are non-local in the weak-coupling limit. Gap functions have more structure in momentum space than is seen in highly correlated systems.

In the weak-coupling limit, the gap function summed over the FS is zero. Sign changing solutions which violate this criterion (as proposed for the pnictides) reflect underlying strong coupling phenomena.

Competition between non-superconducting orders is absent in the weak-coupling limit. In the weak-coupling limit, a well-behaved Fermi liquid is present, and it is destabilized only by a BCS instability.

III. Effect of longer range interactions

Longer range repulsive interactions

Generically, longer ranged, repulsive interactions, V, weaken pairing strengths of the Hubbard model.

Scattering amplitude in the Cooper channel (weak-coupling limit):

$$\Gamma(\hat{k}, \hat{k}') = U + V + aU^2 + bV^2 + cUV + \cdots$$

V directly competes with induced attraction and tend to suppress pairing tendency of the Hubbard model.

At what scale V_0 is superconducting solutions from the Hubbard model suppressed?

2 reasonable guesses:

1) $V_0 \sim T_c$ 2) $V_0 \sim U^\alpha$

Longer range repulsive interactions

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At what scale V_0 is superconducting solutions from the Hubbard model suppressed?

Our conclusion:

$\boxed{\text{2)} \quad V_0 \sim U^\alpha}$

Extended Hubbard model

$$H = H_0 + H_{int}$$

Consider electrons on a square lattice for simplicity.

$$H_0 = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + h.c.$$

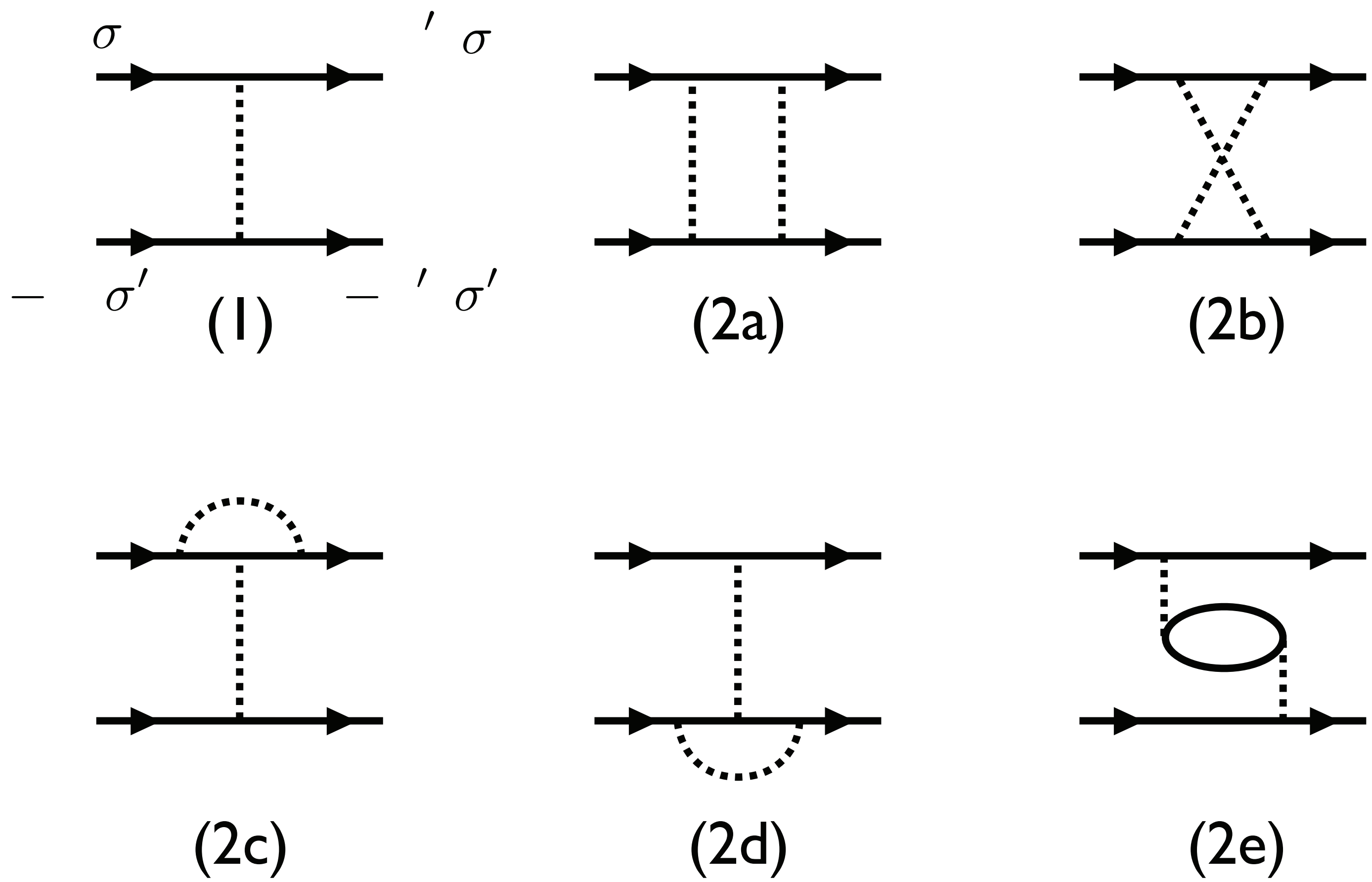
$$H_{int} = U \sum_i n_{i\uparrow} n_{i\downarrow} + V \sum_{\langle ij \rangle} n_i n_j + V' \sum_{\langle\langle ij \rangle\rangle} n_i n_j$$

In the weak-coupling limit: $U \to 0, V \to 0, V' \to 0$

The phase diagram depends sensitively on how this limit is taken.

2 distinct asymptotic regimes: $\begin{cases} 1) V \sim U^2/W \\ 2) V \sim U \end{cases}$

1) V ~ U^2/W



Obtain effective interactions to O(U^2): Only first order correction from V is needed - the next correction is O(V^2) ~ O(U^4) and is negligible.

1) V ~ U^2/W

Bare V repels states with nearest-neighbor pairing.

It competes with the induced attractive interactions ~ O(U^2/W).

dxy, g-wave states are unaffected by it.

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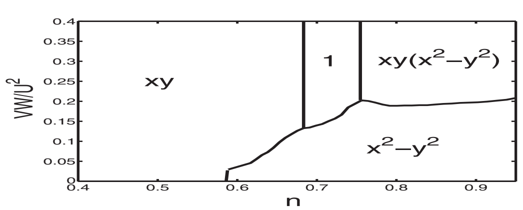


$d_{x^2-y^2}$ pairing survives for a finite V near half-filling. It's pairing strength decreases as V increases.

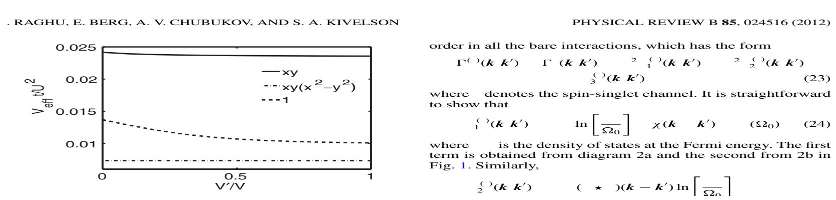
"1": extended s-wave state ~ Real(x+iy)^4.
"xy(x2-y2)": g-wave state ~ Im(x+iy)^4.

1) V ~ U^2/W, V'~ V

The effect of a non-zero second-neighbor repulsive interaction V' ~ V ~ U^2/W. Is surprisingly weak.

Pairing strengths decrease, but superconductivity remains stable.

Closer to half-filling, the dx^2-y^2 state is also stable against V' ~ V ~ U^2/W.



On a lattice, first order repulsion does not preclude more "extended" variants of a given pairing symmetry.

2) V ~ U

Since V~U, in obtaining effective interactions to $O(U^2)$, we must consider all contributions to order V^2.

The induced attractive interactions are unable to overcome V in the weak-coupling limit. All states having nearest-neighbor pairing are disfavored.

When V'=0, the dxy and g-wave states dominate the phase diagram: Cooper pairing occurs at distances larger than the range of V.

With non-zero V', the g-wave state remains the dominant superconducting solution.

Similar results involving Jellium models with screened Coulomb interactions and $r_s << 1$: Kohn Luttinger instability persists at large angular momentum (A. Chubukov and M Yu Kagan, J. Phys. Condens. Matt. **1**, 3135 (1989)).

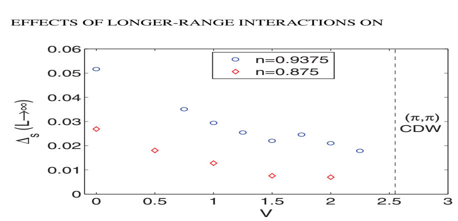
Results at intermediate coupling

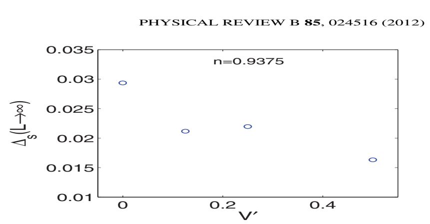
DMRG solution of 2-leg ladders with extended interactions.

$$\Delta_s = E(S=1) - E(S=0) \qquad (L \to \infty)$$

$$U = 8t, V' = 0$$

Spin-gap remains finite in the presence of non-zero V.





Superconducting correlations: power law decay, and “d-wave like”.
Amplitude of Pair-field susceptibility decreases with V.

Longer ranged interactions have a weak effect even at intermediate coupling.

Role of screening in conventional superconductors

Electron-phonon mediated superconductors are insensitive to the range of Coulomb interactions.

2 well-separated energy scales: $1)\ \omega_D,\ 2)\ E_F \quad \omega_D << E_F$

Scattering amplitude in the Cooper channel (weak-coupling limit):

$$\Gamma(\hat{k},\hat{k}';\omega) = (\mu^* - \lambda)/\rho$$

$\mu =$ dimensionless repulsive interaction (instantaneous)

$$\mu^* = \frac{\mu}{1+\mu\log[E_F/\omega_D]}$$

$$\approx 1/\log\left[E_F/\omega_D\right]$$

$\lambda =$ dimensionless attractive interaction (retarded)

Phonon-mediated attraction is local, and largely *insensitive to the range of Coulomb interactions*.

Summary

1) Unconventional superconductivity from repulsive interactions is a robust phenomenon. It is stable against the inclusion of longer (but finite) ranged repulsive interactions.

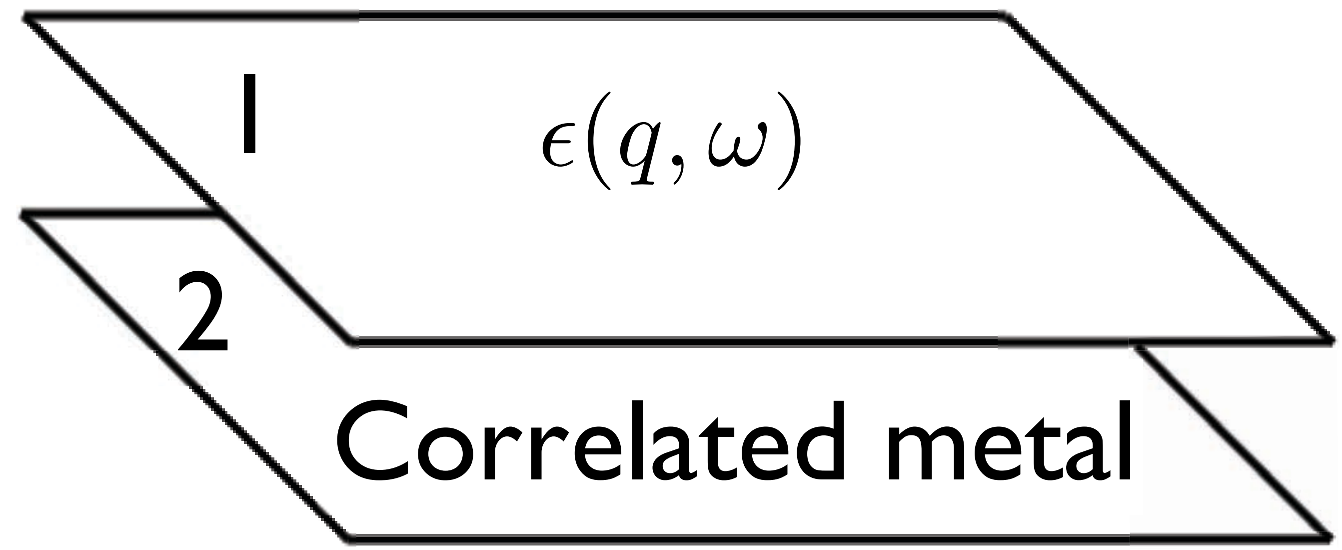
2) Pairing strengths decrease with longer ranged interactions but d-wave pairing superconductivity remains present until V ~ U.

3) By contrast, electron-phonon superconductors are largely insensitive to the presence of longer-range repulsive interactions - due to retardation.

4) This suggests a generic strategy for obtaining higher transition temperatures: screening by a proximate polarizable medium could reduce V, V'..., leading enhancement of T_c.

III. Application: Raising T_c of unconventional superconductors

Screening due to proximate polarizable medium



$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_{int}$$

$$\mathcal{H}_1 = \sum_{k\sigma} E(\boldsymbol{k}) c^\dagger_{1\boldsymbol{k}\sigma} c_{1\boldsymbol{k}\sigma}$$

$$\mathcal{H}_2 = \sum_{k\sigma} E_2(\boldsymbol{k}) c^\dagger_{2\boldsymbol{k}\sigma} c_{2\boldsymbol{k}\sigma} + \sum_{\boldsymbol{q}} V_2(\boldsymbol{q}) n_2(\boldsymbol{q}) n_2(-\boldsymbol{q})$$

$$\mathcal{H}_{int} = \sum_{\boldsymbol{q}} V_{1,2}(\boldsymbol{q}) n_1(\boldsymbol{q}) n_2(-\boldsymbol{q})$$

Purely capacitive coupling

Single electron interlayer tunneling: will introduce disorder into the metal if layer 1 is disordered. We will neglect it (and justify this below).

Enhanced screening by proximate polarizable medium

Integrate out layer 1: obtain effective Hamiltonian for just the metal.

$$\mathcal{H}_{eff} = \sum_{k\sigma} E_2(\boldsymbol{k}) c^{\dagger}_{2\boldsymbol{k}\sigma} c_{2\boldsymbol{k}\sigma} + \sum_{\boldsymbol{q}} V_{eff}(\boldsymbol{q}) n_2(\boldsymbol{q}) n_2(-\boldsymbol{q})$$

$$V_{eff}(\boldsymbol{q}) = V_2(q) - V_{1,2}^2(\boldsymbol{q}) \chi(\boldsymbol{q}, \omega)$$

χ : charge susceptibility of polarizable medium.

In lattice systems, these are effective density-density interactions:

$$\sum_{ij} V_{i,j} n_i n_j$$

To gain qualitative understanding, consider interactions with finite range that are obtained from the screening process.

Consider this effective model in weak and intermediate coupling.

In both cases, pairing strengths are enhanced as the <u>short-distance</u> polarizablity of the medium increases.

Strategies for choosing the polarizable medium

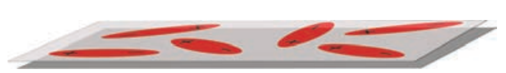
(A) Couple metals capacitively to the correlated metal.

Top gate alone: screens Coulomb interactions to dipolar interactions $V(r) \sim 1/r^3$.

Top and bottom gates: exponential screening of Coulomb interactions.

(B) Interfaces with amorphous dipolar liquids and anti-ferroelectrics.

Screening occurs over short distances.



The experimental test: couple optimally doped cuprate films to polarizable media. If screening of long-range interactions by the polarizable medium occurs, T_c is enhanced beyond optimal doping value.

The end