Symmetric Topological Phases (II)

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Jan. 2014, Theory Winter School - National High Magnetic Field Laboratory

Plan:

- Yesterday:

The introduction of symmetry fractionalization:

(1) AKLT chain

(2) Generalized symmetry fractionalizations for:

topological defects (dislocations in topological insulators)

topological excitations in topologically ordered phases.

- Today:

(1) Quantum spin liquid phases in frustrated magnets, and related experiments in materials

(2) Parton constructions of quantum spin liquids, and symmetry fractionalization

Emergent gauge dynamics

- I was talking about symmetry fractionalized gauge charge/flux excitations, say Z2 charge/fluxes. But is that just math?

  Where does gauge field come from?

  In materials, we start from electrons with interactions.

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- I was talking about symmetry fractionalized gauge charge/flux excitations, say Z2 charge/fluxes. But is that just math?

  Where does gauge field come from?

  In materials, we start from electrons with interactions.

- But even starting from electronic degrees of freedom, which only carry E&M gauge charge, the low energy dynamics of a system could show emergent intrinsic gauge dynamics.

Example: FQHE

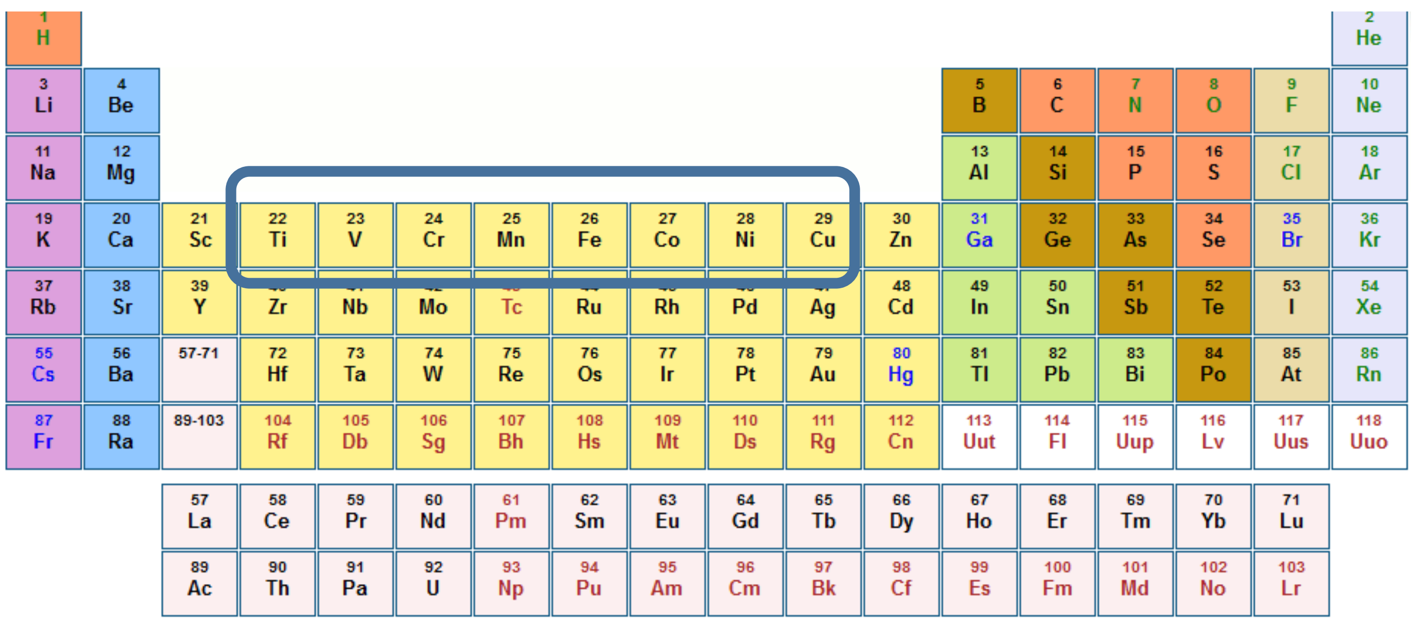
- Quantum spin liquids are quantum phases with such emergent gauge fields.

Basic magnetism

- Mott Insulators – Coulomb repulsion localizes electrons to atomic sites. Only spin degree of freedom.

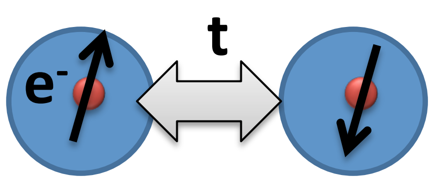
$$H = -t\sum_{<ij>} c_i^\dagger c_j + U\sum_i n_{i\uparrow}n_{i\downarrow}$$

Mott Insulator: U>>t



Basic magnetism

- Only spin degree of freedom. Simplest quantum many body system!

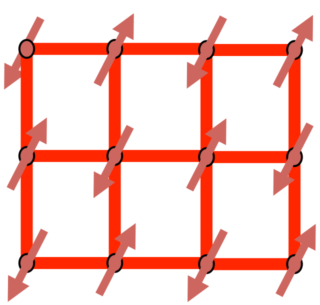


Opposite Spins gain by virtual hopping:
$J \approx t^2/U$; $H = J\, S_1 \bullet S_2$

$$H = -t \sum_{<ij>} c_i^\dagger c_j + U \sum_i n_{i\uparrow} n_{i\downarrow} \Rightarrow H = J \sum_{<ij>} \mathbf{S}_i \cdot \mathbf{S}_j$$

Hubbard → Heisenberg

- Example: La_2**Cu**O_4 (parent compound of cuprates)



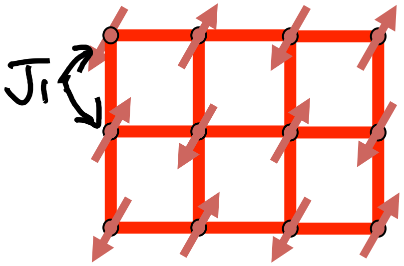
S=1/2 square lattice AntiFerromagnet

J ≈ 1,000Kelvin; t,U ≈10,000 Kelvin

Neel temperature T_N~250Kelvin

Frustrated magnetism

- Even in the presence of quantum fluctuations, an antiferromagnetic Heisenberg-like system often has "obviously" favorable classical magnetic order pattern:



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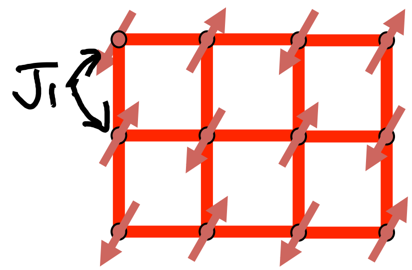
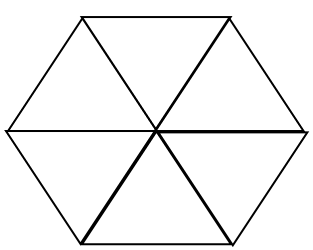
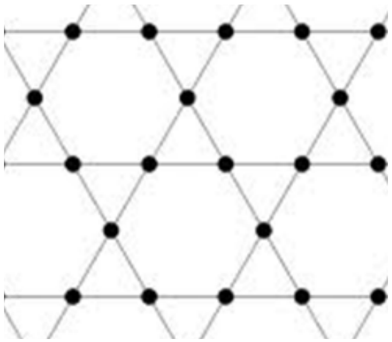


- Geometric frustration → no "obviously" favorable order pattern:

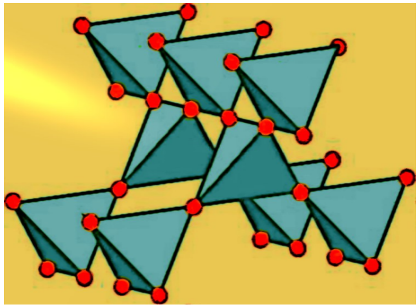
Triangular Lattice:



Kagome Lattice:



Pyrochlore Lattice:



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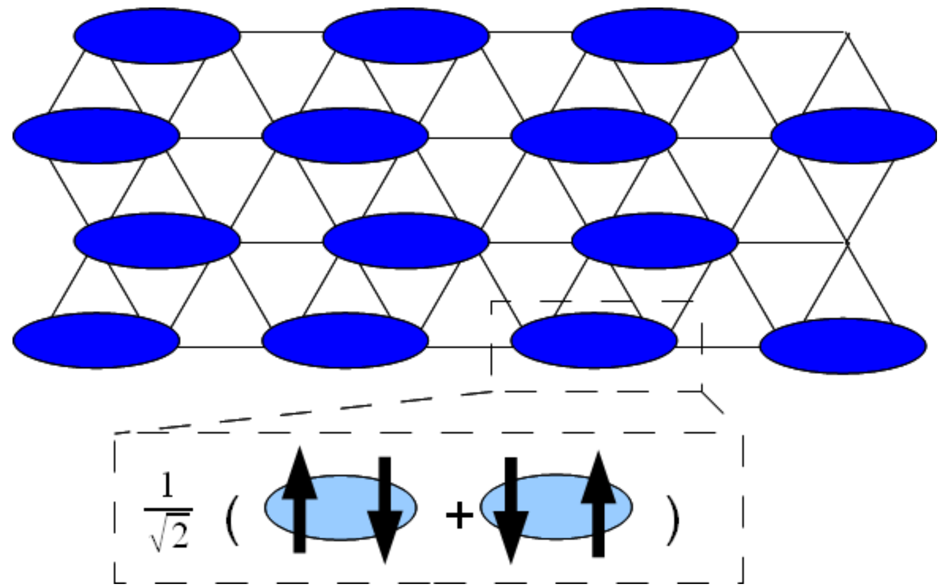
Consider spin-1/2 system:

(half-filled)

One possibility:

valence bond solid (VBS)



Non-magnetic ground state

But breaks translational symmetry

The unit cell is doubled → can be viewed as a trivial band insulator

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Is it possible to have a non-magnetic, fully symmetric ground states?

--- quantum spin liquid

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In one spatial dimension this is not surprising due to Mermin-Wigner theorem.

In higher dimensions, in fact, spin liquids are guaranteed to be exotic phases.

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In higher dimensions, in fact, spin liquids are guaranteed to be exotic phases.

First of all, they are certainly NOT band insulators. (violate Luttinger's theorem)

In addition: A no-go theorem

- Hastings (2004)

Consider a translational symmetric (periodic boundary condition) spin-1/2 system in d-spatial dimensions with finite ranged interactions, with one spin per unit cell.

Theorem: in such a system the ground state is separated from the first excited state by an energy gap that vanishes in the thermodynamic limit:

$E_1 - E_0 < \mathrm{Log}(L)/L$, for a system of linear size L.

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Interpretation:

If the ground state breaks symmetry, this is not surprising. (Goldstone mode…)

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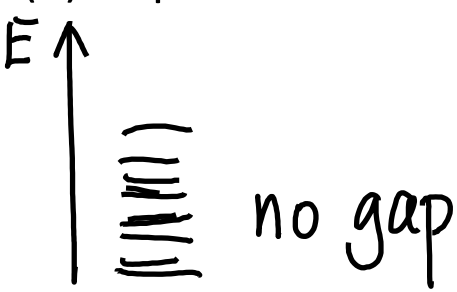
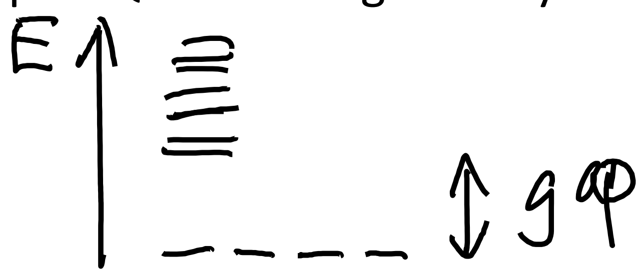
Interpretation:

However if the ground state is a spin liquid (no symmetry breaking), this theorem indicates only two possibilities:

(1) Gapless QSL



(2) Gapped QSL with degeneracy on torus



Quantum spin liquid is a new state of matter

- Gapless QSL

What protects the gapless modes?

In conventional phases, the only two possible ways leading to gapless phases:

(1) Goldstone modes (free boson)

(2) Fermi liquid (free fermion)

But in gapless QSL, none of these mechanism holds.

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- Gapped QSL

What is protecting the ground state degeneracy on torus?

In the absence of symmetry breaking, the only mechanism we know is:

Topological order (e.g., emergent gauge field…)

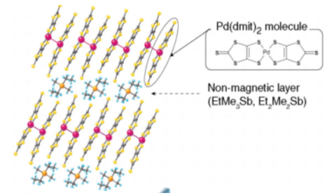
for example:

Laughlin's nu=1/3 state has three-fold ground state deg. on torus.

Quantum spin liquid materials

- In the past decade, a few strong candidate materials are found:

(J>=100K, no magnetic order down to <=50mK)



dmit organic salts

- Spin-1/2 Triangular lattice near Mott transition

  organic salts: ·-(BEDT-TTF)2Cu2(CN)3 (Kanoda's group)

  d-mit

  --- gapless QSL with metallic-like thermal transport!

(Incomplete list)

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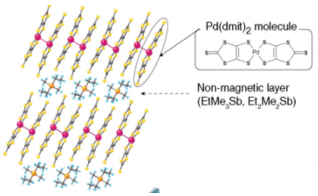

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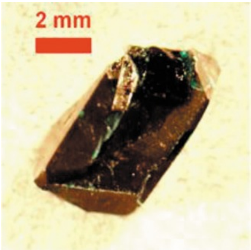
- Spin-1/2 Kagome lattice Heisenberg system
  Herbertsmithite: ZnCu3(OH)6Cl2 (Y. Lee's group …)
  --- gapless or small gapped QSL



Herbertsmithite

- Hyperkagome Iridate: Na4Ir3O8 (Takagi's group…)
  --- gapless QSL?

- Spin-1/2 Pyrochlore lattice quantum spin ice Yb2Ti2O7 (Ross et.al 2009…)

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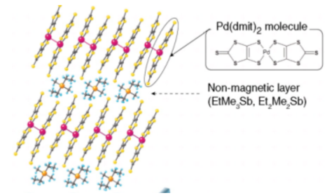

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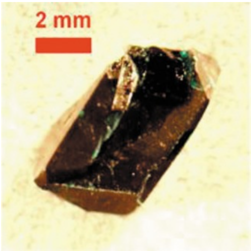

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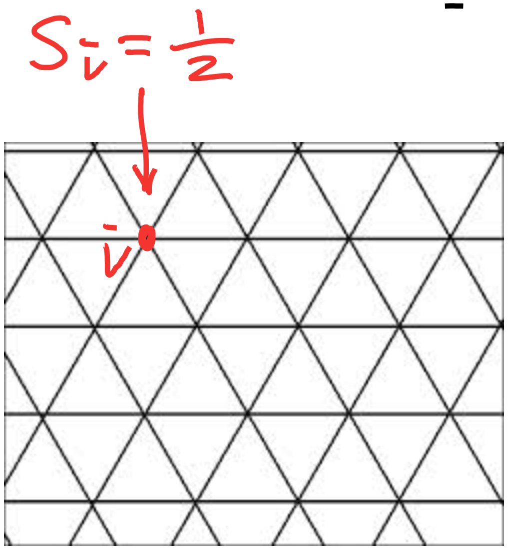
How to describe a QSL?

- In history, we know that explicitly writing down the wavefunction helps a lot!

- Can we write down a QSL wavefunction?

Let's consider triangular spin-1/2 lattice as an example.



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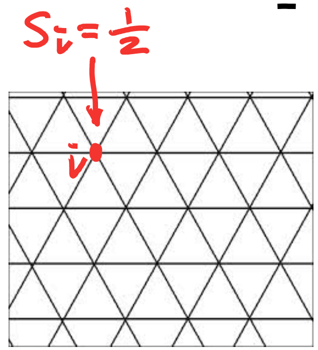

- Can we write down a QSL wavefunction?

The usual way of writing down a spin wavefunction:

$$|\Psi\rangle = |S_1, S_2, \cdots S_N\rangle$$

Intrinsically biased towards magnetic order.

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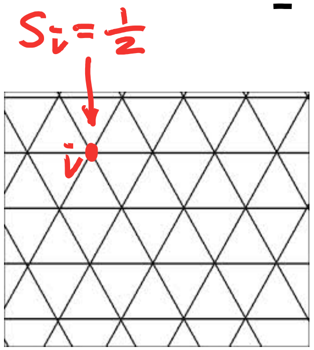

- Can we write down a QSL wavefunction?

The parton construction: (Schwinger-boson method)

(1) Enlarge hilbert space:

Split the spin in to partons:

$$\vec{S}_i = \frac{1}{2} b^{\dagger}_{i\alpha} (\vec{\sigma})_{\alpha\beta} b_{i\beta}$$

$\alpha = \uparrow, \downarrow$, $b_{i\uparrow}$, $b_{i\downarrow}$: spin-$\frac{1}{2}$ bosons.

(Auerbach, Arovas, Read, Sachdev…)

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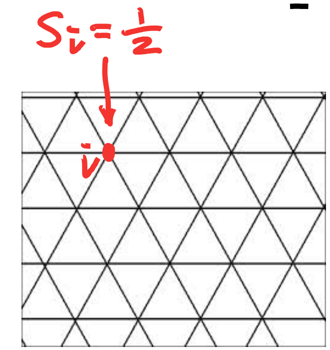

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physical $\mathcal{H}$: $\uparrow \quad \downarrow$.

Enlarged $\mathcal{H}$: $\uparrow \quad \downarrow \quad 0 \quad \uparrow\uparrow \quad \downarrow\downarrow \quad \uparrow\downarrow \quad \cdots$

(0, $\uparrow\uparrow$, $\downarrow\downarrow$, $\uparrow\downarrow$, …: unphysical)

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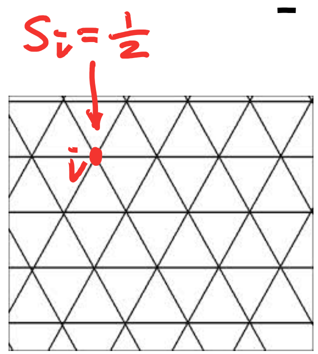

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Why would we do this?

--- Similar to the AKLT-model, these auxillary Schwinger-bosons helps us to compactly writing down an interesting wavefunction.

(Auerbach, Arovas, Read, Sachdev…)

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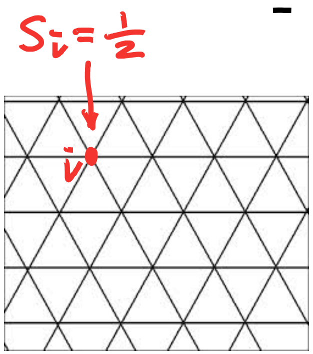

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Split the spin in to partons: $\vec{S}_i = \frac{1}{2} b_{i\alpha}^{\dagger} (\vec{\sigma})_{\alpha\beta} b_{i\beta}$

(2) Write down a spin-rotation sym. boson mean-field state:

$$H_{MF} = \sum_{ij} B_{ij} b_{i\alpha}^{\dagger} b_{j\alpha} + \sum_{ij} A_{ij} (b_{i\alpha}^{\dagger} b_{j\beta}^{\dagger} \varepsilon_{\alpha\beta} + h.c.)$$

$$\Rightarrow \quad |GS\rangle_{MF}$$

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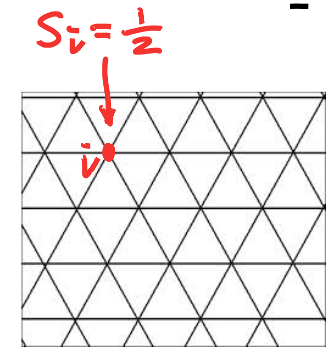

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(3) Project mean-field state back to physical hilbert space:

$$|spin\rangle \equiv P_G |GS\rangle_{MF}$$

P_G: projection

Quantum Spin liquid wavefunction

$\vec{S}_i = \frac{1}{2} b^{\dagger}_{i\alpha} (\vec{\sigma})_{\alpha\beta} b_{i\beta}$

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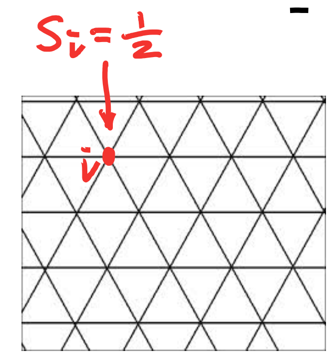

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P_G: projection

- To respect lattice space group symmetry, Aij/Bij cannot be chosen arbitrarily. (will come back on this shortly)

Intuition→ |Aij| all same, |Bij| all same for NN bonds.

But what about the U(1) phases? (will come back.)

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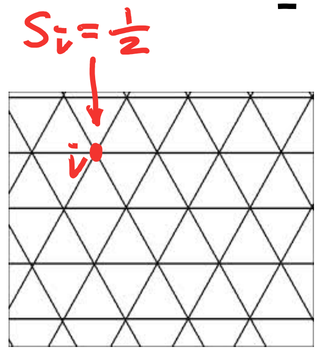

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- Imagine using |spin> as variational wavefunction for some model.

Energetically:

Does the optimal $|GS\rangle_{MF}$ have Schwinger boson condensation ?

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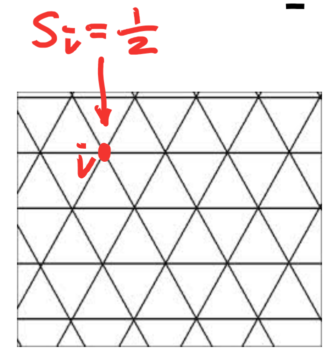

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If YES: magnetically ordered state

If NO: gapped QSL state!

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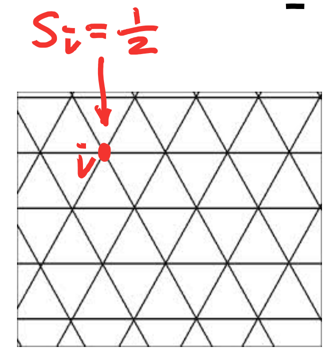

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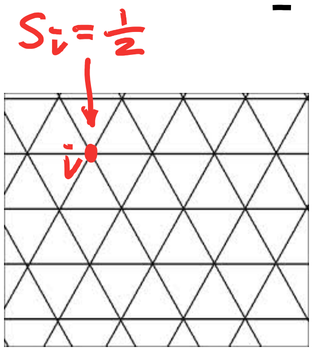
Energetically:

Does the optimal $|GS\rangle_{MF}$ have Schwinger boson condensation ?

Let's assume this for the moment → If NO: gapped QSL state!

Triangular lattice example

- Aij term (singlet pairing) is favored by AF coupling.



For simplicity, let's consider a mean-field state setting Bij=0.

What's the condition for Aij such that lattice symmetry is respected?

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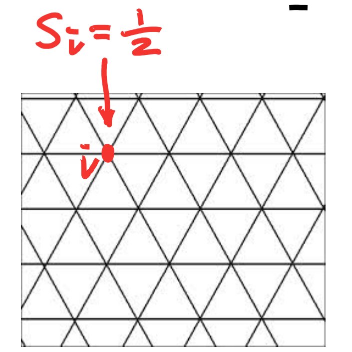

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Note:

$$A_{ij}(b_{i\uparrow}^{+} b_{j\downarrow}^{+} - b_{i\downarrow}^{+} b_{j\uparrow}^{+})$$

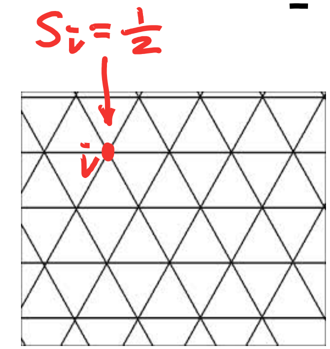
$$= -A_{ij}(b_{j\uparrow}^{+} b_{i\downarrow}^{+} - b_{j\downarrow}^{+} b_{i\uparrow}^{+})$$

$$\Rightarrow \quad A_{ij} = -A_{ji} \qquad (A \text{ is directional})$$

Naively → Aij always break lattice symmetry?!

The gauge structure of QSL wavefunction

$$|spin\rangle \equiv P_G |GS\rangle_{MF} \qquad P_G: \text{projection}$$



- Although |GS>MF breaks lattice symmetry, |spin> may restore it.

Consider two MF states:

$$H^1_{MF}(\{A_{ij}\}) = \sum_{ij} A_{ij}\left(b^\dagger_{i\alpha} b^\dagger_{j\beta} \varepsilon_{\alpha\beta} + h.c.\right)$$

$$H^2_{MF}(\{A_{ij} e^{i\theta_i + i\theta_j}\}) = \sum_{ij} A_{ij}\left(e^{i\theta_i} b^\dagger_{i\alpha} e^{i\theta_j} b^\dagger_{j\beta} \varepsilon_{\alpha\beta} + h.c.\right)$$

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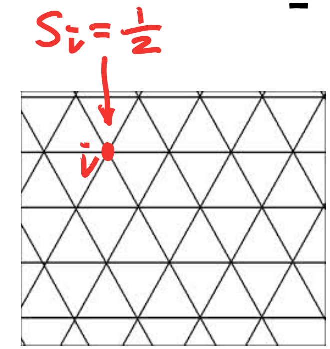

- Although |GS>MF breaks lattice symmetry, |spin> may restore it.

Consider two MF states:

$$H^1_{MF}(\{A_{ij}\}) = \sum_{ij} A_{ij}\left(b^\dagger_{i\alpha} b^\dagger_{j\beta} \varepsilon_{\alpha\beta} + h.c.\right)$$

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$$\Rightarrow \langle 0| b_{1\uparrow} b_{2\uparrow} b_{3\downarrow} \cdots b_{N\uparrow} |GS\rangle^1_{MF}$$

$$= \langle 0| b_{1\uparrow} b_{2\uparrow} b_{3\downarrow} \cdots b_{N\uparrow} |GS\rangle^2_{MF} \cdot \left[e^{-i\sum_{\text{all sites}} \theta_i}\right]$$

← overall phase

The gauge structure of QSL wavefunction

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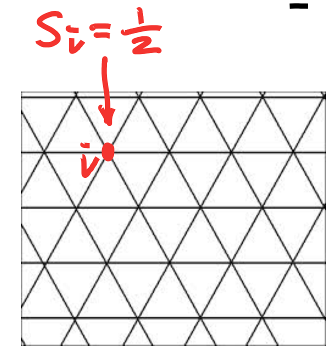

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label the same quantum state!

- Many to one labeling --- the definition of gauge theory.

(gauge "symmetry" is NOT physical symmetry)

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- Imagine we use |spin(Aij)> as variational wavefunctions,

The effective Hamiltonion must be U(1) gauge invariant.

$$H(\{A_{ij}\}) = H(\{A_{ij}e^{i\theta_i+i\theta_j}\})$$

The gauge structure of QSL wavefunction

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The spin state $|\mathrm{spin}\{A_{ij}\}\rangle$ respects lattice symmetry *g* as long as:

$$A_{ij} \xrightarrow{g} A_{g^{-1}(i),g^{-1}(j)} = A_{ij}\, e^{i[\theta_g(i)+\theta_g(j)]}$$

(Wen, 2002, Wang & Vishwanath 2006)

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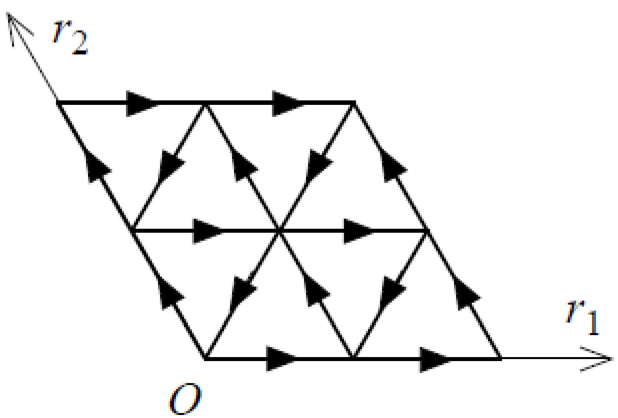
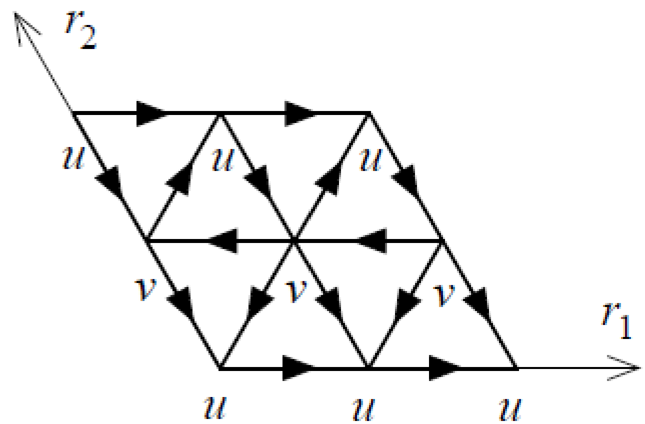


“Zero-flux state”



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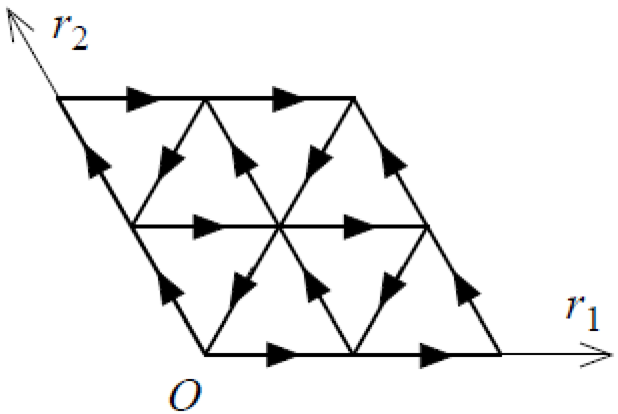

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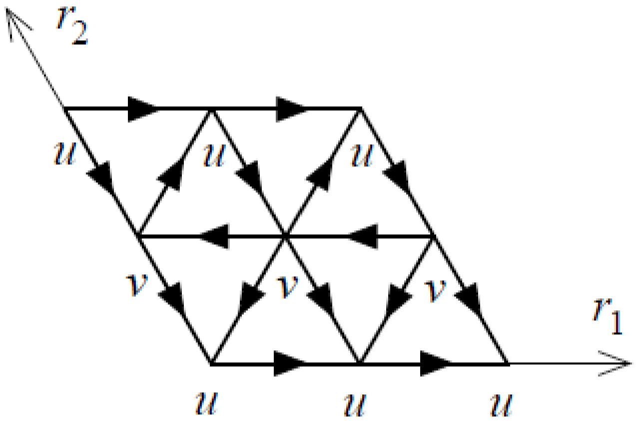

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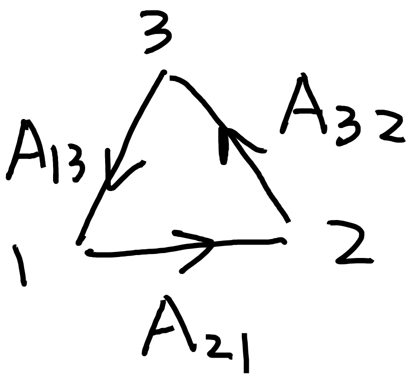
Are these two states different?
(will come back to this shortly)

Low energy excitations?

Low energy excitations

- Higgs mechanism:

The non-zero Aij breaks U(1) gauge redundancy down to Z2:



Consider:

$$P_1 \equiv A_{13} A_{32}^{*} A_{21}$$

gauge transformation: $P_1 \rightarrow P_1 e^{i2\theta_1}$

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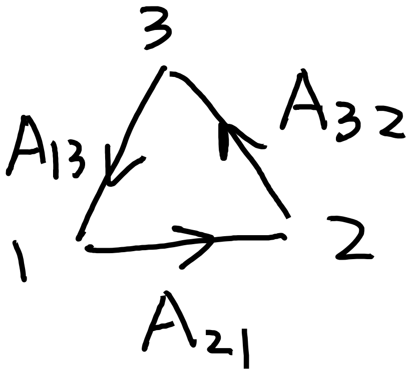

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- Low energy excitations:

gauge charge-1: Schwinger boson(spinon) + pi-gauge-flux: vison

Summary of discussion so far

- By attempting to construct symmetric QSL, we are forced to consider the gauge structure of wavefunctions.

- On triangular lattice, the states we discussed has emergent Z2 gauge dynamics. (gapped QSL)

  gauge charge: spin-1/2 boson (spinon)

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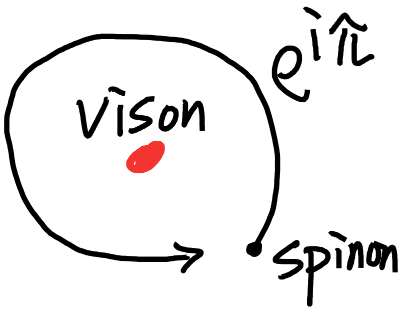
These are anyons!



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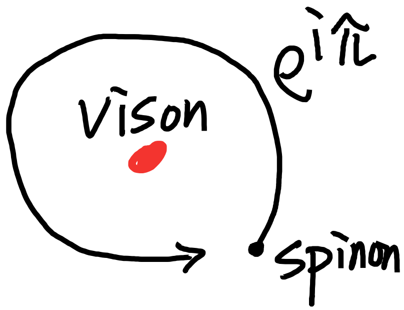

This is striking: another example of symmetry fractionalization.

Physical spectrum only contains integer spin excitations,

but quasiparticle (gauge charge) can be spin-1/2.

(Fractionalization of spin-rotation symmetry.)

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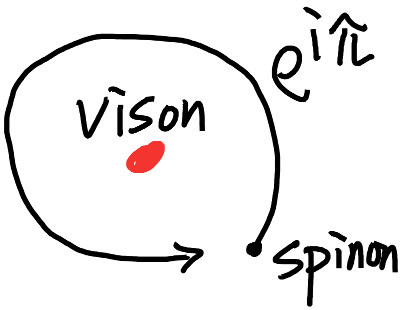

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- Consistent with “no-go theorem”: QSL has to be exotic

Coming back: Examples:

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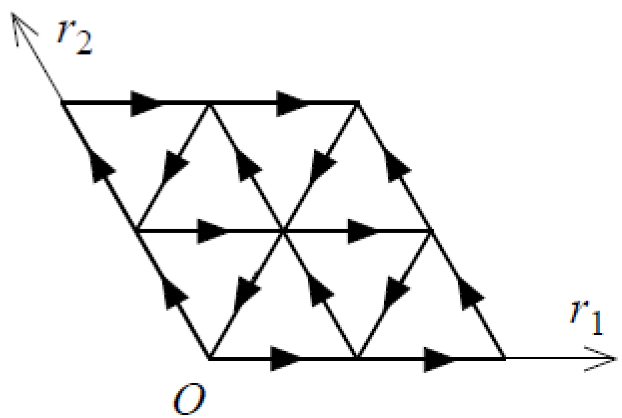

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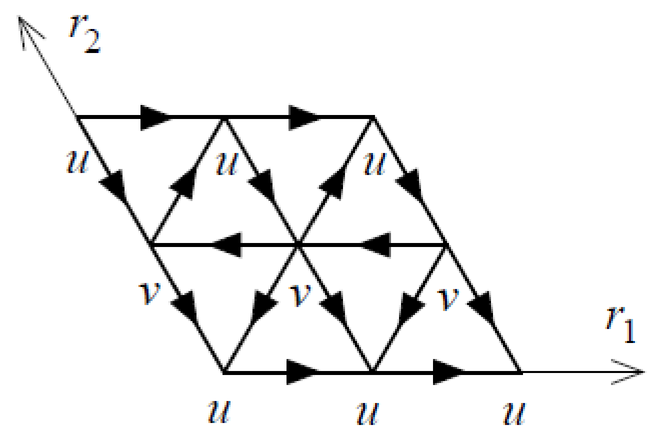

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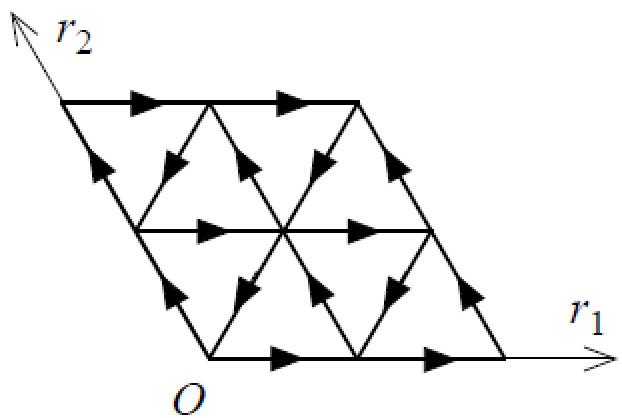

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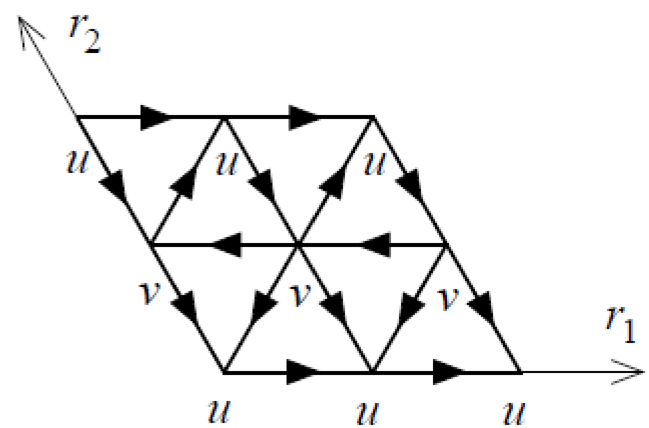


"pi-flux state"

They are different, no way to adiabatically connect.

Deep reason: These two states represent two inequivalent fractionalizations of lattice sym!

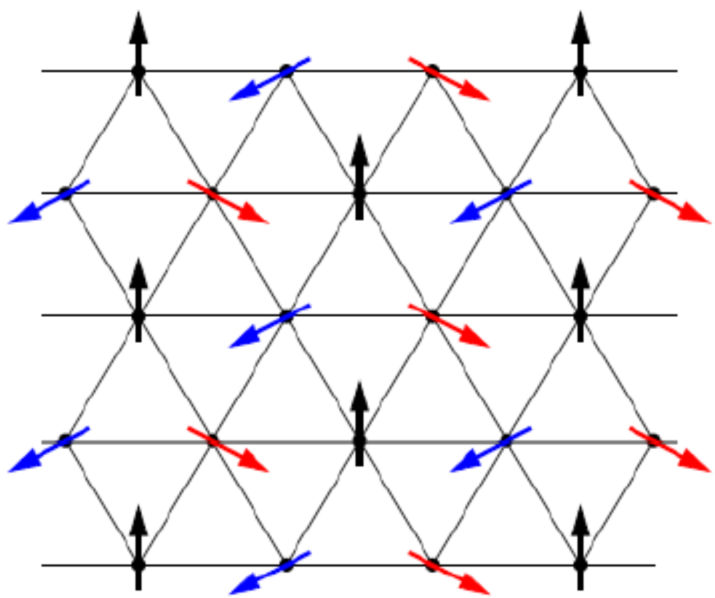
Spinons (gauge charge) can form projective representations of lattice symmetry.
In fact, also generally described by $H^2(\text{lattice SG}, Z_2)$ (Wen, Hermele,…)

Energetics

- I was assuming that energetically optimal $|GS\rangle_{MF}$ does not have Schwinger-boson condensation.

- For triangular lattice spin-1/2 Heisenberg model, this is NOT true.

In fact the optimal state has boson condensation exactly describing the 120-degree magnetic order:

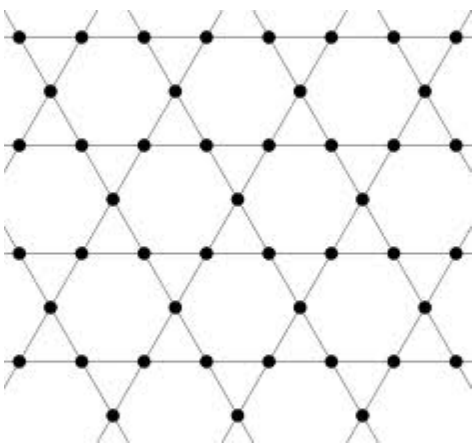


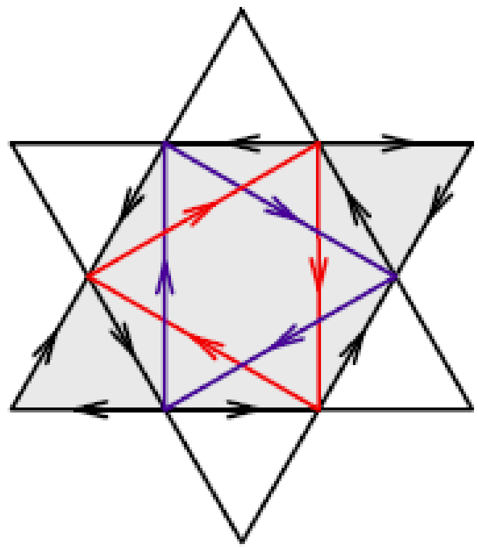
Energetics

- I was assuming that energetically optimal $|GS\rangle_{MF}$ does not have Schwinger-boson condensation.

- But for Kagome spin-1/2 J1-J2 Heisenberg model, the optimal state is found to be fully gapped QSL .

(Tay, Motrunich 2011)



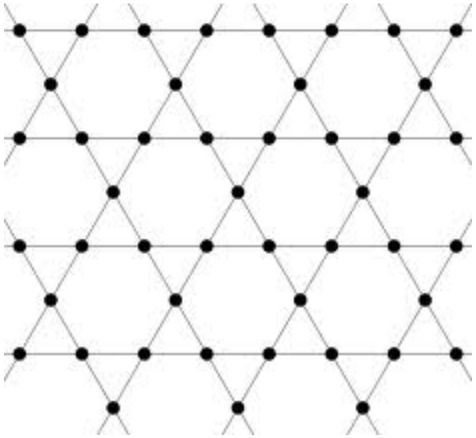


(a) $\mathbf{q} = 0$ SB ansatz

(Sachdev 1992, Wang & Vishwanath 2006)

Kagome spin-1/2 system

- For NN only Heisenberg interaction:



Some numerical works (incomplete list):

Series expansion: VBS (Singh, Huse, 2008)

MERA: VBS (Evenbly, Vidal, 2010)

DMRG: gapped QSL (Yan, Huse, White 2011.....)

Variational Monte Carlo: U(1)-Dirac gapless QSL

(YR, Hermele, Lee, Wen Iqbal, Poilblanc, Becca,.....)

Experiment signatures of QSL?

- Apart from no magnetic ordering:

(1) If gapless → metallic thermal transport in an insulator

(2) If gapped, more difficult to prove by usual probes.

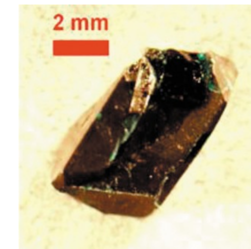
But at least spin excitations should form continuum without quasiparticle peaks.

- Spin-1/2 Kagome lattice Heisenberg system

  Herbertsmithite: $ZnCu_3(OH)_6Cl_2$

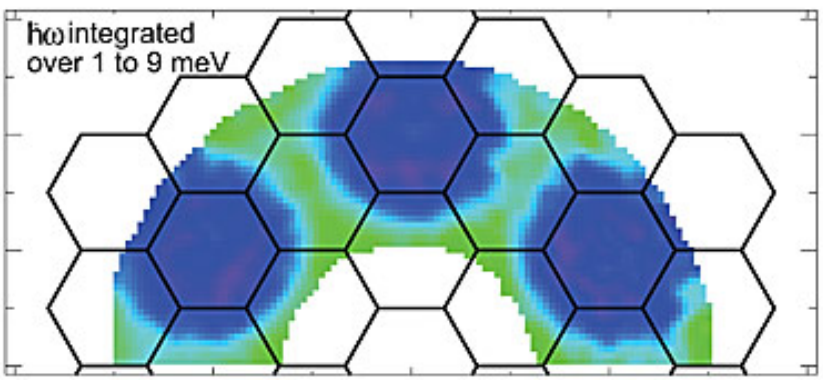
  J~200K, no ordering down to 50mK,

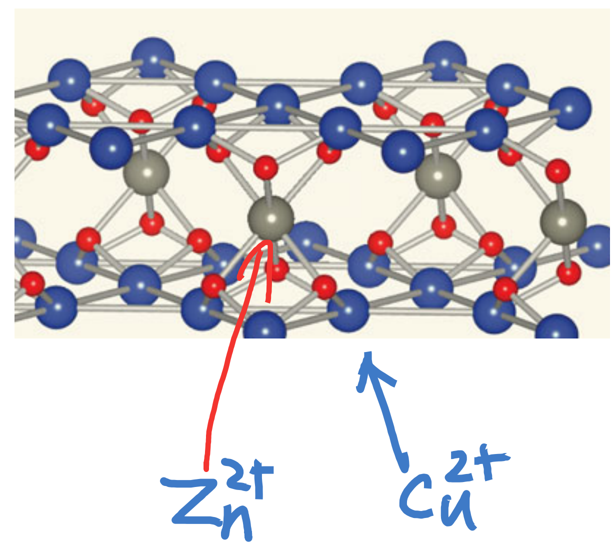
- Experiments support gapless QSL:



Herbertsmithite

e.g, Han et.al, 2012: Inelastic neutron scattering.





A central question for classification of the ground state of herbertsmithite is whether a spin gap exists. One surprising aspect of our data is that the spin excitations seem to be gapless over a wide range of Q positions, at least down to $\hbar\omega = 0.25$ meV. This observation is difficult

Summary

- In frustrated magnets, QSLs may be realized as ground states.

- By constructing QSL wavefunctions, we are forced to realize:
  QSL hosts emergent gauge dynamics.

- Gapped QSL is "topologically ordered".
  topological dynamical excitations: gauge charge/flux (anyons)

- Symmetry fractionalization occurs for gauge charge/flux excitations:
  local symmetries (spin rotation),
  and spatial symmetries (lattice symmetry).

- Thank you!