Quantum Hall Effect: a Paradigm of Topological Order
I

Probing (non-Abelian) Anyons in Quantum Hall Systems

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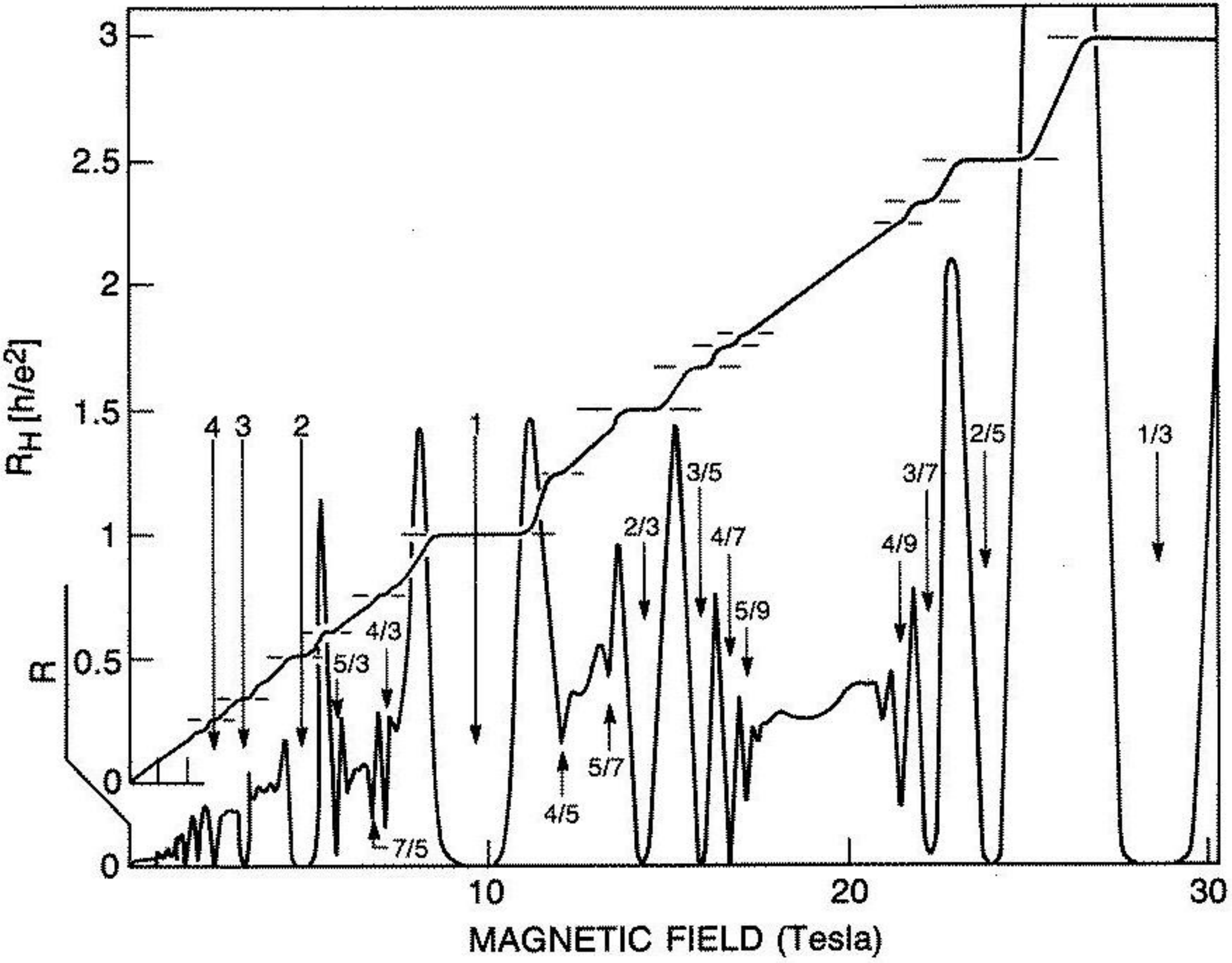
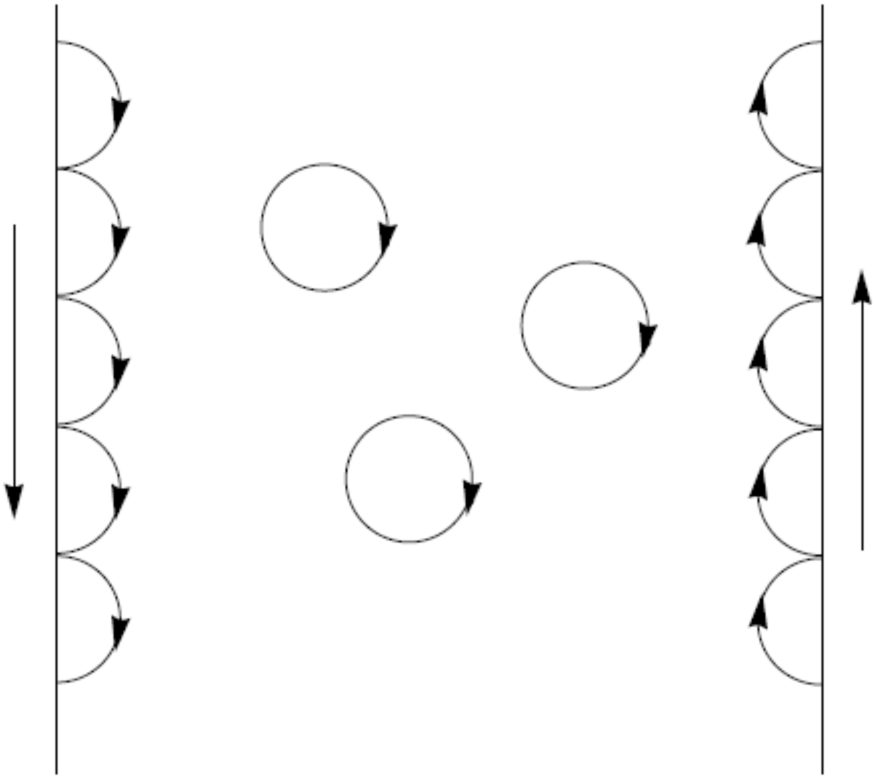
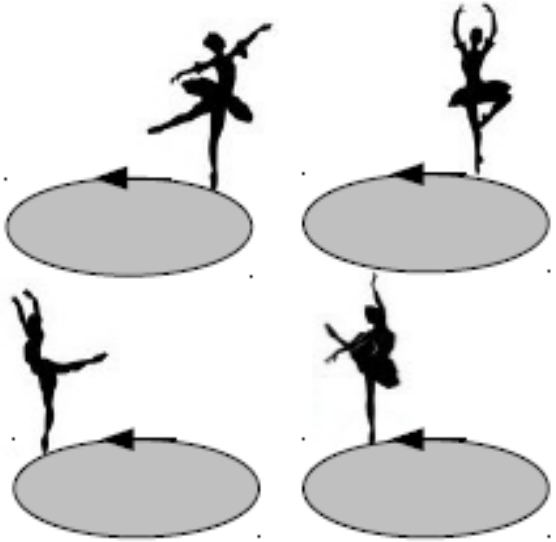
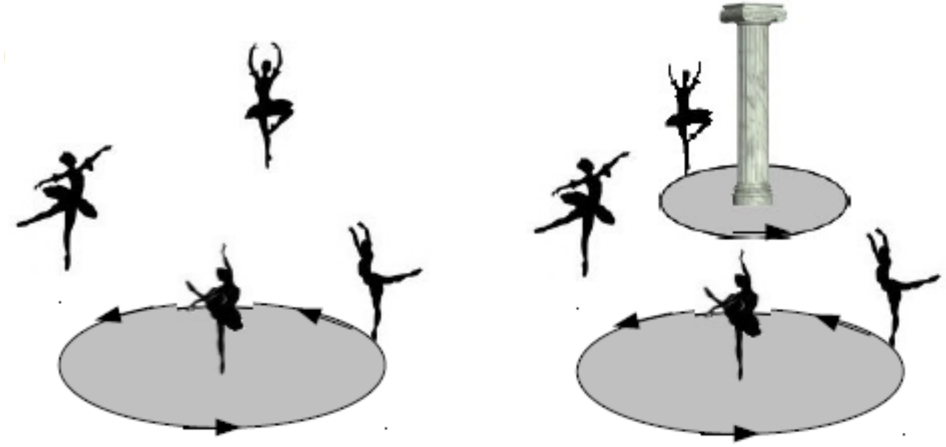


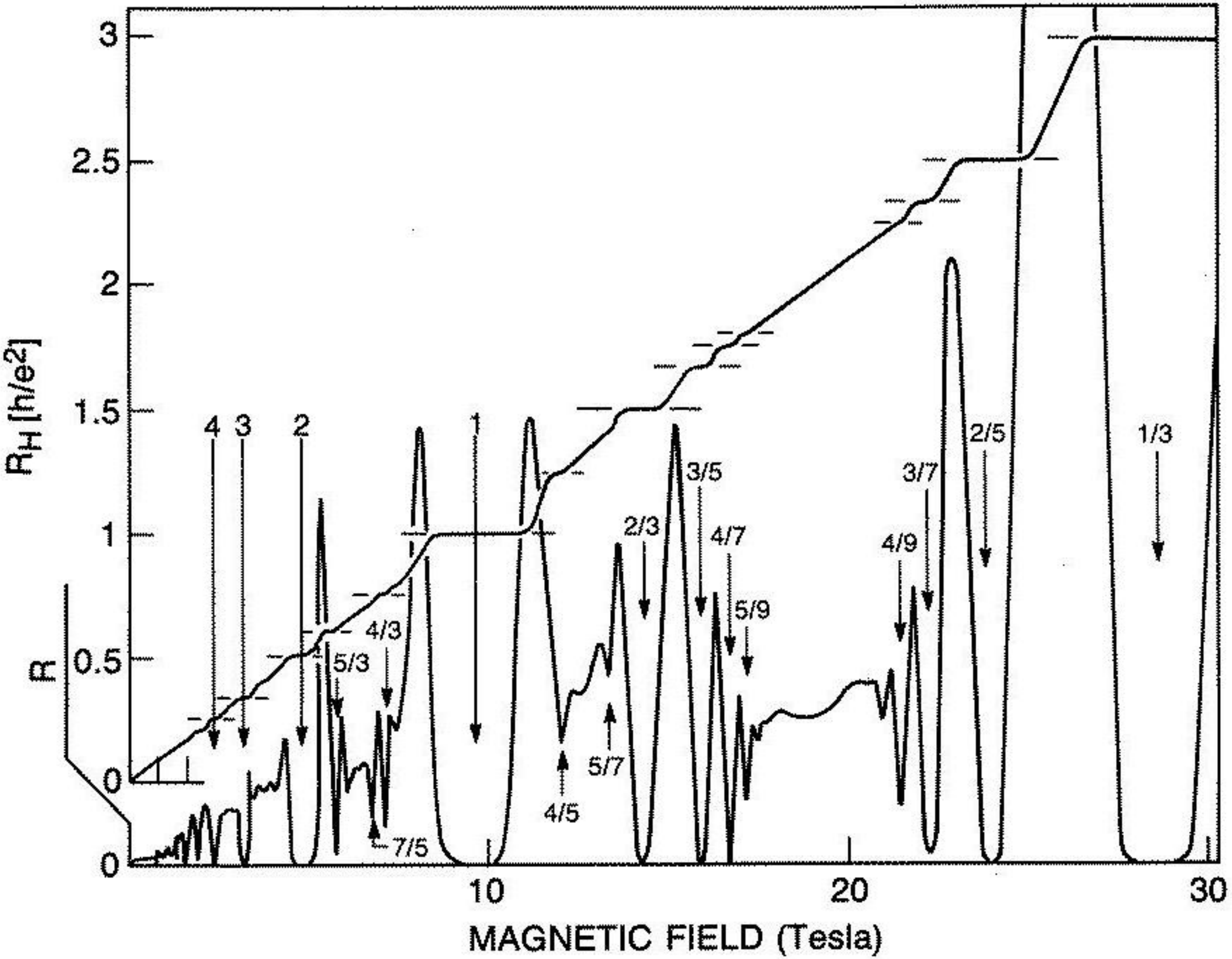
R. Willett and J. Eisenstein





Chiral Edge States Responsible for Dissipationless, Quantized Hall Transport!

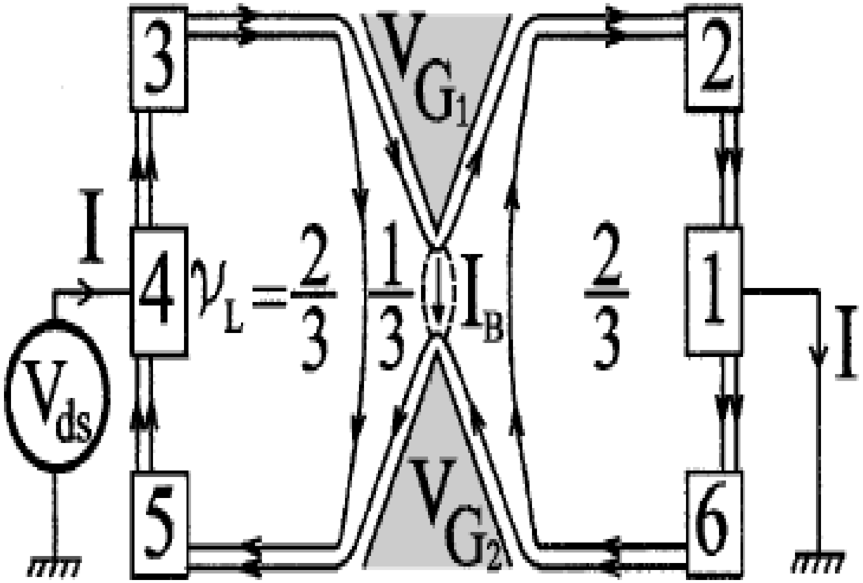


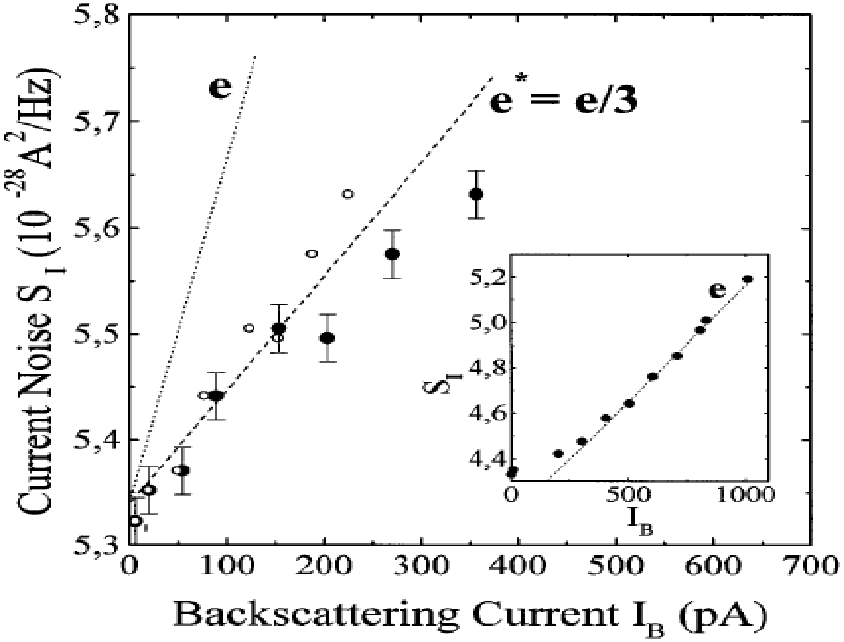


R. Willett and J. Eisenstein

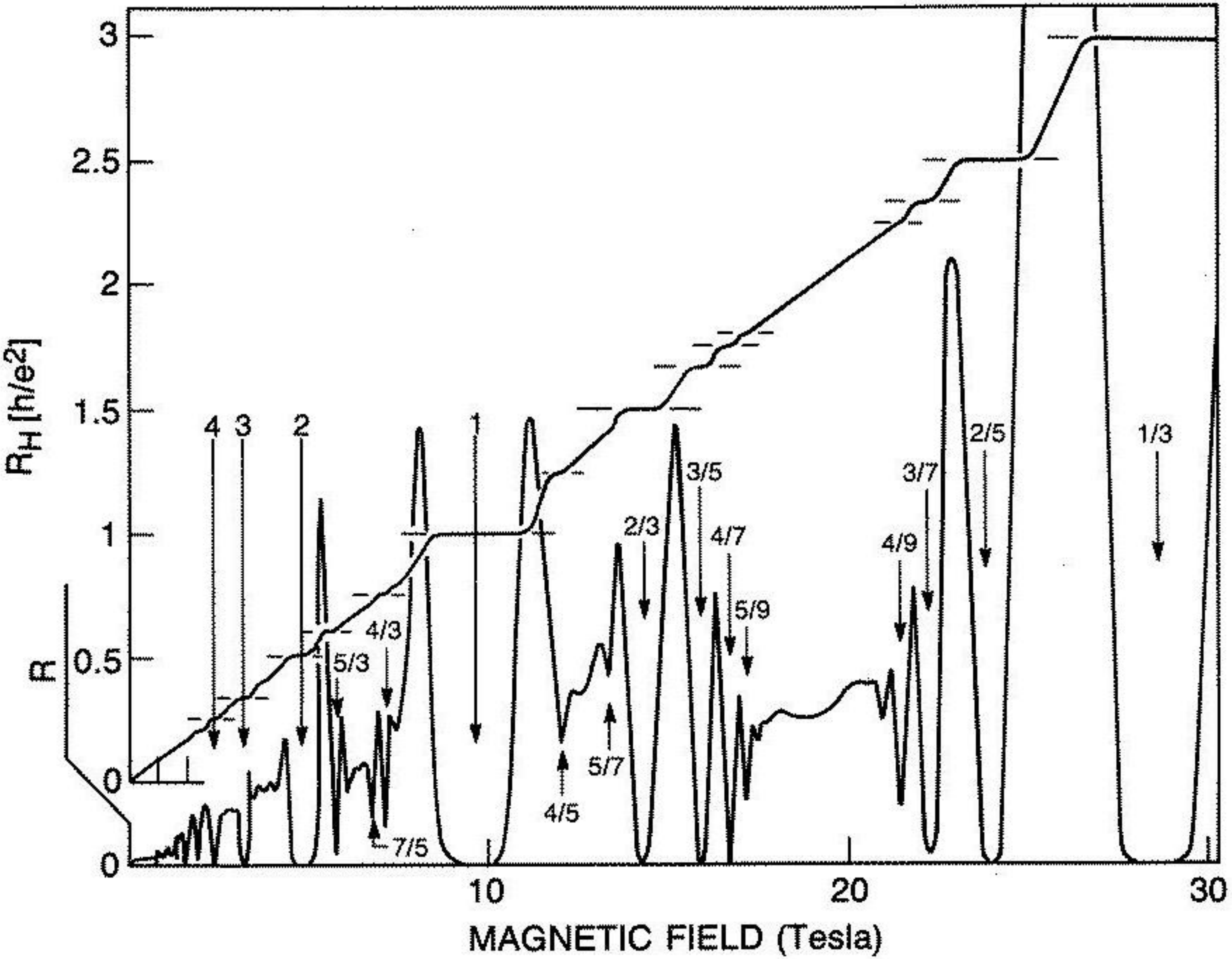
Fundamental difference between 1/3 and IQH states *not* manifested in bulk transport measurements!

Fractional quasiparticle charge probed through noise in (backscattering) *edge* current:





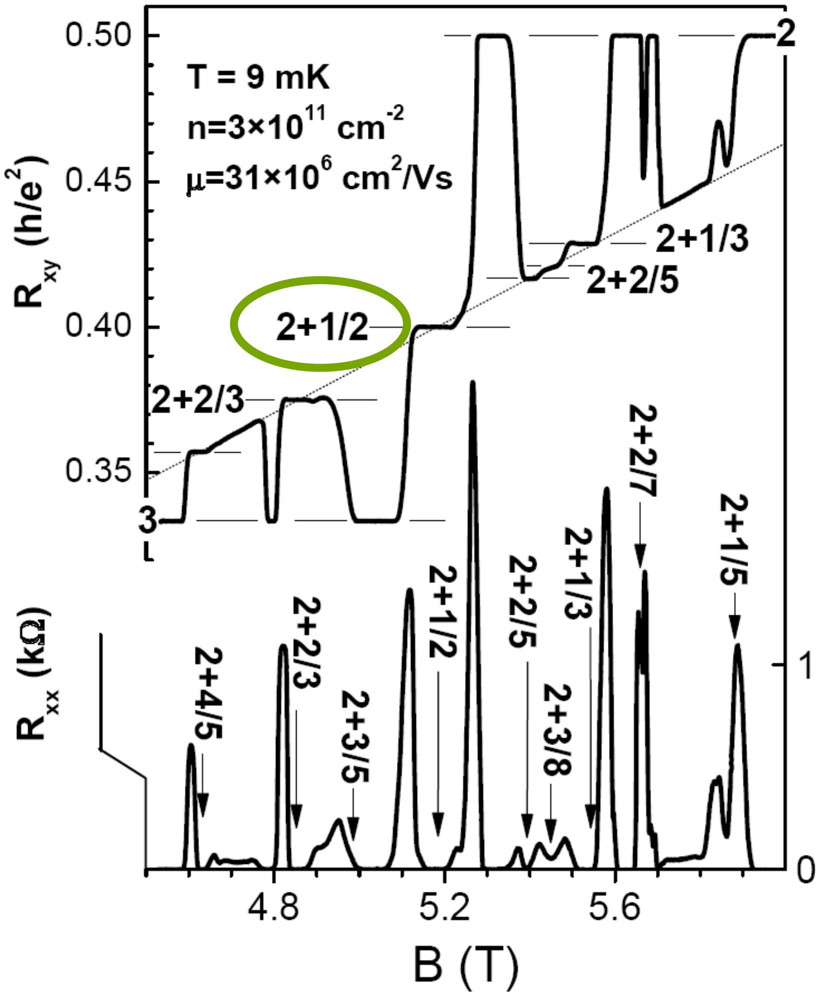
Saminadayar et al. PRL 97; de-Picciotto et al. Nature 97.



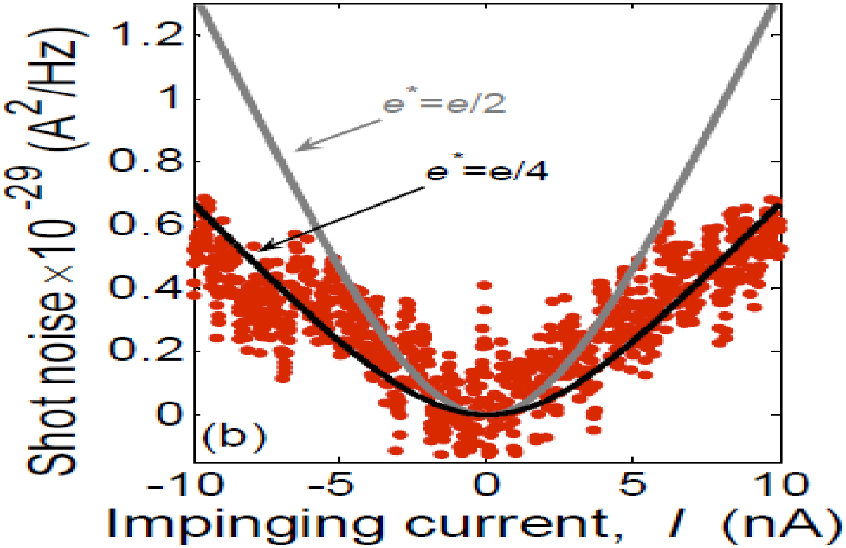
R. Willett and J. Eisenstein

Possible Non-Abelian FQH States

Willett et al. PRL 87;
J.S. Xia et al., PRL (2004).



$\nu = 5/2$: Probable Moore-Read Pfaffian state, or its particle-hole conjugate (anti-Pfaffian). Charge $e/4$ quasiparticles (1/2 of Laughlin qp) obeying non-Abelian statistics.



M. Dolev et al., Nature 08

The Moore-Read Wave Function

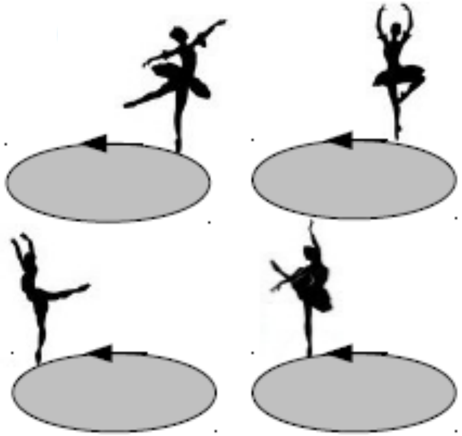
$$\Psi_{\mathrm{MR}}(z_1,z_2,\ldots,z_N)=\mathrm{Pf}\left(\frac{1}{z_i-z_j}\right)\prod_{i<j}(z_i-z_j)^2\exp\left\{-\sum_i\frac{|z_i|^2}{4}\right\}$$

$$\mathrm{Pf}\,M_{ij}=\frac{1}{2^{N/2}(N/2)!}\sum_{\sigma\in S_N}\mathrm{sgn}\sigma\prod_{k=1}^{N/2}M_{\sigma(2k-1)\sigma(2k)}$$

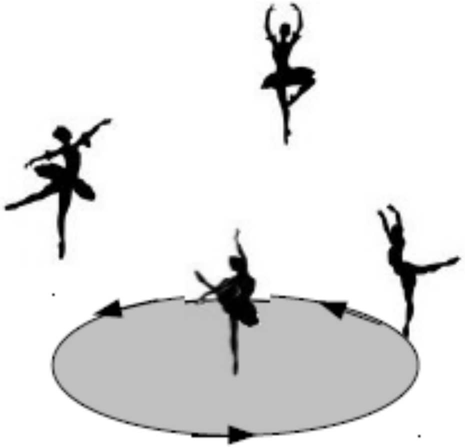
Interpretation: p-wave paired or topological superconducting state of composite fermion; quasiparticle ≈ superconducting vortex; carrying a Majorana fermion degree of freedom!.

$$\Psi_{\mathrm{MR}}^{e/4}(z_1,z_2,\ldots,z_N)=\mathrm{Pf}\left(\frac{z_i+z_j}{z_i-z_j}\right)\prod_{i<j}(z_i-z_j)^2\exp\left\{-\sum_i\frac{|z_i|^2}{4}\right\}$$

Integer ("trivial")
Dancing Pattern



Laughlin (Abelian)
Dancing Pattern



Moore-Read
Dancing Pattern

More complicated dancing pattern corresponds to more sophisticated topological order, and allow for more exotic topological defects/quasiparticles, including ones with non-Abelian statistics.

The Moore-Read Wave Function

$$\Psi_{\mathrm{MR}}(z_1,z_2,\ldots,z_N)=\mathrm{Pf}\left(\frac{1}{z_i-z_j}\right)\prod_{i<j}(z_i-z_j)^2\exp\left\{-\sum_i\frac{|z_i|^2}{4}\right\}$$

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What is non-Abelian Statistics?

Ground state degeneracy grows exponentially with quasiparticle number (when sufficiently far apart), even with fixed positions :

$$D \sim d^{N_q}, \;\; S_d = k_B \log D = k_B N_q \log d + O(1)$$

$d > 1$ is the quantum dimension; for Pfaffian or aPf, $d = \sqrt{2}.$

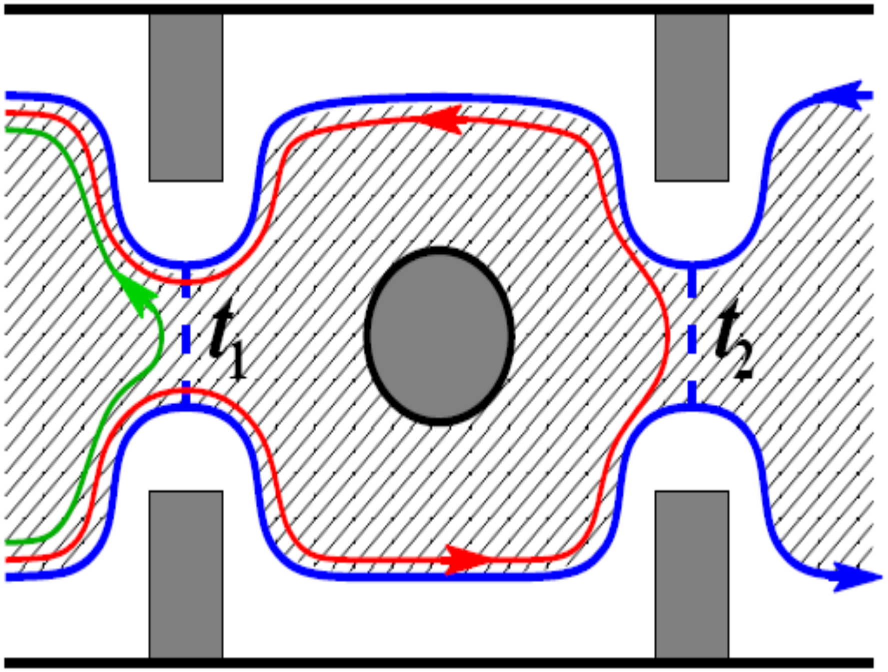
Braiding (and only braiding) induces unitary transformation in the ground state subspace.

Methods to probe non-Abelian statistics:

1. Detecting non-Abelian entropy S_d using bulk thermoelectric or thermodynamic measurements (KY and B. Halperin 09; A. Stern and N. Cooper 09; Gervais + KY 10; Barlas + KY 12; **Eisenstein 10, 13**);

2. Detecting the unitary transformation induced by braiding (edge interferometry; **Willett, Kang**).

Non-Abelian Quasiparticle Interferometry



Chamon, Freed, Kivelson, Sondhi, Wen 97; originally aimed at detecting Abelian fractional statistics.

Better for detection of non-Abelian statistics:

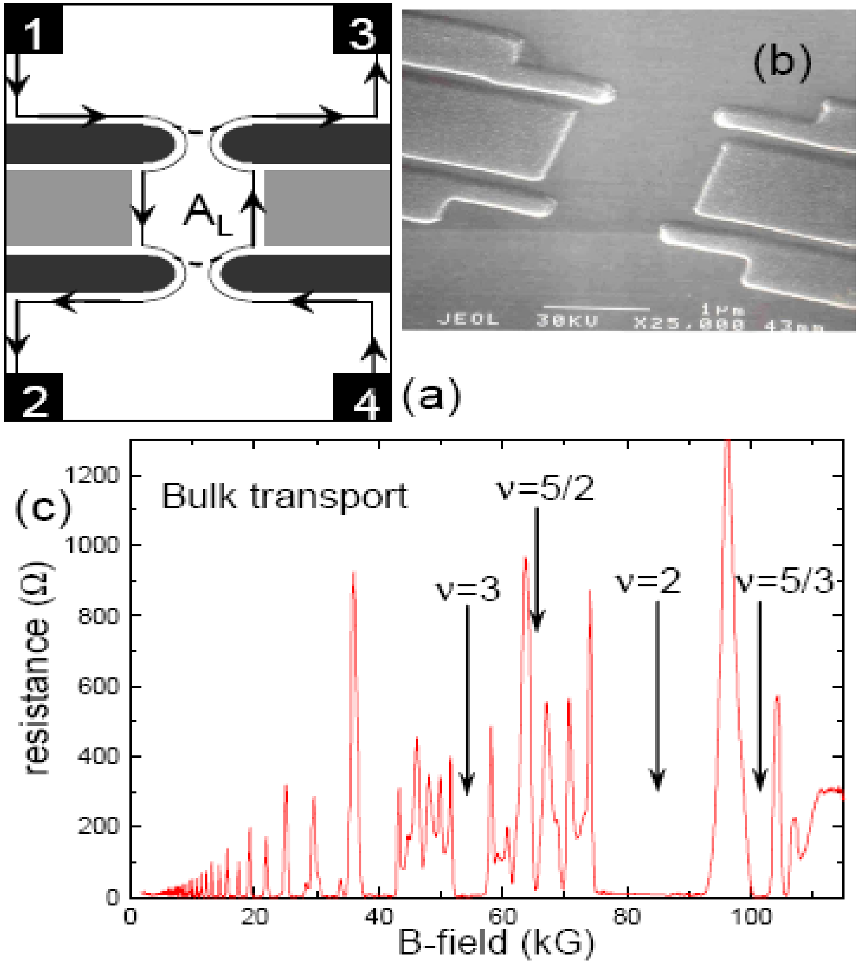
Stern and Halperin 06, Bonderson, Kitaev and Shtengel 06.

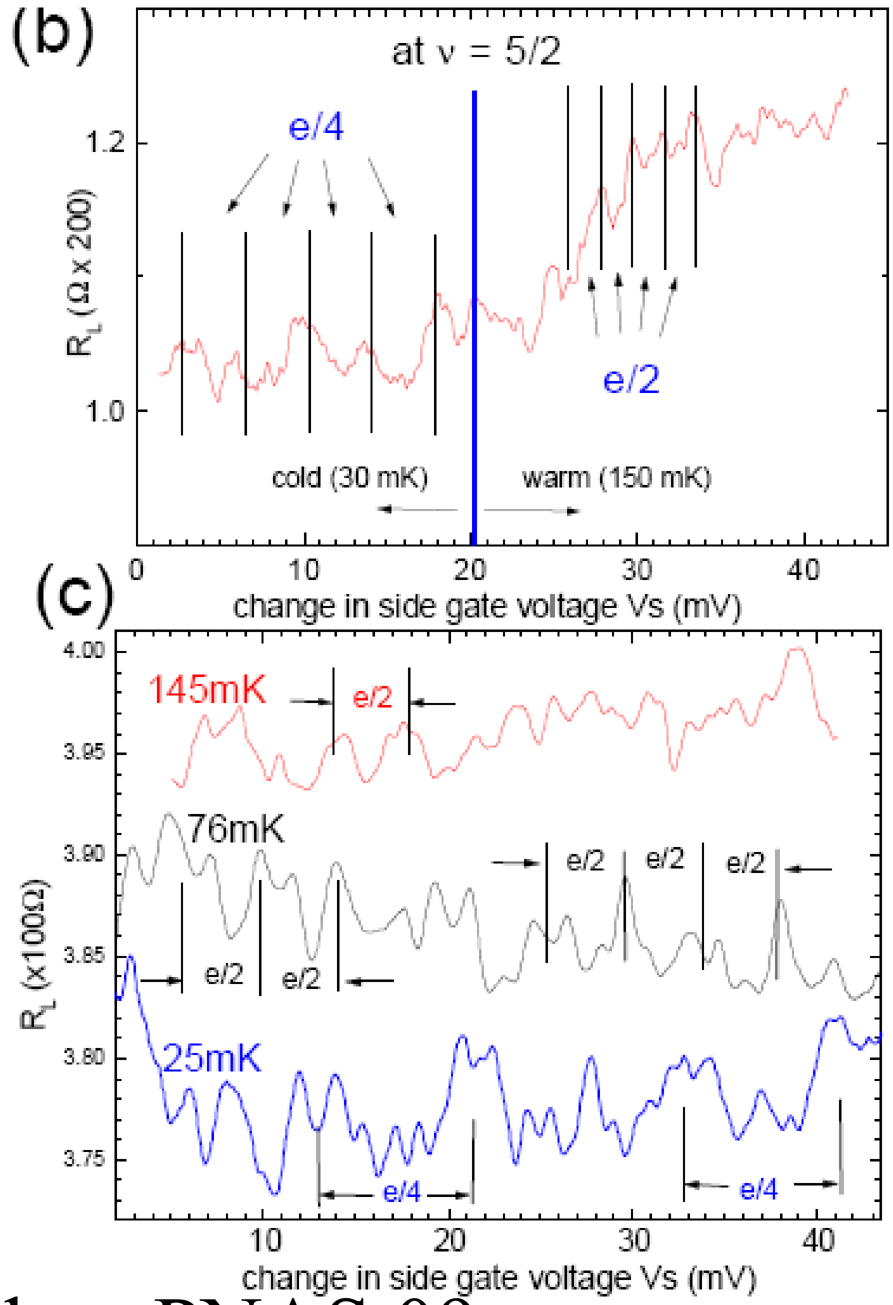
Complications: quasiparticles need to remain coherent in order to be able to interfere; quasipartciles other than 1/4 may also contribute; possible edge reconstruction; competing phases near 5/2 ……

Needed: Quantitative understanding of edge states at 5/2. Perform detailed numerical study using disc geometry.

For numerical work on semi-quantitative understanding of 5/2 edge states, see X. Wan, KY and E. Rezayi, PRL 06; X. Wan, Z. Hu, E. Rezayi, and KY PRB 08; H. Chen, Z. Hu, KY, E. Rezayi and X. Wan; PRB 09; Z. Hu, E. Rezayi, X. Wan and KY, PRB 09.

Some of our qualitative and semi-quantitive predictions, in particular importance of e/2 quasiparticle contribution to interference, are borne out by R. Willett, L. Pfieffer and K. West, PNAS 09, PRB 10, PRB 11 and Arxiv 13.



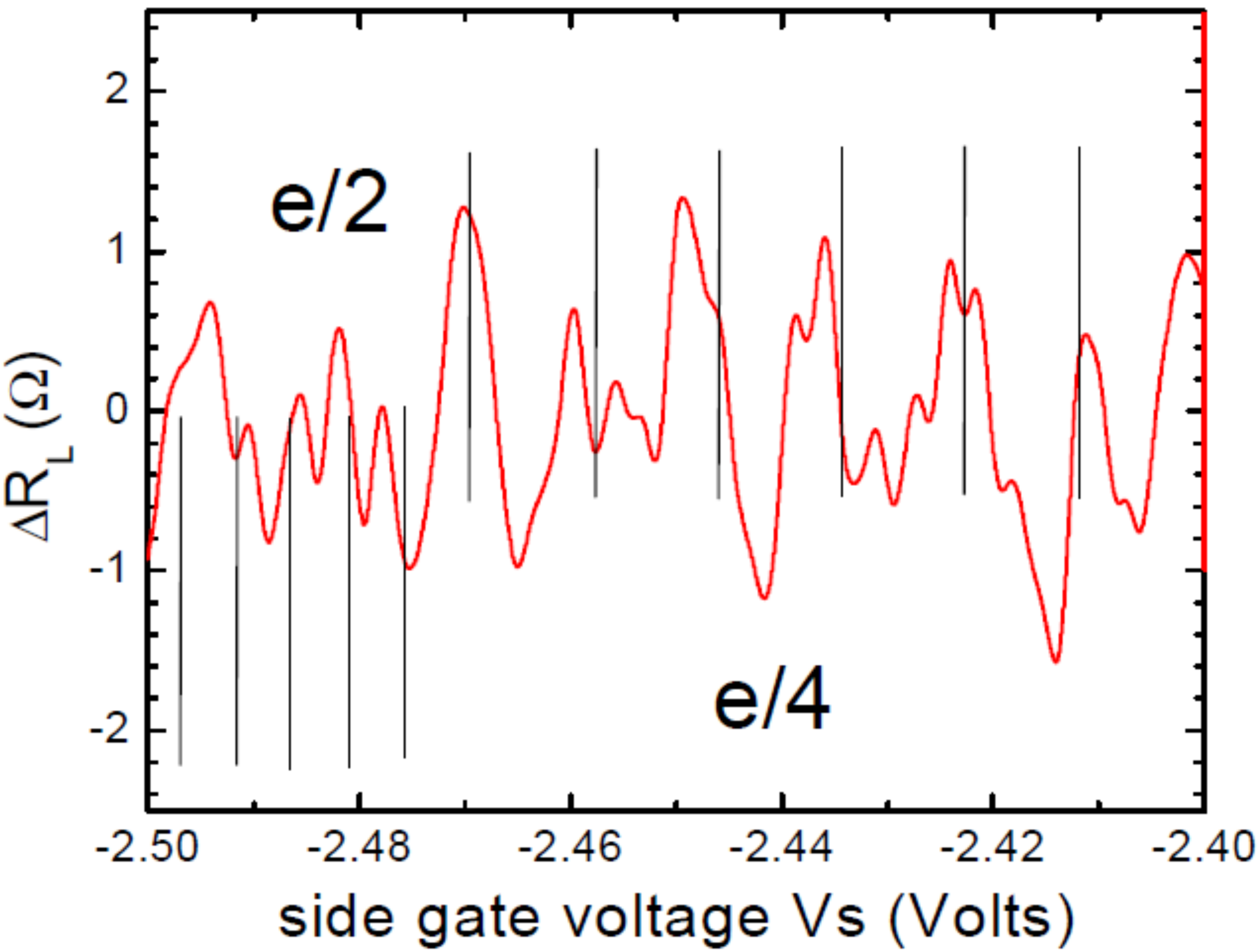


L ~ 1 micron; using estimates of velocities from our numerics, need T < 40 mK to observe interference of e/4 quasiparticle.

Willett, PNAS 09

Interference at 5/2

Area 2



Willett, arxiv 13

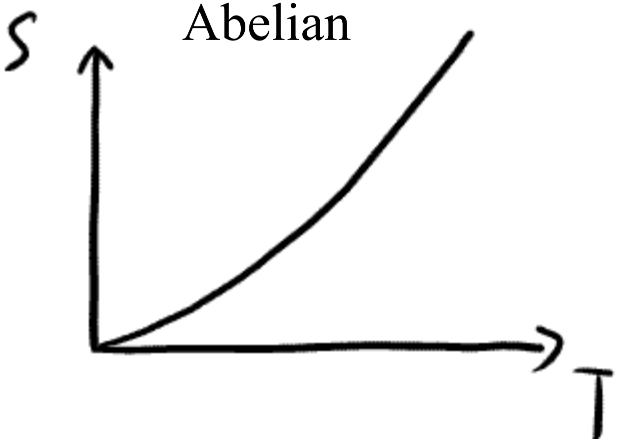
Probing non-Abelian Anyons Using **Bulk** Measurements

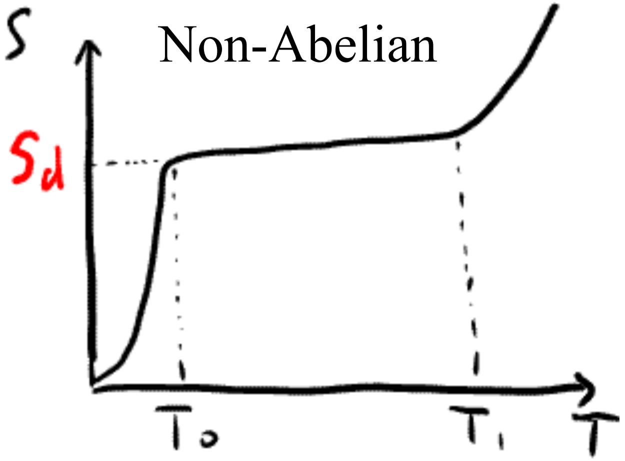
Difference between non-Abelian and Abelian QH states more dramatic than difference between IQH and FQH states; visible in **bulk** properties!

Ground state degeneracy grows exponentially with quasiparticle number (when sufficiently far apart), even when their positions are fixed:

$$D \sim d^{N_q}, \quad S_d = k_B \log D = k_B N_q \log d + O(1)$$

$d > 1$ is the quantum dimension; for Pfaffian or aPf, $d = \sqrt{2}.$





Problem: 2DEG embedded in 3D environment; ***S*** dominated by the latter.

Probing non-Abelian Anyons Using Bulk Measurements

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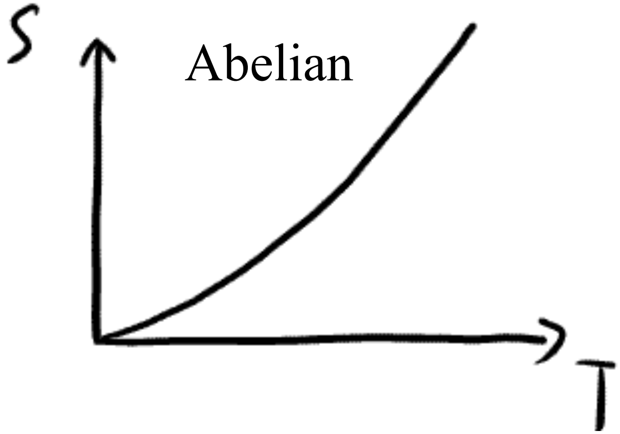
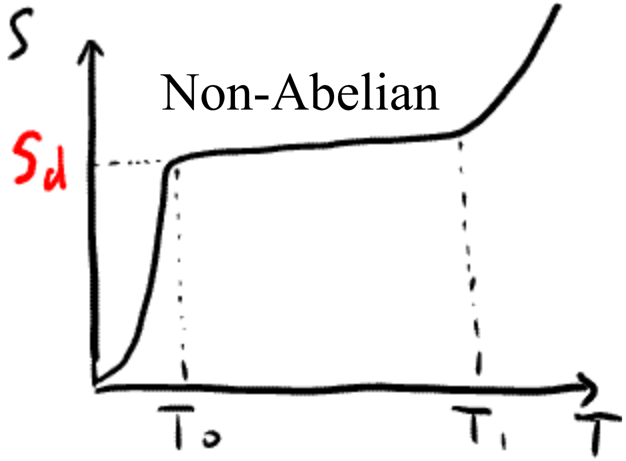




Problem: 2DEG embedded in 3D environment; S dominated by the latter.

Thermopower as a Measure of (2D) Bulk Entropy

2D entropy accessible through thermoelectric response (from mobile electrons only) to gradient of *T*! Reason: entropy conjugate to *T*, just like charge conjugate to potential. So "roughly speaking", thermopower *Q* measures entropy per charge carrier :

$$Q = -\nabla V / \nabla T \approx -S / Ne$$

Rigorously justified in the clean limit ***and*** in the presence of B field in: Obraztsov (65), non-interacting; Cooper, Halperin and Ruzin (97). Justified ***without*** B field in KY and B. I. Halperin, PRB 09.

Clean limit crucial:

$$Q = -S / Ne$$

Mechanical Equilibration: $\nabla P = (\frac{\partial P}{\partial \mu})_T \nabla \mu + (\frac{\partial P}{\partial T})_\mu \nabla T = -n \nabla \phi$

P: pressure; *μ*: local chemical potential; *Φ*: external potential.

Maxwell relations from $d\Omega = -SdT - PdA - N_e d\mu$:

$$(\frac{\partial P}{\partial \mu})_{T,A} \;=\; (\frac{\partial N_e}{\partial A})_{T,\mu} \;=\; N_e/A = n$$

$$(\frac{\partial P}{\partial T})_{\mu,A} = (\frac{\partial S}{\partial A})_{T,\mu} = S/A$$

$\rightarrow$ $n\nabla\mu + (S/A)\nabla T = -n\nabla\phi$ or $\boxed{\nabla\xi/\nabla T = -S/N_e}$

$\xi = \mu + \phi$ electrochemical potential measured by ideal contacts.

(KY and B. I. Halperin, PRB 09)

Thermopower of a non-Abelian QH Liquid

$$Q = -|(B - B_0)/B_0|(k_B/|e^*|)\log d$$

for the temperature range $T_0 \ll T \ll T_1$.

Since quasiparticle charge e^* can be measured independently, Q provides a direct measurement of *quantum dimension* **d**!

T_0 exponentially small with quasiparticle distance. T_1 of order Debye temperature of quasiparticle Wigner crystsal:

$$k_B T_D \sim \frac{e^2}{\epsilon l_B}\sqrt{\frac{e}{|e^*|}}\left[\frac{\nu_0|B - B_0|}{B_0}\right]^{\frac{3}{2}}$$

At edge of 5/2 plateau in best sample, $T_D \sim 100$ mK. Also need to melt WC to get a liquid state; $T_M \sim 7$ mK. **(KY and B. Halperin 09)**

Thermopower of Two-Dimensional Electrons at $\nu = 3/2$ and $5/2$

W. E. Chickering[1], J. P. Eisenstein[1], L. N. Pfeiffer[2] and K. W. West[2]
[1] *Condensed Matter Physics, California Institute of Technology, Pasadena, CA 91125*
[2] *Dept. of Electrical Engineering, Princeton University, Princeton, NJ 08544*
(Dated: March 9, 2010)

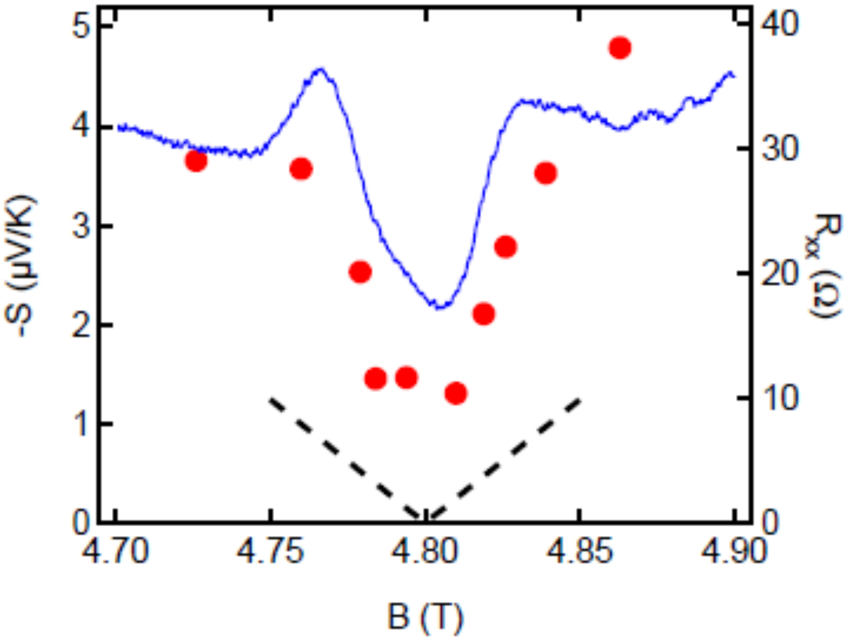


FIG. 8: (color online) Thermopower *vs.* magnetic field (red circles) along with R_{xx} *vs.* magnetic field (blue curve) about $\nu = 5/2$ at $T \approx 82$ mK. The dashed line represents the thermopower of Eq. (3) for $B_0 = 4.80$ T.

One difficulty (among others): long thermal relaxation time below 80 mK.

Messages: (i) Non-Abelian entropy *may* already represent a sizable fraction of total entropy; (ii) at low T, 2DEG (almost) decoupled from environment; *easy to do things adiabatically!*

Thermoelectric response of fractional quantized Hall and re-entrant insulating states in the N=1 Landau level

W.E. Chickering[1], J.P. Eisenstein[1], L.N. Pfeiffer[2], and K.W. West[2]
[1] *Condensed Matter Physics, California Institute of Technology, Pasadena, CA 91125*
[2] *Department of Electrical Engineering, Princeton University, Princeton, NJ 08544*

(Dated: November 16, 2012)

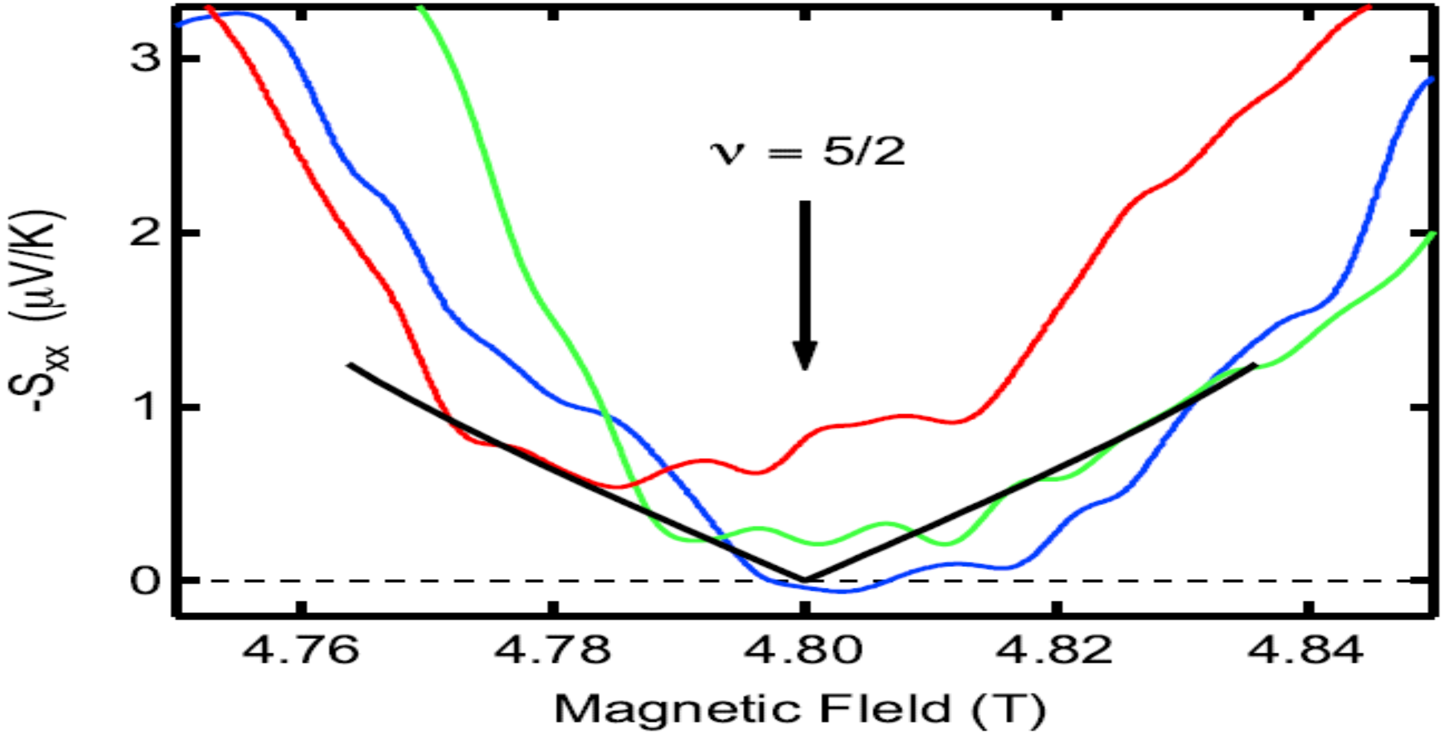
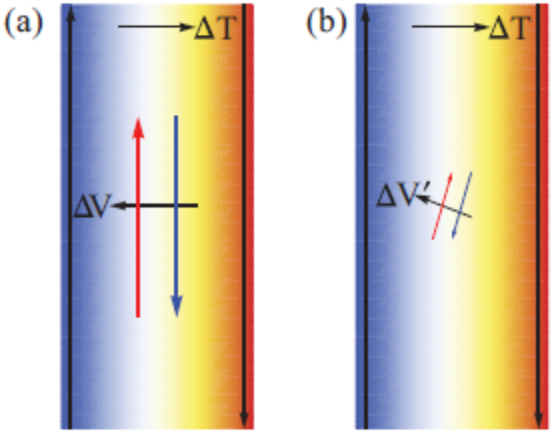


FIG. 4: (color online). S_{xx} near $\nu = 5/2$ at $T = 20$ mK (blue), 28 mK (green) and 41 mK (red). Solid black lines give theoretical prediction of Yang and Halperin [32].

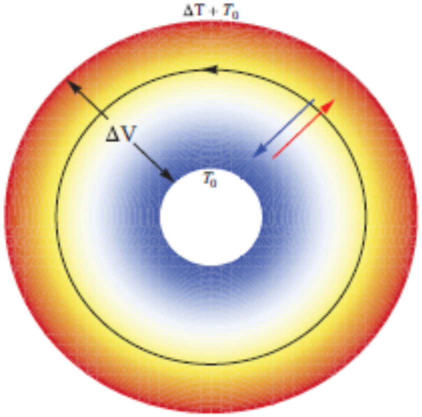
A Potentially Better Way: Thermopower of Corbino Geometry

(Y. Barlas and KY, PRB12)

Hall Bar



Corbino



Advantage compared to Hall bar: Only quasiparticles contribute to both thermo and electric responses.

Central Result: Corbino thermopower measures entropy per quasiparticle divided by its charge (bigger signal and valid in the presence of strong disorder!).

$$Q_C \approx \frac{S_{\text{tot}}}{e^\star N_q}$$

A Potentially Better Way: Thermopower of Corbino Geometry

(Y. Barlas and KY, PRB12)

$$\mathbf{j} = L^{11}\nabla\phi - L^{12}\frac{\nabla T}{T},$$

$$\mathbf{j}_Q = L^{21}\nabla\phi - L^{22}\frac{\nabla T}{T},$$

Hall Bar: $Q_H = \frac{1}{T}L^{12}(L^{11})^{-1}$

Corbino: $Q_C = \frac{1}{T}\frac{L^{12}_{rr}}{L^{11}_{rr}}$

Transport coefficients calculated in IQH regime using self-consistent Born Approximation:

$$L^{11}_{rr} = \frac{e^2\omega_c^2}{8\pi h}\int_{-\infty}^{\infty} d\epsilon\left(-\frac{\partial n_F}{\partial \epsilon}\right) \times \sum_n [(n+1)ImG_n(\epsilon)ImG_{n+1}(\epsilon)],$$

$$L^{12}_{ij}(T,\mu) = \int_{-\infty}^{\infty} d\epsilon \frac{\epsilon-\mu}{e}\left(-\frac{\partial n_F}{\partial \epsilon}\right)L^{11}_{ij}(T=0,\epsilon)$$

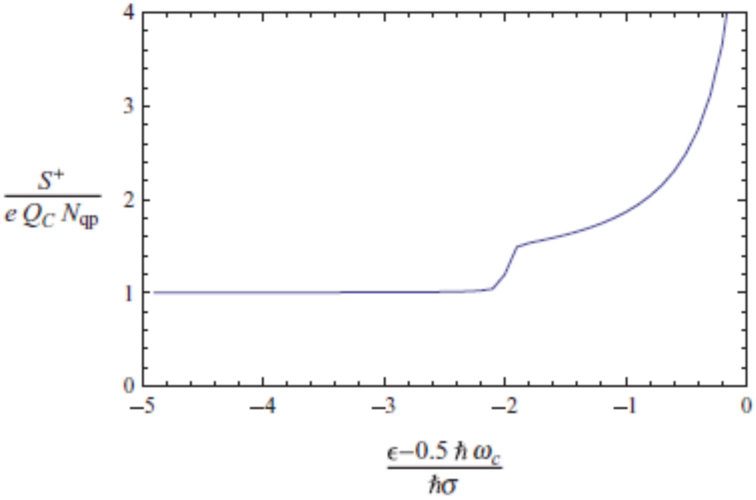


FIG. 6. (Color online) Numerical attained ratio of the *entropy per quasiparticle per quasiparticle charge* $S/(eN_q)$ and the Corbino thermopower Q_C for the quasielectron region of an isolated lowest Landau level for $k_\beta T/\hbar\sigma = 0.01$.

Diffusion Thermopower of Quantum Hall States Measured in Corbino Geometry

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(Received March 2, 2013; accepted March 15, 2013; published online April 4, 2013)

We have measured the diffusion thermopower of a quantum Hall system in a Corbino setup. A concentric electron-temperature gradient is introduced by irradiating microwaves, via a coplanar waveguide, near the outer rim of a circular mesa of a two-dimensional electron gas. The resulting radial thermovoltages exhibit sawtooth-like oscillations with the magnetic field, taking large positive (negative) values just below (above) integer fillings with sign reversal at the center of the quantum Hall plateaus. The behavior is in agreement with a recent theory [Y. Barlas and K. Yang: Phys. Rev. B **85** (2012) 195107], which treats disorder within the self-consistent Born approximation.

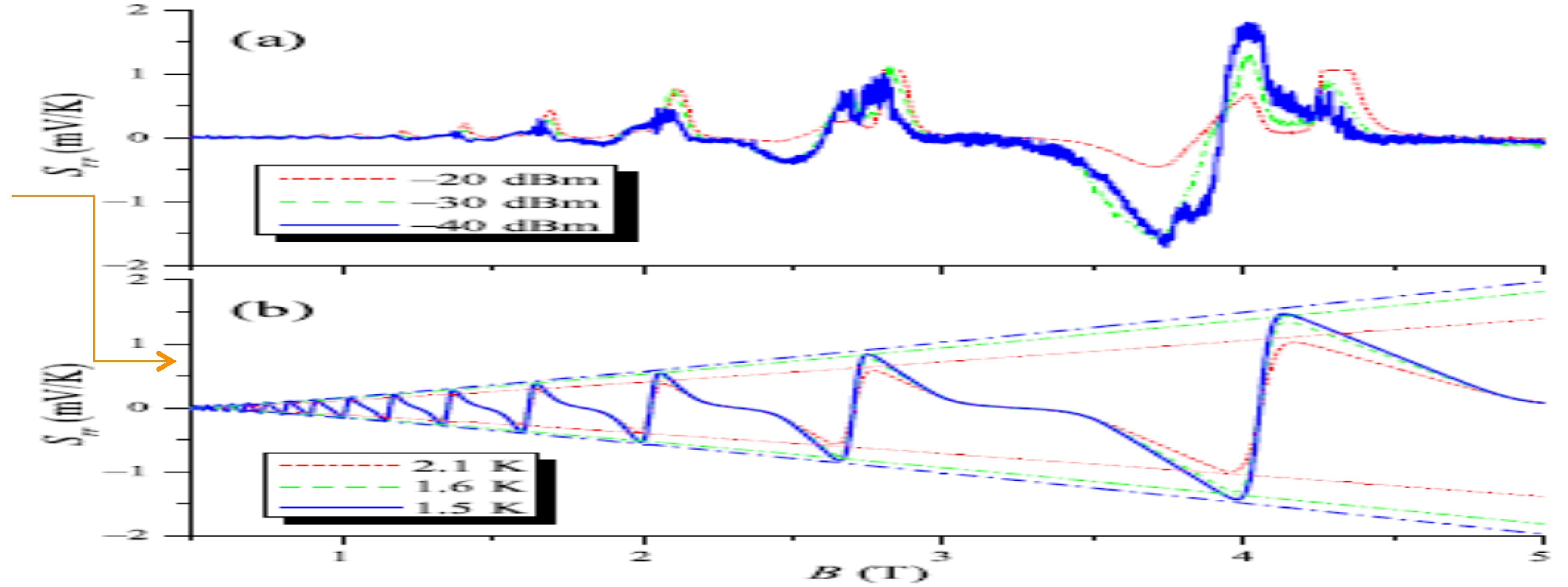

Fig. 4. (Color online) (a) Thermopowers S_{rr} deduced from thermovoltages V_{TP} measured at $T_{\mathrm{bath}} = 1.4$ K with three different microwave powers, $P_{\mathrm{NA}} = -20, -30$, and -40 dBm, corresponding to $T_{\mathrm{e,high}} \simeq 2.8, 1.8, 1.5$ K, respectively. (b) Thermopowers calculated using SCBA for three different

PHYSICAL REVIEW LETTERS

Thermopower in the Quantum Hall Regime

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(Received 10 April 2013; published 25 September 2013)

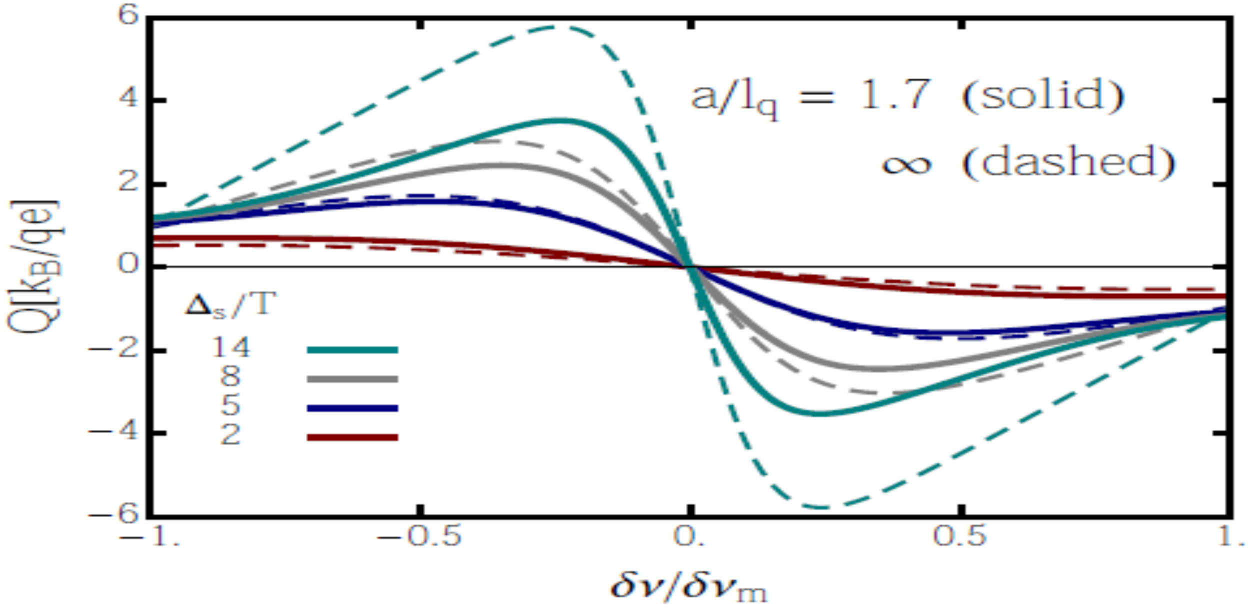
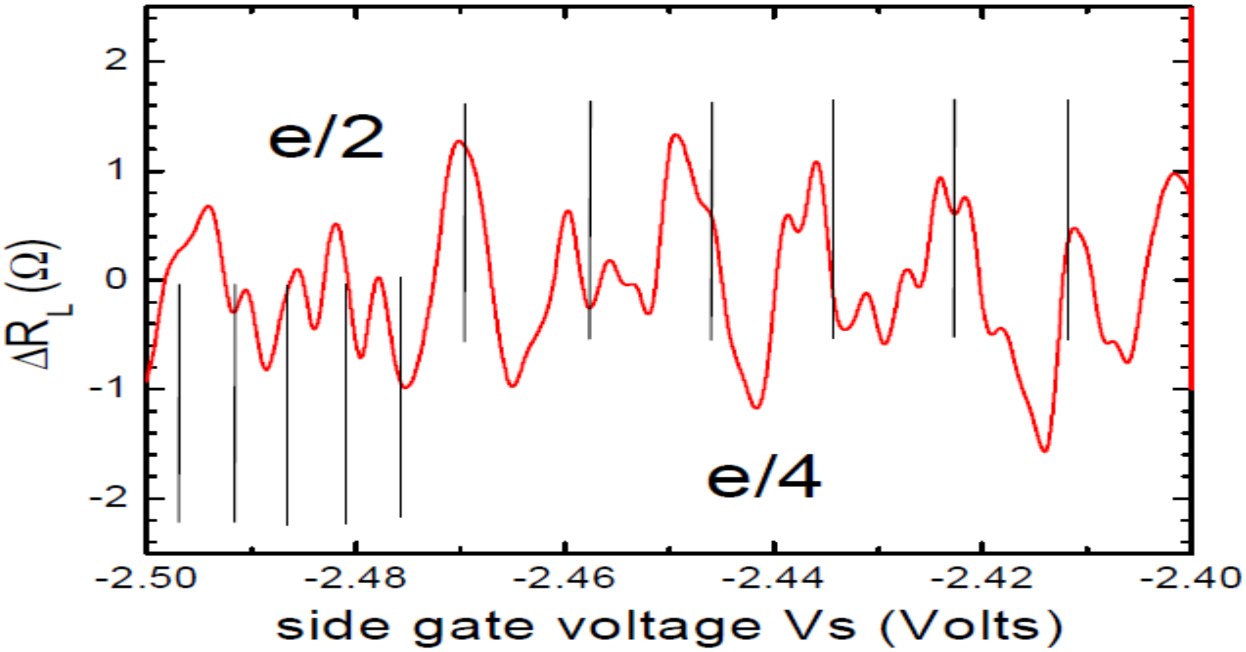


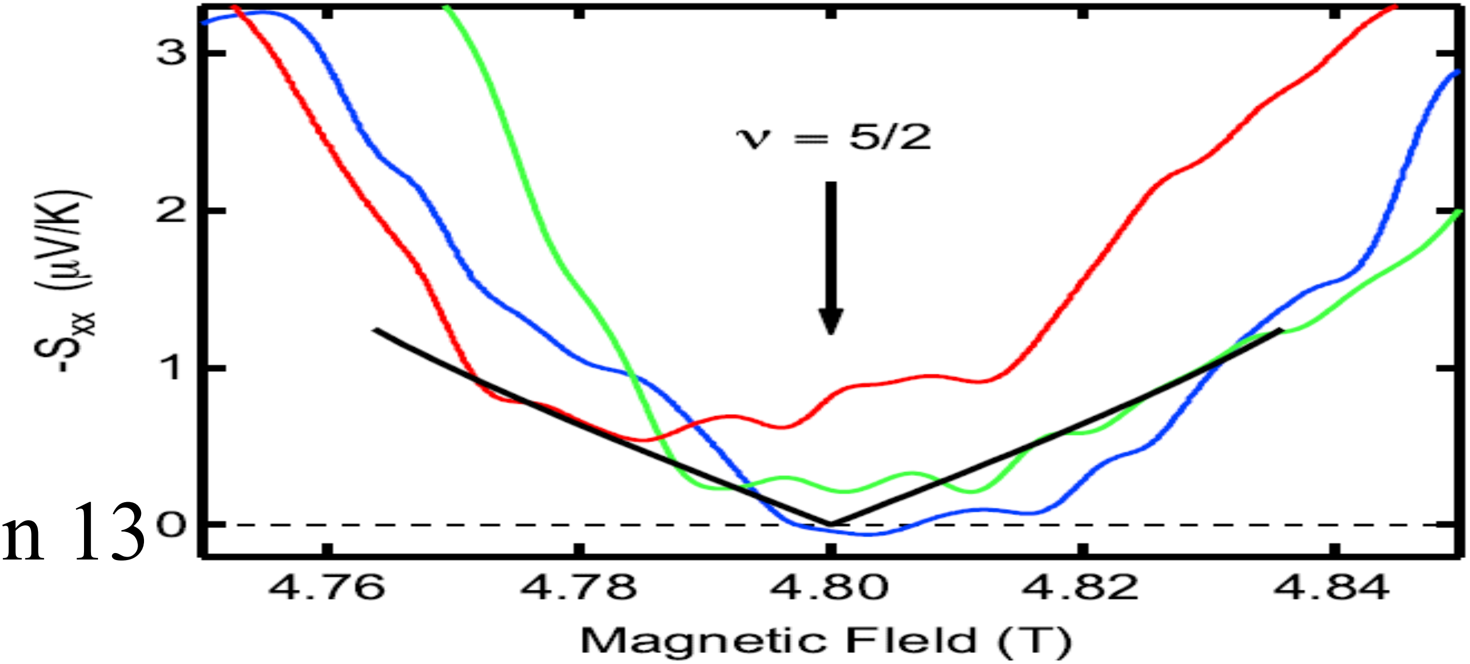
Figure 2. Thermopower for the Corbino geometry in the limit $a/l_q \to \infty$ (dashed line) and for $a/l_q = 1.7$ (solid) as a function of $\delta\nu/\delta\nu_m$ for different values of Δ_s/T. When $a/l_q \to \infty$, there is a discontinuity in Q at $g = 0$. Tunneling removes this and reduces the size of the maximum/minimum. The position of the maximum in $|Q|$ and its value are shown in Fig. 3.

Interference at 5/2

Willett 13



Area 2



Eisenstein 13

So has a Majorana mode now been seen?

My answer: Maybe, but experiment falls short of "smoking gun".

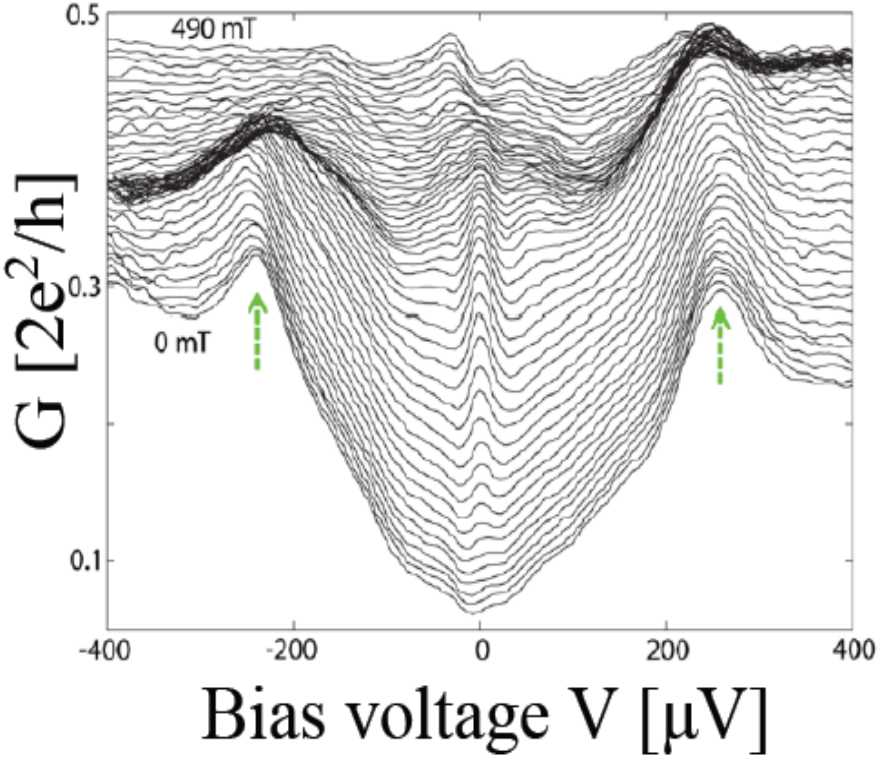
-Agrees qualitatively but not quantitatively with theory (peak height far too small)

-Disorder may lead to similar peaks ***even in a trivial superconductor***

-Gap is "soft", and suggests system is far from clean limit

-No signature of bulk phase transition from trivial to topological phase as magnetic field increases

-Wires are quite small, so finite-size effects may be an issue



Mourik et al., Science 2012

From Alicea's lecture

Good news: New generation of experiments is well underway. Situation likely to be clarified within 1-2 years.

Adiabatic Cooling with Non-Abelian Anyons

(G. Gervais and KY PRL10)

$$S_{tot} = S_D + S_n(T)$$

$S_n(T)$ is the entropy due to normal excitations

Expect quasiparticles to form Wigner crystal at low density:

$$E_{phonon}(k) = E_0(\Delta B)(ka)^{3/2}$$

$$S_n(T) \approx \alpha N_e k_B \frac{e}{e^*} \frac{\Delta B}{B_0} \left[\frac{k_B T}{E_0(\Delta B)} \right]^{\frac{4}{3}}$$

$$E_0(\Delta B) \approx 0.071 \frac{e^2}{\epsilon l_B} \sqrt{\frac{e}{e^*}} \left[\frac{\nu_0 \Delta B}{B_0} \right]^{\frac{3}{2}} \qquad \alpha \approx 0.27$$

Adiabatic Cooling with Non-Abelian Anyons

(G. Gervais and KY PRL 10)

In an adiabatic process: $\frac{dS_{tot}}{d\Delta B} = \frac{dS_D}{d\Delta B} + \frac{dS_n}{d\Delta B} = 0;$

$$\frac{dT}{d\Delta B} = \frac{3T}{4\Delta B}\left\{1 - \frac{\log d}{\alpha}\left[\frac{E_0(\Delta B)}{k_B T}\right]^{\frac{4}{3}}\right\}; \quad \boxed{\frac{dT}{d\Delta B} < 0}$$

as long as $T < T^*, \quad T^* = \frac{E_0(\Delta B)}{k_B}\left(\frac{\log d}{\alpha}\right)^{\frac{3}{4}}$

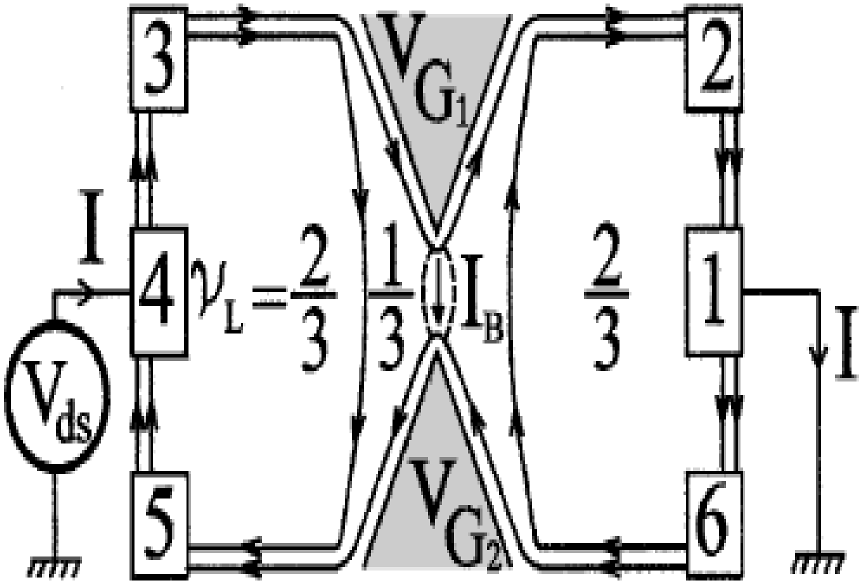
Compare with Abelian case (*d*=1): $\frac{dT}{d\Delta B} = \frac{3T}{4\Delta B} > 0.$

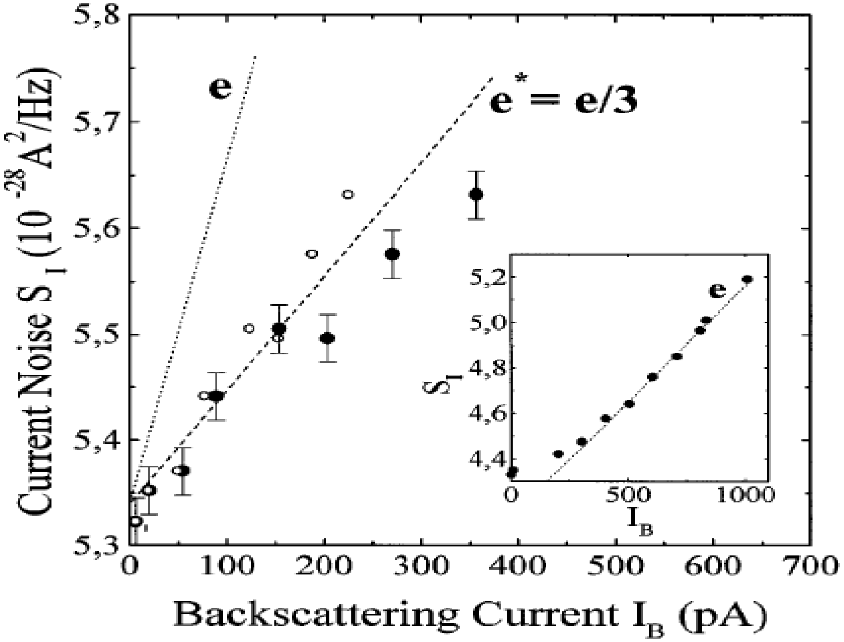
To Do List for Experimentalists

- Increase quasiparticle density by changing filling factor adiabatically.
- Measure (sign of) change of *T*; cooling for non-Abelian and heating for Abelian anyons at low *T*.
- *T* can be measured *in situ* through longitudinal resistivity ("easy"!), among others.

Fundamental difference between 1/3 and IQH states *not* manifested in bulk transport measurements!

Fractional quasiparticle charge probed through noise in (backscattering) *edge* current:





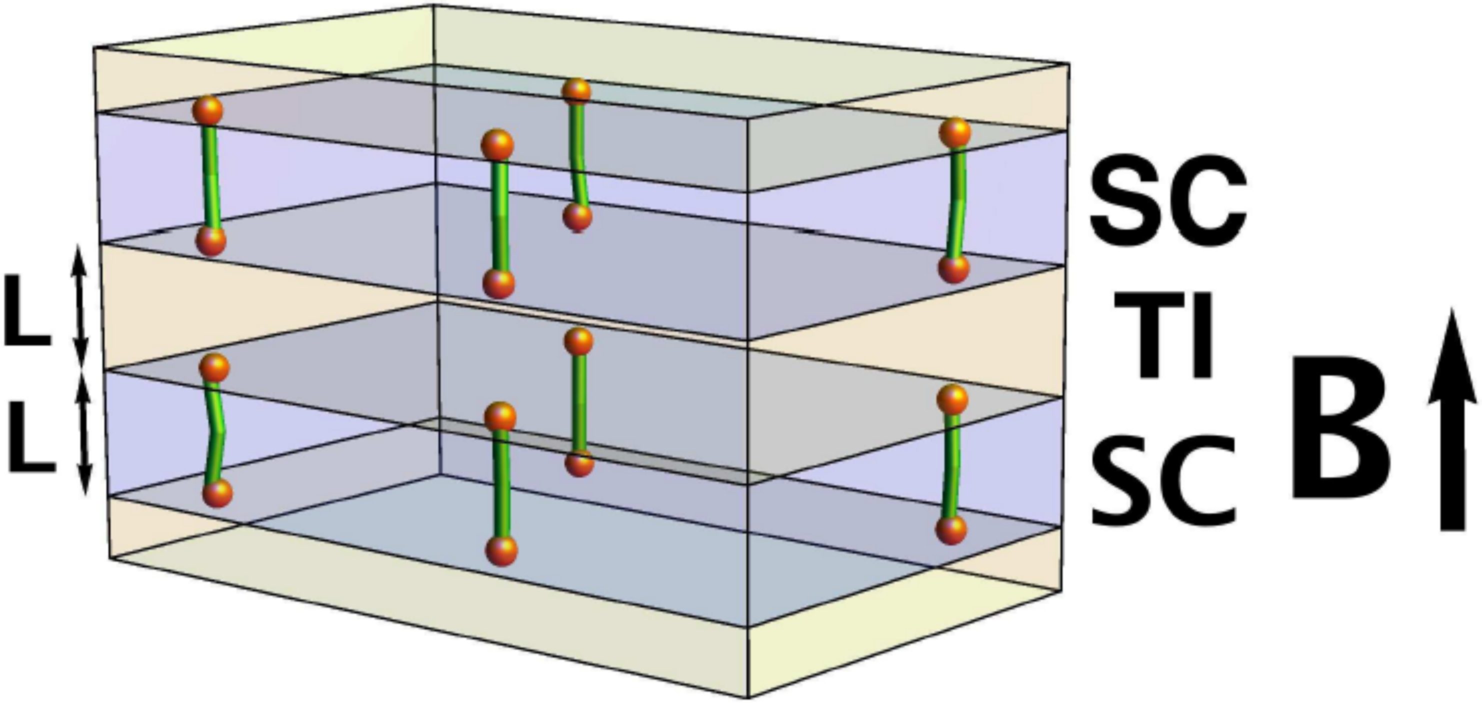
Saminadayar et al. PRL 97; de-Picciotto et al. Nature 97.

Difference between Abelian and non-Abelian QH states *bigger* than difference between integer and fractional QH states, as bulk thermoelectric/entropical probes can tell them apart!

Going beyond 2D: 3D (non-Abelian) "Anyons"

in topological insulator/superconductor hybrid structures!

(Teo+Kane 10; Freedman, Hastings, Nayak, Qi, Walker, Wang 11)



Anyons live at ends of vortex lines, which are centers of hedghogs of $\vec{n} = (\mathrm{Re}\Delta, \mathrm{Im}\Delta, \Delta_{so})$

Difficult to move them around in 3D! But cooling idea still works in principle **(Yamamoto, Freedman and KY 11).**

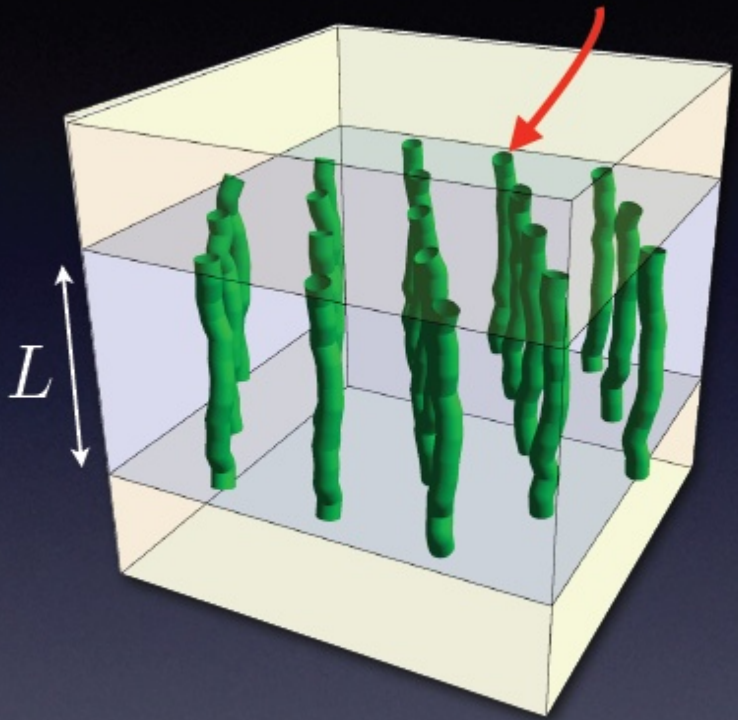
Materials and Structure

(Majorana fermions)

3D non-abelian Ising anyons in hedgehog defects

Teo, Kane PRL (2010)

see also Freedman et al, arXiv: 1005.0583



← TI: n-doped Bi_2Se_3

← SC: p-doped Bi_2Se_3 ($Cu_xBi_2Se_3$)

← TI: n-doped Bi_2Se_3

$$\vec{n} = (\underbrace{\mathrm{Re}\Delta, \mathrm{Im}\Delta}, \Delta_{so})$$

superconducting vortex

$$\Delta(r,\theta) = \Delta(r)e^{in\theta}$$

spin-orbit kink

$$\Delta_{so}(z) = \begin{cases} +\Delta_{so} & |z| < L/2 \\ -\Delta_{so} & |z| > L/2 \end{cases}$$

Model Hamiltonian and Majoranas

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_D & \Delta \\ \Delta^* & -\mathcal{H}_D \end{pmatrix}$$

8x8 Hamiltonian

$$\mathcal{H}_D = \vec{\alpha} \cdot \vec{p} - \epsilon_F - i\gamma^5 \beta \Delta_{so}$$

$\mathcal{H}\Psi = 0$ BdG zero modes

$$\begin{pmatrix} u_{1\uparrow} \\ u_{1\downarrow} \\ u_{2\uparrow} \\ u_{2\downarrow} \end{pmatrix} = \mathcal{N} \underbrace{\begin{pmatrix} J_{(n-1)/2}\left(k_F r\right) e^{-i\pi/4} e^{i(n-1)\theta/2} \\ 0 \\ 0 \\ J_{(n+1)/2}\left(k_F r\right) e^{i\pi/4} e^{i(n+1)\theta/2} \end{pmatrix} \sqrt{\epsilon_F} e^{-\frac{1}{\hbar v_F}\int^r \Delta(r')dr'} e^{-\frac{1}{\hbar v_F}\int^z \Delta_{so}(z')dz'}}_{\text{Majorana Wavefunction!}}$$

upper 4
components

Teo, Kane PRL (2010)
Fukui PRB (2010)
Nishida, Santos, Chamon PRB (2010)

Entropy

$$S = S_{\text{phonons}} + S_{\text{vortices}}$$

$$S_{\text{phonons}}(T) \propto T^3$$

$$S_{\text{vortices}} = S_{\text{CdGM}} + S_{vl} + S_{na}$$

vortex lattice

non-abelian

Caroli, de Gennes, Matricon (1964)

$$S_{\text{CdGM}}(B,T) \propto N_v(B)\sqrt{\frac{\Delta}{2k_B T k_F \xi}}\,e^{-\Delta/2k_B T k_F \xi}$$

Fetter (1967)

$$S_{vl}(B,T) \propto T^{3/2}\left(\frac{R(B)}{\lambda}\right)^{5/2}$$

for triangular lattice

$R(B) = \sqrt{2\phi_0/B\sqrt{3}}$

Gervais, Yang (2010)

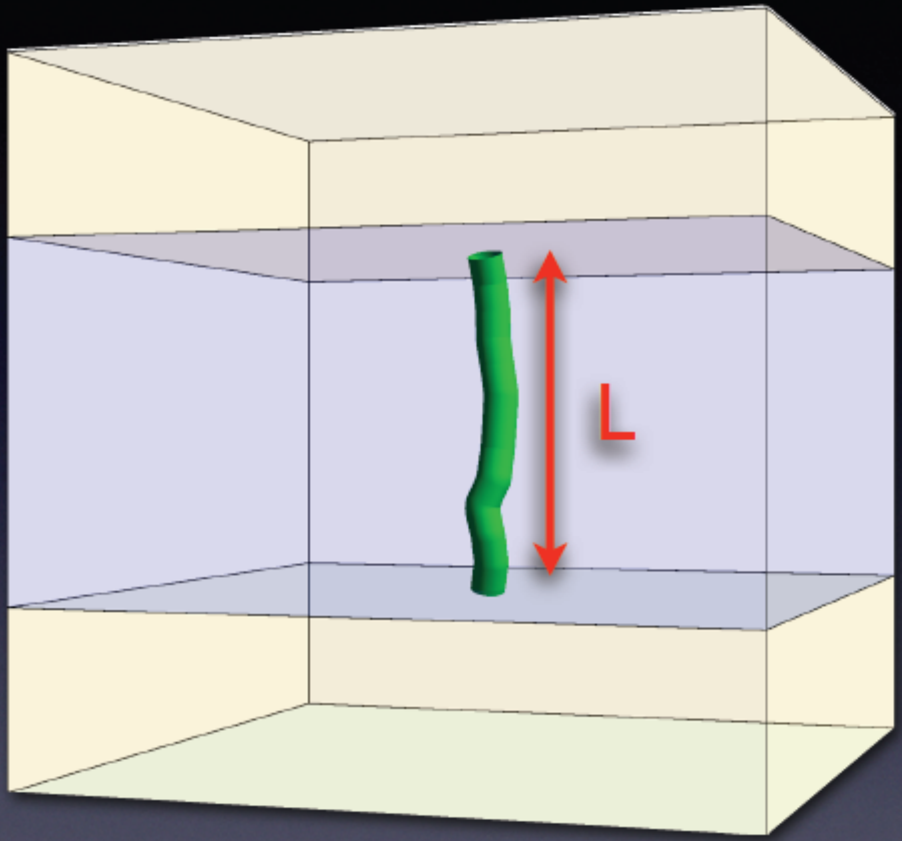
$$S_{na}(B) = N_v(B)\,k_B \log 2$$

responsible for strong cooling!

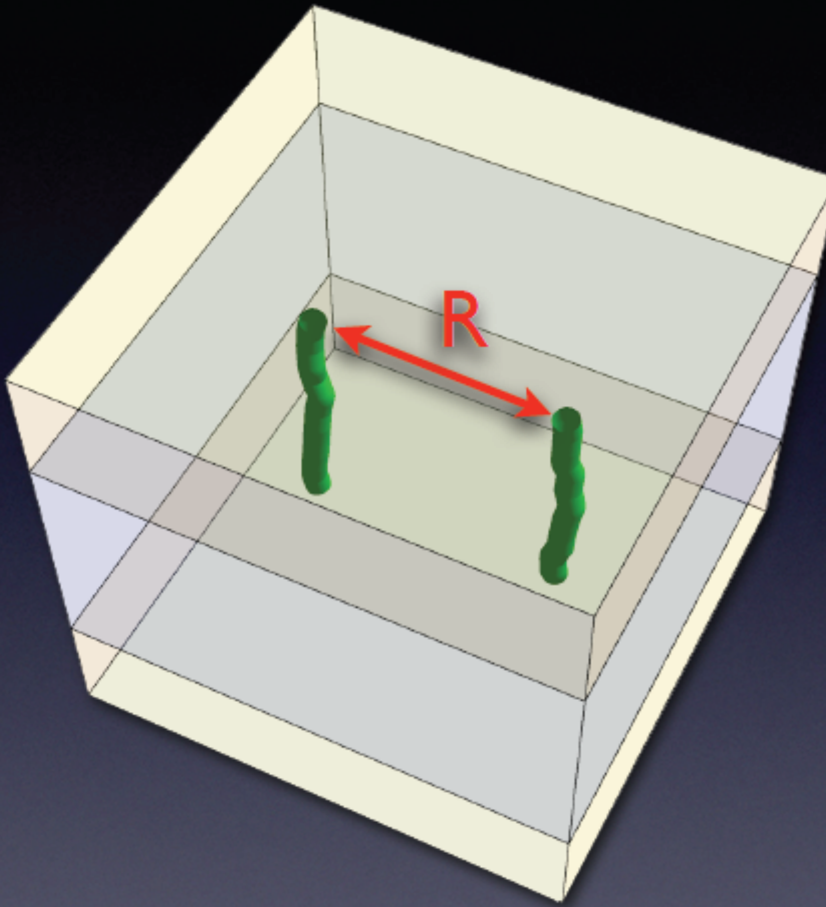
$N_v(B) = BA/\phi_0$

$$\frac{dT}{dB} = -\frac{dS/dB}{dS/dT}$$

Majorana Tunneling



coupling in z-direction



coupling in xy-plane

➡ zero mode degeneracy splitting

Degeneracy Splitting

Majorana zero modes for infinite separation

$$\mathcal{H}\Psi_{1,2} = 0$$

Wavefunctions for non-infinite separation

$$\Psi_\pm = \Psi_1 \pm i\Psi_2$$

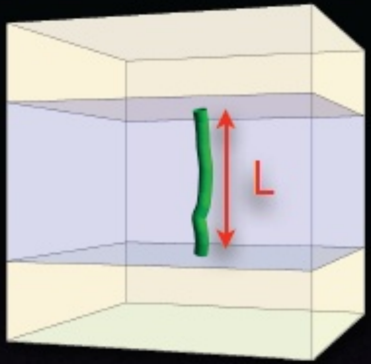
$$\mathcal{H}\Psi_\pm = E_\pm \Psi_\pm$$

<u>Not</u> zero modes. $E_\pm \neq 0$

$$E_+ = \frac{\int_\Sigma \Psi_1^\dagger \mathcal{H}\Psi_+ - \int_\Sigma \Psi_+^\dagger \mathcal{H}\Psi_1}{\int_\Sigma \Psi_1^\dagger \Psi_+}$$

$$E_{\text{split}} = E_+ - E_- = 2E_+$$

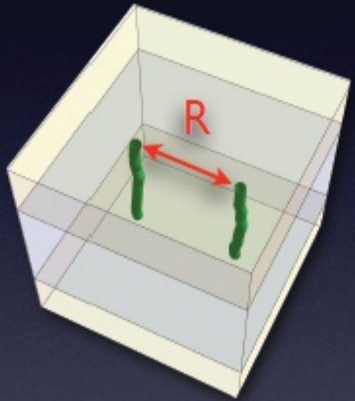
Cheng, Lutchyn, Galitski, Das Sarma PRB (2010)



$$E^{(z)}_{\text{split}}(L) = E^{(z)}_0 \overbrace{\text{sech}^2\left(L/\xi_{so}\right)}^{10^{-68} \text{ for L=100 nm}}$$

← **negligible**

$$E_0^{(z)} \equiv -\frac{4\sqrt{2}\Delta_{so}\left[K(-\epsilon_F^2/\Delta^2) - \frac{E(-\epsilon_F^2/\Delta^2)}{(1+\epsilon_F^2/\Delta^2)}\right]}{[K(-\epsilon_F^2/\Delta^2) - E(-\epsilon_F^2/\Delta^2)]}$$



$$E^{(xy)}_{\text{split}}(R,L) = E_0^{(xy)}(L)\cos\left(\epsilon_F R/\hbar v_F + \alpha\right)\frac{e^{-\frac{R}{\pi\xi}}}{\sqrt{R/\xi}}$$

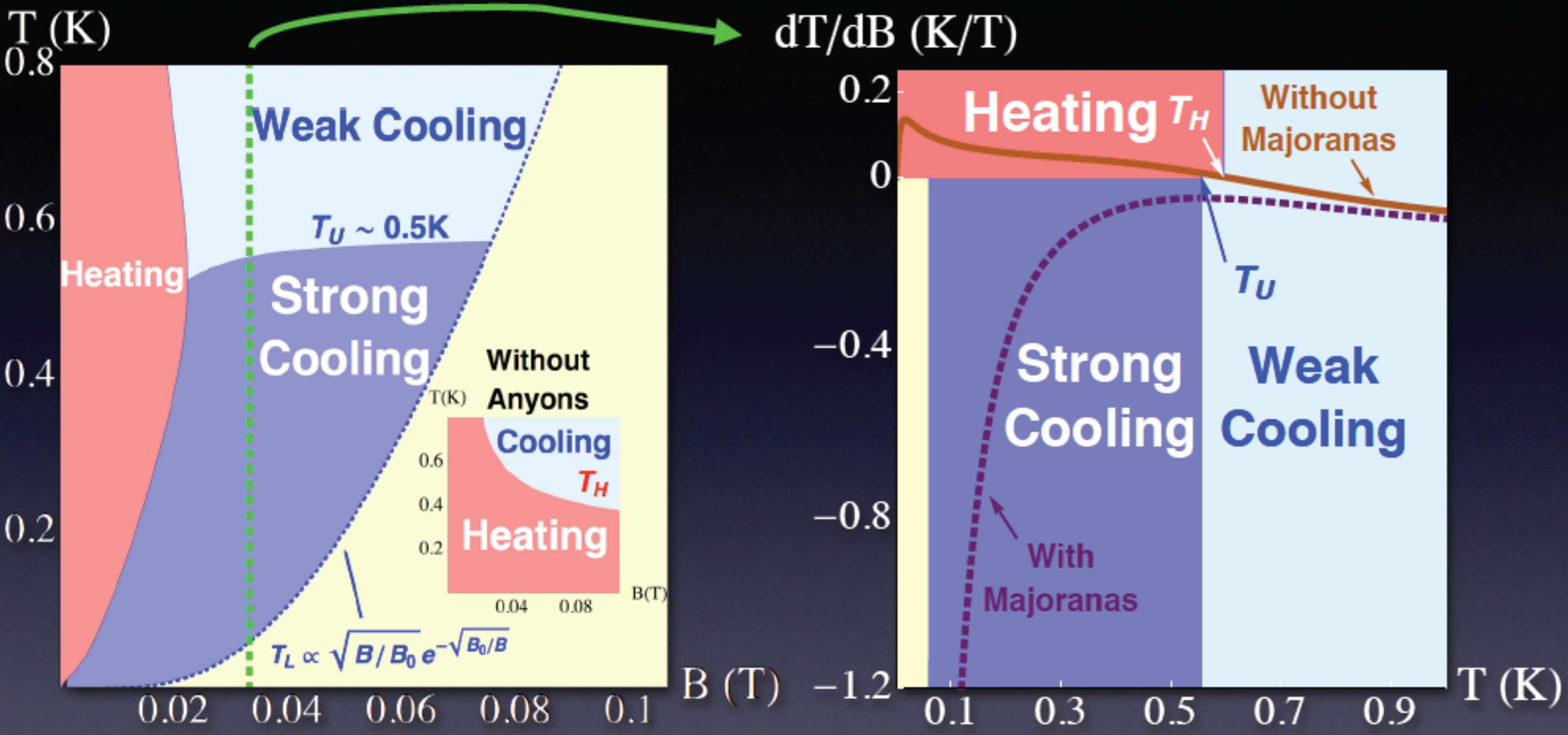
$$E_0^{(xy)}(L) \equiv \frac{8\epsilon_F \tanh(L/\xi_{so})[1+(\epsilon_F/\Delta)^2]^{-1/4}}{[E(-\epsilon_F^2/\Delta^2) - K(-\epsilon_F^2/\Delta^2)]} \qquad R(B) = \sqrt{\frac{\phi_0}{B\sqrt{3}/2}}$$

$$k_B T_{\text{split}} \equiv E_{\text{split}}$$

$T > T_{\text{split}}$ ground state effectively degenerate

$T < T_{\text{split}}$ ground state split (no Majoranas)

Experimental Proposal to Detect 3D Anyons



1. Build a Bi_2Se_3 based SC/TI sandwich with SC thickness L = 100nm (arbitrary)

2. Examine parameter regime:

$$T_L(B) < T < T_U \sim 0.5K$$

$$.02T < B < .06T$$

3. Look for cooling with adiabatic field increase

Summary of Lecture I:

- Possible to measure/probe topological entropy carried by non-Abelian anyons; complementary to transport probes. More generally, “conventional” experimental methods can be used to probe topological phases of matter.

- Might even be possible to manipulate topological entropy for refrigeration, and build a topological quantum refrigerator (perhaps easier than building a topological quantum computer)!

Preview of Lecture II:

Phases fairly well understood theoretically (including many with multi-components due to layer and/or valley degrees of freedom); fascinating experiments with puzzling details; receiving less attention than they deserve these days because not (as) "topological". **But things don't (necessarily) have to be topological to be interesting!**

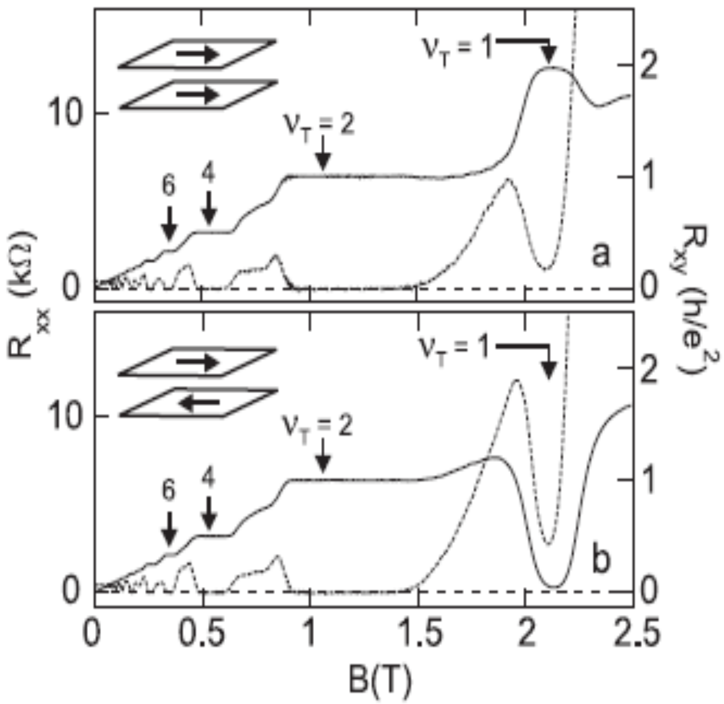


FIG. 2: Hall and longitudinal resistances (solid and dotted traces, respectively) in a low density double layer 2DES at $T = 50$ mK. a) Currents in parallel in the two layers. b) Currents in counterflow configuration. Resistances determined from voltage measurements on one of the layers.

Kellog, Eisenstein, Pfeiffer and West, PRL 04; similar results from Shayegan group.

However, many quantum phase transitions between different quantum Hall states not well understood, either theoretically or experimentally.