Friendly introduction to AdS/CMT

2. Non-equilibrium physics

Andrew Lucas

University of Colorado, Boulder

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Retarded Green's functions

- consider a quantum system in thermal equilibrium:

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  and assume $h(t \to -\infty) \to 0$
- then to first order in h,

$$\langle \mathcal{A}(\mathbf{x},t)\rangle = \int_{-\infty}^{\infty} \mathrm{d}s \; G^{\mathrm{R}}_{\mathcal{A}\mathcal{O}}(\mathbf{x},\mathbf{y},t-s)h(\mathbf{y},s)$$

$$G^{\mathrm{R}}_{\mathcal{A}\mathcal{O}}(\mathbf{x},\mathbf{y},t) = \mathrm{i}\Theta(t)\langle[\mathcal{A}(\mathbf{x},t),\mathcal{O}(\mathbf{y})]\rangle$$

Quasinormal modes dominate response

- singularity structure of correlators dominates response; e.g.

$$G^{\mathrm{R}}(k,\omega) \sim \underbrace{\frac{1}{\omega + \mathrm{i}Dk^2}}_{\text{hydrodynamic diffusion}} + \underbrace{\frac{1}{\omega + \mathrm{i}\tau^{-1}}}_{\text{finite lifetime}} + \cdots$$

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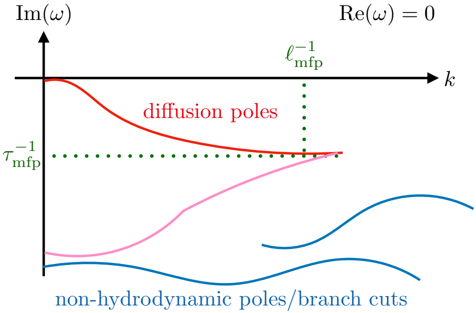
- quasiparticles replaced by these quasinormal modes



- recall in holography, bulk field ϕ obeys

$$\phi = \underbrace{h r^{d+1-\Delta}}_{\text{source}} + \underbrace{\frac{\langle \mathcal{O} \rangle}{2\Delta - d - 1} r^{\Delta}}_{\text{response}} + \cdots$$

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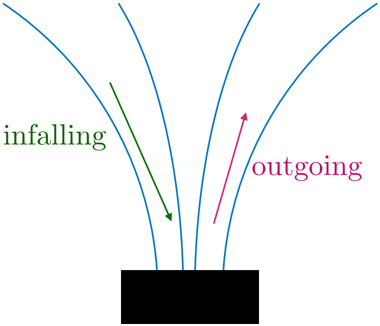
- can generalize to evaluate higher-point (beyond linear response) and far from equilibrium

Infalling boundary conditions

- calculating $G^{\mathrm{R}} \implies$ infalling boundary conditions in the presence of a black hole!



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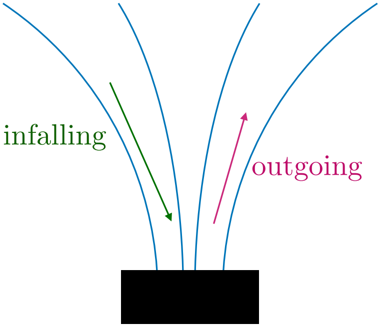

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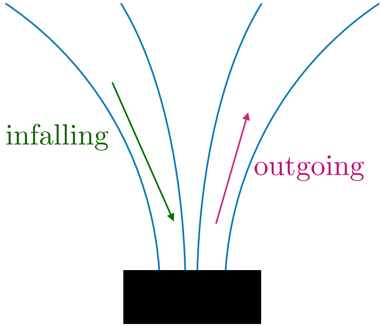

- stuff falling behind event horizon = thermalization
- e.g. scalar field ϕ obeys

$$\phi(k, \omega, r \to r_{\mathrm{h}}) \sim (r_{\mathrm{h}} - r)^{-\mathrm{i}\omega/4\pi T}$$

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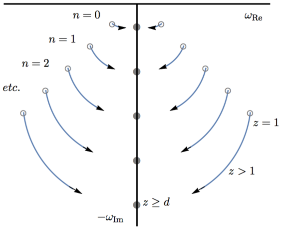

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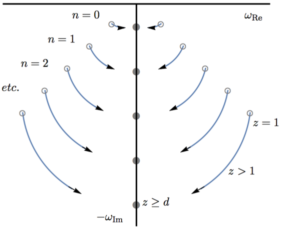

- claim: every gapless holographic model has quasinormal mode at frequency ω_* with

$$\mathrm{Im}(\omega_*) \gtrsim -T$$

(Planckian decay rate)

- a particular quasinormal mode: hydrodynamic diffusion

$$\partial_t \rho = D_{\text{charge}} \nabla^2 \rho = \frac{\sigma}{\chi_{\rho\rho}} \nabla^2 \rho,$$

$$\partial_t \epsilon = D_{\text{energy}} \nabla^2 \epsilon = \frac{T\kappa}{\chi_{\epsilon\epsilon}} \nabla^2 \epsilon,$$

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- transport coefficients σ, κ, η fixed by physics on the horizon! (membrane paradigm) [Iqbal, Liu; 0809.3808]

Viscosity bound

- all holographic models with Einstein gravity + matter have universal shear viscosity η: [Kovtun, Son, Starinets; hep-th/0405231]

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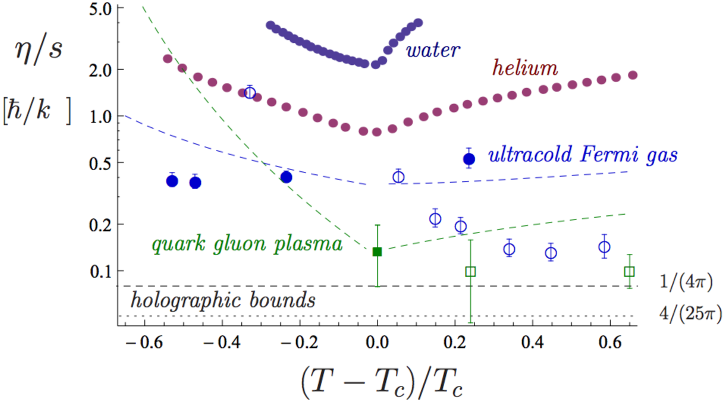
- in a charge neutral system, $\chi_{PP} = Ts$:

$$D_{\mathrm{momentum}} \sim \frac{\eta}{Ts} \sim \frac{\hbar}{k_{\mathrm{B}} T}$$

but at finite density, $\chi_{PP} \neq Ts$...

Viscosity bound in experiment

- bound consistent with experiment: [Adams *et al*; 1205.5180]



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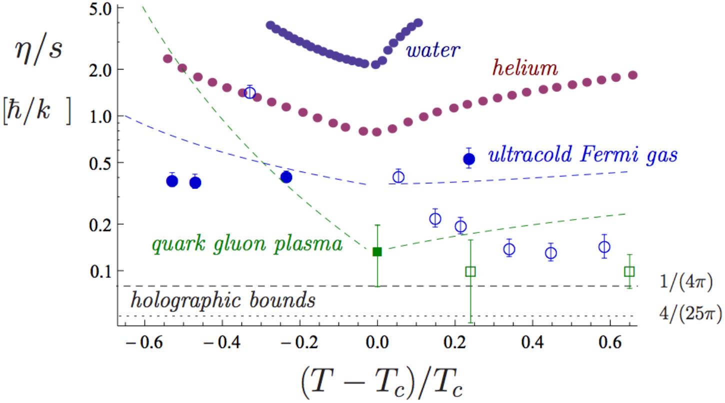

- theoretically, bound has been violated [Brigante *et al*; 0712.0805]

Conductivity at zero density

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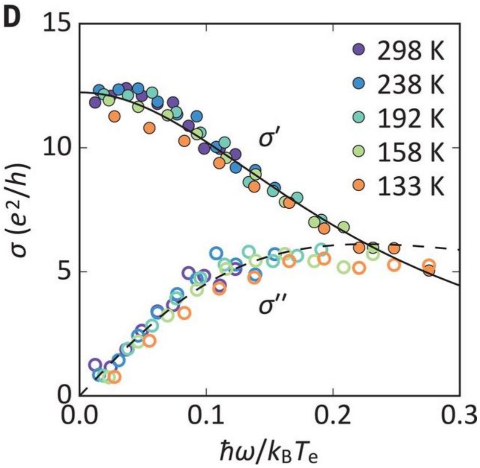
- holographic calculation of F: ODE in Mathematica

$$\partial_r \left(Y_1(r) \partial_r A_x \right) = -\omega^2 Y_2(r) A_x$$

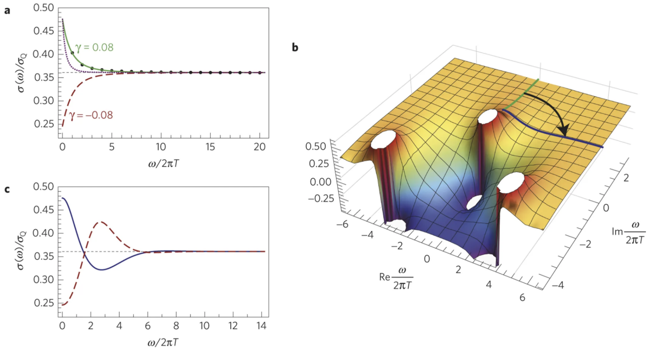
with $Y_{1,2}$ known from bulk geometry

- recent experiment on graphene measured $\sigma = F(\omega/T)$:



[Gallagher *et al*; (2019)]

- using holography to analytically continue F in a 2d lattice model: [Witczak-Krempa *et al*; 1309.2941]



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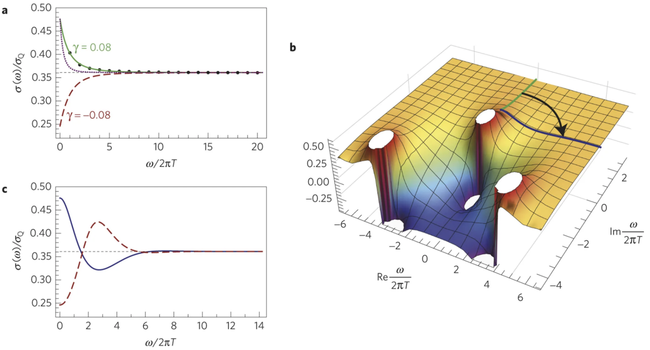


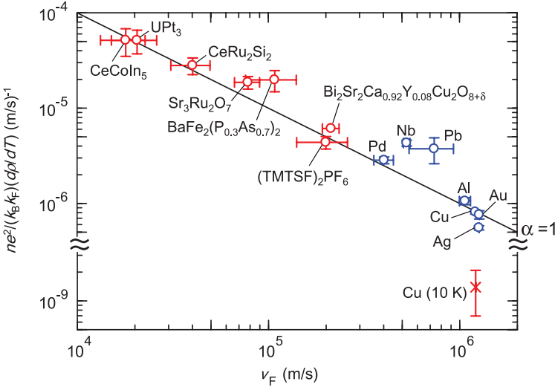
- note: $\omega \gg T$ region also understood with conformal perturbation theory [Lucas *et al*; 1608.02586]

Planckian time in experiment?

- Planckian resistivity observed in many strange metals:

$$\rho \sim \frac{m}{ne^2}\frac{k_{\mathrm{B}}T}{\hbar}$$

[Bruin *et al*; (2013)]



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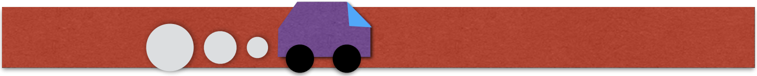

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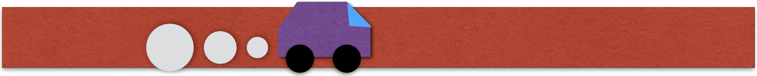

- σ sensitive to how translational symmetry broken

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does this really work? what is τ?

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- holography reproduces this result! [Lucas; 1501.05656]

$$\lim_{\omega\to 0} \frac{\mathrm{Im}\left(G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(\mathbf{k},\omega)\right)}{\omega} \sim \sqrt{g_{\mathrm{horizon}}}\,\phi(r_{\mathrm{h}})^2$$

because horizon physics determines spectral weight!

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- near spin density wave criticality? [Patel, Sachdev; 1408.6549]

$$\rho \sim T^0 + T + T^4$$

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- field theory analogues in U(1)-symmetric theories?

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$$\sigma \geq 1 \quad \text{(conductivity of clean neutral plasma)}$$

$$\kappa \geq \frac{4\pi^2 T}{3}$$

[Grozdanov, Lucas, Sachdev, Schalm; 1507.00003]

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- bounds saturated by mean field model with $m = \infty$

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- the usual “metal-insulator” transition is

$$C_1(T) \sim T^{\alpha}, \quad \alpha \text{ tunable through } 0$$

and is neither Anderson or Mott

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- adding dilaton to bulk action...

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  with $C_{1,2}(T) \sim T^{\alpha_{1,2}}$ having tunable scaling
- the usual "metal-insulator" transition is

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- many mean field models are unstable at large m – is there a sensible endpoint?

[Caldarelli *et al*; 1612.07214]

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$$D_{\text{thermal}} = c\frac{v_{\mathrm{B}}^2}{T}$$

for O(1) constant c, in homogeneous systems

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- this relation fails in inhomogeneous systems:

$$D_{\mathrm{thermal}} \leq c\frac{v_{\mathrm{B}}^2}{T}$$

  (left hand side can be arbitrarily smaller) [Gu *et al*; 1702.08462]

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Nonlinear gravity is hard...

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  - quantum quenches
  - superfluid turbulence [Adams *et al*; 1212.0281]
  - (breakdown of?) Kibble-Zurek scaling [Chesler *et al*; 1407.1862]
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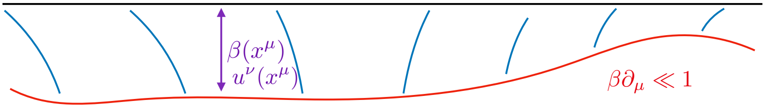
- but ... numerical general relativity = hard!

- long wavelength dynamics of black holes reproduces nonlinear hydrodynamics!

$$\mathrm{d}s^2 = \frac{L^2}{r^2}\left[-2\mathrm{d}r u^\mu(x)\mathrm{d}x_\mu + (1 - f(r,\beta(x)))(u^\mu(x)\mathrm{d}x_\mu)^2 + \mathrm{d}x^\mu \mathrm{d}x_\mu\right]$$



[Bhattacharyya, Hubeny, Minwalla, Rangamani; 0712.2456]

Fluid-gravity correspondence

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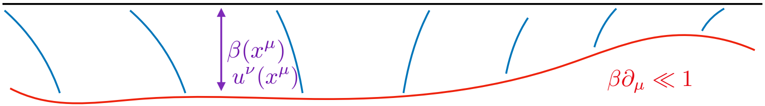

[Bhattacharyya, Hubeny, Minwalla, Rangamani; 0712.2456]

- this method can be used to generate high order corrections to fluid dynamics: e.g.

$$\partial_t \beta = D\nabla^2\beta + D_4\nabla^4\beta + D_6\nabla^6\beta$$

Borel resummability of hydrodynamics can be investigated

[Grozdanov, Kovtun, Starinets, Tadic; 1904.01018]

- consider a rapid change in the Hamiltonian:

$$H = H_{\mathrm{CFT}} + \lambda \mathrm{sech}\frac{t}{\tau}\mathcal{O}$$

with $\mathcal{O}$ an operator of dimension Δ

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- (numerical) correlators tractable in these time-dependent backgrounds

Open questions

- some questions for AdS/CMT?
  - strongly disordered fixed points?
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  - “floating black holes” as a breakdown of thermalization?

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  - strongly disordered fixed points?
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  - "floating black holes" as a breakdown of thermalization?
- questions raised by AdS/CMT:
  - thermal chaos/diffusion in spin chains?
  - "incoherent" metals? resistivity saturation at strong coupling?
  - CFTs at finite T – defects? OPE?