Superconductors without parity symmetry

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Thanks to Paolo Frigeri, Nobuhiko Hayashi, Manfred Sigrist (ETH-Zurich), Huiqiu Yuan, Myron Salamon (UIUC), Akihisha Koga (Osaka University) , Raminder Kaur, Shantanu Mukherjee, and Zhichao Zheng (UWM)

Lecture 2:

Materials

Spin-Orbit Interaction

Superconducting state: Singlet-triplet mixing

Protected spin-triplet pairing state

Physical Observables- Experiments

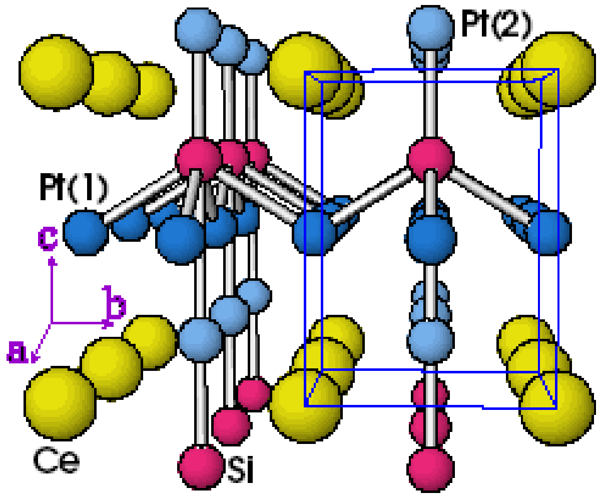
FFLO-like phases

Relevant Materials

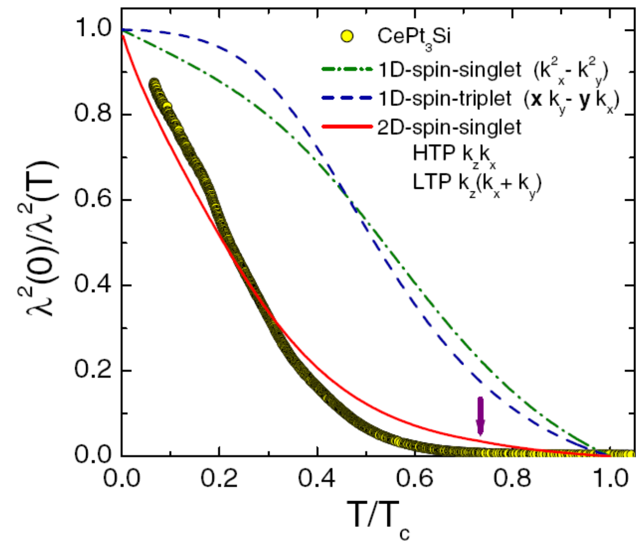
- Many broken inversion superconductors: $Cd_2Re_2O_7$, UIr, Mo_3Al_2C, $La_5B_2C_6$, MoN, Y_2C_3, La_3S_4, LaRhSi, NbReSi, Mo_3P, KOs_2O_6, Li_2Pd_3B, …
- Superconductivity at the surface of a topological insulator
- Interface superconductors $LaTiO_3/SrTiO_3$
- Some of these materials have interesting superconducting properties

$CePt_3Si$, $CeRhSi_3$, $CeIrSi_3$

- $CePt_3Si$: Heavy fermion superconductor: E. Bauer *et al* PRL **92** 027003.



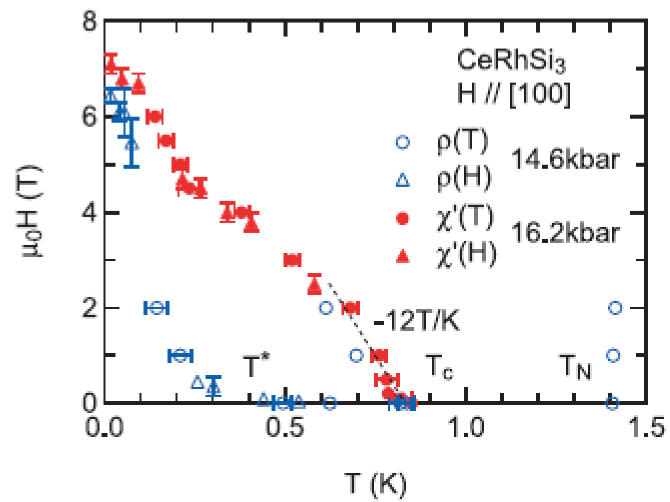
$CePt_3Si$ - P4mm; $CePt_3B$-type



Line Nodes in $CePt_3Si$

Bonalde, PRL 2005

$CeRhSi_3$ and $CeIrSi_3$ also have high H_{c2}



N. Kimura, *et al*, PRL **95**,247004

I.Sugitani, *et al.* JPSJ **75**, 043703

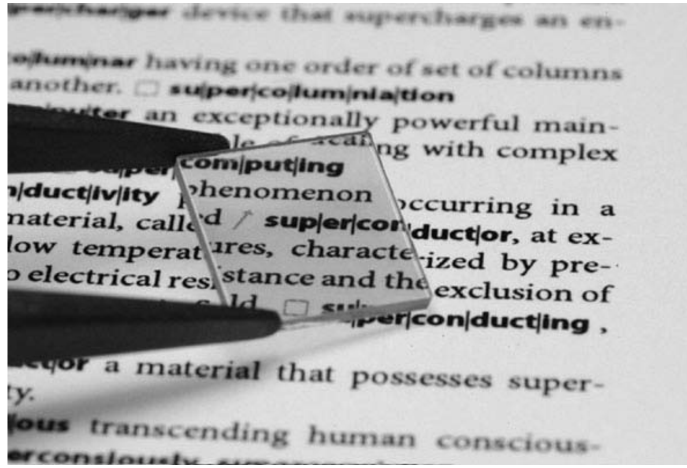
Interface Superconductors

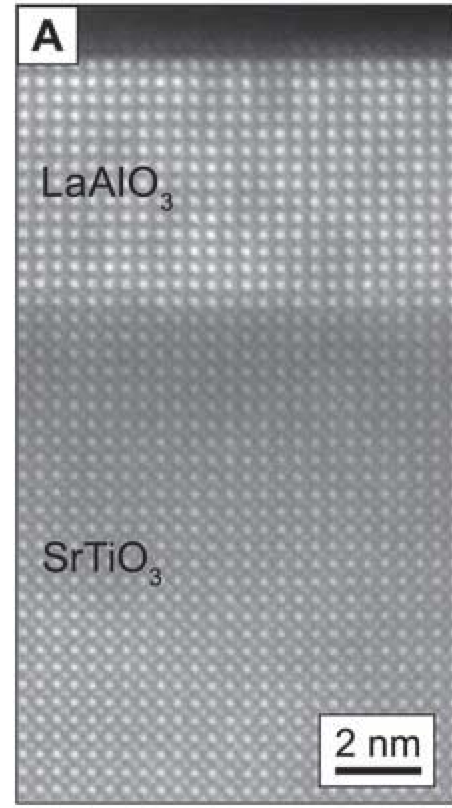
Ohtomo ,Hwang, Nature 427, 423 (2004):

- 2D electron gas at $LaAlO_3$ and $SrTiO_3$ interface

Reyren et al, Science 317, 1196 (2007):

- Superconductivity in the 2D electron gas



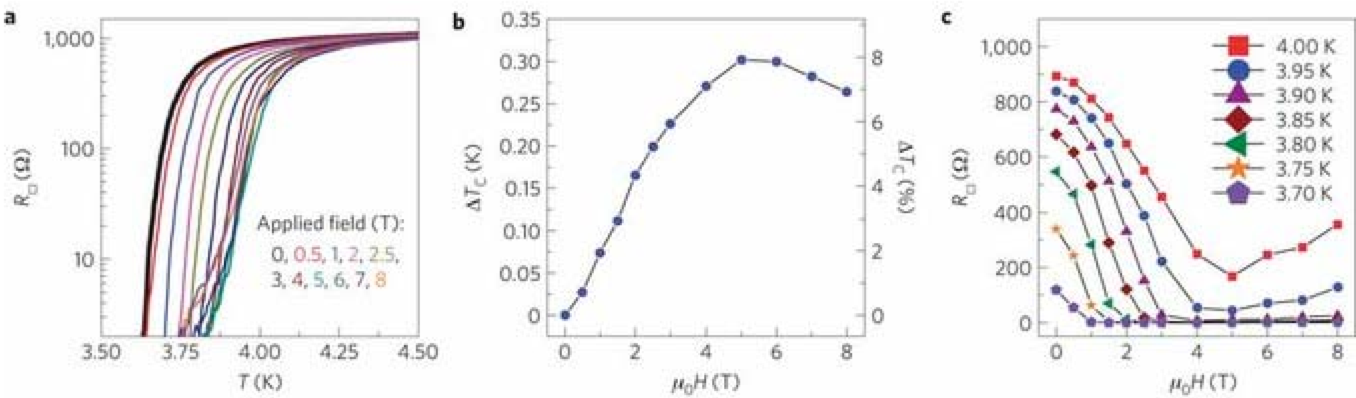


Coexisting superconductivity and ferromagnetism

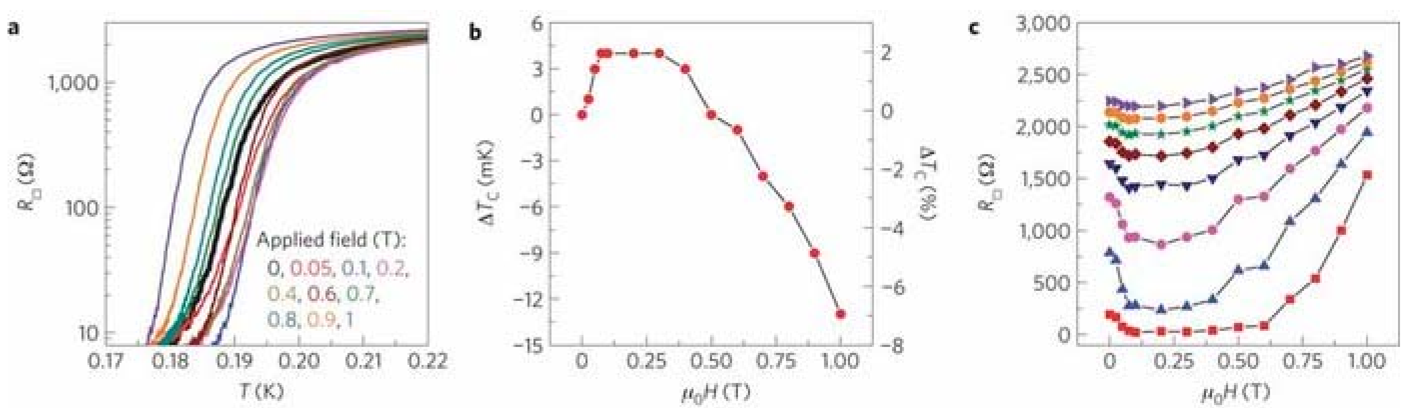
Dikin et al, Phys. Rev. Lett. (2011); Bert et al, Nature Phys. (2011); Li et al, Nature Phys (2011)

Increase of Tc by Magnetic Field

Gardner et al, Nature Physics 7, 895 (2011)



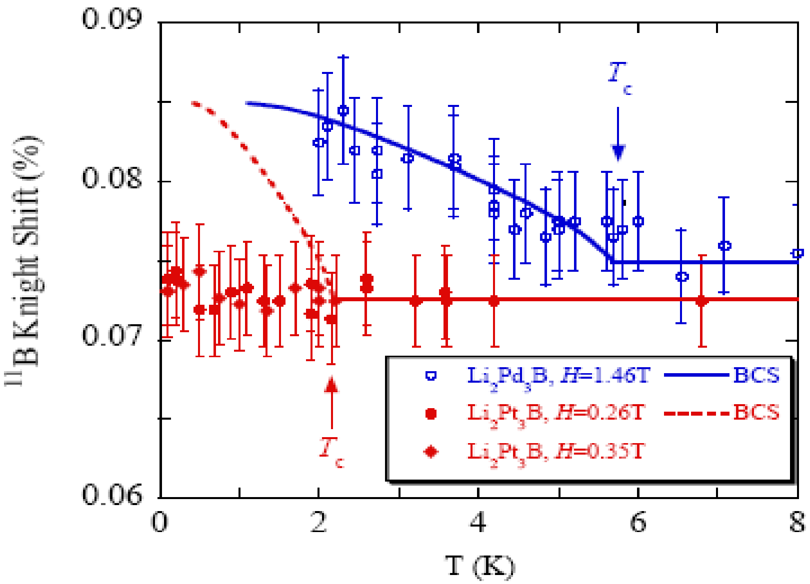
Amorphous Pb film: 21.1 Angstroms (no increased Tc for thicker films)



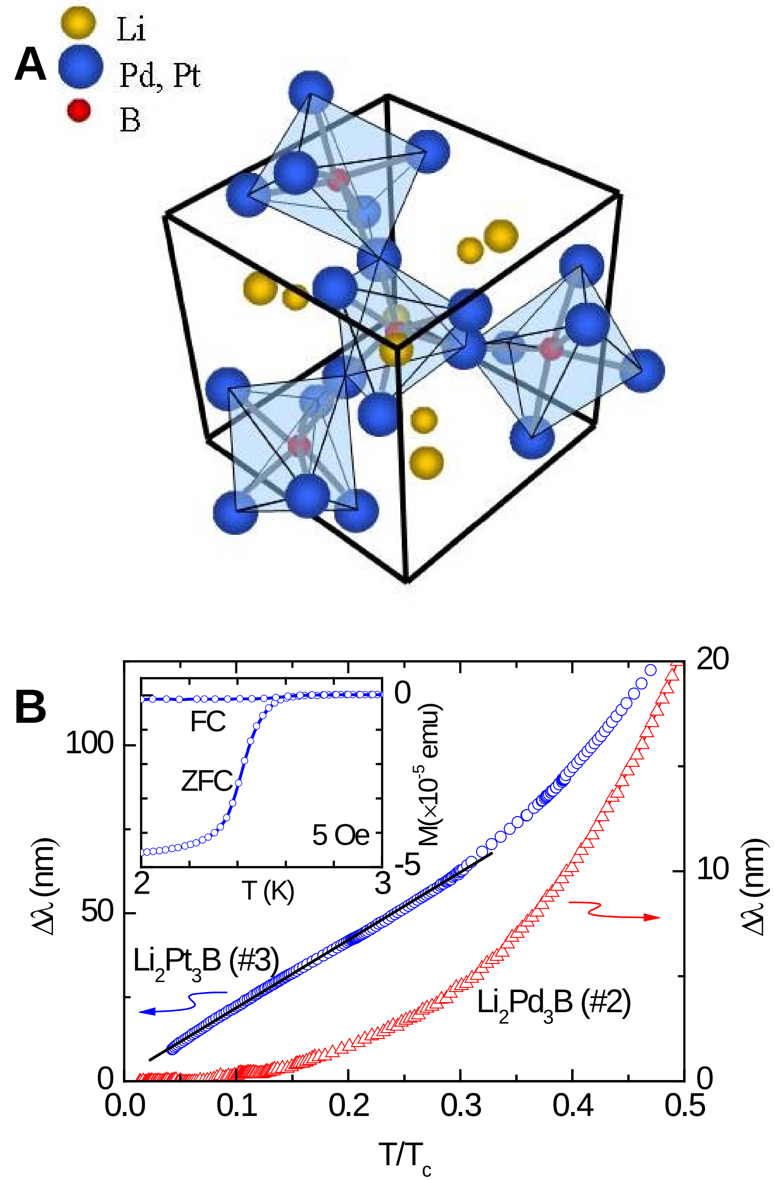
$LaAlO_3/SrTiO_3$ heterostructure

Magnetic fields should decrease Tc.

Li_2Pt_3B and Li_2Pd_3B



Zheng et al, PRL 98 047002 (2007)



Yuan et al , PRL 97, 017006 (2006)

Spin-Orbit Coupling

$$H_p = \alpha \sum_{k,s,s'} \vec{g}_k \cdot \vec{\sigma}_{s,s'} c^t_{ks} c_{ks'}$$

Time Reversal Symmetry implies $\vec{g}_k = -\vec{g}_{-k}$

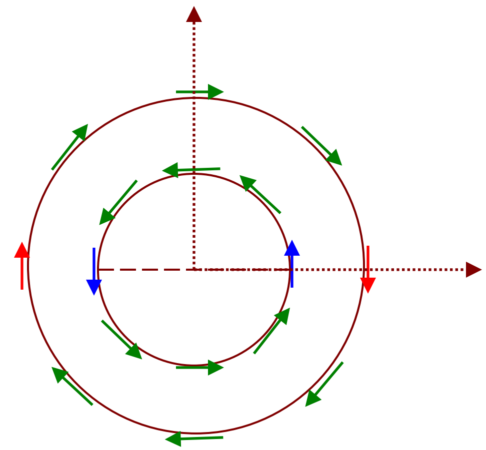
Parity Symmetry implies $\vec{g}_k = \vec{g}_{-k}$

H_p is zero when parity symmetry is also present

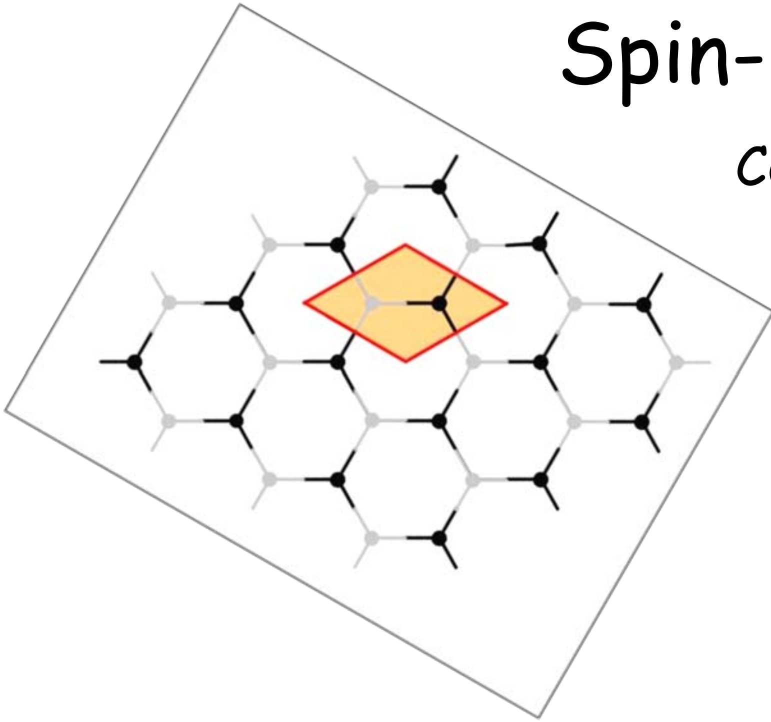
$\varepsilon_k = \xi_k \pm \alpha \, | \, \vec{g}_k \, |$ With spins polarized along $\pm \vec{g}_k$

Example: 2D Rashba

$$\vec{g}_k = \hat{x} k_y - \hat{y} k_x$$



Spin-Orbit Coupling: Example



Consider Pt–As honeycomb lattice (SrPtAs)

Place dxz, dyz orbitals Fe sites
pz orbitals on As sites

Goal is to find an effective Pt d-hopping Hamiltonian

Let

$$|+\rangle = |xz + iyz\rangle$$

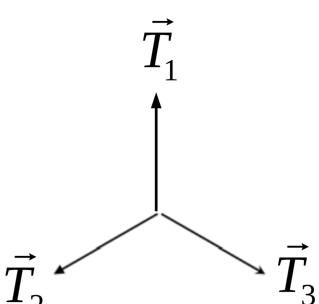
$$|-\rangle = |xz - iyz\rangle$$

Lattice has no inversion symmetry. Include large spin-orbit on Fe sites and only hopping between Fe ands As

$$\vec{L}\cdot\vec{S} = L_z S_z + L_+ S_- + L_- S_+$$

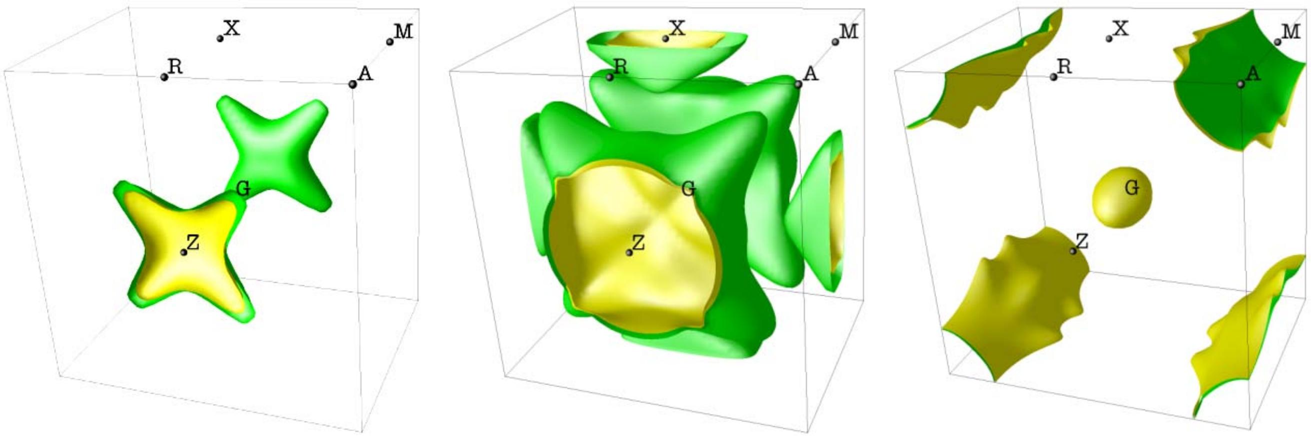
$$|+,\uparrow\rangle, |-,\downarrow\rangle$$

$$|-,\downarrow\rangle, |+,\uparrow\rangle$$



$$\vec{g}_k = \alpha\hat{z}[\sin(\vec{k}\cdot\vec{T}_1) + \sin(\vec{k}\cdot\vec{T}_2) + \sin(\vec{k}\cdot\vec{T}_3)]$$

Band splitting: $CePt_3Si$



Split Fermi surface of

$CePt_3Si$

A.Kozhevnikov, V. Anisimov

$LaAlO_3$ $SrTiO_3$: spin gap is 10 meV and Fermi Energy is 40 meV

Often spin-orbit is a large energy scale with respect to the superconducting gap.

Superconductivity

Symmetry and Cooper pairing

Anderson(1984): degenerate electron states in a $P_{tot}=0$ Cooper Pair;
Which partners are available by symmetry ?

Spin singlet Pairing:

Time reversal sym. T necessary:

$$\left|\vec{k},\uparrow\right\rangle \rightarrow \left|-\vec{k},\downarrow\right\rangle = T\left|\vec{k},\uparrow\right\rangle$$

Spin triplet Pairing:

Time reversal sym. T and parity sym. ***I*** necessary:

$$\left|\vec{k},\uparrow\right\rangle \rightarrow \begin{cases} \left|-\vec{k},\downarrow\right\rangle = T\left|\vec{k},\uparrow\right\rangle \\ \left|-\vec{k},\uparrow\right\rangle = I\left|\vec{k},\uparrow\right\rangle \\ \left|\vec{k},\downarrow\right\rangle = TI\left|\vec{k},\uparrow\right\rangle \end{cases}$$

Superconductivity

$$H = \sum_{k,s} \xi_k c^t_{ks} c_{ks} + \frac{1}{2} \sum_{k,k',s,s'} V(k,k') c^t_{ks} c^t_{-ks'} c_{-k's'} c_{k's}$$

Broken parity appears through:

$$H_p = \alpha \sum_{k,s,s'} \vec{g}_k \cdot \vec{\sigma}_{s,s'} c^t_{ks} c_{ks'} \qquad \vec{g}_k = -\vec{g}_{-k}$$

Linear Gap Equation

$$\Delta_{s,s'}(k) = -\frac{1}{\beta}\sum_{k',s2,s1,n} V(k,k')G^0_{s,s1}(k',\omega_n)\Delta_{s1,s2}(k')G^0_{s',s2}(-k,-\omega_n)$$

$$G^0(k,\omega_n) = G_+(k,\omega_n)\sigma_0 + (\hat{g}(k)\cdot\vec{\sigma})G_-(k,\omega_n)$$

$$G_\pm(k,\omega_n) = \frac{1}{2}\left(\frac{1}{i\omega_n - \xi(k) - \alpha\,|g(k)|} \pm \frac{1}{i\omega_n - \xi(k) + \alpha\,|g(k)|}\right)$$

$$\Delta(k) = \psi(k)i\sigma_y + \vec{d}(k)\cdot\vec{\sigma}i\sigma_y$$

For spin singlet component find:

$$\psi(k) = -k_B T\sum_{n,k'} V(k,k')\{[G_+G_+ + G_-G_-]\psi(k) + [G_+G_- + G_-G_+]\hat{g}(k')\cdot\vec{d}(k')\}$$

Where: $G_iG_j = G_i(k,\omega_n)G_j(-k,-\omega_n)$ Use: $\sum_k f(k) \approx N(0)\int_{-\varepsilon c}^{\varepsilon c} d\xi < f(k) >_{F.S.}$

Has singlet-triplet mixing – for small spin-orbit triplet part is small – for now we will ignore this ($\Delta \ll \alpha \ll \varepsilon c$)

Results on Stability

$$\ln \frac{T_c}{T_{cs}} = O\left(\frac{\alpha^2}{\varepsilon_F^2}\right)$$ spin-singlet

$$\ln \frac{T_c}{T_{ct}} = 2\left\langle \left[|\vec{d}_k|^2 - |\hat{g}_k \cdot \vec{d}_k|^2 \right] f(\rho_k) \right\rangle_k + O\left(\frac{\alpha^2}{\varepsilon_F^2}\right)$$ spin-triplet

$$\rho_k = \frac{\alpha |\vec{g}(k)|}{\pi k_B T_c} \qquad f(\rho) = \sum_n \left(\frac{1}{n + \frac{1}{2} + i\rho} - \frac{1}{n + \frac{1}{2}} \right)$$

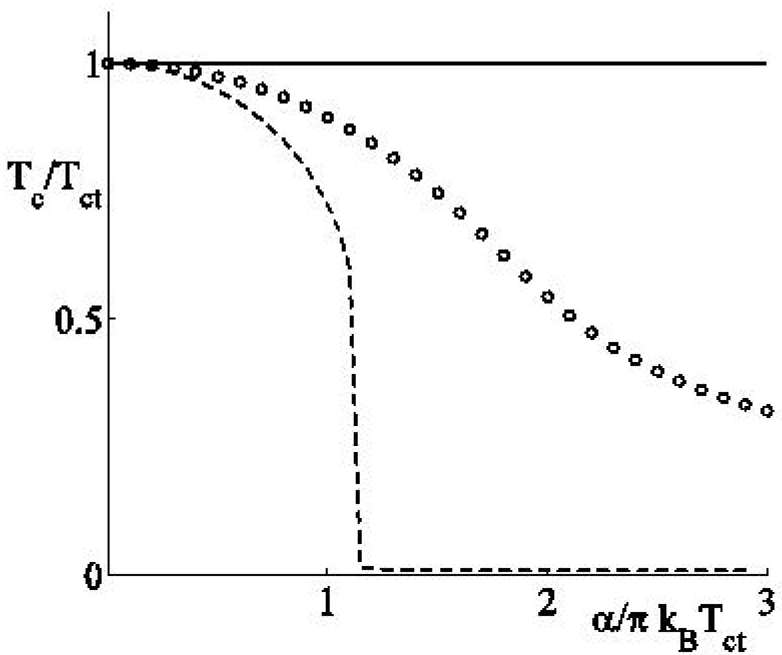
f(ρ) leads to a strong suppression of Tc

T_c for spin-triplet is not suppressed only for a *single protected* ***d*** vector (with ***g*** and ***d*** parallel).

Rashba Case

$$\vec{g}_k = \hat{y}k_x - \hat{x}k_y$$

$$\ln\frac{T_c}{T_{ct}} = 2\left\langle \left[|\vec{d}_k|^2 - |\hat{g}_k \cdot \vec{d}_k|^2 \right] f(\rho_k) \right\rangle_k + O\left(\frac{\alpha^2}{\varepsilon_F^2}\right)$$



$$\vec{d}_k = \hat{y}k_x - \hat{x}k_y$$

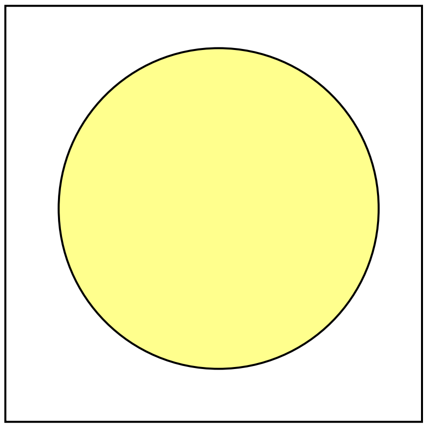
$$\vec{d}_k = \hat{y}k_x + \hat{x}k_y$$

$$\vec{d}_k = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z$$

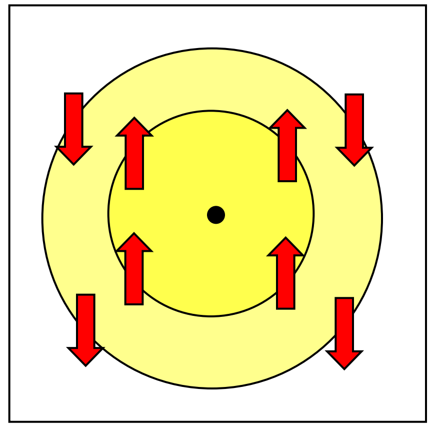
Often, only the protected triplet state (with d||g) can survive

Pauli Paramagnetism

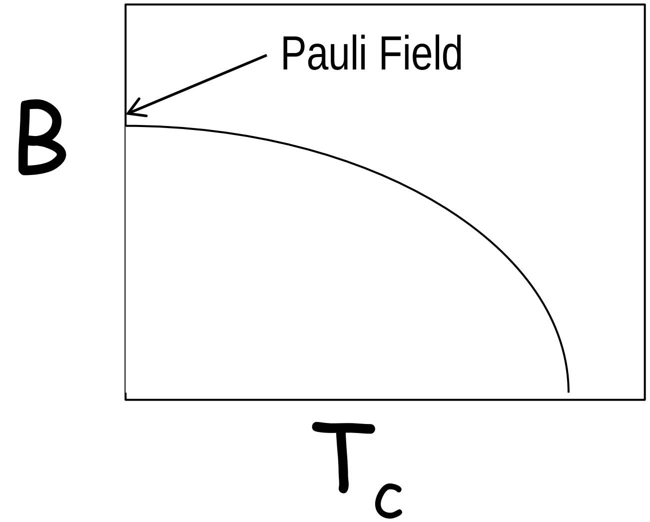
Singlet superconductors with parity symmetry: Pauli Field



No Zeeman

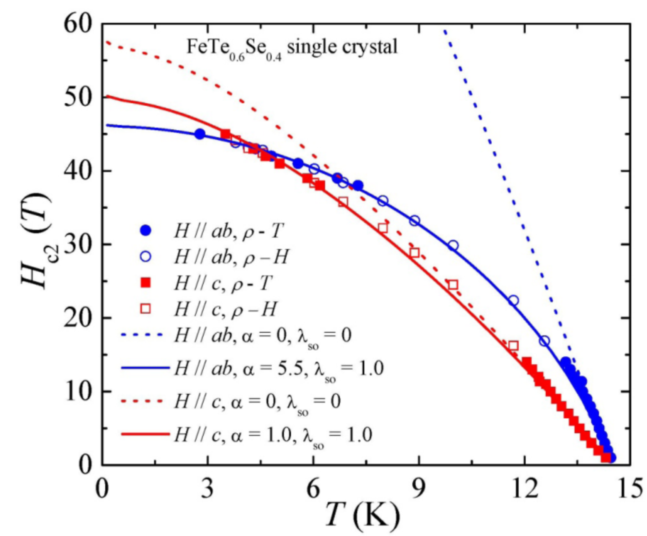


With Zeeman



$$\xi_{\pm}(k) = \varepsilon(k) \pm |\mu_B \vec{B}|$$

Spin Singlet Superconductivity suppressed when $\mu B \sim \Delta$



Khim et al (2010)

Broken parity symmetry case

$$H = \sum_{k,s} \xi_k c_{ks}^t c_{ks} + \frac{1}{2} \sum_{k,k',s,s'} V(k,k') c_{ks}^t c_{-ks'}^t c_{-k's'} c_{k's}$$

$$H_{spin} = \sum_{k,s,s'} (\mu_B \vec{h} + \vec{g}_k) \cdot \vec{\sigma}_{s,s'} c_{ks}^t c_{ks'}$$

Calculate Tc(h):

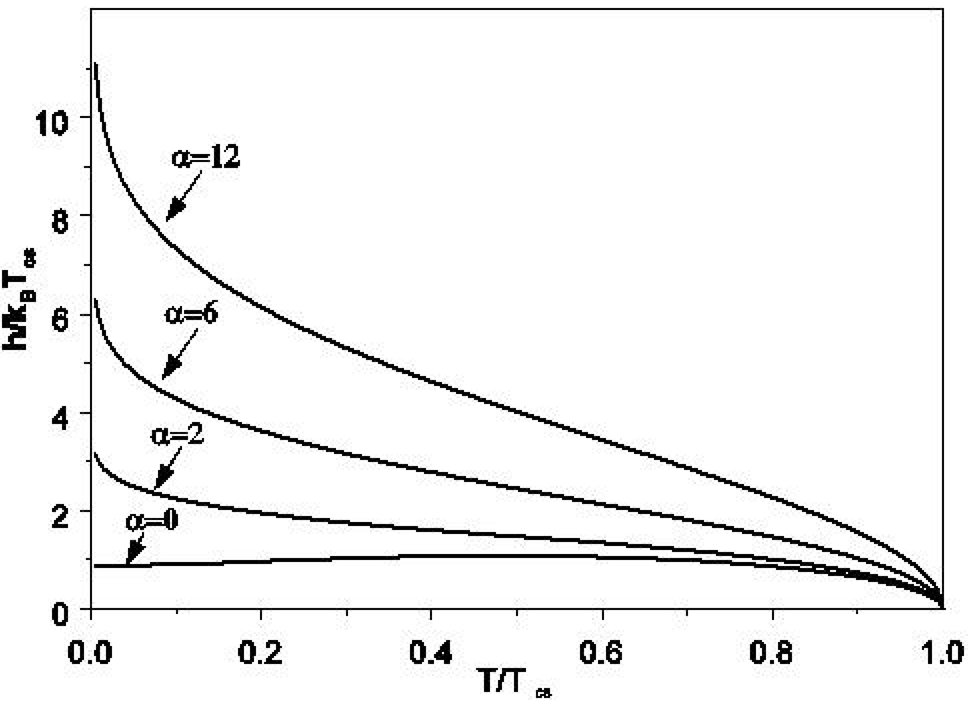
Key result: when $\vec{g} \cdot \vec{h} = 0$ then h diverges at low T:

$$(\mu_B h)^2 \ln \frac{\mu_B h}{\pi k_B T_{cs}} = -\alpha^2 \ln \frac{T_c}{T_{cs}}$$

Will consider $\vec{g} \cdot \vec{h} \neq 0$ later

Suppression of Pauli field: Rashba

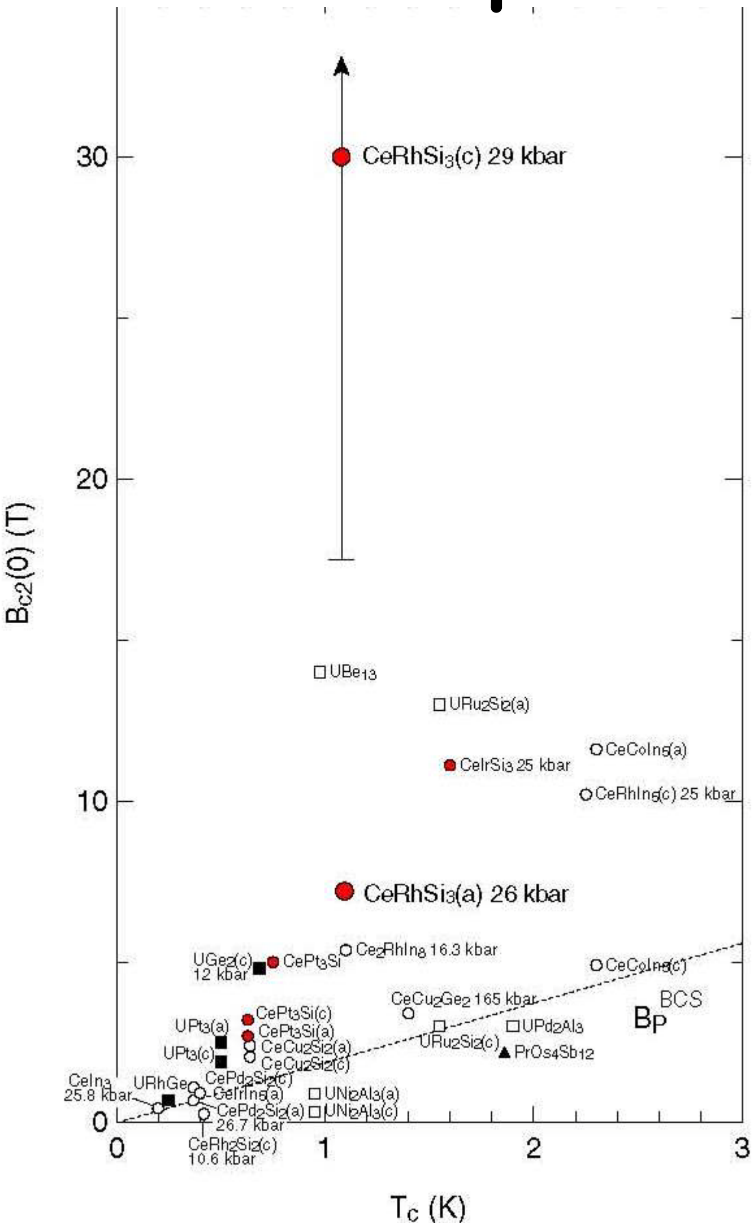
- Pauli field for spin-singlet case is strongly suppressed (also found by Bulaevski):



for $\vec{g} \cdot \vec{h} = 0$

Large $Hc_2(0)$ in $CePt_3Si$ can be due to spin-triplet pairing or spin-singlet pairing with suppressed Pauli field.

Experimental Results



Kimura et al, PRL (2007)

Red dots are non-centrosymmetric materials with a Rashba spin-orbit.

They all surpass the Pauli field.

Critical field determined by vortex physics

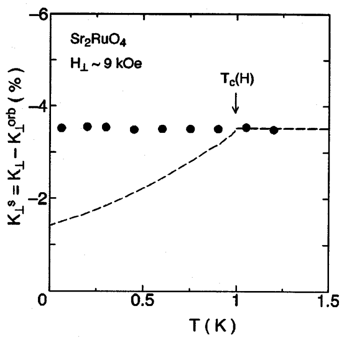
Spin Susceptibility

Example Spin-Triplet: Sr_2RuO_4

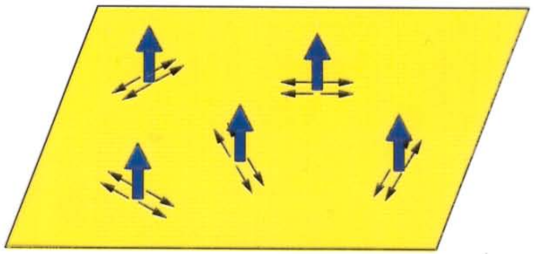
The spin-susceptibility is unchanged when **H**.**d**=0

Spin triplet: Sr_2RuO_4

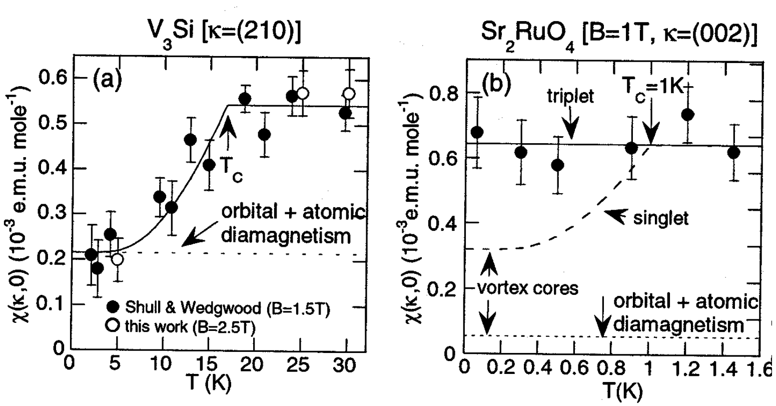
$$\vec{d}(k) = \hat{z}(k_x + ik_y)$$

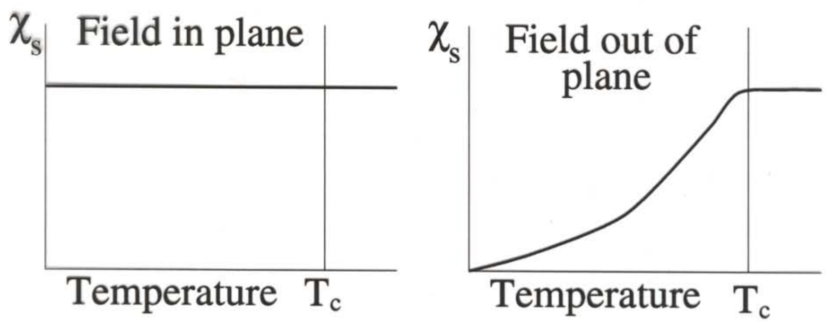


K. Ishida *et al*., Nature 396, 658 (1998).



- Example $\vec{d}$: Cooper pair spins confined to the plane
- Cooper pairs can respond to a field in plane - but not to one out of plane





J.A.Duffy *et al*. , PRL 85, 5412 (2000).

Spin Susceptibilty

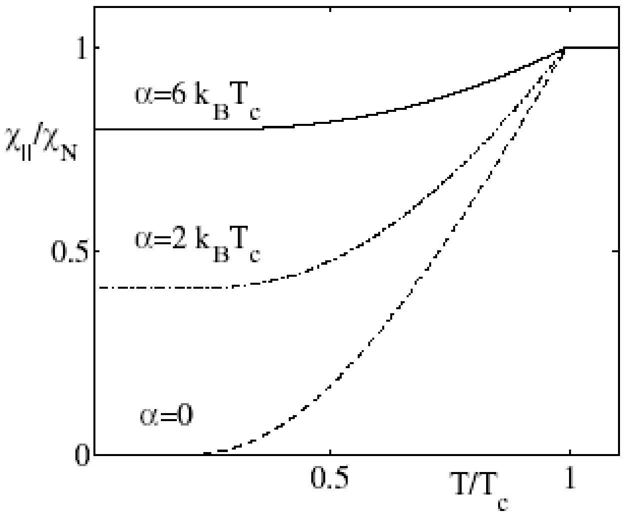
$$\chi_{ij} = -\mu_B^2 k_B T \sum_{k,\omega_n} Tr\Big(\sigma_i G(k,\omega_n)\sigma_j G(k,\omega_n) - \sigma_i F(k,\omega_n)\sigma_j^T F^t(k,\omega_n)\Big)$$

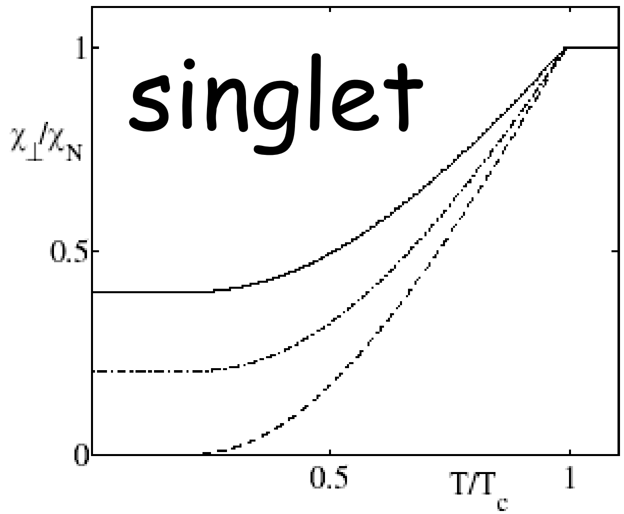
Find spin-triplet susceptibility of protected state is independent of spin-orbit (giving the same result as parity symmetric triplet SC)

Find spin-singlet susceptibility becomes the same as the above spin-triplet protected state when $\alpha >> \Delta$

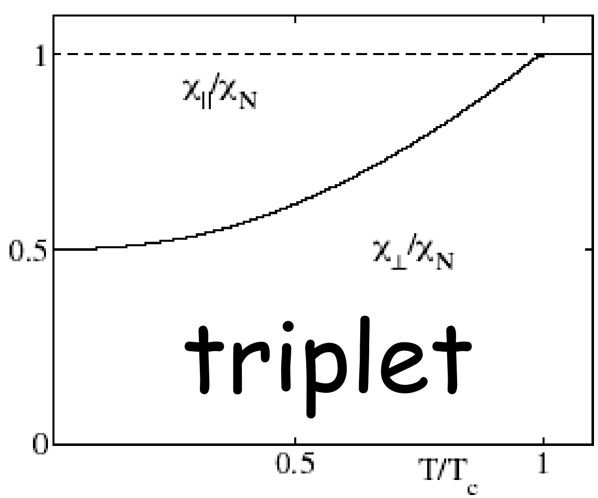
Gor'kov and Rashba (Rashba spin-orbit) PRL 87, 037004 (2001)
See also Yip PRB 65, 144508 (2002)

Spin Susceptibility





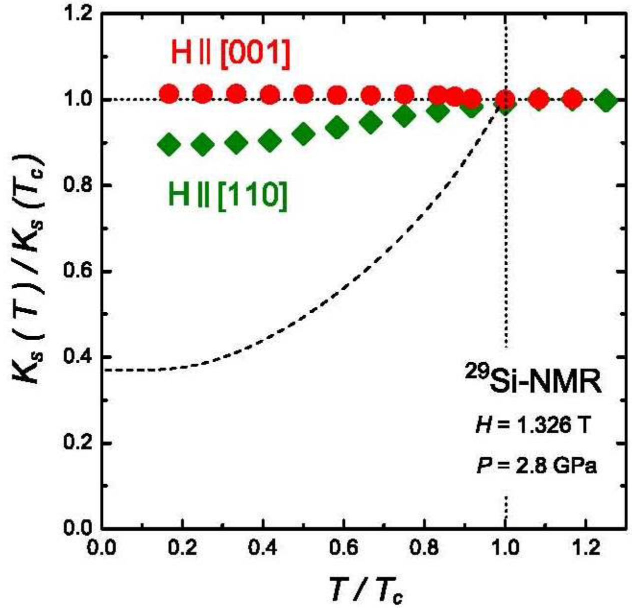
Note $\alpha >> \Delta$ for $CePt_3Si$



Singlet approaches triplet in large α limit. The usual Knight shift measurement is not useful

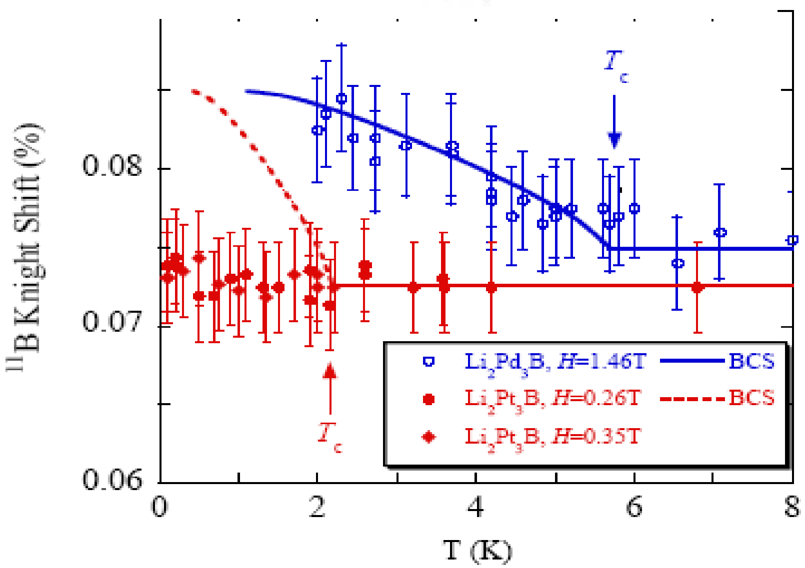
Expect large anisotropy in magnetic response of superconductor.

Experimental Results

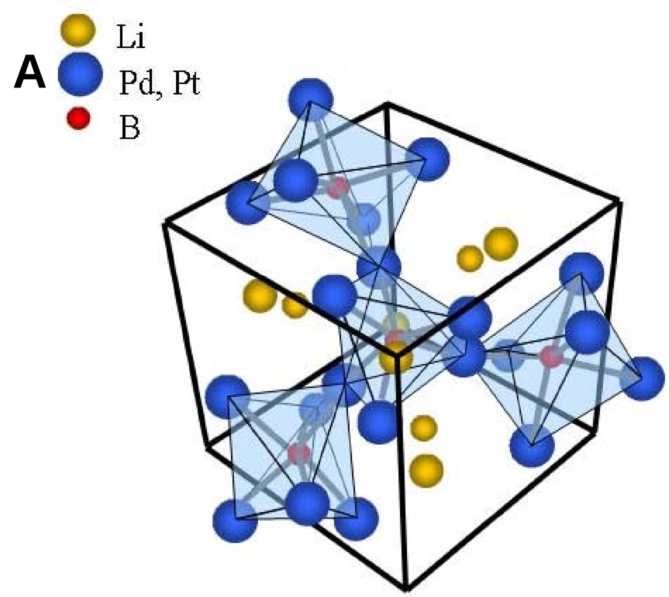


CeIrSi$_3$: Rashba

Mukuda *et al* PRL 2010



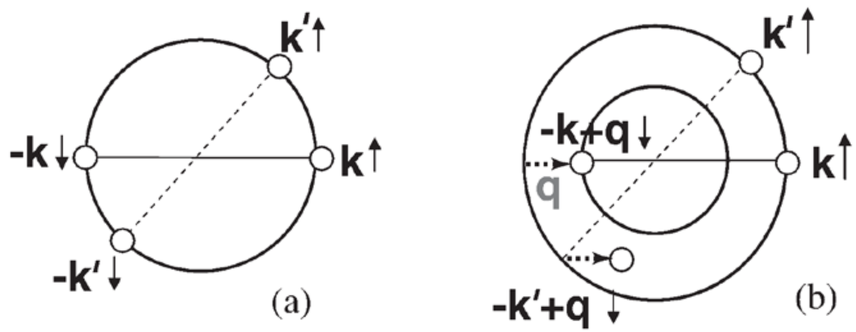
Li$_2$Pt$_3$B and Li$_2$Pd$_3$B: cubic

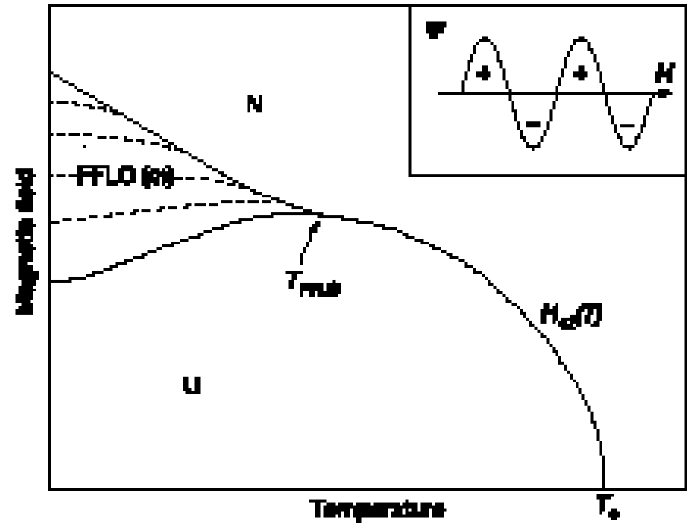


Zheng et al, PRL (2007)

FFLO-like phases

Singlet pairing in a Zeeman field: FFLO





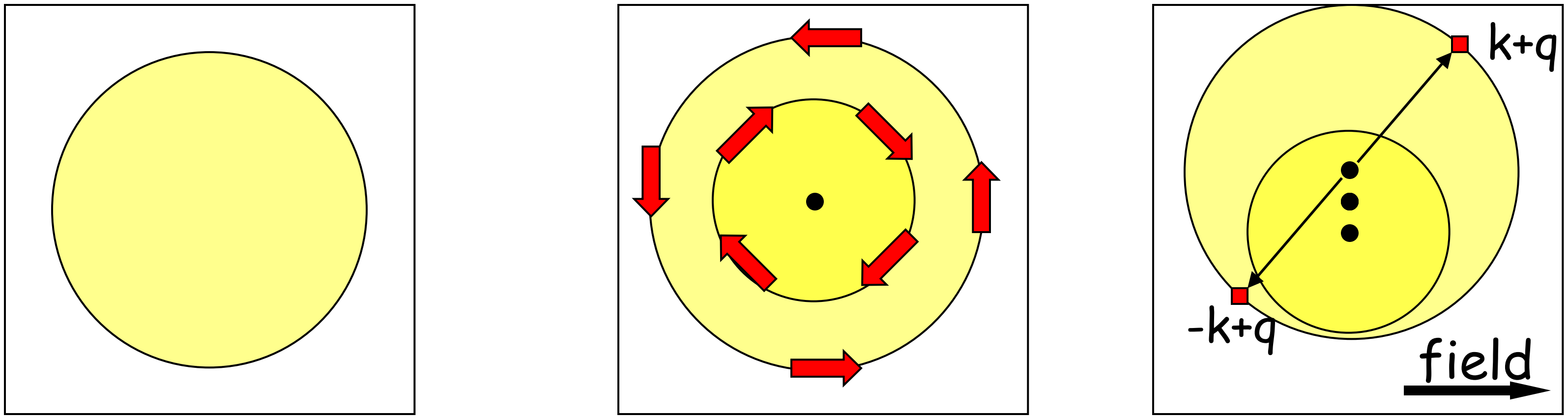
Key Point of FF and LO: Pairing between fermions with different Fermi surfaces leads to "finite momentum" pairing states (FFLO).

$$\Delta(\vec{r}) = \Delta_0 e^{i\vec{q}.\vec{r}} \quad \text{(FF)}$$

$$\Delta(\vec{r}) = \Delta_0 (e^{i\vec{q}.\vec{r}} + e^{-i\vec{q}.\vec{r}})/2 \quad \text{(LO)}$$

Destroyed By Disorder

Parity Broken Case



No ASOC

With ASOC

With ASOC and Zeeman Field

$$\xi_{\pm}(k) = \varepsilon(k) \pm |\alpha \vec{g}(k) + \mu_B \vec{B}|$$

$$\xi_{\pm}(k) \approx \varepsilon(k) \pm \alpha \pm \mu_B \vec{B} \cdot \hat{g}(k)$$

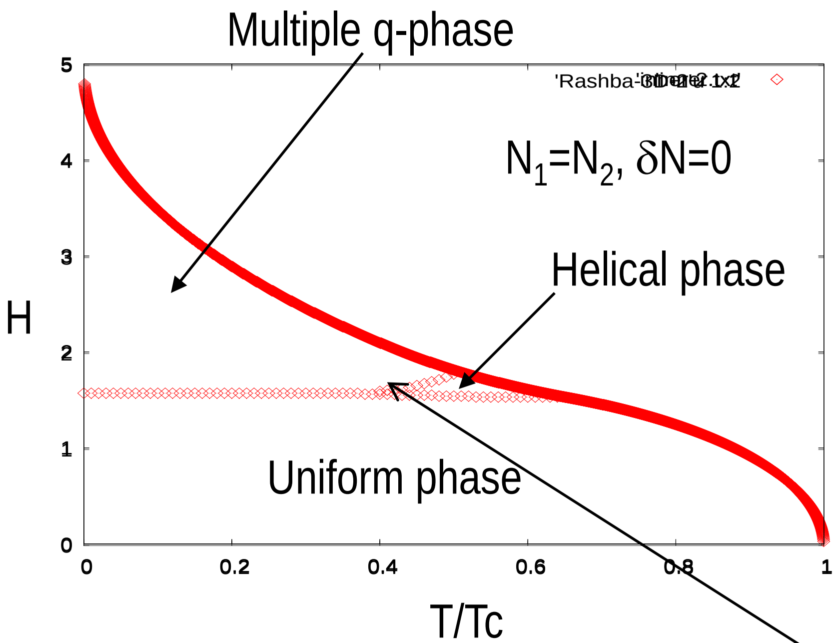
$$\mu_B \vec{B} \cdot \hat{g}(k) = \mu_B B_x k_y / k_F$$

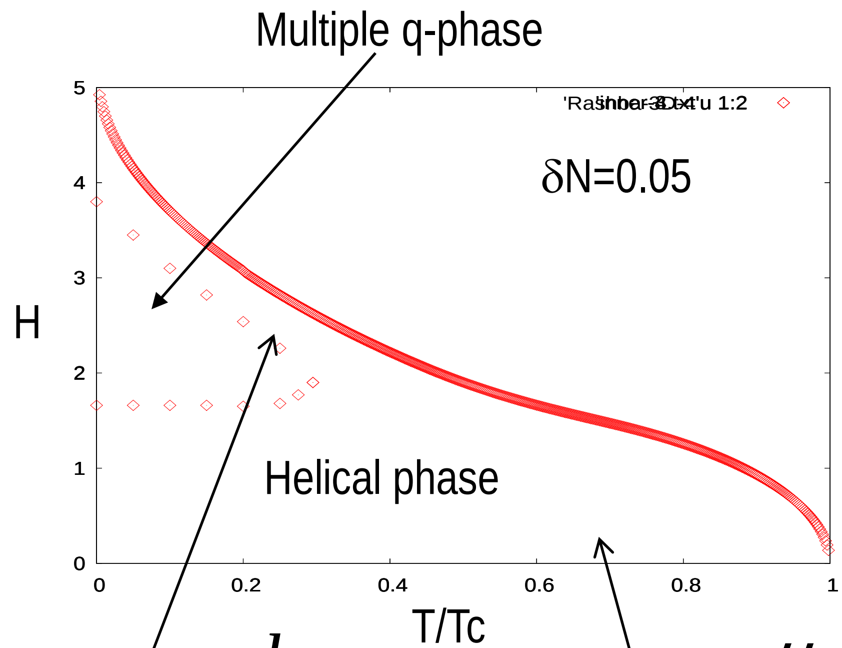
$$\Delta(\vec{r}) = \Delta_0 e^{i 2 \vec{q} . \vec{r}}$$

Can pair every state on one Fermi surface.
The two bands prefer opposite q vectors.

Phase Diagram

$$\delta N = \frac{N_1 - N_2}{N_1 + N_2}$$





$$q \cong \frac{\mu_B h}{v_F}$$

$$q \cong \delta N \frac{\mu_B h}{v_F}$$

Helical phase: $\psi(\vec{r}) = \psi_0 e^{i2\vec{q}.\vec{r}}$

Multiple-q phase: $\psi(\vec{r}) = \psi_1 e^{i2qr} + \psi_2 e^{-2iqr}$

Two different types of q appear - what is the origin?

Ginzburg Landau FFLO/Helical Theories

$$F = \int \left[\alpha(T,h) |\psi|^2 + \beta(h) |\psi|^4 + \kappa(h) |\vec{\nabla}\psi|^2 \right] d^3 r$$

At FFLO transition: $\kappa(h) = \beta(h) = 0$

$$\Rightarrow \psi = \psi_0 \cos(\vec{q} \cdot \vec{r})$$

In broken parity superconductors – new terms appear in the GL theory (Lifschitz invariants)

$$\varepsilon \hat{z} \cdot \vec{B} \times [\psi (i\vec{\nabla}\psi)^* + \psi^* (i\vec{\nabla}\psi)] = \varepsilon \hat{z} \cdot \vec{B} \times \vec{j}_{s,0}$$

Induces a helical solution in a uniform magnetic field

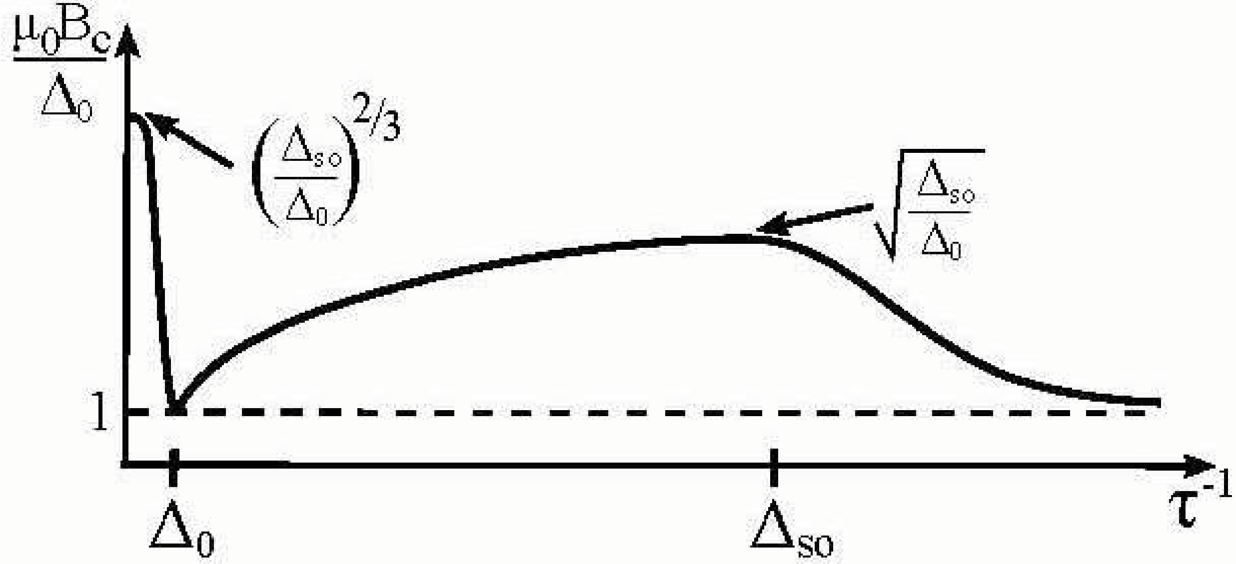
$\Rightarrow \psi = \psi_0 e^{i\vec{q}\cdot\vec{r}}$ with $\vec{q} = -\varepsilon \hat{z} \times \vec{B} / \kappa$

Lifschitz invariant origin suggests helical phase will exist even with impurities. Helical phase has no supercurrent.

Results for 2D Rashba

Barzykin and Gorkov, PRL (2002); DFA, Physica C (2003); Kaur, DFA, Sigrist PRL (2004); DFA and Kaur PRB (2007); Dimitrova and Feigel'man , PRB (2007); Samokhin, PRB (2008); Mineev and Samokhin (2008).

Michaeli, Potter, Lee (2012) ($LaAlO_3/SrTiO_3$ interfaces)



Survives to high Fields in disordered 2D Rashba

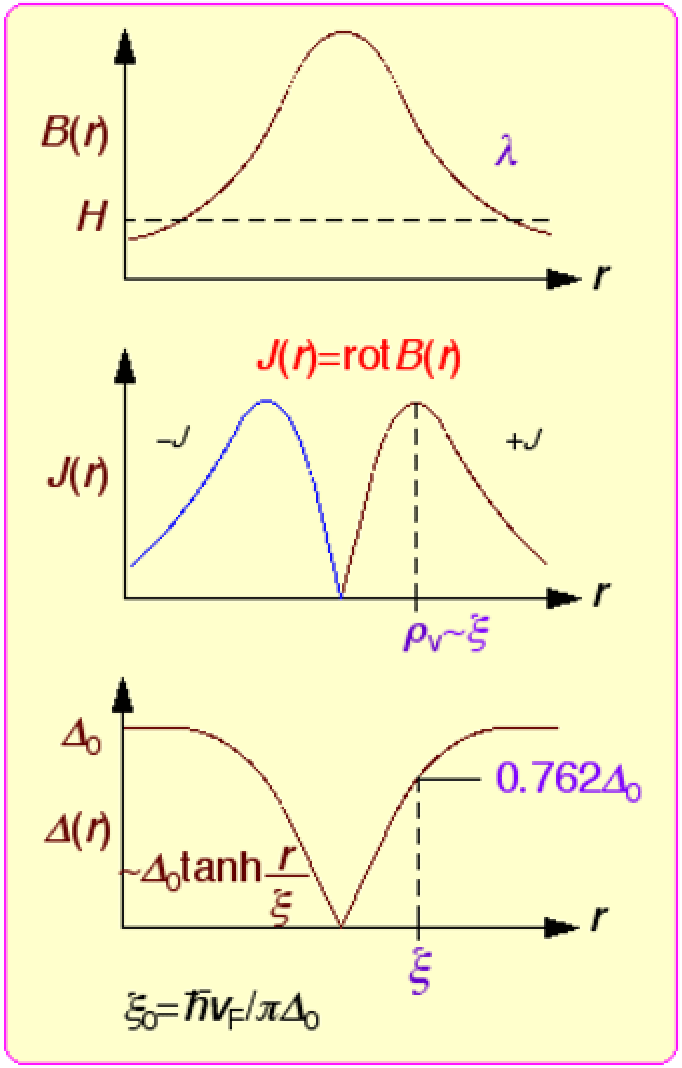
Physics Related to Helical Phase

$$\varepsilon\hat{n}\cdot\vec{B}\times[\psi(\vec{D}\psi)^* + \psi^*(\vec{D}\psi)] = \varepsilon\hat{n}\cdot\vec{B}\times\vec{j}_{s,0}$$

- Edelstein (2D, 1995): in-plane supercurrent will induce a Zeeman field
- Yip (2D, 2002): in-plane Zeeman field will induce a supercurrent. Dimitrova and Feigel'man (2007).
- In Li_2Pt_3B $\varepsilon\vec{B}\cdot\vec{j}_{s,0}$ is allowed by symmetry.
- Levitov, Nazarov, Eliashberg (3D, 1985) magnetic induction jump in Meissner layer at surface and Magnetic field rotates and decays in the superconductor.
- Li_2Pt_3B, the helical *q*-vector will be parallel to the applied magnetic field.

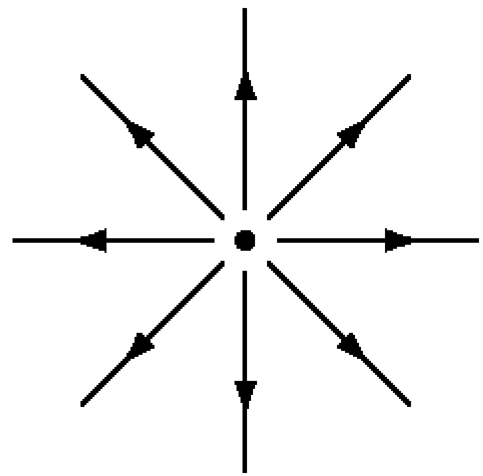
Vortex for c-axis field

$$\varepsilon\hat{n}\cdot\vec{B}\times[\psi(\vec{D}\psi)^{*}+\psi^{*}(\vec{D}\psi)]=\varepsilon\hat{n}\cdot\vec{B}\times\vec{j}_{s,0}$$



$$\vec{D}=i\vec{\nabla}-2e\vec{A}$$

The vortex will develop a *transverse magnetization* that is radial to the applied field.



Fractional Vortices in FFLO-like Phase

Theory of ψ_{Qx} and ψ_{-Qx}

FFLO-like phase: $$\psi = \psi_{Qx} e^{iQx} + \psi_{-Qx} e^{-iQx}$$

$$f = \alpha_1 |\psi_{Qx}|^2 + \alpha_2 |\psi_{-Qx}|^2 + \beta_1 (|\psi_{Qx}|^2 + |\psi_{-Qx}|^2)^2$$

$$+ \beta_2 |\psi_{Qx}|^2 |\psi_{-Qx}|^2 + \frac{1}{2m}(|\nabla \psi_{Qx}|^2 + |\nabla \psi_{-Qx}|^2)$$

$\boldsymbol{U}(1) \times \boldsymbol{U}(1)$ symmetry!

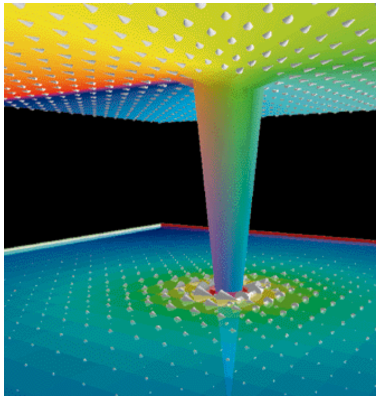
General feature: $$(\psi_{Qx})^n (\psi^*_{Qx})^m (\psi_{-Qx})^p (\psi^*_{-Qx})^k$$

$\boldsymbol{n} - \boldsymbol{m} + \boldsymbol{p} - \boldsymbol{k} = 0$ Gauge invariance $\boldsymbol{n} = \boldsymbol{m}$

$\boldsymbol{n} - \boldsymbol{m} - \boldsymbol{p} + \boldsymbol{k} = 0$ Translational invariance $\boldsymbol{p} = \boldsymbol{k}$

FFLO "Vortices"

$$\psi_Q(\boldsymbol{r},\phi) = |\psi_1(\boldsymbol{r})| \boldsymbol{e}^{in\phi} \quad \psi_{-Q}(\boldsymbol{r},\phi) = |\psi_2(\boldsymbol{r})| \boldsymbol{e}^{im\phi}$$



(n,m) vortices. Consider (1,0) vortex in a phase where the magnitudes of the components far from the vortex core are (ψ_1,ψ_2)

$$\vec{j} = i\hbar m[\psi_1(\nabla\psi_1)^* - \psi_1^*(\nabla\psi_1)] - \frac{2me}{c}(|\psi|_1^2 + |\psi|_2^2)\vec{A}$$

then

$$\oint A \cdot dl = \frac{|\psi_1|^2}{|\psi_1|^2 + |\psi_2|^2}\Phi_0$$

(1,1) vortex is usual Abrikosov vortex with flux Φ_0

Conclusions

- $\mathbf{d}_k$ must be parallel to $\mathbf{g}_k$ for spin-triplet superconductivity
- Paramagnetism is strongly suppressed in spin-singlet superconductors (critical fields are increased)
- Spin susceptibility is the same for both singlet and triplet states when spin-orbit is large.
- In Rashba materials, microscopic arguments imply FFLO/helical phases.
- Broken parity materials have new Lifshitz invariants in the free energy with an associated anomalous magnetization.
- FFLO-like phase will have fractional vortices.