Aalto University
School of Science

Lecture 2: Quantum geometry in moire materials and non-equilibrium flat band transport

Päivi Törmä

Aalto University

Winter theory school: New Frontiers in Superconductivity, Florida 2024

11.1.2024

Super C Room Temperature Superconductivity 2033

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HORIZON2020 Marie Skłodowska-Curie actions

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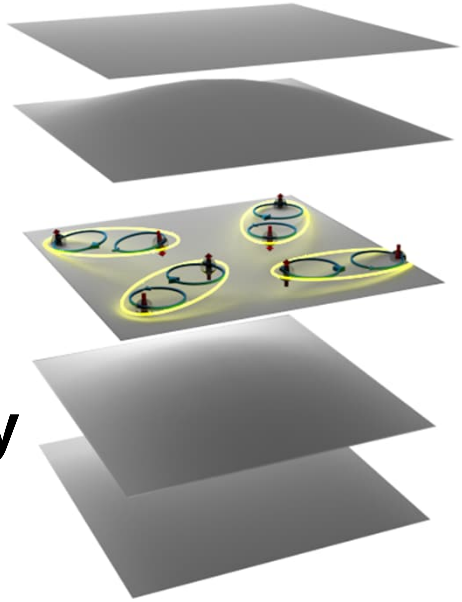

Lecture 1

- **Basics of quantum geometry**
- **Quantum geometry and superconductivity**

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- **Flat band superconductivity and quantum geometry in twisted bilayer graphene (TBG)**
- **Non-Fermi liquid normal states in flat bands**
- **Non-equilibrium transport in flat band superconductors**
- **DC conductivity in a flat band**
- **The many-body quantum metric and the Drude weight**

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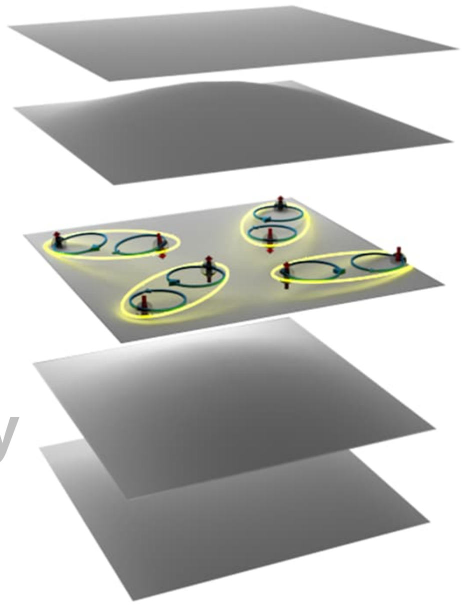

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Twisted Bilayer Graphene (TBG) superconductivity since 2018

Reviews: Balents, Dean, Efetov, Young, Nat Phys 2020

Andrei, Efetov, Jarillo-Herrero, MacDonald, Mak, Senthil, Tutuc, Yazdani, Young, Nat Rev Mater 2021

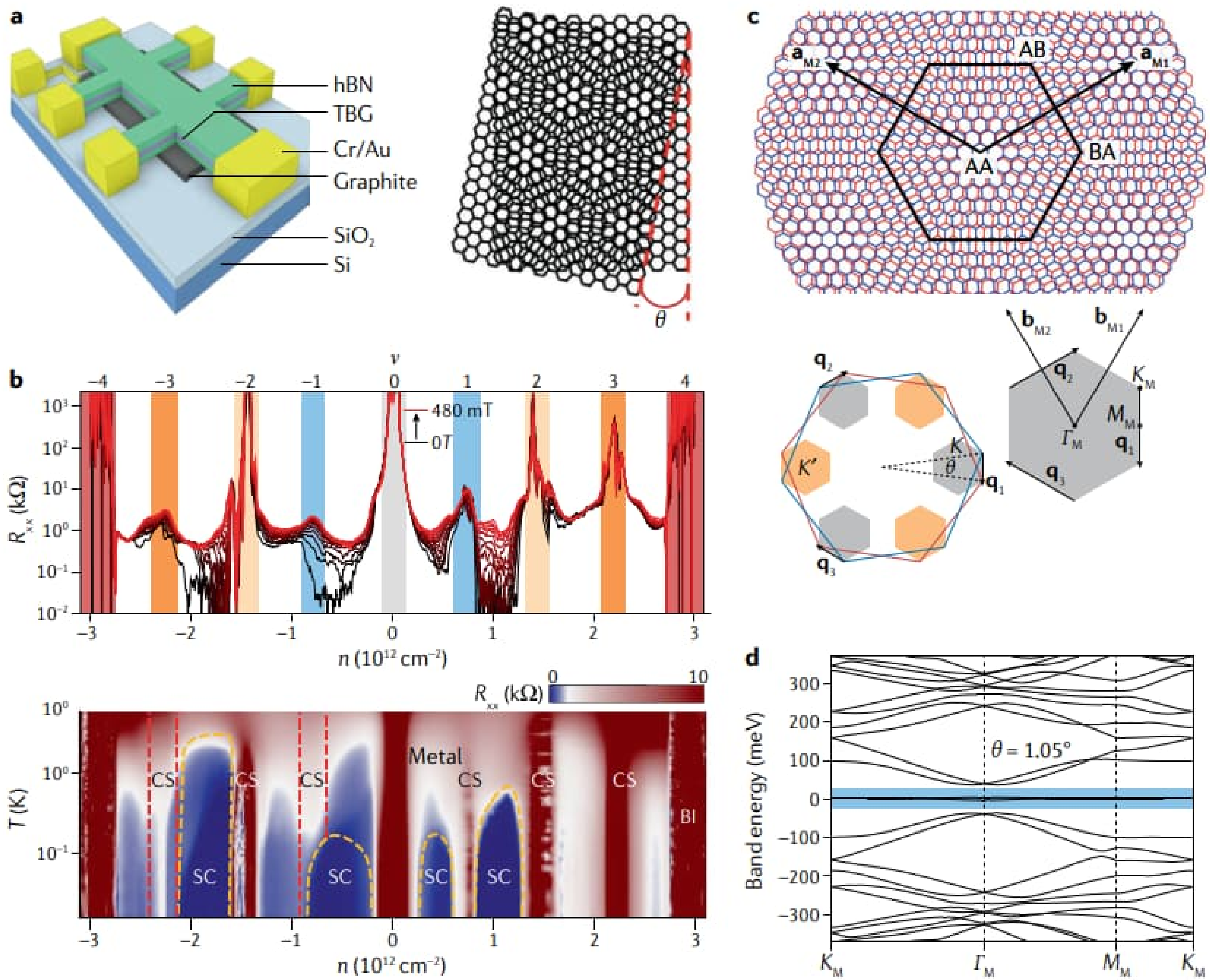
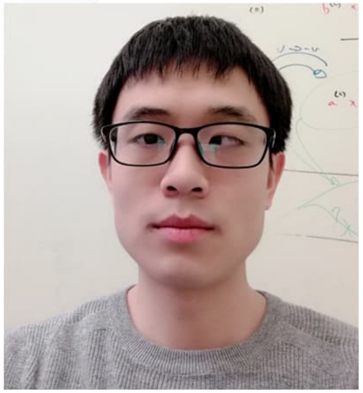


Figure credits see Fig.1 in PT, Peotta, Bernevig, Nat Rev Phys 2022

Geometric contribution in TBG superconductivity

Aleksi Julku

Teemu Peltonen



Long Liang

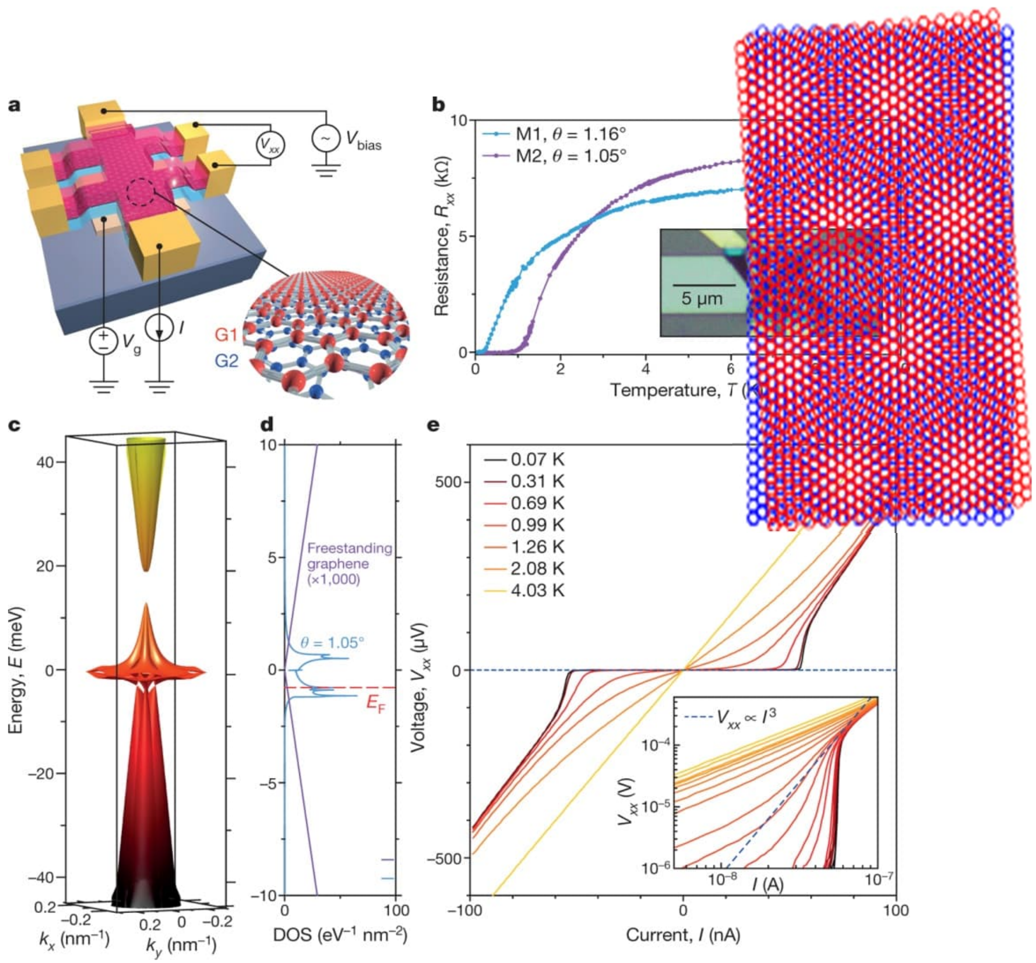
Tero Heikkilä

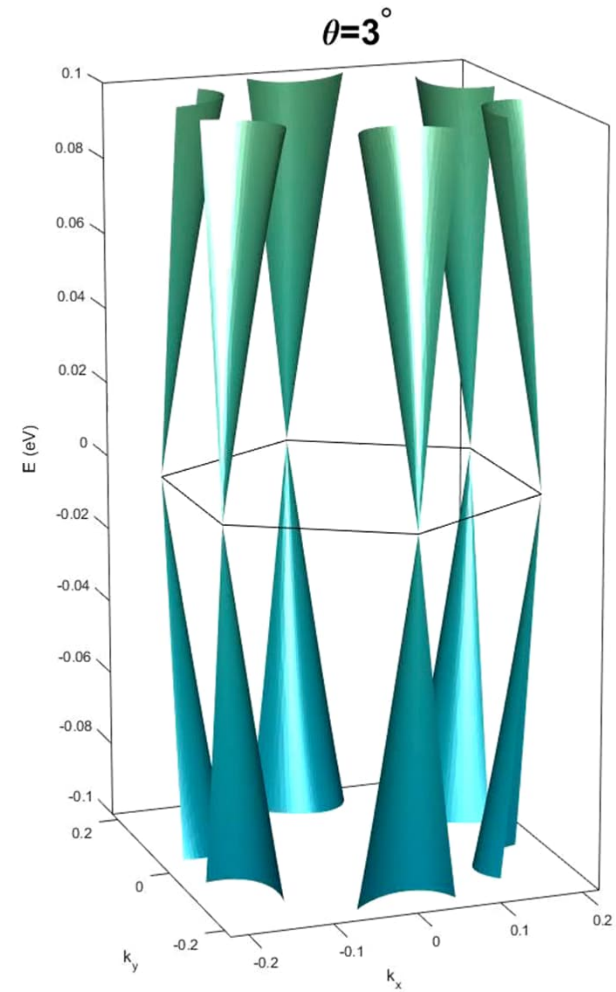
Julku, Peltonen, Liang, Heikkilä, PT, PRB(R) (2020); Editors’ Suggestion

MA-TBG: Magic **A**ngle-**T**wisted **B**ilayer **G**raphene

Twisting graphene layers produces flat bands

(unconventional) superconductivity





Y Cao *et al. Nature* **556**, 43–50 (2018)

Also

Nature 556, 80 (2018)

Science 363, 1059 (2019)

Nature 574, 653-657 (2019))

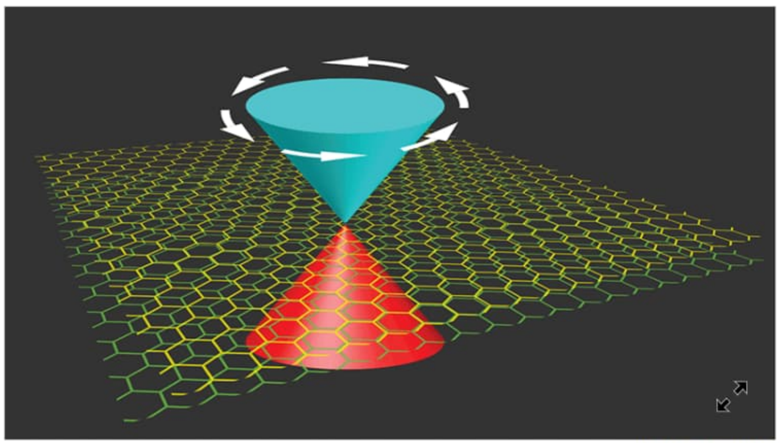
Geometry Rescues Superconductivity in Twisted Graphene

Laura Classen

School of Physics and Astronomy, University of Minnesota, Minneapolis, MN, USA

February 24, 2020 • *Physics* 13, 23

Three papers connect the superconducting transition temperature of a graphene-based material to the geometry of its electronic wave functions.



APS/Alan Stonebraker

Figure 1: Electrons moving through the sheets of twisted bilayer graphene (TBG) have special points in their band structure where two cone-shaped bands meet. The inherent "curvature" of the states in these bands turns out to contribute to the magnitude of TBG'... Show more

On its own, a sheet of graphene is a semimetal—its electrons interact only weakly with each other. But as experimentalists discovered in 2018 [1, 2], the situation changes when two sheets of graphene are stacked together, with a slight ($\sim 1^\circ$) rotation between them (Fig. 1). At this so-called magic twist angle [3] and at low temperatures [1], the electrons become correlated, forming insulating or superconducting phases depending on the carrier density [2–7]. These phases appear to come from a twist-induced flattening of the electronic energy bands, which

Geometric and Conventional Contribution to the Superfluid Weight in Twisted Bilayer Graphene

Xiang Hu, Timo Hyart, Dmitry I. Pikulin, and Enrico Rossi

Phys. Rev. Lett. 123, 237002 (2019)

Published December 5, 2019

Read PDF

Superfluid weight and Berezinskii-Kosterlitz-Thouless transition temperature of twisted bilayer graphene

A. Julku, T. J. Peltonen, L. Liang, T. T. Heikkilä, and P. Törmä

Phys. Rev. B 101, 060505 (2020)

Published February 24, 2020

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Topology-Bounded Superfluid Weight in Twisted Bilayer Graphene

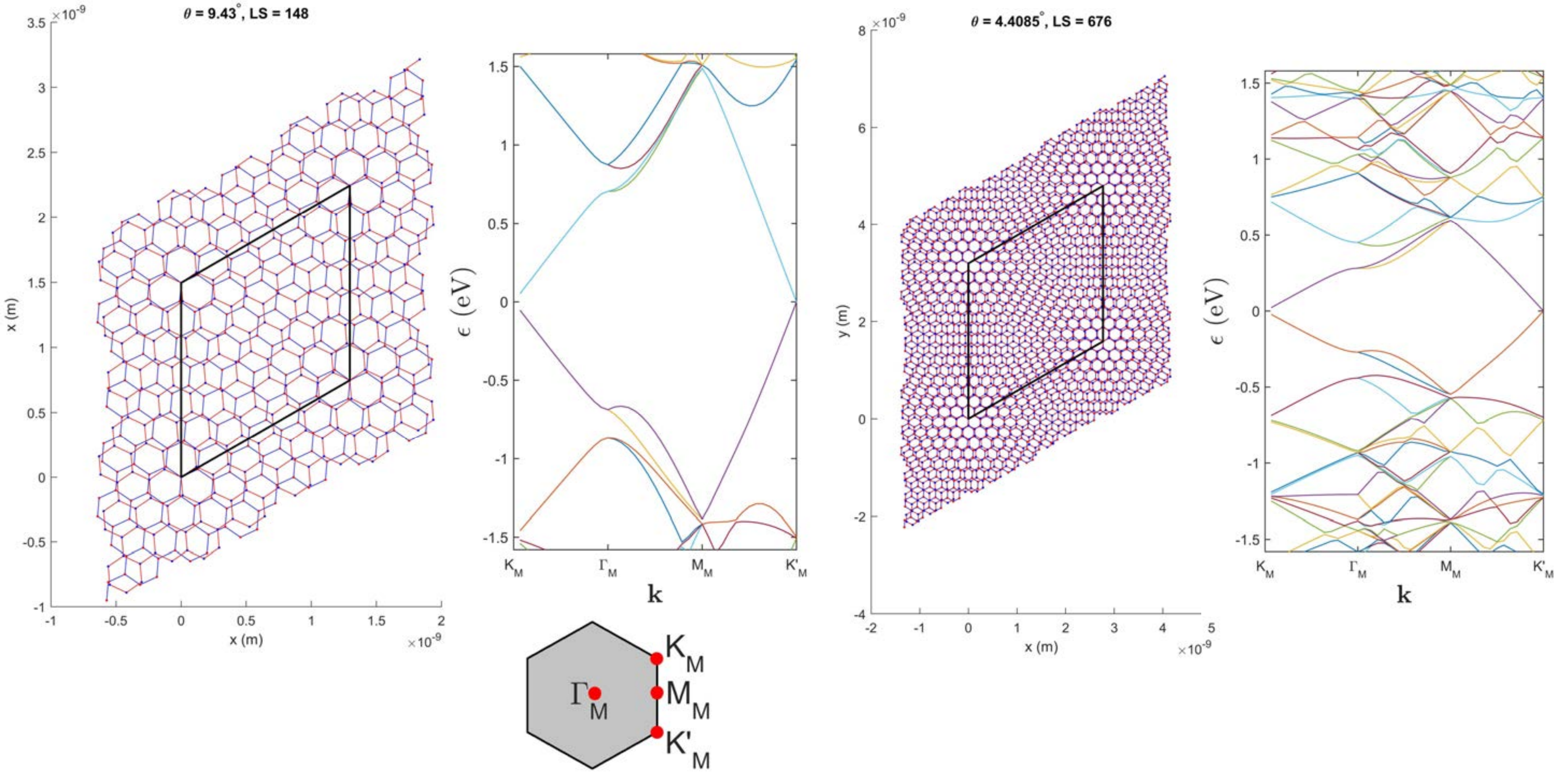
Fang Xie, Zhida Song, Biao Lian, and B. Andrei Bernevig

Phys. Rev. Lett. 124, 167002 (2020)

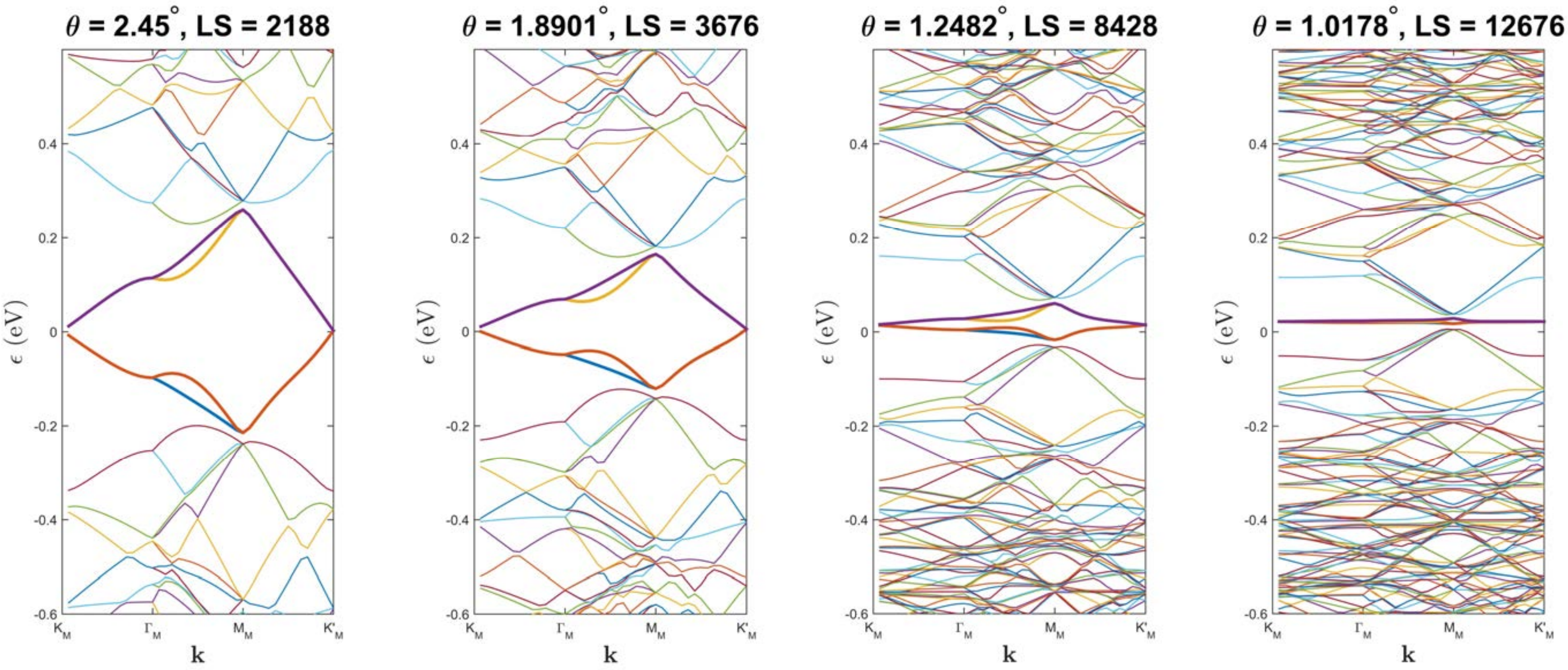
Published April 24, 2020

Read PDF

Non-interacting bands



Non-interacting bands



At magic angle θ ~1 deg, the number of lattice sites per unit cell (LS) around 13 000: numerically still a problem even at the mean-field level

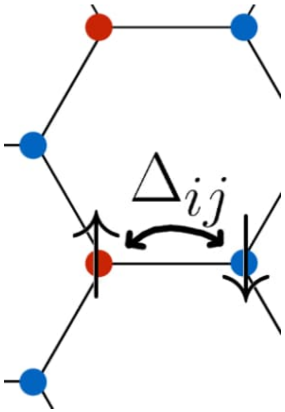
We reduce LS to around 700 by applying a rescaling trick which modifies the twist angle but keeps the Moire periodicity and the Dirac velocity invariant

Fermi-Hubbard lattice model with TBG geometry (600 bands)

$$H = \sum_{ij\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + H_{\text{int}}$$

Two pairing schemes

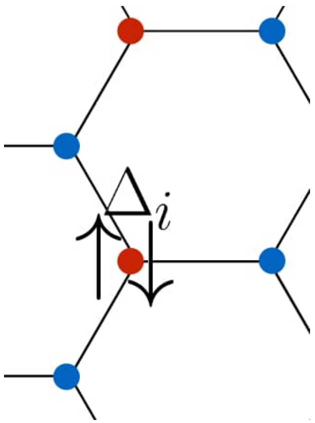
Non-local (RVB) interaction



$$H_{\text{int}} = \frac{J}{2} \sum_{\langle ij \rangle} h^{\dagger}_{ij} h_{ij}$$

$$h_{ij} = c_{i\downarrow} c_{j\uparrow} - c_{i\uparrow} c_{j\downarrow}$$

Local (s-wave) interaction



$$H_{\text{int}} = J \sum_{i} c^{\dagger}_{i\uparrow} c_{i\uparrow} c^{\dagger}_{i\downarrow} c_{i\downarrow}$$

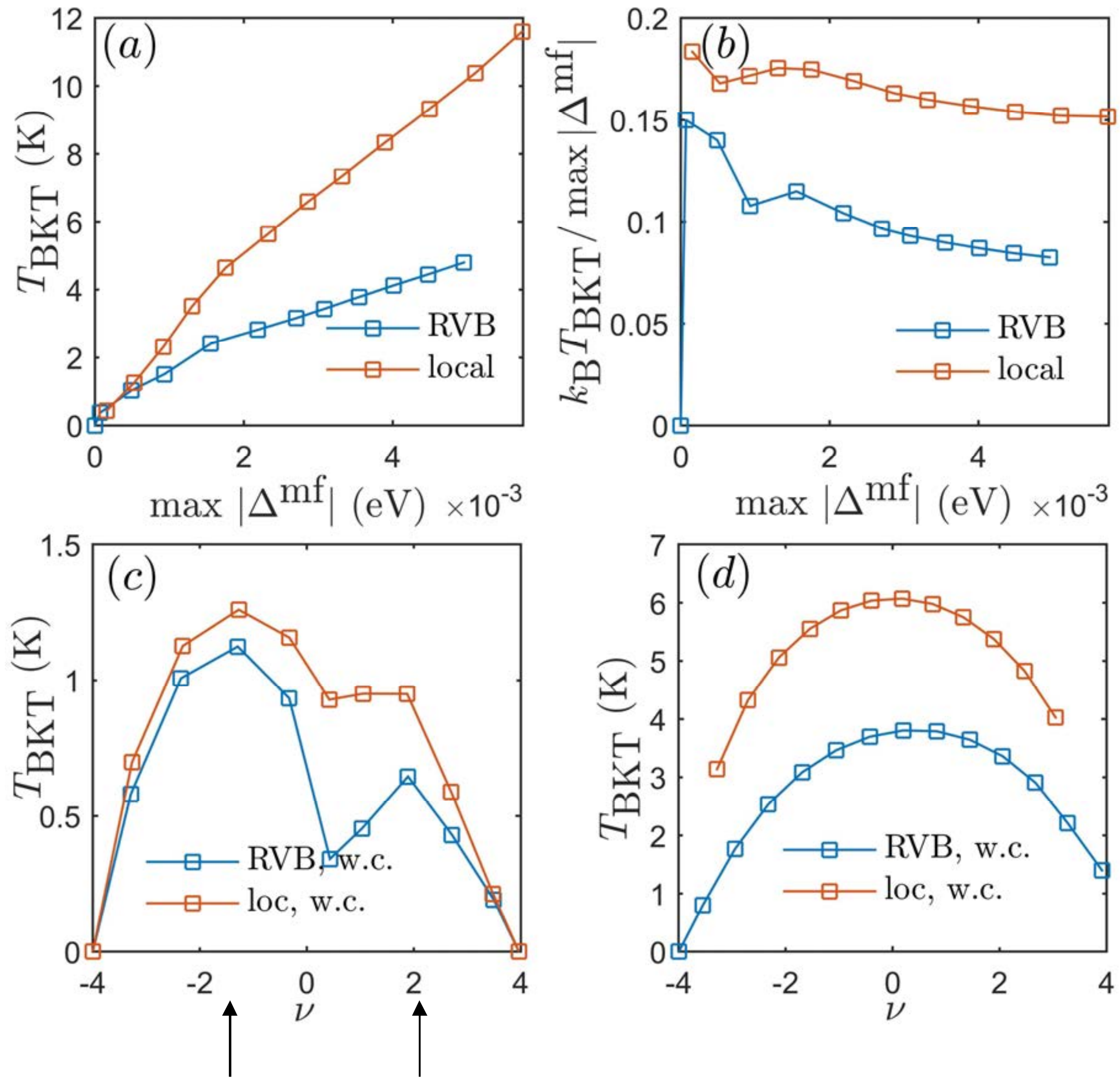
J< 0 is attractive interaction strength

BKT temperature

$$T_{\mathrm{BKT}} = \frac{\pi}{8}\sqrt{\det D^s(T_{\mathrm{BKT}})}$$

For flat band regime local interaction has considerably larger T_{BKT}

Here RVB (resonance valence bond) is the non-local pairing scheme



Small interaction

Large interaction

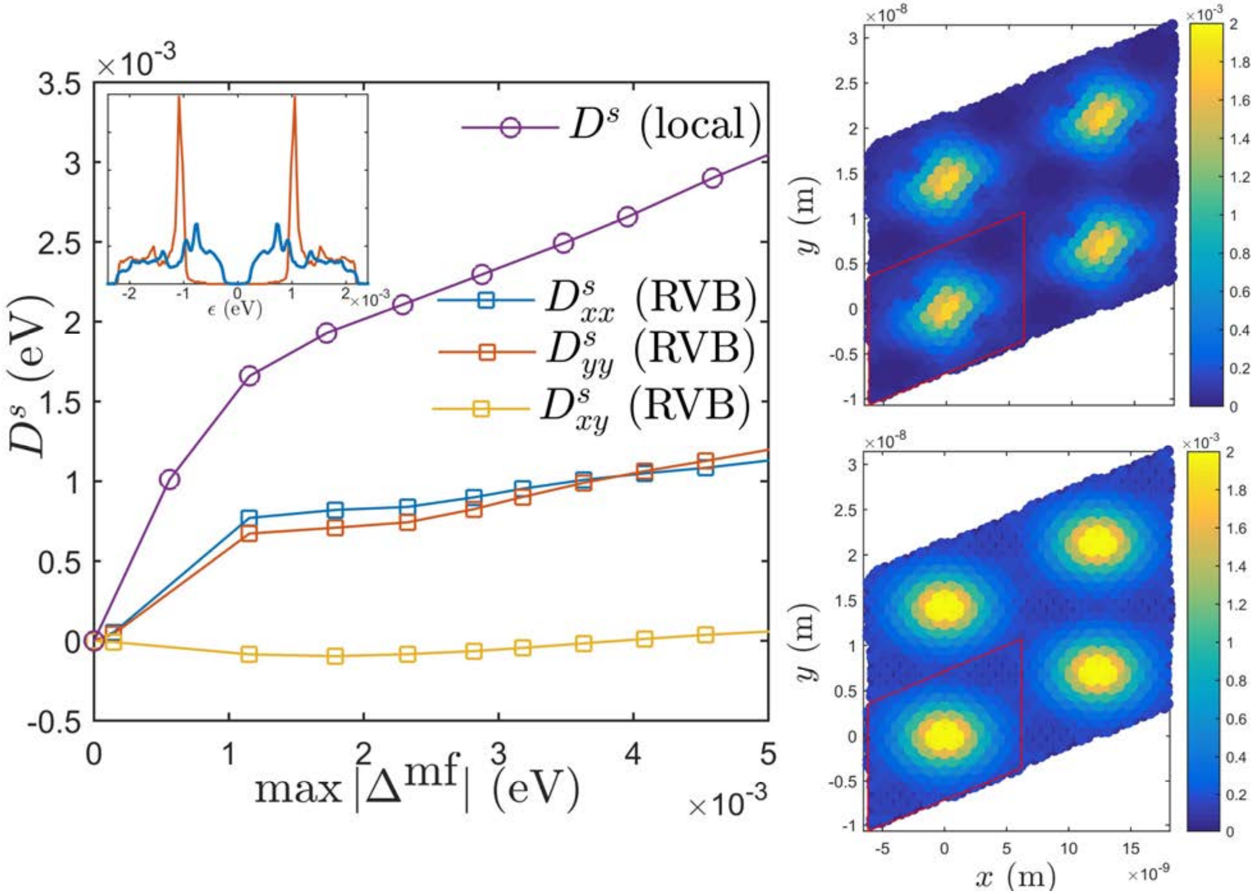
Nematic order parameter for non-local pairing

$$D^s = \begin{bmatrix} D^s_{xx} & D^s_{xy} \approx 0 \\ D^s_{yx} \approx 0 & D^s_{yy}, \end{bmatrix}$$

Local pairing preserves the lattice symmetries and yields isotropic D^s

Non-local pairing breaks the rotational symmetry and yields non-isotropic response

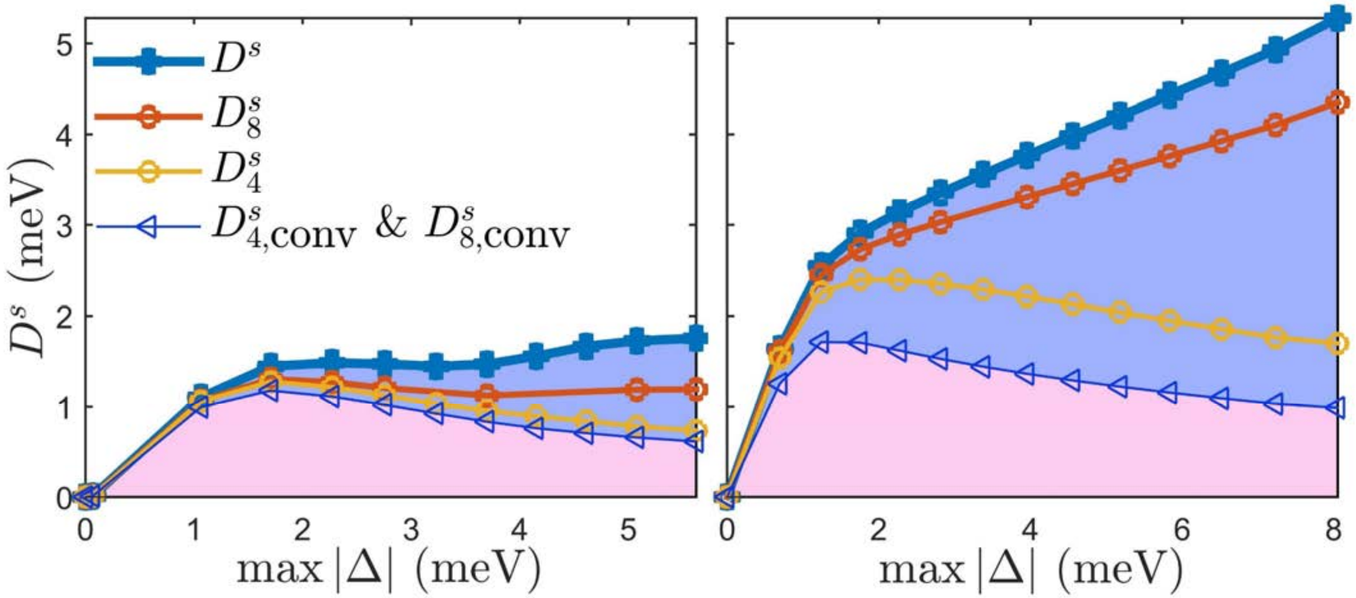
Local pairing has s wave symmetry, non-local yields mixed s+p+d symmetry (d dominant)



Geometric contribution in TBG

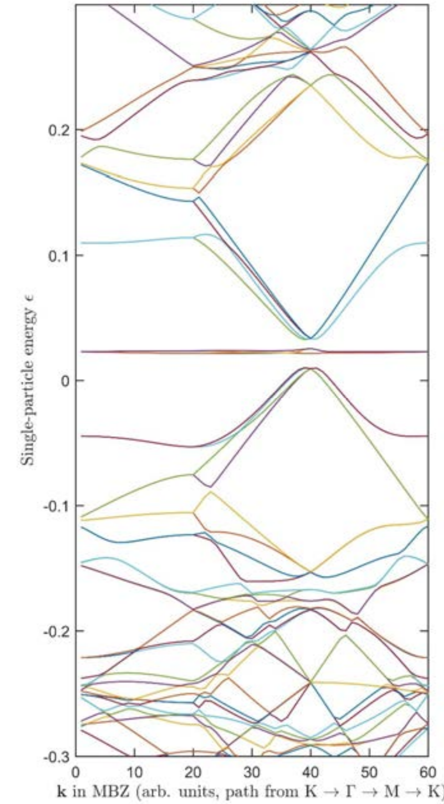
$$D^s = D^s_{\text{conv}} + D^s_{\text{geom}}$$

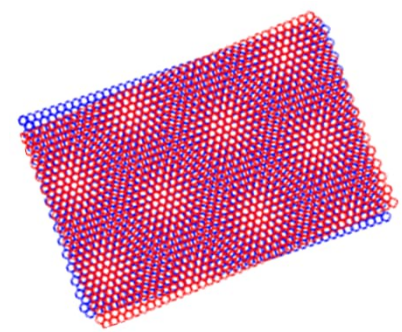
$$T_{\text{BKT}} = \frac{\pi}{8}\sqrt{\det D^s(T_{\text{BKT}})}$$



Non-local (RVB) interaction

Local (s-wave) interaction





Julku, Peltonen, Liang, Heikkilä, PT, PRB(R) (2020); Editors' Suggestion

Confirmed by (only s-wave): Hu, Hyart, Pikulin, Rossi, PRL (2019)

Euler class bound of TBG superconductivity: Xie, Song, Lian, Bernevig, PRL (2020)

TBG theory has advanced since 2020 (e.g. Kang, Vafek, PRB 2023; Vafek, Kang, PRB 2023); quantitative predictions to be revisited

First experiments exploring quantum geometric superconductivity in TBG

Tian, Gao, Che, Xu, Cheung, Watanabe, Taniguchi, Randeria, Zhang, Lau, Bockrath, Nature 2023

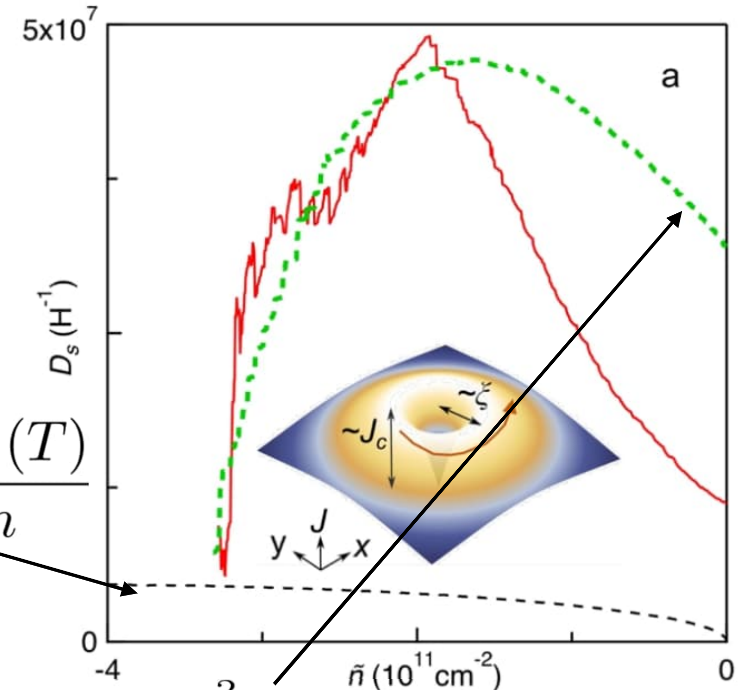
$$\xi = \sqrt{\frac{\Phi_0}{2\pi B_{\mathrm{c2}}}} \quad \Phi_0 = \frac{h}{2e} \quad J_{\mathrm{cs}} = n_s e \frac{\Delta}{\hbar k_{\mathrm{F}}}$$

Critical field and current measured as well as Fermi velocity

Superfluid weight from

$$D_s(0) = \frac{2\pi J_{\mathrm{cs}} \xi}{\Phi_0}$$

$$D_s(T) = \frac{e^2 n_s(T)}{m}$$



Isolated flat band

$$[D_s]_{ij} = \frac{2}{\pi \hbar^2} \frac{\Delta^2}{U N_{\mathrm{orb}}} \mathcal{M}^{\mathrm{R}}_{ij}$$

$$D_s(0, \tilde{n}) \approx b \frac{e^2}{\hbar^2} \Delta(0, \tilde{n})$$

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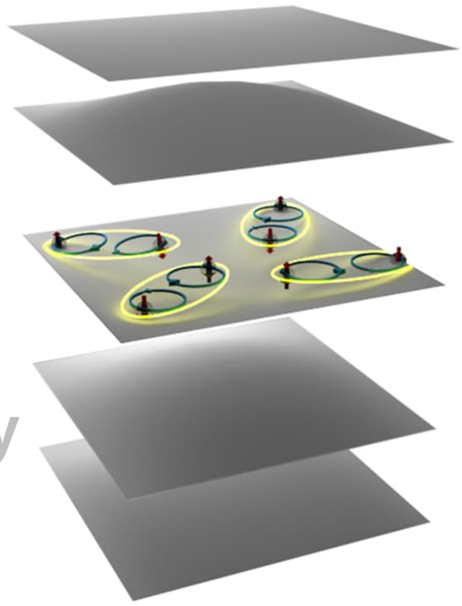

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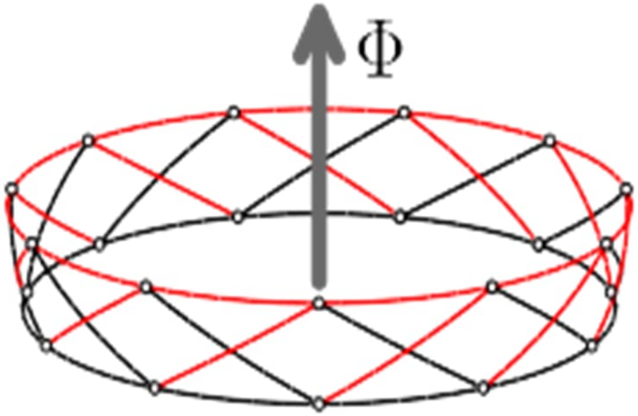
New phenomena also in the flat band <u>normal state</u>

In certain lattice models, only pairs move at any temperature,
Tovmasyan, Peotta, Liang, PT, Huber, PRB 2018



Aharonov-Bohm effect in a ring geometry

Non-Fermi liquid features in double occupancy and entropy (Lieb lattice), Kumar, Peotta, Takasu, Takahashi, PT, PRB(L) 2021

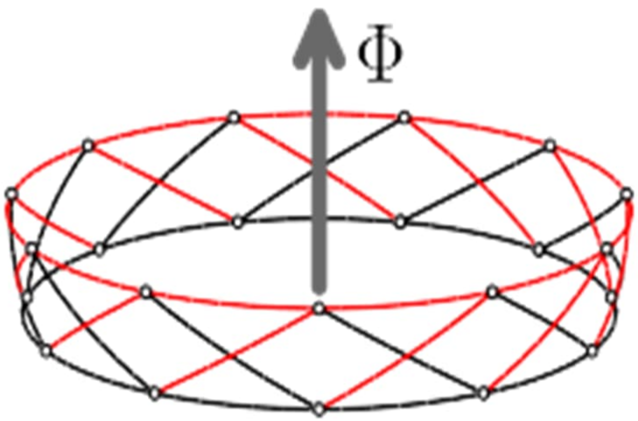
Insulator – pseudogap crossover in the Lieb lattice normal state,
Huhtinen, PT, PRB(L) (2021)

Preformed pairs in a flat band

Tovmasyan, Peotta, Liang, PT, Huber, PRB 2018

What are the charge carriers in the ***normal state*** of a flat band superconductor?
We find: only pairs move (Pi-periodic ground state); non Landau-Fermi liquid.

Ground state energy vs. magnetic flux $E_0(\Phi)$



Aharonov-Bohm effect in a ring geometry

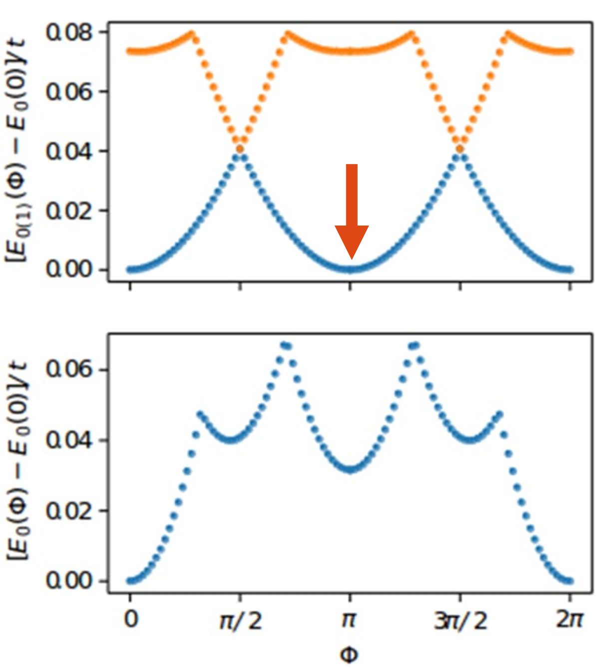
Flat band

$$E_0(\Phi) = E_0(\Phi + \Phi_0/2)$$

$$\Phi_0 = hc/e = 2\pi$$

$$\hbar = c = e = 1$$

Non Flat band



Related to local conserved quantities.

Flat band interacting normal state; Lieb lattice

- **Non-Fermi liquid features in double occupancy and entropy**
- **SU(N) scaling relation**

Pramod Kumar

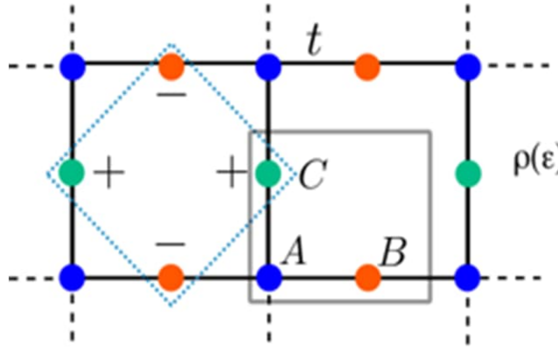
Sebastiano Peotta

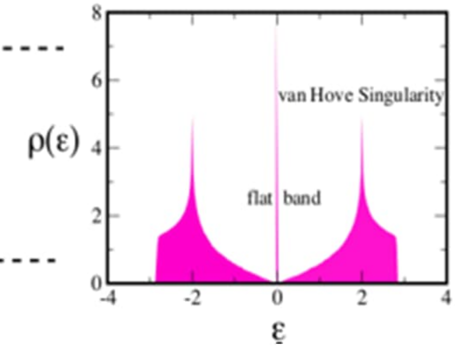
Yosuke Takasu

Yoshiro Takahashi

P Kumar, S Peotta, Y Takasu, Y Takahashi, PT, PRB(L) 2021

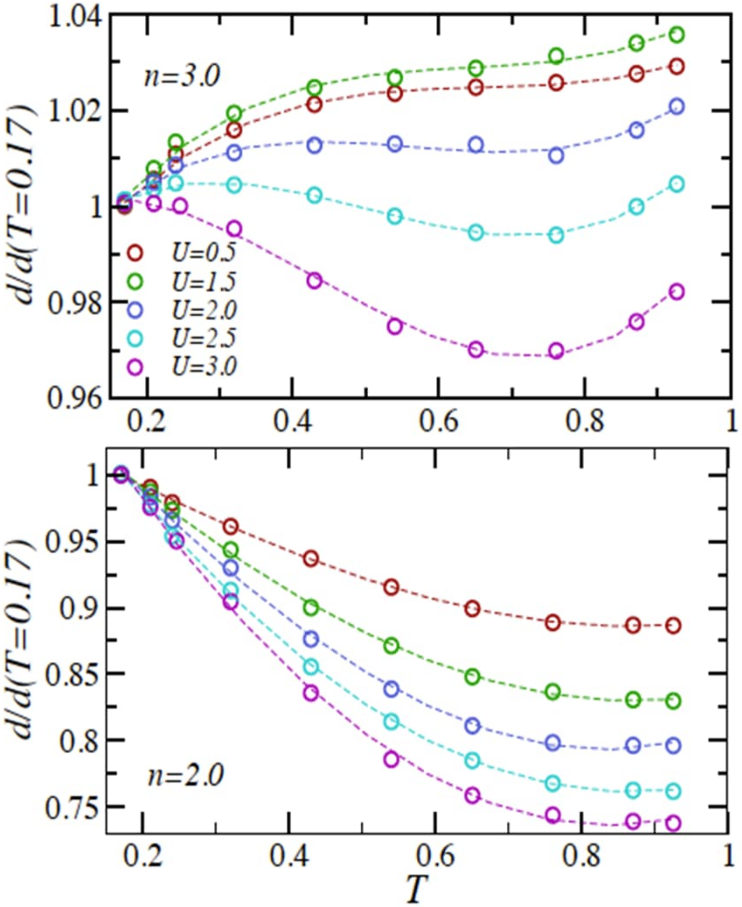
Lieb lattice: repulsive Hubbard model





Normal state properties

average double occupancy (DMFT)



half-filling: flat band significant

Non-Fermi liquid behavior for small interactions at the flat band

lowest band filled

Insulator – pseudogap crossover in the Lieb lattice normal state

Kukka-Emilia Huhtinen

KE Huhtinen, PT, PRB(L) (2021)

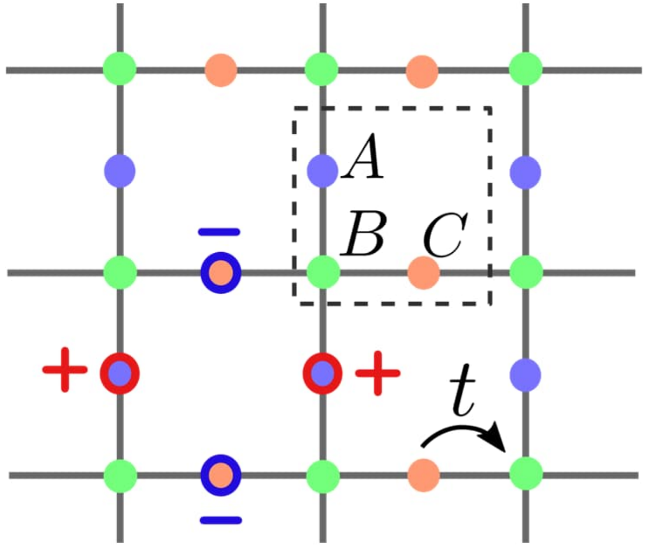
$$\Sigma(\mathbf{k}) \approx \Sigma_{\text{loc}}$$

$$\mathcal{G}(\mathbf{k}) = \left[(\mathcal{G}^0(\mathbf{k}))^{-1} - \Sigma(\mathbf{k})\right]^{-1}$$

$$\mathcal{G}_{\text{loc}} = \left[(\mathcal{G}^0_{\text{loc}})^{-1} - \Sigma_{\text{loc}}\right]^{-1}$$

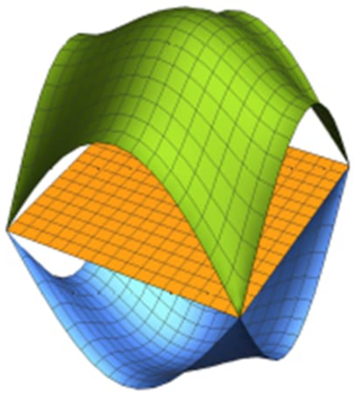
Hubbard model on the Lieb lattice

FOCUS ON THE NORMAL STATE ABOVE SUPERCONDUCTIVITY



Attractive Hubbard model

$$H = \sum_{\sigma} \sum_{i\alpha, j\beta} t_{ij} c^{\dagger}_{\sigma,i\alpha} c_{\sigma,j\beta} - \sum_{\sigma} \sum_{\mathrm{i}\alpha} \mu_{\sigma} n_{\sigma,i\alpha}$$
$$+U \sum_{i\alpha} (n_{\uparrow,i\alpha} - 1/2)(n_{\downarrow,i\alpha} - 1/2)$$



Flat band states reside at A and C sites

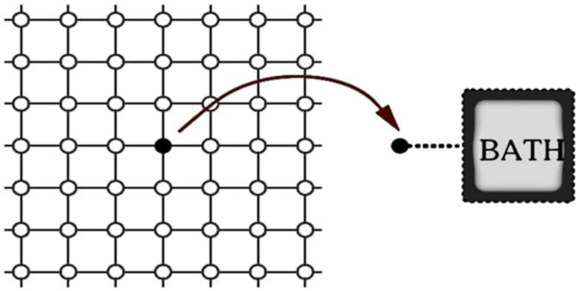
DMFT cluster: A, B and C

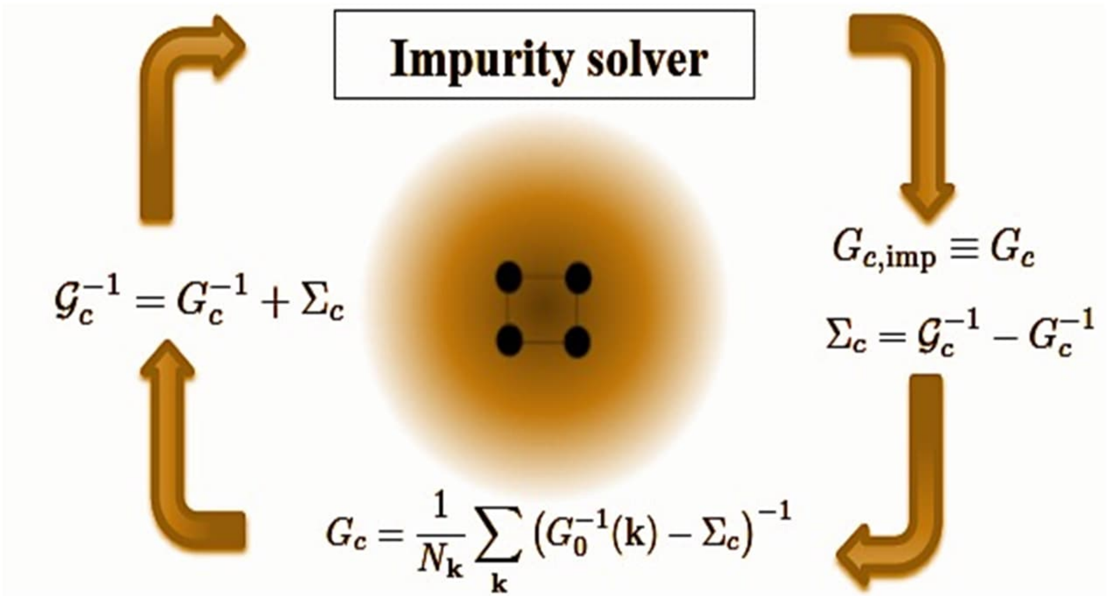
DMFT

Georges, Kotliar, Krauth, Rozenberg, Rev. Mod. Phys. 1996

Kotliar, Savrasov, Haule, Oudovenko, Parcollet, Marianetti, Rev. Mod. Phys. 2006

Dynamical Mean Field Theory (DMFT) to capture quantum effects *beyond mean-field*

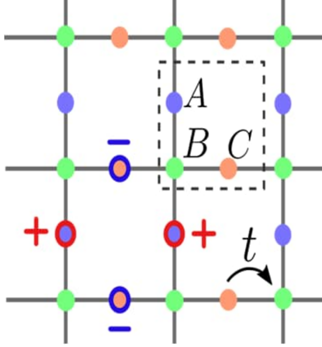


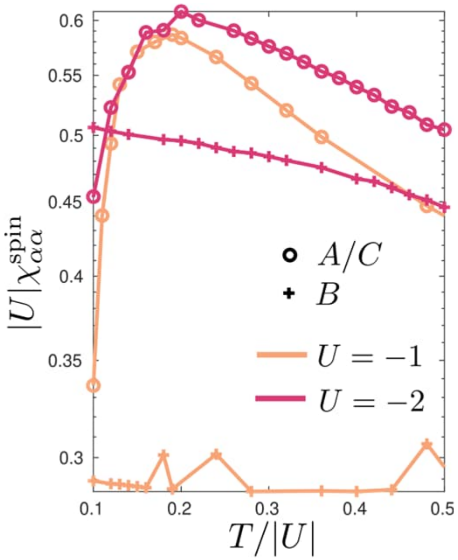


Single site DMFT

Cellular/cluster DMFT; Non-local correlations

Large (U>t) interactions: pseudogap





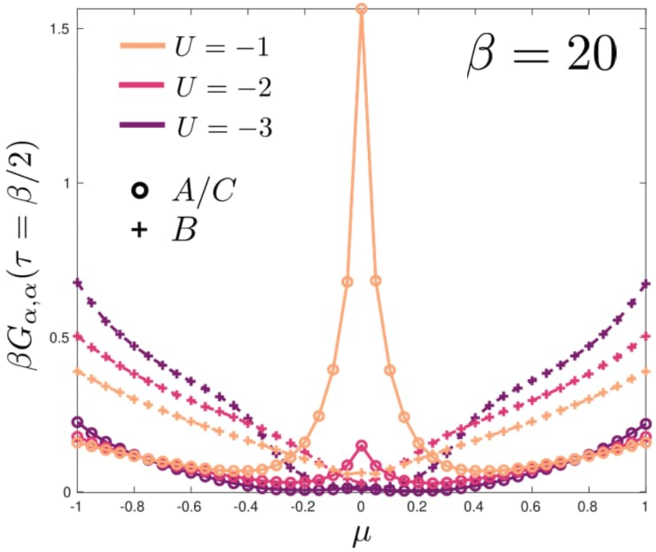
Generalized spin susceptibility:

$$\chi^{\text{spin}}_{\alpha\alpha} = \frac{2}{\beta^2}\sum_{\omega,\omega'}\left(\chi^{\text{ph},\omega,\omega',\nu=0}_{\uparrow\alpha,\uparrow\alpha,\uparrow\alpha,\uparrow\alpha} - \chi^{\text{ph},\omega,\omega',\nu=0}_{\uparrow\alpha,\uparrow\alpha,\downarrow\alpha,\downarrow\alpha}\right)$$

$$\chi_{ijkl}(\tau_1,\tau_2,\tau_3) = G^{(4)}_{ijkl}(\tau_1,\tau_2,\tau_3) - G_{ij}(\tau_1,\tau_2)G_{kl}(\tau_3,0)$$

$$G^{(4)}_{ijkl}(\tau_1,\tau_2,\tau_3) = \langle T_\tau[c_i^\dagger(\tau_1)c_j(\tau_2)c_k^\dagger(\tau_3)c_l(0)]\rangle$$

$$G_{ij}(\tau_1,\tau_2) = \langle T_\tau[c_i^\dagger(\tau_1)c_j(\tau_2)]\rangle$$

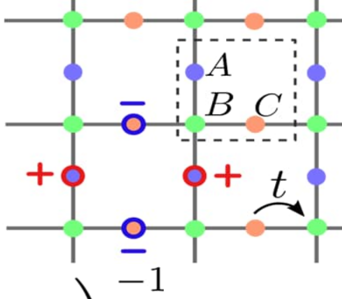


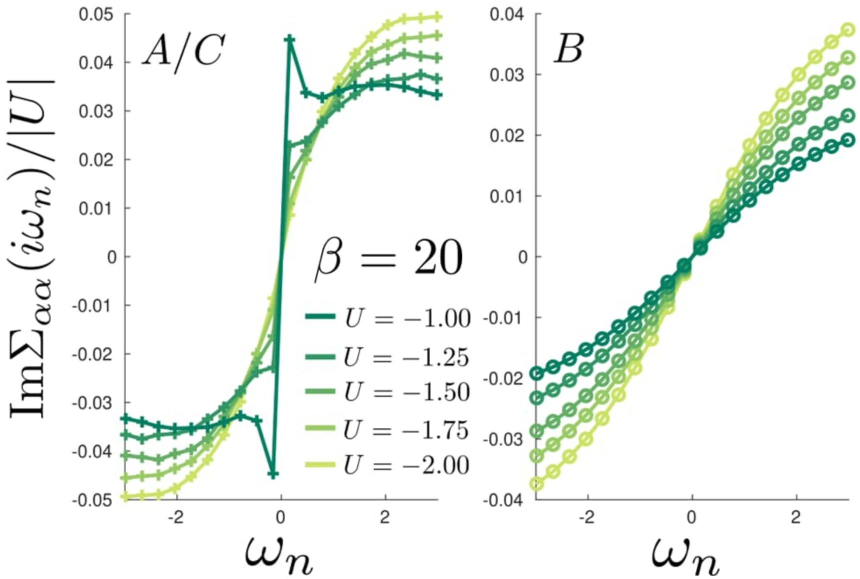
Local contribution to spin susceptibility decreases sharply with temperature at A/C sites.

At low temperatures, $\beta G_{\alpha\alpha}(\beta/2) \approx \mathcal{A}_\alpha(\omega=0)$, where $\mathcal{A}_\alpha$ is the orbital-resolved spectral function.

As interaction is increased, the spectral function becomes depleted around half-filling.

Low interaction (U<t): insulator

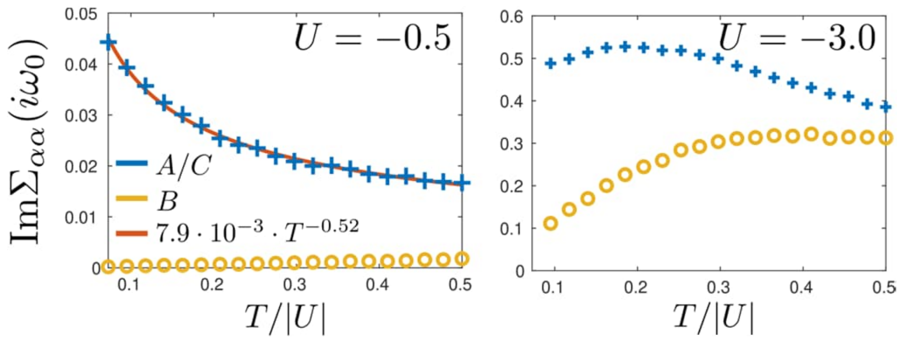




$$Z = \left(1 - \frac{\mathrm{Im}\Sigma(i\omega_n)}{\omega_n}\bigg|_{\omega_n \to 0}\right)^{-1}$$

In DMFT, $Z = m/m^*$, where m is the bare mass and m^* is the effective mass.

The self-energy diverges at low frequencies when the interaction strength is decreased.



The temperature dependence is $T^{-1/2}$ rather than T^{-1} found for Mott insulator.

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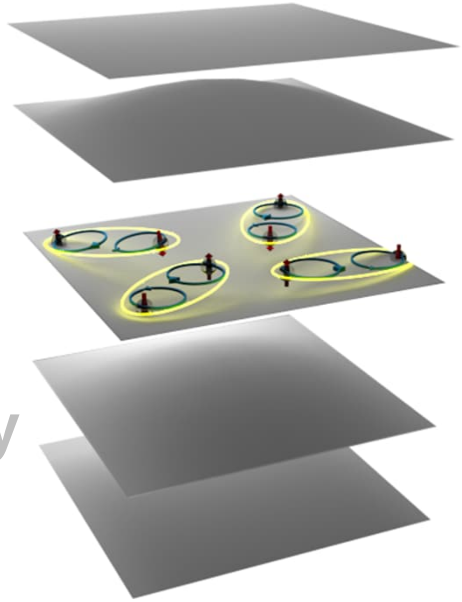

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SYNOPSIS

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Static Electrons in Flat-Band Nonequilibrium Superconductors

May 25, 2023 • *Physics* 16, s76

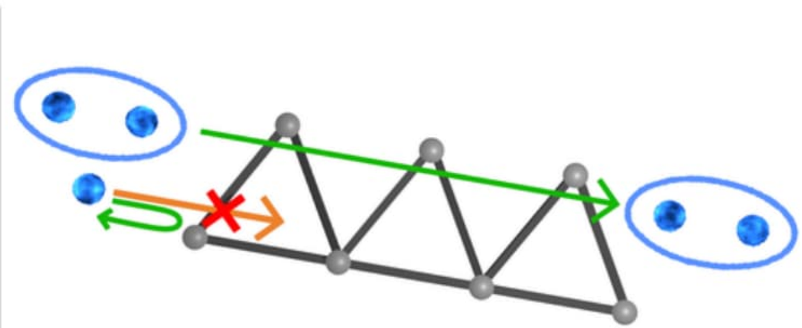
Single electrons stay stationary in superconductors with "flat-band" electronic structures, which could lead to low-energy-consumption devices made from such materials.

Suppression of Nonequilibrium Quasiparticle Transport in Flat-Band Superconductors

Ville A. J. Pyykkönen, Sebastiano Peotta, and Päivi Törmä

Phys. Rev. Lett. 130, 216003 (2023)

Published May 25, 2023



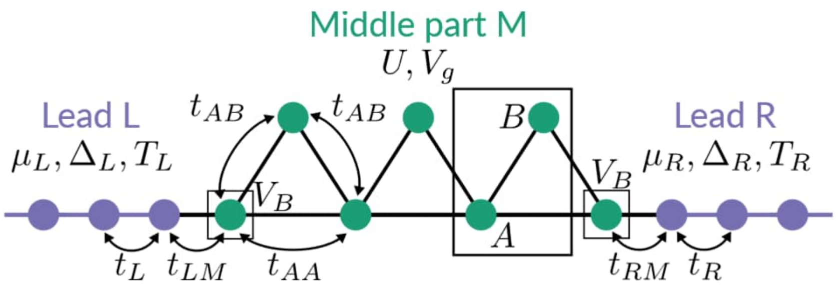
A. Paraoanu/Aalto University

Pyykkönen, Peotta, PT, PRL 2023
Editors' Suggestion

Ville Pyykkönen

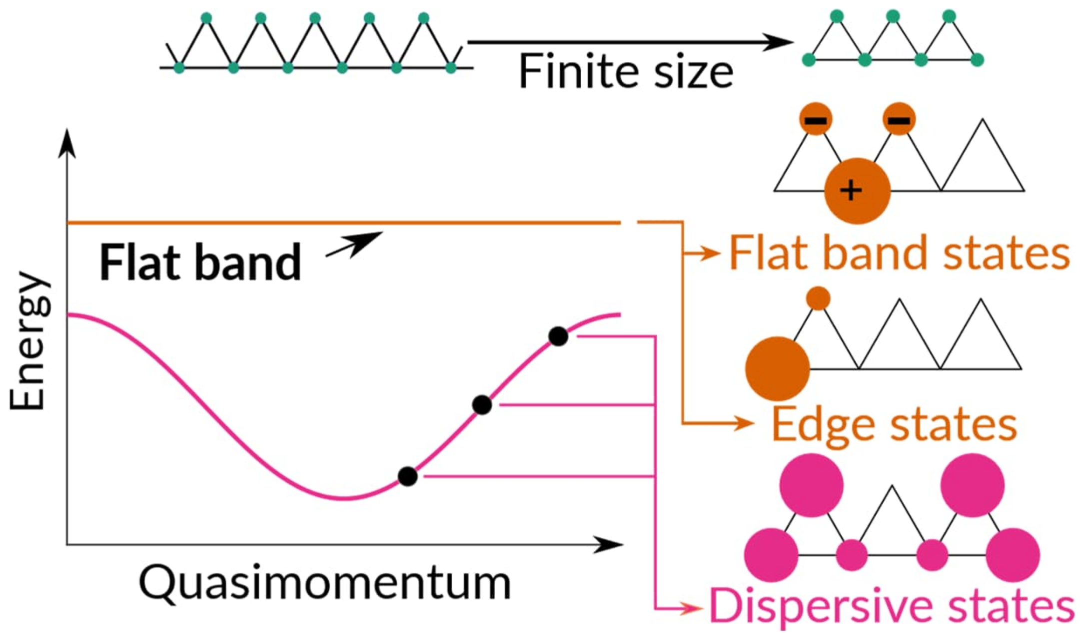
Sebastiano Peotta

Flat, edge and dispersive states in the sawtooth ladder

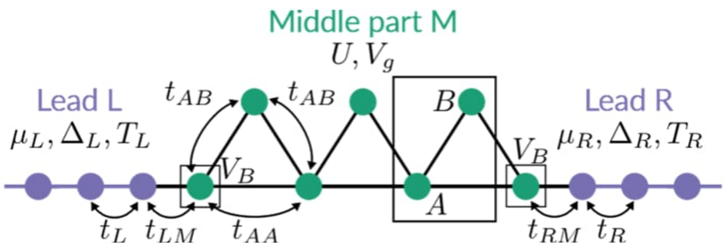


Select a state by gate potential

V_g



Flat band transport in Keldysh formalism



Fermi-Hubbard Hamiltonian

$$\hat{H} = \sum_{\alpha i, \beta j, \sigma} T_{\alpha i, \beta j} \hat{c}^{\dagger}_{\alpha i \sigma} \hat{c}_{\beta j \sigma} + \sum_{\alpha i} U_{\alpha i} \hat{c}^{\dagger}_{\alpha i \uparrow} \hat{c}^{\dagger}_{\alpha i \downarrow} \hat{c}_{\alpha i \downarrow} \hat{c}_{\alpha i \uparrow}$$

$$\alpha, \beta \in \{L, R, M\}$$

Mean-field approximation

$$\hat{H}_{\mathrm{MF}}(t) = \sum_{\alpha i, \beta j} \hat{d}^{\dagger}_{\alpha i} \begin{pmatrix} T_{\alpha i, \beta j} + V_{H, \alpha i}(t) \delta_{\alpha i, \beta j} & \Delta_{\alpha i} \delta_{\alpha i, \beta j} \\ \Delta^{*}_{\alpha i} \delta_{\alpha i, \beta j} & -T^{*}_{\alpha i, \beta j} - V_{H, \alpha i}(t) \delta_{\alpha i, \beta j} \end{pmatrix} \hat{d}_{\beta j}$$

$$\hat{d}_{\alpha i} = \left(\hat{c}_{\alpha i \uparrow}, \hat{c}^{\dagger}_{\alpha i \downarrow} \right)^{\mathrm{T}}$$

Superconducting order parameter

$$\Delta_{\alpha i}(t) = U_{\alpha i} \langle \hat{c}_{\alpha i \downarrow}(t) \hat{c}_{\alpha i \uparrow}(t) \rangle$$

Hartree potential

$$V_{H, \alpha i}(t) = U_{\alpha i} \langle \hat{c}^{\dagger}_{\alpha i \uparrow}(t) \hat{c}_{\alpha i \uparrow}(t) \rangle$$

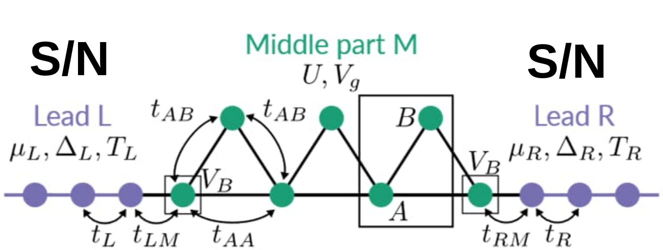
Keldysh formalism, non-equilibrium Green's functions

Dyson equation $G^{R/A}(\omega) = g^{R/A}(\omega) + g^{R/A}(\omega) \Sigma^{R/A}(\omega) G^{R/A}(\omega)$

Kadanoff-Baym kinetic equation

$$G^{<}(\omega) = \left[I + G^{R}(\omega) \Sigma^{R}(\omega) \right] g^{<}(\omega) \left[I + \Sigma^{A}(\omega) G^{A}(\omega) \right] + G^{R}(\omega) \Sigma^{<}(\omega) G^{A}(\omega)$$

Transport



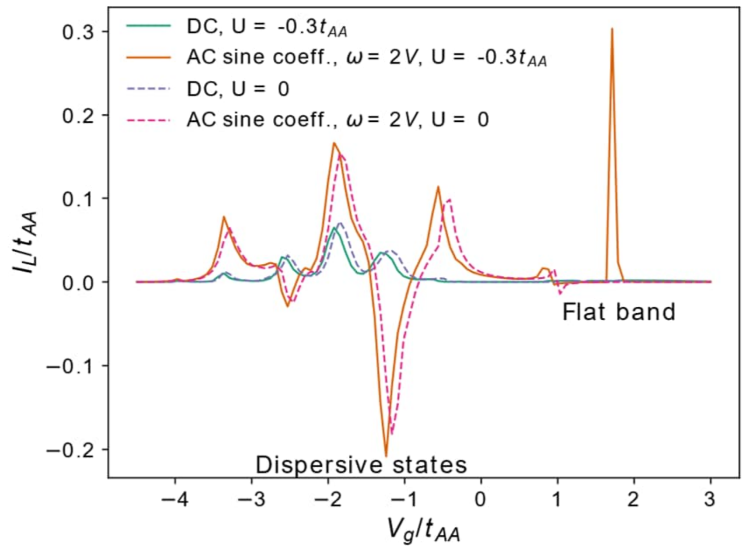
Constant bias

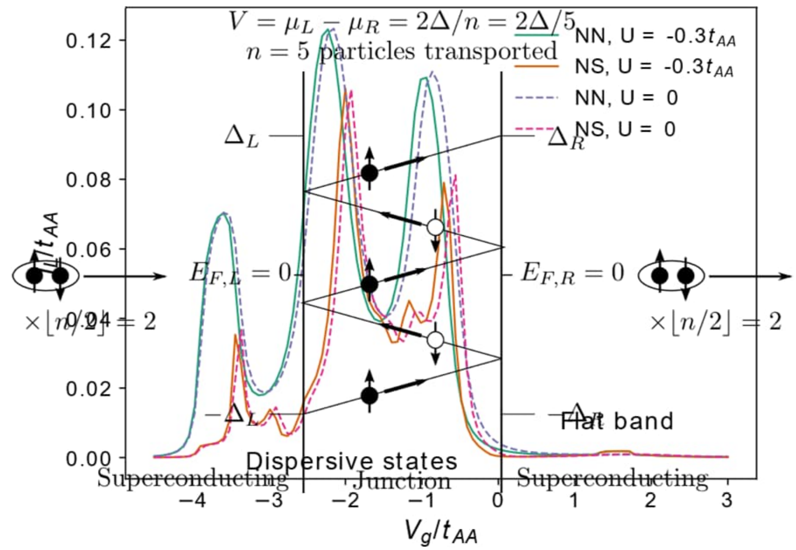
$$V = \mu_L - \mu_R$$

Vary gate potential

$$V_g$$

Current $$I_L(t) = 4\mathrm{Im}\left(t_{ML}\left\langle \hat{c}^\dagger_{M,\text{left edge},\uparrow}(t)\hat{c}_{L,0\uparrow}(t)\right\rangle\right)$$





Superconducting junction: at finite interaction **flat band AC Josephson current is finite** but **DC current (multiple Andreev reflections) quenched**

Normal-normal and normal-superconducting junction: **flat band current is quenched**

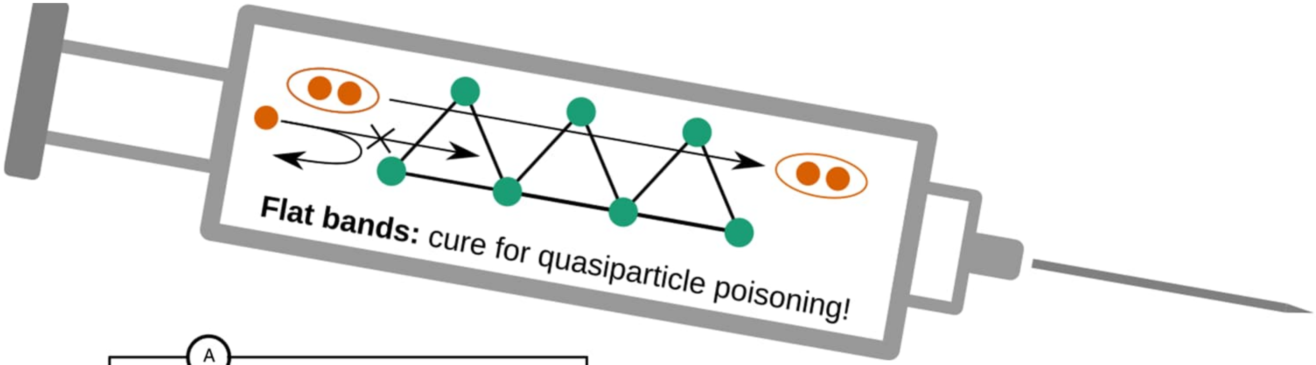
Quasiparticle transport quenched at flat band! Pure supercurrent!

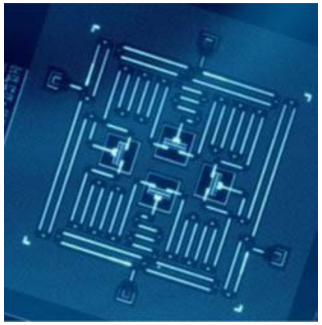
Quasiparticle transport quenched at flat band! Pure supercurrent!

Quasiparticle poisoning

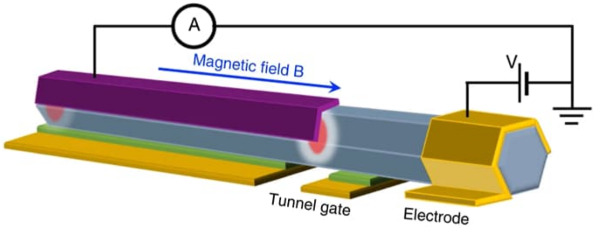
Nonequilibrium Quasiparticles and 2*e* Periodicity in Single-Cooper-Pair Transistors

J. Aumentado, Mark W. Keller, John M. Martinis, and M. H. Devoret
Phys. Rev. Lett. **92**, 066802 – Published 13 February 2004





Four transmons. FJ.M. Gambetta, J.M. Chow, and M. Steffen (npj QuantumInformation 3:2, 2017) by CC BY 4.0 license



Majorana nanowire. H. Zhang, D.E. Liu, M. Wimmer, L.P. Kouwenhoven (Nat Commun 10, 5128, 2019) by CC BY 4.0 license

G. Catelani and J. P. Pekola, Using materials for quasiparticle engineering, Materials for Quantum Technology 2, 013001 (2022)

D. Rainis and D. Loss, Majorana qubit decoherence by quasiparticle poisoning, Phys. Rev. B 85, 174533 (2012)

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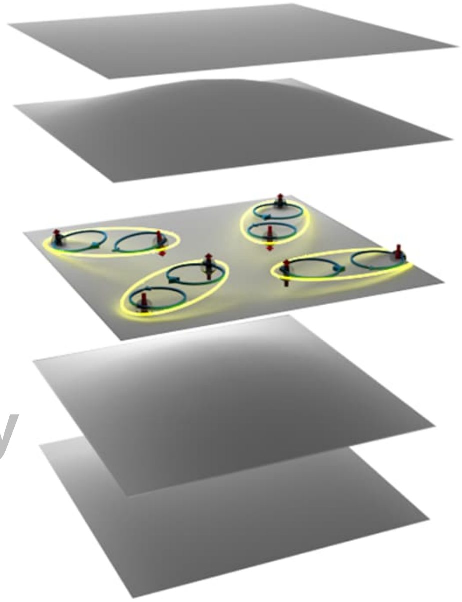


Lecture 1

- Basics of quantum geometry
- Quantum geometry and superconductivity

Lecture 2

- Flat band superconductivity and quantum geometry in twisted bilayer graphene (TBG)
- Non-Fermi liquid normal states in flat bands
- Non-equilibrium transport in flat band superconductors
- **DC conductivity in a flat band**
- The many-body quantum metric and the Drude weight

Conductivity in a flat band

Kukka-Emilia Huhtinen

KE Huhtinen, PT, PRB (2023)

Conductivity in a flat band

Semiclassical Boltzmann theory of transport:

$$\sigma_{\mu\nu}(\omega) = -\frac{e^2}{\hbar}\sum_n \int_{\mathrm{B.z.}} \frac{d^D\boldsymbol{k}}{(2\pi)^D} \frac{\partial n_F(E)}{\partial E}\bigg|_{E=\epsilon_n(\boldsymbol{k})} \partial_\mu \epsilon_n(\boldsymbol{k}) \partial_\nu \epsilon_n(\boldsymbol{k}) \frac{\eta}{(\hbar\omega)^2 + \eta^2} \qquad \partial_\mu = \partial/\partial k_\mu$$

Full Kubo-Greenwood formula:

$$\sigma_{\mu\nu}(\omega) = \frac{e^2}{i\hbar V}\sum_{\boldsymbol{k}}\sum_{mn} \frac{n_F(\epsilon_n(\boldsymbol{k})) - n_F(\epsilon_m(\boldsymbol{k}))}{\epsilon_n(\boldsymbol{k}) - \epsilon_m(\boldsymbol{k})} \frac{[j_\mu(\boldsymbol{k})]_{nm}[j_\nu(\boldsymbol{k})]_{mn}}{\epsilon_n(\boldsymbol{k}) - \epsilon_m(\boldsymbol{k}) + \hbar\omega + i\eta}$$

$$[j_\mu(\boldsymbol{k})]_{mn} = \partial_\mu \epsilon_m(\boldsymbol{k})\delta_{mn} + (\epsilon_m(\boldsymbol{k}) - \epsilon_n(\boldsymbol{k}))\langle \partial_\mu m_{\boldsymbol{k}} | n_{\boldsymbol{k}} \rangle$$

At low temperatures and finite scattering rate η , the interband geometric part is dominant on a flat band.

Insprired by

G. Bouzerar and D. Mayou, Phys. Rev. B 103, 075415 (2021)
J. Mitscherling and T. Holder, Phys. Rev. B 105, 08515 (2022)
B. Mera and J. Mitscherling, Phys. Rev. B 106, 165133 (2022)
G. Bouzerar, Phys. Rev. B 106, 125125 (2022)

Conductivity in a flat band

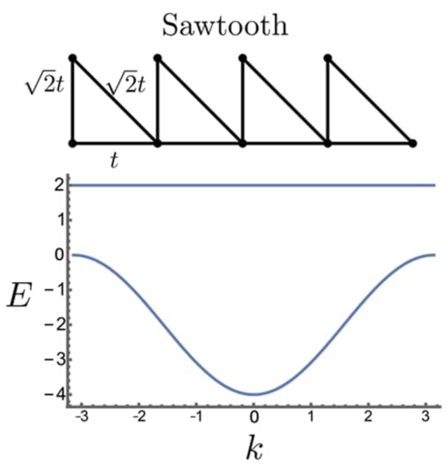
Streda formula:

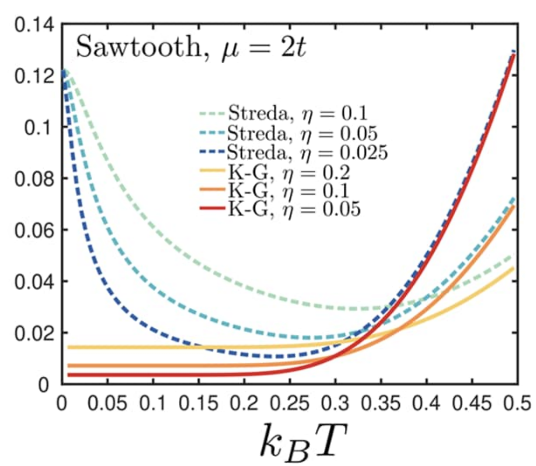
$$\sigma_{\mu\nu}^{\text{sym}}(\omega=0) = -\frac{e^2}{\hbar\pi}\int_{-\infty}^{\infty} \mathrm{d}\epsilon \frac{\partial n_F(\epsilon)}{\partial\epsilon}\mathrm{Tr}[\mathrm{Im}[G_{\boldsymbol{k}}(\epsilon+i\eta)]j_\mu(\boldsymbol{k})\mathrm{Im}[G_{\boldsymbol{k}}(\epsilon+i\eta)]j_\nu(\boldsymbol{k})]$$

$$G_{\boldsymbol{k}}(E) = (E - H_{\boldsymbol{k}})^{-1}$$

This gives a result proportional to the integrated quantum metric in the limit $\eta \to 0^+$ when $T \to 0$ is taken *first.*

This occurs only in *perfectly* (partially) flat bands due to ill-defined terms for states at the Fermi energy. **The Kubo-Greenwood and Streda formulas do not give the same conductivity when a flat band is in the vicinity of the Fermi energy.**





Lack of Fermi surface requires extra care in transport calculations.

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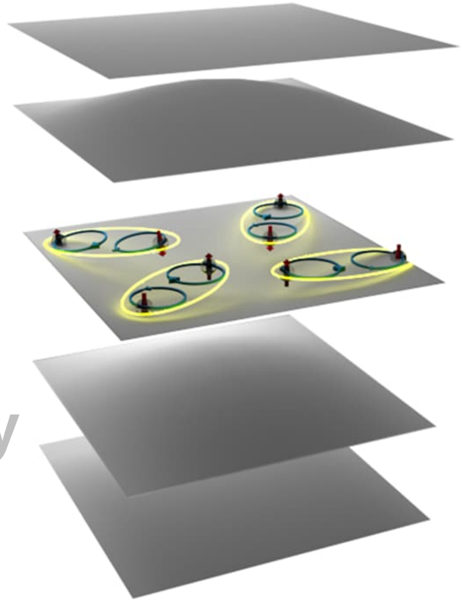

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Drude weight and the many-body quantum metric

Grazia Salerno

Tomoki Ozawa

Salerno, Ozawa, PT, PRB Letter (2023)

The many-body quantum metric (MBQM)

Defined on many-body states with respect to the twisted boundary condition phase

$$\mathfrak{g}(\phi) = \mathrm{Re}\left[\langle \partial_\phi \Psi_0 | \left(1 - |\Psi_0\rangle\langle\Psi_0|\right) |\partial_\phi \Psi_0\rangle\right]$$

determines the ‘‘quantum distance’’ along a given path in ϕ space.

Many-body generalization of the quantum metric

$$\mathfrak{g}(0) = \mathrm{Re}\left[\sum_{m\neq 0} \frac{|\langle\Psi_m|\partial_\phi \hat{H}(\phi)|\Psi_0\rangle|^2}{(E_m(0) - E_0(0))^2}\right]$$

The many-body quantum metric (MBQM)

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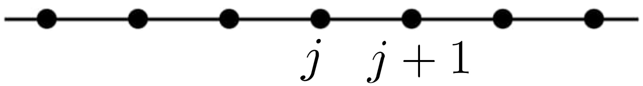
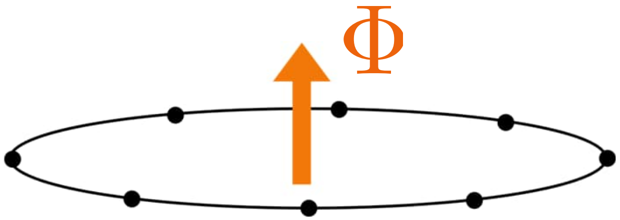
→ Many-body generalization of the quantum metric

$$\mathfrak{g}(0) = \mathrm{Re}\left[\sum_{m\neq 0} \frac{|\langle \Psi_m | \partial_\phi \hat{H}(\phi) | \Psi_0\rangle|^2}{(E_m(0) - E_0(0))^2}\right]$$

$$\partial_\phi \hat{H}(\phi) = L\partial_\Phi \hat{H}(\Phi) = \hat{J} + \mathcal{O}(\Phi)$$

Drude weight and twisted boundary conditions





$$\hat{H}_0 = \hat{H}_{\mathrm{kin}} + \hat{H}_V + \hat{H}_U = (\hat{K} + \hat{K}^\dagger) + \hat{H}_V + \hat{H}_U$$

Superfluid response of the system to a small external flux Φ introduced by the twisted boundary conditions:

$$D_w = \pi L \left. \frac{\partial^2 E(\Phi)}{\partial \Phi^2} \right|_{\Phi=0}$$

$$\hat{H}(\Phi) = \hat{K} e^{i\Phi/L} + \hat{K}^\dagger e^{-i\Phi/L} + \hat{H}_V + \hat{H}_U$$

Drude weight within perturbation theory

$$\hat{H}(\Phi) = \hat{H}_0 + \hat{H}_{\text{pert}} \qquad \text{with} \qquad \hat{H}_{\text{pert}} = \frac{\Phi}{L}\hat{J} - \frac{1}{2}\left(\frac{\Phi}{L}\right)^2 \hat{H}_{\text{kin}}$$

Current operator $\hat{J} = i(\hat{K} - \hat{K}^\dagger)$

$$D_w = 2\pi L \frac{E(\Phi) - E(0)}{\Phi^2} = -\frac{\pi}{L}\langle\Psi_0|\hat{H}_{\text{kin}}|\Psi_0\rangle - \underbrace{\frac{2\pi}{L}\sum_{m\neq 0}\frac{|\langle\Psi_m|\hat{J}|\Psi_0\rangle|^2}{E_m(0) - E_0(0)}}$$

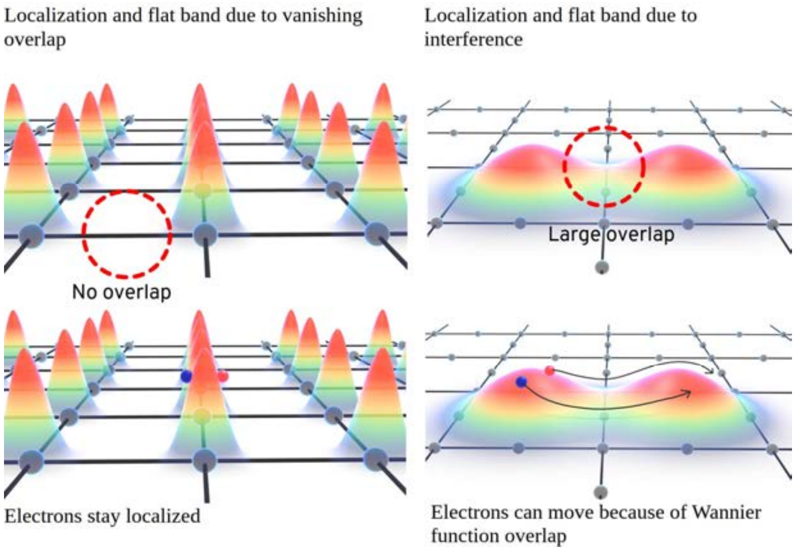
$$> \mathfrak{g}(0) \cdot \varepsilon$$

Can be bounded by the **many-body quantum metric**
if the system has a gap ε

Independent of particle statistics and spatial dimensions!

Summary

Quantum geometry is relevant for any transport or interaction phenomena where overlaps and localization properties of Wannier functions are important – a new viewpoint to condensed matter physics: not only the band structure, but the structure of the Bloch functions



Outlook

Superconductivity at elevated temperatures

Aalto University
School of Science

Super C
Room Temperature
Superconductivity 2033

Institute Q

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HORIZON2020
Marie Skłodowska-
Curie actions

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