MULTI-COMPONENT SUPERCONDUCTIVITY: CHIRAL, NEMATIC, AND CHARGE-4e SUPERCONDUCTING STATES

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References for these lectures

- *Phenomenological superconductivity (review + intro):*

  Sigrist & Ueda, Rev. Mod. Phys. ***63****, 239 (1991)*
  Sigrist, AIP Conf. Proc. ***789****, 165 (2005)*

- *Vestigial orders (review):*

  RMF, Orth, & Schmalian, Annu. Rev. Condens. Matter Phys. ***10****,133 (2019)*

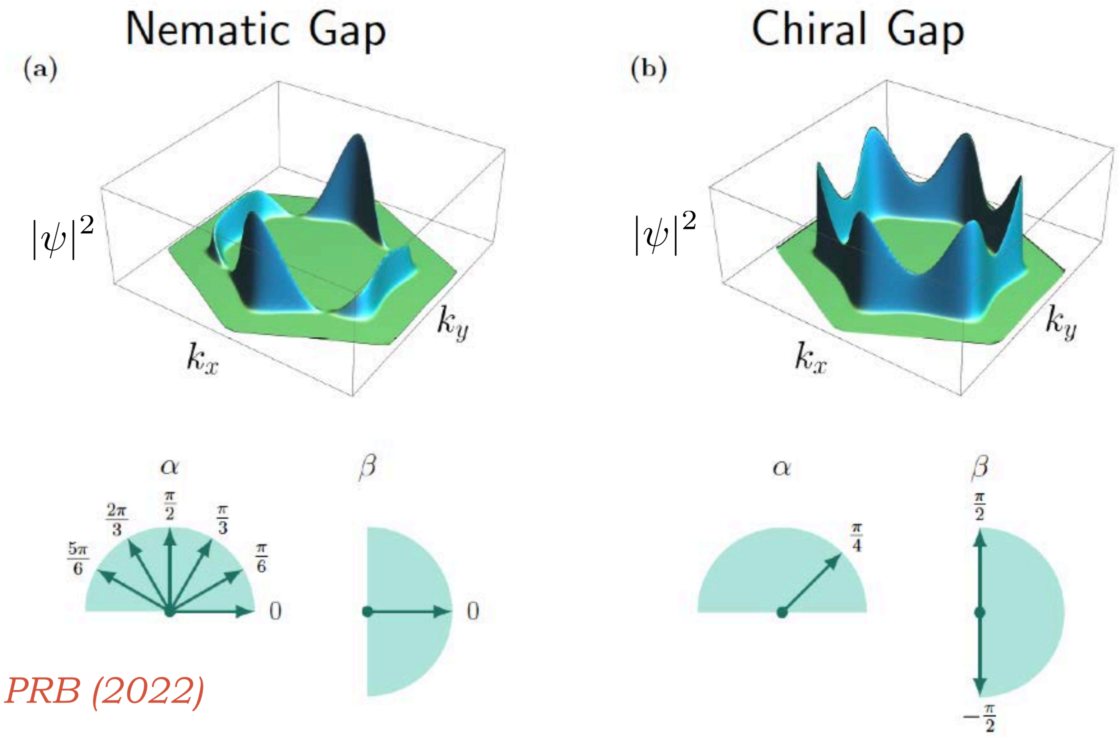
- *Recent results discussed in these lectures:*

  RMF & Fu, Phys. Rev. Lett. ***127****, 047001 (2021)*
  Gali & RMF, Phys. Rev. B ***106****, 094509 (2022)*
  Hecker, Willa, Schmalian, & RMF, Phys. Rev. B ***107****, 224503 (2023)*

Nematic and chiral superconductivity

- Two-component order parameter $\left(d_{x^2-y^2}, d_{xy}\right)$

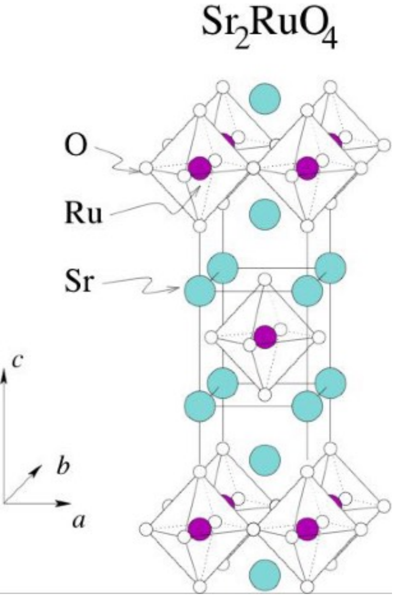
$$\psi = (\psi_1, \psi_2) =$$
$$|\psi| e^{i\varphi} \left(\cos\alpha, e^{i\beta} \sin\alpha\right)$$

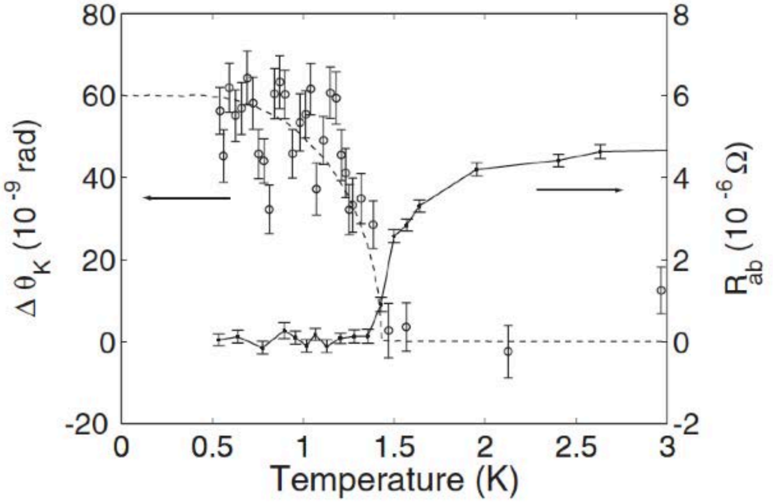


Gali & RMF, PRB (2022)

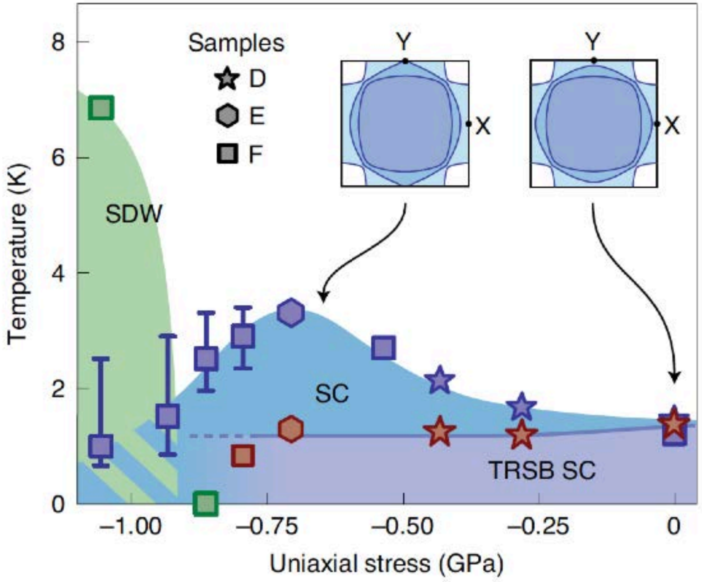
Experimental candidates: chiral SC

- Sr_2RuO_4 (tetragonal): time-reversal symmetry-breaking (TRSB) observed below T_c via Kerr and μSR. Tunable by uniaxial stress.





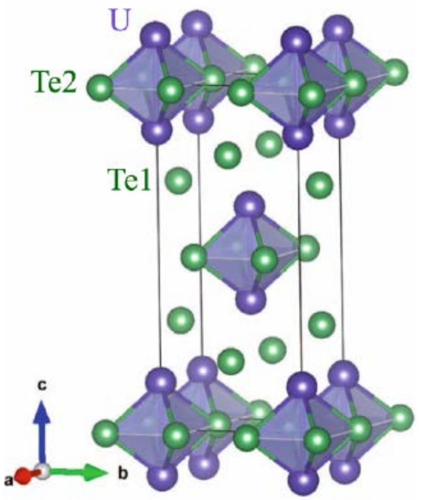
Xia et al, PRL (2006)

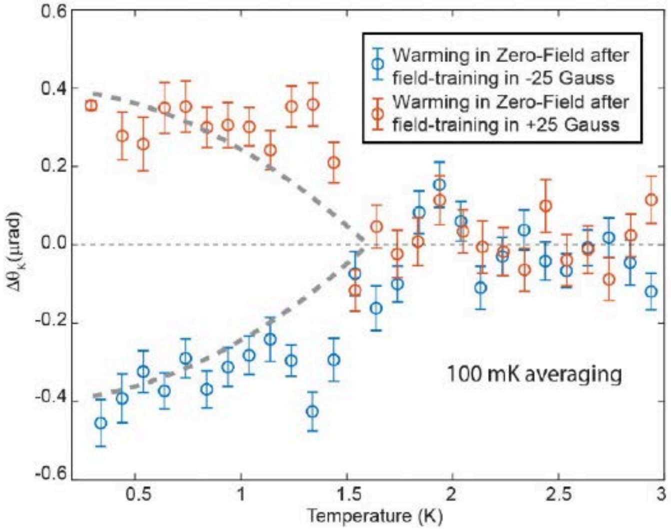


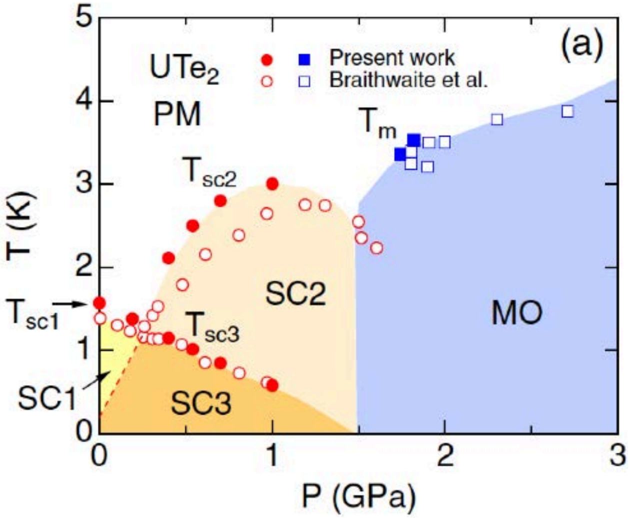
Grinenko et al, Nat Phys (2021)

Experimental candidates: chiral SC

- UTe_2 (orthorhombic): TRSB seen by Kerr (under debate). Thermodynamic evidence for two SC transitions as pressure increases.







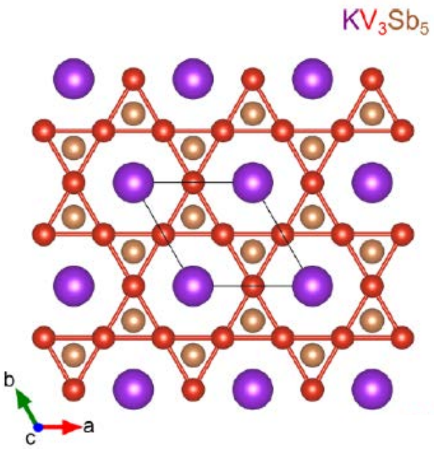
Hayes et al, Science (2021)

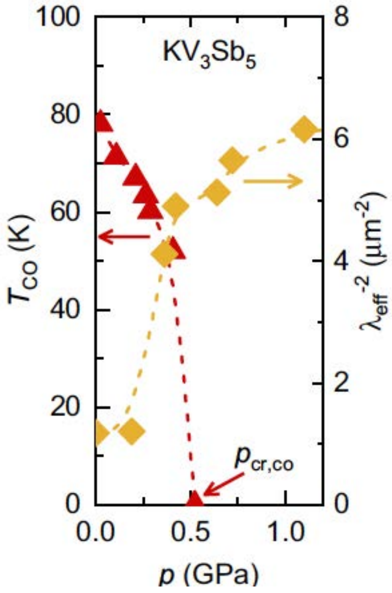
see also: Ajeesh et al, PRX (2023)

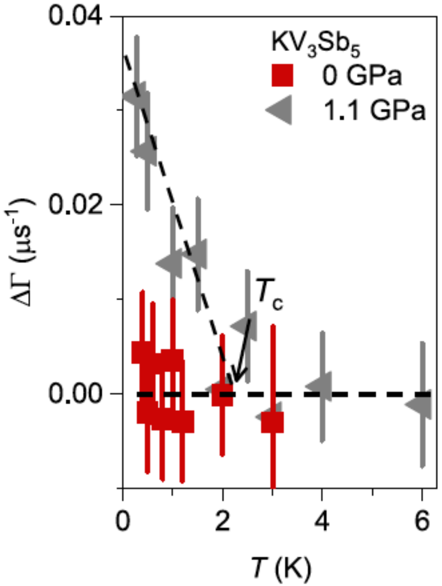
Aoki et al, JPSP (2020)

Experimental candidates: chiral SC

- KV_3Sb_5 under pressure (hexagonal): TRSB seen by μSR.



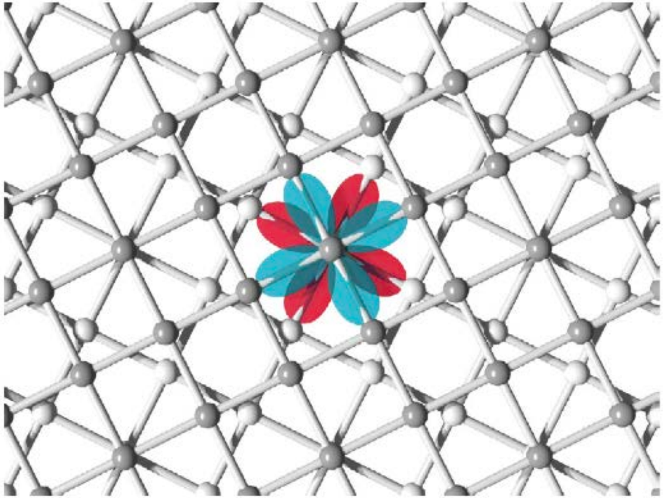


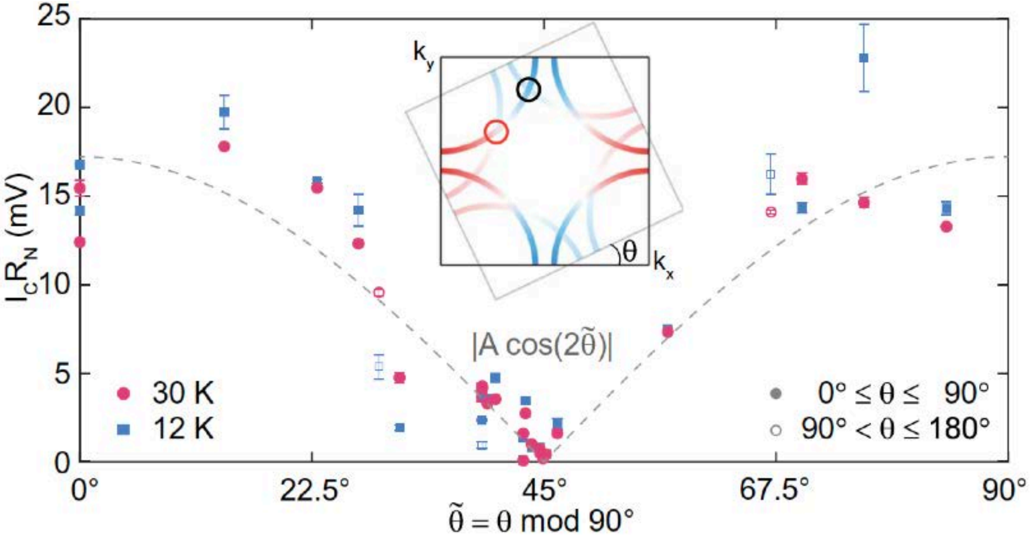


Guguchia et al, Nat Comm (2023)

Experimental candidates: chiral SC

- Twisted $Bi_2Sr_2CaCu_2O_{8+\delta}$ (tetragonal): possible TRSB observed.

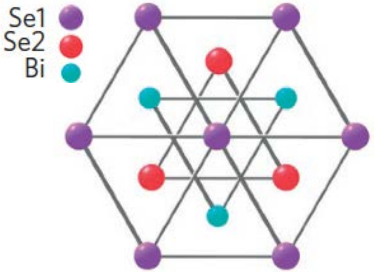


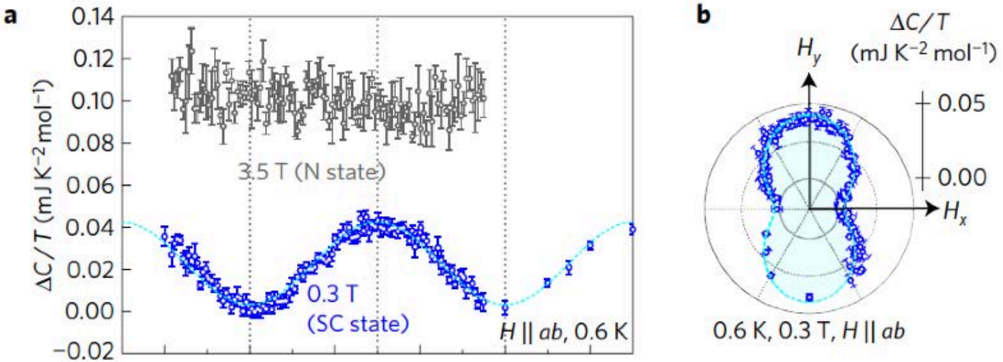


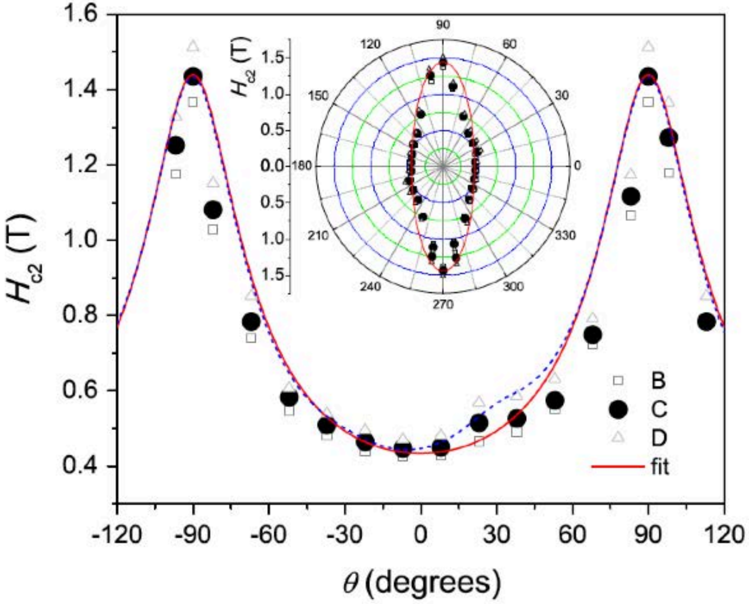
Zhao et al, Science (2023)

Experimental candidates: nematic SC

- Doped Bi_2Se_3 (trigonal): three-fold rotational symmetry breaking observed via H_{c2}, angle-dependent specific heat, and other probes.





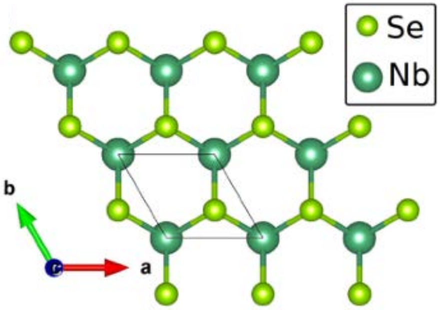


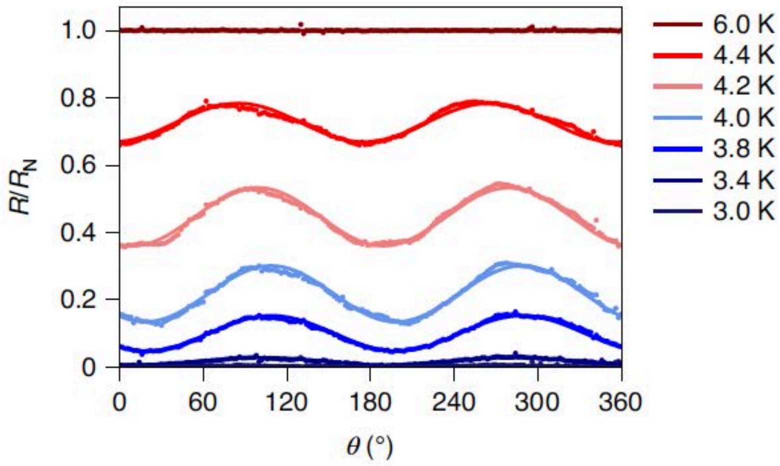
Yonezawa et al, Nat Phys (2016)

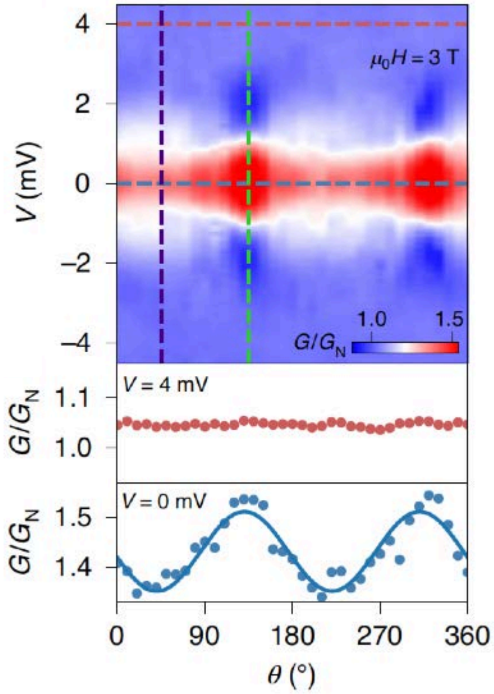
Shen et al, npj Q Mater (2017)

Experimental candidates: nematic SC

- few-layer $NbSe_2$ (hexagonal): three-fold rotational symmetry breaking observed via magneto-resistance, H_{c2} and tunneling data.



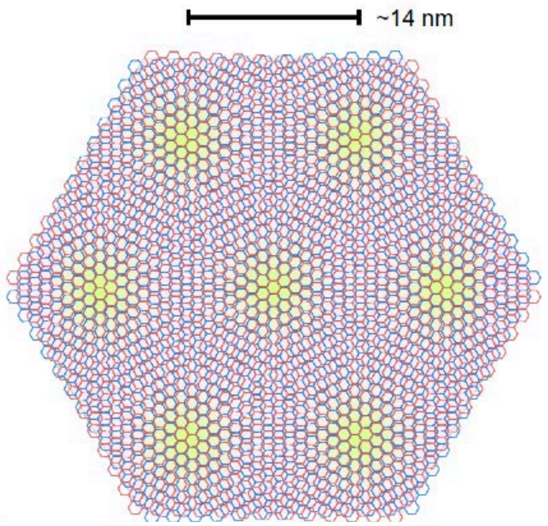


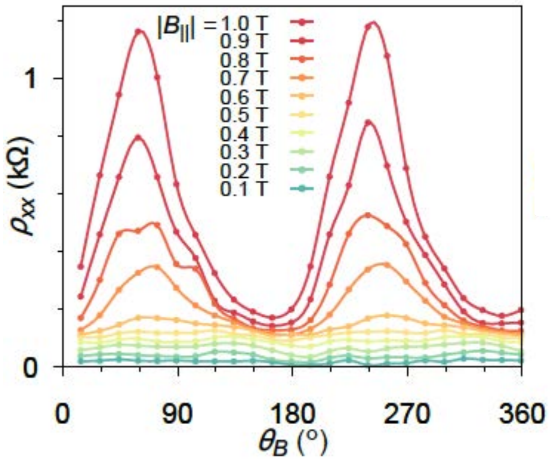


Hamill et al, Nat Phys (2021)
Cho et al, PRL (2022)

Experimental candidates: nematic SC

- Twisted bilayer graphene (hexagonal): three-fold rotational symmetry breaking observed in H_{c2} measurements.

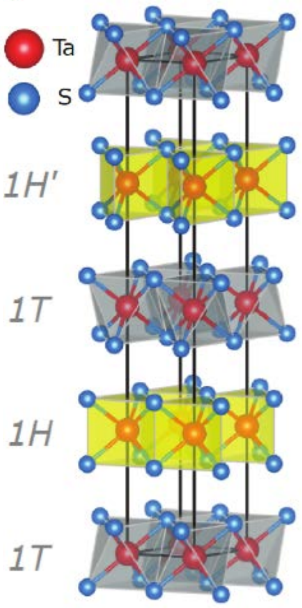


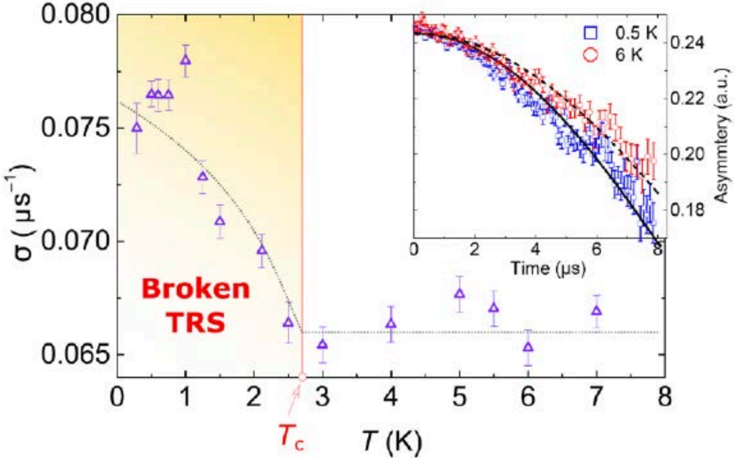


Cao et al, Science (2021)

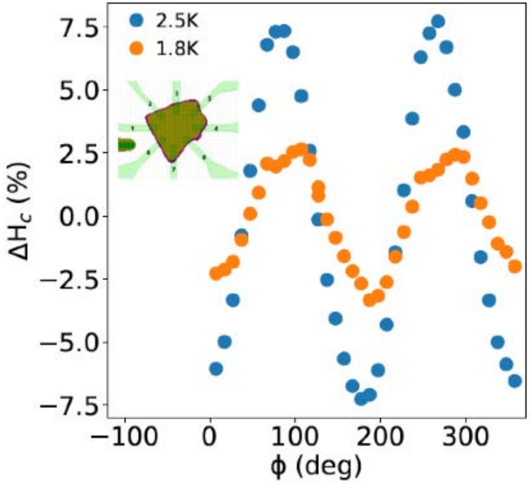
Experimental candidates: nematic + chiral SC

- $4Hb\text{-}TaS_2$ (hexagonal): both three-fold rotational symmetry breaking *and* TRSB are observed (H_{c2} and μSR).





Ribak et al, Sci Adv (2020)



Silber et al, arxiv (2022)

Experimental candidates: challenges

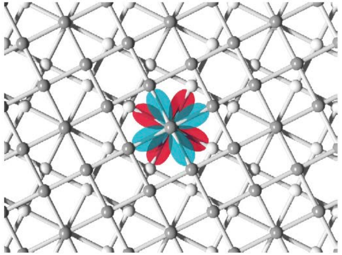
- Are the observed effects extrinsic or intrinsic?
  - ➢Particularly important in the case of nematic SC due to the ubiquitous presence of random strain.

- Why experiments usually observe a single nematic or chiral domain?

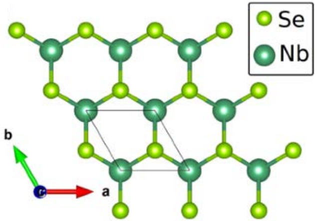
- Do these examples of multi-component superconductivity arise from accidental or symmetry-enforced degeneracies?

- Where to look for *symmetry-enforced* multi-component superconductivity?
- Materials described by non-Abelian point groups, since they have multi-dimensional irreducible representations: cubic > hexagonal > tetragonal.

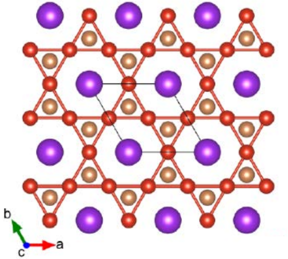
twisted BSCOO



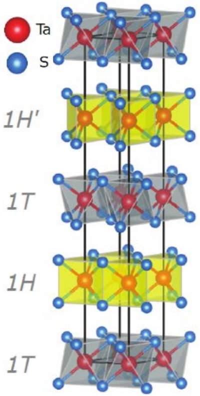
$NbSe_2$



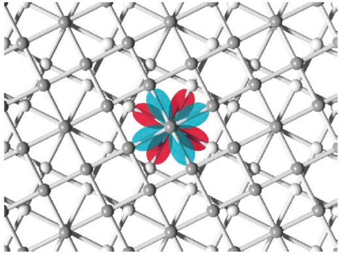
KV_3Sb_5



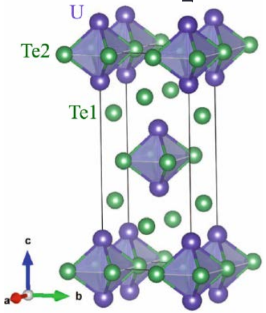
4Hb-TaS_2



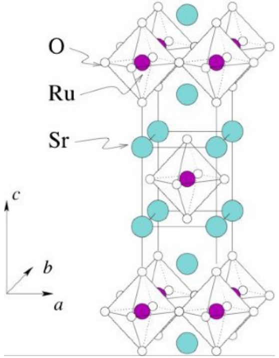
twisted BSCOO



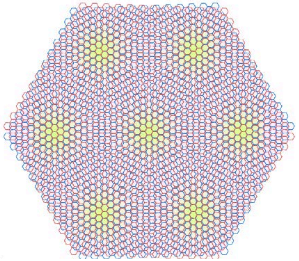
UTe_2



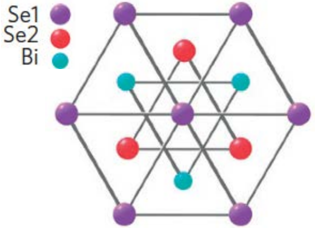
Sr_2RuO_4



MA-TBG

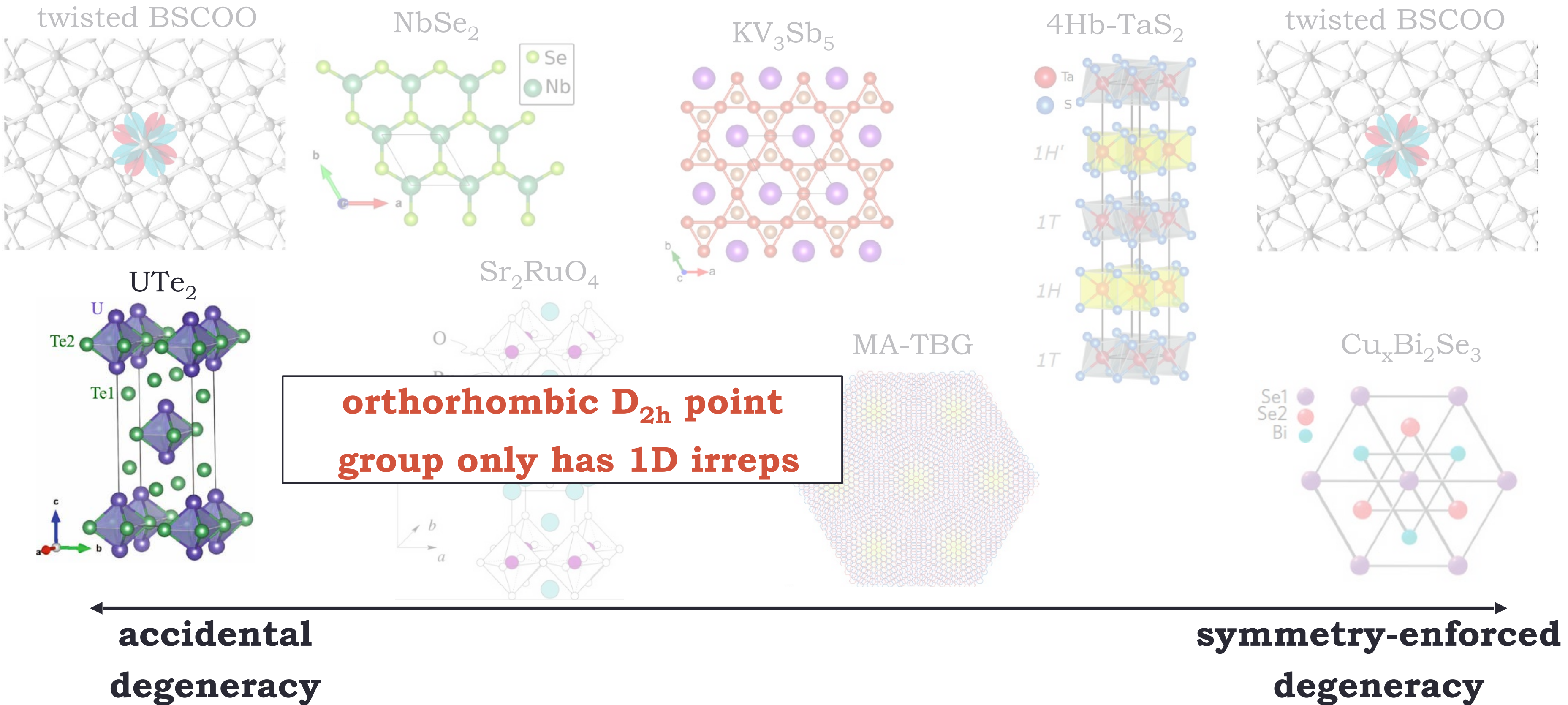


$Cu_xBi_2Se_3$



accidental degeneracy ⟷ **symmetry-enforced degeneracy**

- Where to look for *symmetry-enforced* multi-component superconductivity?
- Materials described by non-Abelian point groups, since they have multi-dimensional irreducible representations: cubic > hexagonal > tetragonal.



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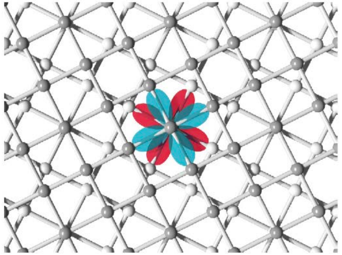
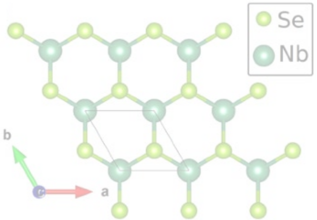
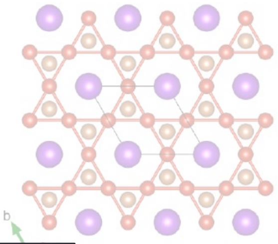


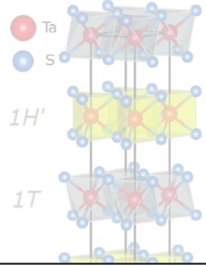
$NbSe_2$



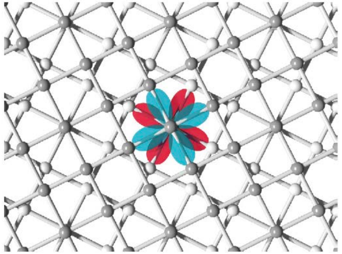
KV_3Sb_5

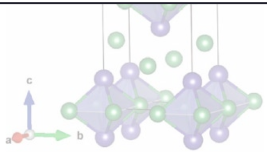


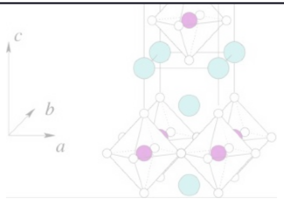
4Hb-TaS_2

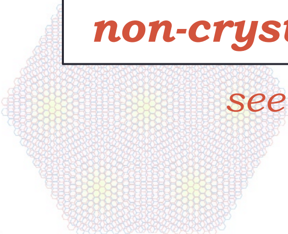


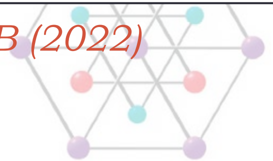
twisted BSCOO











d_{xy} and $d_{x2\text{-}y2}$ gaps belong to different 1D irreps of the crystallographic point group D_{4h}

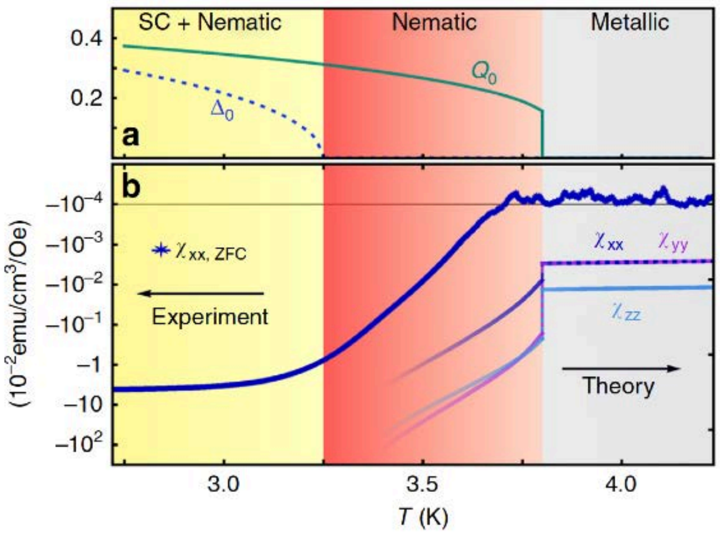
d_{xy} and $d_{x2\text{-}y2}$ gaps belong to the same 2D irrep of the *non-crystallographic* point group D_{4d}

see Haenel et al, PRB (2022)

accidental degeneracy ⟷ **symmetry-enforced degeneracy**

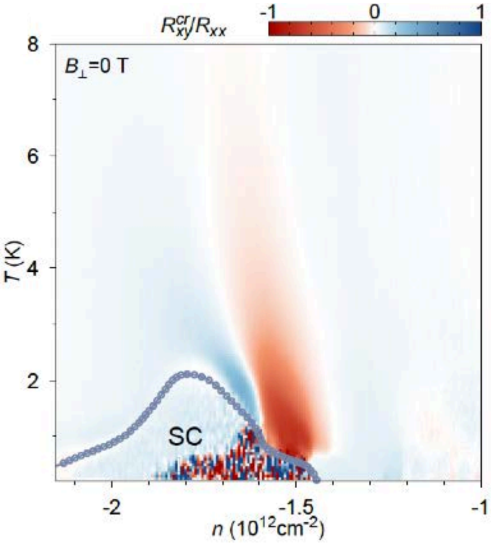
Vestigial superconducting phases: candidates

$Cu_xBi_2Se_3$



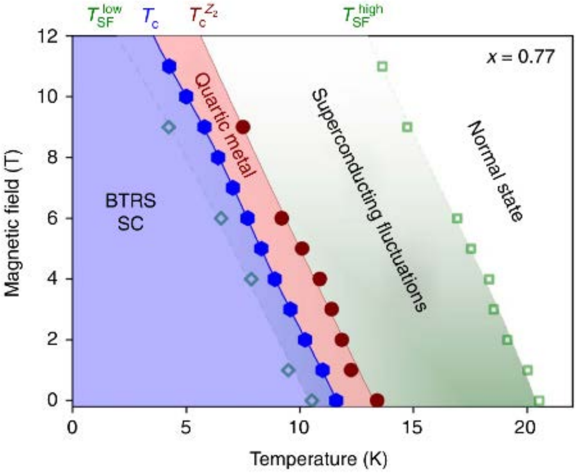
Cho et al, Nat Comm (2020)

MA-TBG



Cao et al, Science (2021)

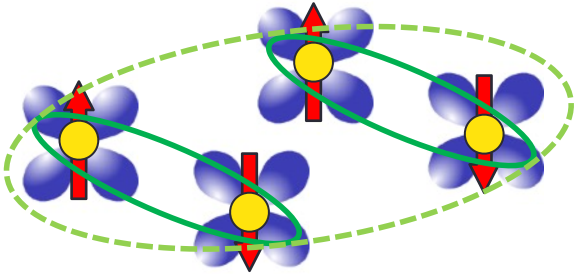
$Ba_{1-x}K_xFe_2As_2$



Grinenko et al, Nat Phys (2021)

Charge-4e superconductivity: theory

- Can electrons form quartets?



a pair of pairs

$$\Delta^{4e} \sim \langle \psi\psi\psi\psi \rangle$$

$$\Delta \to e^{i\theta}\Delta\,, \quad \Delta^{4e} \to e^{2i\theta}\Delta^{4e}$$

$$U(1) \longrightarrow Z_2 : \{1, e^{i\pi}\}$$

residual Z_2 gauge symmetry

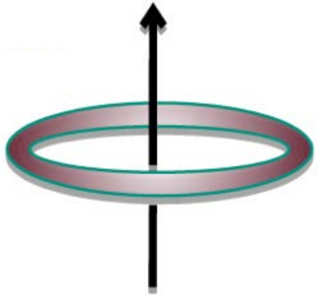
Nozières & Saint James, J. Phys. (1982)

Korshunov, Zh. Eksp. Teor. Fiz (1985)

- Fingerprint of charge-4e order: half flux-quantum.

Berg et al, Nature Phys (2009)

$$\frac{h}{4e} = \frac{1}{2}\Phi_0$$

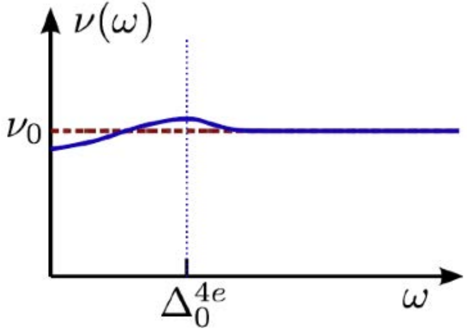


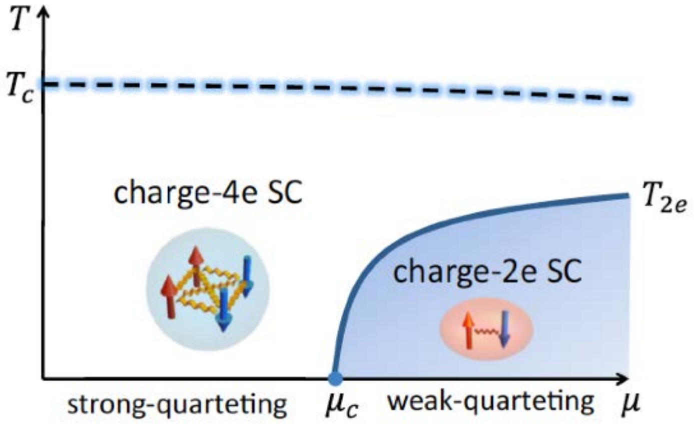
Charge-4e superconductivity: theory

- Properties of the charge-4e superconducting state.
  - ➢Challenging even at mean-field: charge-4e order parameter acts as a 4-fermion effective interaction.

Jiang et al, PRB (2017)

Gnezdilov & Wang, PRB (2022)





- Gapless state with small superfluid density.

where to look for charge-4e SC?

Multi-component SC: vestigial orders

- To capture vestigial orders, we need a method that accounts for fluctuations and goes beyond mean-field: large-N, RG, variational…

- Here we use the ***Gaussian variational approach***, as it allows us to compare different vestigial instabilities.

Fisher & Berg, PRB (2016)
Nie et al, PRB (2017)

Ansatz
$$\mathcal{S}_0 = \frac{1}{2}\hat{\boldsymbol{\Delta}}^\dagger \begin{pmatrix} R_0+\Phi^{B_{1g}} & \Phi^{B_{2g}}-\mathrm{i}\Phi^{A_{2g}} & \phi^{A_{1g}}+\phi^{B_{1g}} & \phi^{B_{2g}} \\ & R_0-\Phi^{B_{1g}} & \phi^{B_{2g}} & \phi^{A_{1g}}-\phi^{B_{1g}} \\ & & R_0+\Phi^{B_{1g}} & \Phi^{B_{2g}}+\mathrm{i}\Phi^{A_{2g}} \\ \text{H.c.} & & & R_0-\Phi^{B_{1g}} \end{pmatrix} \hat{\boldsymbol{\Delta}} + \mathcal{S}_{\text{grad}}$$

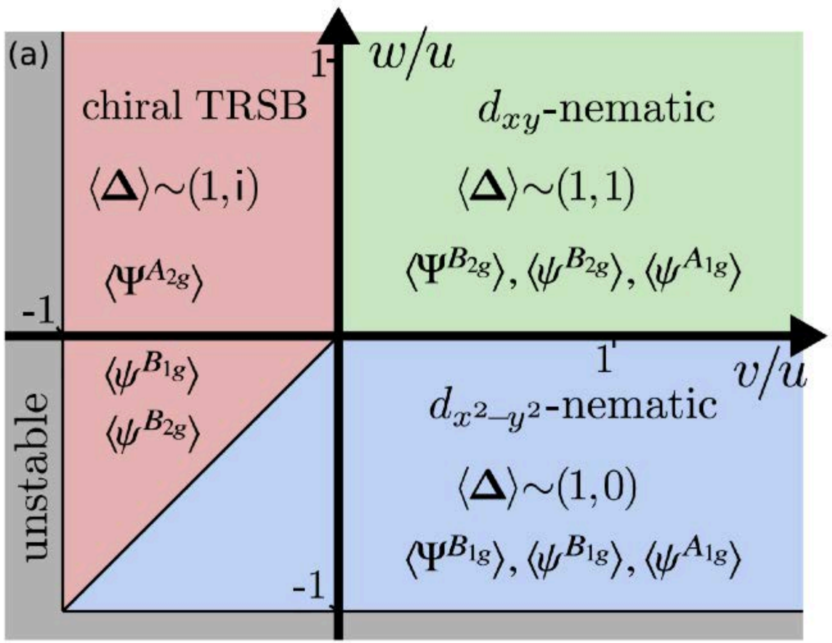
Variational free energy
$$F_v = F_0 + T\langle \mathcal{S} - \mathcal{S}_0 \rangle_0 \quad \leq F$$

Gaussian integrals

Hecker, Willa, Schmalian, & RMF, PRB (2023)

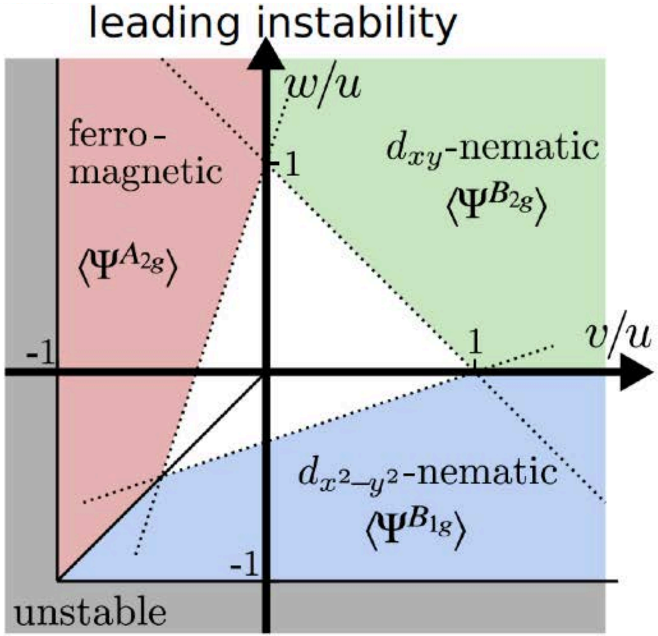
Multi-component SC: tetragonal lattice

- Each SC ground state is compatible with three non-zero bilinears: one real-valued and two complex-valued.



Multi-component SC: tetragonal lattice

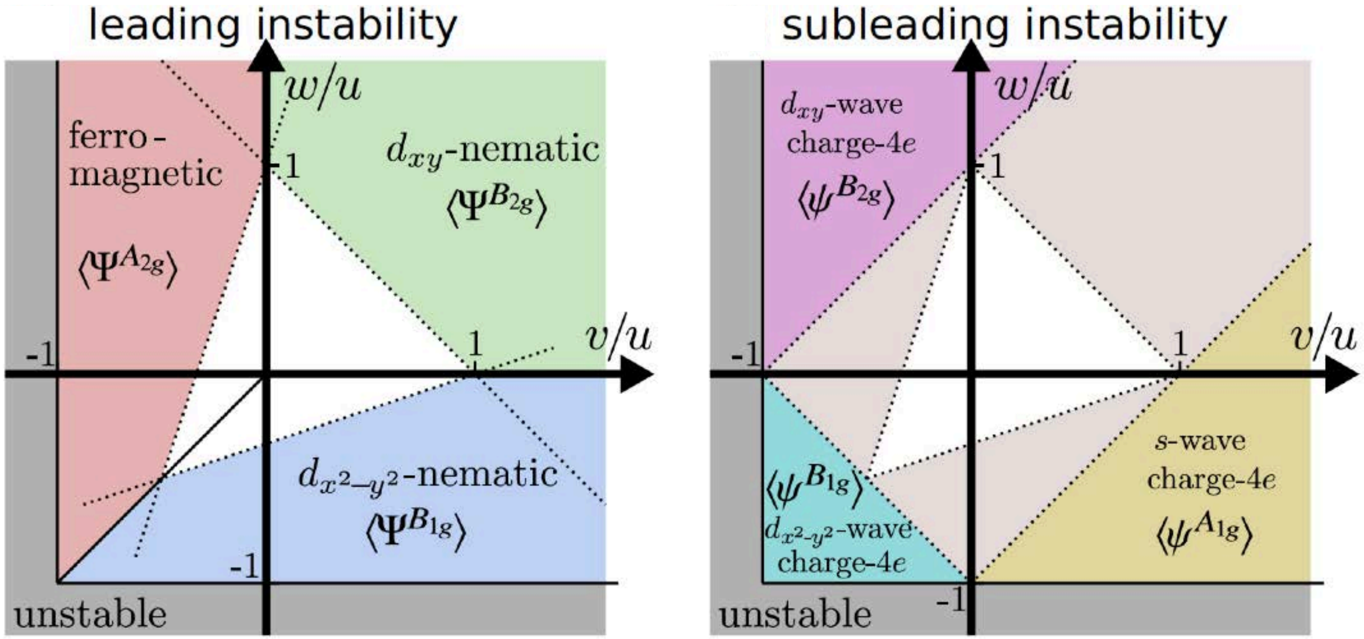
- Leading vestigial orders are the real-valued ones (magnetic or nematic).



see also Fisher & Berg, PRB (2016)

Multi-component SC: tetragonal lattice

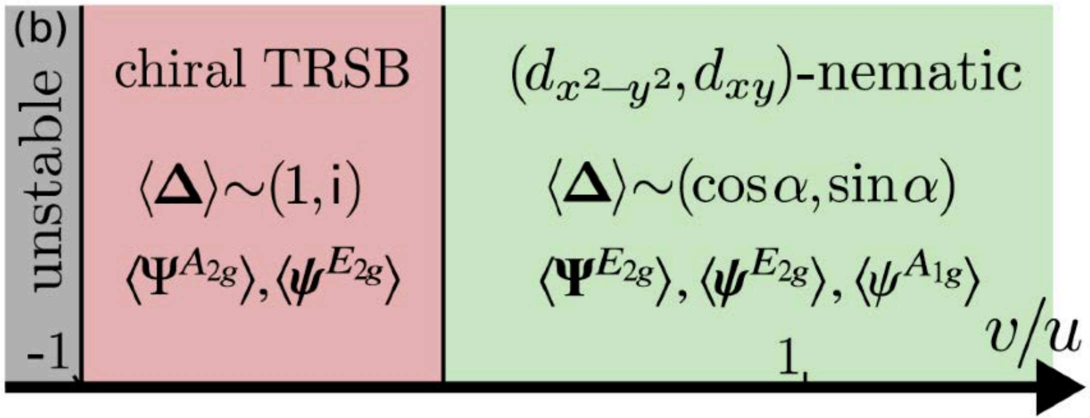
- Leading vestigial orders are the real-valued ones (magnetic or nematic).
- Charge-4e channels are attractive in wide regions of the phase diagrams, but always sub-leading. No indication of coexistence.



Hecker, Willa, Schmalian, & RMF, PRB (2023)

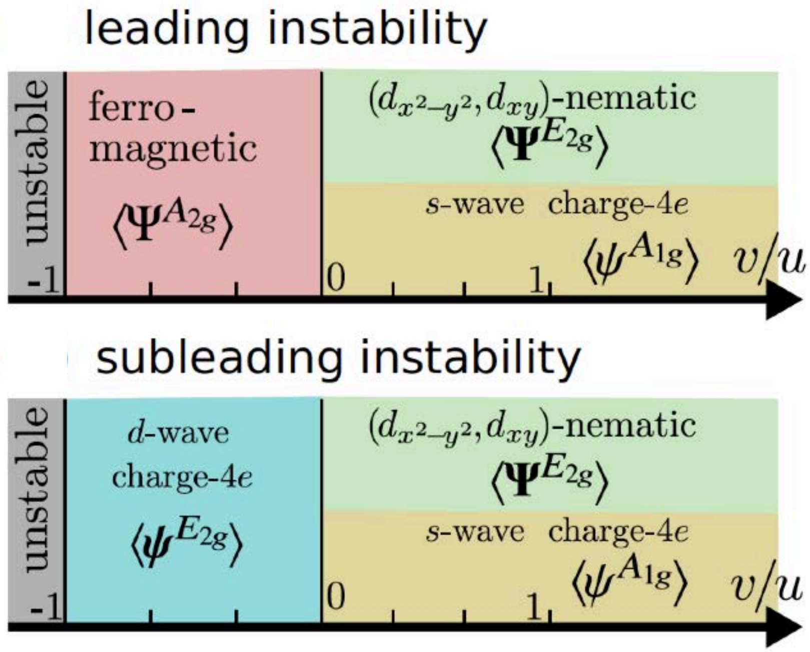
Multi-component SC: hexagonal lattice

- Mean-field phase diagram: chiral and one type of nematic SC state.



Multi-component SC: hexagonal lattice

- Leading vestigial orders are the real-valued ones (magnetic or nematic), but there is a degeneracy between the nematic and s-wave charge-4e instabilities.

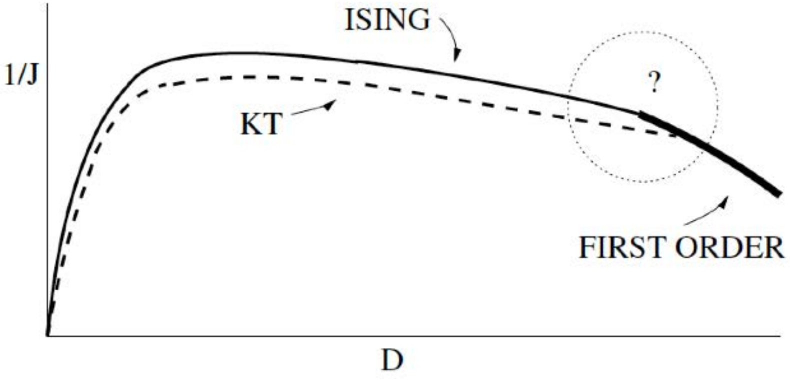


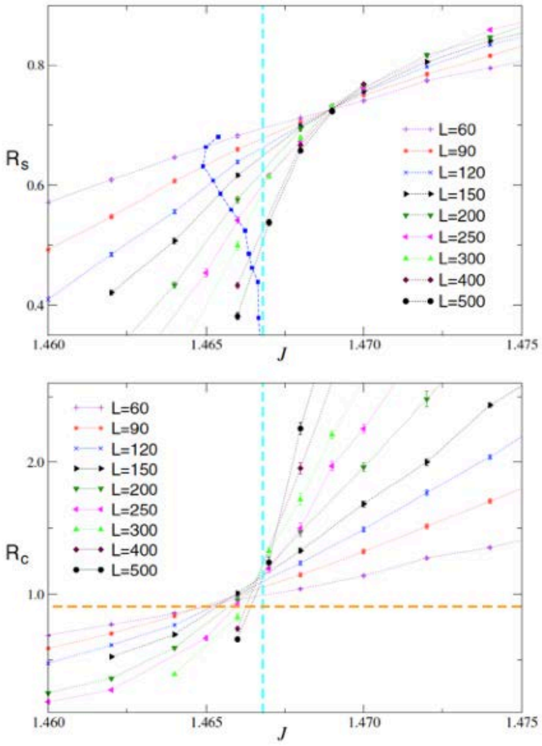
RMF & Fu, PRL (2021); Hecker, Willa, Schmalian, & RMF, PRB (2023)

Vestigial order: numerical results

- Monte Carlo simulations of the O(2) x O(2) Ginzburg-Landau model show split Ising (vestigial) and Kosterlitz-Thouless (SC) transitions.

$$\mathcal{H}_\phi = -J\sum_{\langle xy\rangle,i}\vec{\phi}_{i,x}\cdot\vec{\phi}_{i,y} + \sum_{i,x}\left[\phi_{i,x}^2 + U(\phi_{i,x}^2-1)^2\right] + 2(U+D)\sum_{x}\phi_{1,x}^2\phi_{2,x}^2$$





Hasenbusch, Pelissetto, & Vicari, J. Stat. Mech. (2005)